What can we learn from amplitudes for DVCS?

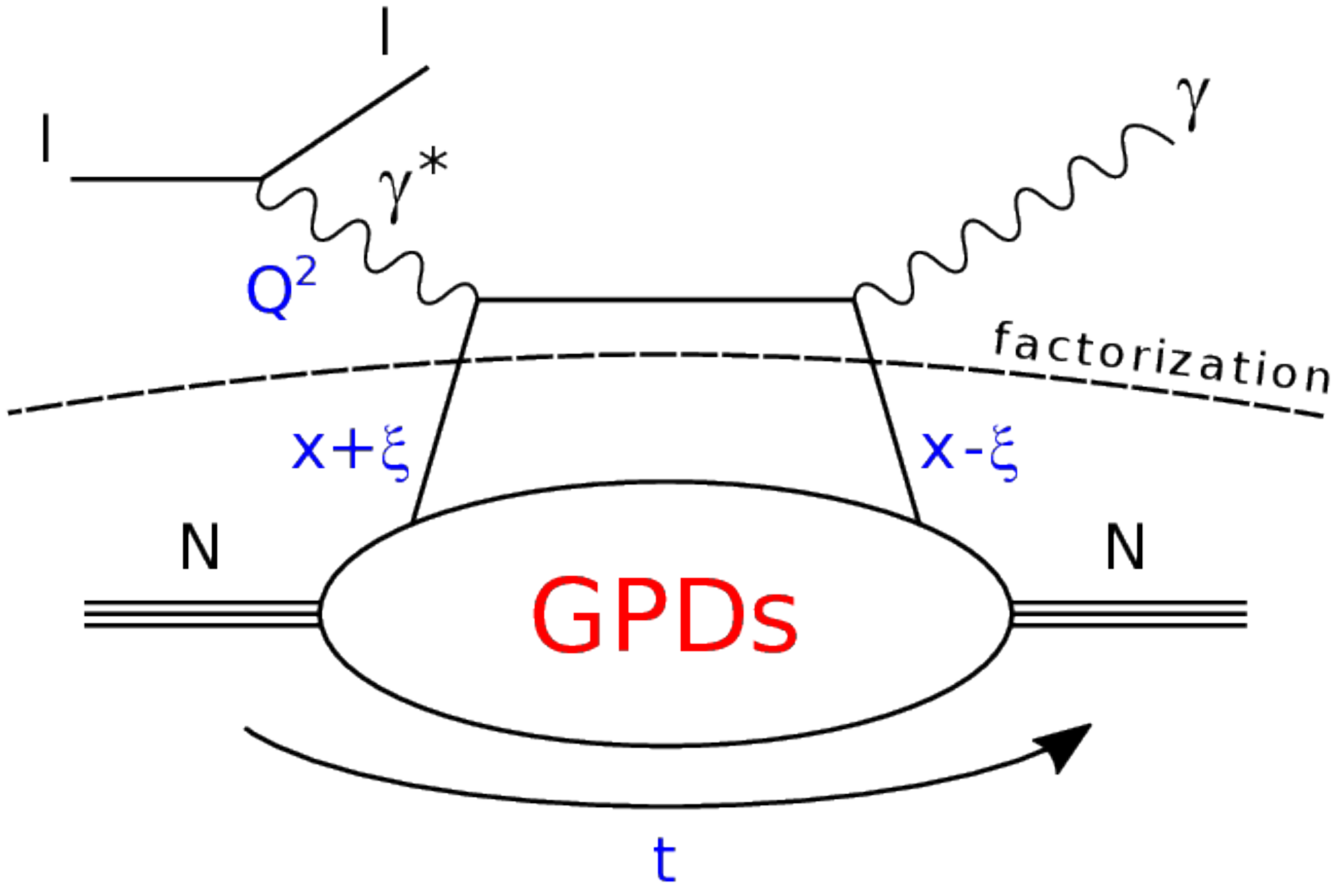
Paweł Sznajder
National Centre for Nuclear Research, Poland

NATIONAL CENTRE FOR NUCLEAR RESEARCH ŚWIERK

RBRC Workshop on GPDs for Nucleon Tomography in the EIC era,
BNL, USA, January 17th, 2024

Outline

Deeply Virtual Compton Scattering (DVCS)



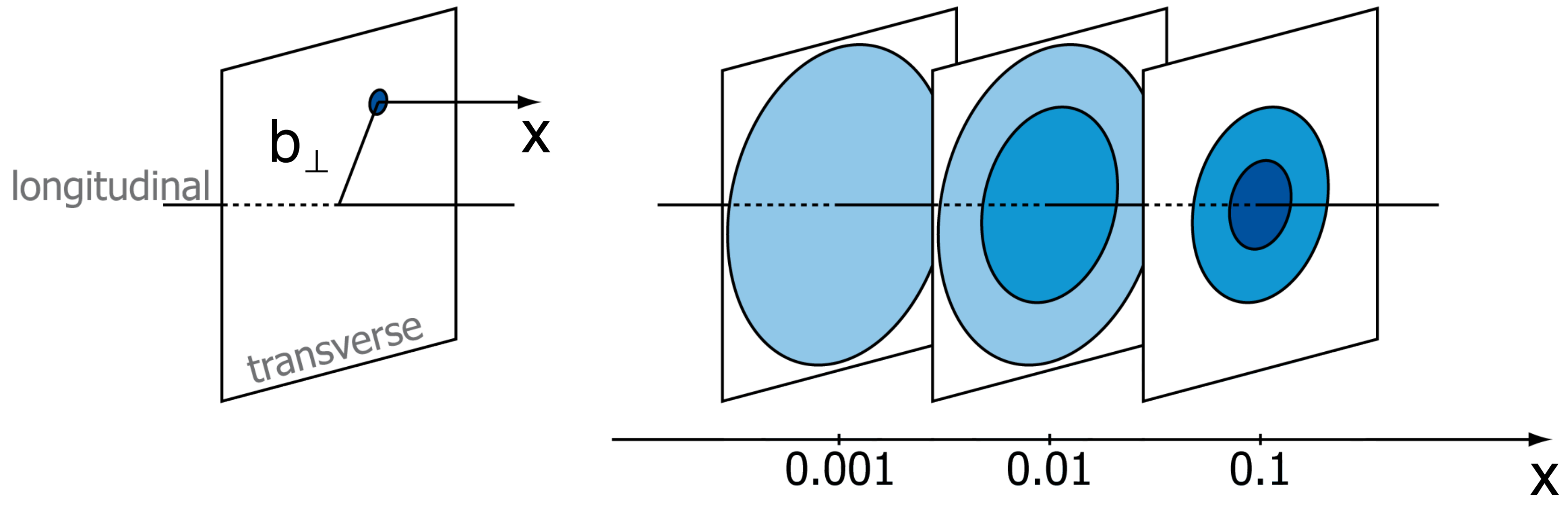
factorisation for $|t|/Q^2 \ll 1$

Chiral-even GPDs:
(helicity of parton conserved)

$H^{q,g}(x,\xi,t)$	$E^{q,g}(x,\xi,t)$	*for sum over parton helicities*
$\widetilde{H}^{q,g}(x,\xi,t)$	$\widetilde{E}^{q,g}(x,\xi,t)$	*for difference over parton helicities*
nucleon helicity conserved	*nucleon helicity changed*	

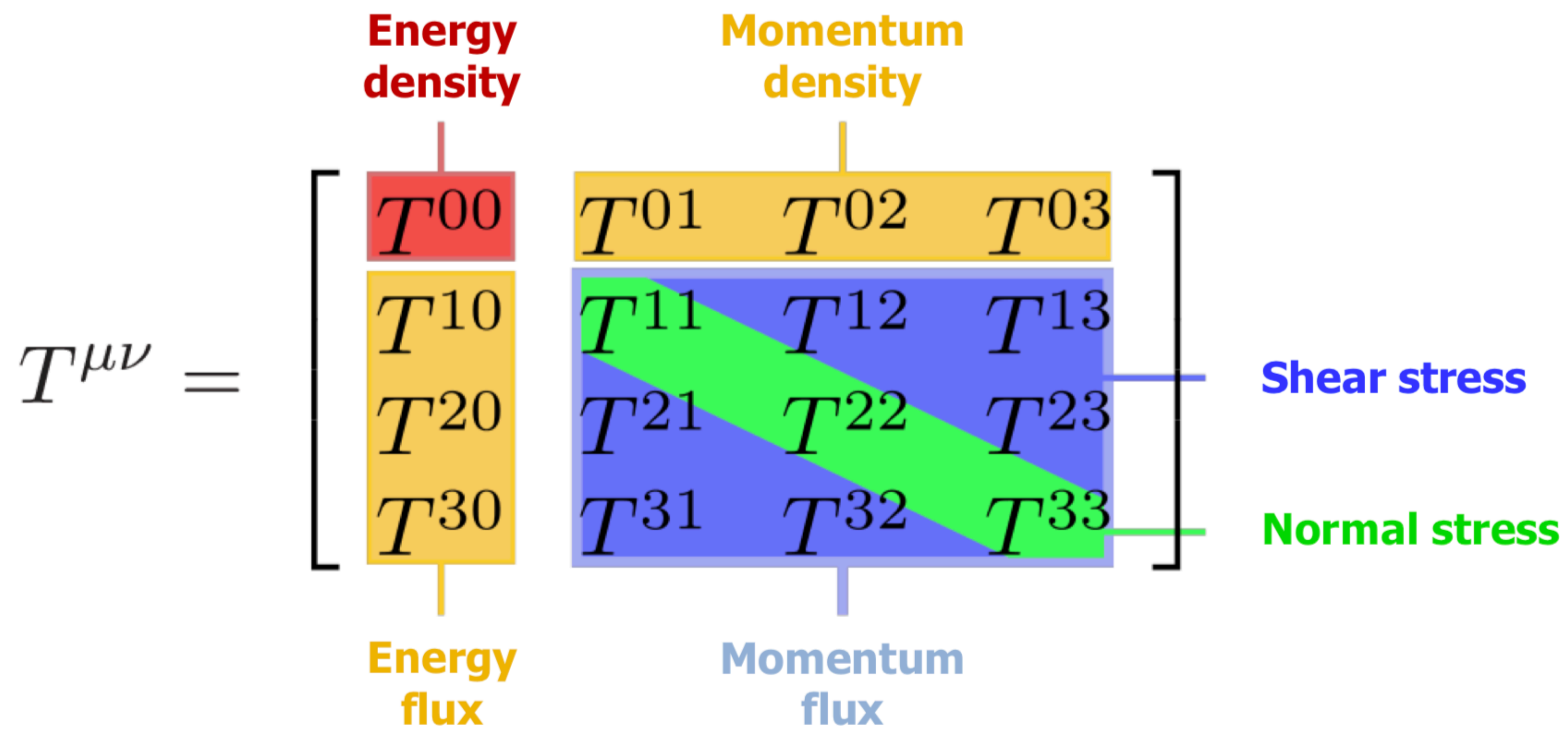
Introduction

Nucleon tomography:



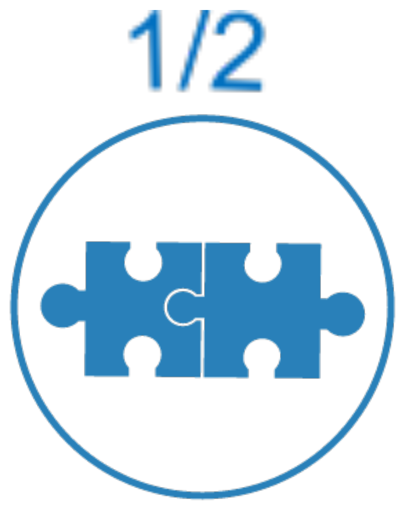
$$q(x, \mathbf{b}_\perp) = \int \frac{\mathrm{d}^2\boldsymbol{\Delta}}{4\pi^2} e^{-i\mathbf{b}_\perp \cdot \boldsymbol{\Delta}} H^q(x, 0, t = -\boldsymbol{\Delta}^2)$$

Energy momentum tensor in terms of form factors (OAM and mechanical forces):



$$\langle p', s' | \widehat{T}^{\mu\nu} | p, s \rangle = \bar{u}(p', s') \left[\frac{P^\mu P^\nu}{M} A(t) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} C(t) + M\eta^{\mu\nu}\bar{C}(t) + \frac{P^\mu i\sigma^{\nu\lambda}\Delta_\lambda}{4M} [A(t) + B(t) + D(t)] + \frac{P^\nu i\sigma^{\mu\lambda}\Delta_\lambda}{4M} [A(t) + B(t) - D(t)] \right] u(p, s)$$

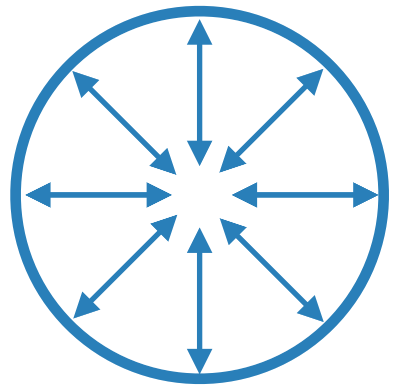
Total angular momentum:



$$A^q(0) + B^q(0) = \int_{-1}^{1} x \left[H^q(x, \xi, 0) + E^q(x, \xi, 0) \right] = 2J^q$$

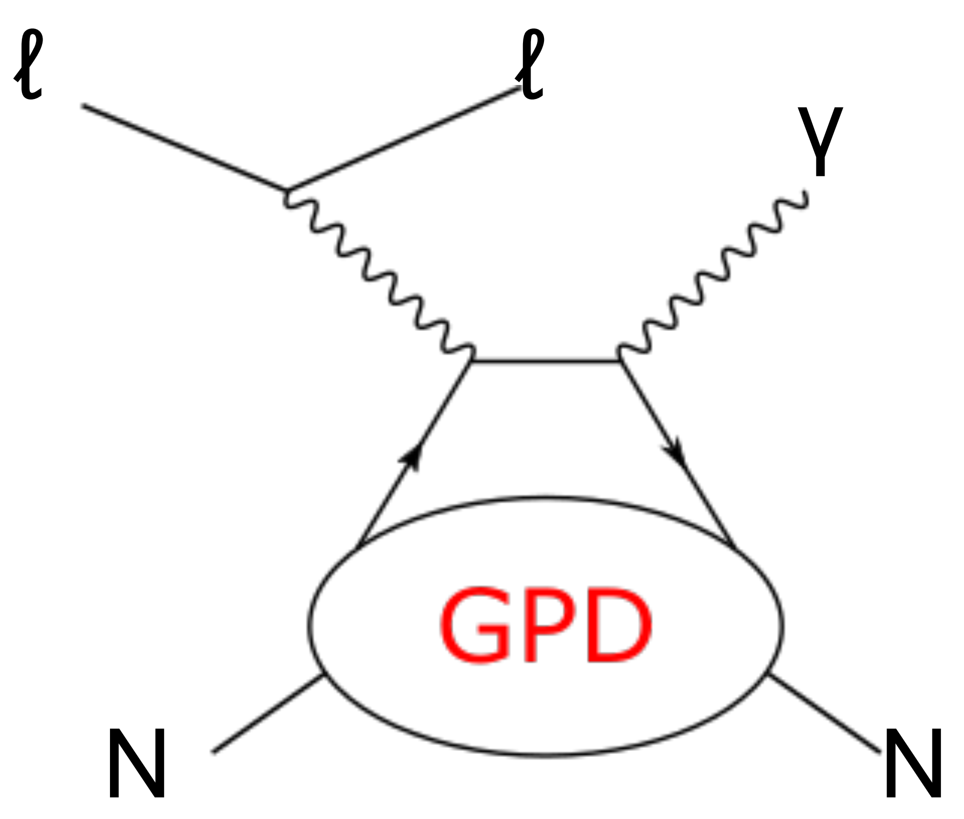
Ji's sum rule

"Mechanical" forces acting on quarks, e.g. pressure in nucleon center:

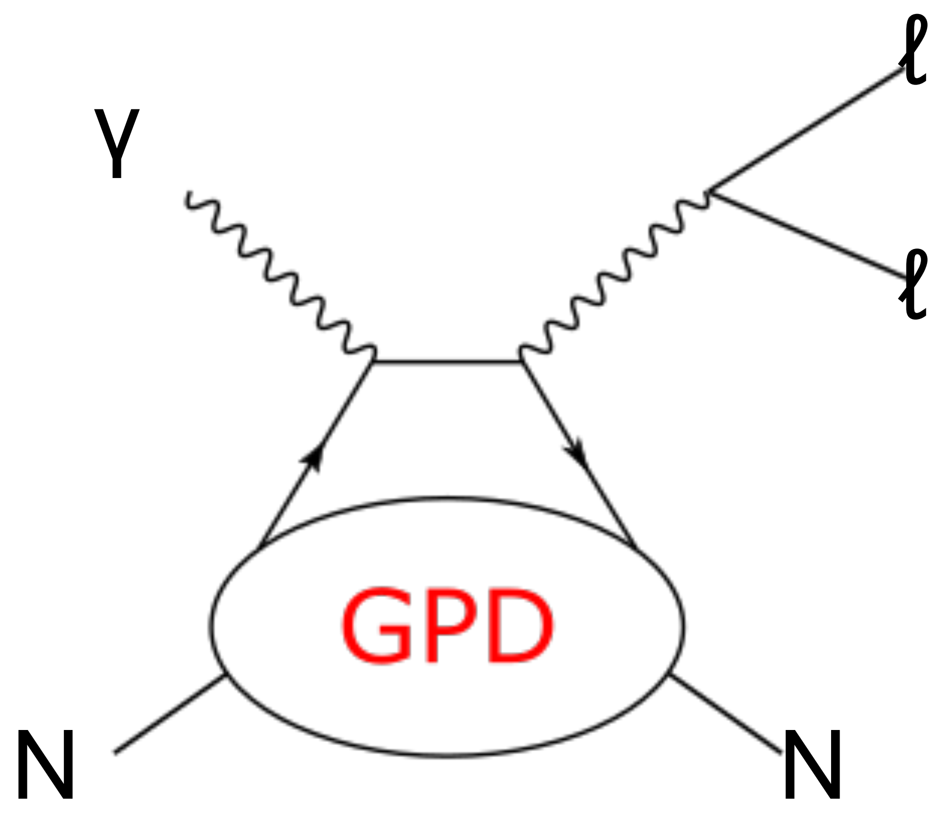


$$p(0) = \frac{1}{6\pi^2 M} \int_{-\infty}^{0} dt \sqrt{-t} t C(t)$$

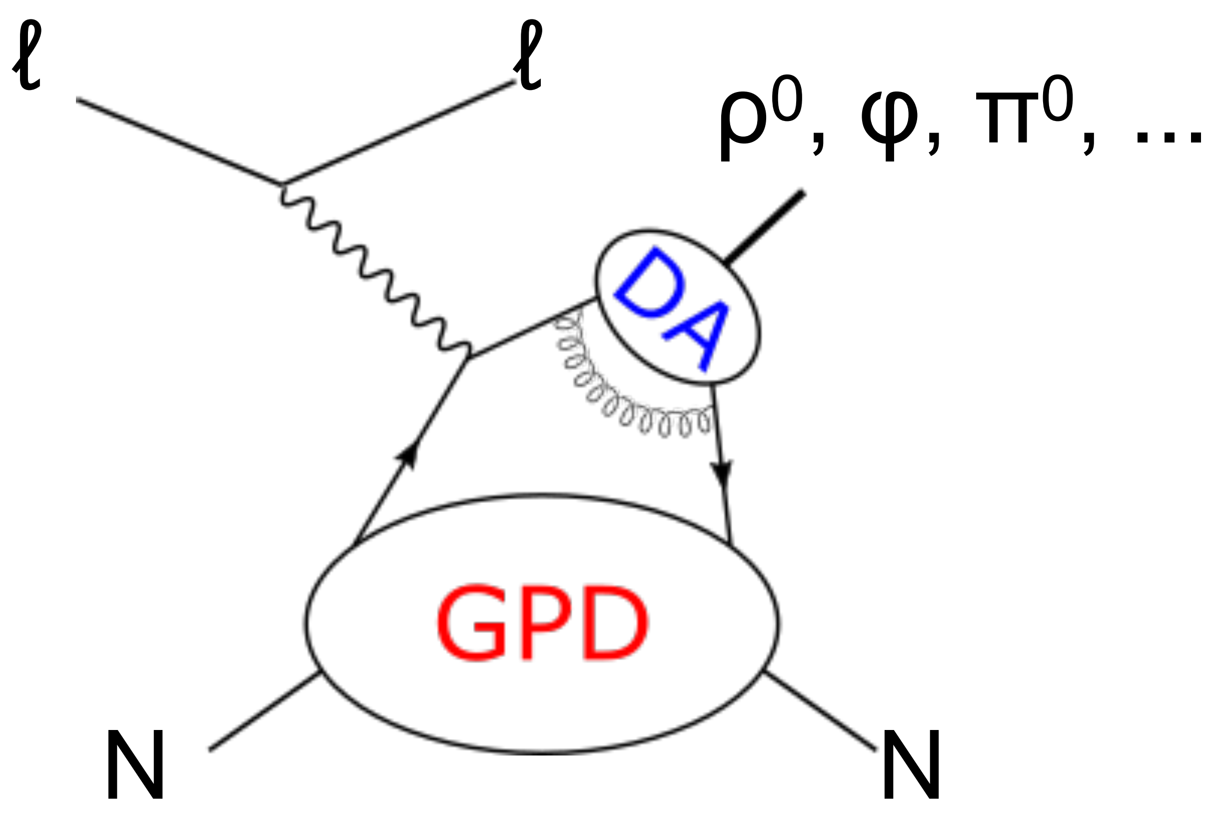
GPDs accessible in various production channels and observables
→ experimental filters



DVCS
Deeply Virtual Compton Scattering



TCS
Timelike Compton Scattering



HEMP
Hard Exclusive Meson Production

more production channels sensitive to GPDs exist!

Shadow GPDs

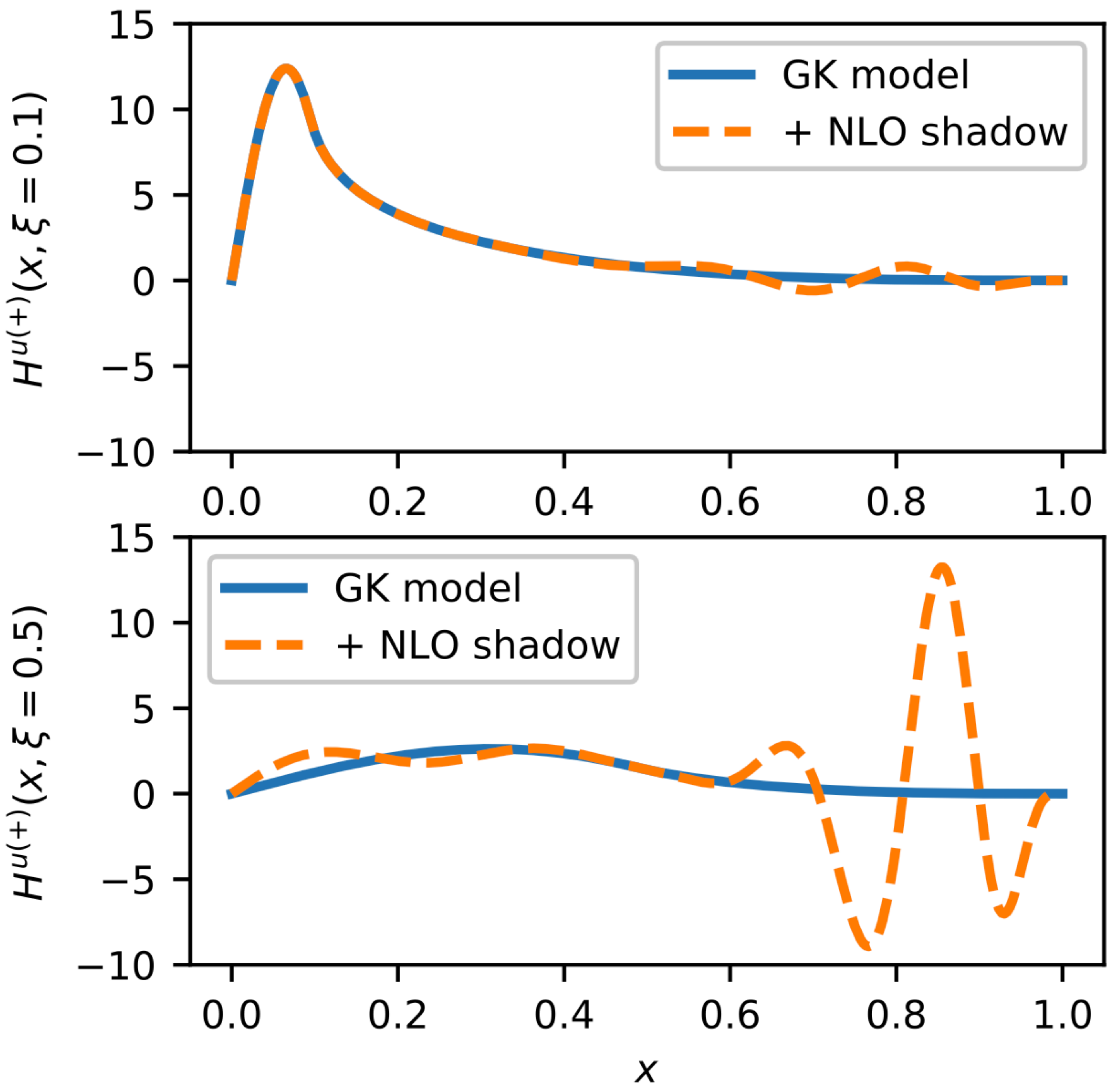
V. Bertone et al.,
Phys. Rev. D 103 (2021) 11, 114019

Shadow GPDs have considerable size, but:

- at arbitrary initial scale do not contribute to PDFs and CFFs
- at other scales contribute negligibly

making the deconvolution of CFFs ill-posed problem

We found such GPDs for DVCS for both LO and NLO
(for discussion see also PRD 108 (2023) 3, 036027)



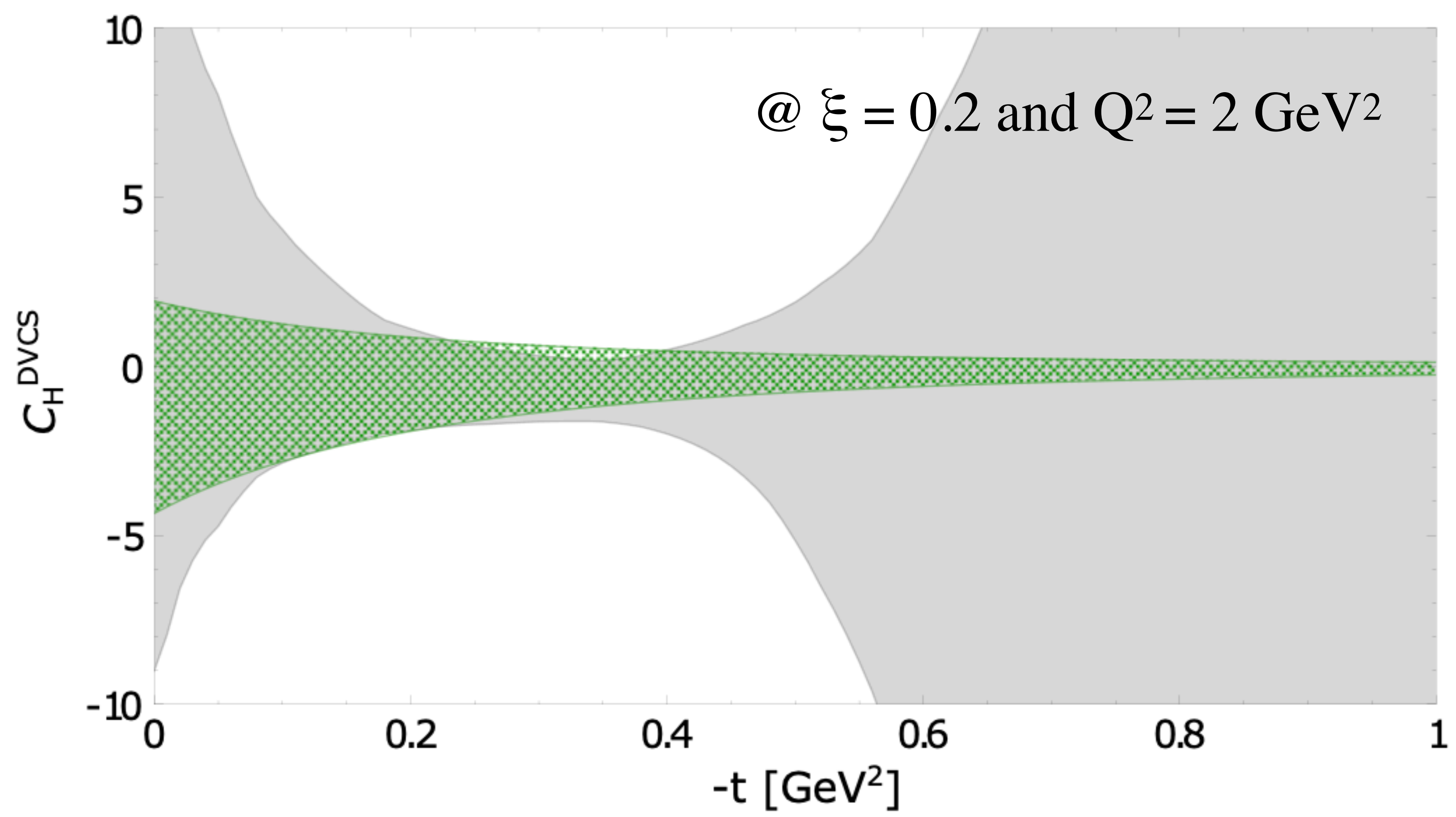
Exceptions

Studies less sensitive to model bias:

- probing nucleon tomography at low-xB (see: *PLB 793 (2019) 188*)

$$\mathrm{d}^3\sigma/(\mathrm{d}x_{\mathrm{Bj}}\,\mathrm{d}Q^2\,\mathrm{d}t) \propto (\mathrm{Im}\mathcal{H}(\xi,t))^2 \propto \left(\sum_q e_q^2 H^{q(+)}(\xi,\xi,t)\right)^2 \propto \left(\sum_q e_q^2 H^{q(+)}(\xi,0,t)\right)^2$$

- extraction of D-term (see: *Nature 570 (2019) 7759, E1*, *EPJC 81 (2021) 4, 300*)

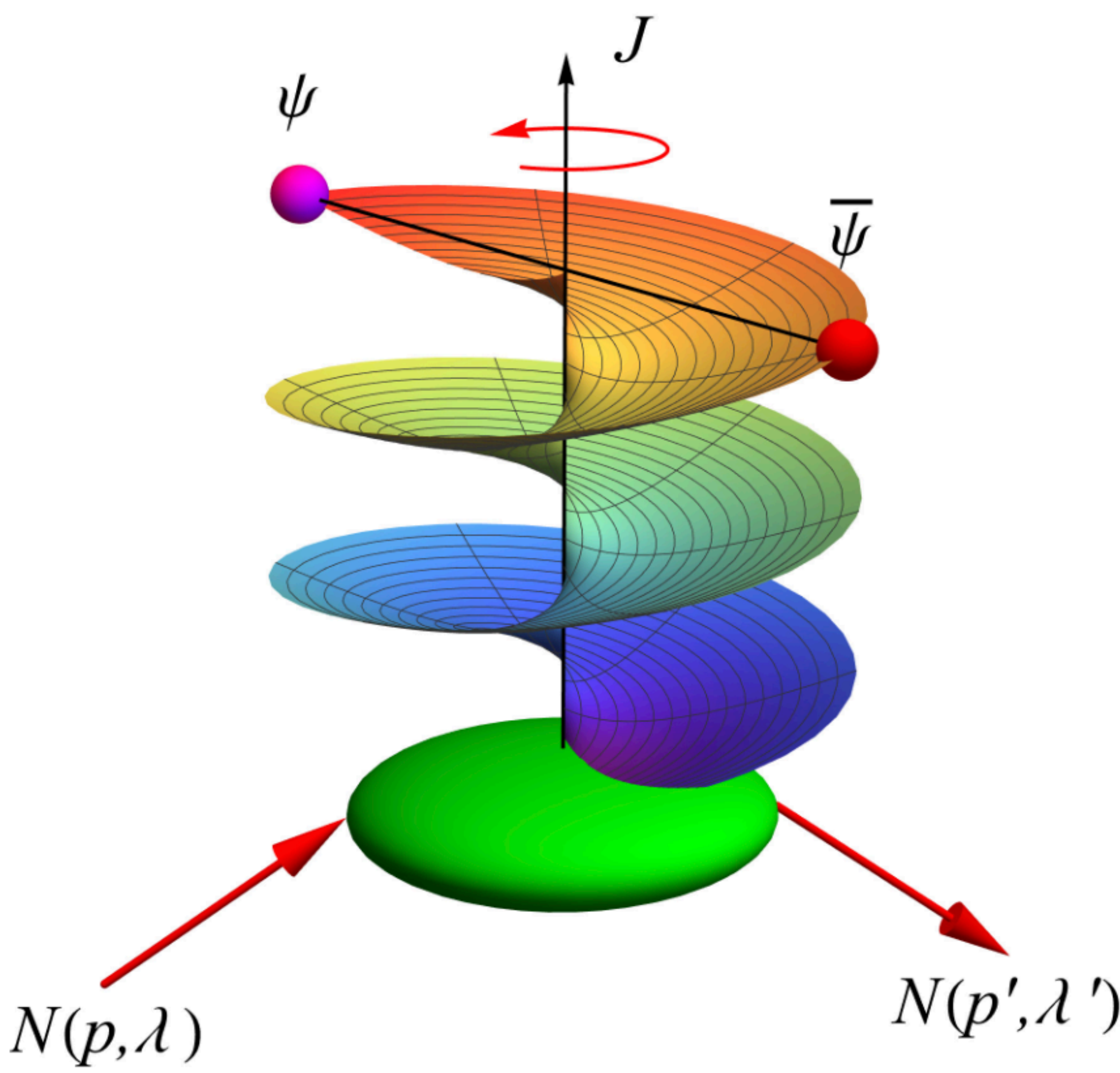


ANN analysis

Model dependent extraction

$$d_1^{uds}(t,\mu_F^2) = d_1^{uds}(\mu_F^2)\left(1-\frac{t}{\Lambda^2}\right)^{-\alpha}$$

$\alpha = 3 \quad \Lambda = 0.8\ \mathrm{GeV}$



- Froissart-Gribov projections (see: *hep-ph/2312.09624 and this talk*)
  (quantification of hadron target response on the string-like QCD probe with fixed angular momentum J)

Froissart-Gribov projections - spin 0

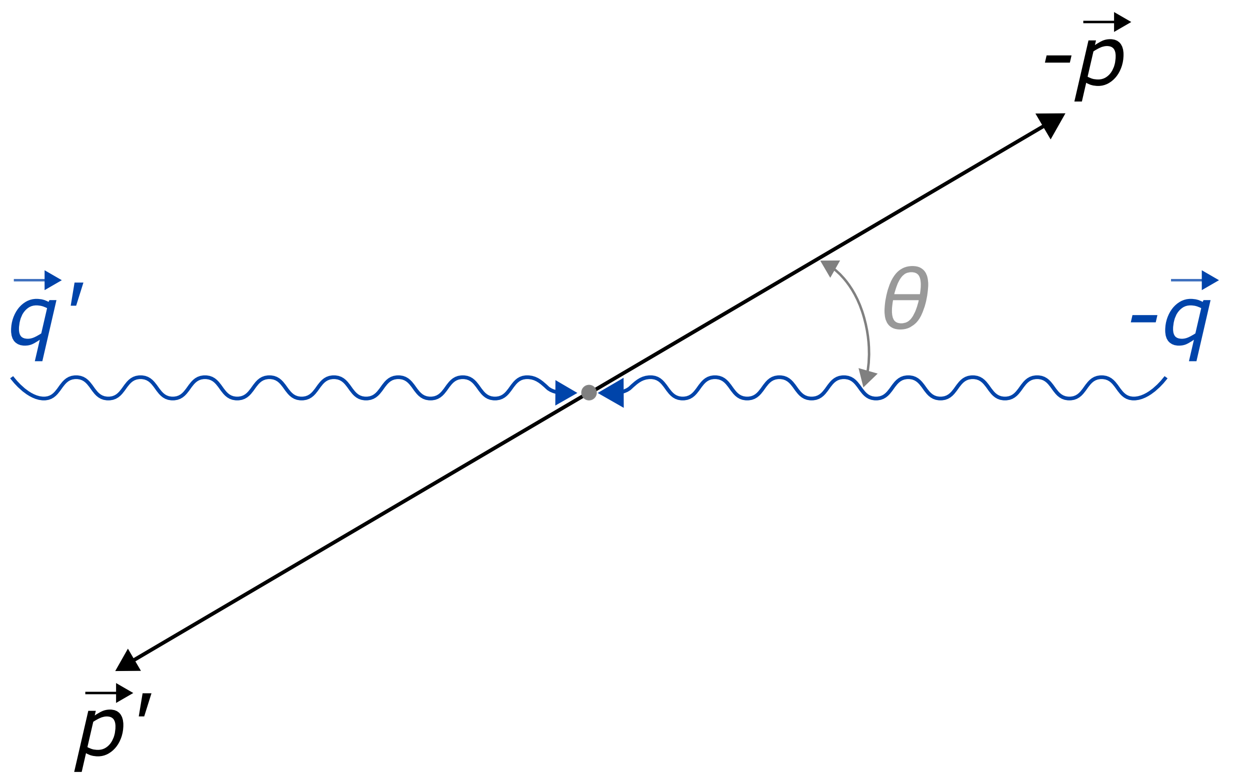
K. Semenov-Tian-Shansky and P.S.
hep-ph/2312.09624

FG projections are obtained by reconstructing cross-channel partial wave expansion amplitudes from the dispersive representation of the amplitude in the direct channel.

In cross-channel: $\gamma^*(q) + \gamma(-q') \to h(p') + \bar{h}(-p)$

Expansion in the cross channel SO(3) partial waves: $$\mathcal{H}_+(\cos\theta_t, t) = \sum_{\substack{J=0 \\ \text{even}}}^{\infty} F_J(t) P_J(\cos\theta_t)$$

which gives: $$F_J(t) = \frac{2J+1}{2} \int_{-1}^{1} d(\cos\theta_t)\, P_J(\cos\theta_t)\, \mathcal{H}_+(\cos\theta_t, t)$$



In direct-channel: $\gamma^*(q) + h(p) \to \gamma(q') + h(p')$

Dispersion relation: $$\mathrm{Re}\,\mathcal{H}_+(\xi, t) = \mathcal{P}\int_0^1 dx \frac{2x H_+(x, x, t)}{\xi^2 - x^2} + 4D(t)$$

where: $$\cos\theta_t \to -\frac{1}{\xi\beta} + \mathcal{O}\left(1/Q^2\right) \qquad \beta = \sqrt{1 - \frac{4m^2}{t}}$$

$\beta = 1$
in the current analysis

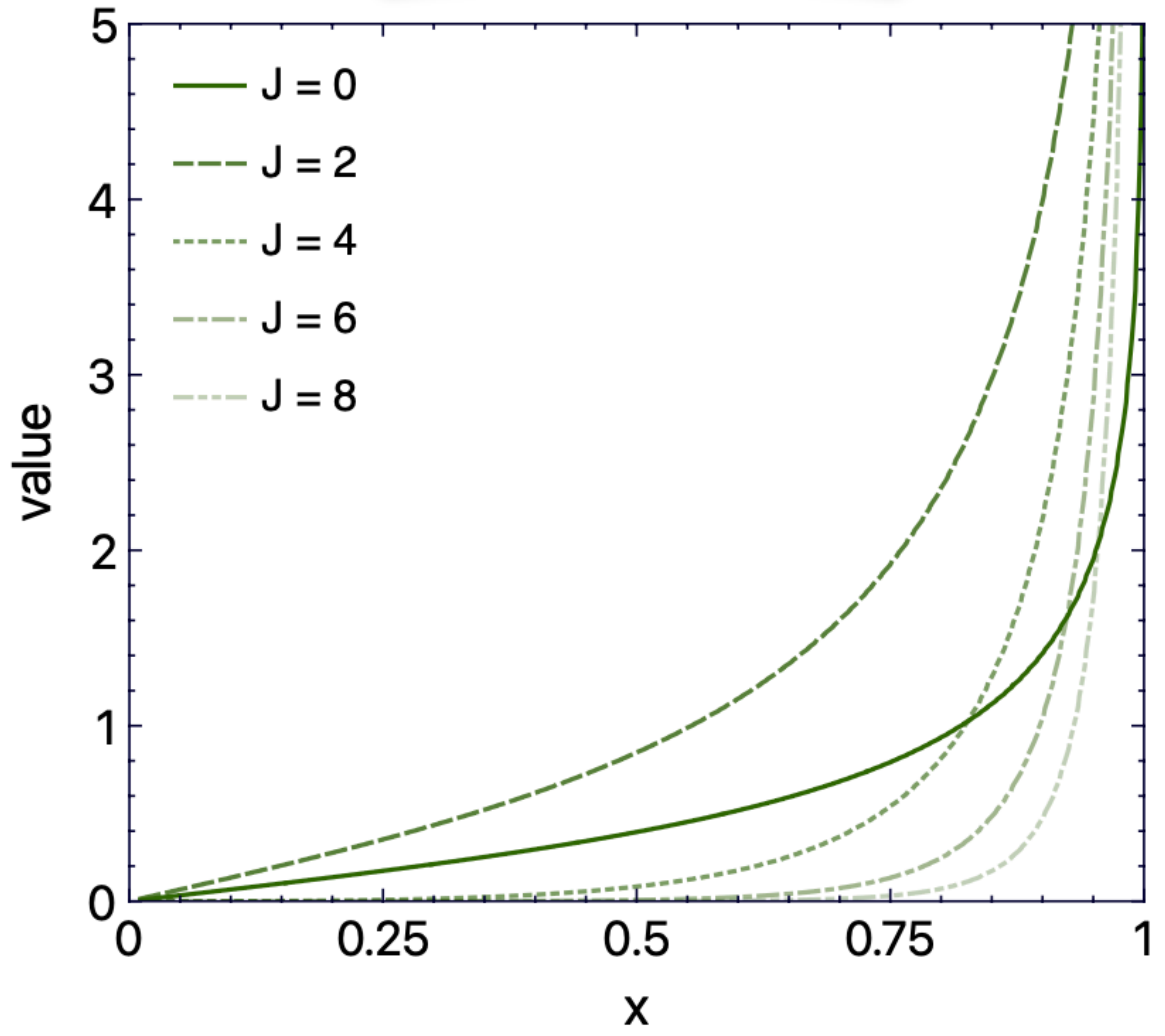
K. Semenov-Tian-Shansky and P.S.
hep-ph/2312.09624

Final result:

$$F_{J=0}(t) = 2\int_0^1 dx \left(\frac{\mathcal{Q}_0(1/x)}{x^2} - \frac{1}{x} \right) H_+(x,x,t) + 4D(t)$$

$$F_{J>0}(t) = 2(2J+1)\int_0^1 dx \frac{\mathcal{Q}_J(1/x)}{x^2} H_+(x,x,t)$$

weight functions



Froissart-Gribov projections - spin 0

K. Semenov-Tian-Shansky and P.S.
hep-ph/2312.09624

Dual parameterisation:

$$H_+(x,\xi,t) = 2\sum_{\substack{n=1\\ \text{odd}}}^{\infty}\sum_{\substack{l=0\\ \text{even}}}^{n+1} B_{n,l}(t)\,\theta\left(1-\frac{x^2}{\xi^2}\right)\left(1-\frac{x^2}{\xi^2}\right)C_n^{3/2}\left(\frac{x}{\xi}\right)P_l\left(\frac{1}{\xi}\right)$$

Coefficients of Mellin moments:

$$\int_0^1 dx x^N H_+(x,\xi,t) = \sum_{\substack{k=0\\ \text{even}}}^{N+1} h_{N,k}(t)\xi^k$$

where:

$$h_{N,k}(t) = \sum_{\substack{n=1\\ \text{odd}}}^{N}\sum_{\substack{l=0\\ \text{even}}}^{n+1} B_{n,l}(t)(-1)^{\frac{k+l-N-1}{2}}\frac{\Gamma\left(1-\frac{k-l-N}{2}\right)}{2^k\Gamma\left(\frac{1}{2}+\frac{k+l-N}{2}\right)\Gamma(2-k+N)}\frac{(n+1)(n+2)\Gamma(N+1)}{\Gamma\left(1+\frac{N-n}{2}\right)\Gamma\left(\frac{5}{2}+\frac{N+n}{2}\right)}$$

E.g.:

$$\int_0^1 dx\, H_+(x,\xi,t) = \frac{6B_{1,2}(t)}{5} + \xi^2\left(\frac{4B_{1,0}(t)}{5} - \frac{2B_{1,2}(t)}{5}\right)$$

Froissart-Gribov projections - spin 0

K. Semenov-Tian-Shansky and P.S.
hep-ph/2312.09624

This gives:

$$F_{J=0}(t) = 4 \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} B_{n,0}(t) = 4 \sum_{\nu=1}^{\infty} B_{2\nu-1,0}(t)$$

$$F_{J>0}(t) = 4 \sum_{\substack{n=J-1 \\ \text{odd}}}^{\infty} B_{n,J}(t) = 4 \sum_{\nu=0}^{\infty} B_{J+2\nu-1,J}(t)$$

The relations allow us to define "sum rules", e.g. for nu = 1:

$$F_{J=0}(t) = 4\left(B_{1,0}(t) + \ldots\right) = \frac{5}{3}h_{1,0}(t) + 5h_{1,2}(t) + \left\{\begin{matrix}\text{contribution of conformal PWs} \\ \text{with } \nu \geq 2\end{matrix}\right\}$$

$$F_{J=2}(t) = 4\left(B_{1,2}(t) + B_{3,2}(t) + \ldots\right) = -\frac{7}{6}h_{1,0}(t) + 9h_{3,0}(t) + \frac{21}{2}h_{3,2}(t) + \left\{\begin{matrix}\text{contribution of conformal PWs} \\ \text{with } \nu \geq 2\end{matrix}\right\}$$

K. Semenov-Tian-Shansky and P.S.
hep-ph/2312.09624

Modified dual parameterisation:

$$H_+(x,\xi,t) = 2\sum_{\substack{n=1\\ \text{odd}}}^{\infty}\sum_{\substack{l=0\\ \text{even}}}^{n+1} \beta^l \bar{B}_{n,l}(t)\,\theta\left(1-\frac{x^2}{\xi^2}\right)\left(1-\frac{x^2}{\xi^2}\right) C_n^{3/2}\left(\frac{x}{\xi}\right) P_l\left(\frac{1}{\xi\beta}\right)$$

$$\beta = \sqrt{1-\frac{4m^2}{t}}$$

Coefficients of Mellin moments:

$$\int_0^1 dx x^N H_+(x,\xi,t) = \sum_{\substack{k=0\\ \text{even}}}^{N+1} h_{N,k}(t)\xi^k$$

$$h_{N,k}(t) = \sum_{\substack{n=1\\ \text{odd}}}^{N}\sum_{\substack{l=0\\ \text{even}}}^{n+1} \beta^{l+k-N-1}\bar{B}_{n,l}(t)(-1)^{\frac{k+l-N-1}{2}} \frac{\Gamma\left(1-\frac{k-l-N}{2}\right)}{2^k\Gamma\left(\frac{1}{2}+\frac{k+l-N}{2}\right)\Gamma(2-k+N)} \frac{(n+1)(n+2)\Gamma(N+1)}{\Gamma\left(1+\frac{N-n}{2}\right)\Gamma\left(\frac{5}{2}+\frac{N+n}{2}\right)}$$

To keep these coefficients regular at t=0 one has to assume:

For $\beta \neq 1$ FG projections get admixture from higher spins

$$B_{n,n+1}(t) = \bar{B}_{n,n+1}(t)$$

$$B_{n,n-1}(t) = \bar{B}_{n,n-1}(t) - (1-\beta^2)\left(\frac{1}{2}-n\right)\bar{B}_{n,n+1}(t)$$

spin J=n-1 spin J=n+1

Froissart-Gribov projections - spin ½

K. Semenov-Tian-Shansky and P.S.
hep-ph/2312.09624

Electric combination:

$$H_{\pm}^{(E)}(x, \cos\theta_t, t) = H_{\pm}(x, \cos\theta_t, t) + \tau E_{\pm}(x, \cos\theta_t, t)$$

$$\tau \equiv t/(4m^2)$$

helicities of $p\bar{p}$ couple to |λ−λ'| = 0

has to be expanded in $P_J(\cos\theta_t)$ rotation function

Magnetic combination:

$$H_{\pm}^{(M)}(x, \cos\theta_t, t) = H_{\pm}(x, \cos\theta_t, t) + E_{\pm}(x, \cos\theta_t, t)$$

helicities of $p\bar{p}$ couple to |λ−λ'| = 1

has to be expanded in $\sin\theta_t P_J'(\cos\theta_t)/\sqrt{J(J+1)}$ rotation function

K. Semenov-Tian-Shansky and P.S.
hep-ph/2312.09624

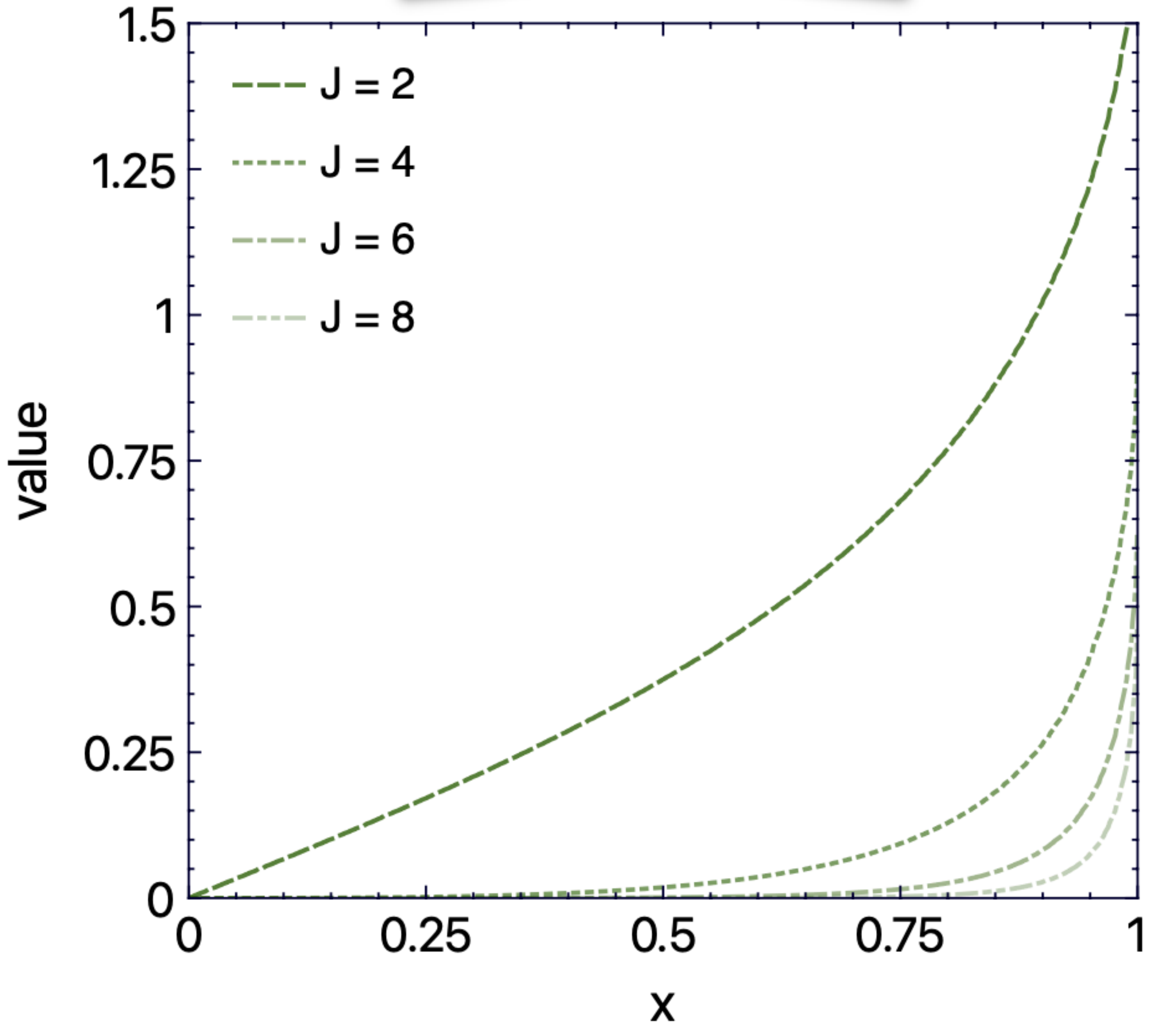
Final result:

$$F^{(E)}_{J=0}(t) = 2\int_0^1 dx \left[\frac{\mathcal{Q}_0(1/x)}{x^2} - \frac{1}{x}\right] H^{(E)}_{+}(x,x,t) + 4(1-\tau)D(t)$$

$$F^{(E)}_{J>0}(t) = 2(2J+1)\int_0^1 dx \frac{\mathcal{Q}_0(1/x)}{x^2} H^{(E)}_{+}(x,x,t)$$

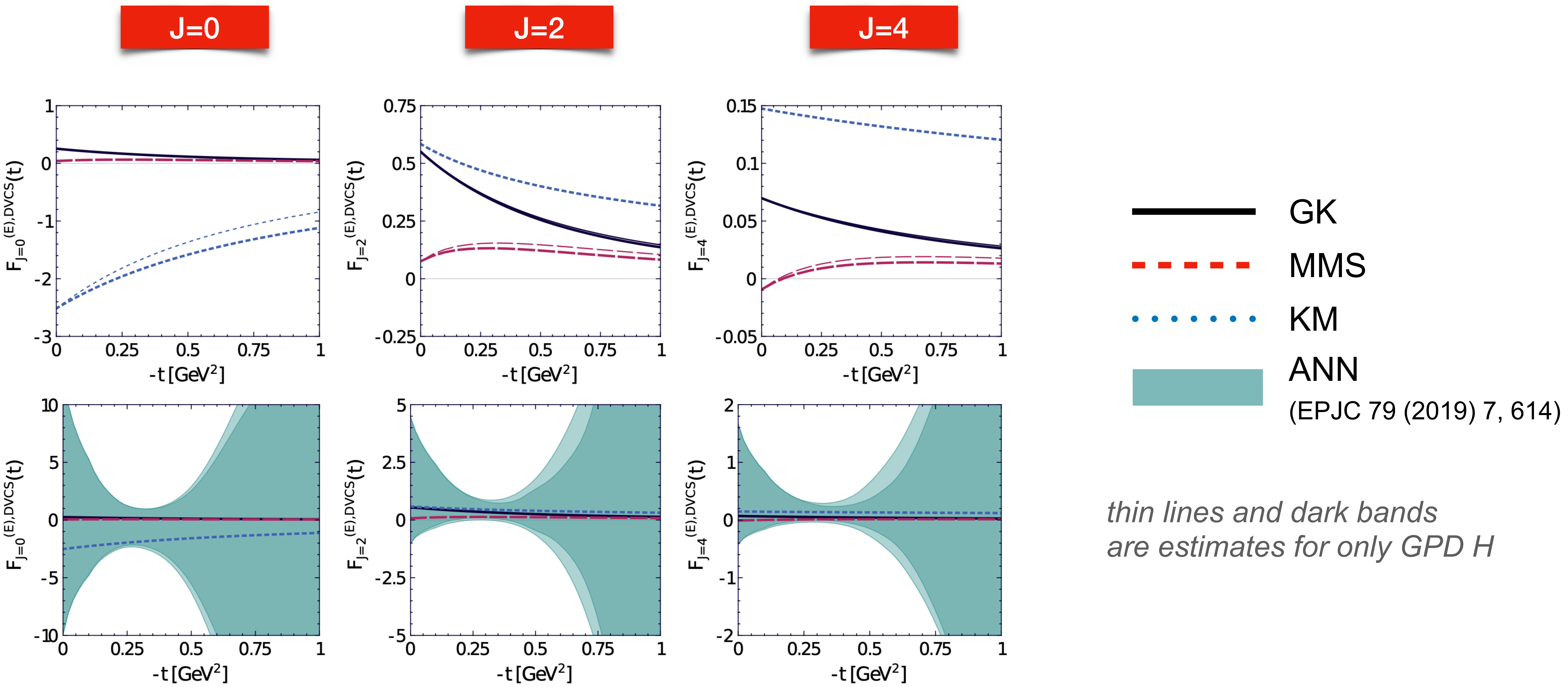
$$F^{(M)}_{J}(t) = 2\int_0^1 dx H^{(M)}_{+}(x,x,t) \frac{2J+1}{J(J+1)} \frac{(-1)}{x} \sqrt{\frac{1}{x^2}-1}\, \mathcal{Q}^1_J(1/x)$$

weight functions (for $F^{(M)}_J$)



K. Semenov-Tian-Shansky and P.S.
hep-ph/2312.09624

Numerical estimates - electric case:

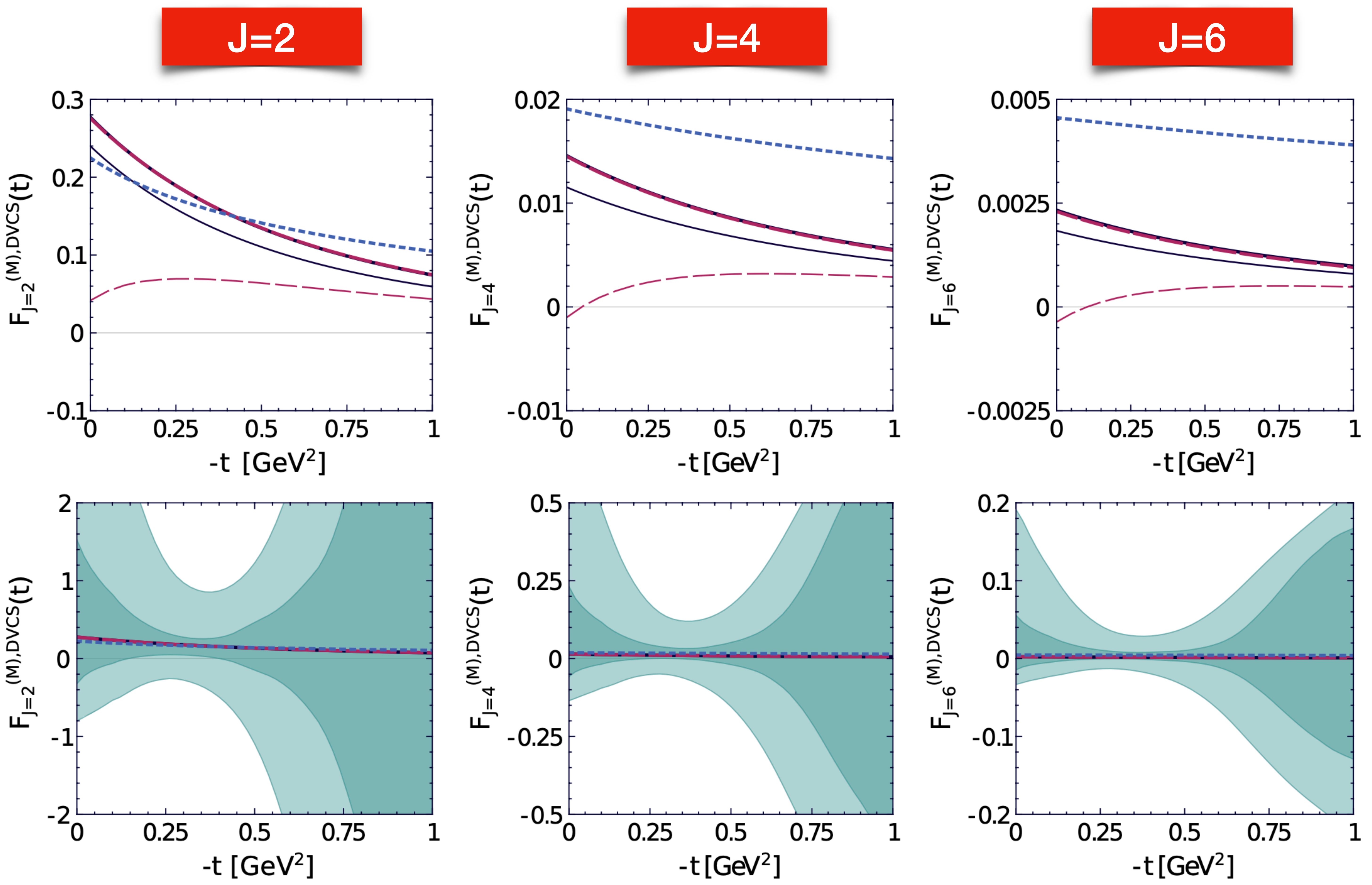


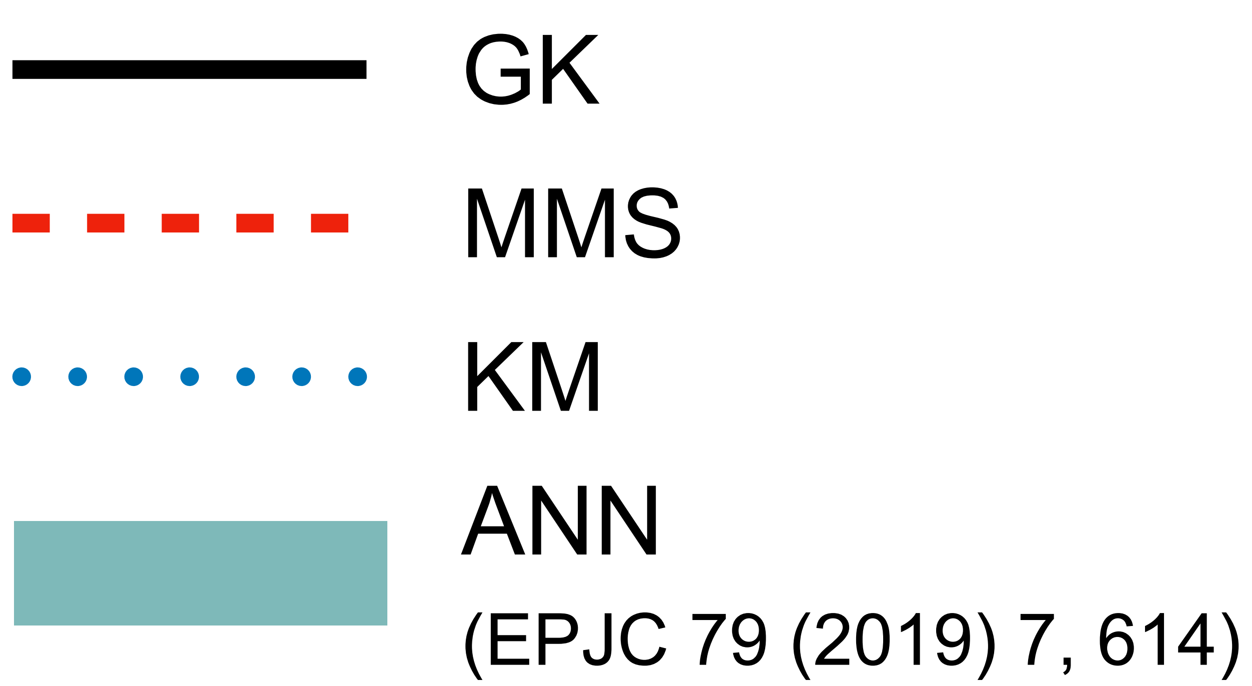
thin lines and dark bands are estimates for only GPD H

Froissart-Gribov projections - spin ½

K. Semenov-Tian-Shansky and P.S.
hep-ph/2312.09624

Numerical estimates - magnetic case:





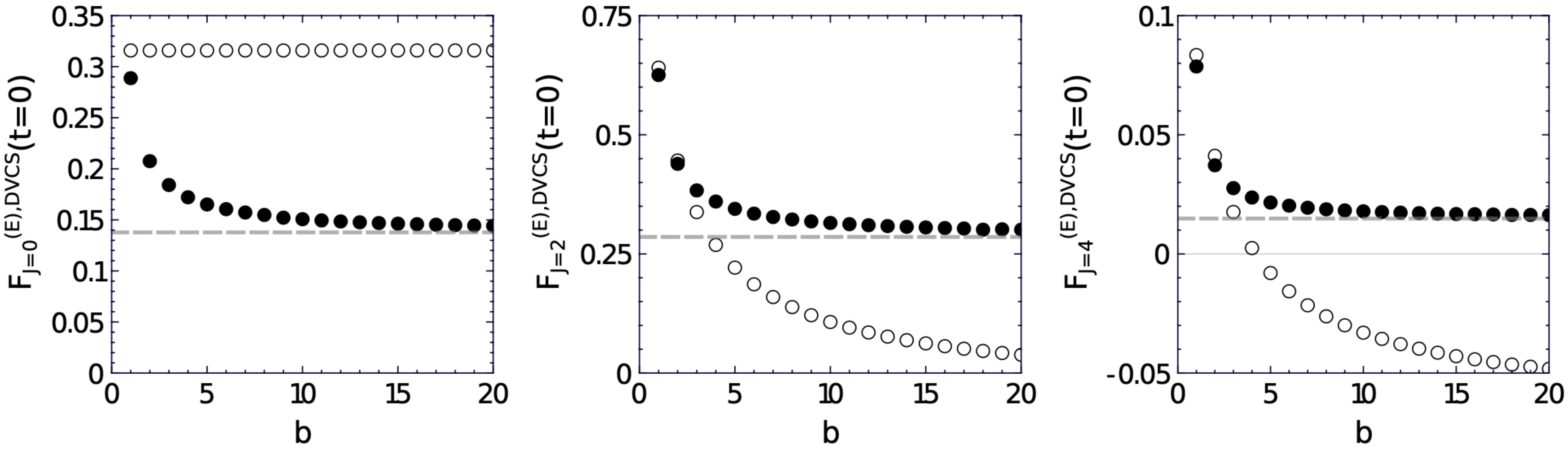
thin lines and dark bands are estimates for only GPD H

Froissart-Gribov projections - spin ½

K. Semenov-Tian-Shansky and P.S.
hep-ph/2312.09624

Sensitivity of FG projections on the shape of DD profile function

$$H(x,\xi,t=0) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\, \delta(\beta+\xi\alpha-x)\, q(\beta) \frac{\Gamma(2b+2)}{2^{2b+1}\Gamma^2(b+1)} \frac{\left((1-|\beta|)^2-\alpha^2\right)^b}{(1-|\beta|)^{2b+1}}$$



● projections from GPD

b → ∞ limit (no skewness effect)

○ projections from sum rules (i.e. from certain Mellin moments) for $\nu = 1$

Double DVCS

K. Deja, V. Martínez-Fernández,
B. Pire, PS, J. Wagner
Phys. Rev. D 107 (2023) 9, 094035

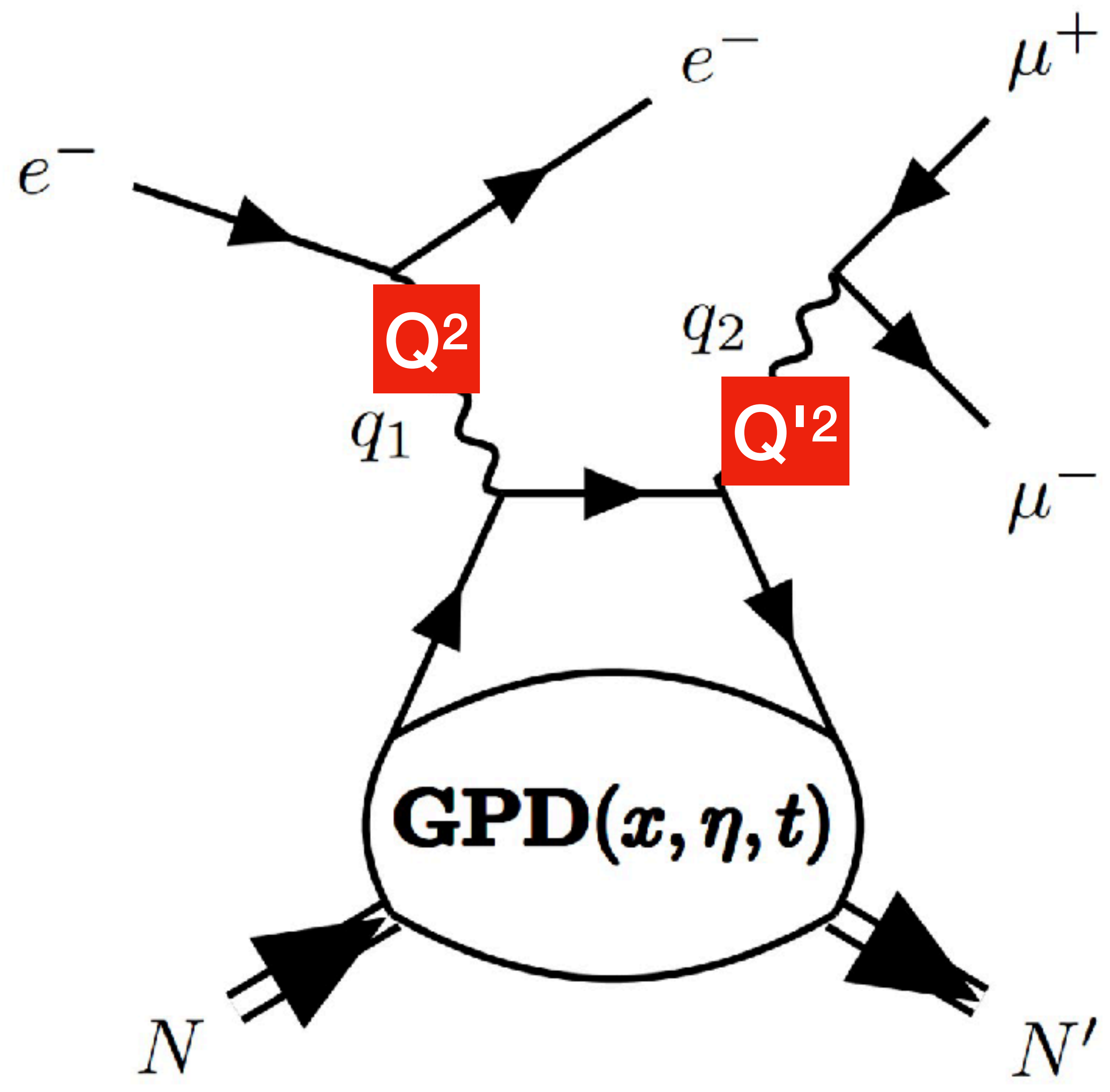
- The process allows to directly probe GPDs outside x=ξ line, but is much more challenging experimentally

$$\xi = \frac{Q^2 + Q'^2}{2Q^2/x_B - Q^2 - Q'^2}$$

$$(\mathcal{H}, \mathcal{E})(\rho, \xi, t) = \sum_{f=\{u,d,s\}} \int_{-1}^{1} dx \; C_f^{(-)}(x, \rho)(H_f, E_f)(x, \xi, t)$$

$$\rho = \xi \frac{Q^2 - Q'^2}{Q^2 + Q'^2}$$

$$C_f^{(\pm)}(x, \rho) \overset{LO}{=} \left(\frac{e_f}{e}\right)^2 \left(\frac{1}{\rho - x - i0} \pm \frac{1}{\rho + x - i0}\right)$$



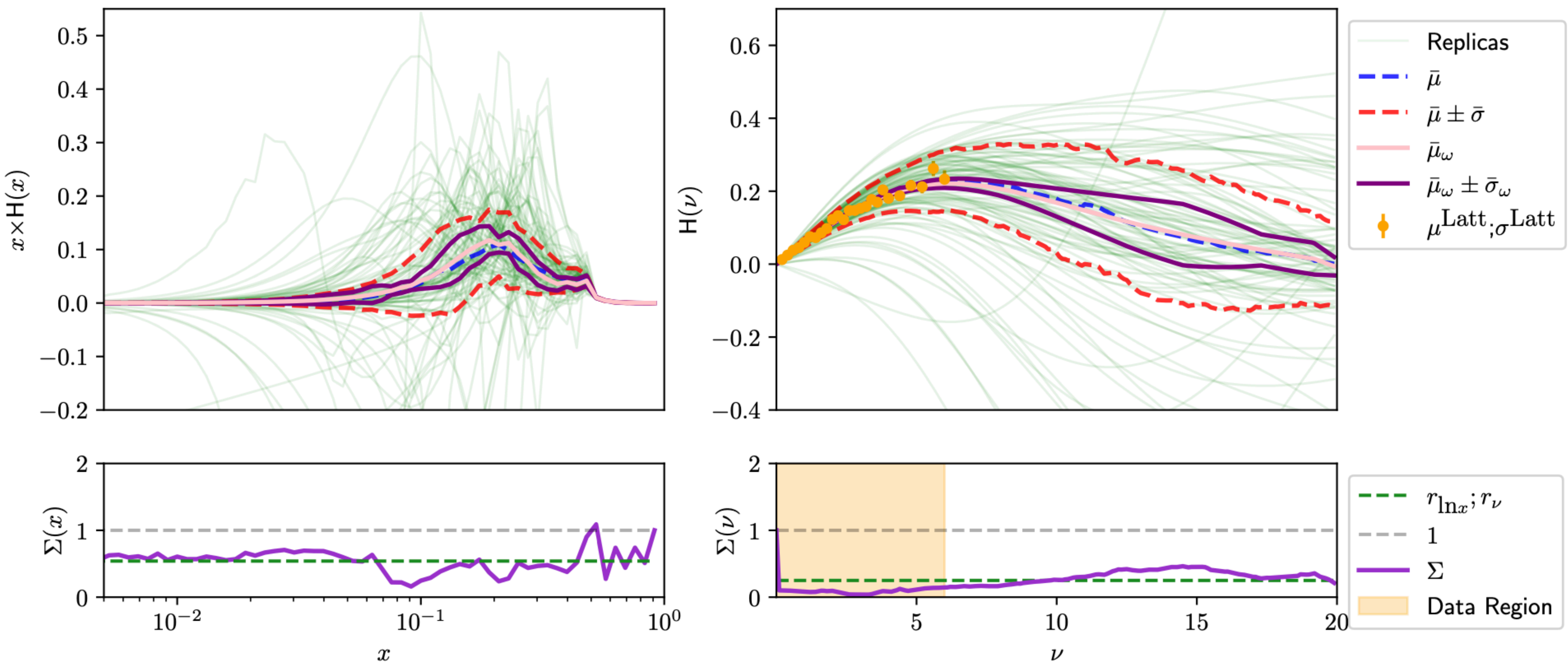
- We revisit DDVCS phenomenology in view of new experiments, including reevaluation of DDVCS and BH cross-sections with Kleiss-Stirling spinor techniques

- Obtained results are available in PARTONS and EpIC MC generator

M. J. Riberdy, H. Dutrieux, C. Mezrag, PS, hep-ph/2306.01647

- Exploratory study to include lattice-QCD results!

Reduction of GPD model uncertainties due to inclusion of pseudo-latticeQCD results



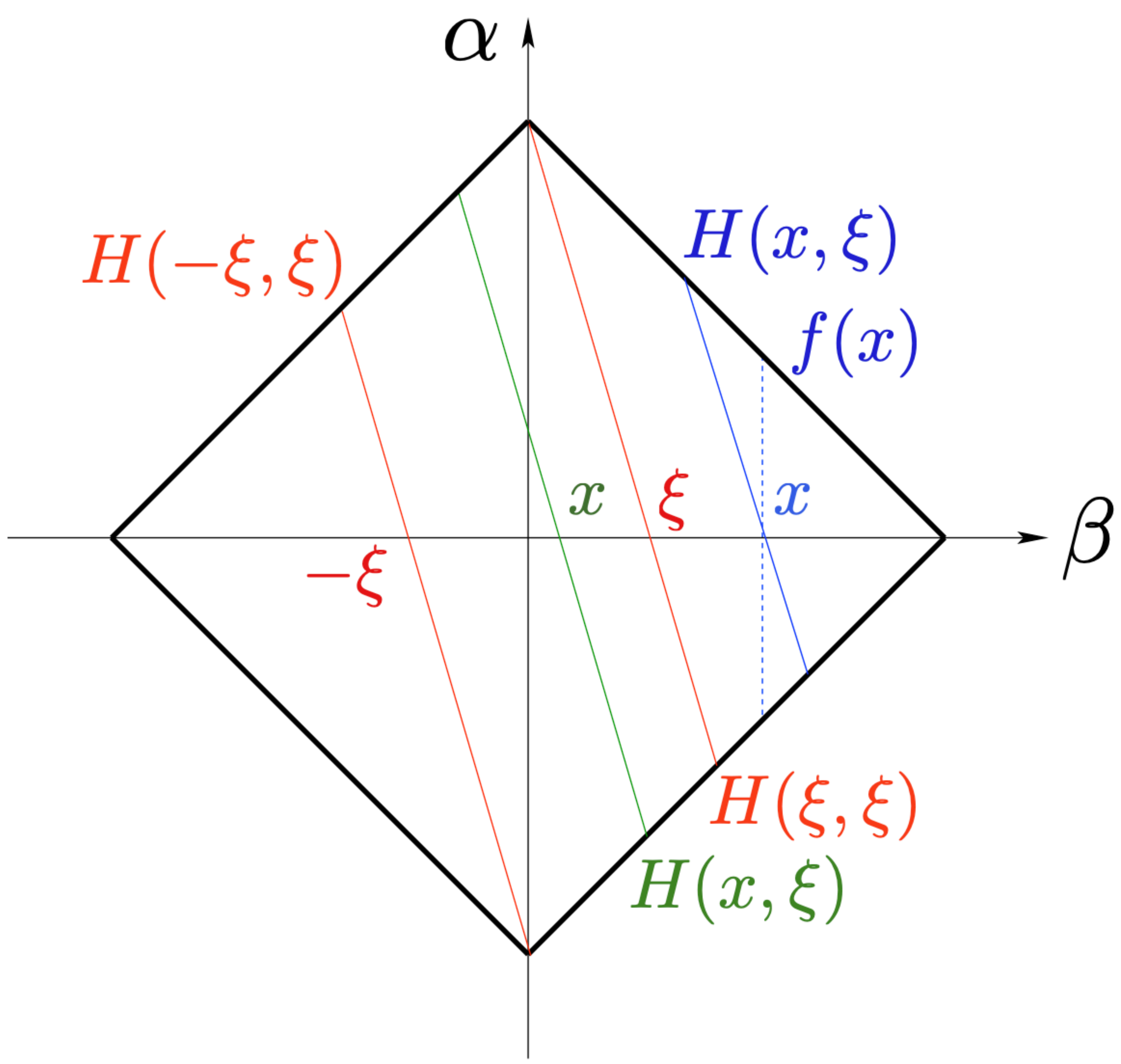
Double distribution:

$$H(x, \xi, t) = \int \mathrm{d}\Omega\, F(\beta, \alpha, t)$$

where:

$$\mathrm{d}\Omega = \mathrm{d}\beta\, \mathrm{d}\alpha\, \delta(x - \beta - \alpha\xi)$$

$$|\alpha| + |\beta| \leq 1$$



from PRD83, 076006, 2011

H. Dutrieux et al.,
Eur. Phys. J. C 82 (2022) 3, 252

Double distribution:

$$(1-x^2)F_C(\beta,\alpha) + (x^2-\xi^2)F_S(\beta,\alpha) + \xi F_D(\beta,\alpha)$$

Classical term:

$$F_C(\beta,\alpha) = f(\beta)h_C(\beta,\alpha)\frac{1}{1-\beta^2}$$

$$f(\beta) = \mathrm{sgn}(\beta)q(|\beta|)$$

$$h_C(\beta,\alpha) = \frac{\mathrm{ANN}_C(|\beta|,\alpha)}{\int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \mathrm{ANN}_C(|\beta|,\alpha)}$$

Shadow term:

$$F_S(\beta,\alpha) = f(\beta)h_S(\beta,\alpha)$$

$$f(\beta) = \mathrm{sgn}(\beta)q(|\beta|)$$

$$h_S(\beta,\alpha)/N_S = \frac{\mathrm{ANN}_S(|\beta|,\alpha)}{\int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \mathrm{ANN}_S(|\beta|,\alpha)} - \frac{\mathrm{ANN}_{S'}(|\beta|,\alpha)}{\int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \mathrm{ANN}_{S'}(|\beta|,\alpha)} .$$

$$\mathrm{ANN}_{S'}(|\beta|,\alpha) \equiv \mathrm{ANN}_C(|\beta|,\alpha)$$

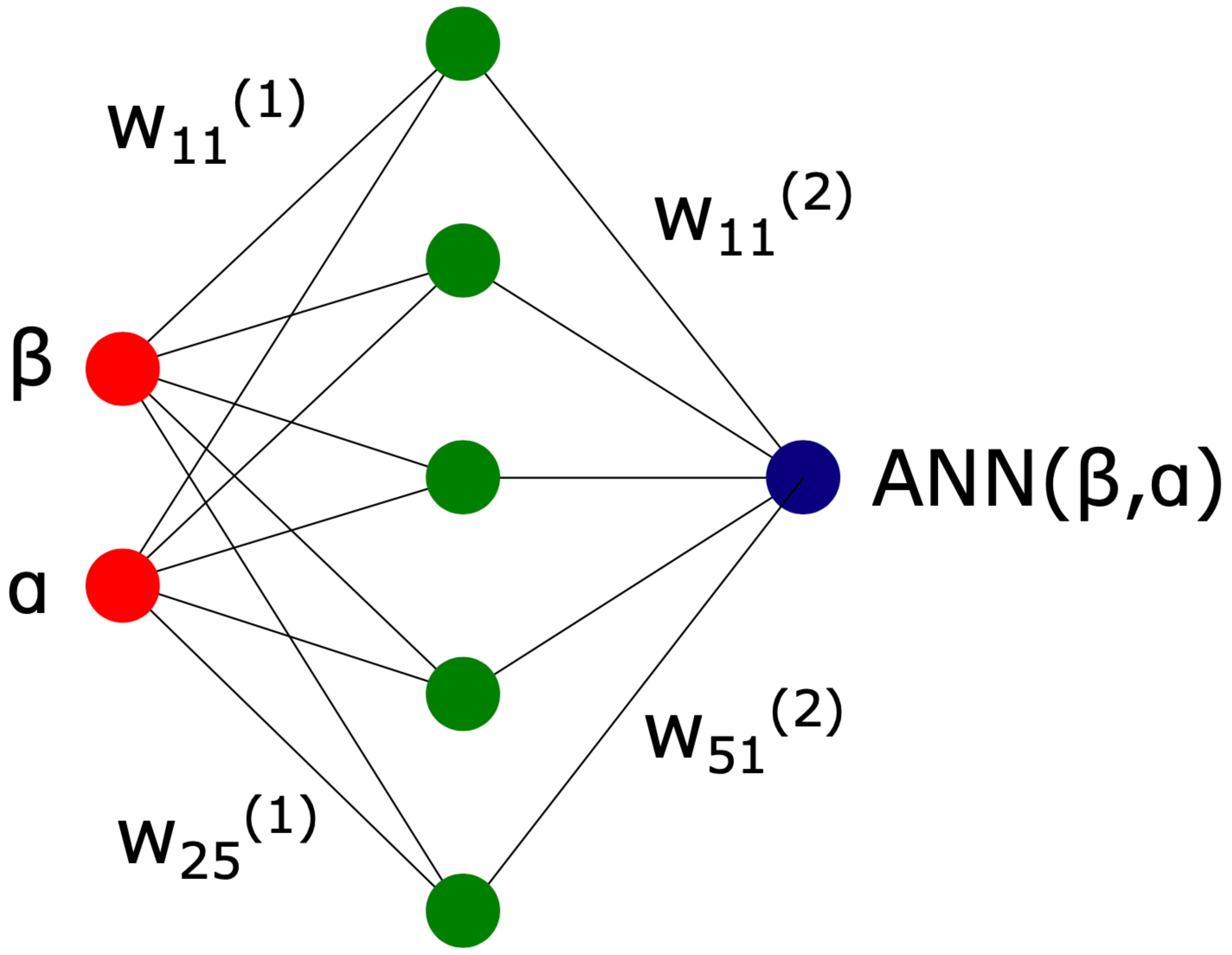
D-term:

$$F_D(\beta,\alpha) = \delta(\beta)D(\alpha)$$

$$D(\alpha) = (1-\alpha^2)\sum_{\substack{i=1 \\ \mathrm{odd}}} d_i C_i^{3/2}(\alpha)$$

H. Dutrieux et al.,
Eur. Phys. J. C 82 (2022) 3, 252

Our ANNs:



Requirements:

symmetric w.r.t. α

symmetric w.r.t. β

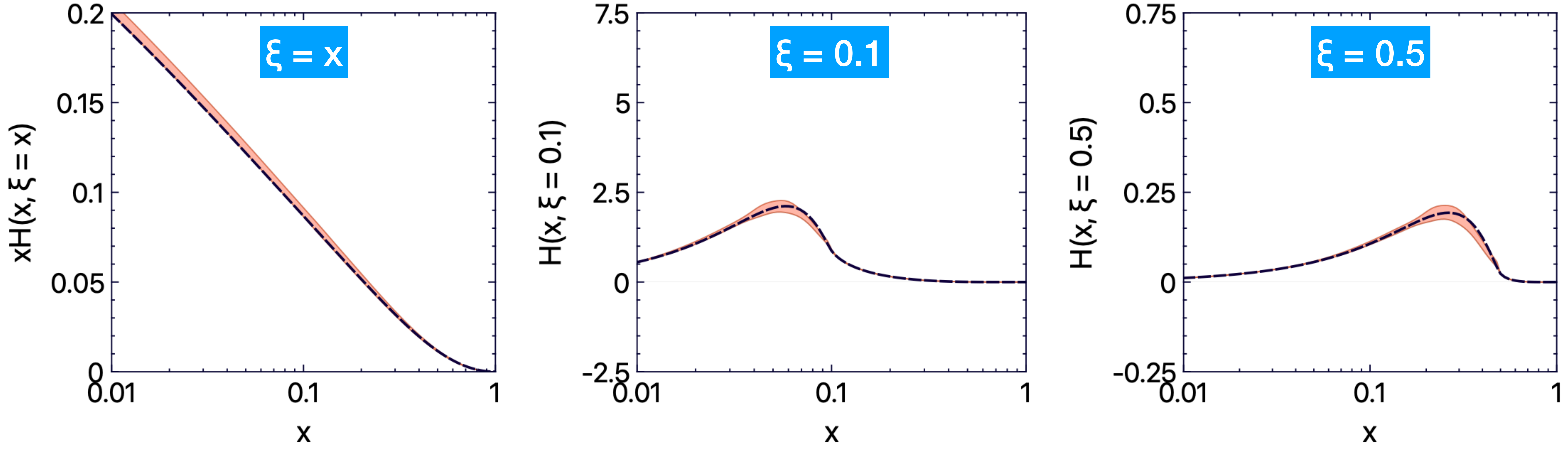
vanishes at | α | + | β | = 1

Activation function:

$$\left(\varphi_i\left(w_i^\beta|\beta| + w_i^\alpha\alpha/(1-|\beta|) + b_i\right) - \varphi_i\left(w_i^\beta|\beta| + w_i^\alpha + b_i\right)\right) + (w^\alpha \to -w^\alpha)$$

Demonstration of results

H. Dutrieux et al.,
Eur. Phys. J. C 82 (2022) 3, 252



Conditions:

- Input: 400 x ≠ ξ points generated with GK model
- Positivity not forced

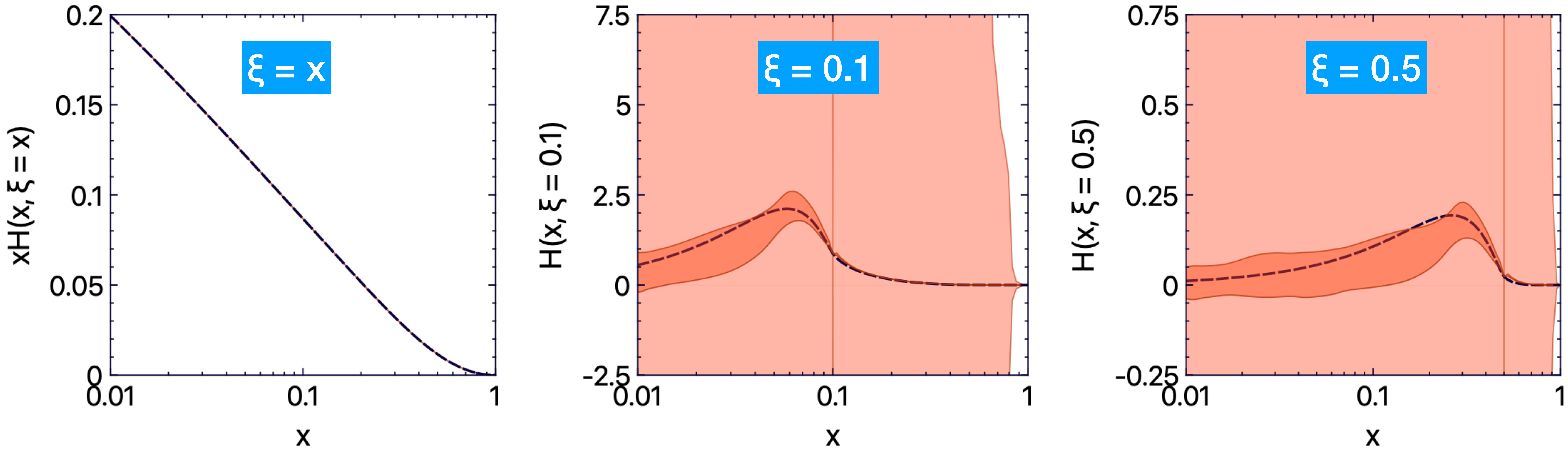
Technical detail of the analysis:

- Minimisation with genetic algorithm
- Replication for estimation of model uncertainties
- “Local” detection of outliers
- Dropout algorithm for regularisation

- - - - GK

ANN model 68% CL $F_C + F_S + F_D$

H. Dutrieux et al.,
Eur. Phys. J. C 82 (2022) 3, 252



GK

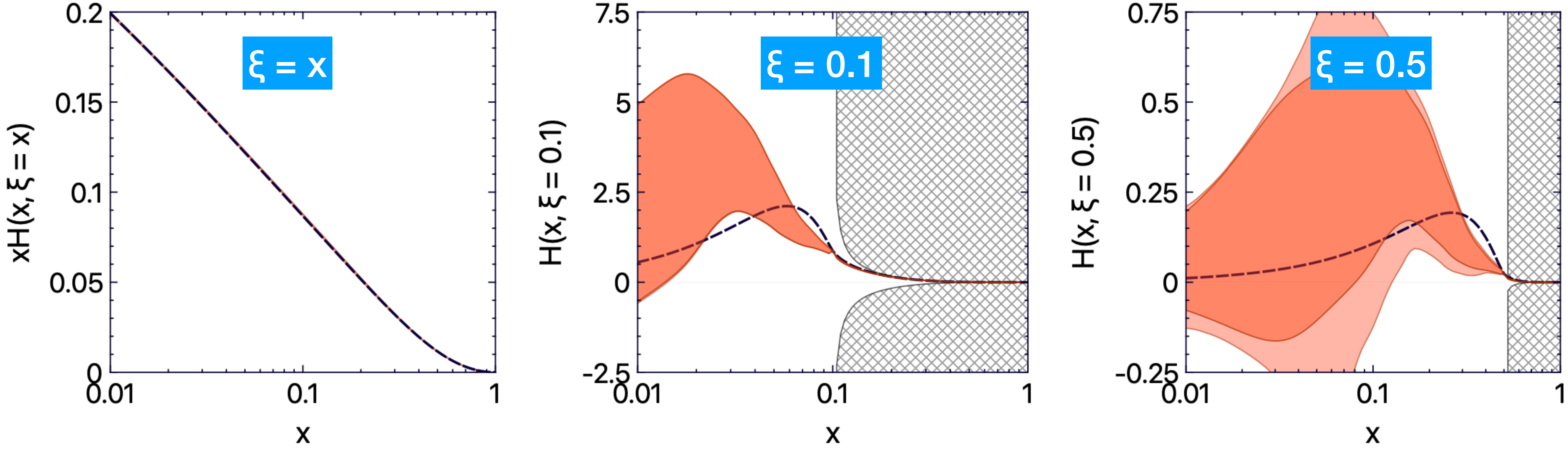
ANN model 68% CL F_C

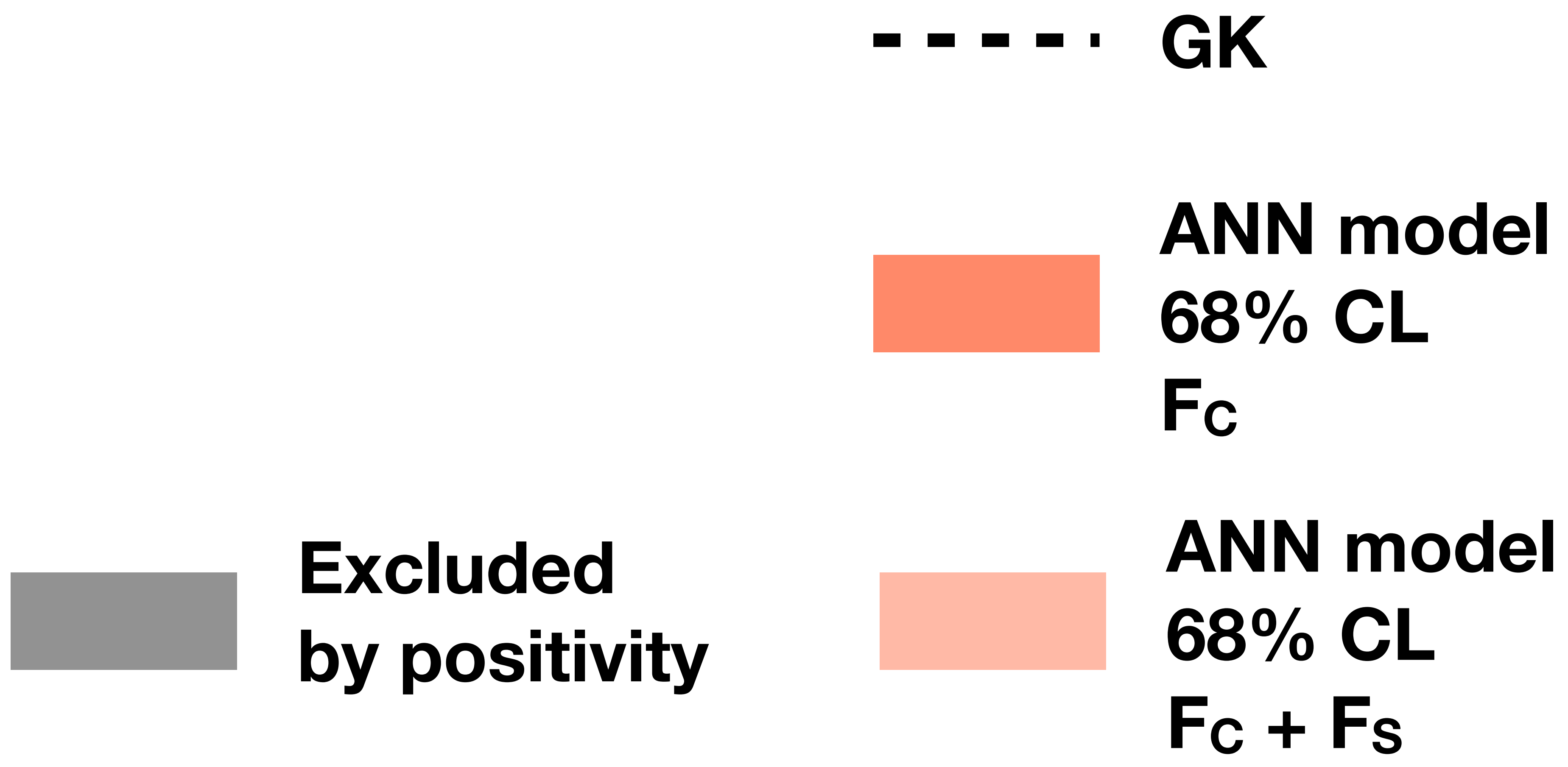
ANN model 68% CL $F_C + F_S$

Conditions:

- Input: 200 x = ξ points generated with GK model
- Positivity not forced

H. Dutrieux et al.,
Eur. Phys. J. C 82 (2022) 3, 252





Conditions:

- Input: 200 x = ξ points generated with GK model
- Positivity **forced**

Summary

- Recent progress in:
  - understanding of fundamental problems, like deconvolution of CFFs
    → important for extraction of GPDs
  - interpretation of GPDs
    → Froissart-Gribov projections quantifying hadron target response on the string-like QCD probe with fixed angular momentum J
  - description of exclusive processes
    → new sources of GPD information
  - modelling of GPD, fulfilling all theory-driven constraints (including positivity)
    → subject not touched enough in the current literature
    → developed in mind for easy inclusion of latticeQCD data