

# Probing the gravitational form factors of the nucleon from near-threshold heavy quarkonium photo-production

Yuxun Guo

Lawrence Berkeley National Laboratory

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[2305.06992]

In collaboration with Xiangdong Ji, Yizhuang Liu, Jinghong Yang, and Feng Yuan

[2308.13006]

Workshop on GPD for Nucleon Tomography in the EIC Era

Brookhaven National Laboratory, NY

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# Outline

- »» Intro – Nucleon structure and gravitational form factors
- »» Quarkonium production and gluonic structure
- »» Approach the GFFs with Compton form factors
- »» Threshold  $J/\psi$  production and proton GFFs
- »» Summary and outlook



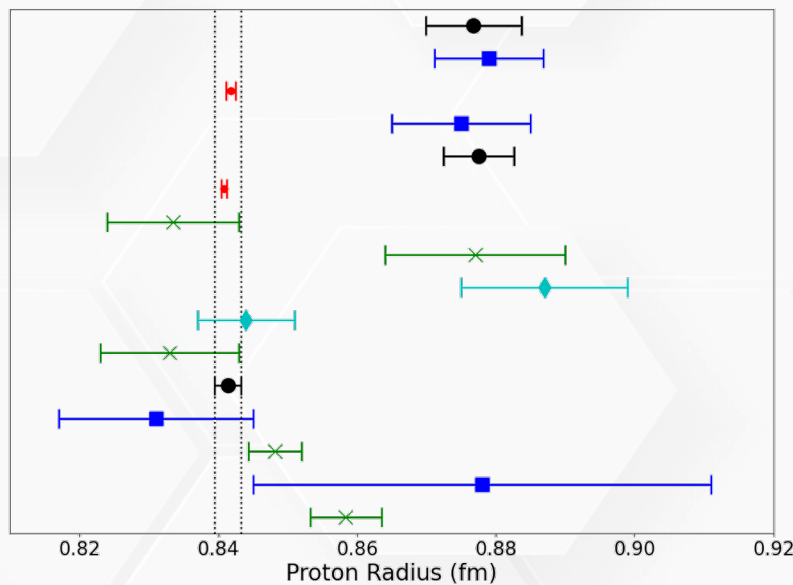
# Mass structure of the nucleon

Even though we can get the nucleon mass itself easily and precisely, counter-intuitively, the **mass structures** of microscopic particles are hard to access.

More than 100 years after we know the proton **charge**, we are still puzzled by its charge radius.

We are doing worse on the proton **mass** structure, as gravitational scattering are much harder, if possible at all.

CODATA 06 (2008)  
Bernauer (2010)  
Pohl (2010)  
Zhan (2011)  
CODATA 10 (2012)  
Antognini (2013)  
Beyer (2017)  
Fleurbaey (2018)  
Sick (2018)  
Alarcon (2019)  
Bezniov (2019)  
CODATA 18 (2019)  
Xiong (2019)  
Grinin (2020)  
Mihovilovic (2021)  
Brandt (2022)



Plot by E. J. Downie at SPIN 2023 at Duke



**WE  
NEED  
THIS!**

# Energy-momentum tensor form factors

The energy-momentum tensor (EMT) is the tool to study the mechanical properties of the nucleon. Its nucleon matrix element can be written as:

$$\langle P' | T_{q,g}^{\mu\nu} | P \rangle = \bar{u}(P') \left[ A_{q,g}(t) \gamma^{(\mu} \bar{P}^{\nu)} + B_{q,g}(t) \frac{\bar{P}^{(\mu} i \sigma^{\nu)\alpha} \Delta_\alpha}{2M_N} + C_{q,g}(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{M_N} + \bar{C}_{q,g}(t) M_N g^{\mu\nu} \right] u(P)$$

X. Ji Phys. Rev. Lett. 78, 610 (1997)

Momentum form factors:

$$A_{q,g}(t)$$

Angular momentum form factors:

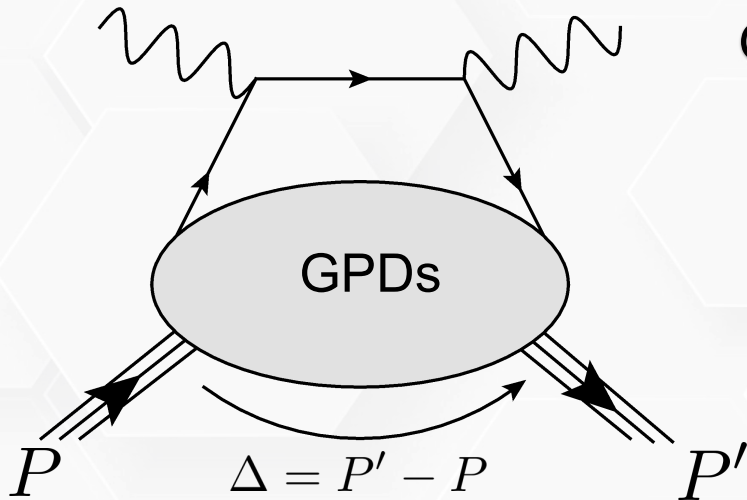
$$J_{q,g}(t) = \frac{1}{2} (A_{q,g}(t) + B_{q,g}(t))$$

Stress tensor form factors:

$$C_{q,g}(t)$$

# Generalized parton distributions (GPDs)

Instead of measuring the whole nucleon, we just knock out one parton at a time.



D. Muller et. al. Fortsch.Phys. 42 101 (1994)

X. Ji Phys. Rev. Lett. 78, 610 (1997)

GPDs are distributions unifying parton distributions and form factors

$$F(x, \Delta^\mu) = F(x, \xi, t)$$

$x$  : average parton momentum fraction

$\xi$  : skewness – longitudinal momentum transfer  $\xi \equiv -n \cdot \Delta / 2$

$t$  : total momentum transfer squared  $t \equiv \Delta^2$

**GPDs are partonic form factors? – at zero skewness**

# Quarkonium Production

# Color dipole probing the gluonic structure

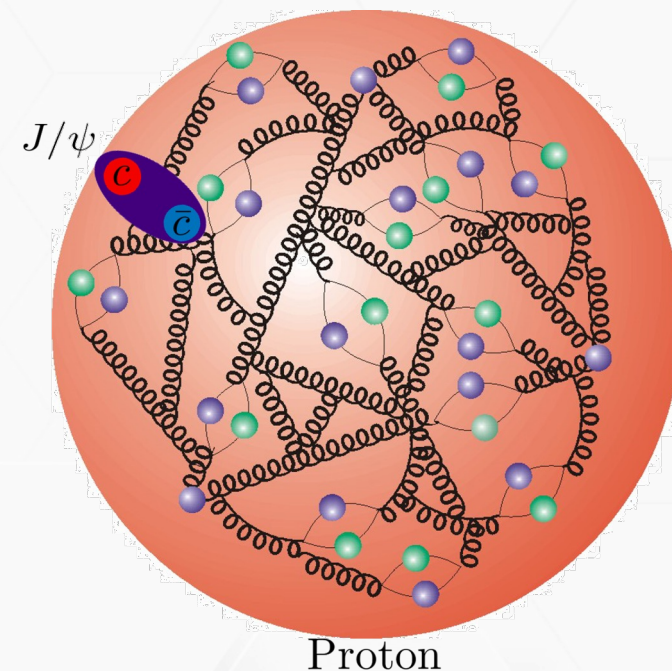
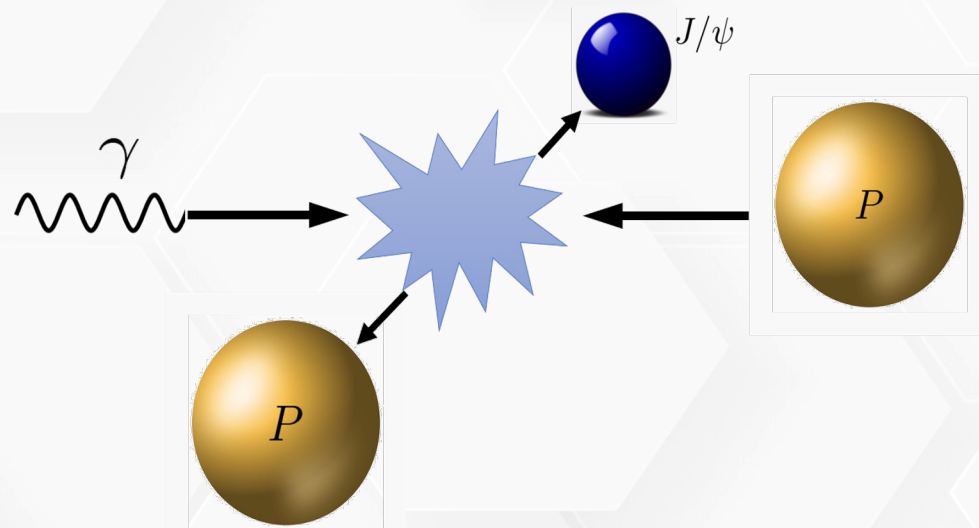
Quadratic Stark effect in QCD – measures the color electric field:

$$\mathcal{H}_{\text{Int}} = \mathbf{E}^a \cdot (\mathbf{r}_c t_c^a - \mathbf{r}_{\bar{c}} t_{\bar{c}}^a)$$

M. B. Voloshin Nucl. Phys. B 154 365-380 (1979)  
M. Luke et al. Phys. Lett. B 288 355-359 (1992)  
D. Kharzeev et al., Eur. Phys. J. C 9 459-462 (1999)

Elastic scattering of J/psi off the nucleon would be ideal.

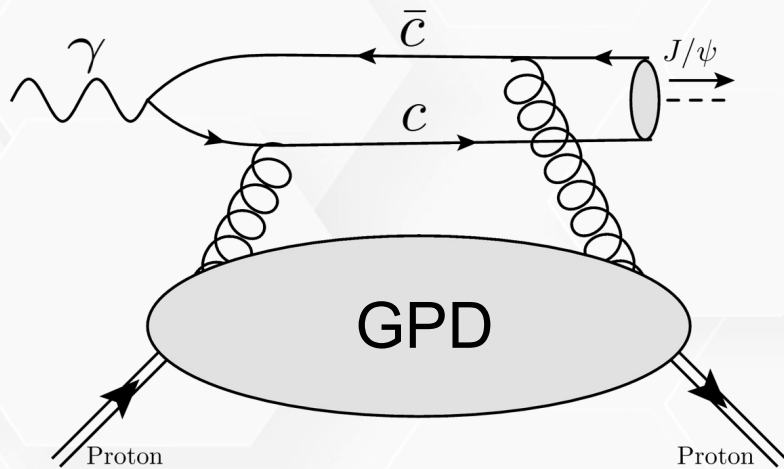
In real life we have photo-/electro- production instead



Heavy meson corresponds to small color dipole --- local gluonic distribution.

# GPD framework for threshold production

Near-threshold exclusive heavy vector meson production in the GPD framework.



Y. Guo et. al. Phys. Rev. D 103 9, 096010 (2021)

- ❑ Leading order factorization with GPDs
- ❑ The same amplitude as the collinear case but with different kinematics;
- ❑ Large momentum transfer/skewness in the heavy quark limit;

$$\frac{d\sigma}{dt} \propto \left[ (1 - \xi^2) |\mathcal{H}_{gC}|^2 - 2\xi^2 \text{Re} [\mathcal{H}_{gC}^* \mathcal{E}_{gC}] - \left( \xi^2 + \frac{t}{4M_p^2} \right) |\mathcal{E}_{gC}|^2 \right],$$

Will be sensitive to the so-called gluonic Compton form factors (gCFFs)

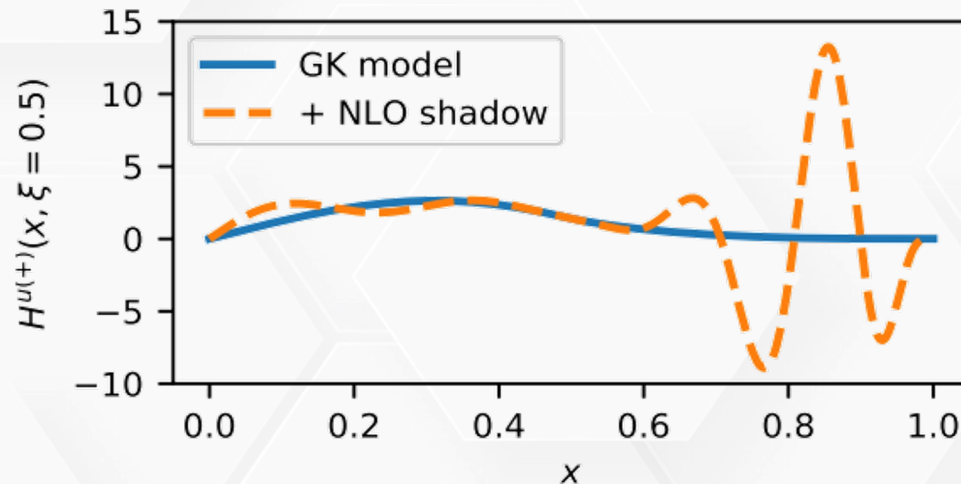


# Inverse problem in exclusive productions

To keep the proton intact in the final state for an exclusive production, the parton will not be explicitly probed / knocked out.

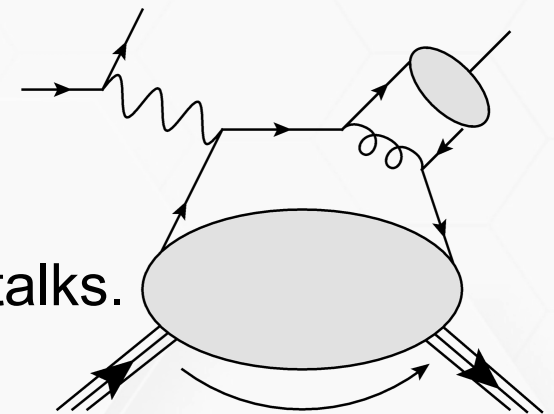
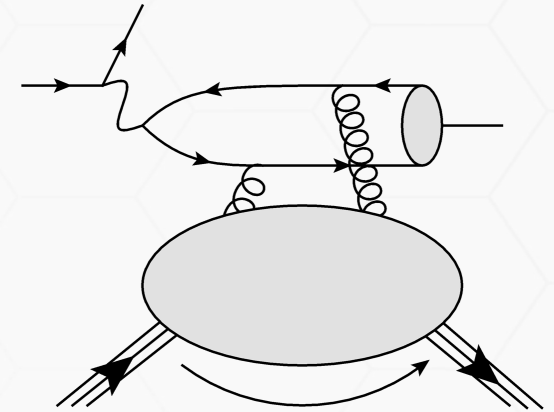
$$\mathcal{H}_{gC}(\xi, t) = \frac{1}{2\xi} \int_{-1}^1 dx \left( \frac{1}{x + \xi - i\epsilon} - \frac{1}{x - \xi + i\epsilon} \right) H_g(x, \xi, t)$$

The amplitude is not sensitive to the parton momentum  $x$



V. Bertone et. al. SciPost Phys.Proc. 8 (2022) 107

Yuxun Guo @ RBRC GPD Workshop



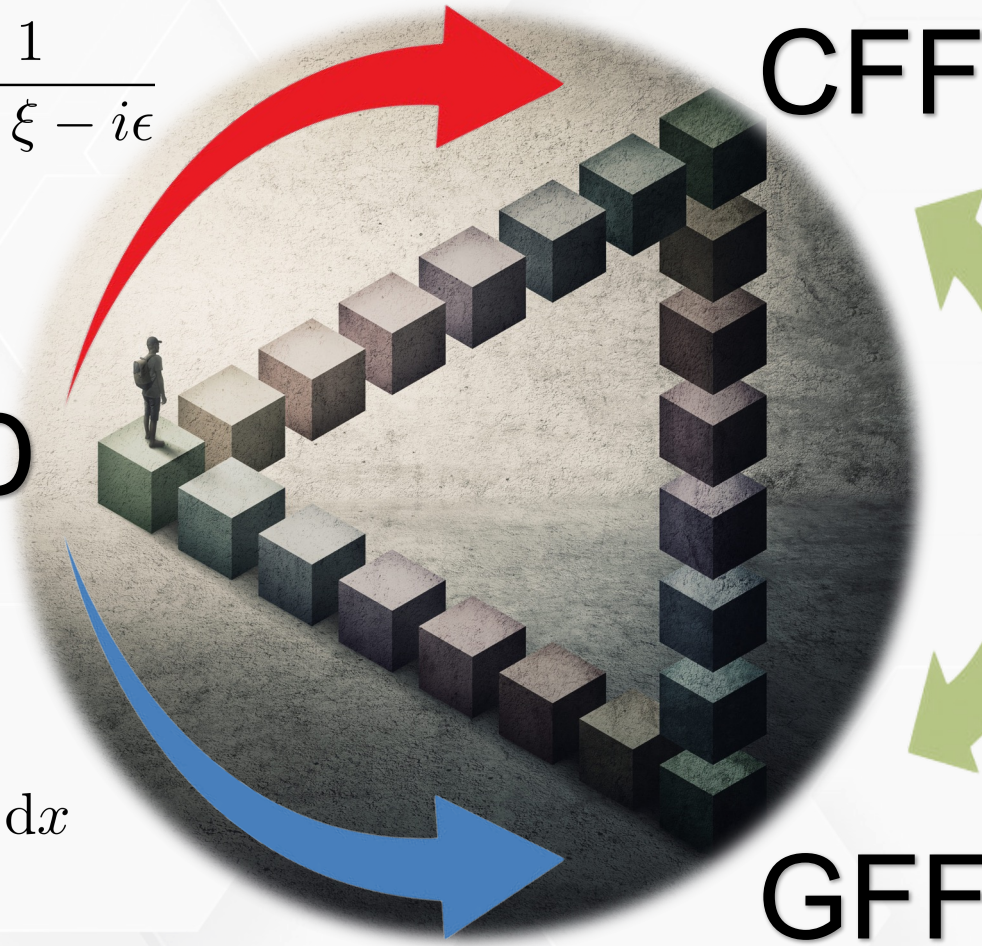
Ref. to the Most of talks.

# GFFs and CFFs

$$\int_{-1}^1 dx \frac{1}{x + \xi - i\epsilon}$$

**GPD**

$$\int_{-1}^1 dx$$



**CFF**



Ref to P. Sznajder's Talk  
and B. Pasquini's Talk

**GFF**

# From CFFs to GFFs

# Large skewness near the threshold

When a heavy quarkonium is produced from a massless real photon near threshold, the momentum transfer must be large. Then consider a Taylor expansion:

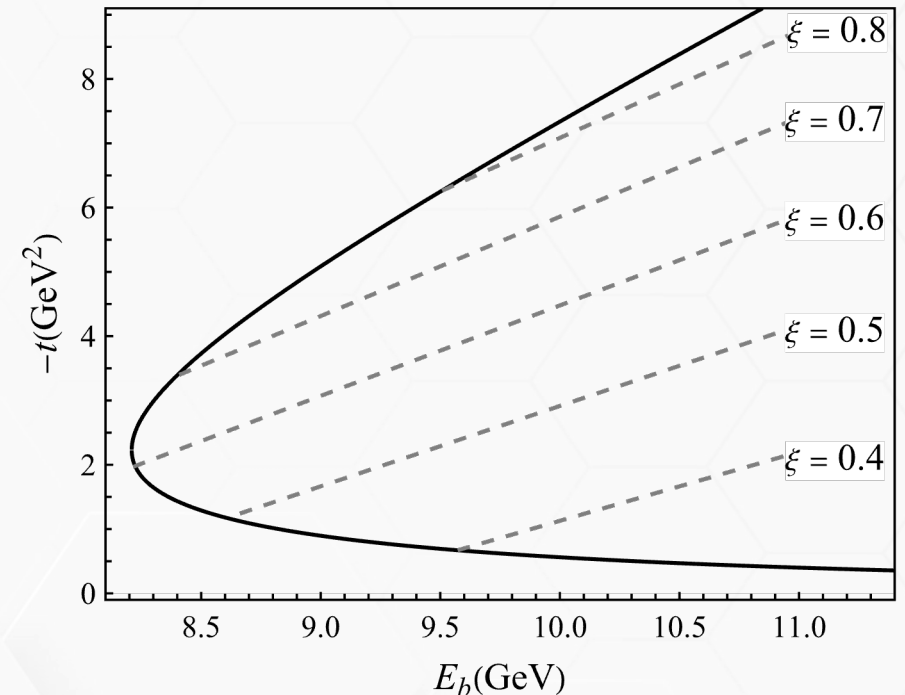
$$\text{Re} \left[ \frac{1}{2\xi} \left( \frac{1}{x + \xi - i\epsilon} - \frac{1}{x - \xi + i\epsilon} \right) \right] = \sum_{n=0}^{\infty} \frac{x^{2n}}{\xi^{2+2n}}$$

The real part of the CFF can then be written as:

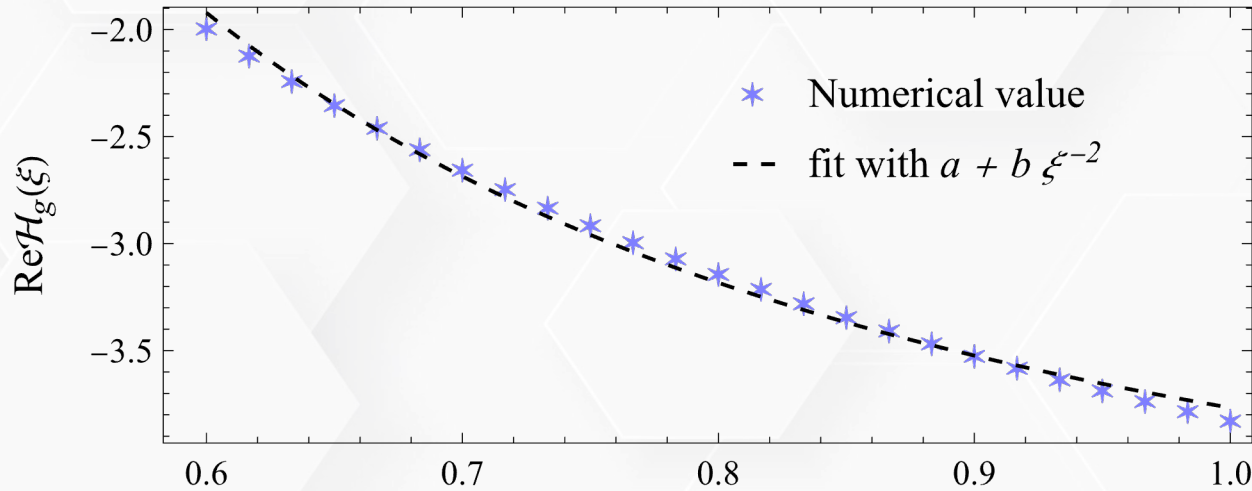
$$\text{Re}\mathcal{H}_{gC}(\xi, t) = \frac{2}{\xi^2} \sum_{n=0}^{\infty} \int_0^1 dx \left( \frac{x}{\xi} \right)^{2n} H_g(x, \xi, t)$$

These x-integrals lead to generalized form factors

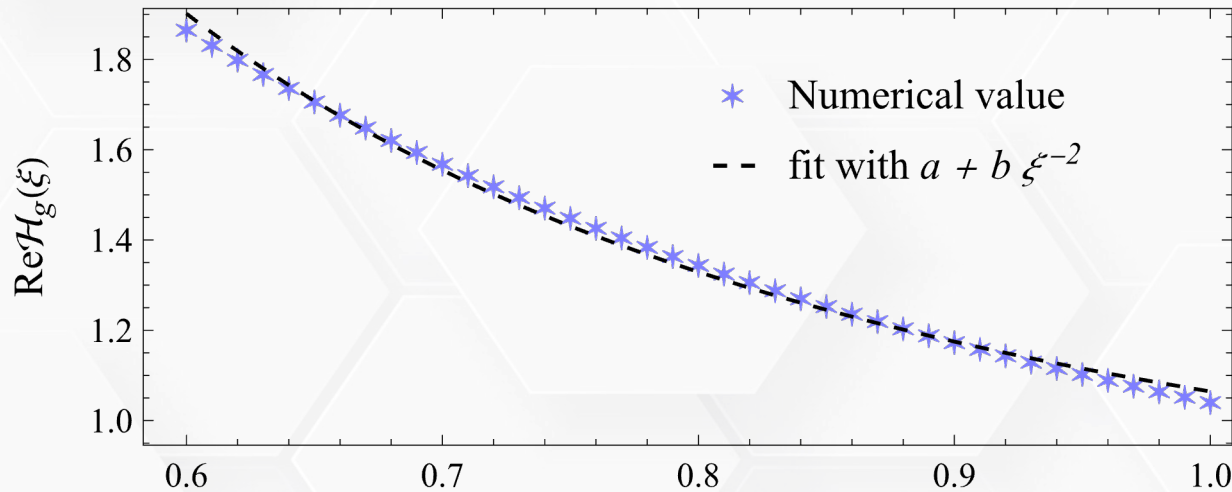
$$\text{Re}\mathcal{H}_{gC}(\xi, t) = C_g(t) + \sum_{n=1}^{\infty} \xi^{-2n} \mathcal{A}_g^{(2n)}(t) \stackrel{?}{\approx} C_g(t) + \xi^{-2} \mathcal{A}_g^{(2)}(t)$$



# Skewness-scaling in the CFFs



Double distribution model



Conformal moment model

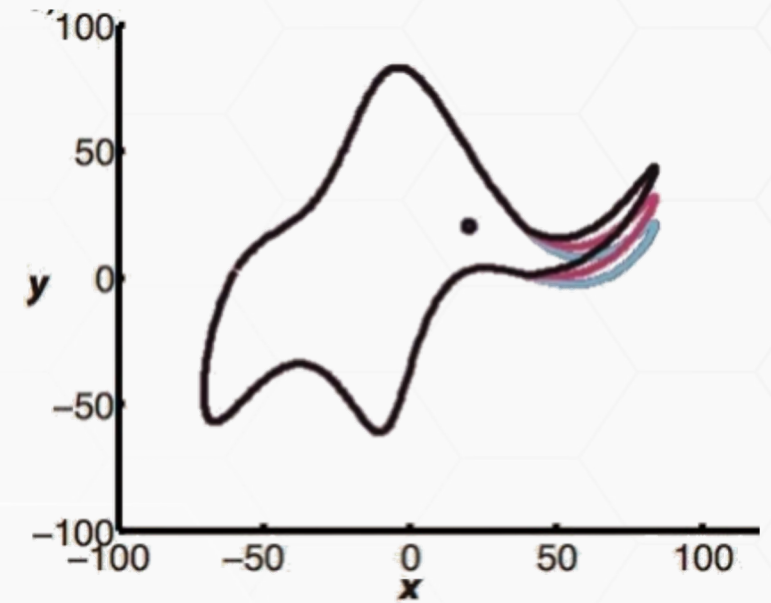
# Fit is meaningless if not physical



With four parameters I can fit an elephant, and with five I can make him wiggle his trunk.

— *John von Neumann* —

AZ QUOTES



# GFFs from CFFs

Let's see if the CFF is related to the GFFs.

With the approximation, the CFF will be

$$\text{Re}\mathcal{H}_{gC}(\xi, t) \approx C_g(t) + \xi^{-2} \mathcal{A}_g^{(2)}(t)$$

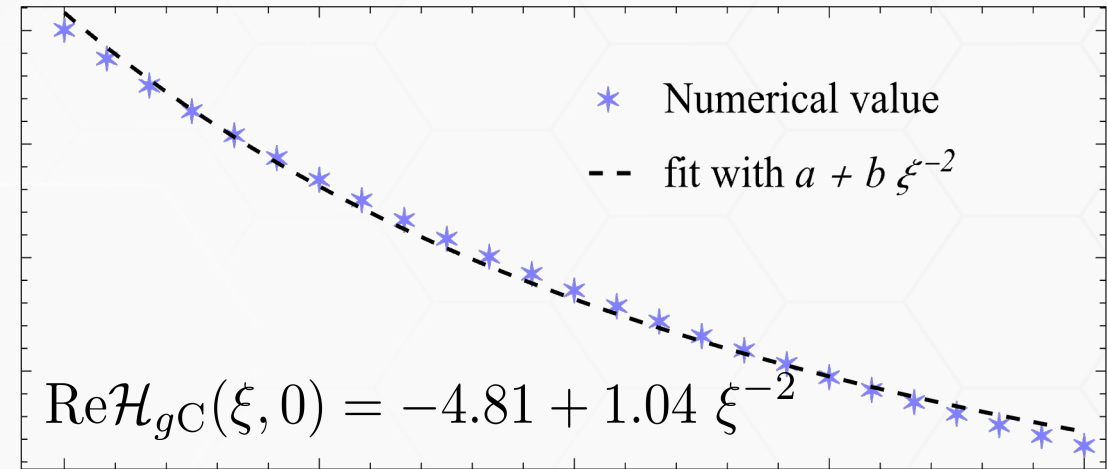
$$\mathcal{A}_g^{(2)}(t) \approx 2A_g(t), C_g(t) \approx 8C_g(t),$$

The input gluon moments are:

$$A_g(0) \approx 0.385, C_g(0) \approx -0.48$$

The extracted values are:

$$\frac{1}{2} \mathcal{A}_g^{(2)}(0) \approx \mathbf{0.52}, \frac{1}{8} C_g(0) \approx \mathbf{-0.60}$$

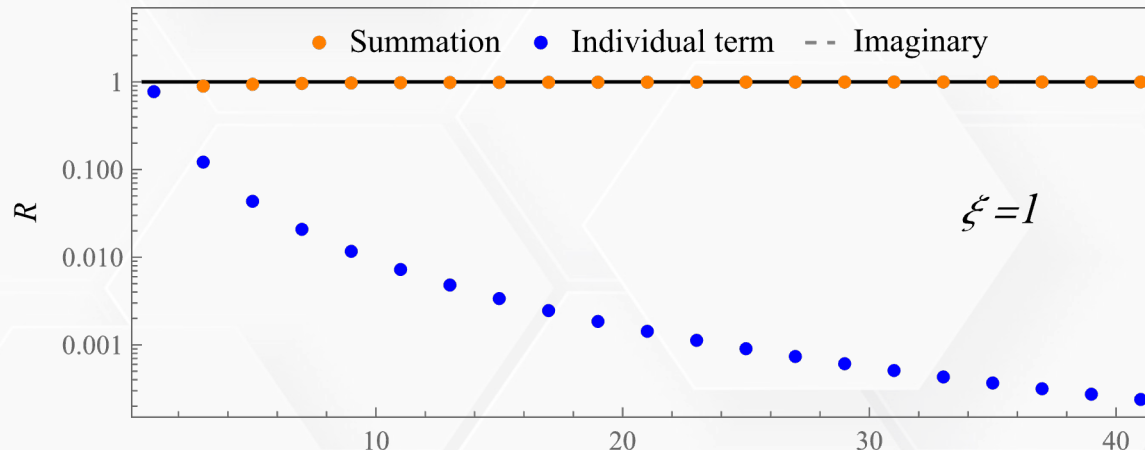


The extractions have 25% excess due to the higher-moment contamination.

# Asymptotic expansion

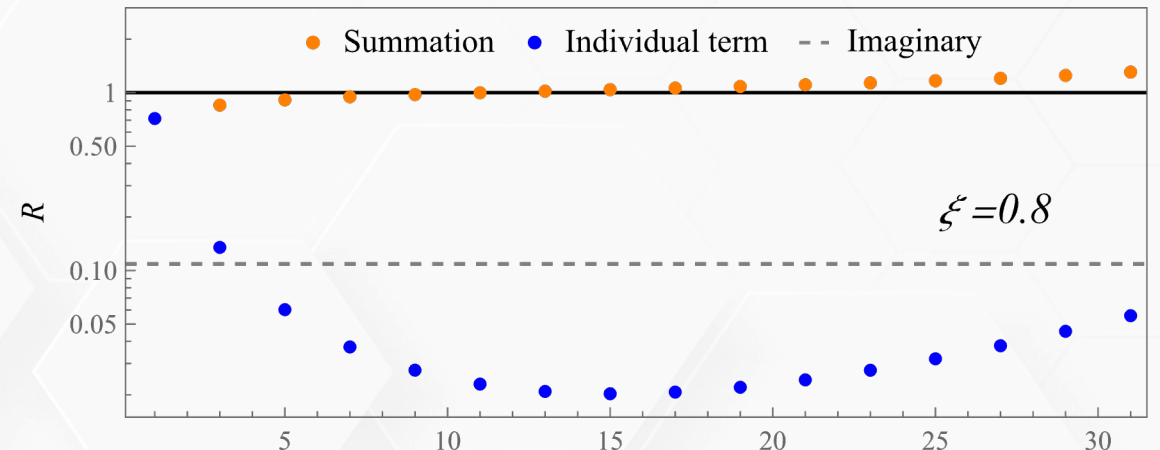
## Convergent series

- ❖ Individual term decreases and vanishes
- ❖ Partial sum eventually approach the analytical result



## Asymptotic series

- ❖ Individual term decreases **then increases**
- ❖ Partial sum eventually diverges
- ❖ Truncation at minimal terms approximated the analytical results well





# Proton GFFs from J/Psi production

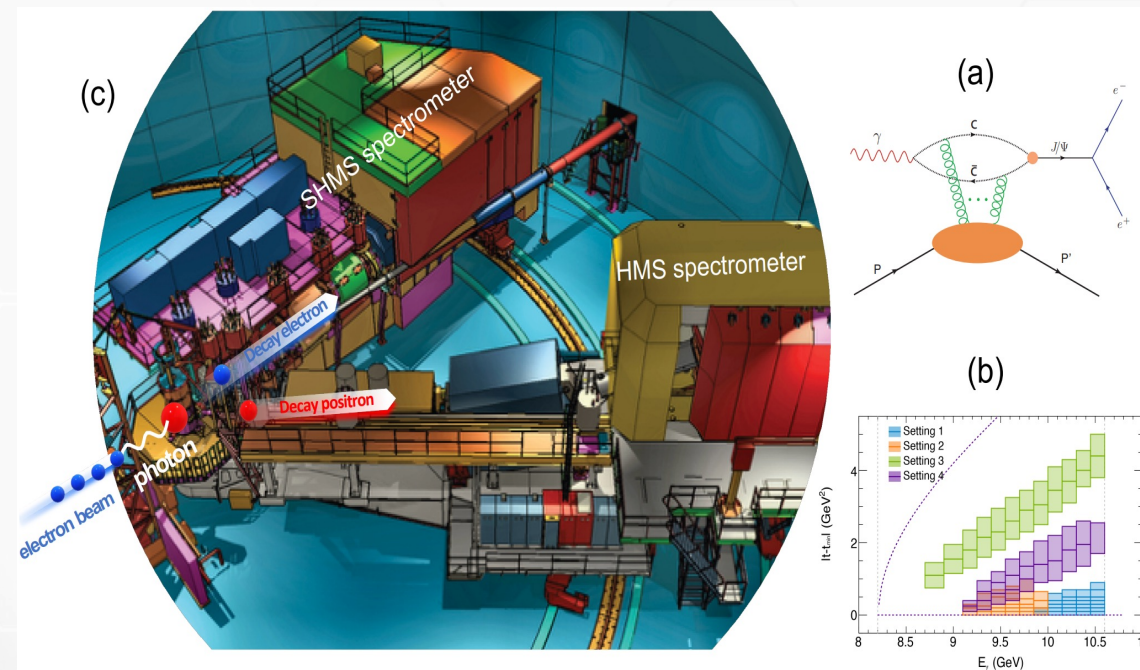
# Near-threshold $J/\psi$ photoproduction

First measurement of near-threshold exclusive  $J/\psi$  photoproduction off the proton by GlueX



GlueX Collaboration Phys. Rev. Lett. 123, 072001 (2019)  
GlueX Collaboration Phys. Rev. C 108 2, 025201 (2023)

Another recent measurement by the  $J/\psi$  007 experiment at JLab Hall C



B. Duran et. al. Nature 615 7954, 813-816 (2023)

# General framework

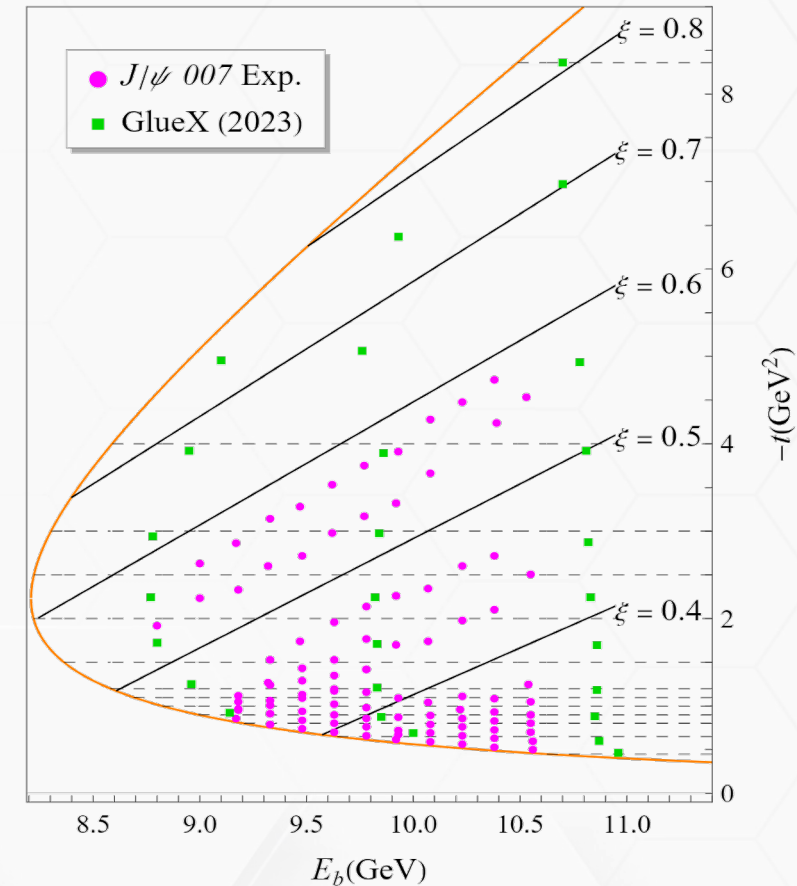
Under the approximation of **large skewness expansion**  
**AND higher-moment suppression**, one has:

$$\mathcal{H}_{gC}(\xi, t) \approx C_g(t) + \xi^{-2} \mathcal{A}_g^{(2)}(t)$$

$$\mathcal{E}_{gC}(\xi, t) \approx -C_g(t) + \xi^{-2} \mathcal{B}_g^{(2)}(t)$$

We can then extract the GFFs from the gCFFs, utilizing the large skewness kinematics in the near-threshold region in the heavy quark limit.

Unfortunately, much more data in the lower  $\xi$  region

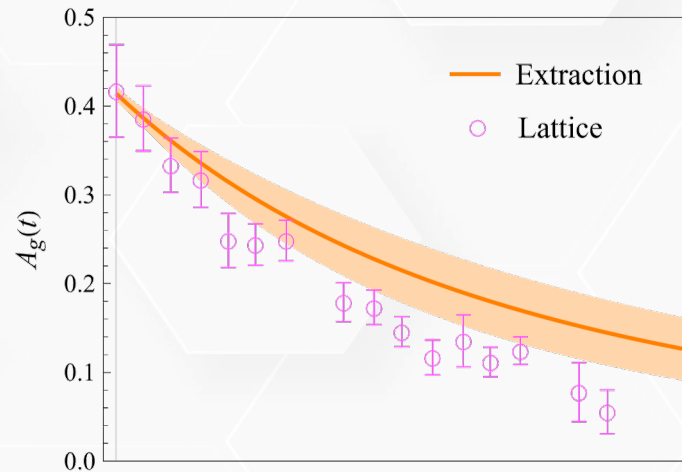


Y. Guo et. al. Phys. Rev. D 108, 034003 (2023)

# Extraction of the nucleon GFFs

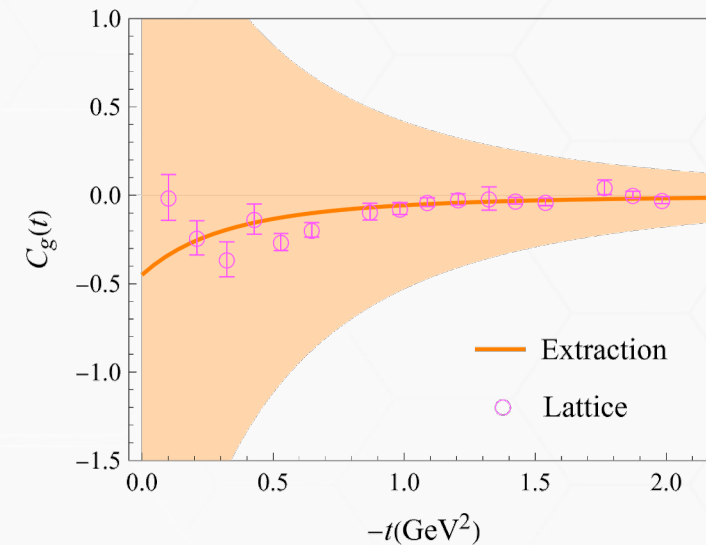
GFFs can be constrained noting the higher-moment contaminations as well as other corrections.

With  $\xi > 0.5$  data



Y. Guo et. al. Phys. Rev. D 108, 034003 (2023)

D. Pefkou et. al. Phys. Rev. D 105 5, 054509 (2022)



Not a completely independent extraction:

- The lower- $t$  region is not constrained well by the data.
- The gluonic momentum fraction from PDF is used as input
- The coverage in  $\xi$  is rather limited.

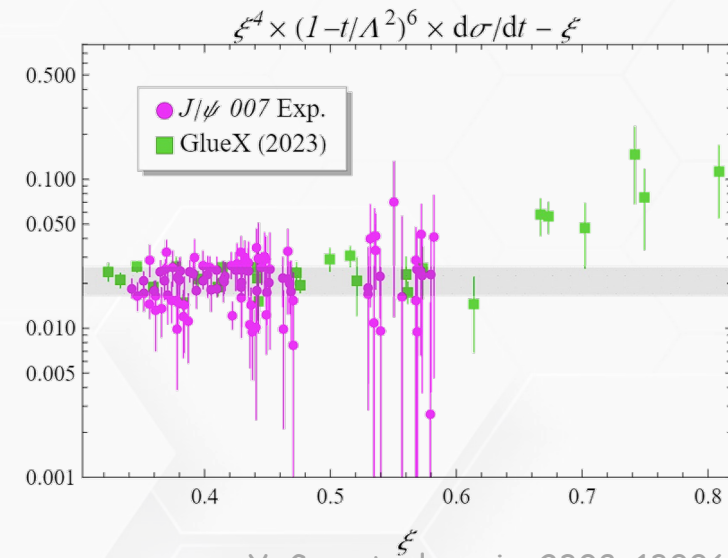
# Utilizing the skewness dependence

How to test the large-skewness framework and quality of the extraction?

$$\begin{aligned} |G(\xi, t)|^2 &= \left[ (1 - \xi^2) |\mathcal{H}_{gC}|^2 - 2\xi^2 \text{Re} [\mathcal{H}_{gC}^* \mathcal{E}_{gC}] - \left( \xi^2 + \frac{t}{4M_p^2} \right) |\mathcal{E}_{gC}|^2 \right] \\ &= \xi^{-4} \left[ G_0(t) + \xi^2 G_2(t) + \xi^4 G_4(t) \right] + \dots \end{aligned}$$

The  $\xi$ -scaling of the cross-section has non-trivial behaviors at large  $\xi$ .

- ❑ The overall  $\xi$ -scaling are consistent with  $\xi^{-4}$
- ❑ Extra  $\xi$ -scaling crucial for the separation of A and C form factors
- ❑ More complicated when consider t-dependence



# What can we do with CFF

Compton-like amplitudes basically measure GPDs at  $x=\xi$

- Imaginary parts given by GPDs at  $x=\xi$

$$\text{Im}\mathcal{H}_{gC}(\xi, t) = \frac{\pi}{\xi} H_g(\xi, \xi, t)$$

- Real parts given by dispersion relation

$$\text{Re}\mathcal{H}_{gC}(\xi, t) = \frac{1}{\pi} \int_0^{\xi_{\text{th}}} d\xi' \frac{2\xi' \text{Im}\mathcal{H}_{gC}(\xi', t)}{(\xi - \xi')(\xi + \xi')} + C_g(t)$$

- Real parts can be formally written as

$$\text{Re}\mathcal{H}_{gC}(\xi, t) = C_g(t) + \sum_{n=1}^{\infty} \xi^{-2n} \mathcal{A}_g^{(2n)}(t)$$

- Imaginary parts – inverse Mellin transform:

$$\text{Im}\mathcal{H}_{gC}(\xi, t) \propto \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \xi^{-s} \mathcal{A}_g^{(s+2)}(t) ds$$

The two statements are mathematically equivalent!

The different applicable regions (near forward/ large skewness) distinguish them

# Summary and outlook

## Summary

- Exclusive heavy quarkonium production as probe of gluonic structures
- Large skewness behavior of gCFFs in near-threshold heavy quarkonium production
- (Potential) extraction of gluonic GFFs in the nucleon with such framework

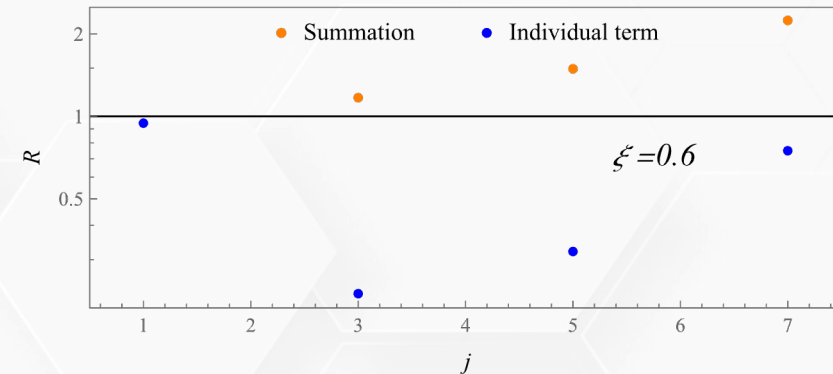
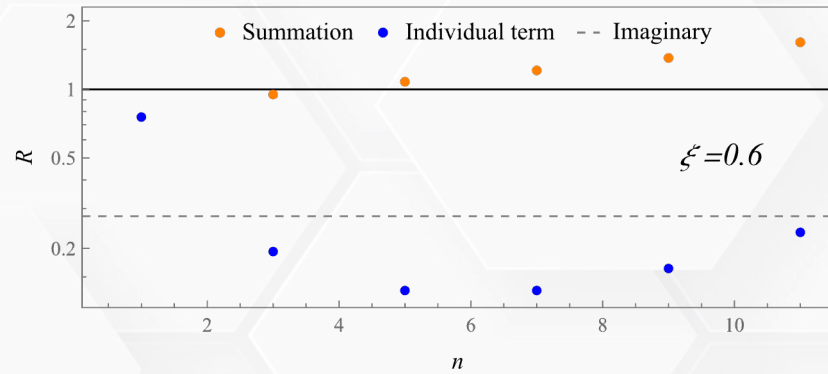
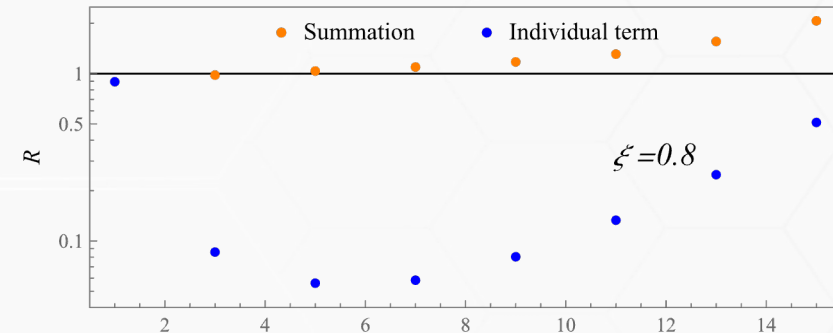
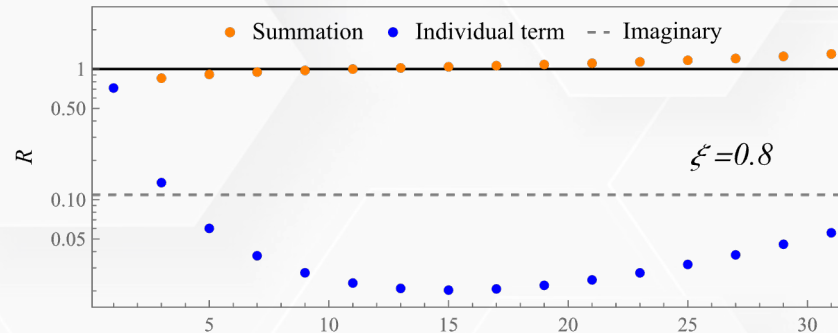
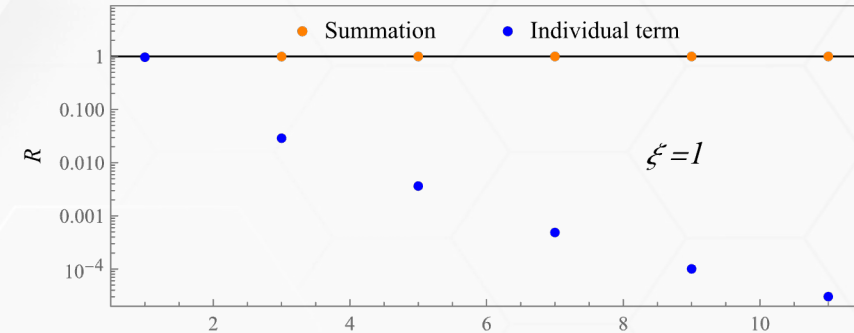
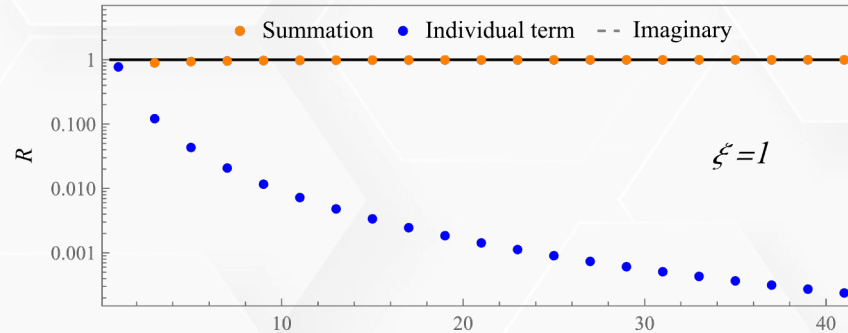
## Outlook

- Next-to-leading order effects in strong coupling and non-relativistic corrections
- Extension to quark GPDs, e.g., time-like Compton scattering for the quark case
- Relations to the other frameworks – collinear NRQCD factorization

**Thank you!**

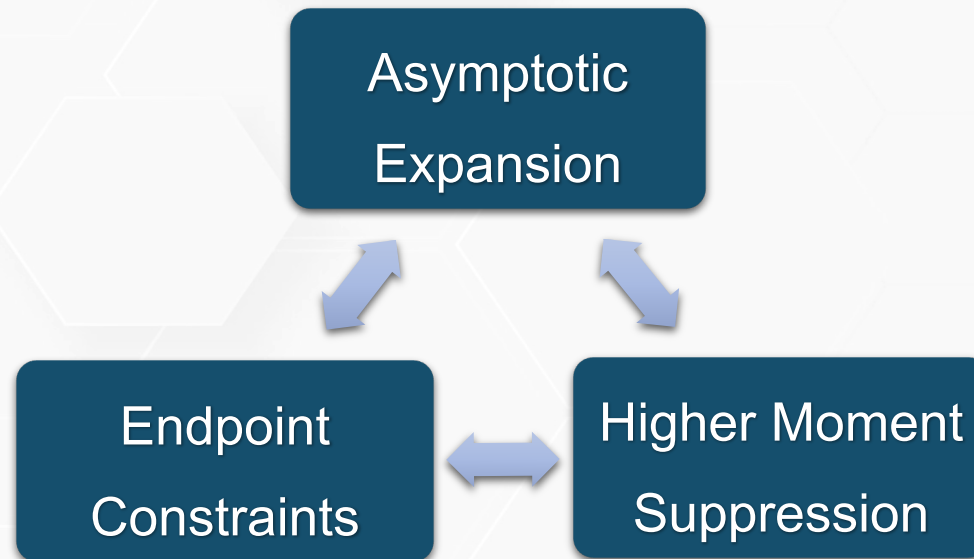


# Conformal moment expansion



# Two more comments

The asymptotic expansion is closely related to the endpoint constraints



The 25% excess from higher moments contamination relate to the conformal ratio?

$$\mathcal{A}_j^{\text{conf}} = \frac{2^{j+2} \Gamma(5/2 + j)}{\Gamma(3/2) \Gamma(4 + j)} \quad \mathcal{A}_1^{\text{conf}} = \frac{5}{4}$$

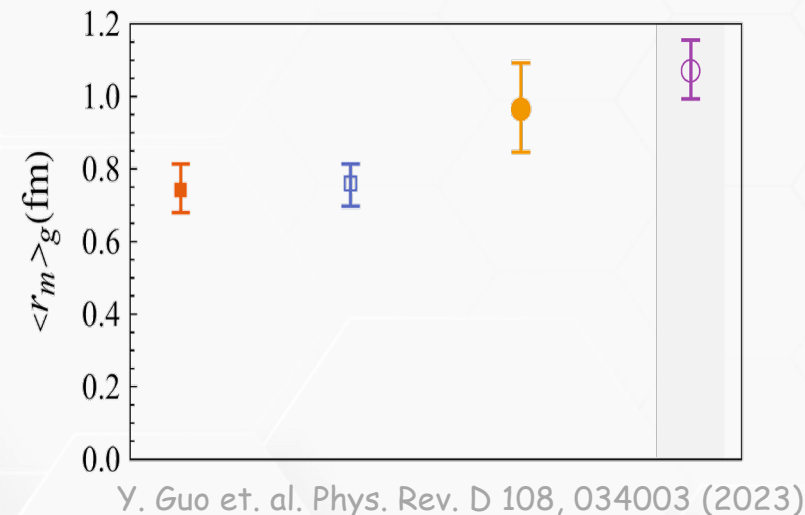
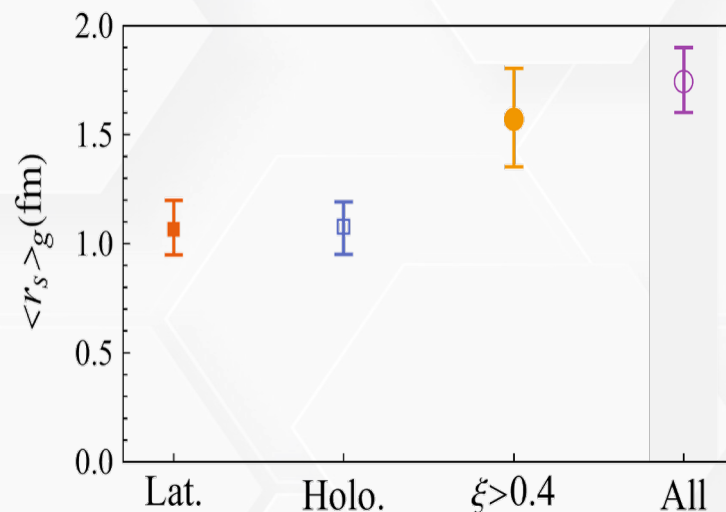
Y. Guo et. al. arxiv: 2308.13006

# Proton mass radius extraction

Proton mass and scalar radii are closely related to the form factors at  $t = 0$

$$\langle r_m^2 \rangle = \left[ 6 \frac{dA(t)}{dt} \Big|_{t=0} - 6 \frac{C(0)}{M_N^2} \right], \quad \langle r_s^2 \rangle = \left[ 6 \frac{dA(t)}{dt} \Big|_{t=0} - 18 \frac{C(0)}{M_N^2} \right],$$

Inclusion of lower- $\xi$  data shows better statistics but introduces massive systematical uncertainties.



Y. Guo et. al. Phys. Rev. D 108, 034003 (2023)

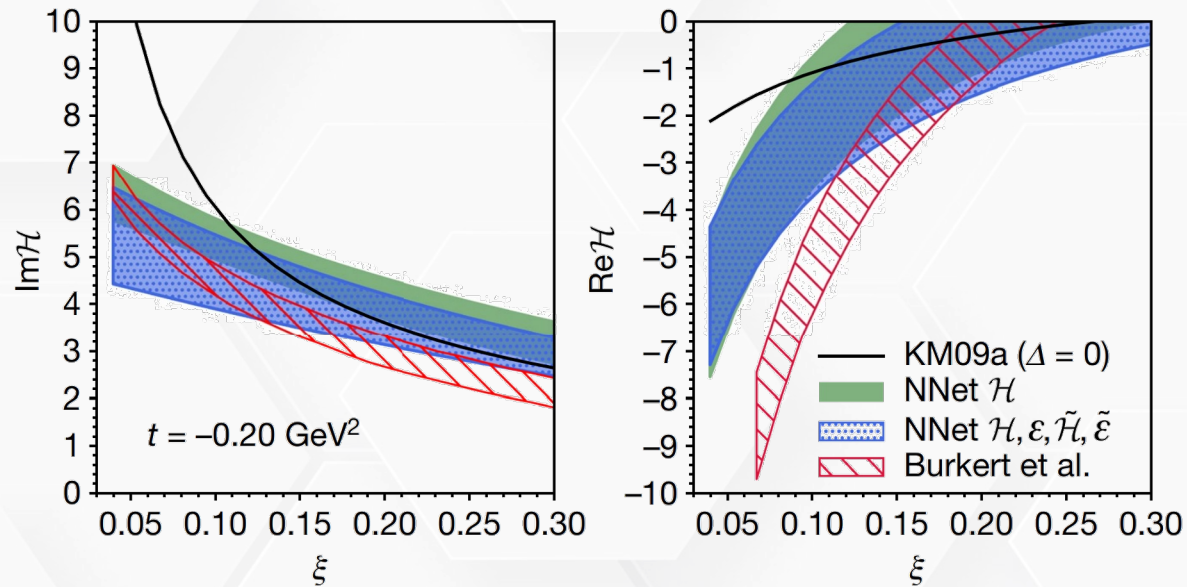
Due to the large- $\xi/t$  nature of this framework, the extraction of proton radii with controlled precision will need more efforts.

# One interesting quark analogy

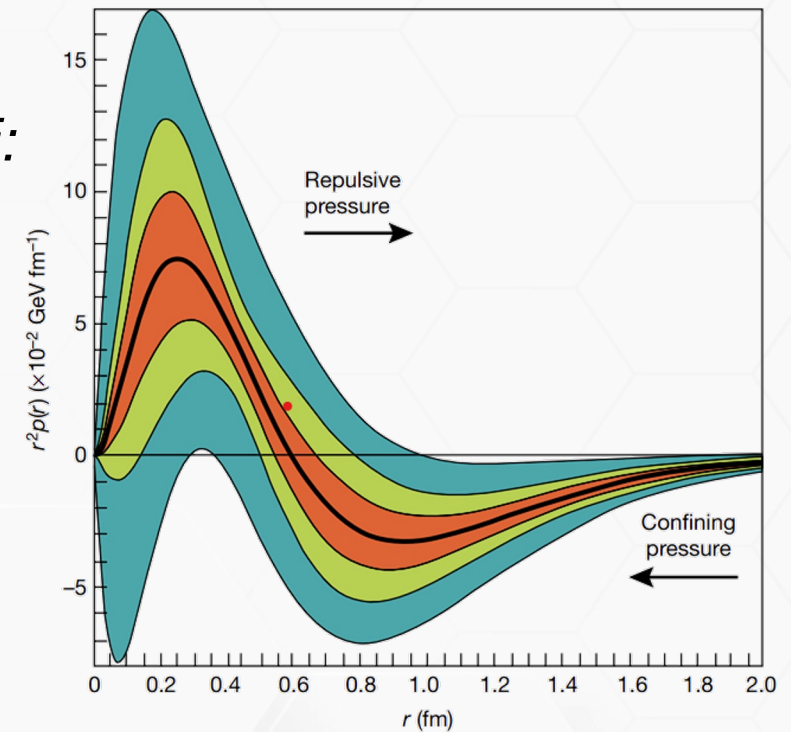
The C form factor can be extracted through the dispersion relation as well:

$$C_g(t) = \text{Re}\mathcal{H}_{gC}(\xi, t) - \frac{1}{\pi} \int_0^{\xi_{\text{th}}} d\xi' \frac{2\xi' \text{Im}\mathcal{H}_{gC}(\xi, t)}{(\xi - \xi')(\xi + \xi')}$$

The extraction seems to be more consistent with increasing  $\xi$ :



K. Kumericki., Nature 570 (2019) 7759, E1-E2



V. D. Burkert et. al., Nature 557 7705, 396-399 (2018)