Renormalons and power corrections to pseudo- and quasi-GPDs

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- Pseudo- and quasi-GPDs
- Renormalon ambiguities and power corrections
- The bubble sum in Borel space
- The $w = 1$ (i.e. leading) renormalon
- Renormalon ambiguity for the quasi-GPD at $x = \xi$
- Summary

- Can calculate hadronic matrix elements of bilocal operators with space-like separation on the lattice.

- E.g. the (quasi-) generalized Ioffe-time distribution

$$\mathcal{I}(\tau,\widetilde{\xi},z^2) = \frac{1}{2P^0}\langle p'|\bar{q}\left(\frac{z}{2}v\right)\gamma^0\left[\frac{z}{2}v,-\frac{z}{2}v\right]q\left(-\frac{z}{2}v\right)|p\rangle,$$

  where we can take $v=(0,0,0,1)$.

- Relevant parameters:
  - average target momentum $P^\mu = \frac{p^\mu + p'^\mu}{2}$
  - (quasi-)skewness $\widetilde{\xi} = \frac{p^z - p'^z}{p^z + p'^z}$
  - (quasi-)Ioffe-time $\tau = zP^z$

- Generalized pseudo distribution

$$\mathcal{P}(x,\widetilde{\xi},z^2) = \int_{-\infty}^{\infty} \frac{d\tau}{2\pi} \mathcal{I}(\tau,\widetilde{\xi},z^2)$$

- Generalized quasi distribution

$$\mathcal{Q}(x,\widetilde{\xi},P^z) = P^z \int_{-\infty}^{\infty} \frac{dz}{2\pi} \mathcal{I}(zP^z,\widetilde{\xi},z^2)$$

- Both reduce to the generalized parton distribution at leading order in $|z|, \frac{1}{P^z}, \alpha_s$.

$$\mathcal{P}(x,\widetilde{\xi},z^2), \mathcal{Q}(x,\widetilde{\xi},P^z) \overset{|z|,\frac{1}{P^z},\alpha_s \to 0}{\longrightarrow} H(x,\xi),$$

where $\xi = \widetilde{\xi}$ at leading order in $\frac{1}{P^z}$.

- All-order factorization theorem (renormalization and factorization scales tacitly implied)

$$\mathcal{I}(\tau, \xi, z^2) = \int_{-1}^{1} dx\, \widetilde{T}(x, \tau, \xi, z^2) H(x, \xi) + O(\lambda)$$
$$= \int_{0}^{1} du\, T(u, \tau, \xi, z^2) I(u\tau, \xi) + O(\lambda)$$

where $\lambda \sim |z|\Lambda \sim \frac{\Lambda}{P^z}$, where $\Lambda \sim \sqrt{-t} \sim \Lambda_{\mathrm{QCD}}$ and $I(\tau, \xi)$ is the Fourier transform ($x \leftrightarrow \tau$) of the GPD.

- Leading order $T = \delta(1-u) + O(\alpha_s)$, $\widetilde{T} = e^{-i\tau x} + O(\alpha_s)$.

- All-order factorization for $\mathcal{P}$ and $\mathcal{Q}$ in terms of H follows immediately

- Generally perturbative series for observables $R = \sum_n r_n \alpha^n$ are asymptotic with zero radius of converge in coupling α
- The best possible approximation to R is obtained by truncating the series at the smallest term, with error $e^{-c/\alpha}$
- Can use Borel summation

$$B[R](w) = \sum_n r_n \frac{w^n}{n!}, \qquad R(\alpha) \text{``} = \text{''} \int_0^\infty dw\, e^{-w/\alpha} B[R](w).$$

Poles of $B[R]$ are called **renormalons**(*)

- Ambiguity of perturbative series as residue (times $e^{-\frac{w}{\alpha_s}}$) at possible pole $w_0 \in \mathbb{R}_{>0}$ of $B[R]$, $\delta_{w_0} R \propto \mathrm{Res}_{w_0} B[R]$. Roughly

$$\delta_{w_0} R \sim e^{-\frac{p}{\beta_0 \alpha_s(Q)}} \sim \left(\frac{\Lambda_{\mathrm{QCD}}^2}{Q^2}\right)^p$$

- In general for factorized expressions

$$R(Q,\Lambda) = C(Q,\mu) \otimes \langle O\rangle(\mu,\Lambda) + O(\Lambda/Q)$$

  the IR renormalon ambiguities in C get cancelled by power-suppressed (UV) contributions in $O(\Lambda_{\mathrm{QCD}}/Q)$

- "UV dominance": An amgibuity term of the form

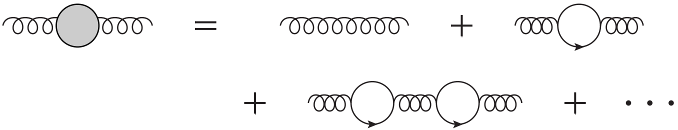
$$\delta_{w_0} C(Q,\mu) \otimes \langle O\rangle(\mu,\Lambda) \sim (\Lambda_{\mathrm{QCD}}/Q)^p$$

  is assumed to reflect the functional form of a higher-power $\sim (\Lambda_{\mathrm{QCD}}/Q)^p$ contribution.

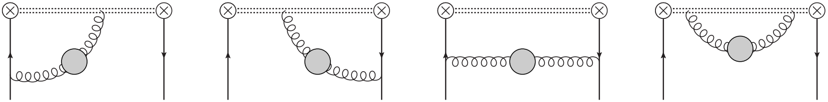
- In the same way renormalon ambiguities in T should be cancelled by $O(\Lambda_{\mathrm{QCD}}^2/(P^z)^2)$

The bubble sum in Borel space

- To investigate renormalons in QCD, we can consider a unique subset of Feynman diagrams to all orders, say the highest power of β_0 contribution at each order in α_s



- In the case of T we have the diagrams

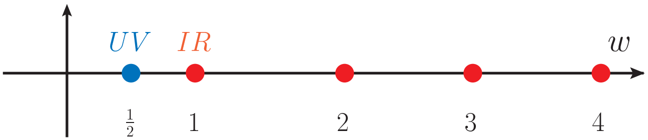


(a) (b) (c) (d)

- Result for the Borel transform of the bubble sum of T

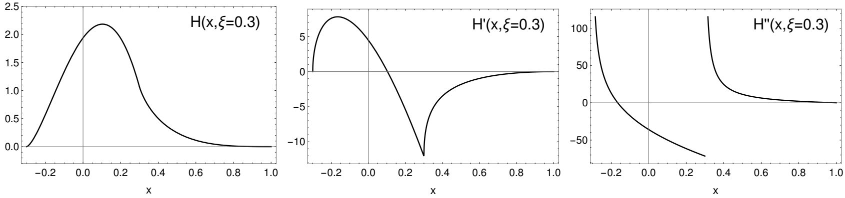
$$B[T(u,\tau,\xi,z^2)](w) = 2C_F e^{5w/3}(z^2\Lambda^2_{\mathrm{QCD}}/4)^w \frac{\Gamma(-w)}{\Gamma(w+1)} \times \left\{ \frac{2}{1+w}\left[u^{1+w}\,{}_2F_1(1,2-w,2+w,u)\right]_+ \cos((1-u)\xi\tau) - \frac{\delta(1-u)}{1+w} - \frac{\delta(1-u)}{1-2w} + (1+w)u^w \frac{\sin((1-u)\xi\tau)}{\tau\xi} \right\},$$

- Get renormalon pole structure



- Additional singularities in the Borel plane might be generated by the Fourier transform. This happens only for the qGPD at $x = \pm\xi$.

- Use simple flavor non-singlet quark GPD model from [Belitsky, Radyushkin, 05]



- Behaviour near the singularities at $x \to \pm\xi$

$$H'(x,\xi) \sim \begin{cases} x \to \xi^+ : & H'(\xi,\xi) + a_1 (x-\xi)^{1/2} + \dots \\ x \to \xi^- : & H'(\xi,\xi) + a_2 (\xi - x)^1 + \dots \\ x \to (-\xi)^+ : & a_3 (x+\xi)^{1/2} \end{cases}$$

- Consequence of the non-analytic behaviour: Fourier transform scales like $I(\tau,\xi) \sim c\,|\tau|^{-5/2}$ as $|\tau| \to \infty$.

- Focus on the $w = 1$ renormalon

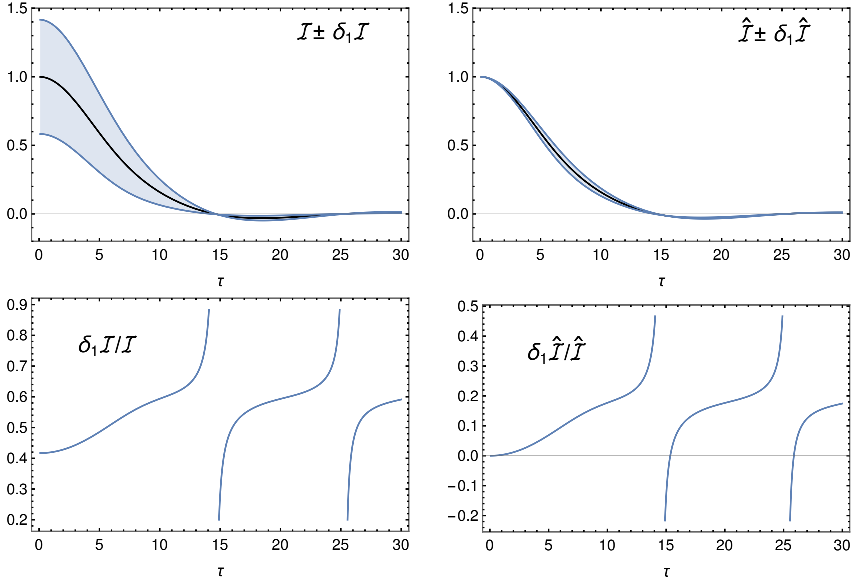
$$\delta_1 T(u, \tau, \xi, z^2) = \kappa z^2 \Lambda^2_{\mathrm{QCD}} \Big\{ (u + (1-u)\log(1-u)) \cos((1-u)\tau\xi) + \frac{u}{\xi\tau} \sin((1-u)\tau\xi) \Big\},$$

where $\kappa = O(1)$ is some constant.

- UV renormalization by zero-momentum subtraction $\widehat{\mathcal{I}}(\tau, \xi, z^2) = \frac{\mathcal{I}(\tau,\xi,z^2)}{\mathcal{I}(0,0,z^2)}$. Effectively adds overall $[...]_+$ (with subtraction at $u = 1$) to $\delta_1 T \Rightarrow$ Simply a shift by $-\frac{5}{12} I(\tau, \xi)$

- $\widehat{\mathcal{P}}$ and $\widehat{\mathcal{Q}}$ are respective Fourier transforms of $\widehat{\mathcal{I}}$.

Take overall prefactor $\kappa z^2 \Lambda_{\rm QCD}^2 = 1$ (it should be much smaller), $\xi = 0.3$



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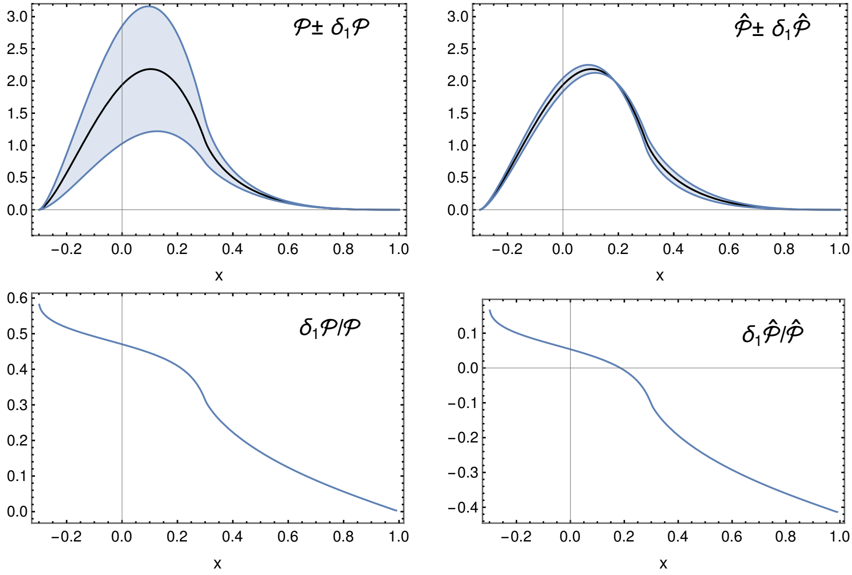
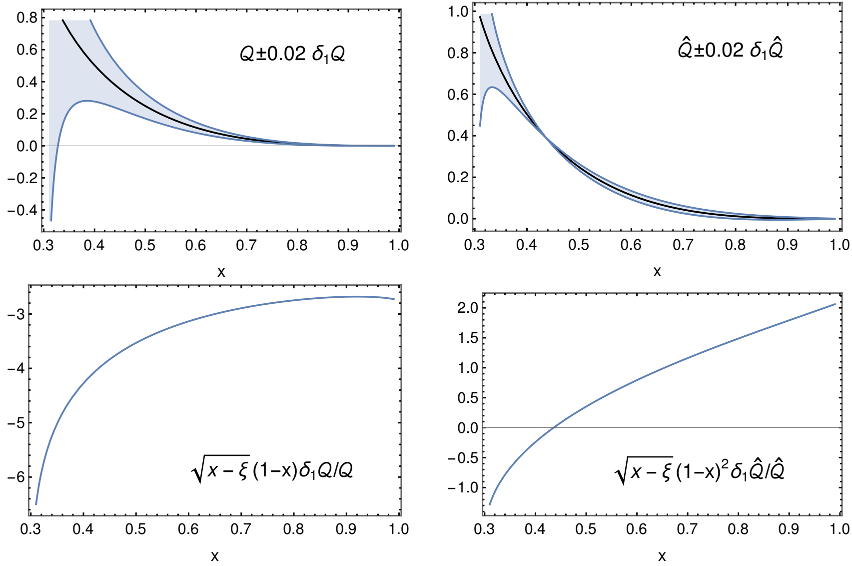


$\kappa\Lambda_{\mathrm{QCD}}^2/(P^z)^2 = 0.02$, corresponding to roughly $P^z \sim 1.5\,\mathrm{GeV}$, $\xi = 0.3$



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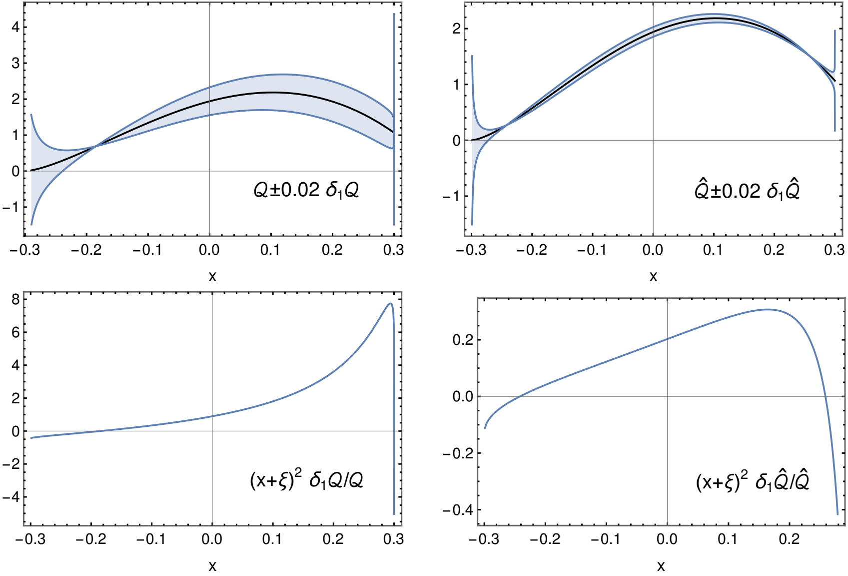

- The behaviour of the renormalon ambiguity for the qGDP at $x = \pm\xi$ is complicated due to the non-analyticity of the GDP. We obtain some interesting properties (only consider $x = +\xi$ for simplicity)
- $\delta_1 \mathcal{Q}(x,\xi)$ diverges like $\frac{1}{\sqrt{x-\xi}}$ as $x \to \xi^+$ and like $\log(x-\xi)$ as $x \to \xi^-$
- There is a delta function contribution

$$\delta_1 \mathcal{Q}(x,\xi) = ... + \left(1 - \frac{\pi^2}{6}\right)\delta(x-\xi)\Big(H'(\xi^+,\xi) - H'(\xi^-,\xi)\Big)$$

- Increasingly singular contributions to the qGPD for higher power terms, since $H^{(2n)}(x,\xi) \sim (x-\xi)^{3/2-2n}$ as $x \to \xi^+$

- What is the reason for the divergence at $x = \xi$?
- The total ambiguity is given by an expansion in powers of $\frac{\Lambda}{P^z}$

$$\delta_R \mathcal{Q}(x,\xi) = \sum_{w \in \{\text{poles of } B[\mathcal{Q}]\}} \underbrace{\delta_w \mathcal{Q}(x,\xi)}_{\sim(\Lambda/P^z)^{2w}} . \qquad (1)$$

- For $x \neq \xi$ we have $\{\text{poles of } B[\mathcal{Q}]\} = \mathbb{N}$.
- The expansion in (1) and the limit $x \to \xi$ do not commute. For x close to ξ the series has to be summed.
- To get the ambiguity at $x = \xi$, $\delta_R \mathcal{Q}(\xi,\xi)$ we need to take $x = \xi$ before taking the residues.

- For $x = \xi$ the Fourier transform produces additional singularities in the Borel plane:

1. A pole at $w = 1/2$ corresponding to a linear power correction $\frac{1}{P^z}$, proportional to $H'(\xi^+, \xi) - H'(\xi^-, \xi)$
2. A pole at $w = \lambda/2$, where $1 < \lambda < 2$ corresponding to to a fractional power correction $\frac{1}{(P^z)^\lambda}$, where λ corresponds to the power in $q(x) \sim x^{\lambda-2}$ as $x \to 0$ where $q(x)$ is the valence-quark GPD
3. A double pole at $w = 1$ corresponding to a power correction $\frac{\ln(P^z)}{(P^z)^2}$

- We have calculated the renormalon ambiguities to pGPDs and qGPDs. These can be used to estimate the sizes of power corrections in the $\frac{1}{P^z}$ expansion.
- Moderately small ambiguities gITD $\mathcal{I}$ and pGDP $\mathcal{P}$. Very small after zero momentum subtraction
- Large ambiguities for the qGPD. Especially at non-analytic points of the GPD $\pm\xi, 1$. Divergence of $\delta_R \mathcal{Q}$ as $x \to \pm\xi$.
- For the qGPD, additional singularities in the Borel plane get generated at $x = \xi$, hinting towards an interesting structure of power corrections for the qGPD at $x = \xi$.