From nonsinglet to singlet GPDs in lattice QCD

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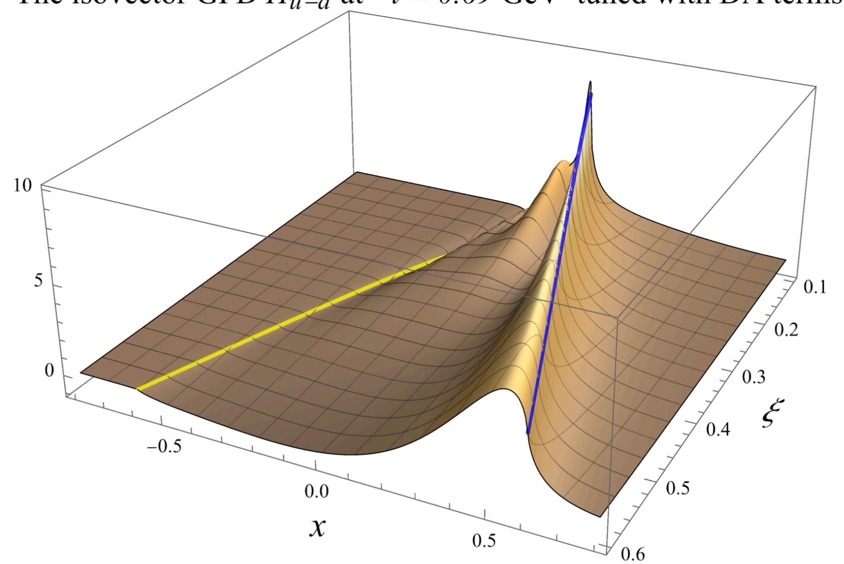
Introduction

- The GPDs as generalizations of the PDFs to **non-forward** kinematics
  - Depend on multiple kinematic variables x, ξ, t
  - Factorization theorem

$$\mathcal{H}\left(\xi, t, Q^2\right) = \int_{-1}^{1} \frac{\mathrm{d}x}{\xi} \sum_{a=g,u,d,\dots} C^a\left(\frac{x}{\xi}, \frac{Q^2}{\mu_F^2}, \alpha_S\left(\mu_F^2\right)\right) H^a\left(x, \xi, t, \mu_F^2\right)$$

- Various models for GPD parametrization have been used for their extraction from experimental data **Kumericki et al, EPJA 16'**
- A recent example: through universal moments parametrization **Guo et al, JHEP 23'**

The isovector GPD H_{u-d} at $-t = 0.69$ GeV^2 tuned with DA terms



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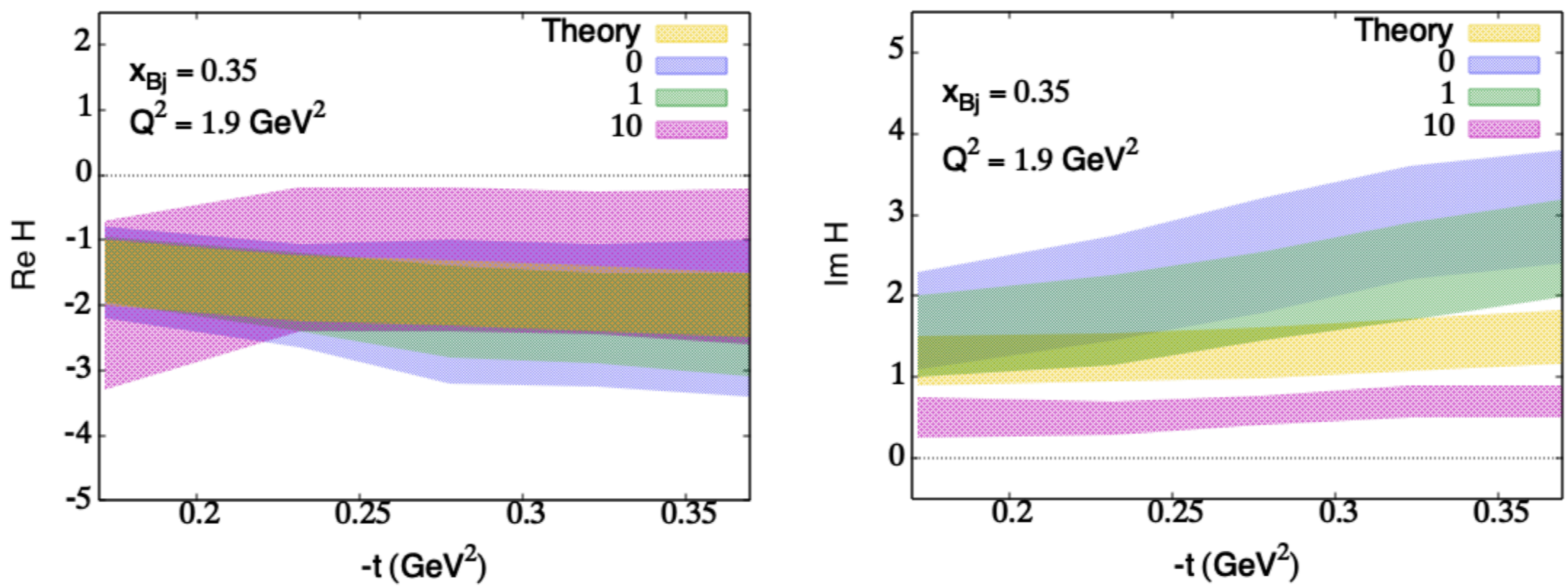
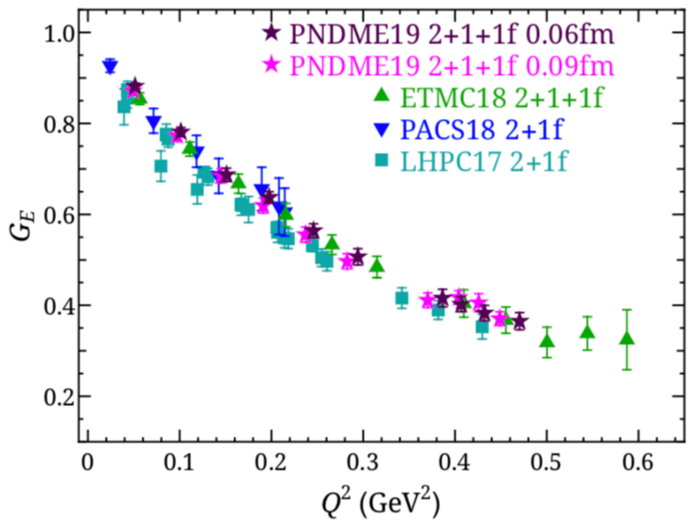
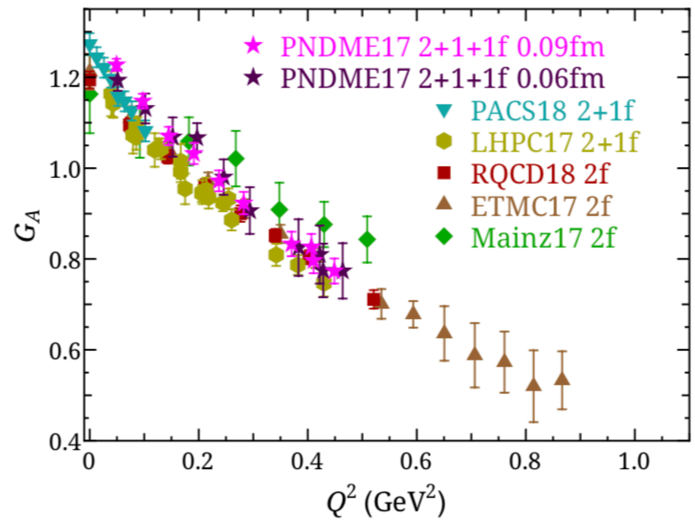
- Various models for GPD parametrization have been used for their extraction from experimental data **Kumericki et al, EPJA 16'**
- ML/DNN applied to analyze exclusive scattering X-sections **Almaeen et al, 22'**

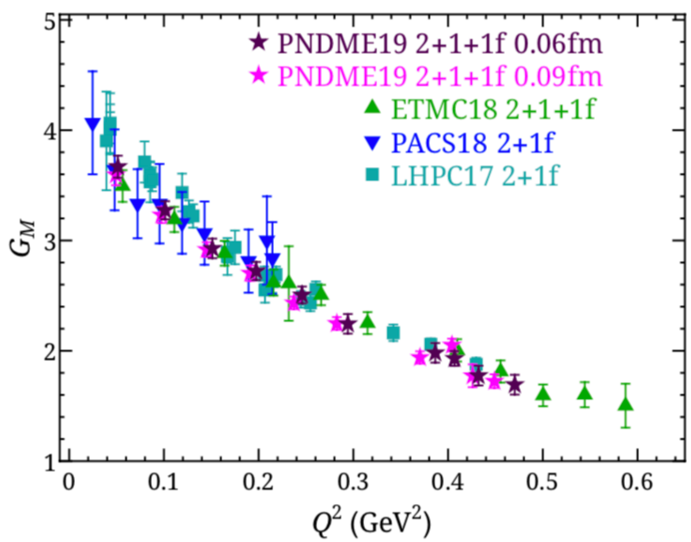


Introduction

- Lattice QCD can provide important complementary inputs — **moments**







$$\langle N(p_f)|V_\mu^+(x)|N(p_i)\rangle =$$
$$\bar{u}^N \left[\gamma_\mu F_1(q^2) + i\sigma_{\mu\nu}\frac{q^\nu}{2M_N}F_2(q^2)\right] u_N e^{iq\cdot x}$$

$$\langle N(p_f)|A_\mu^+(x)|N(p_i)\rangle =$$
$$\bar{u}_N \left[\gamma_\mu\gamma_5 G_A(q^2) + iq_\mu\gamma_5 G_P(q^2)\right] u_N e^{iq\cdot x}$$

Constantinou, JHZ et al, PPNP 21'

Introduction

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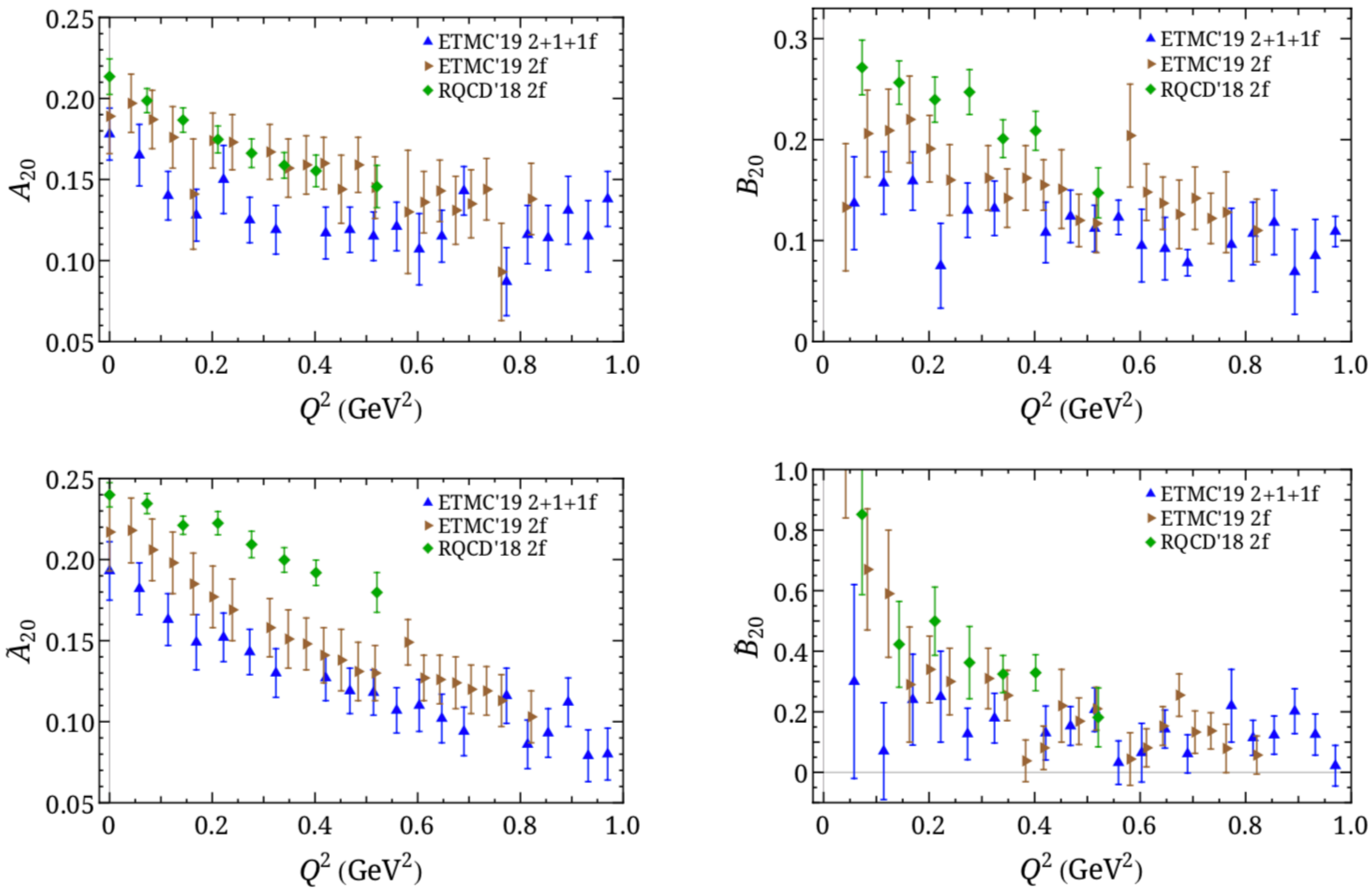
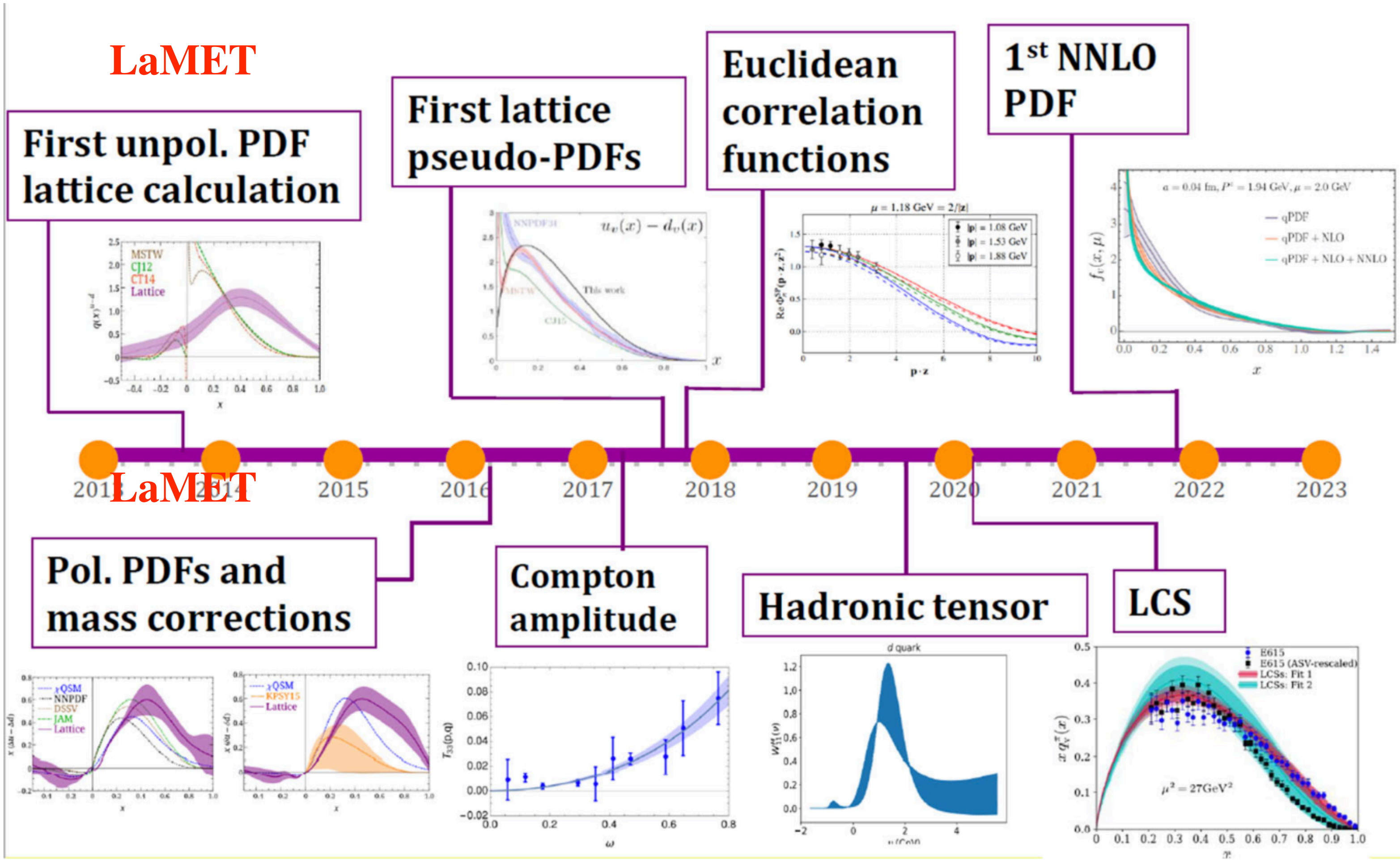


$$\langle N(p',s')|\mathcal{O}_V^{\mu\nu}|N(p,s)\rangle = \bar{u}_N(p',s')\frac{1}{2}\left[A_{20}(q^2)\,\gamma^{\{\mu}P^{\nu\}} + B_{20}(q^2)\,\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m_N} + C_{20}(q^2)\,\frac{1}{m_N}q^{\{\mu}q^{\nu\}}\right]u_N(p,s),$$

$$\langle N(p',s')|\mathcal{O}_A^{\mu\nu}|N(p,s)\rangle = \bar{u}_N(p',s')\frac{i}{2}\left[\tilde{A}_{20}(q^2)\,\gamma^{\{\mu}P^{\nu\}}\gamma^5 + \tilde{B}_{20}(q^2)\,\frac{q^{\{\mu}P^{\nu\}}}{2m_N}\gamma^5\right]u_N(p,s),$$

Introduction

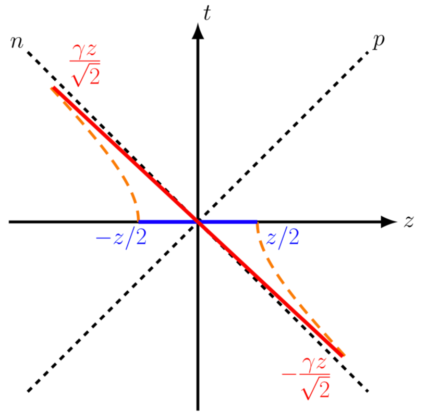
- Lattice QCD can provide important complementary inputs — **x-dependent distributions**
- Significant progress has been achieved along this line



H-W Lin, FBS 23'

Introduction

- Lattice QCD can provide important complementary inputs — **x-dependent distributions**
- **A popular approach: Large-momentum effective theory (LaMET)**
  Ji, PRL 13' & SCPMA 14', Ji, JHZ et al, RMP 21'



$$q(x,\mu) = \int \frac{d\lambda}{4\pi} e^{ix\lambda} \langle P | \bar{\psi}(0) n \cdot \gamma L(0,\lambda n) \psi(\lambda n) | P \rangle, \quad n^2 = 0$$

$$\tilde{q}(y,P^z) = N \int \frac{dz}{4\pi} e^{-iyzP^z} \langle P | \bar{\psi}(0) \gamma^0 L(0,z) \psi(z) | P \rangle$$

$$\tilde{q}(y,P^z) = C\left(\frac{y}{x}, \frac{\mu}{xP^z}\right) \otimes q(x,\mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{(yP^z)^2}, \frac{\Lambda_{QCD}^2}{((1-y)P^z)^2}\right)$$

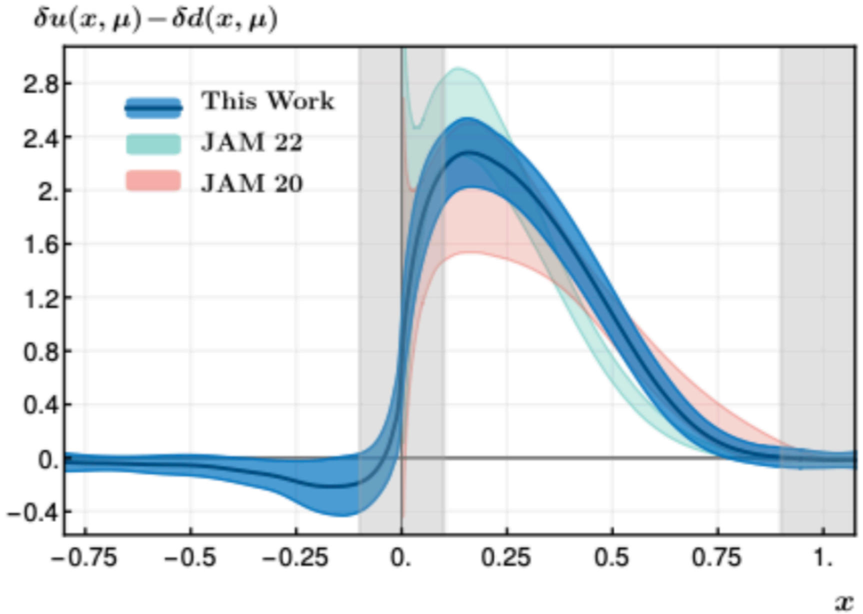
- Theory studies and/or lattice calculations available for
  - Collinear PDFs, distribution amplitudes
  - GPDs, TMDPDFs/wave functions
  - Higher-twist distributions, double parton distributions

A huge number of references…

Lattice results on x-dependent distributions

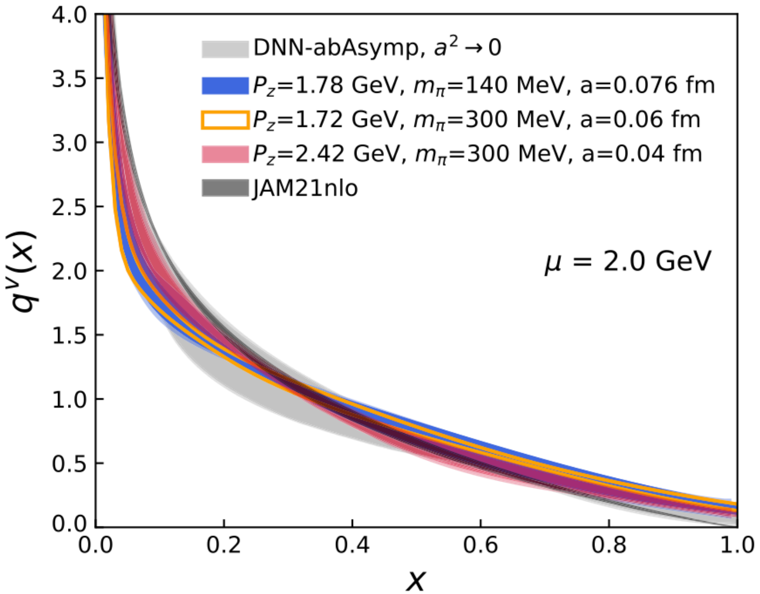
- Lattice QCD can provide important complementary inputs — **x-dependent distributions**
- **A popular approach: Large-momentum effective theory (LaMET)**
  Ji, PRL 13' & SCPMA 14', Ji, JHZ et al, RMP 21'
- Examples of the state-of-the-art:

Nucleon quark transversity PDF



Yao, JHZ et al (LPC) PRL 23'

Pion valence quark PDF



Gao et al, PRD 22'

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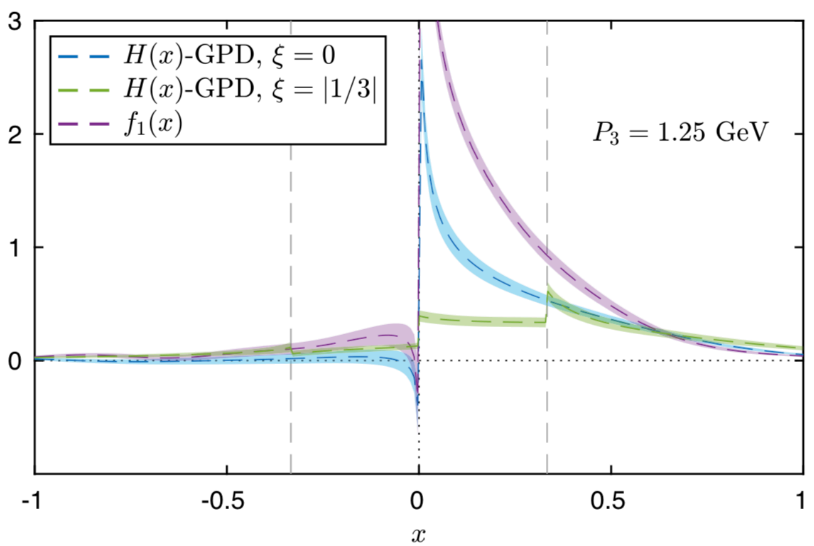
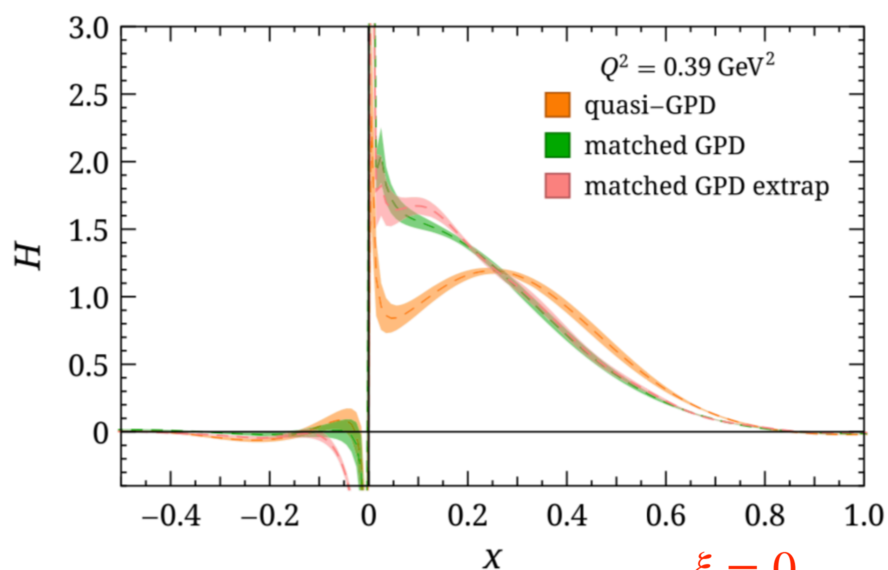
Nucleon quark unpolarized GPD





$\xi = 0$

Alexandrou et al, PRL 20' **Lin, PRL 21'**

Lattice results on x-dependent distributions

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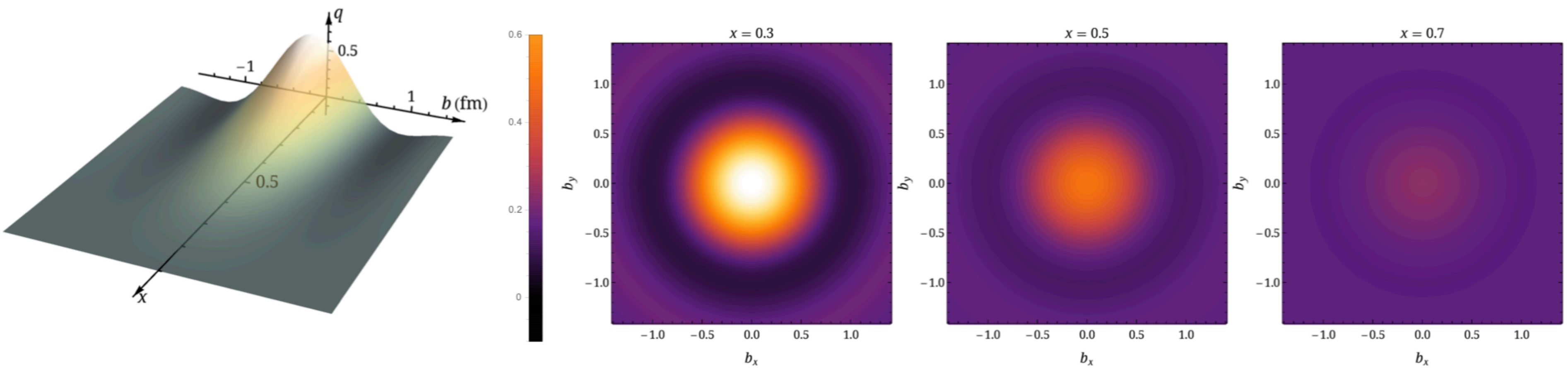
- Examples of the state-of-the-art:

Impact parameter distribution

$$\mathsf{q}(x, b) = \int \frac{d\mathbf{q}}{(2\pi)^2} H(x, \xi = 0, t = -\mathbf{q}^2) e^{i\mathbf{q}\cdot\mathbf{b}}$$



Lin, PRL 21'

Lattice results on x-dependent distributions

- Lattice QCD can provide important complementary inputs — **x-dependent distributions**
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- Examples of the state-of-the-art:

Impact parameter distribution

- Potential improvement:
  - ratio-hybrid renormalization of lattice matrix elements
  - Perturbative matching to be updated
    - **F. Yao, Y. Ji, JHZ, JHEP 23'** (state-of-the-art manual for all collinear leading-twist quantities)
  - Control of power corrections

b_x b_x

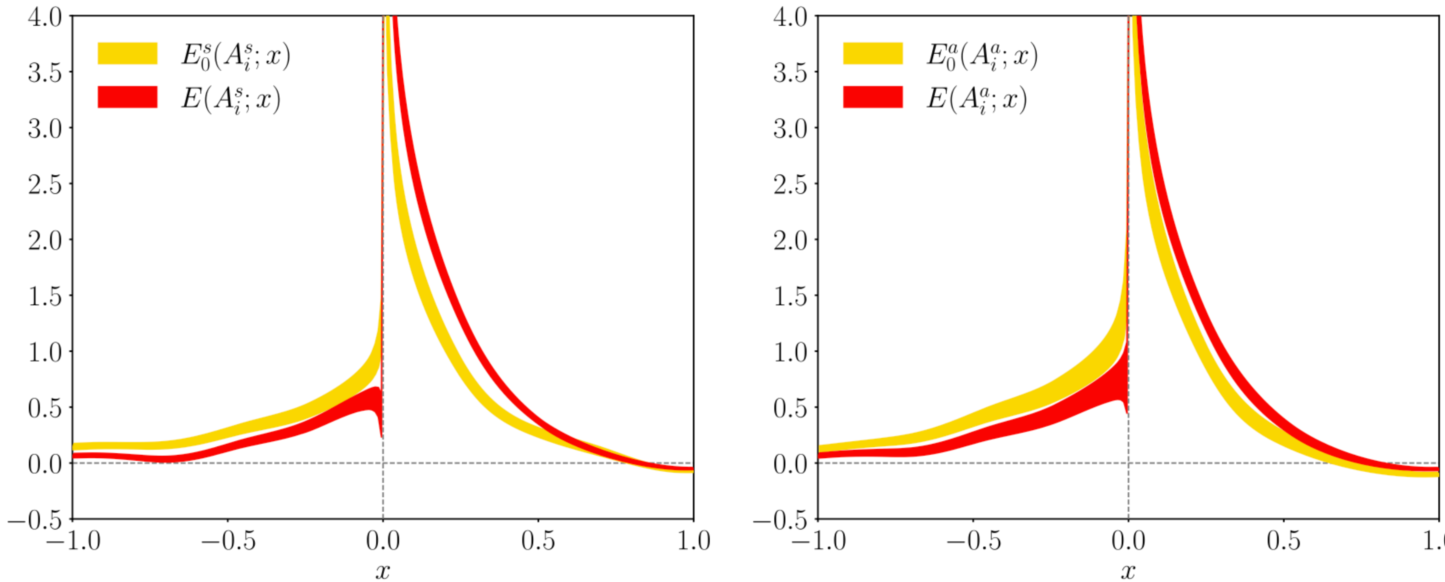
Lin, PRL 21'

Lattice results on x-dependent distributions

- Lattice QCD can provide important complementary inputs — **x-dependent distributions**
- **A popular approach: Large-momentum effective theory (LaMET)** **Ji, PRL 13' & SCPMA 14', Ji, JHZ et al, RMP 21'**
- Examples of the state-of-the-art:

Nucleon quark GPD in an asymmetric frame

Bhattacharya et al, PRD 22'



- Leading-twist violates translational invariance, which can be restored by including kinematic higher-twist contributions **Braun, 23'**

Towards precision control

- Recent developments for collinear PDFs
  - **Higher-order perturbative correction**
    - Unpol. quark PDF@NNLO **Li et al, PRL 21', Chen et al, PRL 21'**
    - Quark TMDPDF@NNLO **Del Rio et al, PRD 23', Ji et al, JHEP 23'**
  - **RG resummation Su, JHZ et al, NPB 23'**
  - **Threshold resummation Gao et al, PRD 21', Ji et al, JHEP 23'**
  - **Power correction, renormalon ambiguity**
    Chen, JHZ et al, PRD 17', Braun, JHZ et al, PRD 19', Liu et al, PRD 21', Zhang et al, PLB 23'
  - **Control of lattice artifacts**
    - ANL-BNL, ETMC, LPC, MSU…
- Shall be extended to GPDs **Holligan et al, 23', Braun et al, 24'**

Theoretical calculations of GPDs

- Calculations so far are focused on **isovector/nonsinglet** combinations that do not mix with gluons

- Renormalization and matching are mostly done in an out-of-date scheme

- There exist discrepancies in results from different groups, from collinear PDFs to GPDs

- It is desirable to have a unified framework for perturbative matching connecting Euclidean to lightcone correlations in a state-of-the-art scheme

- Both for **flavor nonsinglet and singlet**, in **forward and non-forward kinematics**, and **in coordinate and momentum space**

- Provide a manual for extracting all leading-twist GPDs, PDFs and DAs in a state-of-the-art scheme from lattice QCD, and facilitate higher-order perturbative matching calculations

Theoretical calculations of GPDs

- **Quark operator bases** F. Yao, Y. Ji, JHZ, JHEP 23'

$$O_q^{ns}(z_1,z_2)=\frac{1}{2}\left[O_q(z_1,z_2)\pm O_q(z_2,z_1)\right],\; O_q^{s}(z_1,z_2)=\frac{1}{2}\left[O_q(z_1,z_2)\mp O_q(z_2,z_1)\right]$$

- The upper sign corresponds to the unpolarized (vector) case, the low sign corresponds to the helicity (axial-vector) case
- Transversity always takes the + sign, and does not mix with gluons

- **Gluon operator bases**

$$\begin{aligned} O_{g,u}(z_1,z_2)&=g_\perp^{\mu\nu}\mathbf{F}_{\mu\nu}\,,\\ O_{g,h}(z_1,z_2)&=i\epsilon_\perp^{\mu\nu}\mathbf{F}_{\mu\nu}\,,\\ O_{g,t}(z_1,z_2)&=\hat{S}\,\mathbf{F}_{\mu\nu}=\frac{1}{2}\left[\mathbf{F}_{\mu\nu}+\mathbf{F}_{\nu\mu}\right]-\frac{1}{d-2}g_\perp^{\mu\nu}\,\mathbf{F}_\alpha{}^\alpha, \end{aligned}$$

$$\mathbf{F}_{\mu\nu}=z_{12}^\rho F_{\rho\mu}(z_1)[z_1,z_2]F_{\nu\sigma}(z_2)z_{12}^\sigma\equiv F_{z_{12}\mu}(z_1)[z_1,z_2]\,F_{\nu z_{12}}(z_2)$$

- Sandwiching these operators between different external states defines different quasi-observables

Theoretical calculations of GPDs

- **Factorization formula** F. Yao, Y. Ji, JHZ, JHEP 23'

$$\begin{pmatrix} O_q \\ O_g \end{pmatrix} = \begin{pmatrix} C_{qq} & C_{qg} \\ C_{gq} & C_{gg} \end{pmatrix} \otimes \begin{pmatrix} O_q^{l.t.} \\ O_g^{l.t.} \end{pmatrix} + h.t.$$

- $l.t.$ stands for the leading-twist projection which acts as the generating function of leading-twist local operators and gives lightcone quantities

$$\langle P_1 S_1 | O^{l.t.}_{\gamma^+}(z_1^-, z_2^-) | P_2 S_2 \rangle$$
$$= \int_{-1}^{1} dx \, e^{i(x+\xi)P^+ z_1^- - i(x-\xi)P^+ z_2^-} \bar{u}(P_1 S_1) \left[H(x,\xi,t)\gamma^+ + E(x,\xi,t) \frac{i\sigma^{+\mu}\Delta_\mu}{2M} \right] u(P_2 S_2)$$

- The mixing matrix depends on two Feynman parameters in general, for example

$$O_q(z_1, z_2) = \int_0^1 d\alpha \int_0^{\bar{\alpha}} d\beta \left[C_{qq}(\alpha, \beta, \mu^2 z_{12}^2) O_q^{l.t.}(z_{12}^\alpha, z_{21}^\beta) + C_{qg}(\alpha, \beta, \mu^2 z_{12}^2) O_g^{l.t.}(z_{12}^\alpha, z_{21}^\beta) \right.$$
$$\left. + \widetilde{C}_{qq}(\alpha, \beta, \mu^2 z_{12}^2) O_q^{l.t.}(z_{21}^\alpha, z_{12}^\beta) + \widetilde{C}_{qg}(\alpha, \beta, \mu^2 z_{12}^2) O_g^{l.t.}(z_{21}^\alpha, z_{12}^\beta) \right],$$

Theoretical calculations of GPDs

- **Determination of matching matrix** **F. Yao, Y. Ji, JHZ, JHEP 23'**

	$O_q/O_q^{l.t.}$	$O_g/O_g^{l.t.}$
quark	$C_{qq}^{(1)} = \frac{\langle q\lvert O_q\rvert q'\rangle^{(1)} - \langle q\lvert O_q^{l.t.}\rvert q'\rangle^{(1)}}{\langle q\lvert O_q^{l.t.}\rvert q'\rangle^{(0)}}$	$C_{gq}^{(1)} = \frac{\langle q\lvert O_g\rvert q'\rangle^{(1)} - \langle q\lvert O_g^{l.t.}\rvert q'\rangle^{(1)}}{\langle q\lvert O_q^{l.t.}\rvert q'\rangle^{(0)}}$
gluon	$C_{qg}^{(1)} = \frac{\langle g\lvert O_q\rvert g'\rangle^{(1)} - \langle g\lvert O_q^{l.t.}\rvert g'\rangle^{(1)}}{\langle g\lvert O_g^{l.t.}\rvert g'\rangle^{(0)}}$	$C_{gg}^{(1)} = \frac{\langle g\lvert O_g\rvert g'\rangle^{(1)} - \langle g\lvert O_g^{l.t.}\rvert g'\rangle^{(1)}}{\langle g\lvert O_g^{l.t.}\rvert g'\rangle^{(0)}}$

- NLO results in $\overline{\text{MS}}$ scheme

$$C_{qq}^{\overline{\text{MS}}}(\alpha,\beta,\mu^2 z_{12}^2) = \delta(\alpha)\delta(\beta) + 2a_s C_F\left\{\left(A_2 + \left[\frac{\bar\alpha}{\alpha}\right]_+\delta(\beta) + \left[\frac{\bar\beta}{\beta}\right]_+\delta(\alpha)\right)(\mathrm{L_z}-1) + \mathrm{A}_3 - 2\left[\frac{\ln(\alpha)}{\alpha}\right]_+\delta(\beta) - 2\left[\frac{\ln(\beta)}{\beta}\right]_+\delta(\alpha)\right\} + 2a_s C_F(-2\mathrm{L_z}+2)\delta(\alpha)\delta(\beta)\,,$$

$$A_{2,u} = 1\,,\qquad A_{2,h} = 1\,,\qquad A_{2,t} = 0\,,$$

$$A_{3,u} = 2\,,\qquad A_{3,h} = 4\,,\qquad A_{3,t} = 0\,,$$

Theoretical calculations of GPDs

- **Determination of matching matrix** F. Yao, Y. Ji, JHZ, JHEP 23'

	$O_q/O_q^{l.t.}$	$O_g/O_g^{l.t.}$
quark	$C_{qq}^{(1)} = \frac{\langle q\vert O_q\vert q'\rangle^{(1)} - \langle q\vert O_q^{l.t.}\vert q'\rangle^{(1)}}{\langle q\vert O_q^{l.t.}\vert q'\rangle^{(0)}}$	$C_{gq}^{(1)} = \frac{\langle q\vert O_g\vert q'\rangle^{(1)} - \langle q\vert O_g^{l.t.}\vert q'\rangle^{(1)}}{\langle q\vert O_q^{l.t.}\vert q'\rangle^{(0)}}$
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- NLO results in $\overline{\text{MS}}$ scheme

$$
\begin{aligned}
C_{gg}^{\overline{\text{MS}}}(\alpha,\beta,\mu^2 z_{12}^2) =&\delta(\alpha)\delta(\beta) + 2a_s C_A\left\{ \left(E_1 + \left[\frac{\bar{\alpha}^2}{\alpha}\right]_+ \delta(\beta) + \left[\frac{\bar{\beta}^2}{\beta}\right]_+ \delta(\alpha)\right)(\mathrm{L_z}-1) + E_2 \right.\\
&\left. - 2\left[\frac{\ln(\alpha)}{\alpha}\right]_+ \delta(\beta) - 2\left[\frac{\ln(\beta)}{\beta}\right]_+ \delta(\alpha)\right\} + 2a_s C_A\left(-3\,\mathrm{L_z}+2\right)\delta(\alpha)\delta(\beta)\,,
\end{aligned}
$$

$$
\begin{aligned}
E_{1,u} =&4(1-\alpha-\beta+3\alpha\beta)\,, \qquad E_{1,h} = 4(1-\alpha-\beta)\,, \qquad E_{1,t} = 0,\\
E_{2,u} =&\frac{5}{2}E_{1,u} + 6\alpha\beta\,, \qquad E_{2,h} = \frac{3}{2}E_{1,h}\,, \qquad E_{2,t} = 2(1+\alpha+\beta-2\alpha\beta).
\end{aligned}
$$

Theoretical calculations of GPDs

- **Determination of matching matrix** F. Yao, Y. Ji, JHZ, JHEP 23'

	$O_q/O_q^{l.t.}$	$O_g/O_g^{l.t.}$
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- NLO results in $\overline{\text{MS}}$ scheme

$$C_{qg}^{\overline{\text{MS}}}(\alpha,\beta,\mu^2 z_{12}^2) = 4ia_s T_F N_f\, \mathbf{z}_{12}\, B_3\, \text{L}_z\,.$$

$$B_{3,u} = \bar{\alpha}\bar{\beta} + 3\alpha\beta\,, \qquad B_{3,h} = \bar{\alpha}\bar{\beta} - \alpha\beta.$$

$$C_{gq}^{\overline{\text{MS}}}(\alpha,\beta,\mu^2 z_{12}^2) = \frac{-2ia_s C_F}{\mathbf{z}_{12}}\left\{D_3\,(\text{L}_z+1) + 4 - 2\big(\delta(\alpha)+\delta(\beta)\big) + (\text{L}_z + D_4)\,\delta(\alpha)\delta(\beta)\right\},$$

$$D_{3,u} = 2\,, \qquad D_{3,h} = -2\,, \qquad D_{4,u} = 1\,, \qquad D_{4,h} = 0\,.$$

- Results in ratio and hybrid schemes are also available

Theoretical calculations of GPDs

- **Determination of matching matrix** **F. Yao, Y. Ji, JHZ, JHEP 23'**

	$O_q/O_q^{l.t.}$	$O_g/O_g^{l.t.}$
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- NLO results in $\overline{\text{MS}}$ scheme

$$C_{qg}^{\overline{\text{MS}}}(\alpha, \beta, \mu^2 z_{12}^2) = 4ia_s T_F N_f \, \mathbf{z}_{12} \, B_3 \, \mathrm{L_z} \,.$$

$$B_{3,u} = \bar{\alpha}\bar{\beta} + 3\alpha\beta \,, \qquad B_{3,h} = \bar{\alpha}\bar{\beta} - \alpha\beta.$$

$$C_{gq}^{\overline{\text{MS}}}(\alpha, \beta, \mu^2 z_{12}^2) = \frac{-2ia_s C_F}{\mathbf{z}_{12}} \Big\{ D_3 \, (\mathrm{L_z} + 1) + 4 - 2\big(\delta(\alpha) + \delta(\beta)\big) + (\mathrm{L_z} + D_4) \, \delta(\alpha)\delta(\beta) \Big\} \,,$$

$$D_{3,u} = 2 \,, \qquad D_{3,h} = -2 \,, \qquad D_{4,u} = 1 \,, \qquad D_{4,h} = 0 \,.$$

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- Double Fourier transform gives the results for GPDs in momentum space

$$\frac{1}{\mathbf{z_{12}}}\mathrm{e}^{ix\mathbf{z_{12}}} = i\int_{\infty(-1+i\epsilon)}^{x} \mathrm{e}^{ix_1\mathbf{z_{12}}}\, dx_1$$

- The lower limit is chosen for convenience and brings ambiguities
- Removed by requiring that the same moments are generated before and after FT

Theoretical calculations of GPDs

- **Determination of matching matrix** F. Yao, Y. Ji, JHZ, JHEP 23'

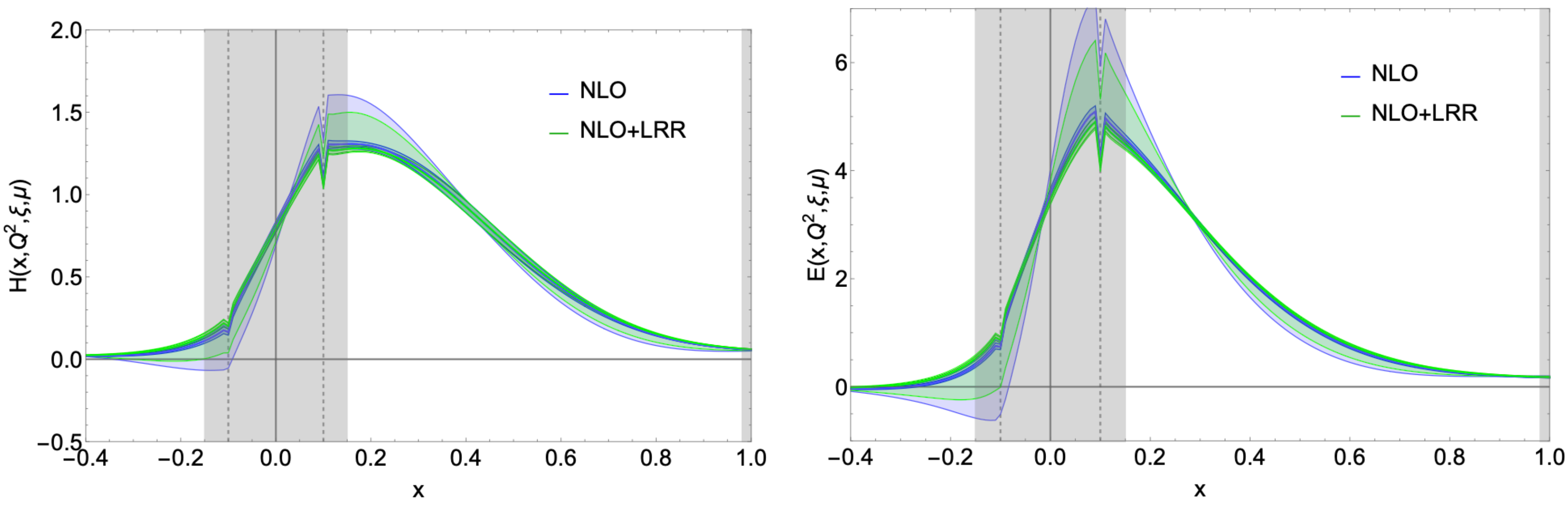
	$O_q/O_q^{l.t.}$	$O_g/O_g^{l.t.}$
quark	$C_{qq}^{(1)} = \frac{\langle q\|O_q\|q'\rangle^{(1)} - \langle q\|O_q^{l.t.}\|q'\rangle^{(1)}}{\langle q\|O_q^{l.t.}\|q'\rangle^{(0)}}$	$C_{gq}^{(1)} = \frac{\langle q\|O_g\|q'\rangle^{(1)} - \langle q\|O_g^{l.t.}\|q'\rangle^{(1)}}{\langle q\|O_q^{l.t.}\|q'\rangle^{(0)}}$
gluon	$C_{qg}^{(1)} = \frac{\langle g\|O_q\|g'\rangle^{(1)} - \langle g\|O_q^{l.t.}\|g'\rangle^{(1)}}{\langle g\|O_g^{l.t.}\|g'\rangle^{(0)}}$	$C_{gg}^{(1)} = \frac{\langle g\|O_g\|g'\rangle^{(1)} - \langle g\|O_g^{l.t.}\|g'\rangle^{(1)}}{\langle g\|O_g^{l.t.}\|g'\rangle^{(0)}}$

- The momentum space results obtained from the FT
  - Agree with direct calculations in momentum space up to terms that vanish upon integration, **except for C_{gq}**

- Expected to be
  - related to the prescription of the pole $1/\mathbf{z}_{12}$
  - connected to the long-standing mismatch in GPD evolution kernel calculations in coordinate and momentum space

Theoretical calculations of GPDs

- Implementation in lattice calculations of GPDs



$\xi = 0.1$

Holligan et al, 23'

Summary

- Lattice QCD can provide complementary information to experimental data on the 3D structure of nucleons
- For simple quantities such as collinear PDFs, lattice calculations of x-dependent distributions have reached a stage where precision control becomes important
- Similar analyses shall be extended to the lattice calculations of GPDs
- A unified framework has been developed for perturbative calculations required for the extraction of all leading-twist collinear parton observables
- Sheds light on the discrepancy in existing results in the literature
- Facilitate perturbative calculations to higher orders