

Color transparency and probing GPDs

Mark Strikman, PSU



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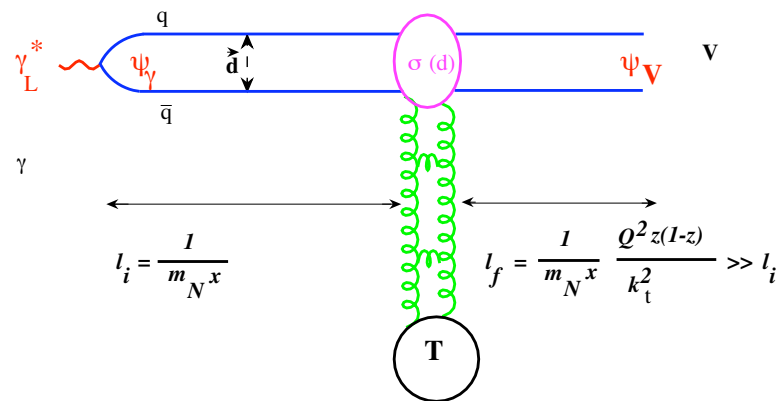
Outline

- ❖ Exclusive VM production in DIS- lessons from HERA
- ❖ Color transparency - necessary (sufficient ?) condition for factorization
How to suppress diffusion of a small wave packet to to a hadron size ones
- ❖ Theory of the Leading twist shadowing, nuclear GPDs
- ❖ Tests of theory of nuclear shadowing
Coherent J/ψ production off heavy and light nuclei
Use of ZDC at LHC

Very briefly

Vector meson diffractive production: Theory and HERA data

Space-time picture of Vector meson production at small x in the target rest frame



\Rightarrow Similar to the $\pi + T \rightarrow 2jets + T$ process, $A(\gamma_L^* + p \rightarrow V + p)$ at $p_t = 0$ is a convolution of the light-cone wave function of the photon $\Psi_{\gamma^* \rightarrow |q\bar{q}\rangle}$, the amplitude of elastic $q\bar{q} - target$ scattering, $A(q\bar{q}T)$, and the wave function of vector meson, ψ_V : $A = \int d^2d \psi_{\gamma^*}^L(z, d) \sigma(d, s) \psi_V^{q\bar{q}}(z, d)$.

The leading twist parameter free answer is BFGMS94

$$\left. \frac{d\sigma_{\gamma^* N \rightarrow VN}^L}{dt} \right|_{t=0} = \frac{12\pi^3 \Gamma_{V \rightarrow e^+e^-} M_V \alpha_s^2(Q) \eta_V^2 \left| \left(1 + i\frac{\pi}{2} \frac{d}{d \ln x}\right) x G_T(x, Q^2) \right|^2}{\alpha_{EM} Q^6 N_c^2}$$

. Here, $\Gamma_{V \rightarrow e^+e^-}$ is the decay width of $V \rightarrow e^+e^-$;

$$\eta_V \equiv \frac{1 \int \frac{dz d^2 k_t}{z(1-z)} \Phi_V(z, k_t)}{2 \int dz d^2 k_t \Phi_V(z, k_t)} \rightarrow 3 \quad |Q^2 \rightarrow \infty$$

Note: In the leading twist $d=0$ in $\psi_V(z, d)$. Finite b effects in the meson wave function is one of the major sources of the higher twist effects.

energy denominator $\frac{1}{Q^2 + \frac{m^2 + k_t^2}{z(1-z)}}$ **operator of interaction** $\left(\frac{1}{Q^2 + \frac{m^2 + k_t^2}{z(1-z)}} \right)^4$
m- quark mass

$\frac{Q^2}{(\mu^2 + Q^2)^4} \rightarrow \frac{1}{Q^6}$
 $\mu^2 \geq m_V^2$

A QCD dipole model of J/ψ production - aims to account more accurately for geometry

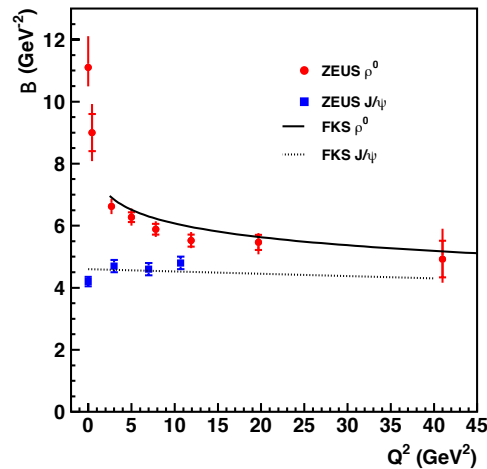
$$A(\gamma + p \rightarrow J/\psi + p) = \int d^2d \psi_{\gamma \rightarrow c\bar{c}}(z, d) \sigma_{tot}(c\bar{c}, p) \psi_{J/\psi}(z, d)$$

Slow onset of the LT for cross section both for light and heavy mesons

Slow squeezing of dipole size for light mesons, but early dominance of small dipoles for J/ψ

- Universal t-slope: process is dominated by the scattering of quark-antiquark pair in a small size configuration - t-dependence is predominantly due to the transverse spread of the gluons in the nucleon - two gluon nucleon form factor/ diagonal gluon GPD $F_g(x,t)$. $d\sigma/dt \propto F_g^2(x,t)$. Onset of universal regime FKS[Frankfurt,Koepf, MS,97] **early for J/ψ late for ρ**

$$r_T \propto \frac{1}{Q} \left(\frac{1}{m_c} \right) \ll r_N$$



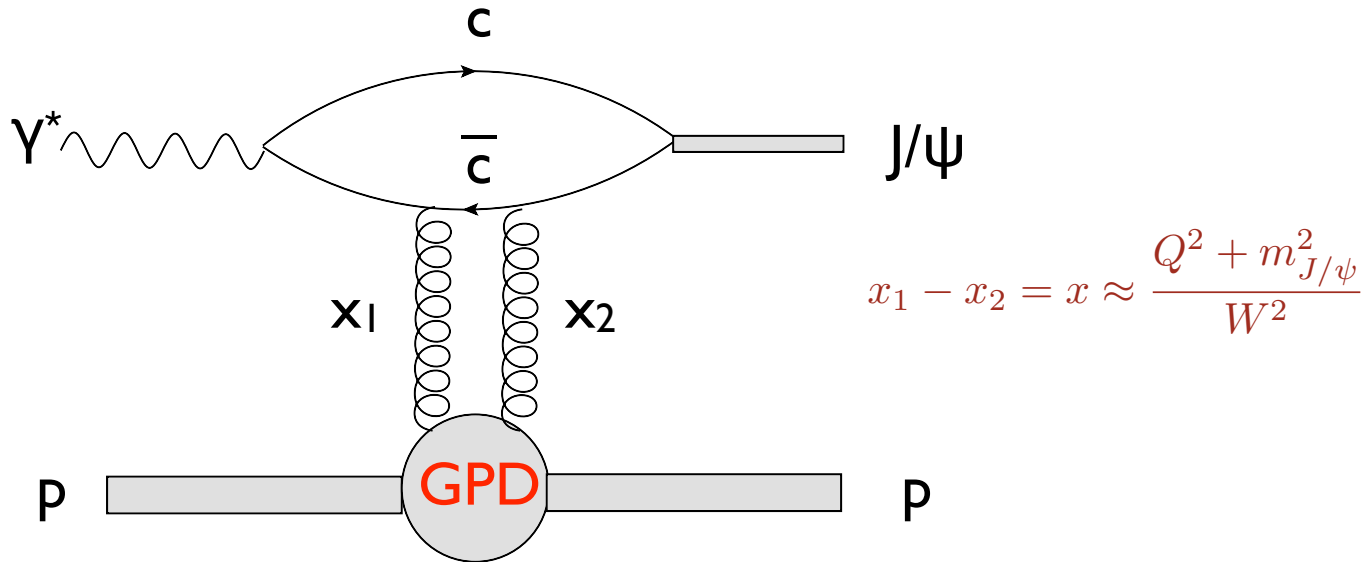
Convergence of the t-slopes, B - $\frac{d\sigma}{dt} = A \exp(Bt)$ ρ -meson electroproduction to the slope of J/ψ photo(electro)production.

⇒ Transverse distribution of gluons GPD) can be extracted from $\gamma + p \rightarrow J/\psi + N$

Correction for finite J/ψ size is ~ 10%.

Reminder: transverse spread of gluons enters into description of jet production in pp collisions at the LHC energies

Caviate: experimentally one can measure only nondiagonal GPD



Analysis of the overlapping integral including Fermi motion of quarks in J/ψ (Koepf et al)

$$x_1/x_2 \sim 2 \div 3$$

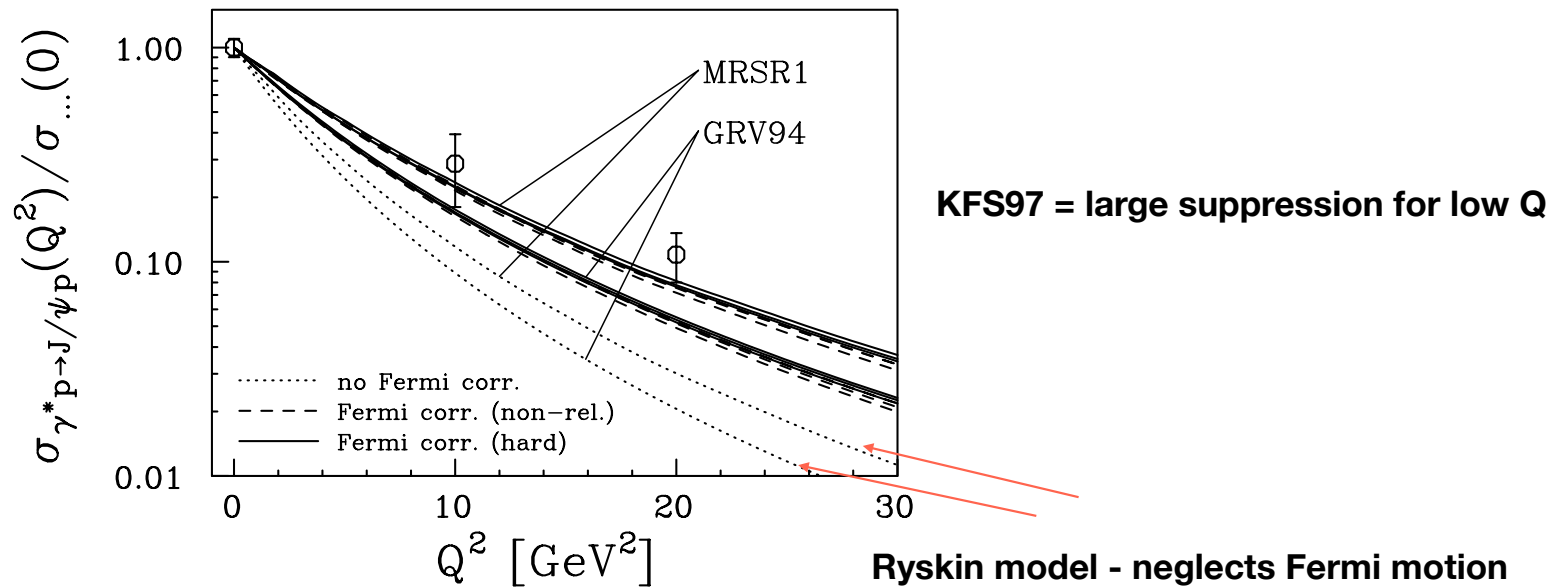
$$x_{eff} = (x_1 + x_2)/2 \sim x$$

In many models Fermi motion is neglected and x_2 is assumed to be 0.

Open questions in exclusive J/psi production

a) How safe it is to neglect Fermi motion of quarks

- Confirmation of the presence of the Fermi suppression factor $T(Q^2)$ in Q^2 dependence of J/ψ production:



Leading $\ln x$ (plus energy conservation), vs leading $\ln Q^2$ approximations (DGLAP)

Preferable at LHC & Top RHIC energies

b) Relation between NR and LC wave functions LC wave function of quarkonium

Normalization of light cone wave function through f_V does not contain terms (Brodsky & Lepage), while in nonrelativistic model there is a Barbieri factor $\propto \alpha_s$

$$1 - 16\alpha_s/3\pi \sim 0.5$$

suggests presence of large $c\bar{c}g$ component in charmonia

c) what is the value of m_c and how it evolves with resolution?

Charmonium models: $m_c > m_{J/\psi}/2$; pQCD $< m_{J/\psi}/2$

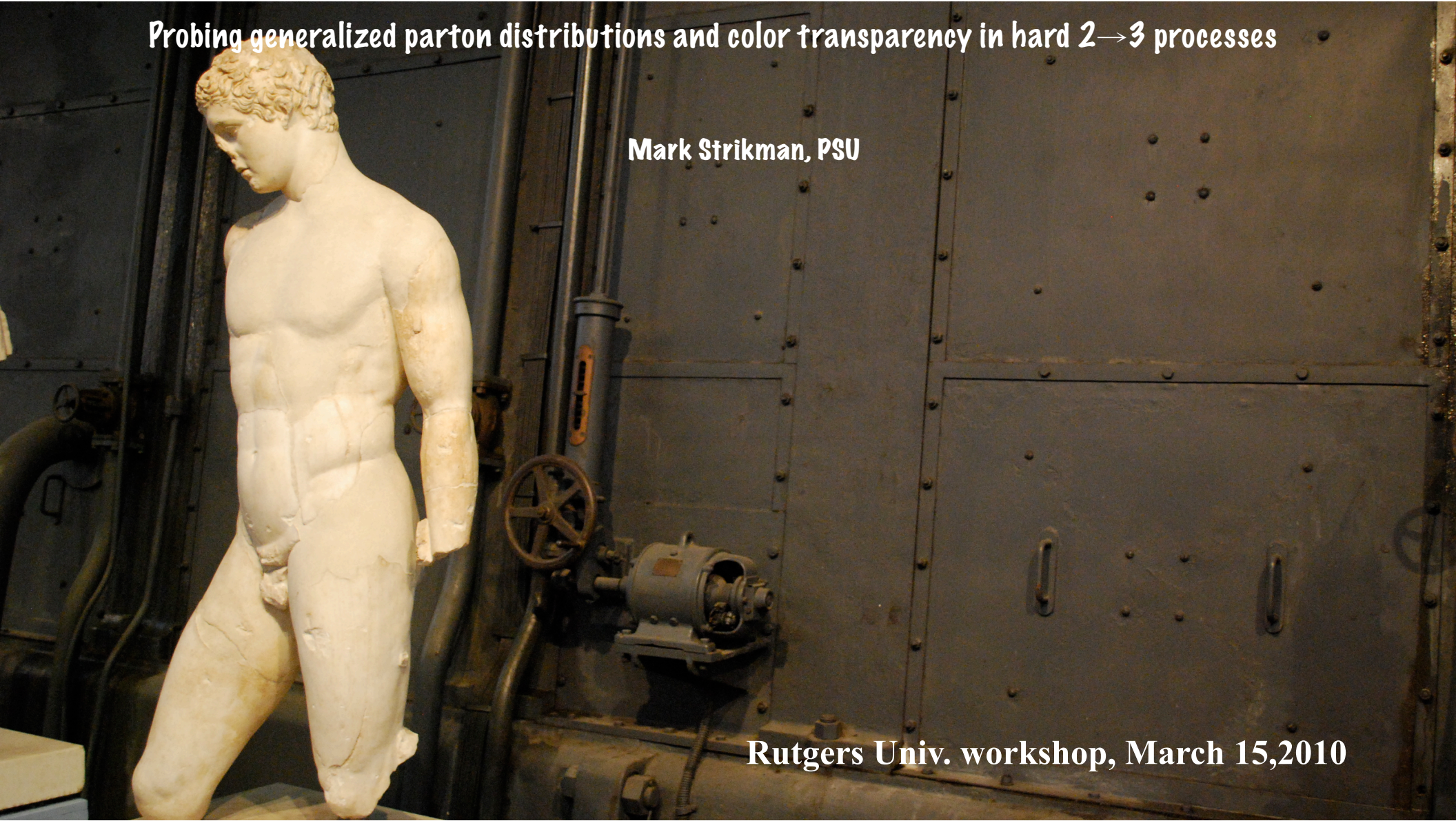


These processes cannot be used so far for extraction of the absolute value of gluon density (need much larger Q, m). However since J/ψ is a compact probe, ratios for different targets are mostly unaffected. The t -dependence also can be trusted

Probing generalized parton distributions and color transparency in hard $2 \rightarrow 3$ processes

Mark Strikman, PSU

Rutgers Univ. workshop, March 15, 2010



Starting at what t $2 \rightarrow 2$ large angle process allow to do analog of DIS -
select point - like configurations in hadrons?

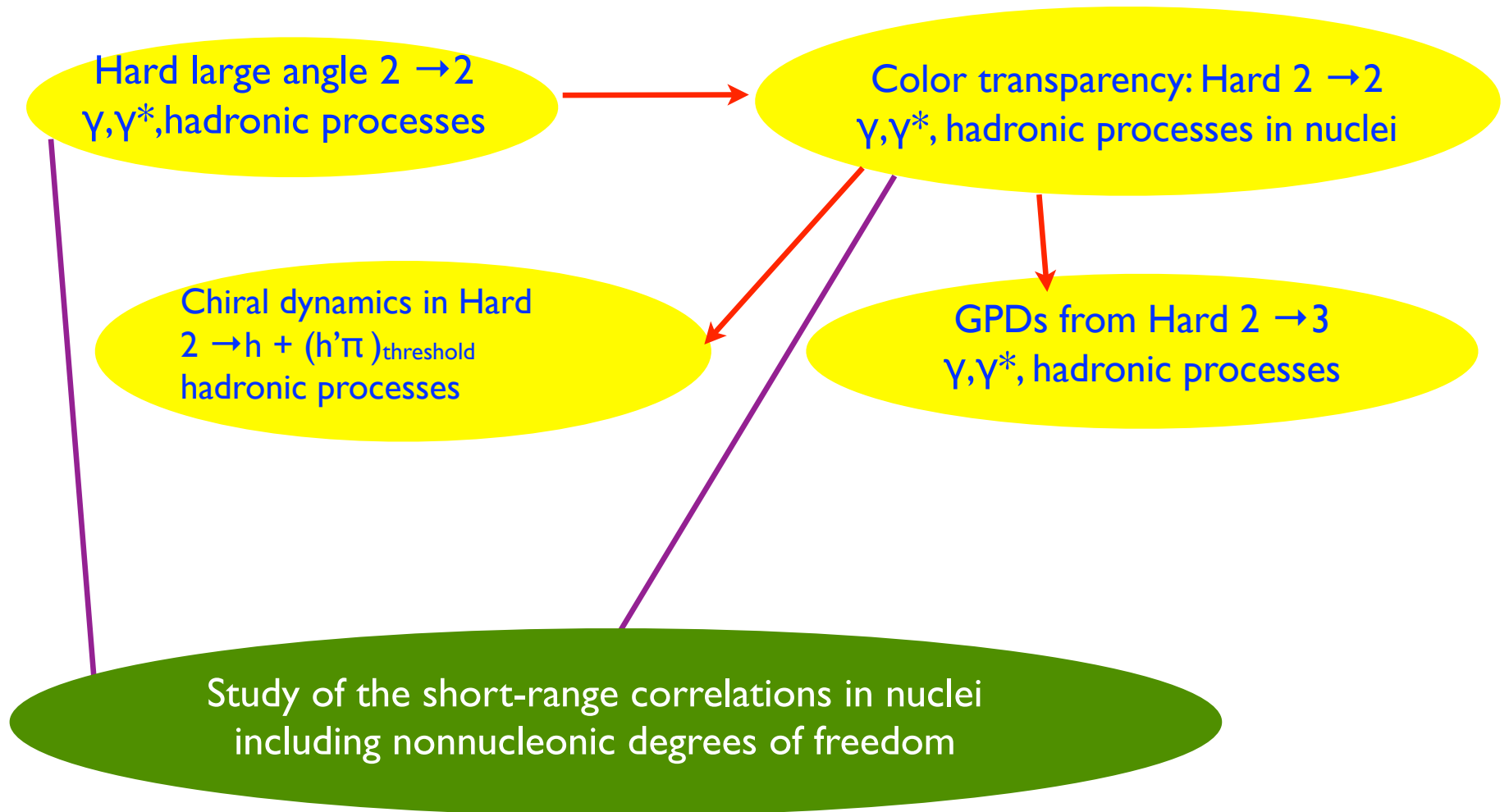
Hard large angle $2 \rightarrow 2$
 γ, γ^* , hadronic processes

Color transparency: Hard $2 \rightarrow 2$
 γ, γ^* , hadronic processes in nuclei

Chiral dynamics in Hard
 $2 \rightarrow h + (h'\pi)_{\text{threshold}}$
hadronic processes

GPDs from Hard $2 \rightarrow 3$
 γ, γ^* , hadronic processes

Study of the short-range correlations in nuclei
including nonnucleonic degrees of freedom



Main tool for exclusive processes is color coherence (CC) property of QCD and resulting **Color transparency (CT)**

CT phenomenon plays a dual role:

- ✘ probe of the high energy dynamics of strong interaction
- ✘ probe of minimal small size components of the hadrons

at intermediate energies also a unique probe of the space time evolution of wave packages

Basic tool of CT: suppression of interaction of small size color singlet configurations = CC

For a dipole of transverse size d :

$\sigma \approx cd^2$ in the lowest order in α_s (two gluon exchange **F.Low 75**)

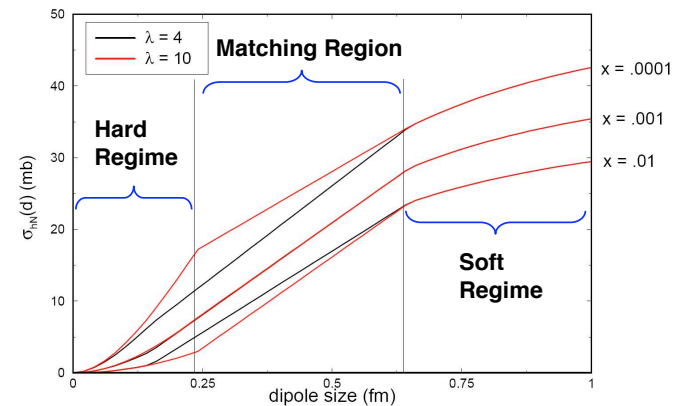
$$\sigma(d, x_N) = \frac{\pi^2}{3} \alpha_s(Q_{eff}^2) d^2 [x_N G_N(x_N, Q_{eff}^2) + 2/3 x_N S_N(x_N, Q_{eff}^2)]$$

$Q^2 = 3.0 \text{ GeV}^2$

Important at $E_{\text{dipole}} < 10 \text{ GeV}$

Here **S** is sea quark distribution for quarks making up the dipole.

(Baym et al 93, FS&Miller 93 & 2000)



Brief Summary of CT dynamics ingredients: *squeeze and freeze*

Squeezing: (a) *high energy CT*

* Select special final states: diffraction of pion into two high p_t jets: $d_{q\bar{q}} \sim 1/p_t$

* Select a small initial state: γ^*_L - $d_{q\bar{q}} \sim 1/Q$ in $\gamma^*_L + N \rightarrow M + B$

QCD factorization theorems are valid for these processes with the proof based on the CT property of QCD

(b) *Intermediate energy CT*

* Nucleon form factor

* γ^*_L (γ^*_T ?) + $N \rightarrow M + B$

* Large angle ($t/s = \text{const}$) two body processes: $a + b \rightarrow c + d$ Brodsky & Mueller 82

↑ Problem: *strong*
| *correlation* between
| t (Q) and lab
↓ momentum of
| produced hadron

Freezing: Main challenge: $|qqq\rangle$ ($|qq\bar{q}\rangle$ is not an eigenstate of the QCD Hamiltonian. So even if we find an elementary process in which interaction is dominated by small size configurations - they are not frozen. They evolve with time - expand after interaction to average configurations and contract before interaction from average configurations

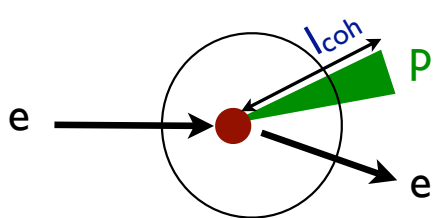
(FFLS88)

$$|\Psi_{PLC}(t)\rangle = \sum_{i=1}^{\infty} a_i \exp(iE_i t) |\Psi_{it}\rangle = \exp(iE_1 t) \sum_{i=1}^{\infty} a_i \exp\left(\frac{i(m_i^2 - m_1^2)t}{2P}\right) |\Psi_{it}\rangle$$

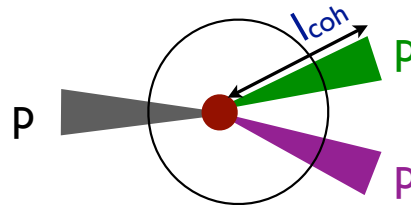
$$\sigma^{PLC}(z) = \left(\sigma_{hard} + \frac{z}{l_{coh}} [\sigma - \sigma_{hard}] \right) \theta(l_{coh} - z) + \sigma \theta(z - l_{coh})$$

Quantum Diffusion model of expansion

$l_{coh} \sim (0.4 - 0.8) \text{ fm } E_h[\text{GeV}]$ actually incoherence length



$eA \rightarrow ep$ (A-1) at large Q



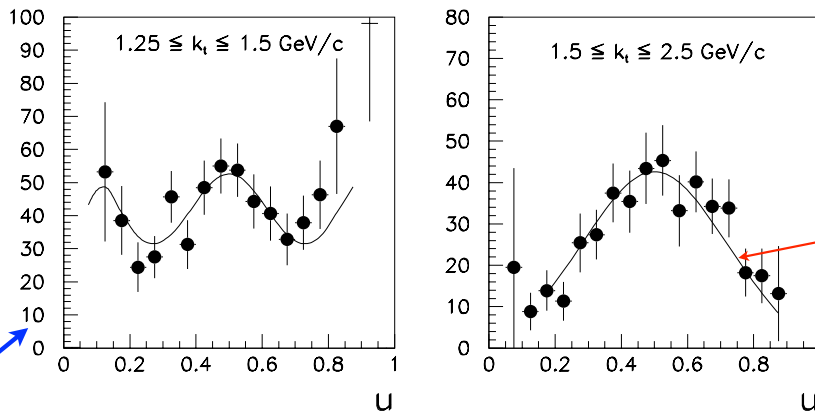
$pA \rightarrow pp$ (A-1) at large t and intermediate energies

Note - one can use multihadron basis with build in CT (Miller and Jennings) or diffusion model - numerical results for σ^{PLC} are very similar.

High energy color transparency is well established

At high energies weakness of interaction of point-like configurations with nucleons - is routinely used for explanation of DIS phenomena at HERA.

First experimental observation of high energy CT for pion interaction (Ashery 2000): $\pi + A \rightarrow \text{''jet''} + \text{''jet''} + A$. Confirmed predictions of pQCD (Frankfurt, Miller, MS93) for A -dependence, distribution over energy fraction, u carried by one jet, dependence on $p_t(\text{jet})$, etc. Factorization is proven,



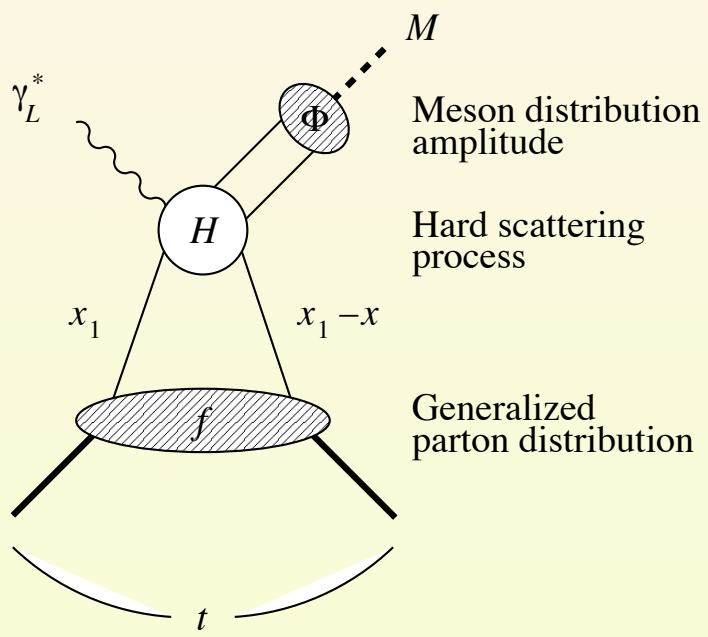
Squeezing occurs already before the leading term $(1-z)z$ dominates!!!

prediction
(π wave funct)²

$$Q^2(\pi \text{ f.f.}) \sim 4k_t^2(\text{jet})$$

↓
strong squeezing in π form factor
for $Q^2=6 \text{ GeV}^2$

High energy CT = QCD factorization theorem for DIS exclusive meson processes (Brodsky, Frankfurt, Gunion, Mueller, MS 94 - vector mesons, small x ; general case Collins, Frankfurt, MS 97). The prove is based (as for dijet production) on the CT property of QCD not on closure like the factorization theorem for inclusive DIS.



No interaction between partons
to form M and baryon system due
to CT /squeezing

- ⇒⇒⇒ Presence of small size $q\bar{q}$ Fock components in light mesons is unambiguously established
- ⇒⇒⇒ At transverse separations $d \leq 0.3$ fm pQCD reasonably describes “small $q\bar{q}$ - dipole”- nucleon interaction for $10^{-4} < x < 10^{-2}$
- ⇒⇒⇒ Color transparency is established for the interaction of small dipoles with nucleons and with nuclei (for $x \sim 10^{-2}$)

Intermediate energies

Main issues

- At what Q^2 / t particular processes select PLC - for example interplay of end point and LT contributions in the e.m. form factors, exclusive meson production.
- $l_{\text{coh}} = (0.4 \div 0.8 \text{ fm}) p_h [\text{GeV}] \rightarrow p_h = 6 \text{ GeV}$ corresponds $l_{\text{coh}} = 4 \text{ fm} \sim 1/\sigma_{NN}\rho_0$

need high energies to see large CT effect even if squeezing is effective at $E \sim$ few GeV

Experimental situation

- ☀ Energy dependence of transparency in (p,2p) is observed for energies corresponding to $l_{\text{coh}} \geq 3$ fm. Such dependence is impossible without freezing. But not clear whether effect is CT or something else? Needs independent study & new approaches.

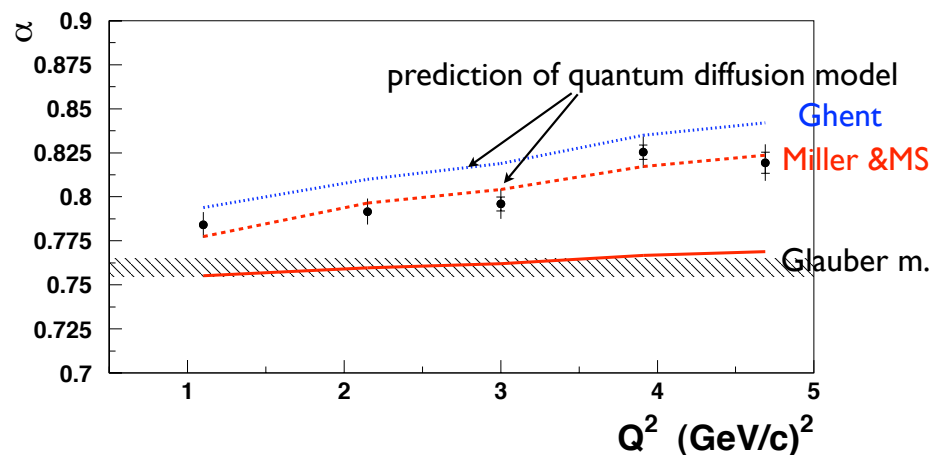
- ☀ $\gamma^* + A \rightarrow \pi A^*$ evidence for increase of transparency with Q (Dutta et al 07)

Note that elementary reaction for Jlab kinematics is dominated by ERBL term so

$\gamma^* N$ interaction is local. γ^* does not transform to $q\bar{q}$ distance $1/m_N X$ before nucleon

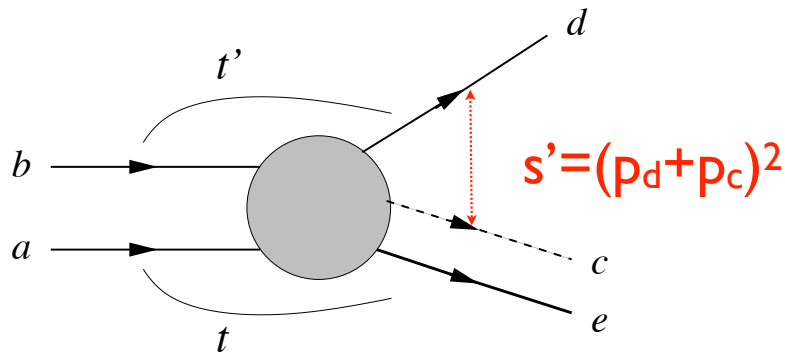
A- dependence checks not only squeezing but **small** l_{coh} as well

Also Jlab and HERMES ρ meson production data & FNAL J/ψ data indicate CT



Idea is to consider **new type of hard hadronic processes** - branching exclusive processes of large c.m. angle scattering on a “cluster” in a target/projectile (MS94)

to study both CT of $2 \rightarrow 2$ and hadron GPDs



Limit:

$$-t' > \text{few GeV}^2, -t'/s' \sim 1/2$$

$$-t = \text{const} \sim 0$$

$$\implies s'/s = y < 1,$$

$$t_{\min} = [m_a^2 - m_b^2 / (1-y)]y$$

T Kumano, MS, and Sudoh PRD 09;

Kumano & MS arXiv:0909.1299, Phys.Lett. 2010

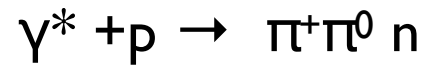
For hadron induced processes two kinematics - different detector strategies

“a” at rest - “d” and “c” in forward spectrometer, “e” in recoil detector

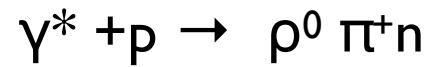
\implies can use neutron (^2H)/ transversely polarized target

“b” at rest - “d”, “c” and “e” in forward spectrometer \implies can use neutron target

For e p collider possible processes



current fragmentation



nucleon fragmentation

For fixed meson - meson system mass,...)

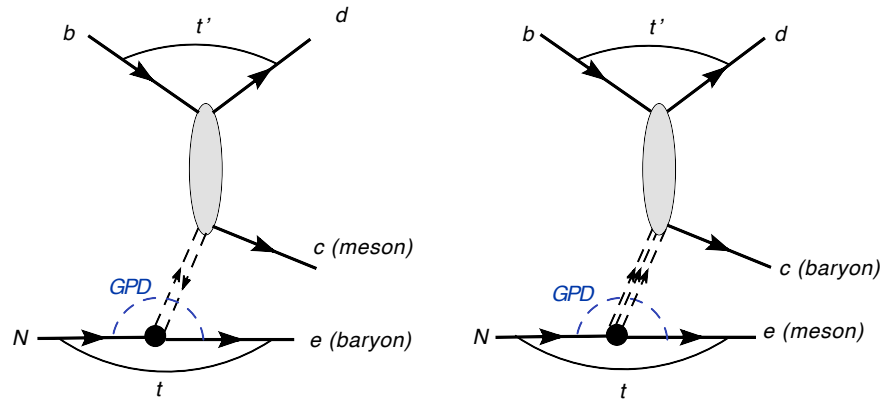
a rather fast decrease of the cross section with s:

$$\sigma \propto s^{-1 \div 2}$$

How practical for collider kinematics (recent papers of Qui and Zhite Yu and earlier today)

- requires further studies - resolution, acceptance rates for neutron, proton

Factorization:



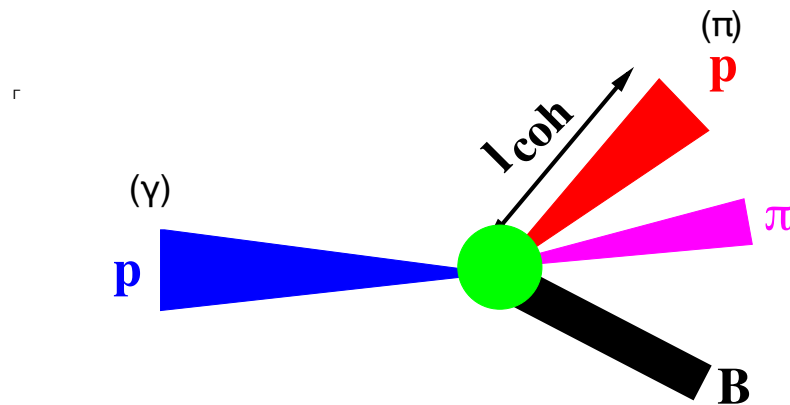
If the upper block is a hard ($2 \rightarrow 2$) process, “b”, “d”, “c” are in small size configurations as well as exchange system (qq, qq \bar{q}). Can use CT argument as in the proof of QCD factorization of meson exclusive production in DIS (Collins, LF, MS 97)



$$\mathcal{M}_{NN \rightarrow N\pi B} = GPD(N \rightarrow B) \otimes \psi_b^i \otimes H \otimes \psi_d \otimes \psi_c$$

Minimal condition for factorization:

$$l_{coh} > r_N \sim 0.8 \text{ fm}$$



Time evolution of the 2 → 3 process

$$l_{coh} = (0.4 \div 0.6 \text{ fm}) \cdot p_h / (\text{GeV}/c)$$

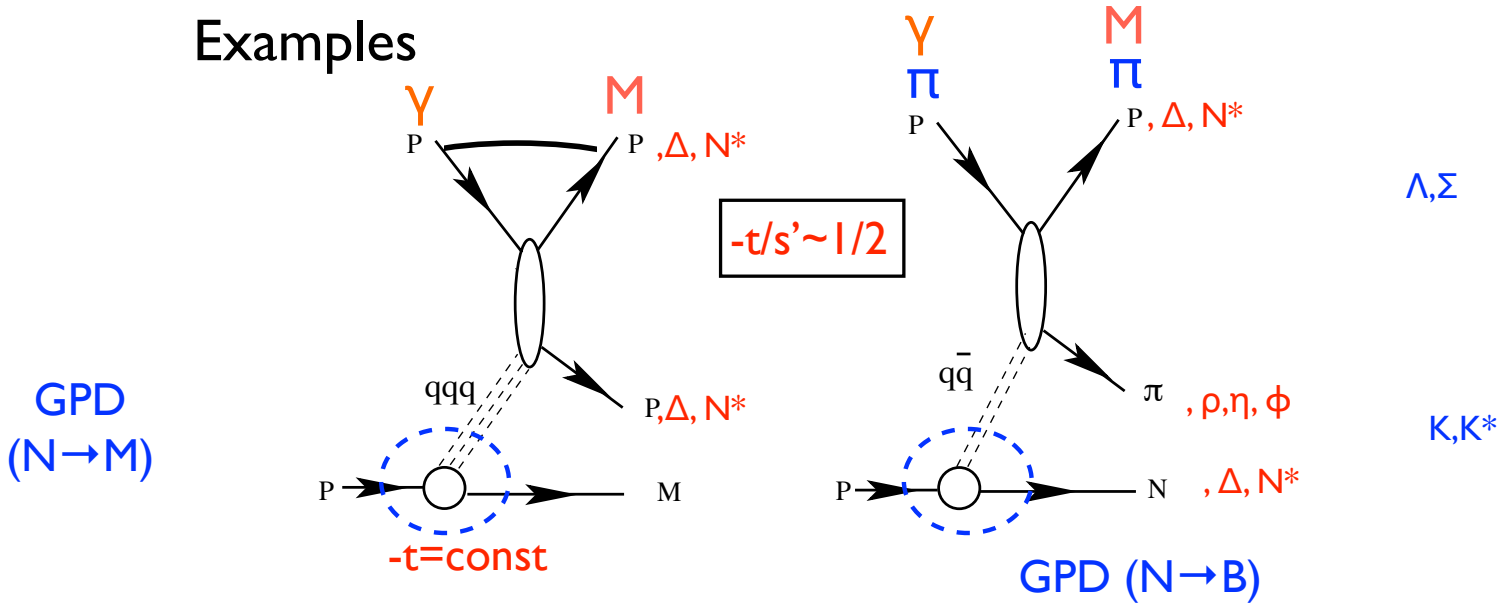
easy to satisfy at EIC

$$p_c \geq 3 \div 4 \text{ GeV}/c, \quad p_d \geq 3 \div 4 \text{ GeV}/c$$

$$p_b \geq 6 \div 8 \text{ GeV}/c$$

easier to reach than in CT reactions with nuclei

Examples



$$pp \rightarrow pN + M(\pi, \eta, \pi\pi)$$

$$pp \rightarrow p\Delta + M(\pi, \eta, \pi\pi)$$

$$pp \rightarrow p\Lambda + K^+$$

$$\pi^- p \rightarrow p\pi + M$$

$$\pi^- p \rightarrow \pi^- \pi^- \Delta^{++},$$

$$\pi^- p \rightarrow \pi^- \pi^+ \Delta^0,$$

$$\pi^- p \rightarrow \pi^- \pi^0 p,$$

$$\pi^- p \rightarrow \pi^- p + (\pi^0 \pi^0 - \text{forward low } p_t)$$

COMPASS

J-PARC if beams of pions with energies 20 -40 GeV are doable

Study of Hidden/Intrinsic Strangeness & Charm in hadrons

$$\gamma p \rightarrow M + \Lambda_{sp} \text{ (any other strange baryon)} + K^+(K^*)$$

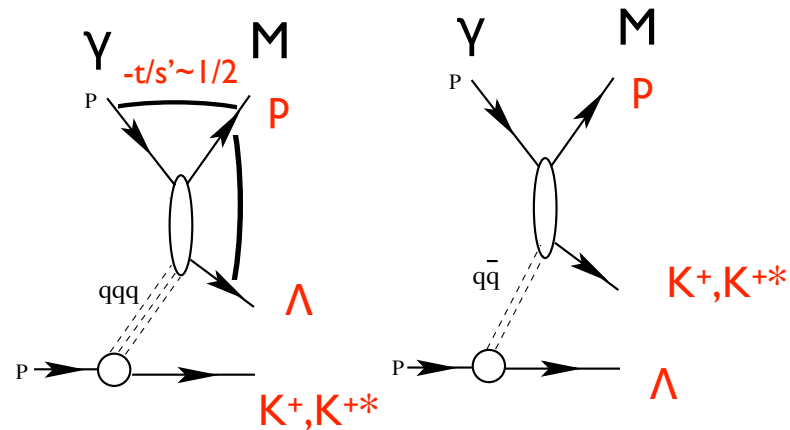
$$pp \rightarrow K(K^*)_{sp} + \Lambda + p$$

$$pp \rightarrow \phi_{sp} + p + p$$

$$pp \rightarrow \bar{D}_{sp} + \Lambda_c + p$$

$$\gamma p \rightarrow M + \bar{D}_{sp} + \Lambda_c$$

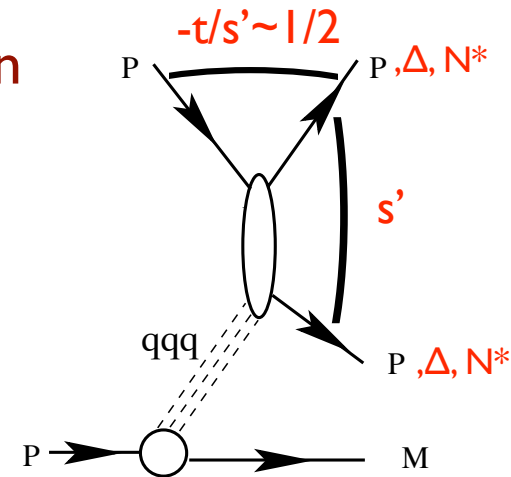
BNL experiment: EVA has few candidate events



Study of the spin structure of the nucleon

use of polarized beams and/or targets

Can one gain from electron polarization?



$$\vec{p}\vec{p} \rightarrow \Lambda_{sp} \text{ (any other strange baryon)} + K^+(K^*) + p$$

$$\vec{p}\vec{p} \rightarrow K^+(K^*)_{sp} + \Lambda \text{ (any other strange baryon)} + p$$

$$\vec{p}\vec{p} \rightarrow \Delta_{sp} \text{ (any other strange baryon)} + \text{meson} + p$$

study of the $N\Delta$ GPDs - more GPDs than for NN case - QCD chiral model - selection rules; single transverse spin asymmetries

Frankfurt, Pobilitza, Polyakov, MS 98

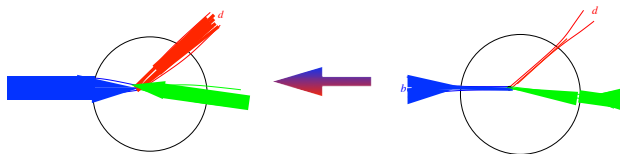
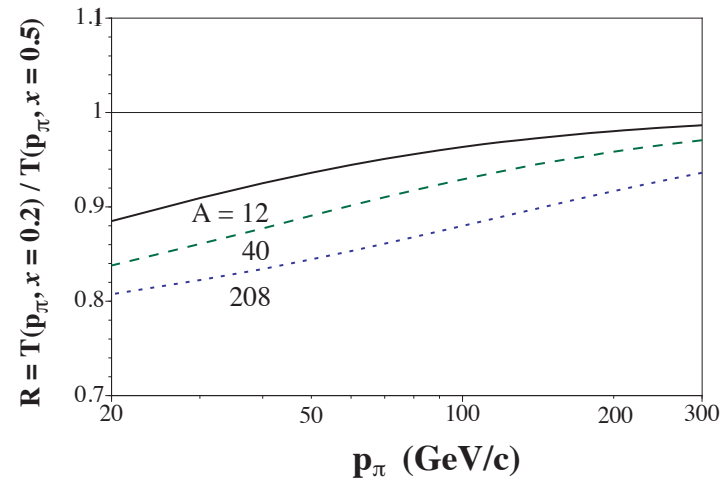
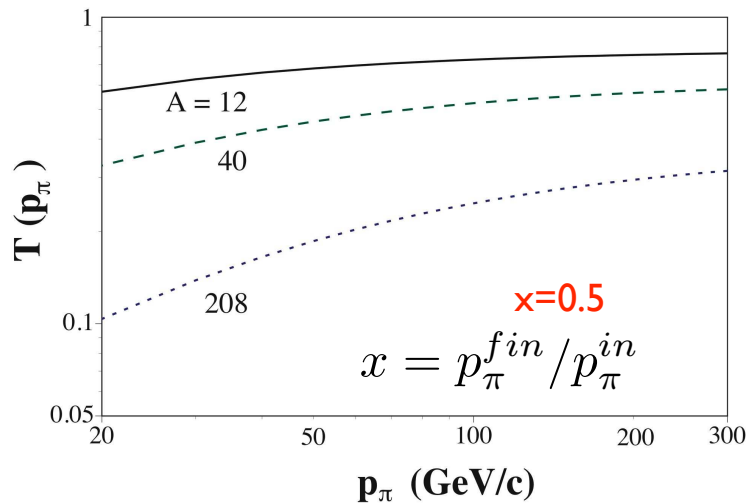
Defrosting point like configurations - energy dependence for fixed s',t'

$$\sigma^{PLC}(z) = \left(\sigma_{hard} + \frac{z}{l_{coh}} [\sigma - \sigma_{hard}] \right) \theta(l_{coh} - z) + \sigma \theta(z - l_{coh})$$

Quantum Diffusion model of expansion

Use $l_{coh} \sim 0.6 \text{ fm } E_h[\text{GeV}]$

which describes well CT for pion electroproduction

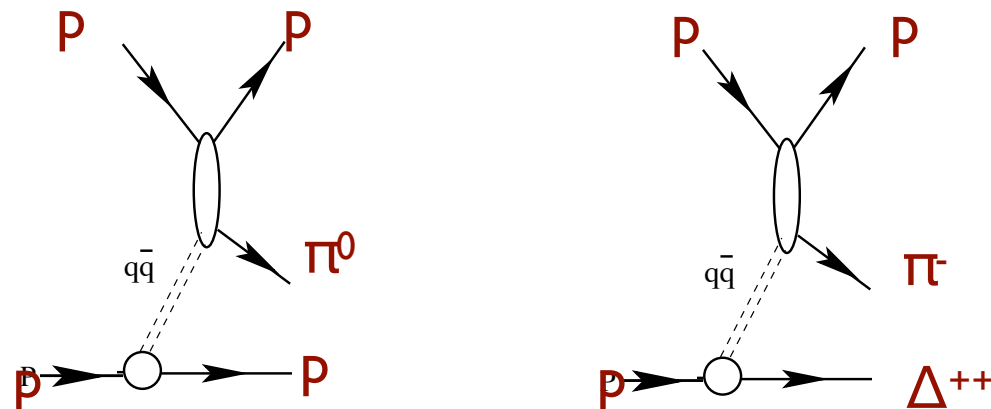


Higher energy - smaller the angle

Huge CT effect

A detailed theoretical study of the reactions $pp \rightarrow NN\pi$, $N\Delta\pi$ was recently completed.
Factorization based on squeezing

Kumano, Strikman, and Sudoh 09



It appears that it is much easier to squeeze meson than proton (few constituents)

J.W. Qui - parametric statement CT for meson-meson

hard block collision, no CT for collisions involving baryons

Discussed processes will allow (in the CT regime)

- ✱ to discover that pattern of interplay of hard and soft physics in one of the most fundamental hadronic processes of large angle scattering
- ✱ compare wave function of different mesons and baryons
- ✱ map the space-time evolution of small wave packets at distances $1 < z < 6$ fm
- ✱ test the role of chiral degrees of freedom in hard interactions

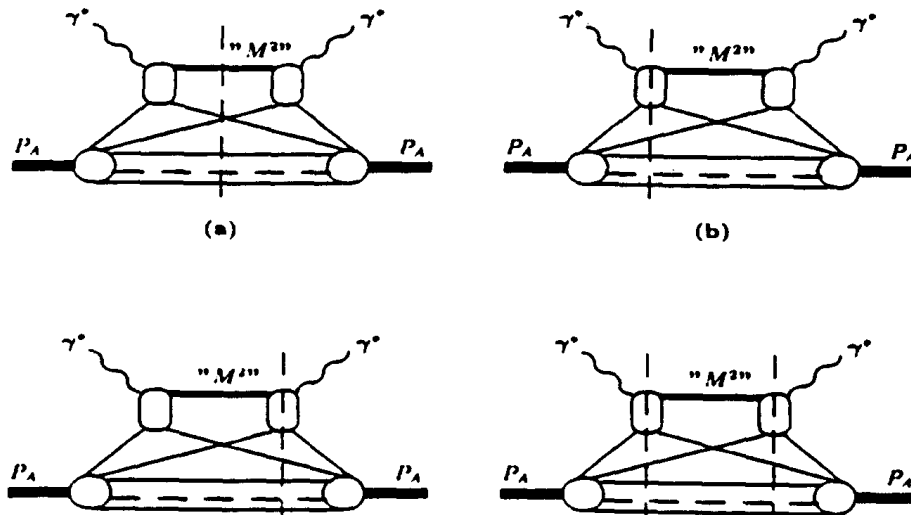
Program which can be performed at COMPASS and also J-PARC (complementary - different beams, higher energies, etc).

EIC can follow up this program at higher energies and address issues of both the hadron and photon structure.

Ultrapерipheral collisions (UPC) at the LHC is clearly a forerunner of EIC. - data now not in 20 years.

For example, in extracting small x nuclear gluon gpdfs

For many UPC flagship reactions, EIC would have to make emphasis on few % precision studies of the A - dependence.



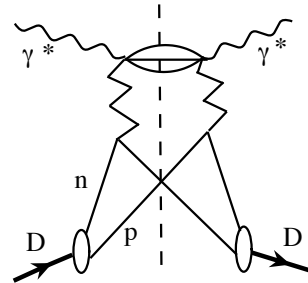
Cuts of double scattering diagram corresponding to diffraction (a),
 Screening of the scattering of a single nucleon (b/c), double multiplicity (d)

Unitarity relates these cuts - Abramovski, Gribov, Kancheli

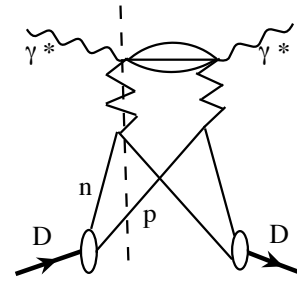
$$\sigma_{tot} = \sigma_{impulse} + \sigma_{shad}$$

$$\sigma_{shad} = \sigma_{dif} + \sigma_{single} + \sigma_{double} = -\sigma_2$$

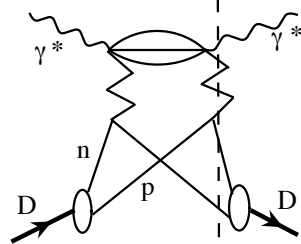
AGK relation between cross sections of different channels:



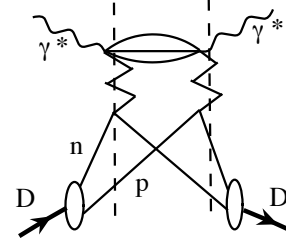
$$\sigma_{dif} = \sigma_2$$



$$\sigma_{single} \text{ "p"} = -2\sigma_2$$



$$\sigma_{single} \text{ "n"} = -2\sigma_2$$



$$\sigma_{double} = 2\sigma_2$$

Using AGK we re-derived original Gribov result for nuclear shadowing extending it to include the real part effects. This approach does not require separation of diffraction into leading twist and higher twist parts.

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AGK allows to rewrite sign alternating series as a series all positive terms

$$\sigma_{N_{coll}}^{(1)} = \sigma_1 - 4\sigma_2; \sigma_{N_{coll}}^{(2)} = 2\sigma_2; \sigma_{N_{coll}}^{(diff)} = \sigma_2;$$

Observable": N_{coll} (or number of neutrons in ZDC) vs x_A . Const for $x_A > 0.02$, graduate increase with decrease of x_A , decrease of the effect with increase of p_{tT} of charm, p_T of leading pion in current fragmentation region.

Looking for tail corresponding to 3 - 5 wounded nucleons.

These measurements would complement measurements of gluon gpdfs from coherent J/psi production

Consistency check of leading twist approximation (LTA) of shadowing

For example, N=1, N=2 values test interaction in the rim region of a nucleus.

$$N=1 \propto \int d^2b \cdot y(b) e^{-y(b)} \quad y(b) = \sigma_{eff}(T(b))$$

σ_{eff} includes fluctuations of diffractive cross section

Note that in difference from models based on fitting the data LTA first calculates nuclear GPDs and gets pdfs by integrating over b calculates diagonal GPDS

$$x f_{i/A}(x, b, Q_0^2) =$$

$$A x f_{i/N}(x, Q_0^2) T_A(\vec{b}) - 8\pi A(A-1) \Re e \frac{(1-i\eta)^2}{1+\eta^2} \int_x^{0.1} dx_{\mathbb{P}} \beta f_i^{D(4)}(\beta, Q_0^2, x_{\mathbb{P}}, t=0)$$

$$\times \int_{-\infty}^{\infty} dz_1 \int_{z_1}^{\infty} dz_2 \rho_A(\vec{b}, z_1) \rho_A(\vec{b}, z_2) e^{i(z_1-z_2)x_{\mathbb{P}}m_N} e^{-\frac{A}{2}(1-i\eta)\sigma_{soft}^i(x, Q_0^2) \int_{z_1}^{z_2} dz' \rho_A(\vec{b}, z')},$$

Applications of LTA to the processes with lightest nuclei

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Coherent J/ψ Electroproduction on ${}^4\text{He}$ and ${}^3\text{He}$ at the Electron-Ion Collider: Probing Nuclear Shadowing One Nucleon at a Time

Vadim Guzey , Matteo Rinaldi , Sergio Scopetta, Mark Strikman ,and Michele Viviani

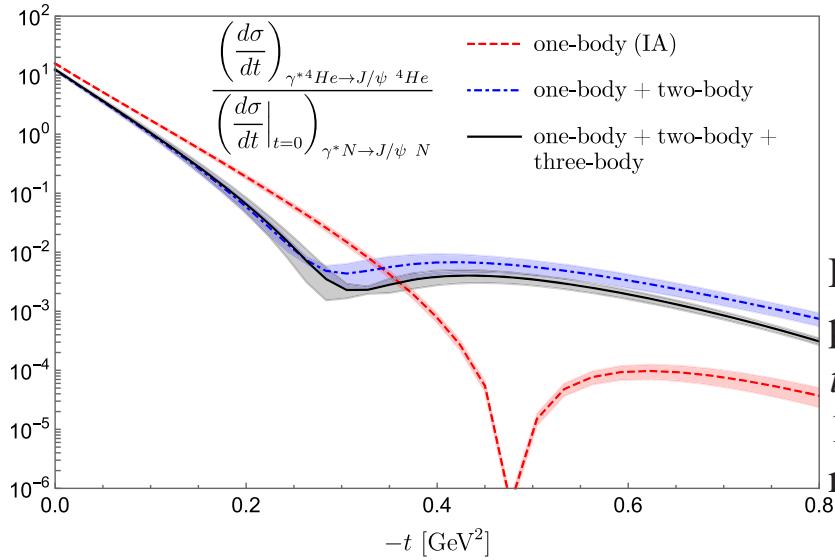


FIG. 3. Ratio of the differential cross section for J/ψ coherent production on ${}^4\text{He}$ to the same quantity for the nucleon target at $t = 0$ as a function of $-t$ at $x = 10^{-3}$. Relative errors of 10% and 15% have been considered on the quantities B_0 and σ_2 , respectively (see text and the Supplemental Material [31]).

Supplemental slides

Strategy of the first numerical analysis:

- account for contributions of GPDs corresponding to $q\bar{q}$ pairs with $S=1$ and 0
- Approximate the ERBL configurations by the pion and ρ -meson poles
- Use experimental information about
 $\pi^- p \rightarrow \pi^- p, \pi^- p \rightarrow \rho^- p$
 $\pi^+ p \rightarrow \pi^+ p, \pi^+ p \rightarrow \rho^+ p$

$$d\sigma = \frac{S}{4\sqrt{(p_a \cdot p_b)^2 - m_N^4}} \overline{\sum} \lambda_a, \lambda_b \sum \lambda_d, \lambda_e |\mathcal{M}_{NNN\pi B}|^2$$

$$\times \frac{1}{2E_c} \frac{d^3 p_c}{(2\pi)^3} \frac{1}{2E_d} \frac{d^3 p_d}{(2\pi)^3} \frac{1}{2E_e} \frac{d^3 p_e}{(2\pi)^3} (2\pi)^4 \delta^4(p_a + p_b - p_c - p_d - p_e)$$

$$\frac{d\sigma}{d\alpha d^2 p_{BT} d\theta_{cm}} = f(\alpha, p_{BT}) \phi(s', \theta_{cm})$$

$$\alpha \equiv \alpha_{spec} = (1 - \xi)/(1 + \xi)$$

$$s' = (1 - \alpha)s$$

$$\phi(s', \theta_{cm}) \approx (s')^n \gamma(\theta_{cm})$$

$$\begin{aligned}
\mathcal{M}_N^V &= \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle N, p_e | \bar{\psi}(-\lambda n/2) \not{n} \psi(\lambda n/2) | N, p_a \rangle \\
&= I_N \bar{\psi}_N(p_e) \left[H(x, \xi, t) \not{n} + E(x, \xi, t) \frac{i\sigma^{\alpha\beta} n_\alpha \Delta_\beta}{2m_N} \right] \psi_N(p_a)
\end{aligned}$$

$$I_N = \langle 1/2 | \tilde{T} | 1/2 \rangle \langle \frac{1}{2} M_N : 1m | \frac{1}{2} M'_N \rangle / \sqrt{2}$$

$$\begin{aligned}
\mathcal{M}_N^A &= \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle N, p_e | \bar{\psi}(-\lambda n/2) \not{n} \gamma_5 \psi(\lambda n/2) | N, p_a \rangle \\
&= I_N \bar{\psi}_N(p_e) \left[\tilde{H}(x, \xi, t) \not{n} \gamma_5 + \tilde{E}(x, \xi, t) \frac{n \cdot \Delta \gamma_5}{2m_N} \right] \psi_N(p_a)
\end{aligned}$$

