Flavor Decomposition of the proton gravitational form factors & mechanical structure of the proton

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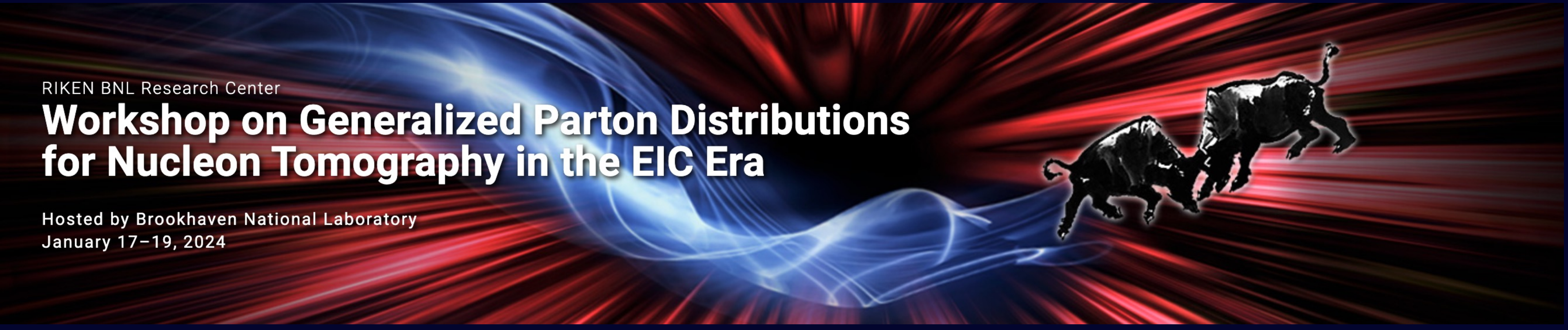

Gravitational form factors
of
the proton

Energy-Momentum Tensor

Hilbert-Einstein Action (Hilbert, 1915)

$$S = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\;R + \int d^4x\sqrt{-g}L_M$$

Changing the metric in the long-wave approximation

$$g^{\mu\nu} = \eta^{\mu\nu} + \delta g^{\mu\nu}(\boldsymbol{r}) \qquad\qquad \lambda_{\rm grav} \gg M_N^{-1}$$

we find the Energy momentum Tensor that characterizes the response of the nucleon to the static variation of the space-time metric:

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S_M}{\delta g_{\mu\nu}}$$

$$\partial_\mu T^{\mu\nu} = 0$$

Gravitational (EMT) form factors

- EMT current in QCD & GFFs

Kobzarev et al. 1962; Pagels, 1966

$$T_q^{\mu\nu} = \frac{1}{4}\overline{\psi}_q\left(-i\overleftarrow{\mathcal{D}}^\mu\gamma^\nu - i\overleftarrow{\mathcal{D}}^\nu\gamma^\mu + i\overrightarrow{\mathcal{D}}^\mu\gamma^\nu + i\overrightarrow{\mathcal{D}}^\nu\gamma^\mu\right)\psi_q - g^{\mu\nu}\overline{\psi}_q\left(-\frac{i}{2}\overleftarrow{\not{\mathcal{D}}} + \frac{i}{2}\overrightarrow{\not{\mathcal{D}}} - m_q\right)\psi_q,$$

$$T_g^{\mu\nu} = F^{a,\mu\eta}\,F^{a,}{}_\eta{}^\nu + \frac{1}{4}g^{\mu\nu}F^{a,\kappa\eta}\,F^{a,}{}_{\kappa\eta}.$$

D(Druck)-term Weiss & Polyakov, 1999

$$\langle p'|T^{\mu\nu}(0)|p\rangle = \bar{u}(p')\left[A^a(t)\frac{P^\mu P^\nu}{M_N} + J^a(t)\frac{iP^{\{\mu\sigma_\nu\}\rho}\Delta_\rho}{2M_N} + D^a(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M_N} + M_N\bar{c}^a(t)g^{\mu\nu}\right]u(p)$$

δg^{00} δg^{0i} δg^{ij}

Non-conservation of EMT pieces (cosmological constant)

O. V. Teryaev, Front. Phys. 11 (2016)
K.-F. Liu, PRD 104 (2021)

$\sum_a A^a(0) = 1$ Mass

Spin

$\sum_a J^a(0) = \frac{1}{2}$

Deformation of space
= mechanical properties of the nucleon

Pressure & Shear-force distributions (pressure anisotropy)

$$\partial_\mu T^{\mu\nu} = 0 \to \sum_{q,g}\bar{c}^{q,g} = 0$$

Flavor decomposition

- To decompose the GFFs, we need to compute the generalized EMT form factors for the flavor triplet & octet.

$$F_B^{\chi=0} = F_B^u + F_B^d + F_B^s,$$
$$F_B^{\chi=3} = F_B^u - F_B^d,$$
$$F_B^{\chi=8} = \frac{1}{\sqrt{3}}\left(F_B^u + F_B^d - 2F_B^s\right)$$

$$\sum_{a=q,g} F_B^a(t) = F_B(t), \quad \bar{c}_B(t) = 0$$

- The effective EMT current

$$\hat{T}^{\text{eff}}_{\mu\nu,\chi}(x) = \frac{i}{4}\bar{\psi}(x)\left(\gamma_\mu \overrightarrow{\partial}_\nu + \gamma_\nu \overrightarrow{\partial}_\mu - \gamma_\mu \overleftarrow{\partial}_\nu - \gamma_\nu \overleftarrow{\partial}_\mu\right)\lambda_\chi \psi(x)$$

- The GFFs can be regarded as the second moments of the vector GPDs.

XD Ji, PRL 78 (1997)

H. Y. Won, HChK, and J.-Y. Kim, PRD **108** (2023)

Mass distribution

- 00 component of the EMT

M.V. Polyakov, PLB 555 (2003)) for 3D densities

Each component gets a contribution from $\bar{c}^a_B$

$$\varepsilon^a_B(r)\delta_{J_3'J_3} := T^{a,B}_{00}(\boldsymbol{r}, J_3', J_3)$$
$$= M_B \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} \left[\boxed{A^a_B(t) + \bar{c}^a_B(t)} - \frac{t}{4M_B^2}\left(A^a_B(t) - 2J^a_B(t) + D^a_B(t)\right)\right] \delta_{J_3'J_3}.$$

- The mass of a baryon

$$\int d^3r \sum_{a=q,g} \varepsilon^a_B(r) = M_B A_B(0) = M_B$$

: The A form factor is normalized to 1.

- The mass radius of a baryon

$$\langle r^2_{\mathrm{mass}}\rangle_B = \frac{\int d^3r\, r^2 \varepsilon_B(r)}{\int d^3r\, \varepsilon_B(r)} = 6\frac{d}{dt}\left[A_B(t) - \frac{t}{4m_B^2} D_B(t)\right]_{t=0}$$

H. Y. Won, HChK, and J.-Y. Kim, 2310.04670

H. Y. Won, HChK, and J.-Y. Kim, PRD **108** (2023)

Angular momentum distribution

- 0i component of the EMT

$$J_i^{a,B}(\boldsymbol{r},J_3',J_3) := \epsilon_{ijk} r_j T_{0k}^{a,B}(\boldsymbol{r},J_3',J_3)$$
$$= 2\left(\hat{S}_j\right)_{J_3' J_3} \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} \left[\left(J_B^a(t) + \frac{2}{3} t \frac{dJ_B^a(t)}{dt}\right)\delta_{ij} + \left(\Delta_i \Delta_j - \frac{1}{3}\boldsymbol{\Delta}^2 \delta_{ij}\right)\frac{dJ_B^a(t)}{dt}\right]$$

- The angular momentum of a baryon

$$\rho_{J,B}^a(r) := \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} \left[\left(J_B^a(t) + \frac{2}{3} t \frac{dJ_B^a(t)}{dt}\right)\right] \qquad \rho_{J,B}(r) = \sum_{a=q,g} \rho_{J,B}^a(r)$$

$$\int d^3r \sum_{a=q,g} J_i^{a,B}(\boldsymbol{r},J_3',J_3) = 2\left(\hat{S}_i\right)_{J_3' J_3} J_B(0) = \left(\hat{S}_i\right)_{J_3' J_3},$$

H. Y. Won, HChK, and J.-Y. Kim, 2310.04670

H. Y. Won, HChK, and J.-Y. Kim, PRD **108** (2023)

Mechanical Properties

- ij component of the EMT

$$T_{ij}^{a,B}(\boldsymbol{r}, J_3', J_3) = p_B^a(r)\delta^{ij}\delta_{J_3' J_3} + s_B^a(r)\left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right)\delta_{J_3' J_3}$$

Pressure density

Shear-force density

$$p_B^a(r) = \frac{1}{6M_B}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\tilde{D}_B^a(r) - M_B\int\frac{d^3\Delta}{(2\pi)^3}e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}}\bar{c}_B^a(t),$$

$$s_B^a(r) = -\frac{1}{4M_B}r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\tilde{D}_B^a(r)$$

- D-term and cbar form factors

$$D_B^a(t) = 4M_B\int d^3r\,\frac{j_2(r\sqrt{-t})}{t}s_B^a(r),$$

$$\bar{c}_B^a(t) - \frac{t}{6M_B^2}D_B^a(t) = -\frac{1}{M_B}\int d^3r\, j_0(r\sqrt{-t})p_B^a(r)$$

H. Y. Won, HChK, and J.-Y. Kim, 2310.04670

H. Y. Won, HChK, and J.-Y. Kim, PRD **108** (2023)

Stability conditions (Equilibrium eqs)

- Conservation of the EMT

$$\sum_{a=q,g} \partial^i T_{ij}^{a,B} = \sum_{a=q,g} \frac{r_j}{r}\left[\frac{2}{3}\frac{\partial s_B^a(r)}{\partial r} + \frac{2 s_B^a(r)}{r} + \frac{\partial p_B^a(r)}{\partial r}\right] = \sum_{q=u,d,s} f_{B,j}^q + f_{B,j}^g = 0$$

- Internal force fields

M. V. Polyakov & P. Schweitzer, IJMPA33 (2018)

$$f_{B,j}^a = -M_B \frac{\partial}{\partial r^j}\int \frac{d^3\Delta}{(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} \bar{c}_B^a(t)$$

- Equilibrium Equation

$$\frac{\partial}{\partial r}\left(\frac{2}{3}s_B(r) + p_B(r)\right) + \frac{2 s_B(r)}{r} = 0$$

H. Y. Won, HChK, and J.-Y. Kim, 2310.04670

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Stability conditions (Equilibrium eqs)

- Von Laue condition

$$\int_0^\infty dr\; r^2 p_B(r) = 0$$

⇒ Pressure density must have at least one nodal point.

- Local stability condition

IA Perevalova, MV Polyako, P Schweitzer, PRD 94 (2016).

$$\frac{2}{3}s_B(r) + p_B(r) > 0$$

See also Lorce et al. EPJC 79 (2019) for other stability conditions.

- Mechanical radius

$$\langle r^2_{\rm mech}\rangle_B = \frac{\int d^3r\; r^2\left(\frac{2}{3}s_B(r)+p_B(r)\right)}{\int d^3r\;\left(\frac{2}{3}s_B(r)+p_B(r)\right)} = \frac{6D_B(0)}{\int_{-\infty}^0 D_B(t)dt}$$

H. Y. Won, HChK, and J.-Y. Kim, 2310.04670

H. Y. Won, HChK, and J.-Y. Kim, PRD **108** (2023)

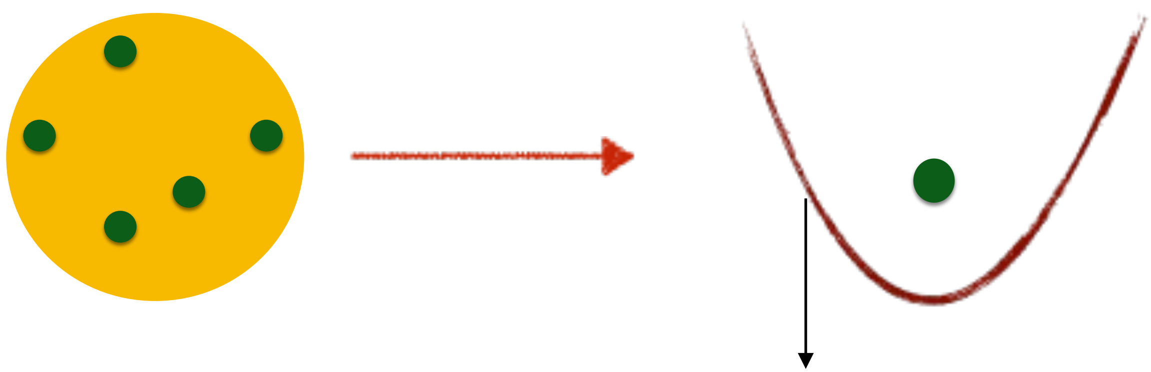
Pion Mean-field approach

Mean fields

Given action $S[\phi]$,

$$\left.\frac{\delta S}{\delta \phi}\right|_{\phi=\phi_0} = 0$$: Solution of this saddle-point equation ϕ_0

This classical solution is regarded as a mean field.



Mean-field potential that is produced by all other particles.

- Nuclear shell models
- Ginzburg-Landau theory for superconductivity
- Quark potential models for baryons

Pion mean-field approach (Chiral Quark-Soliton model)

* Baryons as a state of Nc quarks bound by mesonic mean fields.

E. Witten (1979)

Effective chiral action:

$$S_{\rm eff}[\pi^a] = -N_c {\rm Tr}\log\left(i\partial\!\!\!/ + iMU^{\gamma_5} + i\hat{m}\right)$$

D. Diakonov & V. Petrov (1986)

D. Diakonov, V. Petrov, P. Pobylitsa (1988)

* Key point: Hedgehog Ansatz

$$\pi^a(\boldsymbol{r}) = \begin{cases} n^a F(r),\ n^a = x^a/r, & a = 1,\,2,\,3 \\ 0, & a = 4,\,5,\,6,\,7,\,8. \end{cases}$$

It breaks spontaneously $\mathrm{SU}(3)_{\rm flavor} \otimes \mathrm{O}(3)_{\rm space} \to \mathrm{SU}(2)_{\rm isospin+space}$

Witten's trivial embedding

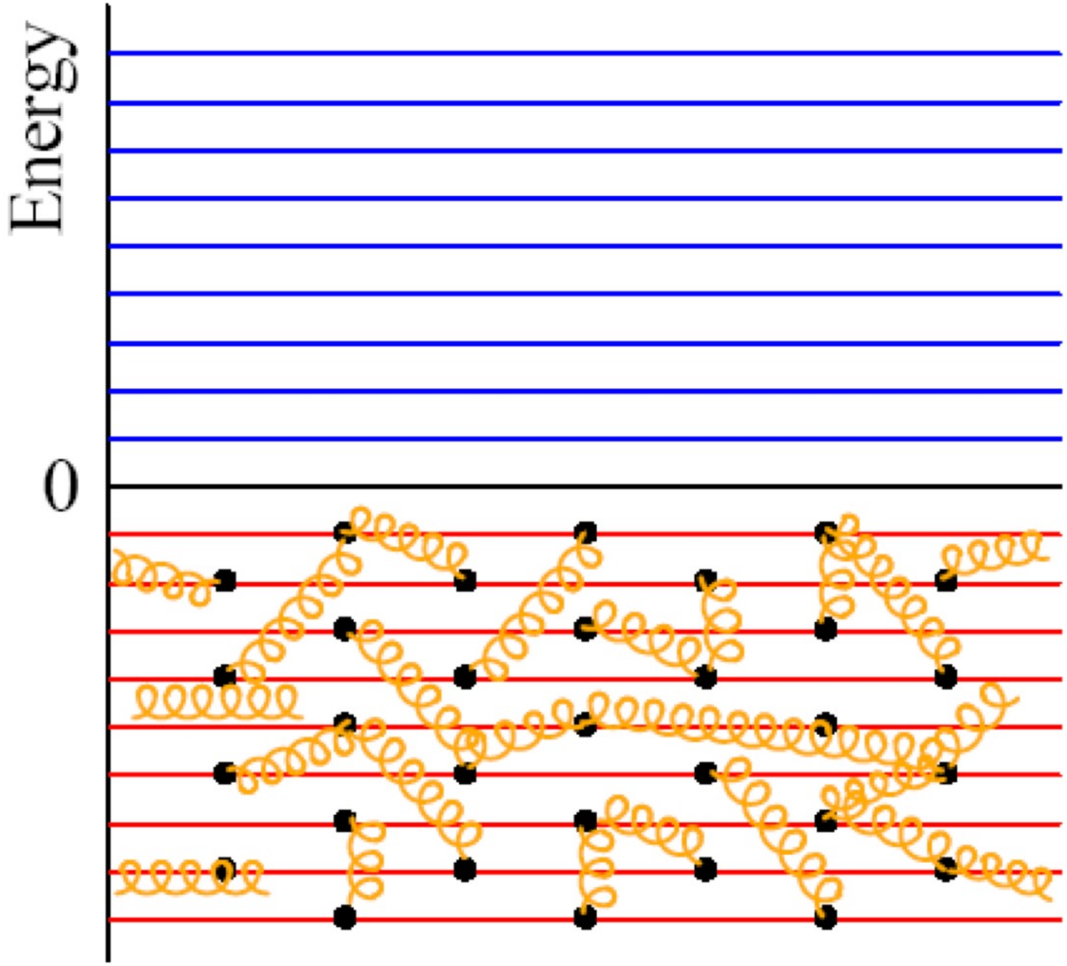
$$U_o = \begin{pmatrix} e^{i\boldsymbol{n}\cdot\boldsymbol{\tau}P(r)} & 0 \\ 0 & 1 \end{pmatrix}$$

Ch. Christov, HChK, K. Goeke et al. PPNP (1996)

D. Diakonov hep-ph/9802298

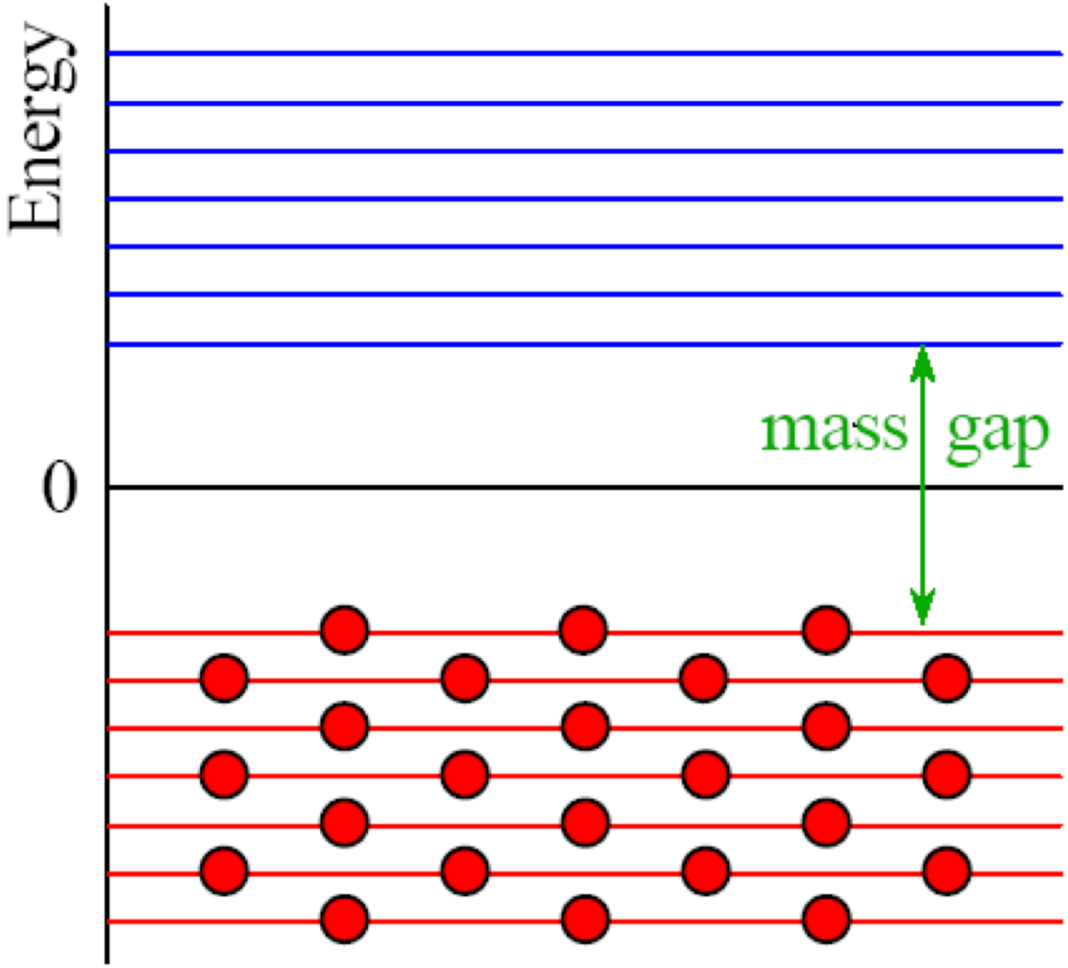
Schematic view on the XQSM

Interacting vacuum



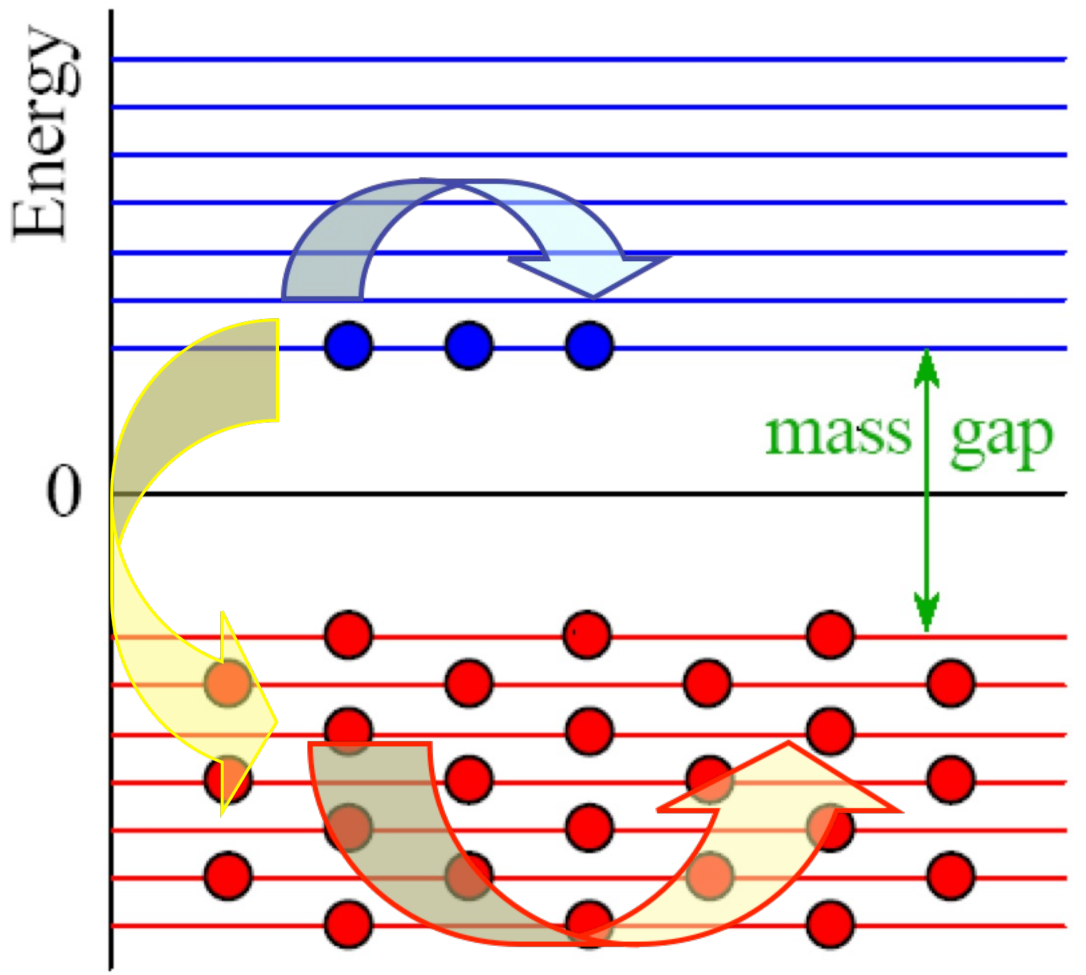
Schematic view on the XQSM

Spontaneous breakdown of chiral symmetry



Schematic view on the XQSM

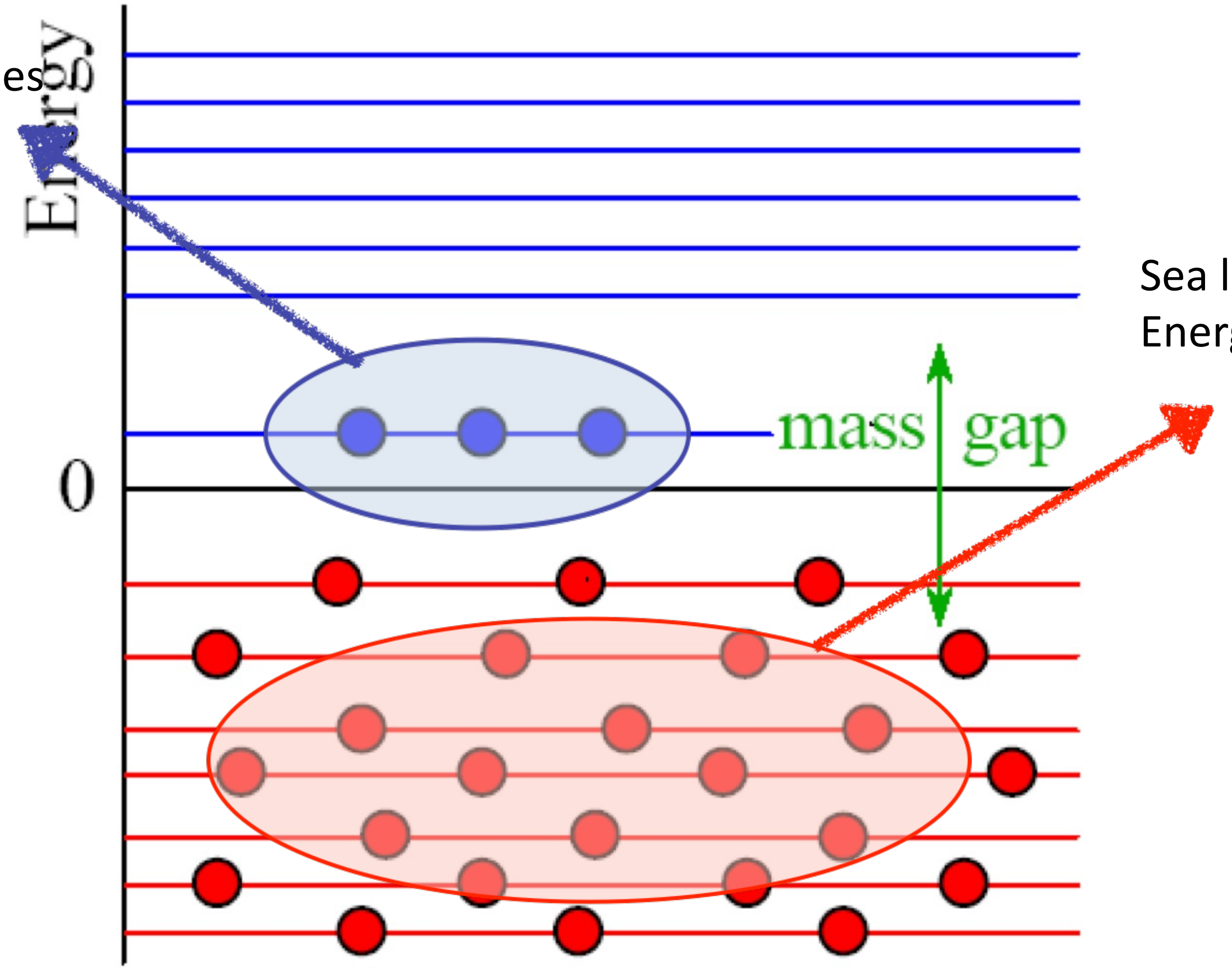
Interaction between quarks and pion background fields



Schematic view on the XQSM

Nc quarks are bounded by the pion mean fields self-consistently.

Valance level:
Energy decreases

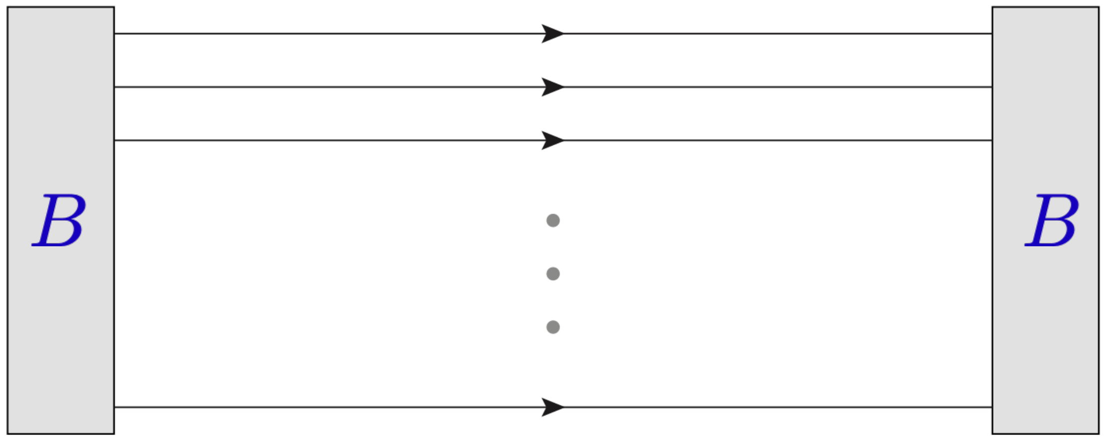


Sea level:
Energy increase

The system is stabilized!

Baryon correlation function

Baryon as Nc valence quarks bound by pion mean fields



$$\langle J_B J_B^\dagger \rangle_0 \sim e^{-N_c E_{\mathrm{val}} T}$$

$$\Pi_N(\vec{x},t) = \Gamma_N^{\{f\}}\Gamma_N^{\{g\}*}\frac{1}{Z}\int dU \prod_{i=1}^{N_c}\left\langle 0,T/2\left|\frac{1}{D(U)}\right|0,-T/2\right\rangle_{f,g} e^{-S_{\mathrm{eff}}}$$

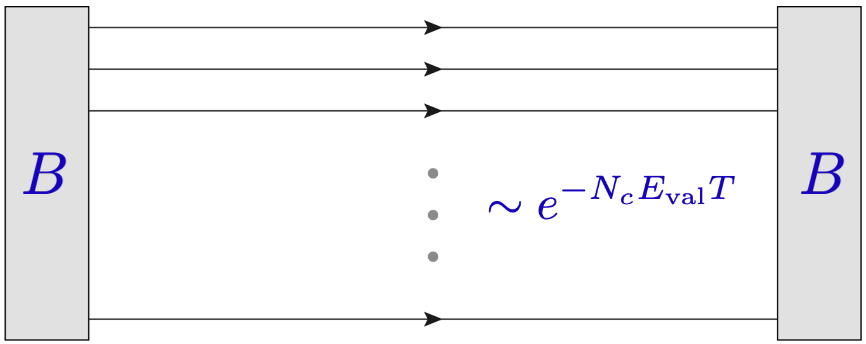
Presence of Nc quarks will polarize the vacuum or create mean fields.

Nc valence quarks → Vacuum polarization or meson mean fields

Ch. Christov, HChK, K. Goeke et al. PPNP (1996)
D. Diakonov hep-ph/9802298

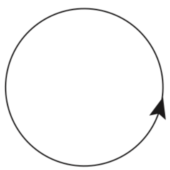
Baryon correlation function

Baryon as Nc valence quarks bound by pion mean fields



$$\sim e^{-N_c E_{\rm val} T}$$

$$E_{\rm cl} = N_c E_{\rm val} + E_{\rm sea}$$



$$\sim e^{-E_{\rm sea} T}$$

Classical Nucleon mass is described by the Nc valence-quark energy and sea-quark energy.

Ch. Christov, HChK, K. Goeke et al. PPNP (1996)

$$\frac{\delta E_{cl}}{\delta U} = 0 \longrightarrow M_{\rm cl} \longrightarrow P(r)$$

P(r): Soliton profile function
or Soliton field

Zero-mode(collective) quantization

- Rotational & Translational zero modes

$$\int \mathcal{D}U \mathcal{F}[U(\boldsymbol{x})] \to \int d^3\boldsymbol{X} \int \mathcal{D}A\, \mathcal{F}\left[TAU_{\mathrm{cl}}(R\boldsymbol{x})A^\dagger T^\dagger\right]$$

- Collective Hamiltonian & Wavefunctions in flavor SU(3) symmetry

$$H_{\mathrm{coll}} = M_{\mathrm{sol}} + \frac{1}{2I_1}\sum_{i=1}^{3}\hat{J}_i^2 + \frac{1}{2I_2}\sum_{p=4}^{7}\hat{J}_p^2$$

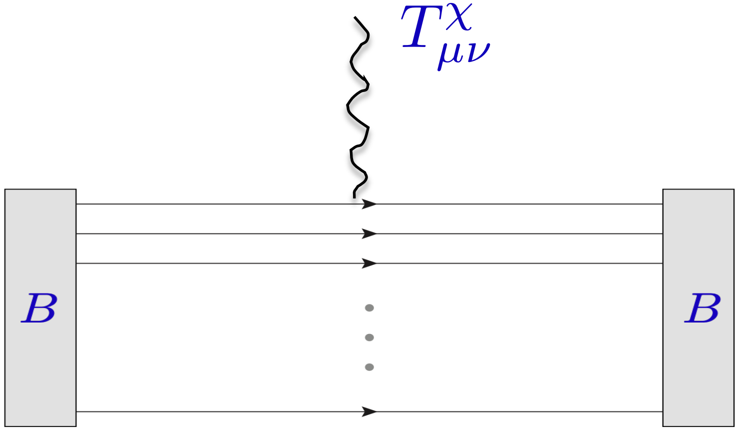
$$\Psi^{(\mu)}_{(YTT_3)(Y_R J J_3)}(A) = \sqrt{\dim(\mu)}(-1)^{J_3 - Y_R/2} D^{(\mu)*}_{(YTT_3)(Y_R J - J_3)}(A)$$

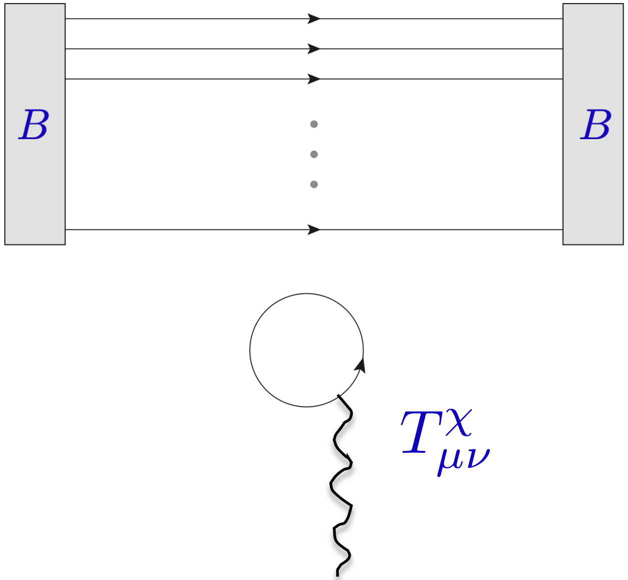
Ch. Christov, HChK, K. Goeke et al. PPNP (1996)

D. Diakonov hep-ph/9802298

GFFs from the XQSM

- Rotational & Translational zero modes





$$\langle B(p', J_3')|\hat{T}^{\mathrm{eff}}_{\mu\nu,\chi}(0)|B(p, J_3)\rangle = \lim_{T\to\infty}\frac{1}{Z_{\mathrm{eff}}}\mathcal{N}^*(p')\mathcal{N}(p)e^{ip_4\frac{T}{2}-ip_4'\frac{T}{2}}\int d^3\boldsymbol{x}\, d^3\boldsymbol{y} e^{(-i\boldsymbol{p}'\cdot\boldsymbol{y}+i\boldsymbol{p}\cdot\boldsymbol{x})}$$
$$\times\int \mathcal{D}\psi\mathcal{D}\psi^\dagger\mathcal{D}U\, J_B(\boldsymbol{y}, T/2)\hat{T}^{\mathrm{eff}}_{\mu\nu,\chi}(0)J_B^\dagger(\boldsymbol{x}, -T/2)\exp\left[-S_{\mathrm{eff}}\right]$$

For detailed results, see the refs. H. Y. Won, HChK, J.-Y. Kim 2310.04670 & PRD 108 (2023)

Results & Discussion

Mass distributions

- A form factors at t=0: $\chi = 0,\ 3,\ 8$

$$A_p^\chi(0) + \bar{c}_p^\chi(0) = \frac{1}{M_{\rm sol}} \int d^3r\, \varepsilon_p^\chi(r)$$

$$A_p^0(0) + \bar{c}_p^0(0) = 1, \qquad A_p^3(0) + \bar{c}_p^3(0) = 0.25, \quad A_p^8(0) + \bar{c}_p^8(0) = 0.47, \quad [\,\mathrm{SU}(3)\,]$$

$$A_p^0(0) + \bar{c}_p^0(0) = 1, \qquad A_p^3(0) + \bar{c}_p^3(0) = 0.24. \quad [\,\mathrm{SU}(2)\,]$$

$$\varepsilon_p^{u,d,s}(r) > 0$$

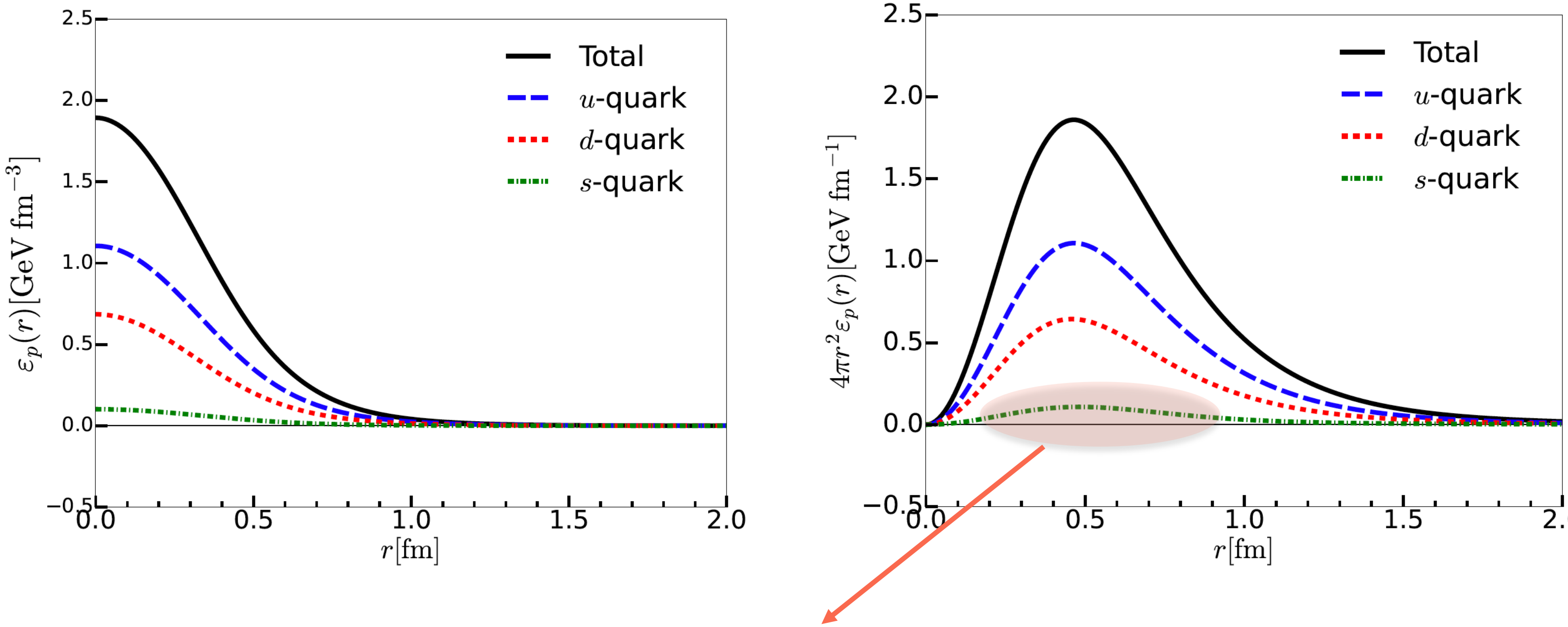
$\varepsilon_p^u(0) = 1.11\ \mathrm{GeV/fm^3},\quad \varepsilon_p^d(0) = 0.69\ \mathrm{GeV/fm^3},\quad \varepsilon_p^s(0) = 0.10\ \mathrm{GeV/fm^3}$ at the origin of the proton

$\varepsilon_p^u(0) > \varepsilon_p^d(0) > \varepsilon_p^s(0)$ The up-quark contribution is the most dominant.

$$\varepsilon_p^u(0) = \varepsilon_n^d(0),\ \varepsilon_n^u(0) = \varepsilon_p^d(0),\ \varepsilon_p^s(0) = \varepsilon_n^s(0)$$

H.W. Won, J.-Y. Kim, HChK, PRD 108 (2023)

Mass distributions



Contribution from the s quark is negligible.

H.W. Won, J.-Y. Kim, HChK, PRD 108 (2023)

LF Momentum distribution

- The second moment of the unpolarized PDF: LF momentum distribution

$$\langle x \rangle_q = \int dx\, x f_1^q(x) = A_p^q(0), \quad \int dx\, x \sum_q f_1^q(x) = 1$$

$$A_p^u(0) = 0.65, \quad A_p^d(0) = 0.34, \quad A_p^s(0) = 0.01, \quad [\,\mathrm{SU}(3)\,]$$
$$A_p^u(0) = 0.66, \quad A_p^d(0) = 0.34, \quad [\,\mathrm{SU}(2)\,]$$

- Mass distribution from the GFFs

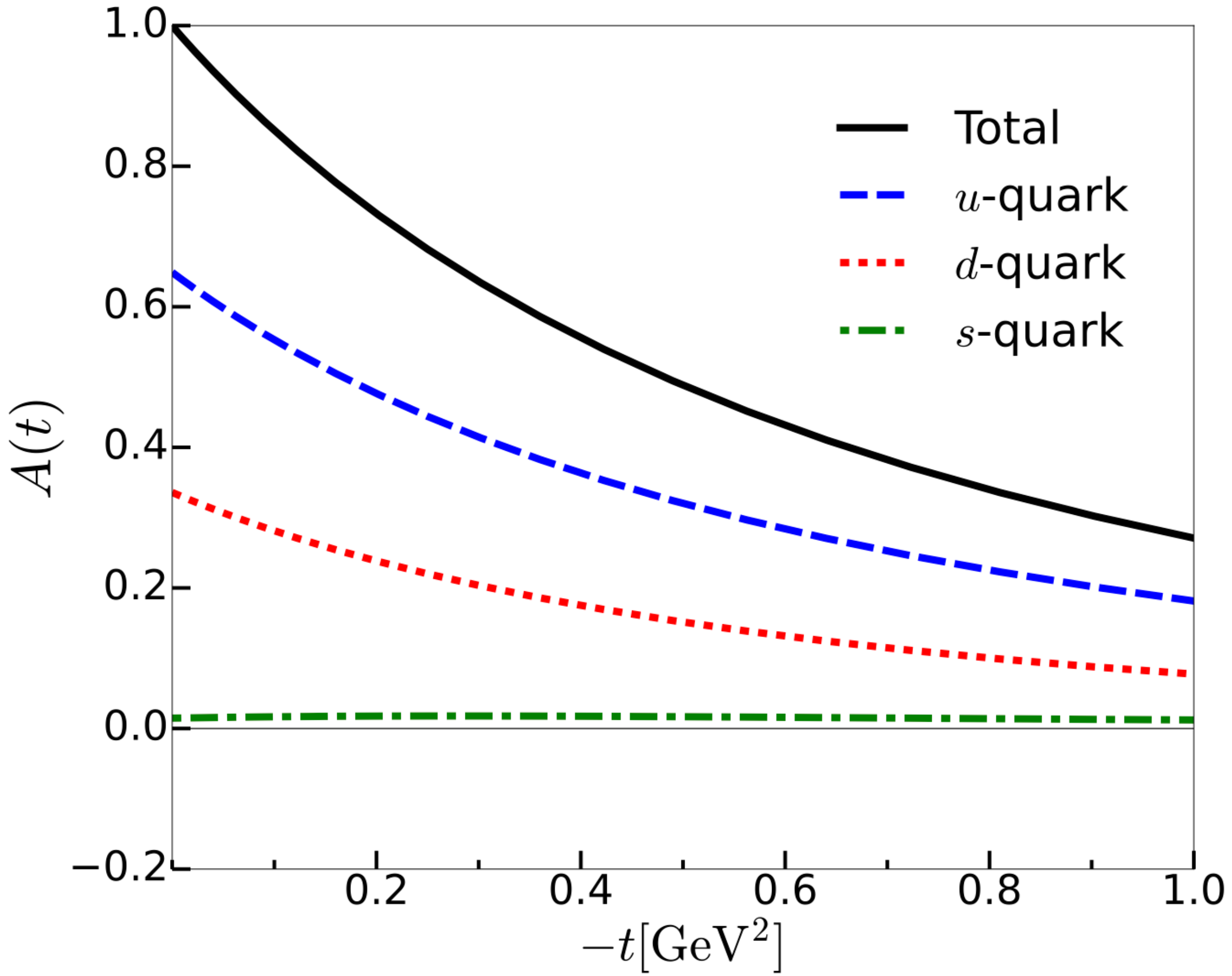
$$A_p^u(0) + \bar{c}_p^u = 0.59, \quad A_p^d(0) + \bar{c}_p^d = 0.35, \quad A_p^s(0) + \bar{c}_p^s = 0.06, \quad [\,\mathrm{SU}(3)\,]$$
$$A_p^u(0) + \bar{c}_p^u = 0.62, \quad A_p^d(0) + \bar{c}_p^d = 0.38, \quad [\,\mathrm{SU}(2)\,]$$

$$M_p^u/M_p \ \ [59.5\%] < \langle x \rangle_u \ \ [64.9\%],$$
$$M_p^d/M_p \ \ [34.5\%] > \langle x \rangle_d \ \ [33.6\%],$$
$$M_p^s/M_p \ \ [6.0\%] > \langle x \rangle_s \ \ [1.5\%]$$

$$\bar{c}_p^a(0) > 0 \quad \rightarrow \quad M_p^a/M_p > \langle x \rangle_a,$$
$$\bar{c}_p^a(0) < 0 \quad \rightarrow \quad M_p^a/M_p < \langle x \rangle_a$$

H.W. Won, J.-Y. Kim, HChK, PRD 108 (2023)

Flavor-decomposed A form factors



$\langle r^2_{\mathrm{mass}} \rangle_p = 0.54\ \mathrm{fm}^2$

H.W. Won, J.-Y. Kim, HChK, PRD 108 (2023)

Angular momentum distribution

$$J_p^0(0) = \int d^3r\, \rho^0_{J,p}(r) = \frac{1}{2}$$

$$J_p^0 = 0.50, \quad J_p^3 = 0.58, \quad J_p^8 = 0.22, \quad [\,\mathrm{SU(3)}\,].$$
$$J_p^0 = 0.50, \quad J_p^3 = 0.55, \quad [\,\mathrm{SU(2)}\,]$$

Strange quark contribution is negligible.

$$J_p^u = 0.52, \quad J_p^d = -0.06, \quad J_p^s = 0.04, \quad [\,\mathrm{SU(3)}\,].$$
$$J_p^u = 0.53, \quad J_p^d = -0.03, \quad [\,\mathrm{SU(2)}\,]$$

$$J = \frac{1}{2}\sum_q \Delta q + \sum_q L^q \quad \Rightarrow \quad \frac{1}{2} = \frac{1}{2}\sum_q \Delta q + \sum_q L^q = 0.23 + 0.27$$

H.W. Won, J.-Y. Kim, HChK, PRD 108 (2023)

Problem of the naïve decomposition

- Decomposition of the isotriplet J

$$J_p^{u-d} = L_p^{u-d} + S_p^{u-d} + \boxed{\delta J_p^{u-d}}$$

M. Wakamatsu & H. Nakakoji, PRD 71 (2005)

Violation of Ji's sum rule (X.D. Ji, PRL 78 (1997))

- Origin of δJ_p^{u-d} : role of gluons

- The second moment of the chiral-odd twist-3 quark distribution

$$\int_{-1}^{1} dx\, x\, e^{u+d}(x) = \frac{m}{M_N} N_c + \boxed{\frac{M}{M_N}\beta}$$

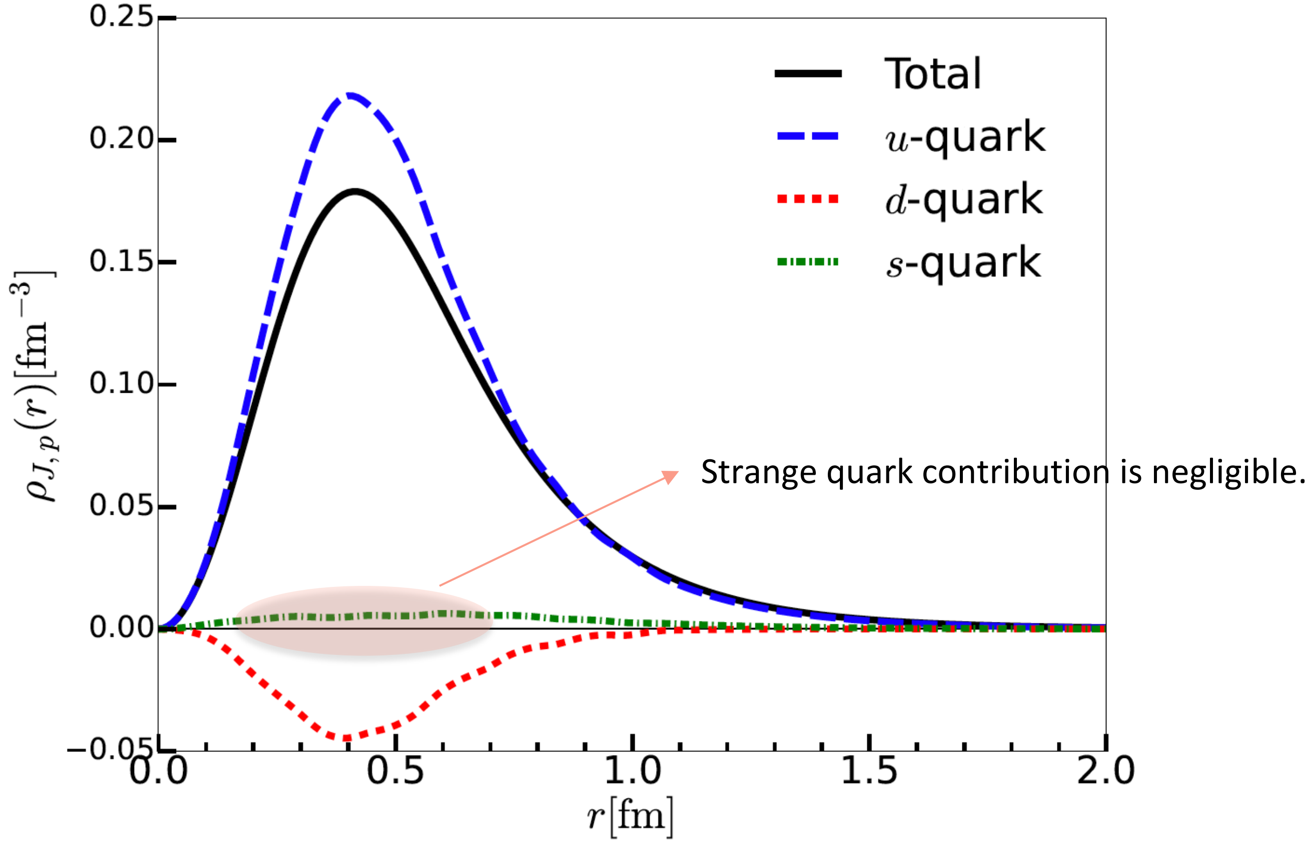
P. Schweitzer, PRD 67 (2005)
Ohnishi & M. Wakamatsu, PRD 69 (2004)

This makes the second moment deviate from QCD.

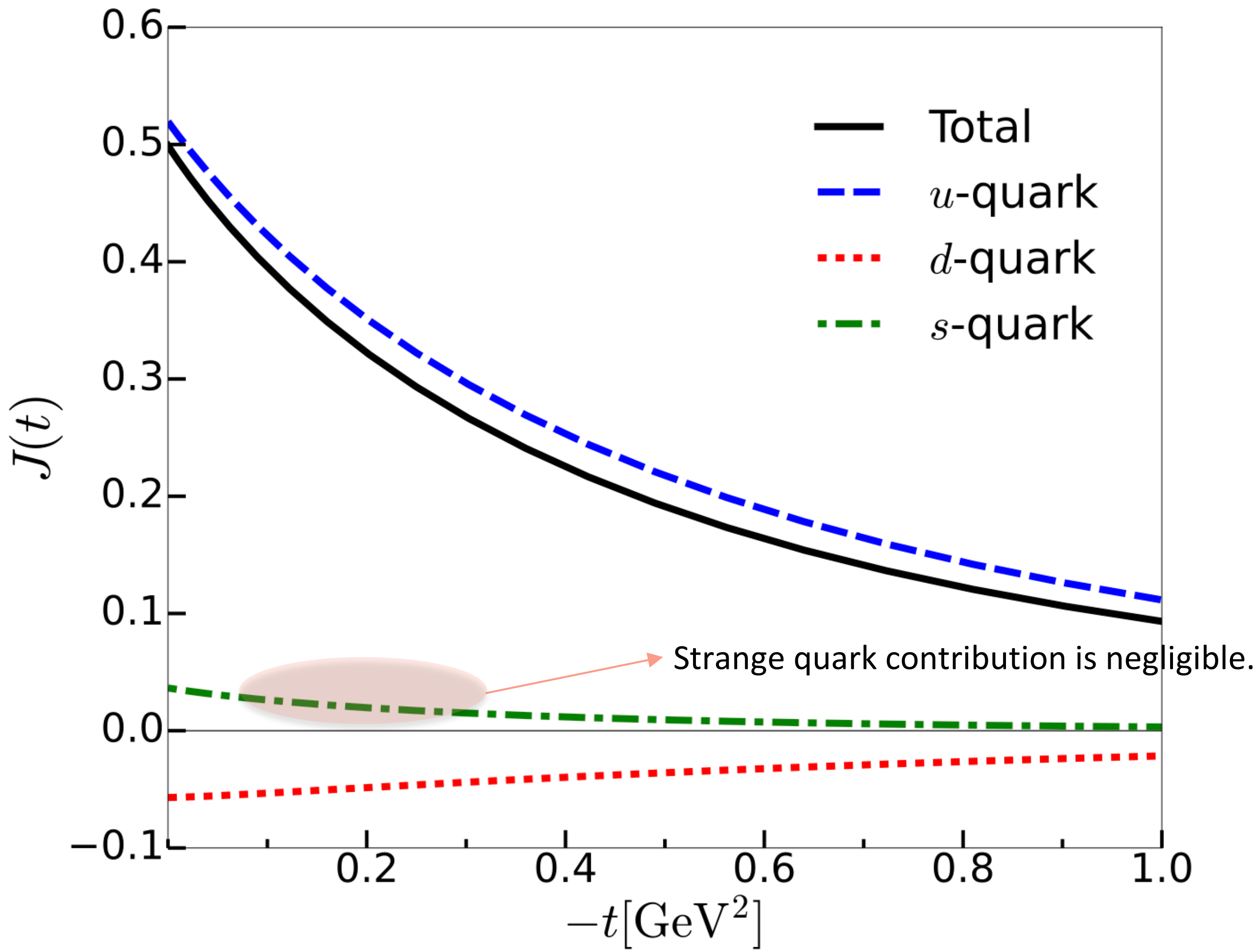
➡ Indication: If the covariant derivatives had been used, these discrepancies would have been resolved.

H.W. Won, J.-Y. Kim, HChK, PRD 108 (2023)

Angular momentum distribution



Flavor-decomposed J form factors



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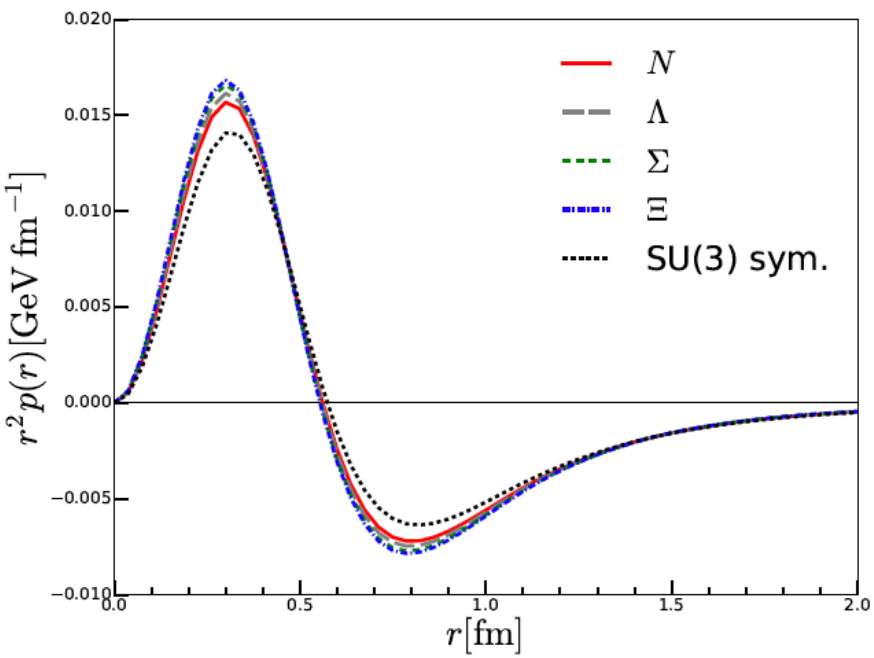
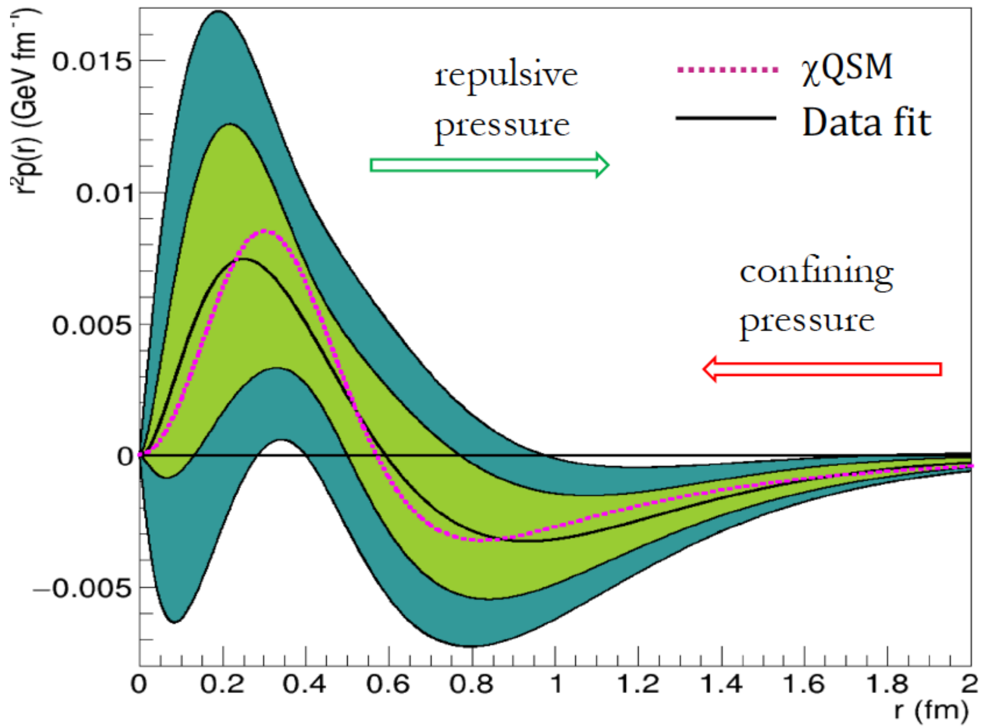
The 3D BF pressure density



H.W. Won, J.-Y. Kim, HChK, PRD 106 (2022)

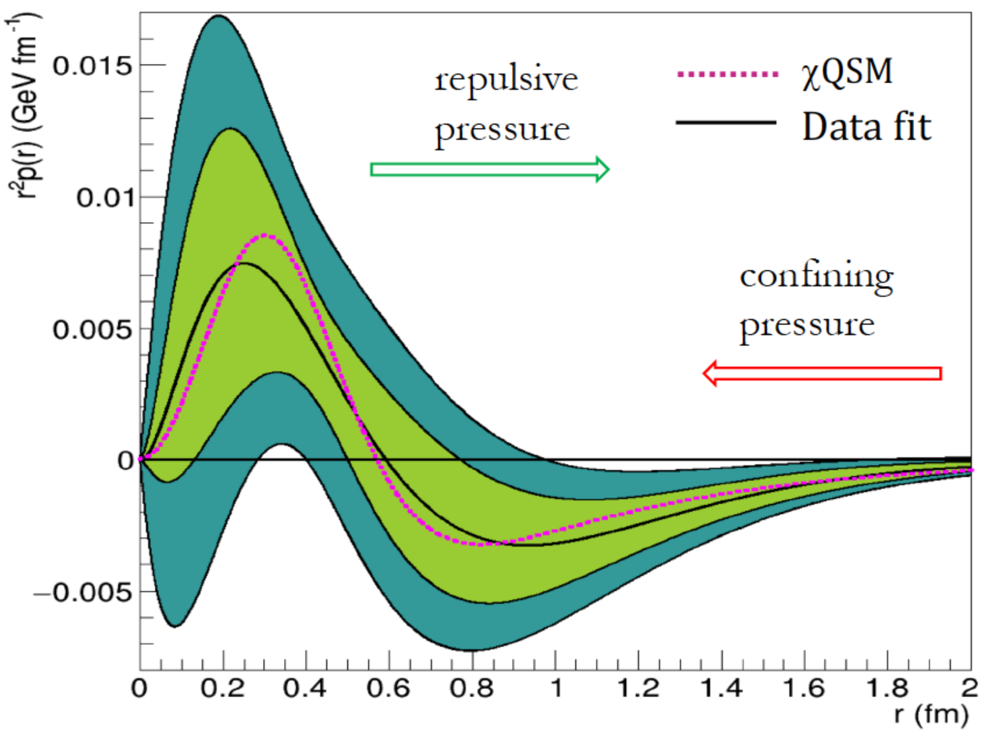


V.B., L. Elouadrhiri, F.X. Girod, Nature 557 (2018) 7705, 396

K. Goeke et al., PRD 75 (2007)

It can possibly be more precisely extracted at EIC.

The 3D BF pressure density



V.B., L. Elouadrhiri, F.X. Girod, Nature 557 (2018) 7705, 396

Burkert et al. assumed the flavor blindness.

$$D^{u-d}(0) \approx 0$$

$D^{u-d}(0) = 0.29$ in SU(2)

$D^{u-d}(0) = 0.062$ in SU(3)

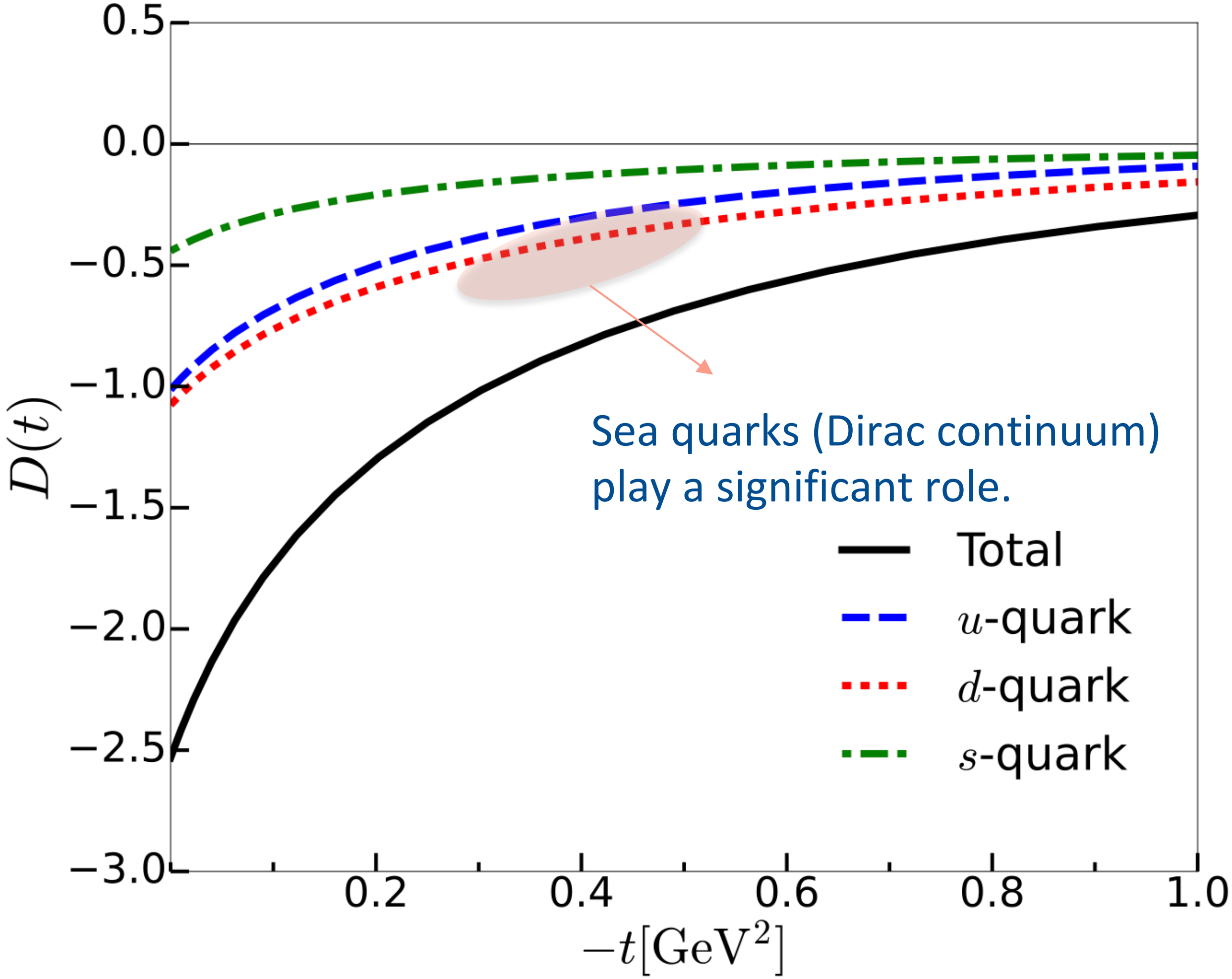
The flavor blindness is only valid in SU(3)!

The strange quarks should essentially be considered in the proton!

Lattice QCD arrives at a similar conclusion.
(D. Hackett et al. 2310.08484)

H.W. Won, J.-Y. Kim, HChK, PRD 108 (2023)

Flavor-decomposed D-term form factors



Similar situation in the EM transitions of the delta isobar

$$\sqrt{\langle r^2 \rangle_{\mathrm{mech}}} = 0.73\,\mathrm{fm}$$

The strange-quark contributions are essential for flavor blindness!

H.W. Won, J.-Y. Kim, HChK, PRD 108 (2023)

Radii

$$\langle r^2\rangle_{\rm mech} = \frac{\int d^3r\ r^2\ \left[\frac{2}{3}s(r)+p(r)\right]}{\int d^3r\ \left[\frac{2}{3}s(r)+p(r)\right]} = \frac{6D}{\int_{-\infty}^{0} dt\ D(t)}$$

$$\sqrt{\langle r^2\rangle_{\rm mech}} = 0.69\,{\rm fm}$$

$$\sqrt{\langle r^2\rangle_{\rm mech}} = (0.63 \pm 0.06 \pm 0.13)\,{\rm fm}$$

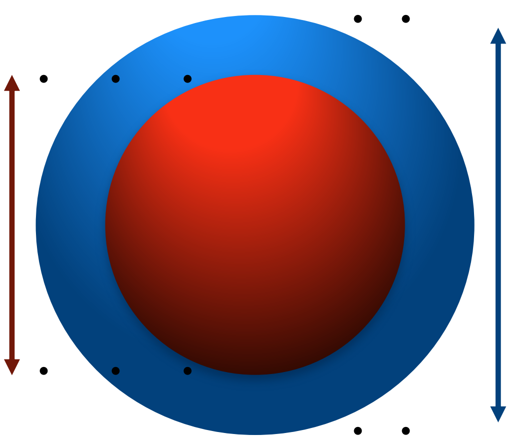
V. Burkert et al. (2022)

$$\sqrt{\langle r^2\rangle_{\rm mech}} = 0.73\,{\rm fm}$$ in SU(3)

$$\sqrt{\langle r^2\rangle_{\rm mech}} < \sqrt{\langle r^2\rangle_{\rm ch}}$$

Mechanical radius

$\sim 0.6\,{\rm fm}$

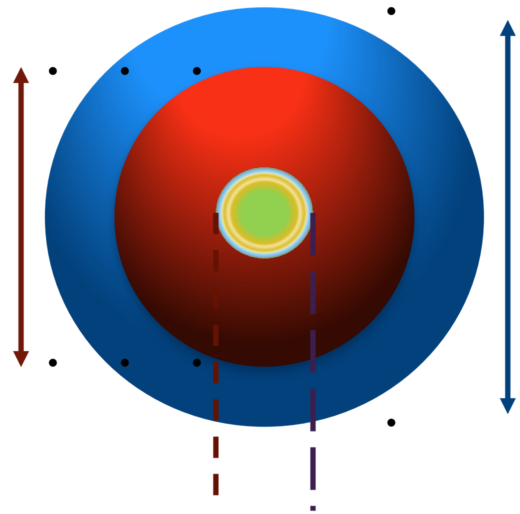


Charge radius

$\sim 0.8\,{\rm fm}$

Radii

Mechanical radius $\sim 0.6\,\mathrm{fm}$

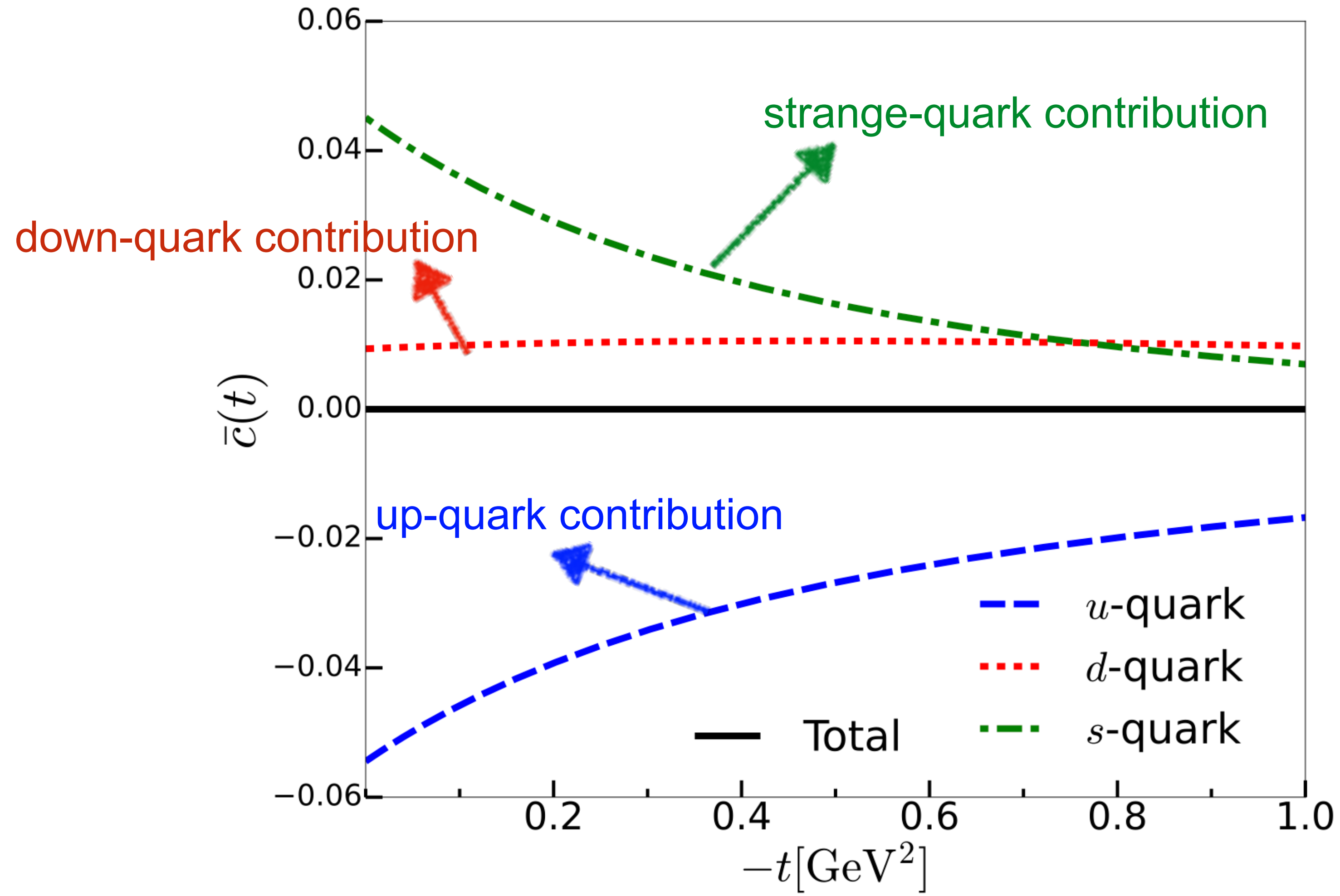


Charge radius $\sim 0.8\,\mathrm{fm}$

Mass radius $\sim 0.5\,\mathrm{fm}$

$$\sqrt{\langle r^2\rangle_{\mathrm{mass}}} < \sqrt{\langle r^2\rangle_{\mathrm{mech}}} < \sqrt{\langle r^2\rangle_{\mathrm{ch}}}$$

Flavor-decomposed cbar form factors



The down & strange-quark contributions exactly cancel out the up-quark contribution!

H.W. Won, J.-Y. Kim, HChK, 2310.04670 [hep-ph] (2023)

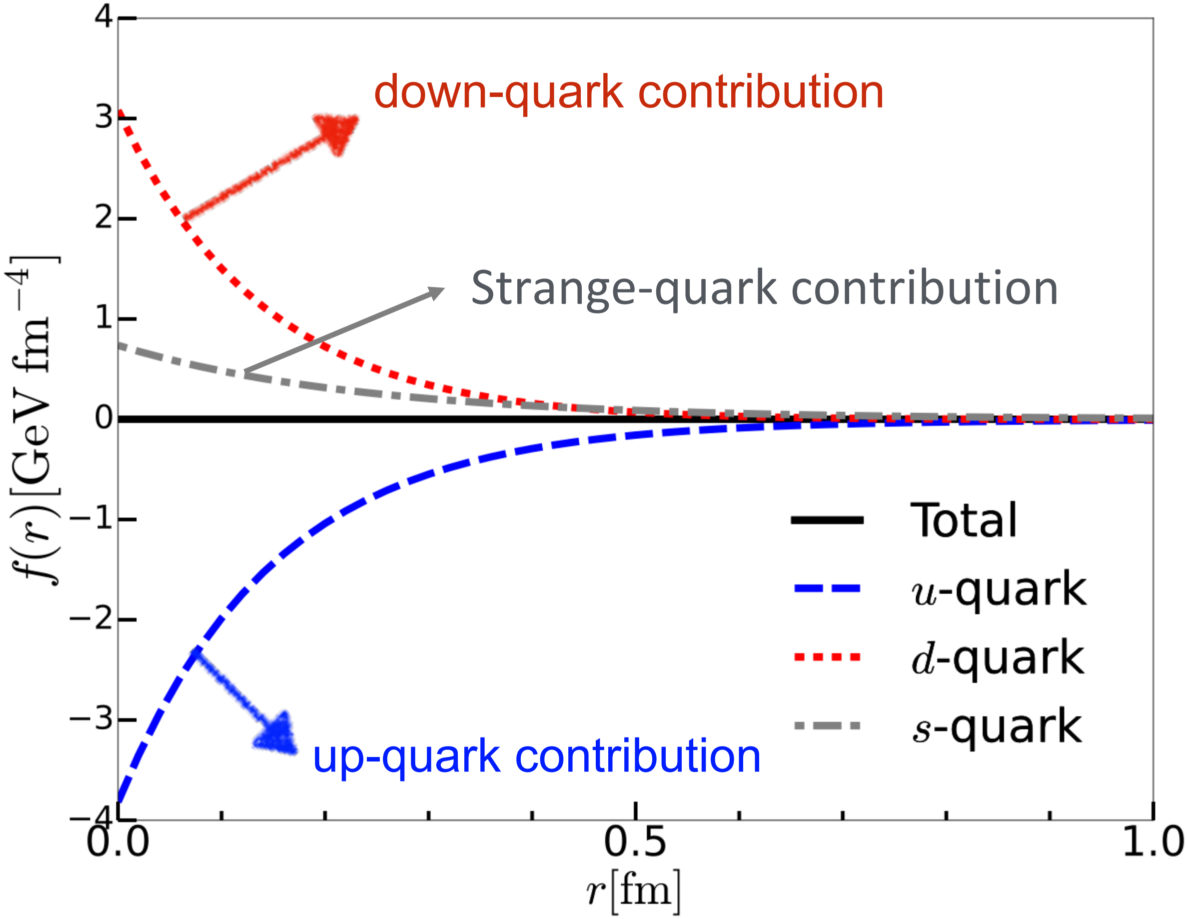
Summary of the results

In the forward limit $t = 0$

B	$A_B^u(0)$	$A_B^d(0)$	$A_B^s(0)$	$J_B^u(0)$	$J_B^d(0)$	$J_B^s(0)$	$D_B^u(0)$	$D_B^d(0)$	$D_B^s(0)$	$\bar{c}_B^u(0)$	$\bar{c}_B^d(0)$	$\bar{c}_B^s(0)$
p	0.649	0.336	0.015	0.520	−0.057	0.036	−1.014	−1.076	−0.441	−0.054	0.009	0.045
n	0.336	0.649	0.015	−0.057	0.520	0.036	−1.076	−1.014	−0.441	0.009	−0.054	0.045
Λ	0.335	0.335	0.331	0.055	0.055	0.390	−0.960	−0.960	−0.611	0.005	0.005	−0.009
Σ^+	0.649	0.015	0.336	0.520	0.036	−0.057	−1.014	−0.441	−1.076	−0.054	0.045	0.009
Σ^0	0.332	0.332	0.336	0.278	0.278	−0.057	−0.727	−0.727	−1.076	−0.005	−0.005	0.009
Σ^-	0.015	0.649	0.336	0.036	0.520	−0.057	−0.441	−1.014	−1.076	0.045	−0.054	0.009
Ξ^0	0.336	0.015	0.649	−0.057	−0.036	0.552	−1.076	−0.441	−1.014	0.009	0.045	−0.054
Ξ^-	0.015	0.336	0.649	−0.036	−0.057	0.520	−0.441	−1.076	−1.014	0.045	0.009	−0.054

Flavor-decomposed cbar form factors

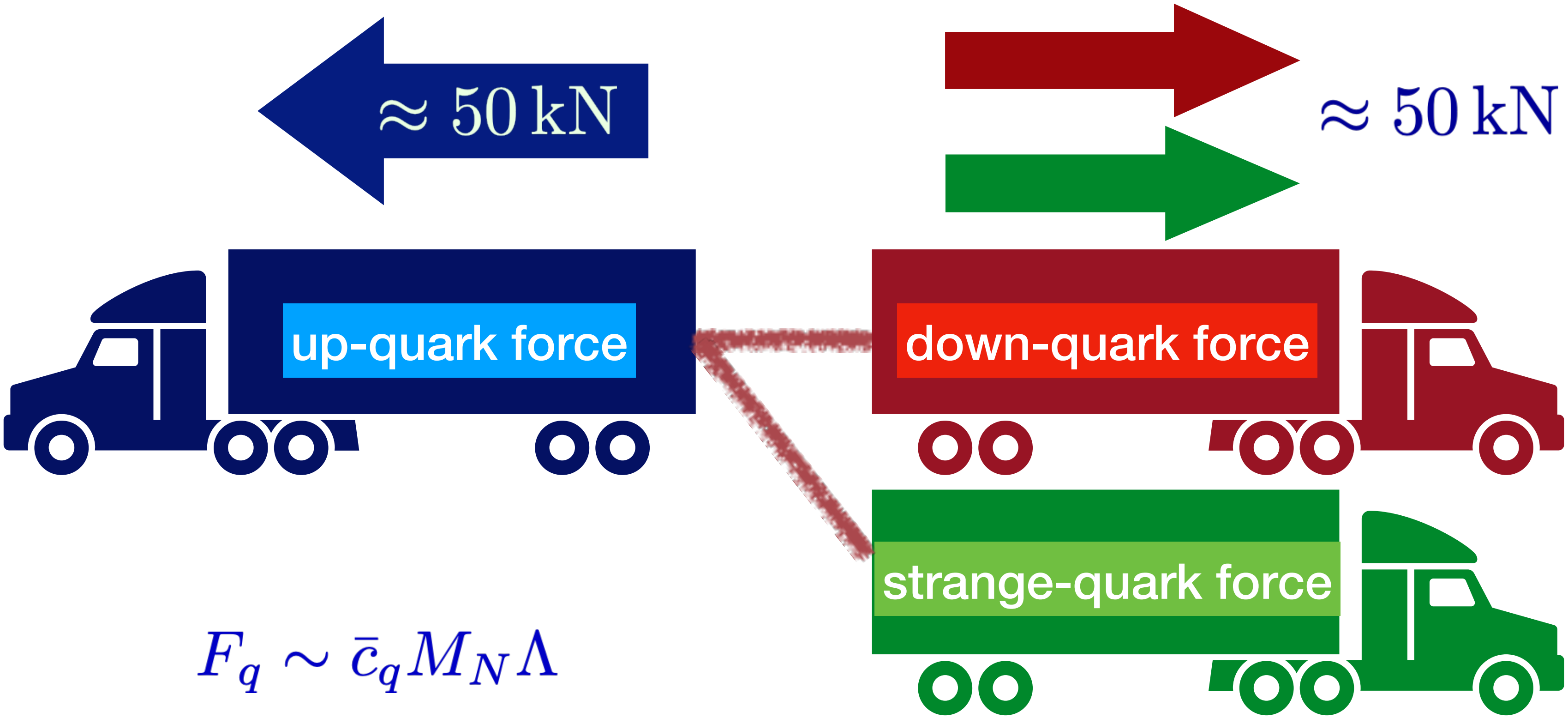
- Force field densities inside the nucleon: $f_j^q = -M_N \frac{\partial}{\partial r^j} \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} \bar{c}^q(t)$



H.W. Won, J.-Y. Kim, HChK, 2310.04670 [hep-ph] (2023)

Cancelation of the force fields from cbar

The up-quark contribution is balanced with the down & strange-quark contributions.



Conclusions & Outlook

Lessons we have learned

1. The mass distributions are different from the momentum distribution by the cbar densities.
2. Decomposition of the flavor triplet J should be carefully considered with the spin-orbit correlation.
3. The flavor blindness is only valid in SU(3).
4. The charge radius is the largest, the mechanical radius is the next, and the mass radius is the smallest.

Gluonic observables of the Nucleon?

Y.W. Choi, HChK, in preparation (2024)

- Nucleon from the instanton vacuum

Strong QCD → Low-Energy effective QCD

Instanton Vacuum — Partition function

➤ Momentum-dependent Dynamical quark mass
➤ "Effective Gluon operators"

- Merits
  - ✓ No free parameters except for $\Lambda_{\rm QCD}$
  - ✓ It is possible to deal with gluons
  - ✓ Quantum field-theoretic method with natural normalization point
  - ✓ Light & singly heavy baryons on an equal footing

→ Reliable and proper theory for hadrons in EIC era

Gluonic observables of the Nucleon?

J. Y. Kim & Ch. Weiss PLB 848 (2024)

Ch. Weiss' talk on Friday

- Spin-Orbit Correlation

Twist-three operators

$$O^{\alpha\beta}(x) \equiv \frac{1}{2}\bar{\psi}(x)\gamma^{[\alpha} i \overleftrightarrow{\nabla}^{\beta]} \tau \psi(x),$$

$$O_5^{\alpha\beta}(x) \equiv \frac{1}{2}\bar{\psi}(x)\gamma^{[\alpha}\gamma_5 \, i \overleftrightarrow{\nabla}^{\beta]} \tau \psi(x)$$

through the instanton vacuum

$$O^{\alpha\beta}(x) = \bar{\psi}(x)\left\{\frac{1}{2}\gamma^{[\alpha} i \overleftrightarrow{\partial}^{\beta]}\tau + \frac{iM}{4}\sigma^{\alpha\beta}[\tau, U^{\gamma_5}(x)]\right\}\psi(x)$$

$$O_5^{\alpha\beta}(x) = \bar{\psi}(x)\left(\frac{1}{2}\gamma^{[\alpha}\gamma_5 \, i \overleftrightarrow{\partial}^{\beta]}\tau - \frac{iM}{4}\sigma^{\alpha\beta}\gamma_5\{\tau, U^{\gamma_5}(x)\}\right)\psi(x)$$

The results will soon come out.

H.Y. Won, J. Y. Kim, HChK, Ch. Weiss, in preparation (2024)

Though this be madness, yet there is method in it.

Hamlet Act 2, Scene 2

by Shakespeare

Many thanks to J-Y. Kim and H-Y. Won.

Thank you very much for the attention!

Backup slide