Meson production at NLO and higher-twist revisited

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Outline

1. DVMP at NLO: NLO global DIS+DVCS+DVρ^0P fits

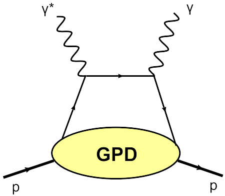
2. DVMP at twist-3: DVπ^0P

[Čuić, Duplančić, Kumerički, P-K. '23]

[Duplančić, Kroll, P-K., Szymanowski '24]

GPDs from deeply virtual exclusive processes

DVCS



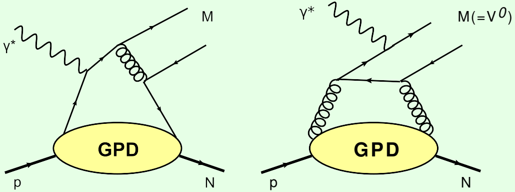
$$\gamma_T^* N \to \gamma N$$

H^q, E^q, $\widetilde{H}^q$, $\widetilde{E}^q$

NLO:

H^G, E^G, $\widetilde{H}^G$, $\widetilde{E}^G$, F_T^G

DVMP



$$\gamma_L^* N \to M N'$$

$M = V_L$: H^{q_i}, E^{q_i}; H^G, E^G

$M = PS$: $\widetilde{H}^{q_i}$, $\widetilde{E}^{q_i}$

$$\gamma_T^* N \to M N'$$

$M_{\text{twist-3}} \Rightarrow F_T^q$

Meson Production status

- DV (V_L) P:
  - data show dominance of γ_L^* contributions

    $\Rightarrow$ twist-2 predictions can describe the data

- DVπP:
  - data show suppression of γ_L^* contributions

    $\Rightarrow$ twist-3 γ_T^* contributions with transversity GPDs in 2-body ($\pi = q\bar{q}$) approximation [Goloskokov, Kroll '10, Goldstein, Hernandez, Liuti '13]

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    $\Rightarrow$ global DIS+DVCS+DVV_LP fits at NLO

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  - similar behaviour for wide-angle pion electroproduction ($t \gg$) but necessary 3-body tw3 contributions ($\pi = q\bar{q}g$) determined; info on tw3 pion DA from photoproduction fits [Kroll, P-K. 18', '21]

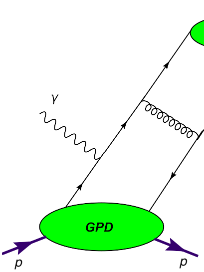
    $\Rightarrow$ full (2- and 3-body) twist-3 contributions confronted with data

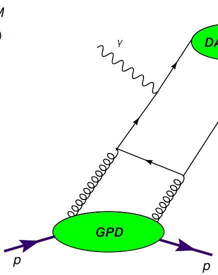
    [Duplančić, Kroll, P-K., Szymanowski '24]

DVMP at twist-2 NLO

DVMP to NLO

$$\gamma^* q \to (q\bar{q})q,\ \gamma^* g \to (q\bar{q})g$$



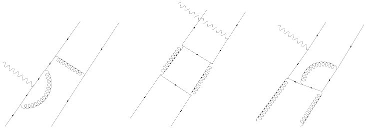


NLO DV PS$^+$ prod.: [Belitsky and Müller '01]

NLO DV V$_L$ prod.: [Ivanov et al '04,]

NLO DV V$_L$ (corr.), PS, (S, PV$_L$) prod.:

[Duplančić, Müller, P-K. '17]



DVMP to NLO

- NLO corrections important $\Leftarrow$ reduction of dependence on the scales and schemes
- only few DVMP phenomenological analysis to NLO:
  [Belitsky and Müller '01], [Diehl, Kugler '07]
  and [Müller, Lautenschläger, P-K., Schäfer '14, Duplančić, Müller, P-K., '17],
  [Lautenschlager, Müller, Schäfer '13, unpublished]
- large NLO corrections and model dependence
- GPD evolution important
- NLO global DIS+DVCS+DVMP fits needed

From momentum fraction to CPaW formalism

DVCS: Compton form factors

$$\mathcal{F}^a(\xi, t, Q^2) = \int \mathrm{d}x \, T^a(x, \xi, Q, \mu_F; \mu_R) \, F^a(x, \xi, t, \mu_F)$$ $a = q, G$ or NS,S

DVMP: Transition form factors

$$\mathcal{F}^a_M(\xi, t, Q^2) = \int \mathrm{d}x \int \mathrm{d}u \, T^{M,a}(x, \xi, u, \ldots) \, F^a(x, \xi, t, \mu_F) \, \phi_M(u, \mu_\varphi)$$

F^a ... GPD, ϕ_M ... DA, T^a ... hard-scattering amplitude

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- conformal partial wave expansion: $C_n^{3/2}(x)$ (quarks), $C_n^{5/2}(x)$ (gluons)

$$F_j^q(\xi, \ldots) = \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} \mathrm{d}x\, \xi^{j-1} C_j^{3/2}(x/\xi) F^q(x, \xi, \ldots),\ \ldots,\ T_j^a,\ T_{j,k}^{M,a}$$

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- series summed using Mellin-Barnes integral over complex j

$$\int_{-1}^{1} \frac{dx}{2\xi} \to 2 \sum_{j=0}^{\infty} \xi^{-j-1} \to \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dj\, \xi^{-j-1} \left[i \pm \left\{ \begin{matrix} \tan \\ \cot \end{matrix} \right\} \left(\frac{\pi j}{2} \right) \right] \equiv \overset{j}{\otimes}$$

[Müller '06, Müller, Schäfer '06]

CPaW formalism for DVCS and DVMP to NLO

[Kumerički, Müller, P-K., Schäfer '07:

"Towards a fitting procedure for DVCS at NLO and beyond"]

$$\mathcal{F}^a(\xi, t, Q^2) = T^a_j(Q, \mu) \overset{j}{\otimes} \mathbb{E}^a_{jl}(\mu, \mu_0; \xi) \overset{l}{\otimes} F^a_l(\xi, t, \mu_0)$$

- T^a_j and GPD evolution ($\mathbb{E}$) to NLO
- application to NNLO ready
- modeling GPD moments: t-chanel SO(3) partial waves

[Müller, Lautenschläger, P-K., Schäfer '14:

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- $T^{M,a}_{j,k}$ to NLO for $M = V_L, P, (S, PV_L)$

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$\rightarrow$ Gepard software [Kumerički '22 on github]

$\rightarrow$ compendium [Čuić, Duplančić, Kumerički, P-K. '23]

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- $T_{j,k}^{M,a}$ to NLO for $M = V_L, P, (S, PV_L)$

$\rightarrow$ Gepard software [Kumerički '22 on github]

$\rightarrow$ compendium [Čuić, Duplančić, Kumerički, P-K. '23]

Advantages: easy evolution, interesting GPD modeling, moments accessible on lattice, stable numerics and efficient fitting

$\rightarrow$ see also X. Ji and G. Santiago talks

Global NLO fits (DIS+DVCS+DVV$_L$P)

small-x global fits to HERA collider data (ρ_0)

- only NLO predecessor: [Lautenschlager, Müller, Schäfer '13 unpublished]
- hard scattering amplitude corrected [Duplančić, Müller, P-K. '17]
- new NLO fit [Čuić, Duplančić, Kumerički, P-K. '23]:
  improved treatment of experimental data

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GPD model: [Kumerički, Müller, P-K., Schäfer '07, Kumerički, Müller '10]

- $H_j^a(\xi, t) = q_j^a \dfrac{1 + j - \alpha_0^a}{1 + j - \alpha_0^a - \alpha'^a t} \left(1 - \dfrac{t}{m_a^2}\right)^{-2} \left(1 + s_2^a \, \xi^2 + s_4^a \, \xi^4\right)$

$$q_j^a = N_a \frac{B(1 - \alpha_0^a + j, \beta^a + 1)}{B(2 - \alpha_0^a, \beta^a + 1)}$$

- small-x kinematics $\Rightarrow a \in \{\mathsf{sea}, \mathsf{G}\}$, only dominant H GPD

Fit parameters:

- DIS: $\{N_{\mathsf{sea}}, \, \alpha_0^{\mathsf{sea}}, \, \alpha_0^{\mathsf{G}}\}$
- DVCS+DVMP: $\{\alpha'^{\mathsf{sea}}, \, \alpha'^{\mathsf{G}}, \, m^2_{\mathsf{sea}}, \, m^2_{\mathsf{G}}, \, s_2^{\mathsf{sea}}, \, s_2^{\mathsf{G}}, \, s_4^{\mathsf{sea}}, \, s_4^{\mathsf{G}}\}$

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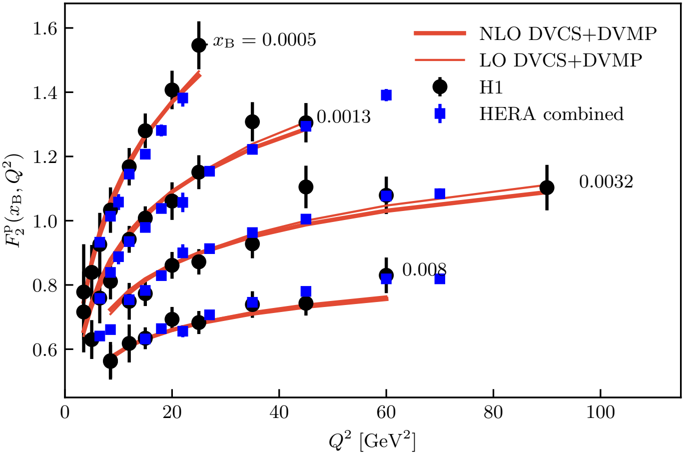
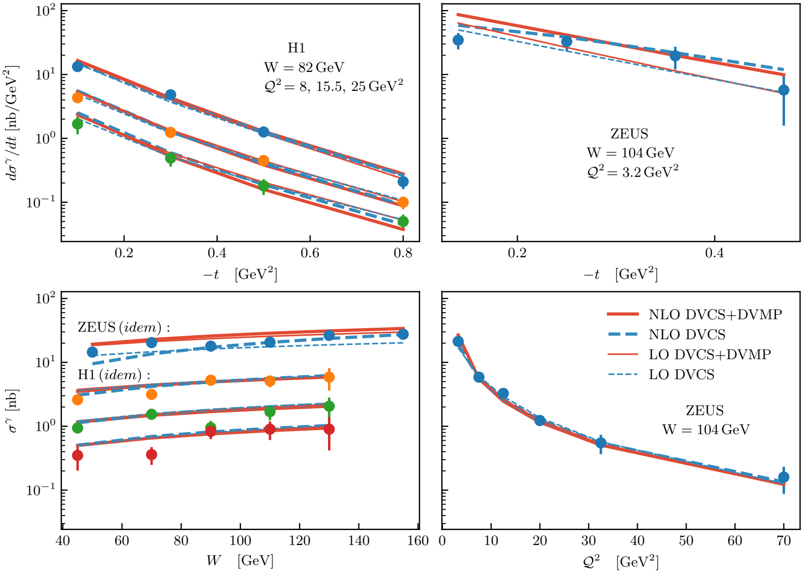
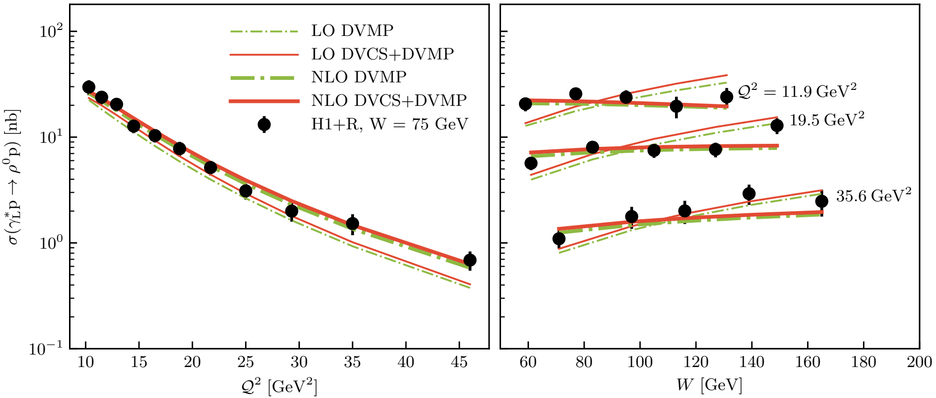


- may seem trivial, but not all popular models describe DIS

Global NLO fits (DIS+DVCS+DVV$_L$P)

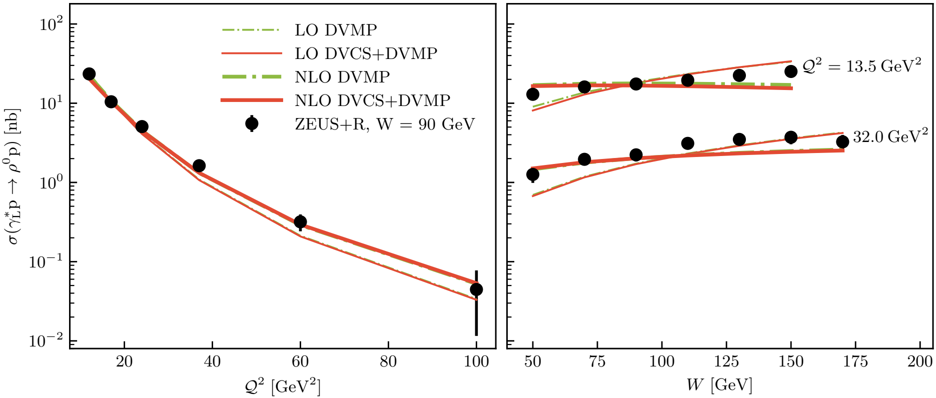


Global NLO fits (DIS+DVCS+DVV_LP)



$$\boxed{R \equiv \frac{\sigma_L^{\rho^0}}{\sigma_T^{\rho^0}}} \to R(W, \mathcal{Q}^2) = \frac{\mathcal{Q}^2}{m_{\rho^0}^2}\left(1 + a\frac{\mathcal{Q}^2}{m_{\rho^0}^2}\right)^{-p}\left(1 + b\frac{\mathcal{Q}^2}{W}\right) \text{ fit}$$

Global NLO fits (DIS+DVCS+DVV$_L$P)



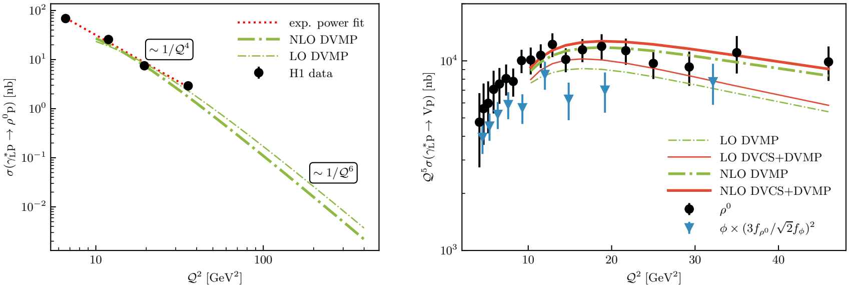
Global NLO fits (DIS+DVCS+DVV$_L$P)

Dataset	Refs.	n_{pts}	LO- DVCS	LO- DVMP	LO- DVCS-DVMP	NLO- DVCS	NLO- DVMP	NLO- DVCS-DVMP
DIS	[90]	85	0.6	0.6	0.6	0.8	0.8	0.8
DVCS	[92–95]	27	0.4	$\gg 1$	0.6	0.6	$\gg 1$	0.8
DVMP	[88, 89]	45	$\gg 1$	3.1	3.3	$\gg 1$	1.5	1.8
Total		157	$\gg 1$	$\gg 1$	1.4	3.7	$\gg 1$	1.1

Table 3. Values of χ^2/n_{pts} for each LO or NLO model (columns) for the total DIS + DVCS + DVMP dataset and for subsets corresponding to different processes (rows). (The values denoted by $\gg 1$ are greater than 10.).

- NLO DVCS-DVMP fit describes the data well

Global NLO fits (DIS+DVCS+DVV$_L$P)



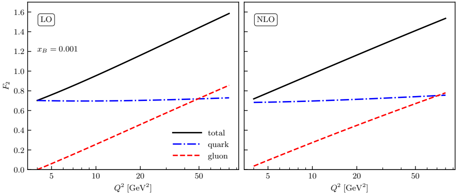
σ_L asymptoticaly: $\dfrac{1}{Q^6}$

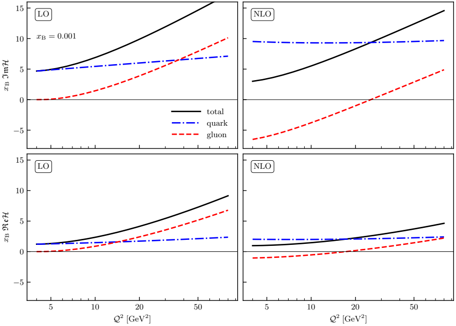
experimental data for fixed x_B: $\approx \dfrac{1}{Q^4}$, for fixed W: $\approx \dfrac{1}{Q^5}$

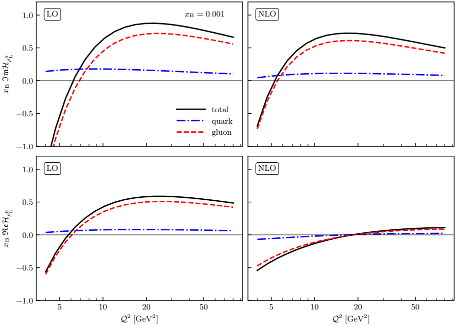
- successful description of Q^2 dependence

Global NLO fits (DIS+DVCS+DVV_LP)

quark-gluon structure?

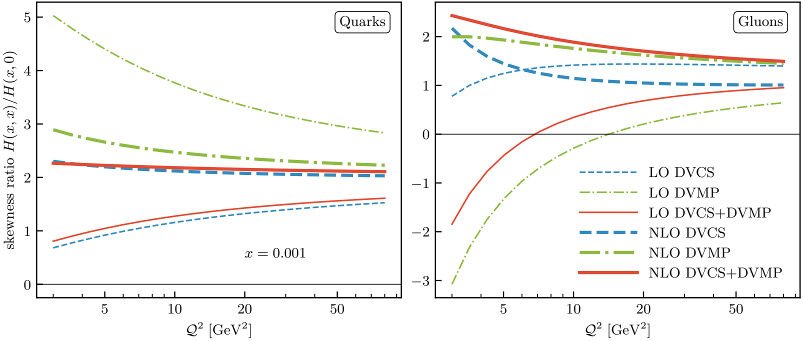






Global NLO fits (DIS+DVCS+DVV$_L$P)

Skewness ratio $r = \dfrac{H(x,x)}{H(x,0)}$



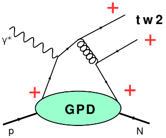
- conformal (Shuvaev) values (PDFs completely specified by GPDs): $r^q \approx 1.65$, $r^G \approx 1$,
- r measures goodness of GPD extraction $\Rightarrow$ NLO fit successful

DVMP at higher-twist

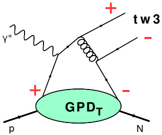
DVMP to twist-3

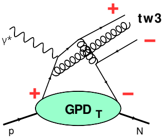
μ photon helicity, $\lambda \ldots$ quark helicities

$\mathcal{H}^{\pi}_{0\lambda,\mu\lambda}$ ... non-flip subprocess amplitudes (twist-2)



$\mathcal{H}^{\pi}_{0-\lambda,\mu\lambda}$ ... flip subprocess amplitudes (twist-3)





Note: just meson DA tw-3 contributions ($\mu_\pi = 2$ GeV)

$\rightarrow$ see J. Zhou talk

distribution amplitudes (DAs):

twist-2 ($q\bar{q}$) : ϕ_π

2-body ($q\bar{q}$) twist-3 $\phi_{\pi p}$, $\phi_{\pi\sigma}$ 3-body ($q\bar{q}g$) twist-3 $\phi_{3\pi}$

$\rightarrow$ connected by equations of motion (EOMs)

Subprocess amplitudes: twist-3

[Duplančić, Kroll, P-K., Szymanowski '24]

$$\begin{aligned}
\mathcal{H}^{\pi,tw3} &= \mathcal{H}^{P,tw3,q\bar{q}} + \mathcal{H}^{P,tw3,q\bar{q}g} \\
&= \big(\mathcal{H}^{P,\phi_{Pp}} + \underbrace{\mathcal{H}^{P,\phi_{P2}^{EOM}}\big) + \big(\mathcal{H}^{P,q\bar{q}g,C_F}}\, + \mathcal{H}^{P,q\bar{q}g,C_G}\big) \\
&= \mathcal{H}^{\pi,\phi_{Pp}} \quad + \qquad \mathcal{H}^{\pi,\phi_{3P},C_F} \qquad + \mathcal{H}^{\pi,\phi_{3P},C_G}
\end{aligned}$$

- 2- and 3-body contributions necessary for gauge invariance

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- 2- and 3-body contributions necessary for gauge invariance
- end-point singularities in $\mathcal{H}^{\pi,\phi_{\pi p}}$: $\int_0^1 \frac{d\tau}{\bar{\tau}} \phi_{\pi p}(\tau) \, \frac{1}{(x-\xi+i\epsilon)^2} \overset{x}{\otimes} H_T(\bar{E}_T)$

$$\phi_{Pp}(\tau) = 1 + a_{Pp}\, C_2^{1/2}(2\tau - 1) + \ldots$$

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  $\phi_{Pp}(\tau) = 1 + a_{Pp}\, C_2^{1/2}(2\tau - 1) + \ldots$

$\Rightarrow$ modified perturbative approach (MPA)

(with $k_\perp$ quark transverse momenta) as in [Goloskov, Kroll, '10])

$\Rightarrow$ pure collinear picture with effective m_g^2

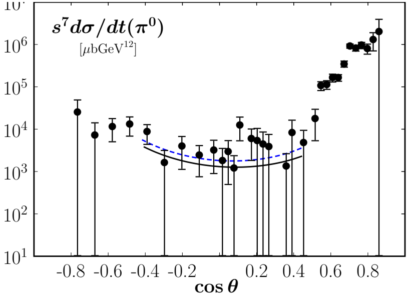
$$\boxed{\int_0^1 d\tau \phi_{\pi p}(\tau) \frac{1}{((x-\xi)\bar{\tau} - m_g^2(2\xi)/Q^2 + i\epsilon)} \frac{1}{(x-\xi+i\epsilon)} \overset{x}{\otimes} H_T(\bar{E}_T)}$$

Results from photoproduction (π)

- complete tw-3 prediction for π_0 photoproduction fitted to CLAS data

  $\Rightarrow \phi_{3\pi}$ coefficients $\omega_{1,0}$, $\omega_{2,0}$, $\omega_{1,1}$ (set 2)

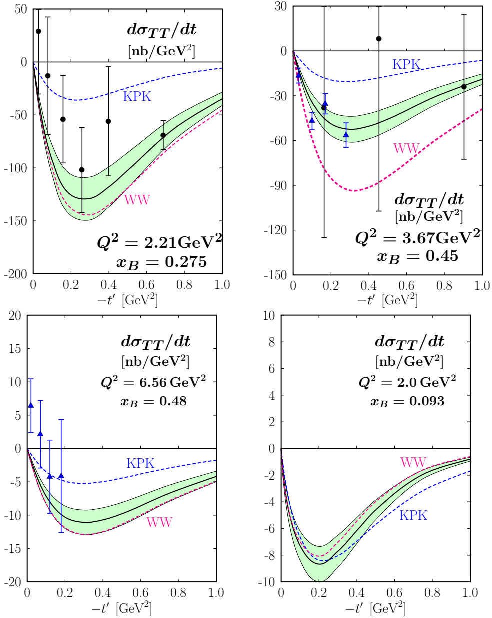
[Kroll, P-K '21]



solid curve: tw2+(tw3 $\phi_{3\pi}$ set1)
dashed curve: tw2+(tw3 $\phi_{3\pi}$ set2)

exp data:
full circles [SLAC '76]

MPA $d\sigma_{TT}$



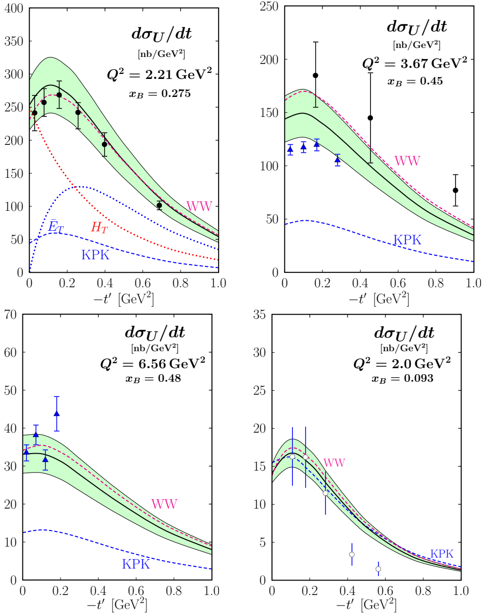
solid curves:
tw3 set 1
dashed curves:
tw3 set 2 ("KPK")

exp data:
full circles [CLAS '14]
triangles [Hall A '20]

$$\left|\frac{d\sigma_{TT}}{dt}\right| \leq \frac{d\sigma_T}{dt}$$

$$\frac{d\sigma_T}{dt} : H_T, \bar{E}_T \qquad \frac{d\sigma_{TT}}{dt} : \bar{E}_T$$

MPA $d\sigma_U$

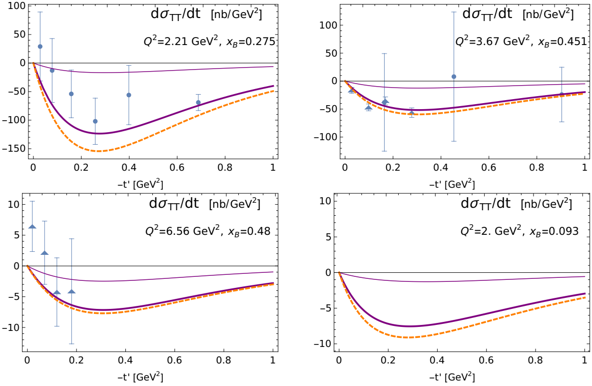


solid curves:
  tw3 set 1
dashed curves:
  tw3 set 2 ("KPK")

exp data:
full circles [CLAS '14]
triangles [Hall A '20]
open circles [COMPASS '19]

$$\frac{d\sigma_U}{dt} = \frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt}$$

Collinear approach with m_g^2: $d\sigma_{TT}$

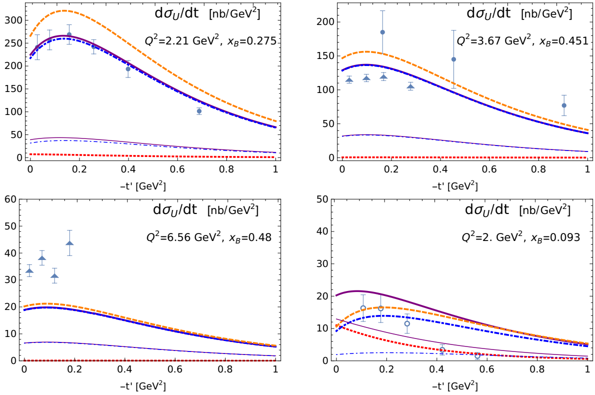


purple solid curves:
  tw3 set 1
thin solid curves:
  tw3 set 2 ("KPK")
orange dashed curves:
  tw3 WW

exp data:
full circles [CLAS '14]
triangles [Hall A '20]

$$m_g^2(Q^2) = \frac{m_0^2}{1 + (Q^2/M^2)^{1+p}}$$

Collinear approach with m_g^2: $d\sigma_U$



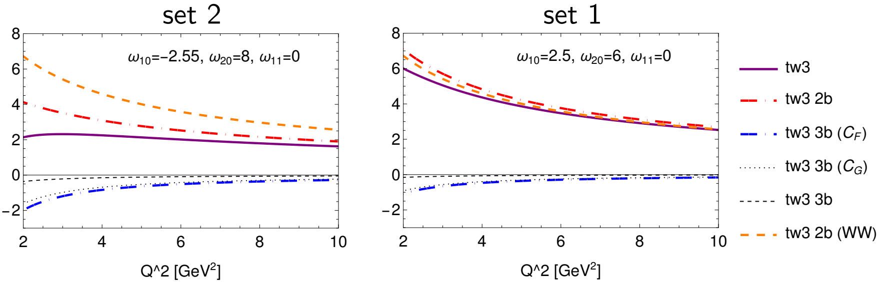
purple solid curves:
tw2 + tw3 set 1
thin solid curves:
tw2 + tw3 set 2
orange dashed curves:
tw2 + tw3 WW

red curves: tw2
blue curves: tw3

exp data:
full circles [CLAS '14]
triangles [Hall A '20]
open circles [COMPASS '19]

- twist-2 contribution significant for COMPASS kinematics (small-x)

Collinear approach with m_g^2



- 3-body contributions smaller but may influence the Q^2 behaviour

Concluding remarks

- DVρ_L^0P
  - Twist-2 NLO contributions can describe the data.
  - Global DIS+DVCS+DVMP fits show importance of NLO.
  - DVMP can only be described at NLO.
- DVπ^0P
  - The improved twist-3 analysis (2- and 3-body meson Fock states included) shows that twist-3 dominates except for COMPASS kinematics (small x_B).
- Meson production promising in accessing information about GPDs.
- Meson DA additional nontrivial nonperturbative input.
- Clear separation of σ_L and σ_T needed from experiment.

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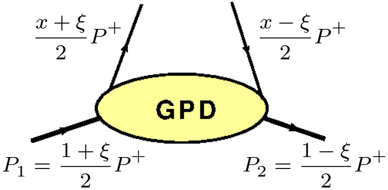
Thank you.

Generalized Parton Distributions



$$\frac{x+\xi}{2}P^+ \qquad \frac{x-\xi}{2}P^+$$

$$P_1 = \frac{1+\xi}{2}P^+ \qquad P_2 = \frac{1-\xi}{2}P^+$$

$$P = P_1 + P_2$$

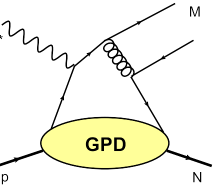
$$\Delta = P_2 - P_1$$

x	parton's "average" longitudinal momentum fraction
$\xi = -\frac{\Delta^+}{P^+}$	longitudinal momentum transfer (skewness)
$\Delta^2 = t$	momentum transfer

GPDs: $\boxed{F^a(x, \xi, t; \mu),\ a \in \{q, G\}}$ $\mu \ldots$ factorization scale

- vector (H^a, E^a) and axial-vector GPDs ($\widetilde{H}^a$, $\widetilde{E}^a$)
- transversity GPDs (H_T^a, E_T^a, $\widetilde{H}_T^a$, $\widetilde{E}_T^a$)

Meson Production: handbag factorization



$\gamma^{(*)} N \to M N'$

DEEPLY VIRTUAL

$Q^2 >>, -t <<$

DVMP

[Collins, Frankfurt, Strikman '97]

- factorization
  $\mathcal{H}^a \otimes GPD$
- GPDs at small $(-t)$

WIDE ANGLE

$-t, -u, s >>$

WAMP

[Huang, Kroll '00]

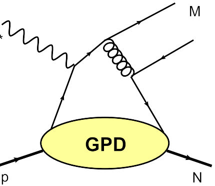
- arguments for factorization
  $\mathcal{H}^a(1/x \otimes GPD(\xi = 0))$
- GPDs at large $(-t)$

$\mathcal{H}^a$ ... parton subprocess helicity amplitudes

$\Rightarrow \mathcal{M}$ ... hadron helicity amplitudes

$\Rightarrow$ observables (cross sections, asymmetries)

Meson Production: handbag factorization



$\gamma^{(*)} N \to M N'$

DEEPLY VIRTUAL	WIDE ANGLE
$Q^2 >>, -t <<$	$-t, -u, s >>$
DVMP	WAMP
[Collins, Frankfurt, Strikman '97]	[Huang, Kroll '00]

DVMP:

- factorization $\mathcal{H}^a \otimes GPD$
- GPDs at small $(-t)$
- tw2: γ_L^*, tw3: γ_T^*

WAMP:

- arguments for factorization $\mathcal{H}^a(1/x \otimes GPD(\xi = 0))$
- GPDs at large $(-t)$

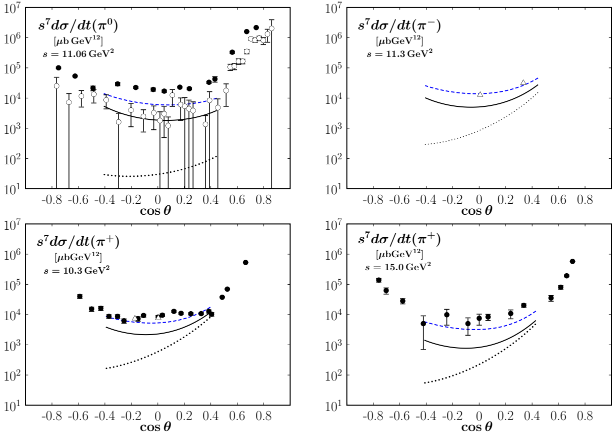
large scale $\mathcal{Q}^2$ ($\mathcal{Q}^2$, s or ...)

- twist expansion: $\langle \mathcal{H} \rangle^{tw2} + \dfrac{\langle \mathcal{H} \rangle^{tw3}}{\mathcal{Q}} + \ldots$
- α_S expansion for each twist: $\alpha_S(\mathcal{Q}) \langle \mathcal{H} \rangle^{LO} + \alpha_S^2(\mathcal{Q}) \langle \mathcal{H} \rangle^{NLO}$

Photoproduction (π)

- complete tw-3 prediction for π_0 photoproduction fitted to CLAS data and obtained predictions for $\pi^{\pm}$

[Kroll, P-K '21]



solid curves:
complete twist-3
dotted curves: twist-2

exp data:
full circles [SLAC '76]
open circles [CLAS '17]
triangles [JLab, Hall A '05]

- twist-2 prediction well below the data

Subprocess amplitudes: twist-3

General structure:

$$\begin{aligned}
\mathcal{H}^{P,tw3} &= \mathcal{H}^{P,tw3,q\bar{q}} + \mathcal{H}^{P,tw3,q\bar{q}g} \\
&= \left(\mathcal{H}^{P,\phi_{Pp}} + \underbrace{\mathcal{H}^{P,\phi_{P2}^{EOM}}\right) + \left(\mathcal{H}^{P,q\bar{q}g,C_F}}\, + \mathcal{H}^{P,q\bar{q}g,C_G}\right) \\
&= \mathcal{H}^{P,\phi_{Pp}} \quad + \qquad \mathcal{H}^{P,\phi_{3P},C_F} \qquad + \mathcal{H}^{P,\phi_{3P},C_G}
\end{aligned}$$

- 2- and 3-body contributions necessary for gauge invariance

- WAMP
  - photoproduction $(Q \to 0)$: $\mathcal{H}^{P,\phi_{Pp}} = 0$ [Kroll, P-K '18]
  - no end-point singularities for $\hat{t} \neq 0$!

Meson Production status

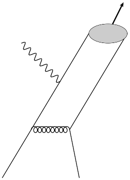
- DV (V_L) P:
  - tw-2 predictions ($\underline{\gamma_L^* N \to V_L N'}$) can describe the data
  - tw-3 calculations ($\underline{\gamma_T^* N \to V_{L,T} N'}$) [Anikin, Teryaev '02], [Golosk., Kroll '13]
- DV (PS) P:
  - tw-2 predictions ($\underline{\gamma_L^* N \to \pi N'}$) bellow the data [HERMES '09] [JLab '12,'16, '20] [COMPAS '19] $\Rightarrow$ importance of $\gamma_T^* N \to \pi N'$

  $\Rightarrow$ tw-3 calculations ($\underline{\gamma_T^* N \to \pi N'}$) with transversity (chiral-odd) GPDs (H_T^q...) [Goloskokov, Kroll '10] (2-body, i.e., WW approximation), [Ahmad, Goldstein Liuti '09, Goldstein, Hernandez, Liuti '13]
- WA (PS) P:
  - tw-2 results [Huang, Kroll '00] bellow the data [SLAC '76], [JLab '05, '18] for photoproduction ($Q^2 = 0$)
  - tw-3 2-body π photoproduction vanishes [Huang, Jakob, Kroll, P-K '03]

  $\Rightarrow$ tw-3 (2- and 3-body) prediction to π_0 photoproduction [Kroll, P-K '18] fitted to CLAS data [CLAS '18]; photoproduction of η, η' mesons [Kroll, P-K. '22] [preliminary GlueX '20]

  $\Rightarrow$ tw-3 prediction for $\pi^{\pm}, \pi^0$ photo- and electroproduction ($Q^2 < -t$) [Kroll, P-K. '21]; extension to DV (PS) P

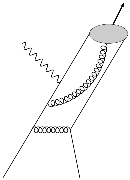
Subprocess amplitudes $\mathcal{H}$: projectors



$q\bar{q} \to \pi$ projector [Beneke, Feldmann '00]

$$(\tau q' + k_\perp) + (\bar{\tau} q' - k_\perp) = q'$$

$$\begin{aligned}\mathcal{P}_2^\pi \quad \sim \quad & f_\pi \Big\{ \gamma_5 \, \not{q}' \phi_\pi(\tau, \mu_F) \\ & + \mu_\pi(\mu_F) \Big[\gamma_5 \, \phi_{\pi p}(\tau, \mu_F) \\ & - \frac{i}{6} \gamma_5 \, \sigma_{\mu\nu} \, \frac{q'^\mu n^\nu}{q' \cdot n} \, \phi'_{\pi\sigma}(\tau, \mu_F) \\ & + \frac{i}{6} \gamma_5 \, \sigma_{\mu\nu} \, q'^\mu \phi_{\pi\sigma}(\tau, \mu_F) \frac{\partial}{\partial k_{\perp\nu}} \Big] \Big\}_{k_\perp \to 0}\end{aligned}$$



$q\bar{q}g \to \pi$ projector [Kroll, P-K '18]

$$\tau_a q' + \tau_b q' + \tau_g q' = q'$$

$$\mathcal{P}_3^\pi \sim f_{3\pi}(\mu_F) \, \frac{i}{g} \, \gamma_5 \, \sigma_{\mu\nu} q'^\mu g_\perp^{\nu\rho} \, \frac{\phi_{3\pi}(\tau_a, \tau_b, \tau_g, \mu_F)}{\tau_g}$$

$\mu_\pi = m_\pi^2/(m_u + m_d) \cong 2$ GeV, $f_{3\pi} \sim \mu_\pi$

NLO for DV V_L production

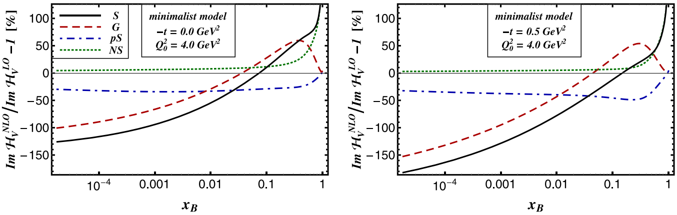


Fig. 6. Relative NLO corrections to the imaginary part of the flavor singlet TFF $\mathcal{F}_V^S$ (solid) broken down to the gluon (dashed), pure singlet quark (dash-dotted) and 'non-singlet' quark (dotted) at $t = 0$ GeV^2 (left panel) and $t = -0.5$ GeV^2 (right panel) at the initial scale $\mathcal{Q}_0^2 = 4$ GeV^2.

[Müller, Lautenschlager, P-K., Schäfer '14]

- large NLO corrections for small x_B, i.e., ξ