# Probing quark OAM in exclusive $\pi^0$ production in ep collisions

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# Outline:

- Background
- Probing Quark OAM via exclusive $\pi^0$ production
- Constraining the gluon GTMD via exclusive $\pi^0$ production
- Summary

# Spin decompositions of proton

- "Spin crisis" $\Delta\Sigma(Q^2 = 10.7GeV^2) = 0.060 \pm 0.047 \pm 0.069$ 1988, EMC

- Jaffe-Manohar decomposition $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + l_q + \Delta g + l_g$ Jaffe, Manohar, 1990

  $\Delta\Sigma \approx 0.3$ $\Delta g \approx 0.2$

  **Canonical OAM**

- Ji decomposition $\frac{1}{2} = J_q + J_g = \frac{1}{2}\Delta\Sigma + \mathrm{L}_q + J_g$ X.D. Ji, 1996

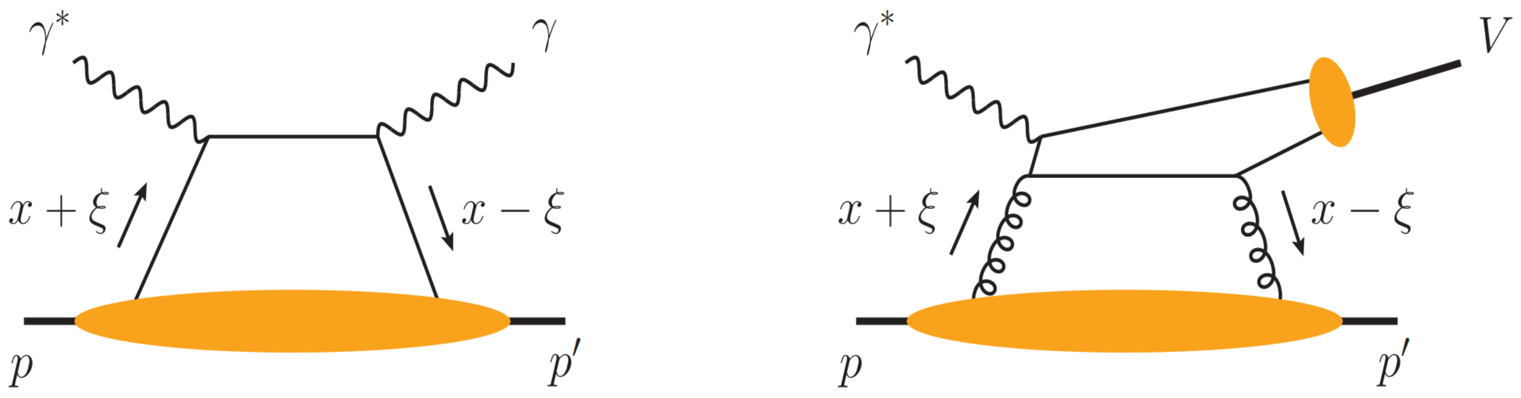
  **Kinematic OAM**

# Probing kinematic parton OAM

- Probing kinematic OAM: $$J_{q,g} = \frac{1}{2}\int_{-1}^{1} dx x[H_{q,g}(x,\xi,0) + E_{q,g}(x,\xi,0)]$$ X.D. Ji, 1996

  - Subtracting the spin contribution(helicity distribution)

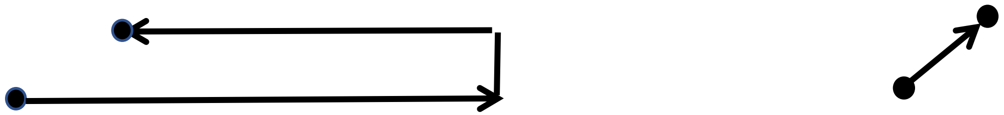
- GPDs, accessible in exclusive processes



# Canonical parton OAM and the GTMD

➢ Canonical OAM and GTMD: $$L^q(x,\xi) = -\int d^2k_\perp \frac{k_\perp^2}{M^2} F_{1,4}^q(x,k_\perp,\xi,\Delta_\perp = 0)$$

C. Lorce, B. Pasquini, 2011; Y. Hatta, 2012

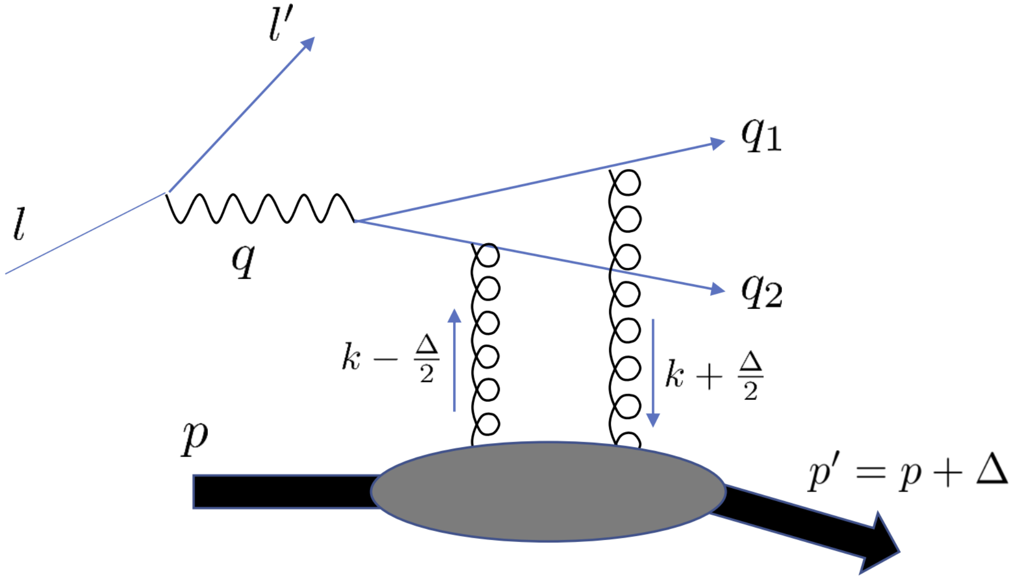


Canonical V.S. Kinematical

Mv+eA Mv

◆Probe F 1,4 in diffractive di-jet production

X. Ji, F. Yuan, and Y. Zhao, 2017; Y. Hatta, Y. Nakagawa, F. Yuan, Y. Zhao, and B. Xiao, 2017; S. Bhattacharya, R. Boussarie, and Y. Hatta 2022



# Generalized TMDs

$$W^{q\,[\Gamma]}_{\lambda,\lambda'}(P,\Delta,x,\vec{k}_\perp) = \int \frac{dz^-\,d^2\vec{z}_\perp}{2(2\pi)^3}\, e^{ik\cdot z}\, \langle p',\lambda'|\,\bar{q}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\,q(\tfrac{z}{2})\,|p,\lambda\rangle\Big|_{z^+=0}$$

S. Meissner, A. Metz, M. Schlegel and K. Goeke, 2008

- which is the Fourier transform of Wigner distribution:

$$\rho^{[\Gamma]}(\vec{b}_\perp,\vec{k}_\perp,x,\vec{S}) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\, e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp}\, W^{[\Gamma]}(\vec{\Delta}_\perp,\vec{k}_\perp,x,\vec{S})$$

## Parametrization:

$$W^{q\,[\gamma^+]}_{\lambda,\lambda'} = \frac{1}{2M}\,\bar{u}(p',\lambda')\left[F^q_{1,1} + \frac{i\sigma^{i+}k^i_\perp}{P^+}F^q_{1,2} + \frac{i\,\sigma^{i+}\Delta^i_\perp}{P^+}F^q_{1,3} + \frac{i\sigma^{ij}k^i_\perp\Delta^j_\perp}{M^2}F^q_{1,4}\right]u(p,\lambda)$$

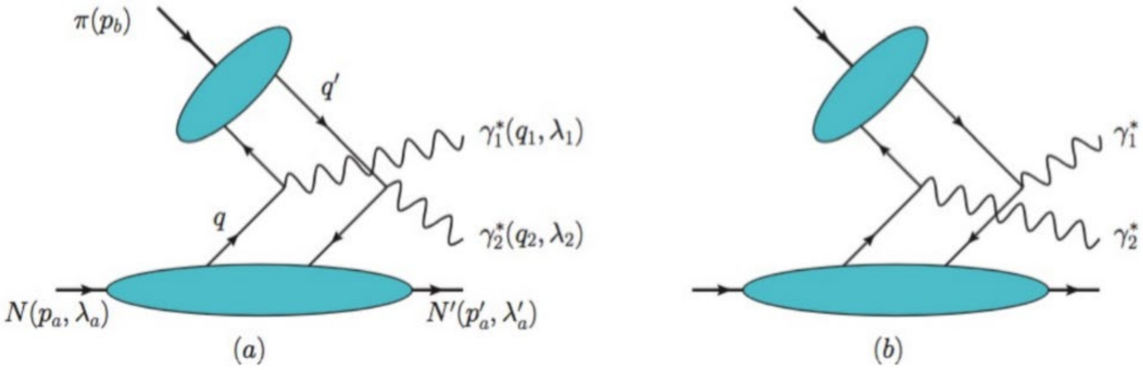
$b_\perp \times k_\perp$ Orbital angular momentum

$$W^{q\,[\gamma^+\gamma_5]}_{\lambda,\lambda'} = \frac{1}{2M}\,\bar{u}(p',\lambda')\left[-\frac{i\varepsilon^{ij}_\perp k^i_\perp\Delta^j_\perp}{M^2}G^q_{1,1} + \frac{i\sigma^{i+}\gamma_5 k^i_\perp}{P^+}G^q_{1,2} + \frac{i\sigma^{i+}\gamma_5\Delta^i_\perp}{P^+}G^q_{1,3} + i\sigma^{+-}\gamma_5\,G^q_{1,4}\right]u(p,\lambda)$$

# How to measure parton Wigner distribution

- Quark case: exclusive double DY process

Only ERBL region!




Bhattacharya, Metz, ZJ 2017

- Gluon case: exclusive double quarknioum production.

$$\frac{1}{2}(\tau_{UL} + \tau_{LU})$$

$$\approx \; 2\,\mathrm{Im}\Big\{ -\frac{1}{M^2}\,(\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j)\, C\Big[\vec{\beta}_\perp \cdot \vec{k}_{a\perp}\, F_{1,4}(x_a, \vec{k}_{a\perp})\, F_{1,1}(x_b, \vec{k}_{b\perp})\Big]\, C\Big[F_{1,1}^*(x_a, \vec{p}_{a\perp})\, F_{1,1}^*(x_b, \vec{p}_{b\perp})\Big]$$

$$+ \; \frac{1}{M^2}\,(\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j)\, C\Big[\vec{\beta}_\perp \cdot \vec{k}_{a\perp}\, G_{1,1}(x_a, \vec{k}_{a\perp})\, G_{1,4}(x_b, \vec{k}_{b\perp})\Big]\, C\Big[G_{1,4}^*(x_a, \vec{p}_{a\perp})\, G_{1,4}^*(x_b, \vec{p}_{b\perp})\Big]\Big\}$$

$$\approx \; 2\,\mathrm{Im}\Big\{ -\frac{1}{M^2}\,(\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j)\, C\Big[\vec{\beta}_\perp \cdot \vec{k}_{a\perp}\, F_{1,4}(x_a, \vec{k}_{a\perp})\, F_{1,1}(x_b, \vec{k}_{b\perp})\Big]\, C\Big[F_{1,1}^*(x_a, \vec{p}_{a\perp})\, F_{1,1}^*(x_b, \vec{p}_{b\perp})\Big]\Big\}$$

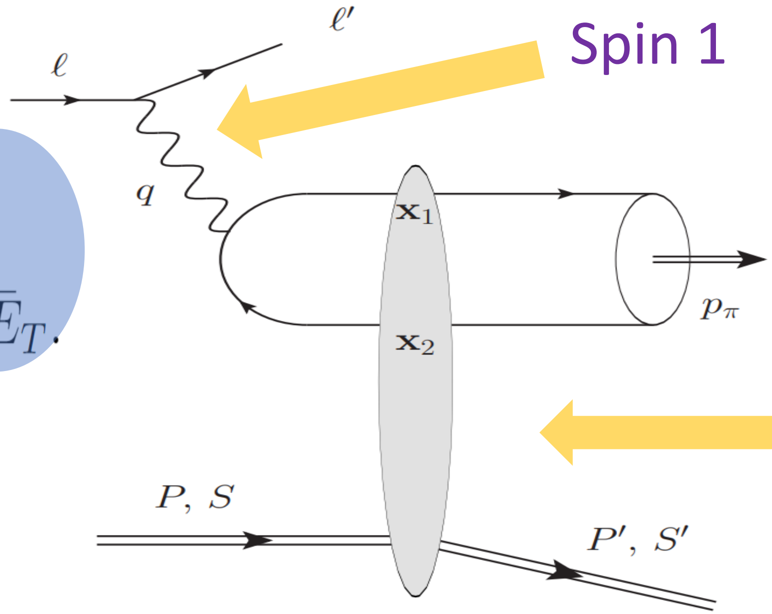
Bhattacharya, Vikash, Metz, Jeng, ZJ 2018

# Exclusive $\pi^0$ production in ep collisions

- In the forward limit, helicity flip process!

$$M_{0-,++} = \langle H_T \rangle \propto \int_{-1}^{1} d\bar{x}\mathcal{H}_{0-,++}(\bar{x},...)\, H_T;$$

$$M_{0+,++} = \langle \bar{E}_T \rangle \propto \frac{\sqrt{-t'}}{4m} \int_{-1}^{1} d\bar{x}\mathcal{H}_{0-,++}(\bar{x},...)\, \bar{E}_T.$$




Spin 1

Spin 0

Spin 1 exchange

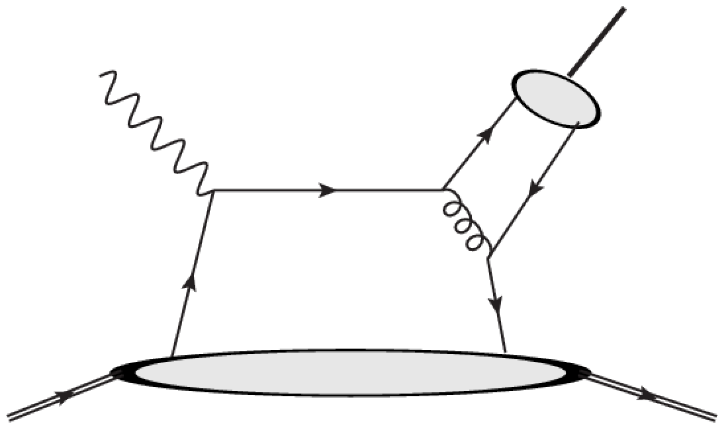
Give access to the chiral odd GPDs

L. Frankfurt, P. Pobylitsa, M. Polyakov, and M. Strikman, 1999

Twist-3 correction to DA, G. Duplančić, P. Kroll, K. Passek-K., and L. Szymanowski, 2024

# Helicity non-flip production: quark channel

- Vanishes at leading twist!



- Non-forward region, allowing pion to carry 1 unit OAM,  twist-3 effect.

- Collinear expansion to isolate twist-3 contribution:

$$H(k_\perp,\Delta_\perp) = H(k_\perp = 0,\Delta_\perp = 0) + \frac{\partial H(k_\perp,\Delta_\perp = 0)}{\partial k_\perp^\mu}\Big|_{k_\perp=0} k_\perp^\mu + \frac{\partial H(k_\perp = 0,\Delta_\perp)}{\partial \Delta_\perp^\mu}\Big|_{\Delta_\perp=0} \Delta_\perp^\mu + \ldots$$

# Angular correlations

➢ Scattering amplitudes
depending on different correlations

$$\mathcal{M}_1 = \frac{g_s^2 e f_\pi}{2\sqrt{2}} \frac{(N_c^2-1)2\xi}{N_c^2\sqrt{1-\xi^2}} \delta_{\lambda\lambda'} \frac{\epsilon_\perp \times \Delta_\perp}{Q^2} \{\mathcal{F}_{1,1} + \mathcal{G}_{1,1}\},$$

$$\mathcal{M}_2 = \frac{g_s^2 e f_\pi}{2\sqrt{2}} \frac{(N_c^2-1)2\xi}{N_c^2\sqrt{1-\xi^2}} \delta_{\lambda,-\lambda'} \frac{M\epsilon_\perp \cdot S_\perp}{Q^2} \{\mathcal{F}_{1,2} + \mathcal{G}_{1,2}\}$$

$$\mathcal{M}_4 = \frac{i g_s^2 e f_\pi}{2\sqrt{2}} \frac{(N_c^2-1)2\xi}{N_c^2\sqrt{1-\xi^2}} \lambda\delta_{\lambda\lambda'} \frac{\epsilon_\perp \cdot \Delta_\perp}{Q^2} \{\mathcal{F}_{1,4} + \mathcal{G}_{1,4}\} ($$

$$S_\perp^\mu = (0^+, 0^-, -i, \lambda)$$

Shohini Bhattacharya, Duxin Zheng, ZJ, 2024

$$\mathcal{F}_{1,1} = \int_{-1}^{1} dx \frac{x^2 \int d^2k_\perp F_{1,1}^{u+d}(x,\xi,\Delta_\perp,k_\perp)}{(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times \int_0^1 dz \frac{\phi_\pi(z)(1+z^2-z)}{z^2(1-z)^2}, \quad (8)$$

$$\mathcal{G}_{1,1} = \int_{-1}^{1} dx \int_0^1 dz \frac{\phi_\pi(z)(x^2+2x^2z+\xi^2)}{z^2(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times \int d^2k_\perp \frac{k_\perp^2}{M^2} G_{1,1}^{u+d}(x,\xi,\Delta_\perp,k_\perp), \quad (9)$$

$$\mathcal{F}_{1,2} = \int_{-1}^{1} dx x \frac{\xi(1-\xi^2)\int d^2k_\perp k_\perp^2 F_{1,2}^{u+d}(x,\xi,\Delta_\perp,k_\perp)}{M^2(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times \int_0^1 dz \frac{\phi_\pi(z)(1+z^2-z)}{z^2(1-z)^2}, \quad (10)$$

$$\mathcal{G}_{1,2} = \int_{-1}^{1} dx \int_0^1 dz \frac{\phi_\pi(z)(x^2+2x^2z+\xi^2)(1-\xi^2)}{z^2(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times \int d^2k_\perp \frac{k_\perp^2}{M^2} G_{1,2}^{u+d}(x,\xi,\Delta_\perp,k_\perp), \quad (11)$$

$$\mathcal{F}_{1,4} = \int_{-1}^{1} dx \frac{x\xi\int d^2k_\perp k_\perp^2 F_{1,4}^{u+d}(x,\xi,\Delta_\perp,k_\perp)}{M^2(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times \int_0^1 dz \frac{\phi_\pi(z)(1+z^2-z)}{z^2(1-z)^2}, \quad (12)$$

$$\mathcal{G}_{1,4} = \int_{-1}^{1} dx \int_0^1 dz \frac{x(4\xi^2z+\xi^2-2x^2z+x^2)}{z^2\xi(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2}\phi_\pi(z) \times \int d^2k_\perp G_{1,4}^{u+d}(x,\xi,\Delta_\perp,k_\perp). \quad (13)$$

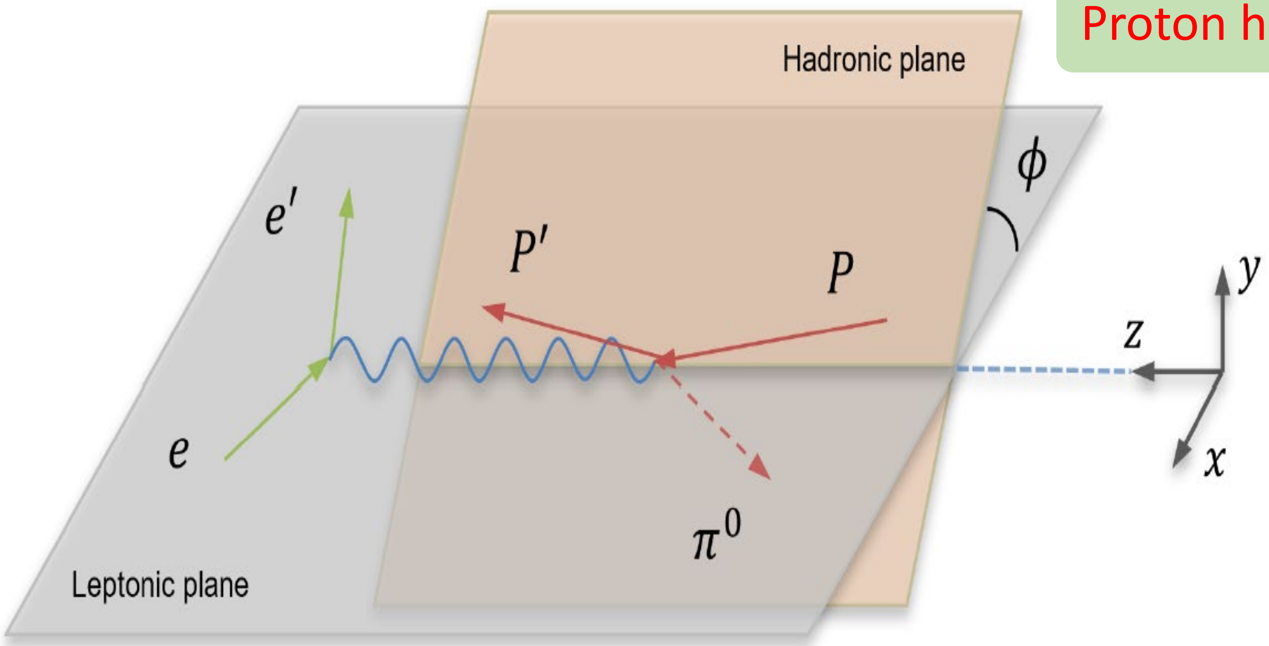
# Azimuthal dependent cross section

$$\frac{d\sigma}{dtdQ^2dx_Bd\phi} = \frac{(N_c^2-1)^2\alpha_{em}^2\alpha_s^2f_\pi^2\xi^3\Delta_\perp^2}{2N_c^4(1-\xi^2)Q^{10}(1+\xi)}\left[1+(1-y)^2\right]\times\left\{\left[|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2+|\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2+2\frac{M^2}{\Delta_\perp^2}|\mathcal{F}_{1,2}+\mathcal{G}_{1,2}|^2\right]\right.$$

$$\left.+\cos(2\phi)a\left[-|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2+|\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2\right]+\lambda\sin(2\phi)\,2a\,\mathrm{Re}\left[(i\mathcal{F}_{1,4}+i\mathcal{G}_{1,4})\left(\mathcal{F}_{1,1}^*+\mathcal{G}_{1,1}^*\right)\right]\right\}$$

Proton helicity

Distinguished experimental signature of $F_{1,4}$




$$\phi = \phi_{l_\perp} - \phi_{\Delta_\perp}$$

$$\int d^2k_\perp \mathrm{Re}[F_{1,1}(x,\xi,\Delta_\perp,k_\perp)] \approx H(x,\xi,\Delta_\perp)$$

$$\int d^2k_\perp \mathrm{Re}[G_{1,4}(x,\xi,\Delta_\perp,k_\perp)] \approx \tilde{H}(x,\xi,\Delta_\perp)$$

# Model input for numerical estimations

- Endpoint singularity and discontinuity:

$$\mathcal{F}_{1,4} = \int_{-1}^{1} dx \frac{x\xi \int d^2k_\perp k_\perp^2 F_{1,4}^{u+d}(x,\xi,\Delta_\perp,k_\perp)}{M^2(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \int_0^1 dz \frac{\phi_\pi(z)(1+z^2-z)}{z^2(1-z)^2}$$

- Model dependent method

$$\int_{\langle p_\perp^2\rangle/Q^2}^{1-\langle p_\perp^2\rangle/Q^2} dz$$

S. V. Goloskokov and P. Kroll, 2005

$$\frac{1}{(x-\xi+i\epsilon)^2} \Rightarrow \frac{1}{(x-\xi-\langle p_\perp^2\rangle/Q^2+i\epsilon)^2}$$

I. V. Anikin, O. V. Teryaev, 2003

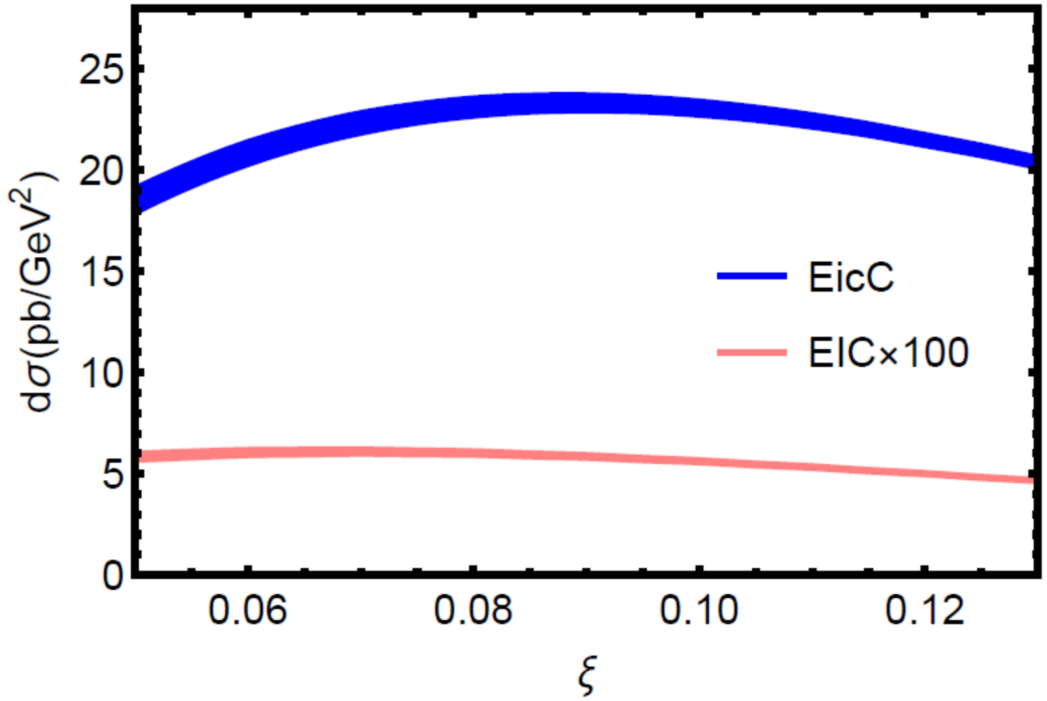
- Wandzura-Wilczek(WW) approximation:

$$L_q(x) \approx x\int_x^1 \frac{dx'}{x'}q(x') - x\int_x^1 \frac{dx'}{x'^2}\Delta q(x')$$

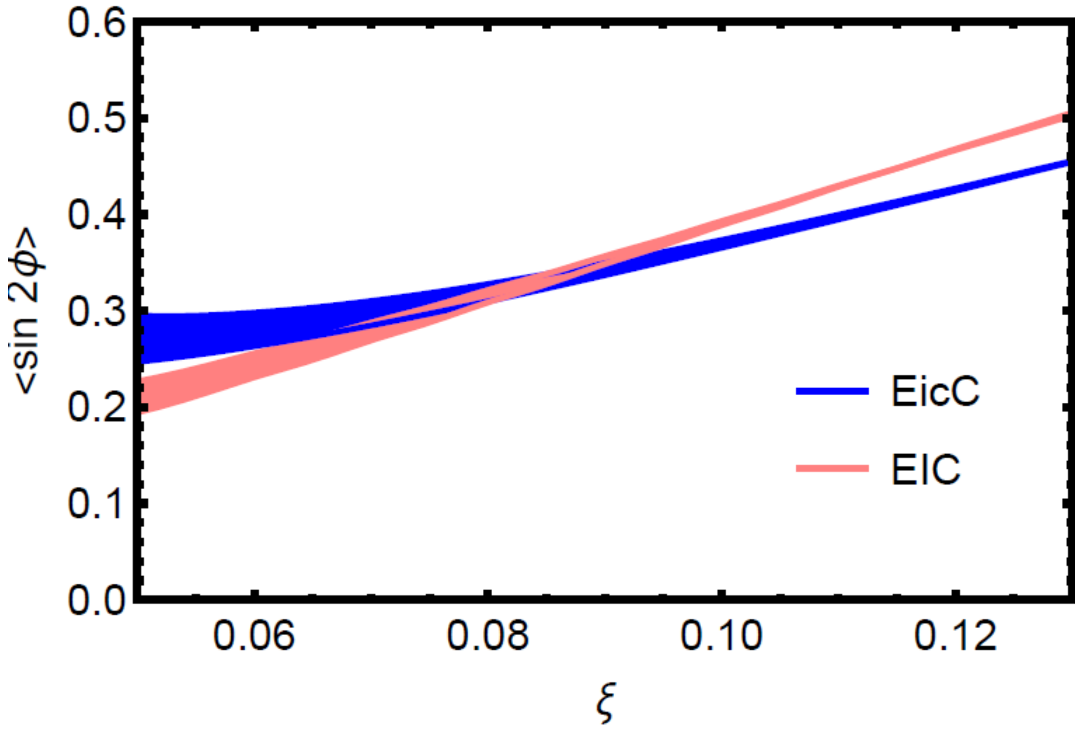
Y. Hatta and S. Yoshida, 2012

# Numerical results

## Unpolarized cross section:




## sin2Φ azimuthal asymmetry




EIC: $Q^2$=10,  EicC: $Q^2$=3

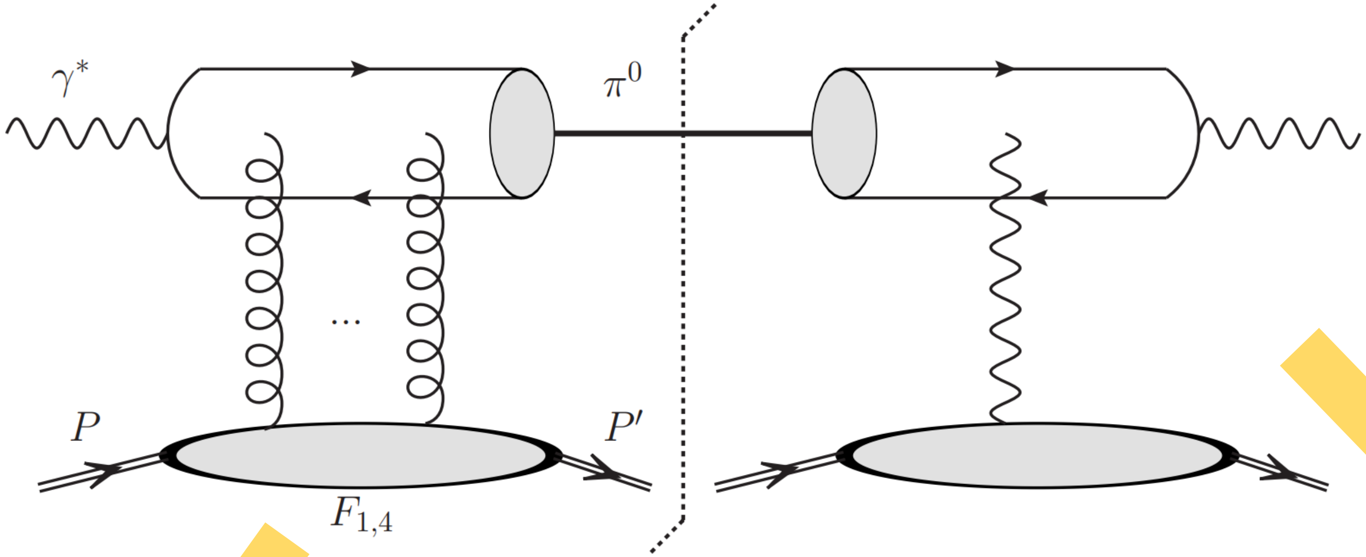
$$\xi = \frac{p^+ - p'^+}{p^+ + p'^+}$$

- We focus on the large zeta region to suppress gluon contribution.

# The gluon channel

When $\xi = \dfrac{p^+ - p'^+}{p^+ + p'^+}$ is small, gluon channel dominates

When $t = (p' - p)^2$ is small, the Primakoff process is important




$$\mathcal{M}_R = \delta_{\lambda\lambda'} e^3 \sqrt{2} f_\pi \frac{\sqrt{1-\xi^2}}{1+\xi} \frac{(\epsilon_\perp^{\gamma^*} \times \Delta_\perp)}{x_B \Delta_\perp^2} \mathcal{F}(t) \int_0^1 dz \frac{\phi_\pi(z)}{6z(1-z)}$$

$$\mathcal{M}_L = i \frac{g_s^2 e f_\pi}{N_c \sqrt{2}} \lambda \delta_{\lambda\lambda'} \frac{\epsilon_\perp \cdot \Delta_\perp}{M^2 Q^2} \frac{\xi^2(1+\xi)}{\sqrt{1-\xi^2}} \int_{-1}^1 dx \frac{F_{1,4}^{(1)}(x,\xi,\Delta_\perp)}{(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \int_0^1 dz \frac{\phi_\pi(z)}{z(1-z)}$$

# Polarization dependent cross section

$$\frac{d\Delta\sigma}{dtdQ^2dx_Bd\phi} = -\sin(2\phi)\frac{\alpha_{em}^3\alpha_s f_\pi^2(1-y)\xi x_B\mathcal{F}(t)}{3Q^8N_c}\left[\int_0^1 dz\frac{\phi_\pi(z)}{z(1-z)}\right]^2 \mathrm{Im}\left[\int_{-1}^1 dx\frac{F_{1,4}^{(1)}(x,\xi,\Delta_\perp)/M^2}{(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2}\right]$$

- Unfortunately, due to symmetry properties:

$$F_{1,4}^{(1)}(x,\xi,\Delta_\perp) = -F_{1,4}^{(1)}(-x,\xi,\Delta_\perp) \quad \Rightarrow \quad \mathrm{Im}\int_{-1}^1 dx\frac{\mathrm{Re}F_{1,4}^{(1)}(x,\xi,\Delta_\perp)}{(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} = 0$$

- Only probe the imaginary part of gluon $F_{1,4}$

# Unpolarized cross section in the near forward region

- Gluon Sivers contribution

$$\frac{d\sigma^{odderon}}{dtdQ^2dx_B} \approx \frac{\pi^5\alpha_{em}^2\alpha_s^2 f_\pi^2}{8x_B N_c^2 M^2 Q^6}\left[1-y+\frac{y^2}{2}\right]\left[\int_0^1 dz \frac{\phi_\pi(z)}{z(1-z)}\int d^2k_\perp \frac{k_\perp^2 x f_{1T}^{\perp g}(x,k_\perp^2)}{k_\perp^2+z(1-z)Q^2}\right]^2$$

R. Boussarie, Y. Hatta, L. Szymanowski, and S. Wallon, 2020

- The Primakoff process

$$\frac{d\sigma^{Pri}}{dtdQ^2dx_B} \approx \frac{\alpha_{em}^4(2\pi)[1+(1-y)^2]f_\pi^2}{x_B Q^6 \Delta_\perp^2}\frac{1-\xi}{1+\xi}\mathcal{F}^2(t)\left[\int_0^1 \frac{dz}{6z(1-z)}\phi_\pi(z)\right]^2$$

◆ The interference channel does not contribute to azimuthal averaged cross section.

# Summary

- Probe quark OAM via exclusive $\pi^0$ production
- Clean background, no final state soft gluon radiations
- The asymmetry is not power suppressed
- Give the access to the DGLAP region

- Endpoint singularity and discontinuity
- Twist-3 quark-gluon correlation function is not included

Thank you for your attention!