On the Lepton Angular Distribution in Drell-Yan, W/Z and Quarkonium Production

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INTT mini-Workshop, National Central University

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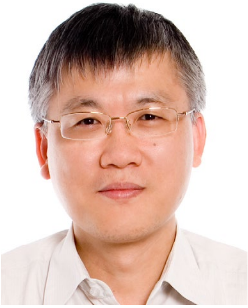
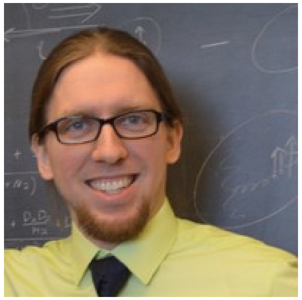
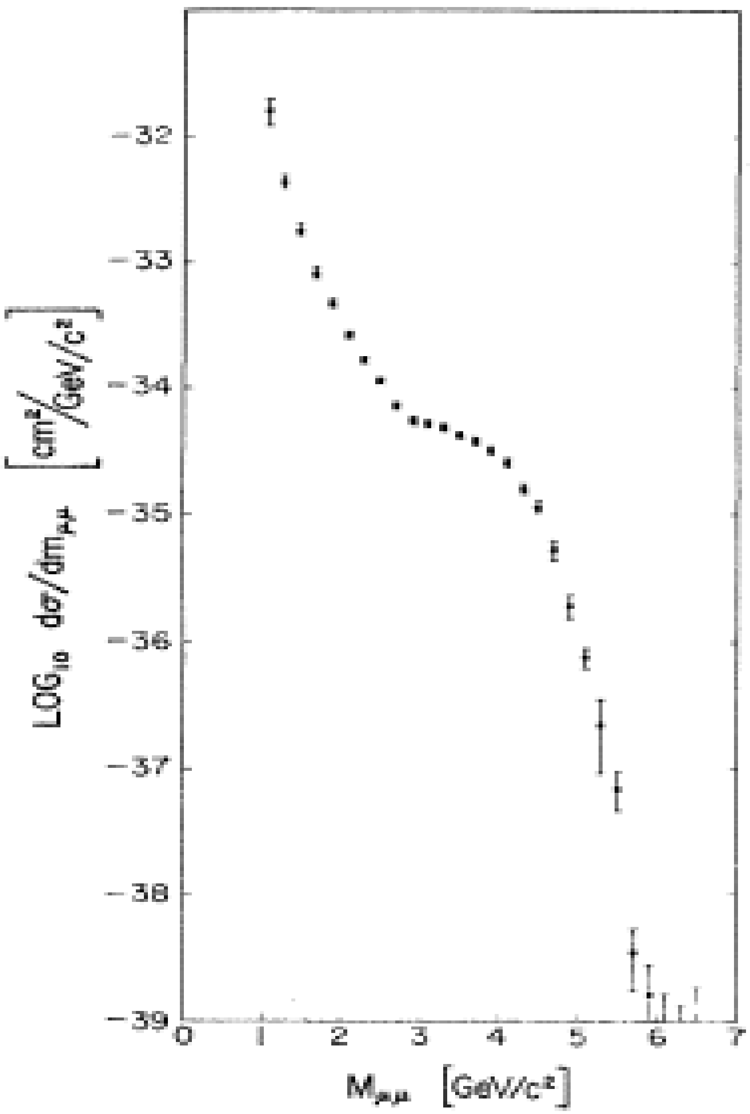
Based on the papers with Wen-Chen Chang,
Evan McClellan, Oleg Teryaev
(and Daniel Boer for one paper)

Phys. Lett. B758 (2016) 384;
Phys. Rev. D 96 (2017) 054020;
Phys. Lett. B789 (2019) 356;
Phys. Rev. D 99 (2019) 014032
Phys. Lett. B797 (2019) 134895
Phys. Rev. D 103 (2021) 034011





First High-mass Dimuon Experiment at BNL



Lederman et al. PRL 25 (1970) 1523

$p + U \rightarrow \mu^+ + \mu^- + X$ 29 GeV proton

Experiment originally designed to search for neutral weak boson (Z^0)

Missed the J/Ψ signal !

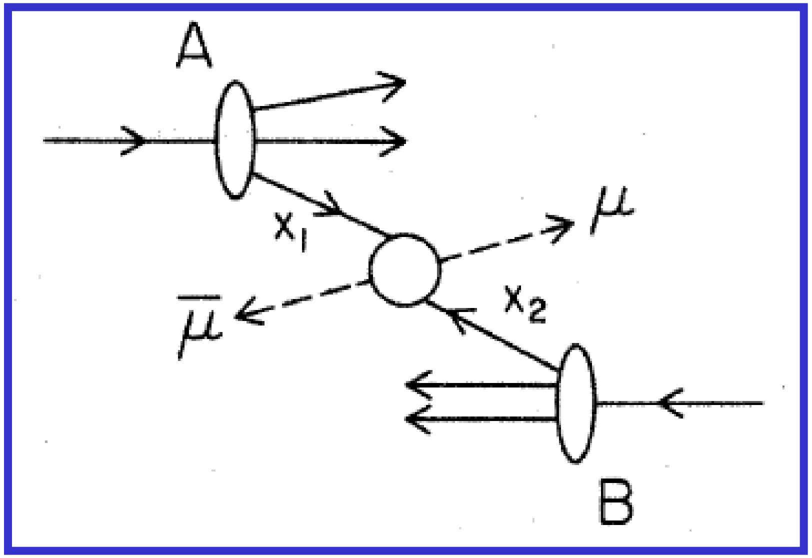
"Discovered" the Drell-Yan process

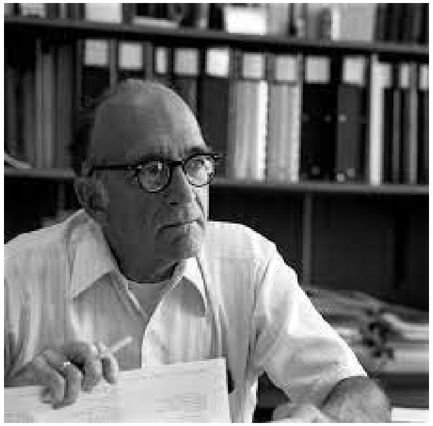
The Drell-Yan Process

MASSIVE LEPTON-PAIR PRODUCTION IN HADRON-HADRON COLLISIONS AT HIGH ENERGIES*

Sidney D. Drell and Tung-Mow Yan
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 25 May 1970)

Cited ~1800 times





$$\left(\frac{d^2\sigma}{dx_1 dx_2}\right)_{D.Y.} = \frac{4\pi\alpha^2}{9sx_1x_2}\sum_a e_a^2\left[q_a(x_1)\bar{q}_a(x_2) + \bar{q}_a(x_1)q_a(x_2)\right]$$

Naive Drell-Yan and Its Successor*

T-M. Yan
Floyd R. Newman Laboratory of Nuclear Studies
Cornell University
Ithaca, NY 14853

February 1, 2008

Abstract

We review the development in the field of lepton pair production since proposing parton-antiparton annihilation as the mechanism of massive lepton pair production. The basic physical picture of the Drell-Yan model has survived the test of QCD, and the predictions from the QCD improved version have been confirmed by the numerous experiments performed in the last three decades. The model has provided an active theoretical arena for studying infrared and collinear divergences in QCD. It is now so well understood theoretically that it has become a powerful tool for new phyiscs information such as precision measurements of the W mass and lepton and quark sizes.

*Talk given at the Drell Fest, July 31, 1998, SLAC on the occasion of Prof. Sid Drell's retirement.

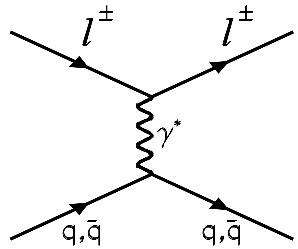
"… our original crude fit did not even remotely resemble the data. Sid and I went ahead to publish our paper because of the model's simplicity…"

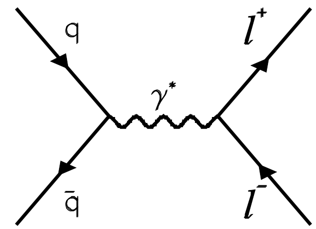
"… the successor of the naïve model, the QCD improved version, has been confirmed by the experiments…"

"The process has been so well understood theoretically that it has become a powerful tool for precision measurements and new physics."

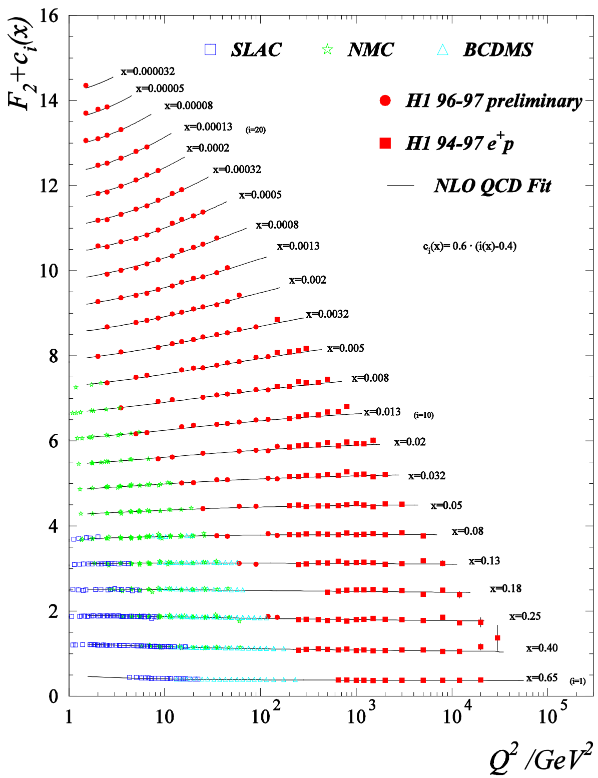
Complementality between DIS and Drell-Yan

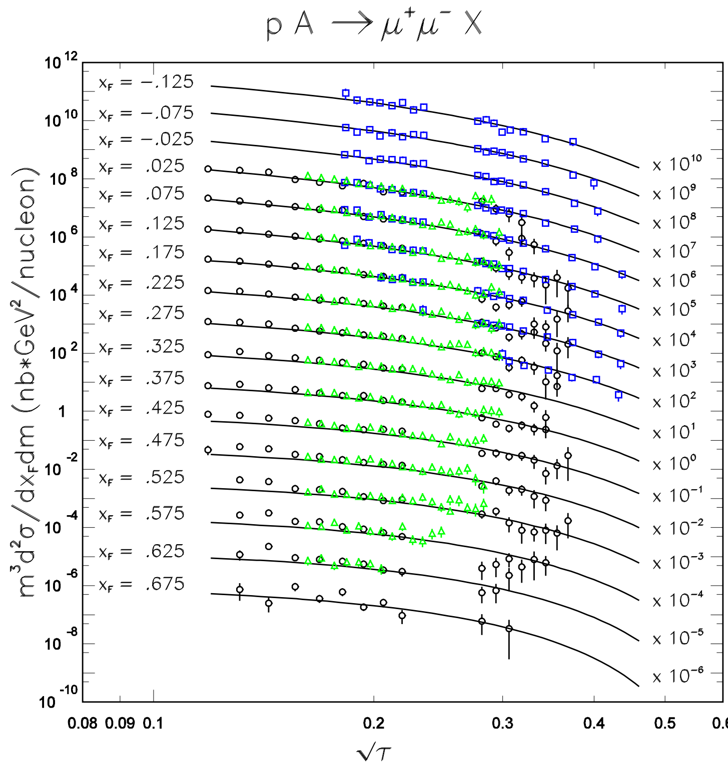
DIS





Drell-Yan





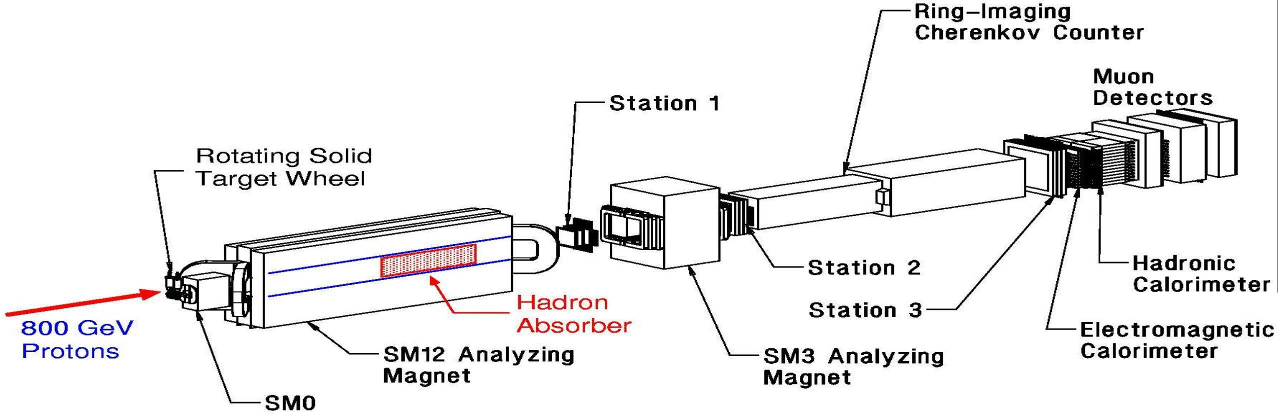
Ann.Rev.Nucl. Part. Sci. 49 (1999) 217;

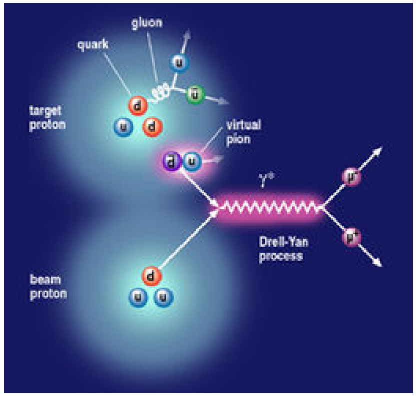
Peng and Qiu, Prog. Part. Nucl. Phys. 76 (2014)43

Both DIS and Drell-Yan process are tools to probe the quark and antiquark structure in hadrons (factorization, universality)

Fermilab Dimuon Spectrometer

(E605 / 772 / 789 / 866 / 906 /1039)



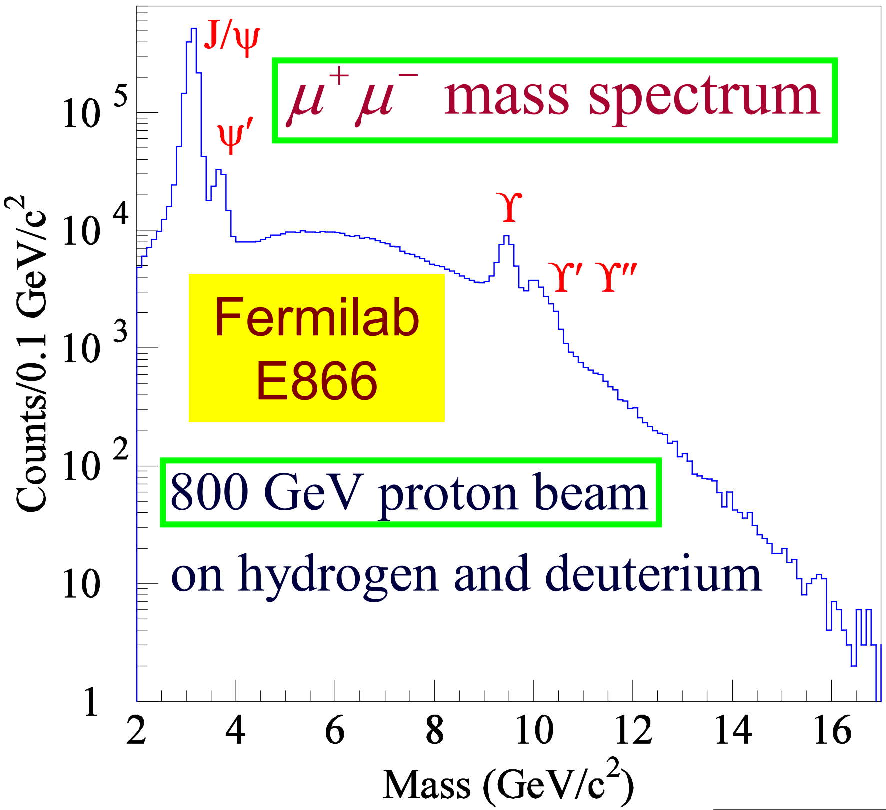


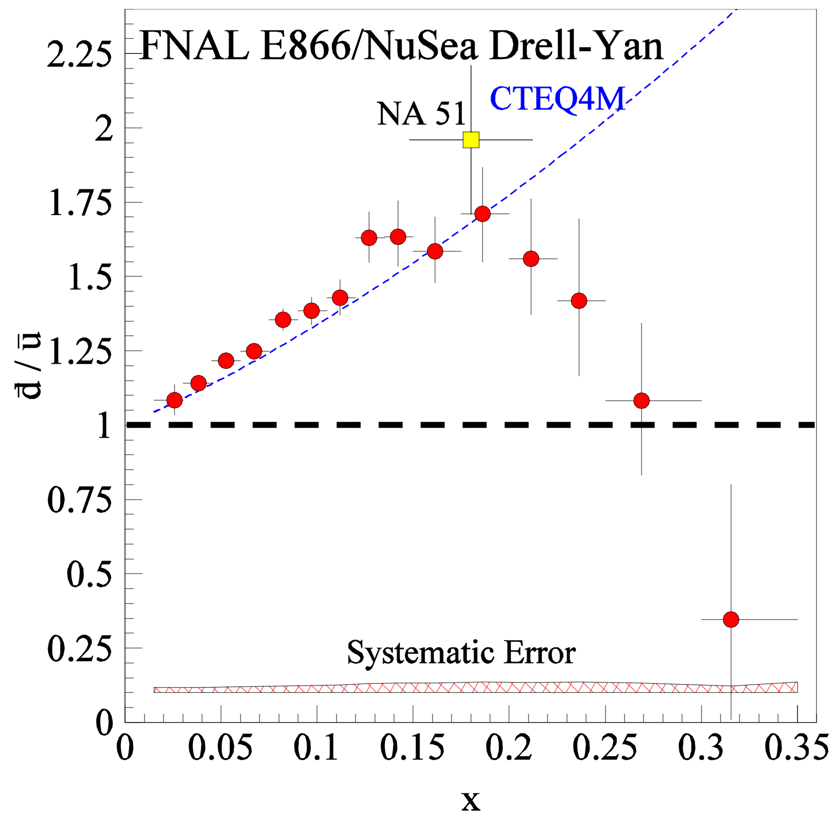
1) Fermilab E772 (proposed in 1986 and completed in 1988)
   "Nuclear Dependence of Drell-Yan and Quarkonium Production"
2) Fermilab E789 (proposed in 1989 and completed in 1991)
   "Search for Two-Body Decays of Heavy Quark Mesons"
3) Fermilab E866 (proposed in 1993 and completed in 1996)
   "Determination of $\bar{d} / \bar{u}$ Ratio of the Proton via Drell-Yan"
4) Fermilab E906/SeaQuest (proposed in 1999, completed in 7/2017)
   "Drell-Yan with the FNAL Main Injector"
5) Fermilab E1039/SpinQuest (proposed in 2017, beam expected 2024)
   "Drell-Yan with Transversely Polarized Target"

EXPERIMENT E789- Moving Cable at Meson.
"The Snake". (1990)

$\bar{d}\,/\,\bar{u}$ flavor asymmetry from Drell-Yan

$$\left(\frac{d^2\sigma}{dx_1 dx_2}\right)_{D.Y.} = \frac{4\pi\alpha^2}{9sx_1x_2}\sum_a e_a^2\left[q_a(x_1)\bar{q}_a(x_2)+\bar{q}_a(x_1)q_a(x_2)\right]$$





$at\;\; x_1 > x_2:$ Drell-Yan: $\boxed{\sigma^{pd}/2\sigma^{pp} \sim \frac{1}{2}(1+\bar{d}(x_2)/\bar{u}(x_2))}$

Angular Distribution in the "Naïve" Drell-Yan

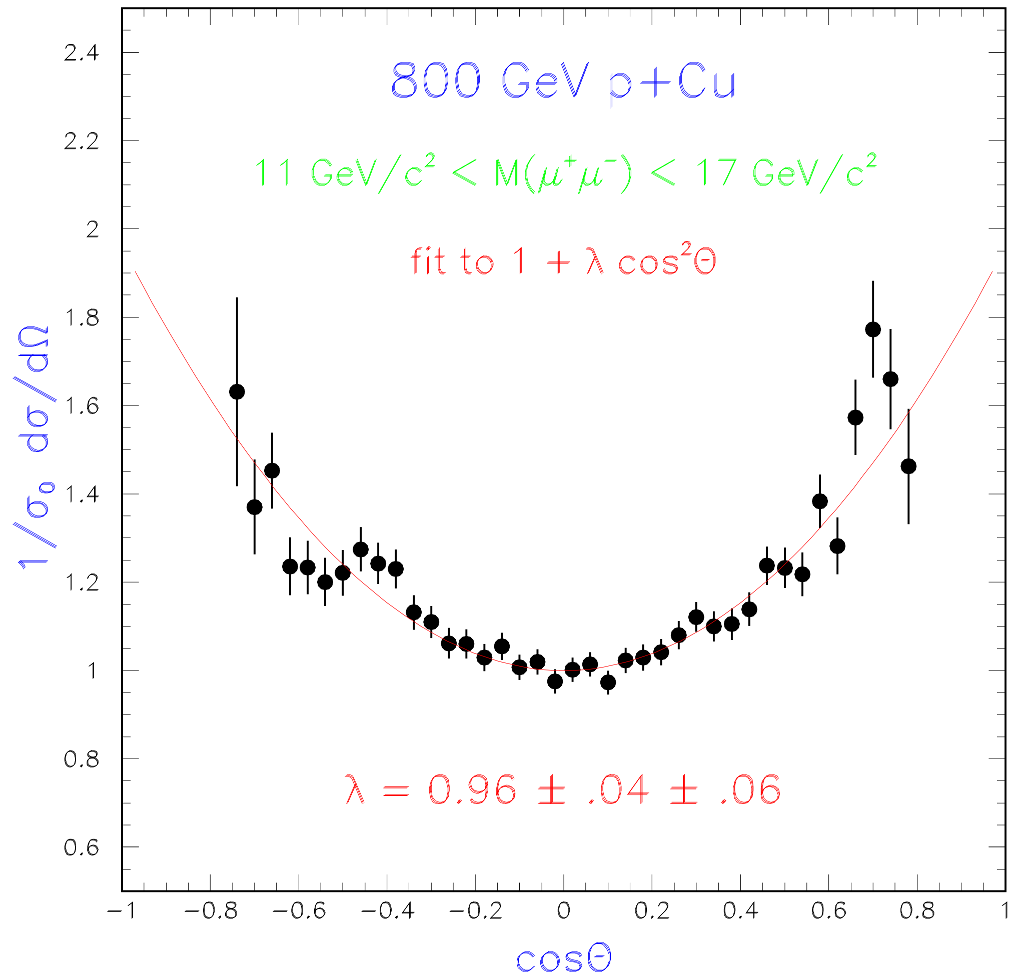
Volume 25, Number 5 PHYSICAL REVIEW LETTERS 3 August 1970

(3) The virtual photon will be predominantly transversely polarized if it is formed by annihilation of spin-$\frac{1}{2}$ parton-antiparton pairs. This means a distribution in the di-muon rest system varying $\boxed{\text{as } (1+\cos^2\theta) \text{ rather than } \sin^2\theta}$ as found in Sakurai's[10] vector-dominance model, where θ is the angle of the muon with respect to the time-like photon momentum. The model used in Fig.

Drell-Yan angular distribution

Lepton Angular Distribution of "naïve" Drell-Yan:

$$\frac{d\sigma}{d\Omega} = \sigma_0(1+\lambda\cos^2\theta); \qquad \lambda = 1$$

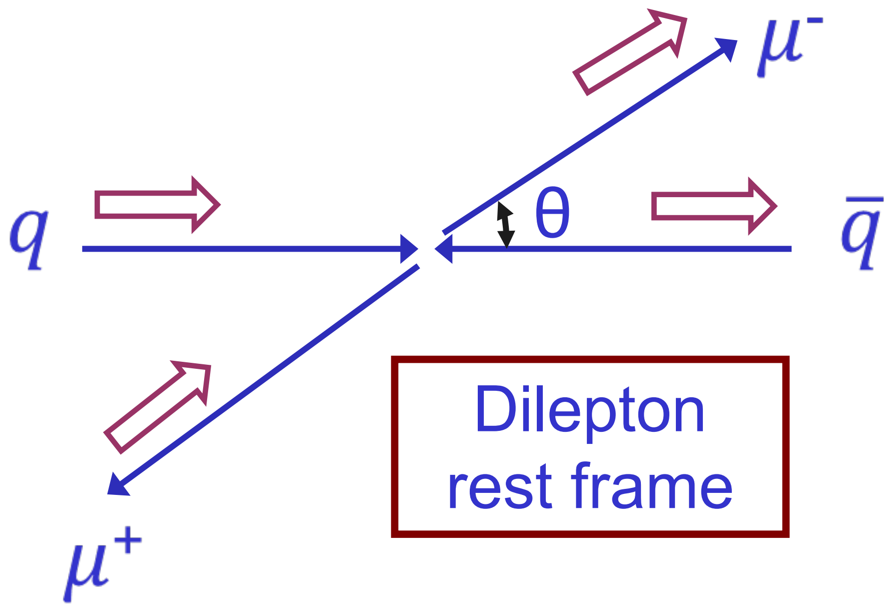


Data from Fermilab E772

(Ann. Rev. Nucl. Part. Sci. 49 (1999) 217-253)

Why is the lepton angular distribution $1 + \cos^2\theta$?

Helicity conservation and parity



$RL \rightarrow RL$

$$d\sigma \sim (1+\cos\theta)^2$$

$RL \rightarrow LR$

$$d\sigma \sim (1-\cos\theta)^2$$

$LR \rightarrow LR$

$$d\sigma \sim (1+\cos\theta)^2$$

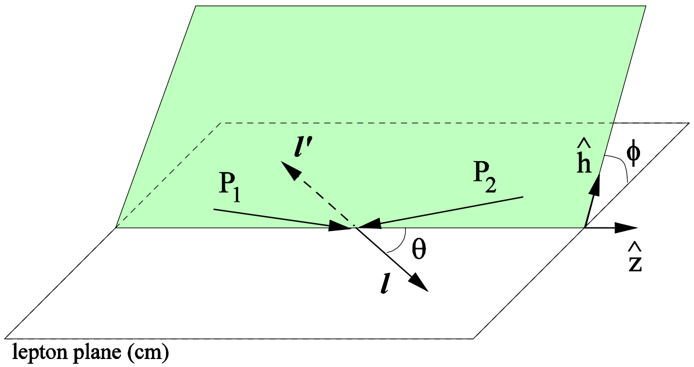
$LR \rightarrow RL$

$$d\sigma \sim (1-\cos\theta)^2$$

Adding all four helicity configurations:

$$d\sigma \sim 1+\cos^2\theta$$

Drell-Yan lepton angular distributions for $p_T > 0$



Θ and Φ are the decay polar and azimuthal angles of the μ^- in the dilepton rest-frame

Collins-Soper frame

A general expression for Drell-Yan decay angular distributions:

$$\left(\frac{1}{\sigma}\right)\left(\frac{d\sigma}{d\Omega}\right)=\left[\frac{3}{4\pi}\right]\left[1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi\right]$$

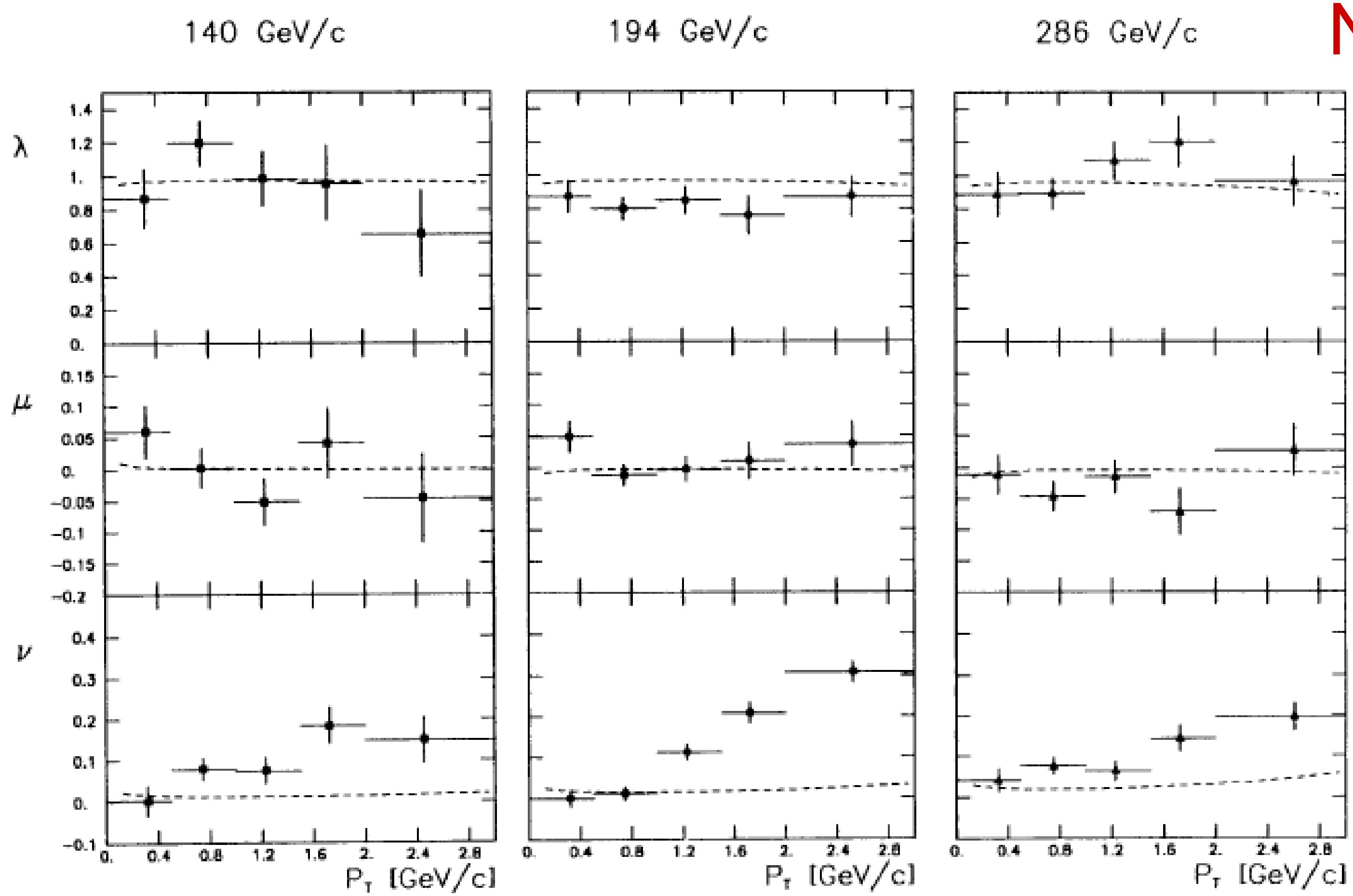
Lam-Tung relation: $1-\lambda=2\nu$

- Reflect the spin-1/2 nature of quarks (analog of the Callan-Gross relation in DIS)
- Insensitive to QCD - corrections

Decay angular distributions in pion-induced Drell-Yan

$$\left(\frac{1}{\sigma}\right)\left(\frac{d\sigma}{d\Omega}\right)=\left[\frac{3}{4\pi}\right]\left[1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi\right]$$

NA10 $\pi^- + \mathbf{W}$



Z. Phys.

37 (1988) 545

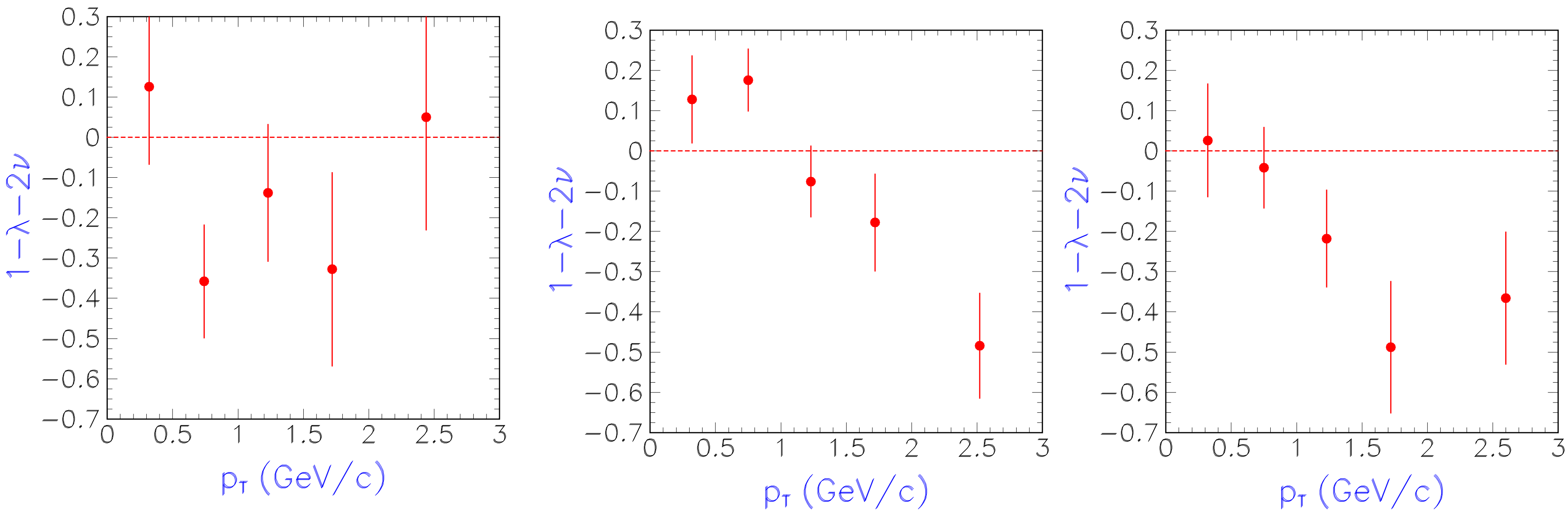
Dashed curves are from pQCD calculations

$\nu \neq 0$ and ν increases with p_T

Decay angular distributions in pion-induced Drell-Yan

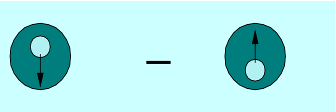
Is the Lam-Tung relation ($1-\lambda-2\nu=0$) violated?

140 GeV/c 194 GeV/c 286 GeV/c



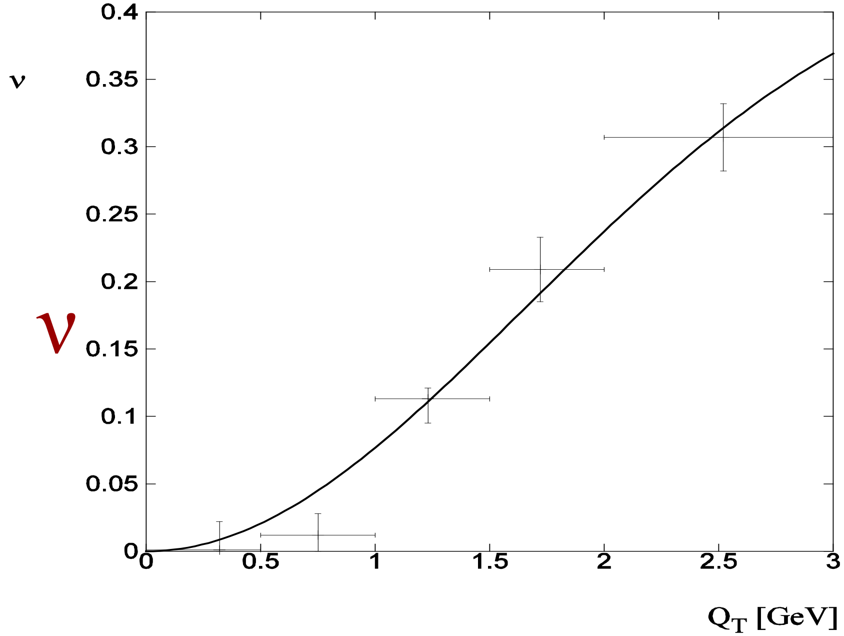
Data from NA10 (Z. Phys. 37 (1988) 545)

Violation of the Lam-Tung relation suggests interesting new origins (Brandenburg, Nachtmann, Mirkes, Brodsky, Khoze, Müller, Eskolar, Hoyer,Väntinnen, Vogt, etc.)

Boer-Mulders function $h_1^\perp$ 

- Boer pointed out that the cos2ϕ dependence can be caused by the presence of the Boer-Mulders function.

- $h_1^\perp$ can lead to an azimuthal dependence with $\nu \propto \left(\frac{h_1^\perp}{f_1}\right)\left(\frac{\overline{h}_1^\perp}{\overline{f}_1}\right)$



Boer, PRD 60 (1999) 014012

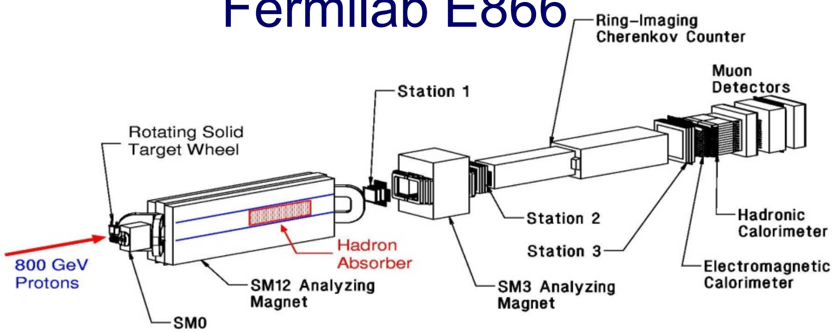
The violation of the Lam-Tung relation is due to the presence of the Boer-Mulders TMD function

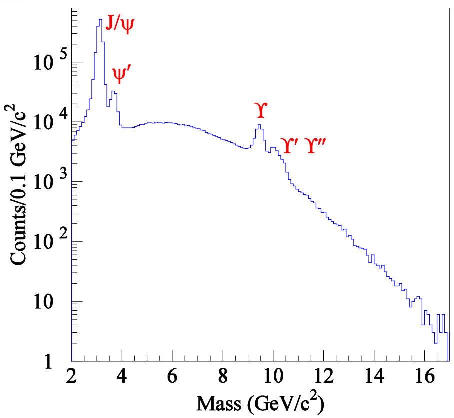
The puzzle is resolved. It also leads to the first extraction of the Boer-Mulders function

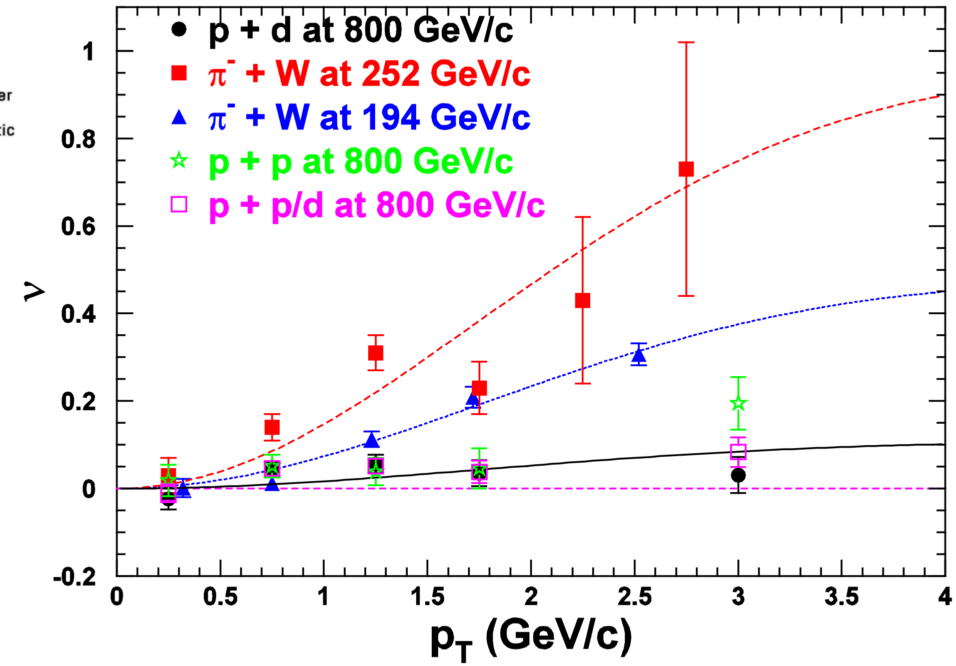
Azimuthal cos2Φ Distribution in p+d Drell-Yan

Lingyan Zhu, JCP et al., PRL 99 (2007) 082301; PRL 102 (2009) 182001

Fermilab E866







With Boer-Mulders function $h_1^{\perp}$:

$\nu(\pi^- W \rightarrow \mu^+\mu^- X) \sim$ [valence $h_1^{\perp}(\pi)$] * [valence $h_1^{\perp}(p)$]

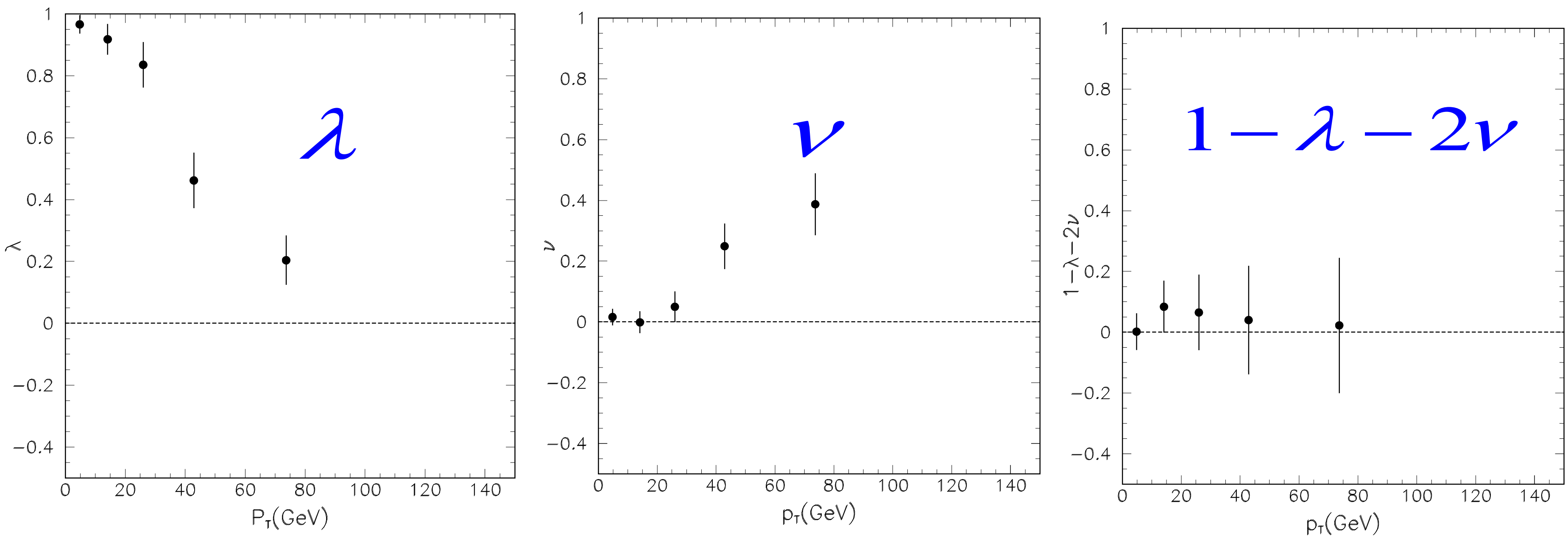
$\nu(pd \rightarrow \mu+\mu- X) \sim$ [valence $h_1^{\perp}(p)$] * [sea $h_1^{\perp}(p)$]

Sea-quark BM function is much smaller than valence BM function

Angular distribution data from CDF Z-production

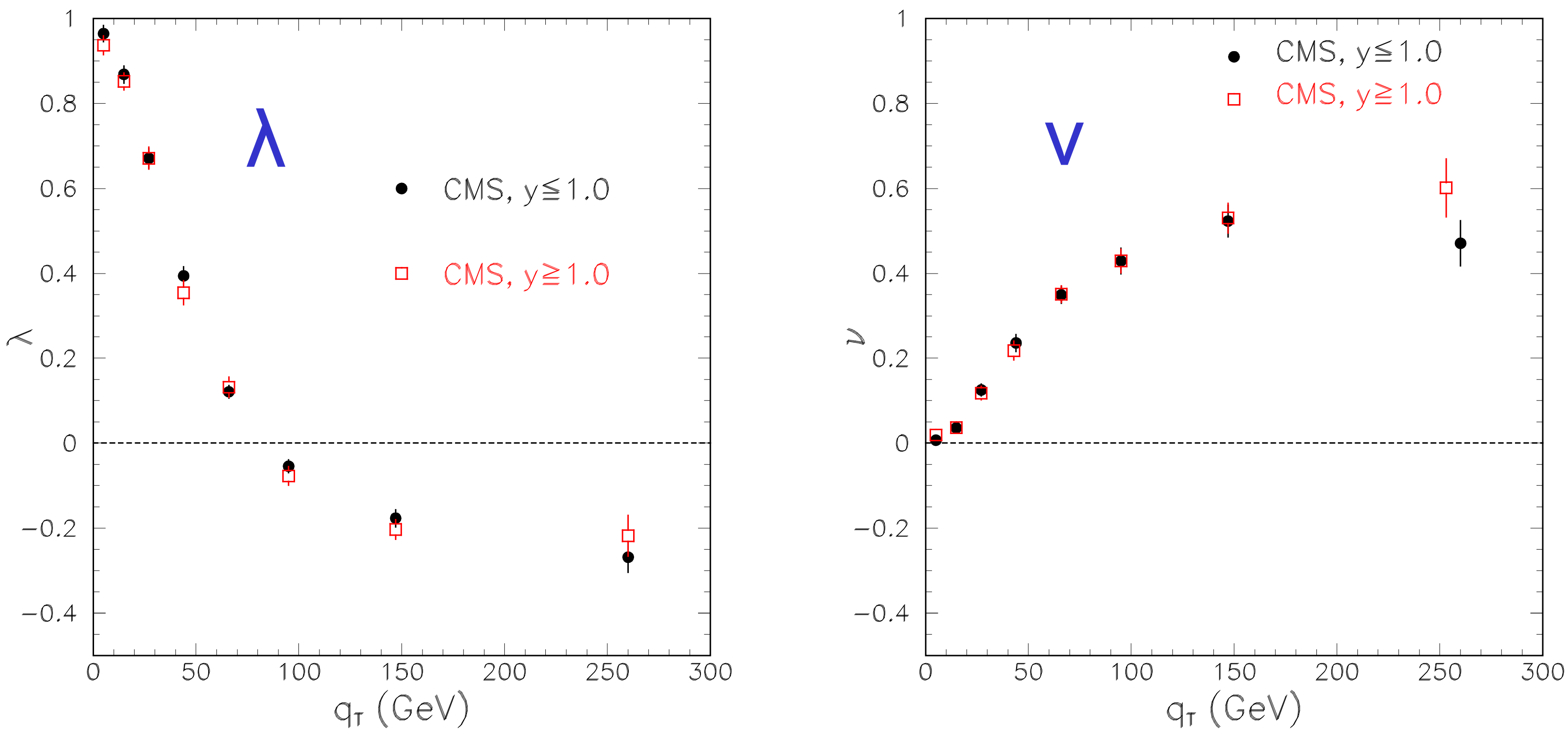
$$p+\overline{p}\rightarrow e^{+}+e^{-}+X \;\; \text{at} \sqrt{s}=1.96\,\text{TeV}$$

arXiv:1103.5699 (PRL 106 (2011) 241801)



- Strong p_T (q_T) dependence of λ and ν
- Lam-Tung relation (1-λ = 2ν) is satisfied within experimental uncertainties (TMD is not expected to be important at large p_T)

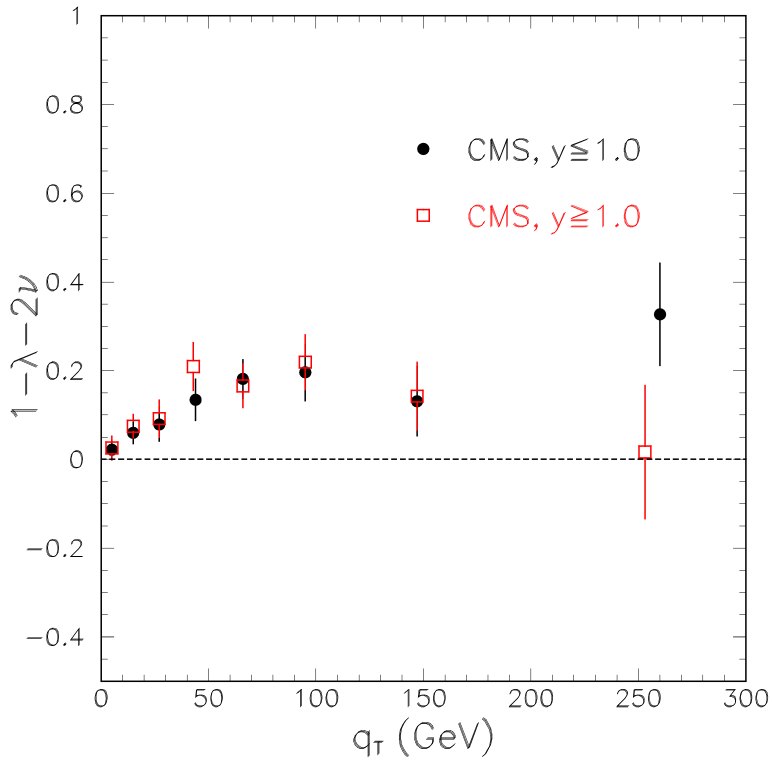
CMS (ATLAS) data for *Z*-boson production in *p*+*p* collision at 8 TeV



(arXiv:1504.03512, PL B 750 (2015) 154)

- Striking q_T (p_T) dependencies for λ and ν were observed at two rapidity regions
- Is Lam-Tung relation violated?

Recent data from CMS for *Z*-boson production in *p*+*p* collision at 8 TeV



- Yes, the Lam-Tung relation is violated ($1-\lambda > 2\nu$)!
- Can one understand the origin of the violation of the Lam-Tung relation (It cannot be due to the Boer-Mulders function)?

Interpretation of the CMS Z-production results

$$\frac{d\sigma}{d\Omega} \propto (1+\cos^2\theta) + \frac{A_0}{2}(1-3\cos^2\theta) + A_1 \sin 2\theta \cos\phi$$
$$+ \frac{A_2}{2}\sin^2\theta\cos 2\phi + A_3 \sin\theta\cos\phi + A_4\cos\theta$$
$$+ A_5 \sin^2\theta \sin 2\phi + A_6 \sin 2\theta \sin\phi + A_7 \sin\theta\sin\phi$$

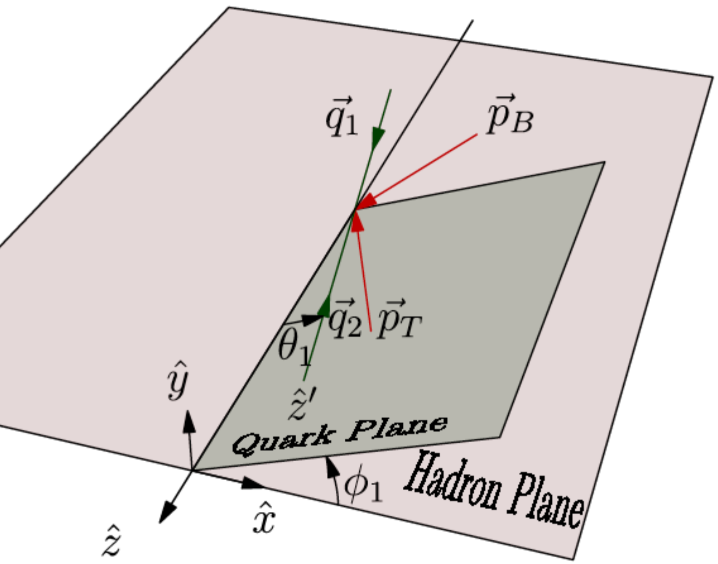
Questions:

- How is the above expression derived?
- Can one express $A_0 - A_7$ in terms of some quantities?
- Can one understand the q_T dependence of A_0, A_1, A_2, etc?
- Can one understand the origin of the violation of Lam-Tung relation?

$$\boxed{\lambda = \frac{2-3A_0}{2+A_0}; \quad \nu = \frac{2A_2}{2+A_0}; \quad \text{L-T relation, } 1-\lambda = 2\nu, \text{ becomes } A_0 = A_2}$$

How is the angular distribution expression derived?

Define three planes in the Collins-Soper frame



1) Hadron Plane

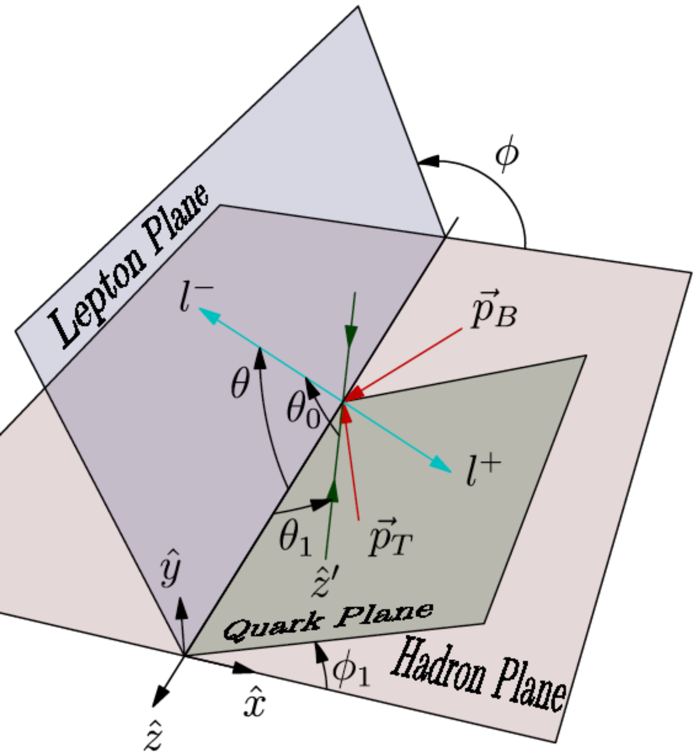
- Contains the beam $\vec{P}_B$ and target $\vec{P}_T$ momenta
- Angle β satisfies the relation $\tan\beta = q_T / Q$

2) Quark Plane

- q and $\overline{q}$ have head-on collision along the $\hat{z}'$ axis
- $\hat{z}'$ and $\hat{z}$ axes form the quark plane
- $\hat{z}'$ axis has angles θ_1 and ϕ_1 in the C-S frame

How is the angular distribution expression derived?

Define three planes in the Collins-Soper frame



1) Hadron Plane

- Contains the beam $\vec{P}_B$ and target $\vec{P}_T$ momenta
- Angle β satisfies the relation $\tan\beta = q_T / Q$

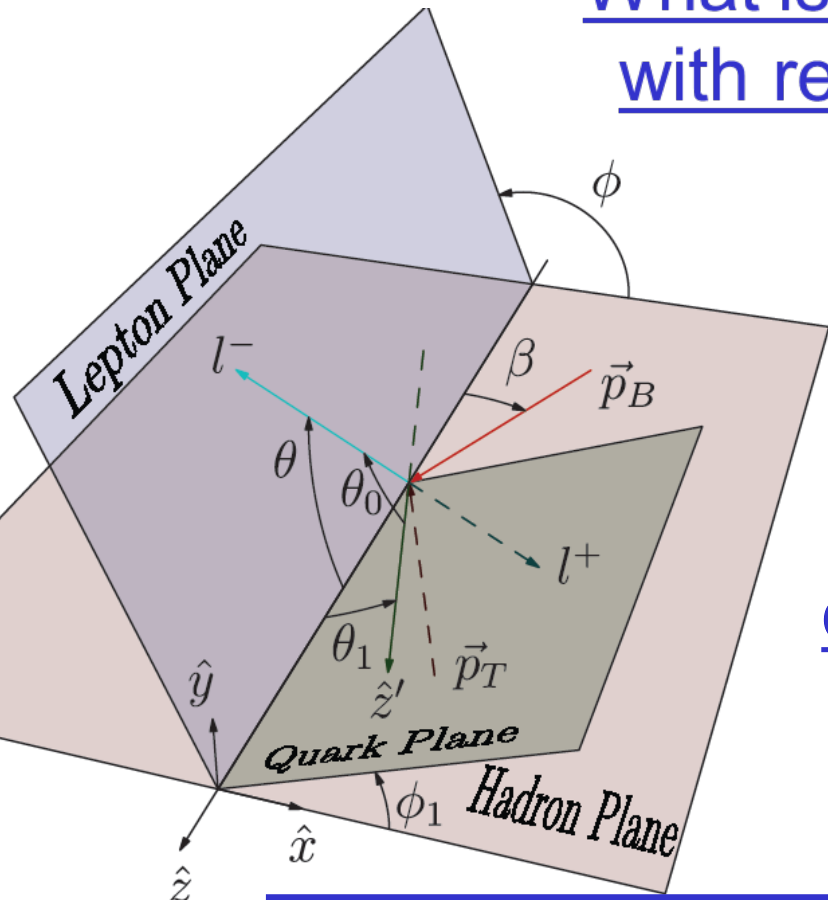
2) Quark Plane

- q and $\overline{q}$ have head-on collision along the $\hat{z}'$ axis
- $\hat{z}'$ axis has angles θ_1 and ϕ_1 in the C-S frame

3) Lepton Plane

- l^- and l^+ are emitted back-to-back with equal $|\vec{P}|$
- l^- and $\hat{z}$ form the lepton plane
- l^- is emitted at angle θ and ϕ in the C-S frame

How is the angular distribution expression derived?



What is the lepton angular distribution with respect to the $\hat{z}'$ (natural) axis?

$$\frac{d\sigma}{d\Omega} \propto 1 + a\cos\theta_0 + \cos^2\theta_0$$

Azimuthally symmetric !

How to express the angular distribution in terms of θ and ϕ?

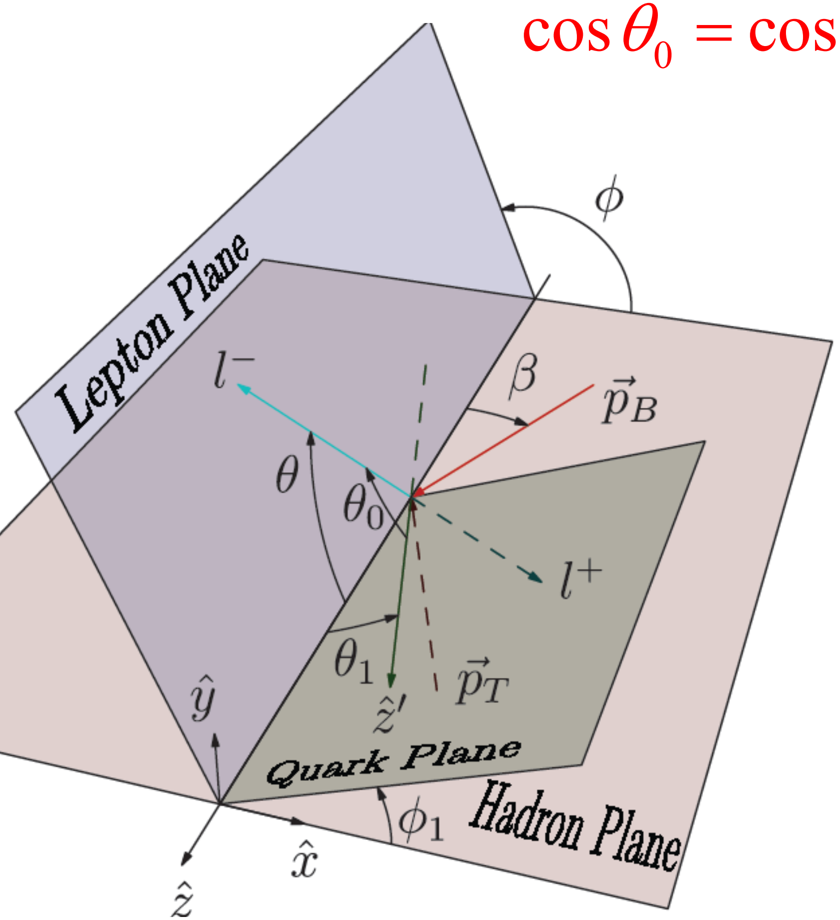
Use the following relation (addition theorem):

$$\boxed{\cos\theta_0 = \cos\theta\cos\theta_1 + \sin\theta\sin\theta_1\cos(\phi - \phi_1)}$$

How is the angular distribution expression derived?

$$\frac{d\sigma}{d\Omega} \propto 1 + a\cos\theta_0 + \cos^2\theta_0$$

$$\cos\theta_0 = \cos\theta\cos\theta_1 + \sin\theta\sin\theta_1\cos(\phi - \phi_1)$$



$$\begin{aligned}\frac{d\sigma}{d\Omega} \propto\ & (1+\cos^2\theta) + \frac{\sin^2\theta_1}{2}(1-3\cos^2\theta) \\ & + (\frac{1}{2}\sin 2\theta_1\cos\phi_1)\sin 2\theta\cos\phi \\ & + (\frac{1}{2}\sin^2\theta_1\cos 2\phi_1)\sin^2\theta\cos 2\phi \\ & + (a\sin\theta_1\cos\phi_1)\sin\theta\cos\phi + (a\cos\theta_1)\cos\theta \\ & + (\frac{1}{2}\sin^2\theta_1\sin 2\phi_1)\sin^2\theta\sin 2\phi \\ & + (\frac{1}{2}\sin 2\theta_1\sin\phi_1)\sin 2\theta\sin\phi \\ & + (a\sin\theta_1\sin\phi_1)\sin\theta\sin\phi.\end{aligned}$$

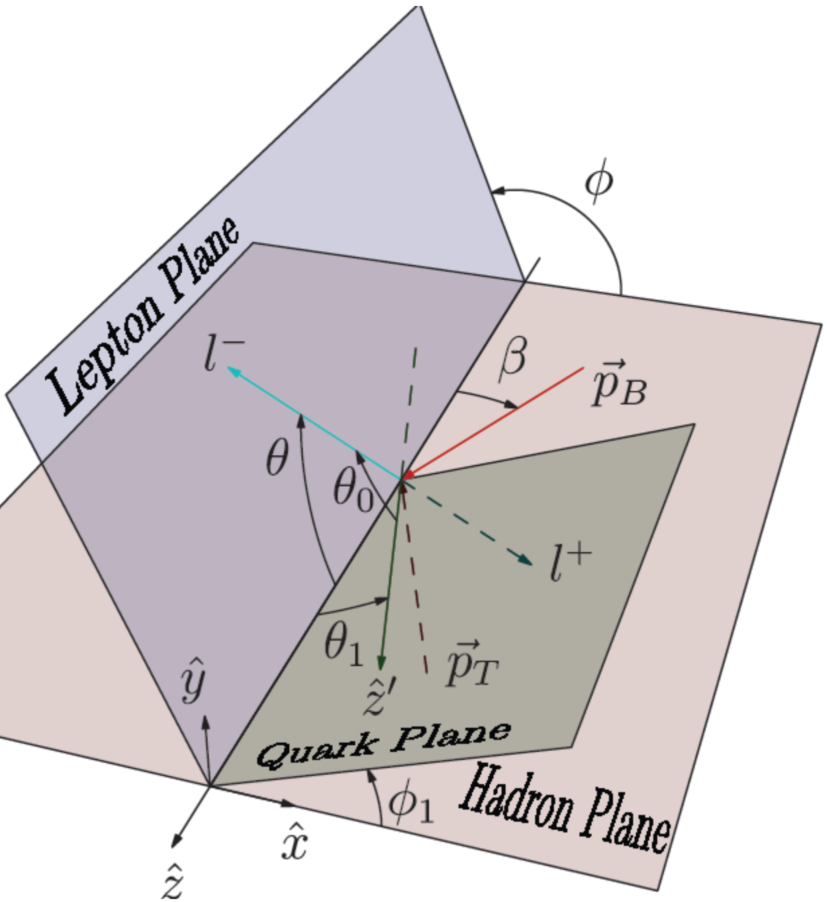
All eight angular distribution terms are obtained!

$$
\begin{aligned}
\frac{d\sigma}{d\Omega} \propto\ & (1+\cos^2\theta) + \frac{\sin^2\theta_1}{2}(1-3\cos^2\theta) \\
& + (\frac{1}{2}\sin 2\theta_1 \cos\phi_1)\sin 2\theta\cos\phi \\
& + (\frac{1}{2}\sin^2\theta_1 \cos 2\phi_1)\sin^2\theta\cos 2\phi \\
& + (a\sin\theta_1\cos\phi_1)\sin\theta\cos\phi + (a\cos\theta_1)\cos\theta \\
& + (\frac{1}{2}\sin^2\theta_1 \sin 2\phi_1)\sin^2\theta\sin 2\phi \\
& + (\frac{1}{2}\sin 2\theta_1 \sin\phi_1)\sin 2\theta\sin\phi \\
& + (a\sin\theta_1\sin\phi_1)\sin\theta\sin\phi .
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\sigma}{d\Omega} \propto\ & (1+\cos^2\theta) + \frac{A_0}{2}(1-3\cos^2\theta) \\
& + A_1 \sin 2\theta\cos\phi \\
& + \frac{A_2}{2}\sin^2\theta\cos 2\phi \\
& + A_3\sin\theta\cos\phi + A_4\cos\theta \\
& + A_5\sin^2\theta\sin 2\phi \\
& + A_6\sin 2\theta\sin\phi \\
& + A_7\sin\theta\sin\phi
\end{aligned}
$$

$A_0 - A_7$ are entirely described by θ_1, ϕ_1 and a

Angular distribution coefficients $A_0 - A_7$



$$A_0 = \left\langle \sin^2\theta_1 \right\rangle$$

$$A_1 = \frac{1}{2}\left\langle \sin 2\theta_1 \cos\phi_1 \right\rangle$$

$$A_2 = \left\langle \sin^2\theta_1 \cos 2\phi_1 \right\rangle$$

$$A_3 = a\left\langle \sin\theta_1 \cos\phi_1 \right\rangle$$

$$A_4 = a\left\langle \cos\theta_1 \right\rangle$$

$$A_5 = \frac{1}{2}\left\langle \sin^2\theta_1 \sin 2\phi_1 \right\rangle$$

$$A_6 = \frac{1}{2}\left\langle \sin 2\theta_1 \sin\phi_1 \right\rangle$$

$$A_7 = a\left\langle \sin\theta_1 \sin\phi_1 \right\rangle$$

Some implications of the angular distribution coefficients $A_0 - A_7$

$$A_0 = \langle \sin^2\theta_1 \rangle$$

$$A_1 = \frac{1}{2}\langle \sin 2\theta_1 \cos\phi_1 \rangle$$

$$A_2 = \langle \sin^2\theta_1 \cos 2\phi_1 \rangle$$

$$A_3 = a\langle \sin\theta_1 \cos\phi_1 \rangle$$

$$A_4 = a\langle \cos\theta_1 \rangle$$

$$A_5 = \frac{1}{2}\langle \sin^2\theta_1 \sin 2\phi_1 \rangle$$

$$A_6 = \frac{1}{2}\langle \sin 2\theta_1 \sin\phi_1 \rangle$$

$$A_7 = a\langle \sin\theta_1 \sin\phi_1 \rangle$$

- $A_0 \geq A_2$ (or $1 - \lambda - 2\nu \geq 0$)
- Lam-Tung relation ($A_0 = A_2$) is satisfied when $\phi_1 = 0$
- Forward-backward asymmetry, a, is reduced by a factor of $\langle \cos\theta_1 \rangle$ for A_4
- A_5, A_6, A_7 are odd function of ϕ_1 and must vanish from symmetry consideration
- Some equality and inequality relations among $A_0 - A_7$ can be obatined

Some implications of the angular distribution coefficients $A_0 - A_7$

$$A_0 = \langle \sin^2\theta_1 \rangle$$

$$A_1 = \frac{1}{2}\langle \sin 2\theta_1 \cos\phi_1 \rangle$$

$$A_2 = \langle \sin^2\theta_1 \cos 2\phi_1 \rangle$$

$$A_3 = a\langle \sin\theta_1 \cos\phi_1 \rangle$$

$$A_4 = a\langle \cos\theta_1 \rangle$$

$$A_5 = \frac{1}{2}\langle \sin^2\theta_1 \sin 2\phi_1 \rangle$$

$$A_6 = \frac{1}{2}\langle \sin 2\theta_1 \sin\phi_1 \rangle$$

$$A_7 = a\langle \sin\theta_1 \sin\phi_1 \rangle$$

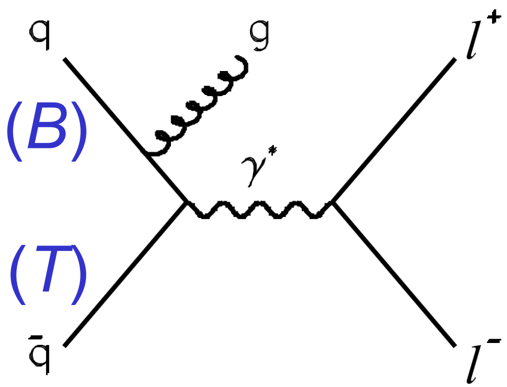
Some bounds on the coefficients can be obtained

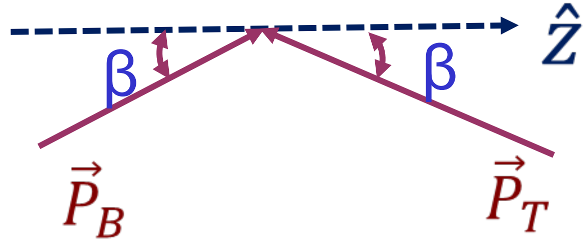
$$\boxed{\begin{array}{l} 0 < A_0 < 1 \\ -1/2 < A_1 < 1/2 \\ -1 < A_2 < 1 \\ -a < A_3 < a \\ -a < A_4 < a \end{array}}$$

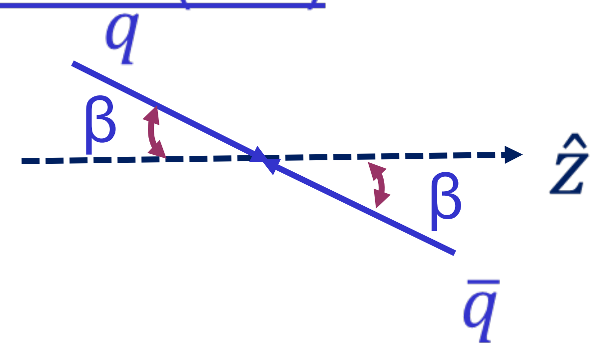
What are the values of θ_1 and ϕ_1 at order α_s?

1) $q\bar{q} \to \gamma^*(Z^0)g$

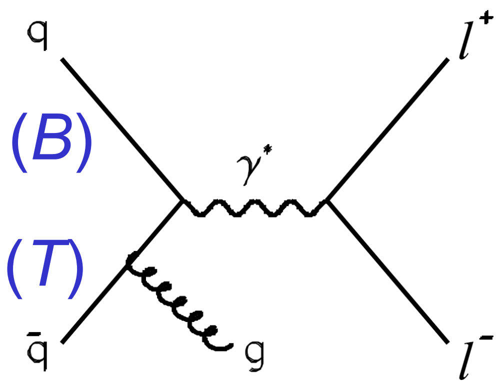
In γ^* rest frame (C-S)

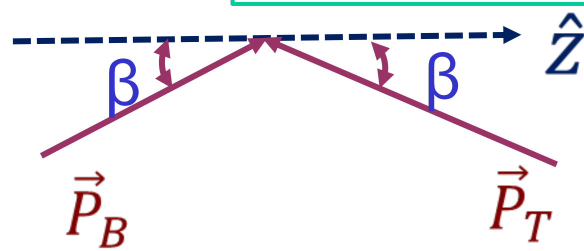


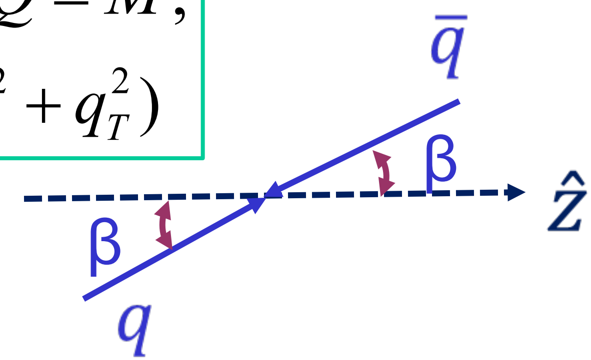




$$\boxed{\begin{aligned}&\tan\beta = q_T/Q;\ \ Q = M;\\ &\sin^2\beta = q_T^2/(Q^2+q_T^2)\end{aligned}}$$







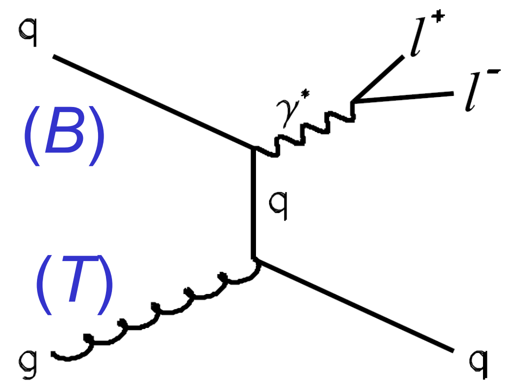
$\boxed{\theta_1 = \beta \ \text{ and } \ \phi_1 = 0;}$ $A_0 = A_2 = \sin^2\beta$

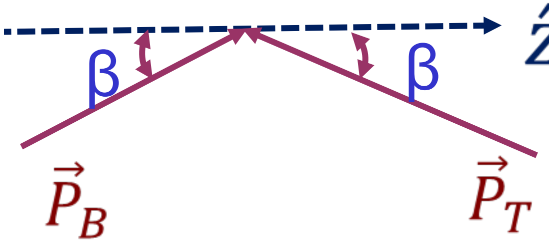
$$\lambda = \frac{2-3A_0}{2+A_0} = \frac{2Q^2-q_T^2}{2Q^2+3q_T^2}; \qquad \nu = \frac{2A_2}{2+A_0} = \frac{2q_T^2}{2Q^2+3q_T^2}$$

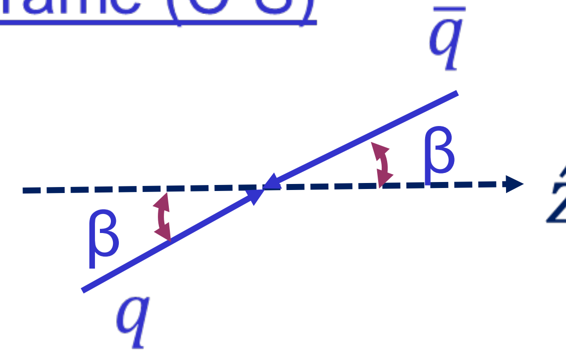
What are the values of θ_1 and ϕ_1 at order α_s?

2) $qg \rightarrow \gamma^*(Z^0)q$

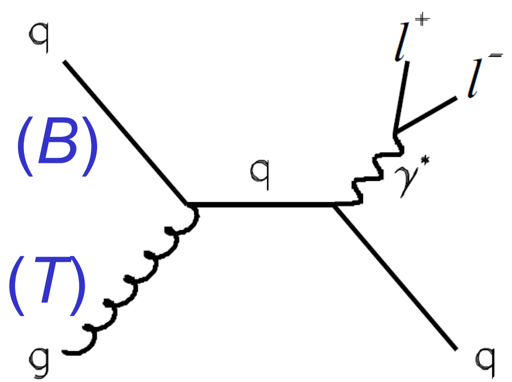
In γ^* rest frame (C-S)

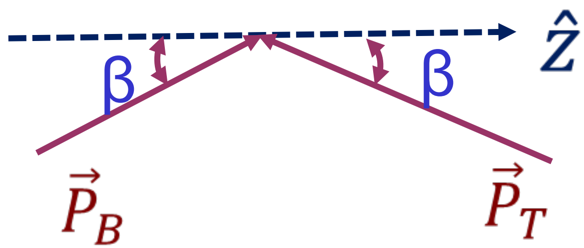


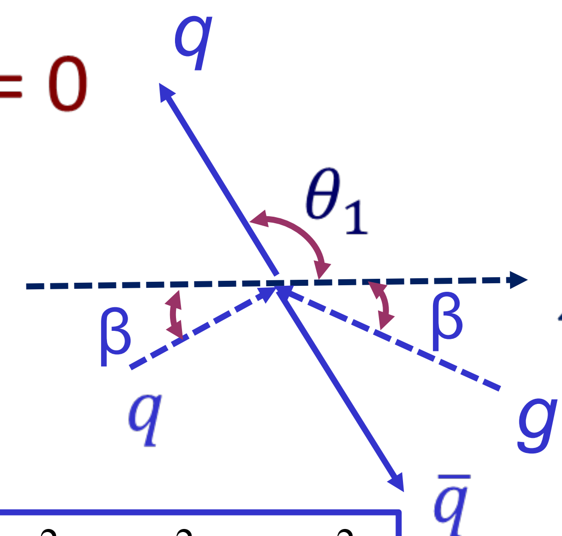




$\theta_1 = \beta$ and $\phi_1 = 0$





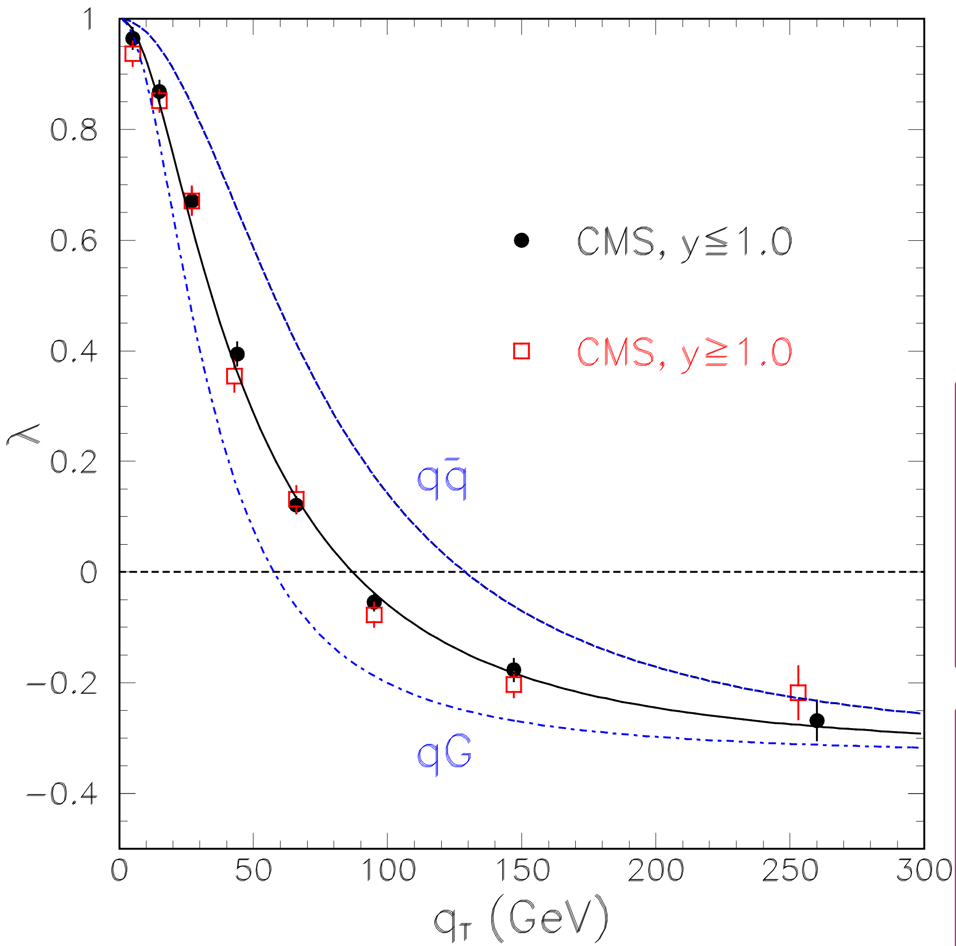


$$\boxed{\theta_1 > \beta \text{ and } \phi_1 = 0; \quad A_0 = A_2 \approx 5q_T^2/(Q^2 + 5q_T^2)}$$

$$\lambda = \frac{2 - 3A_0}{2 + A_0} = \frac{2Q^2 - 5q_T^2}{2Q^2 + 15q_T^2}; \qquad \nu = \frac{2A_2}{2 + A_0} = \frac{10q_T^2}{2Q^2 + 15q_T^2}$$

Compare with CMS data on λ

(*Z* production in *p*+*p* collision at 8 TeV)



$$\lambda = \frac{2Q^2 - q_T^2}{2Q^2 + 3q_T^2} \quad \text{for} \quad q\bar{q} \to Zg$$

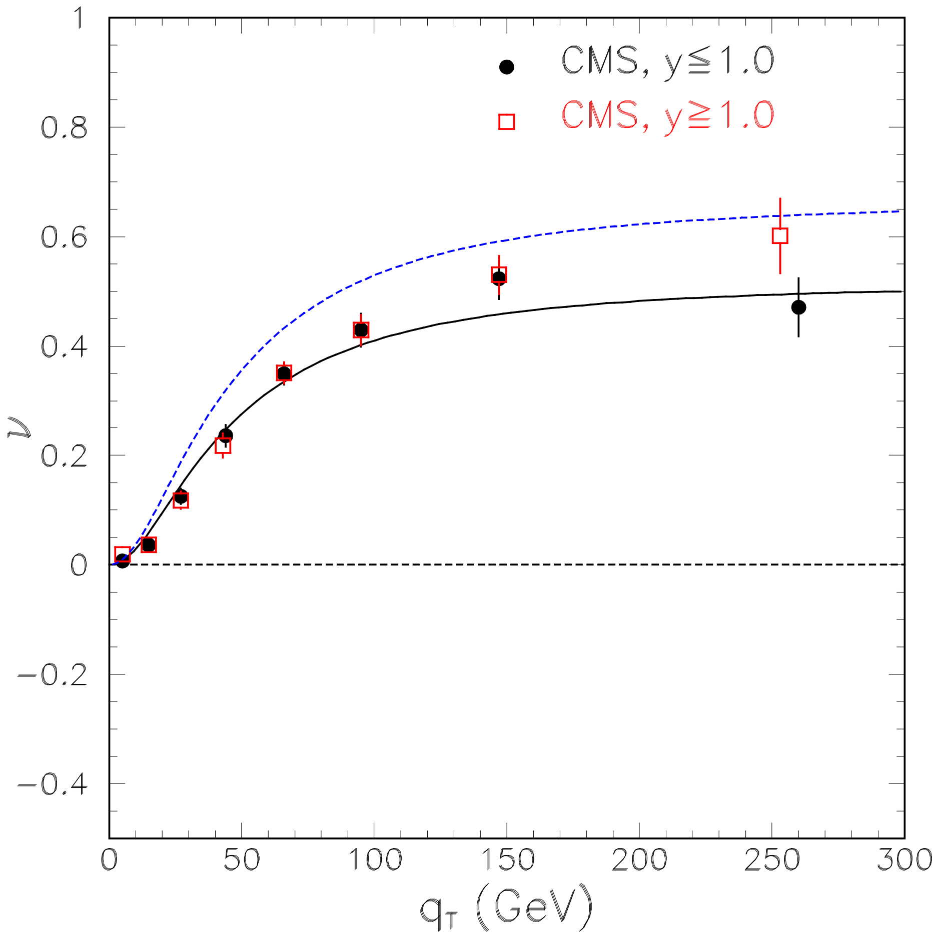
$$\lambda = \frac{2Q^2 - 5q_T^2}{2Q^2 + 15q_T^2} \quad \text{for} \quad qG \to Zq$$

For both processes
$\lambda = 1$ at $q_T = 0$ ($\theta_1 = 0^o$)
$\lambda = -1/3$ at $q_T = \infty$ ($\theta_1 = 90^o$)

Data can be well described with a mixture of 58.5% qG and 41.5% $q\bar{q}$ processes

Compare with CMS data on ν

(*Z* production in *p*+*p* collision at 8 TeV)



$$\nu = \frac{2q_T^2}{2Q^2 + 3q_T^2} \quad \text{for} \quad q\bar{q} \to Zg$$

$$\nu = \frac{10q_T^2}{2Q^2 + 15q_T^2} \quad \text{for} \quad qG \to Zq$$

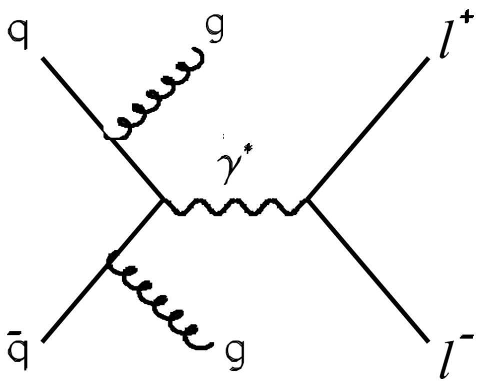
Dashed curve corresponds to a mixture of 58.5% qG and 41.5% $q\bar{q}$ processes

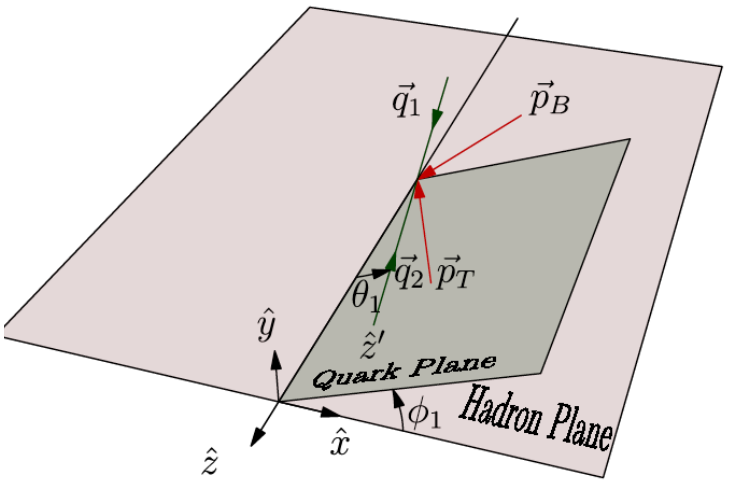
Solid curve corresponds to $\langle \sin^2\theta_1 \cos 2\phi_1 \rangle / \langle \sin^2\theta_1 \rangle = 0.77$

$q - \bar{q}$ axis is non-coplanar relative to the hadron plane

Origins of the non-coplanarity

1) Processes at order α_s^2 or higher

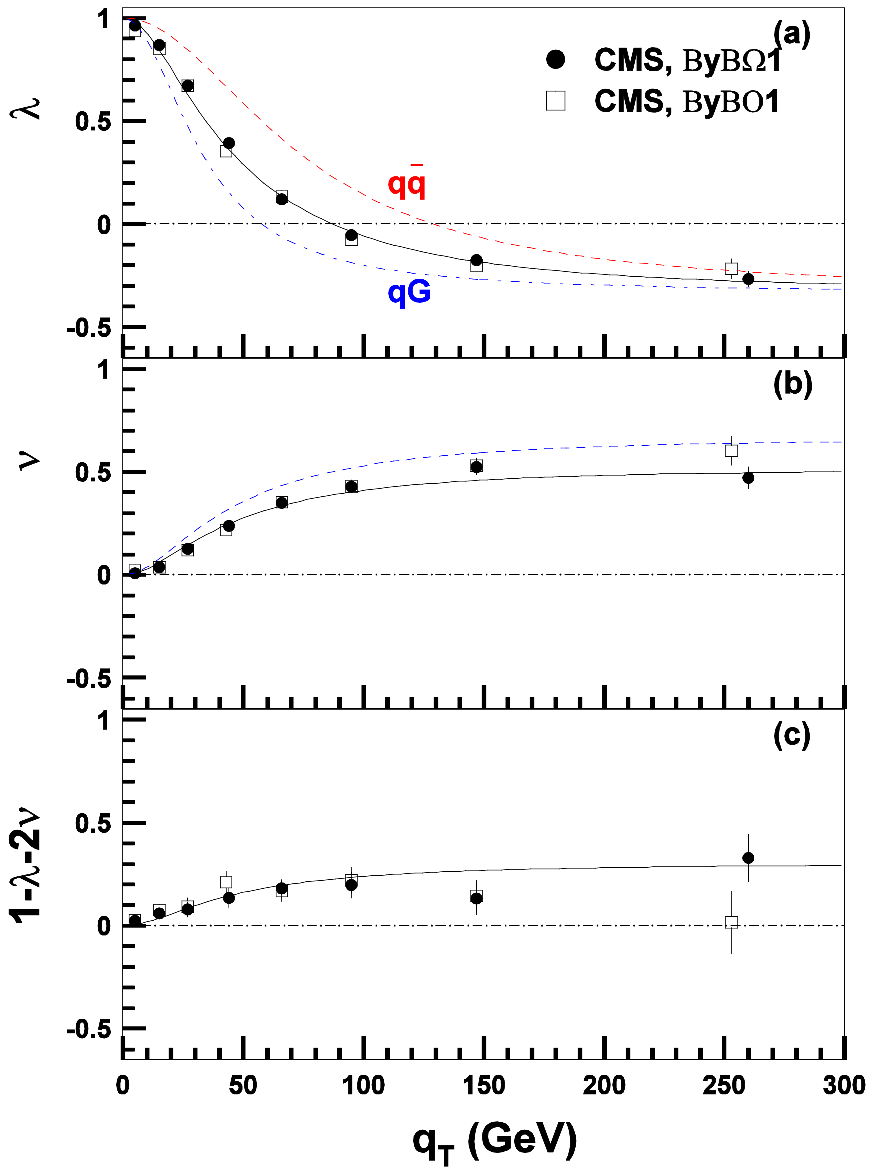




2) Intrinsic k_T from interacting partons

(Boer-Mulders functions in the beam and target hadrons)

Compare with CMS data on Lam-Tung relation

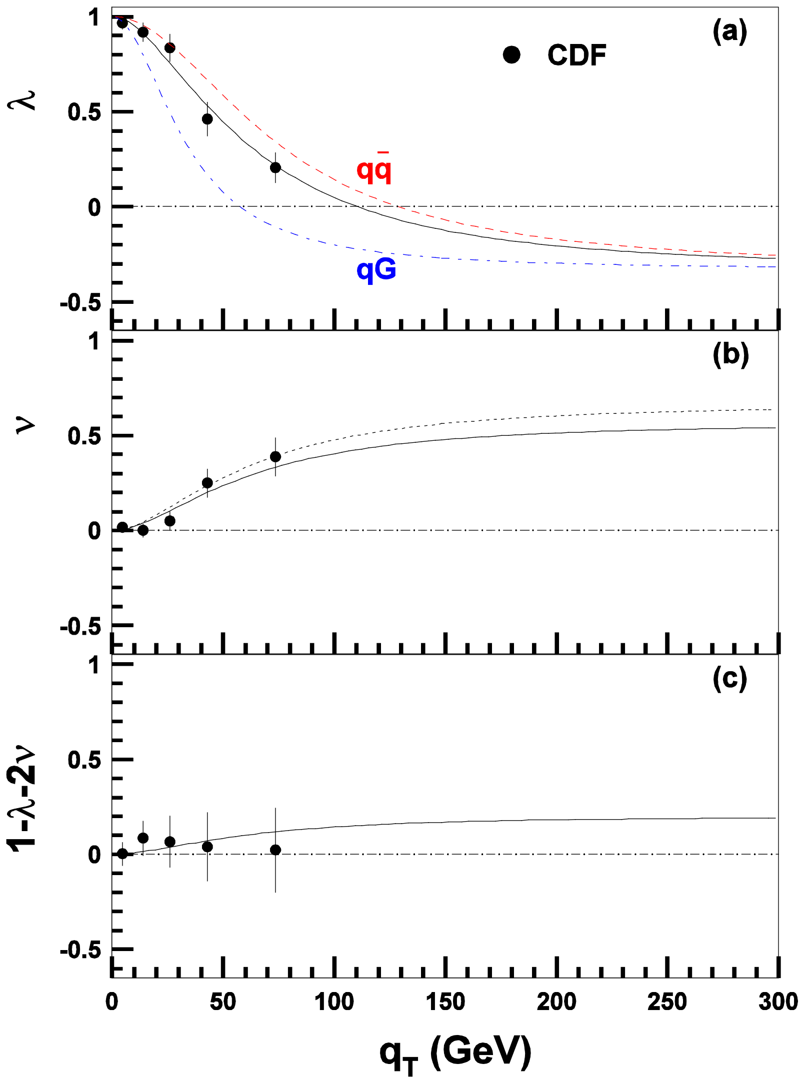


Solid curves correspond to a mixture of 58.5% qG and 41.5% $q\bar{q}$ processes, and $\langle \sin^2\theta_1 \cos 2\phi_1 \rangle / \langle \sin^2\theta_1 \rangle = 0.77$

Violation of Lam-Tung relation is well described

Compare with CDF data

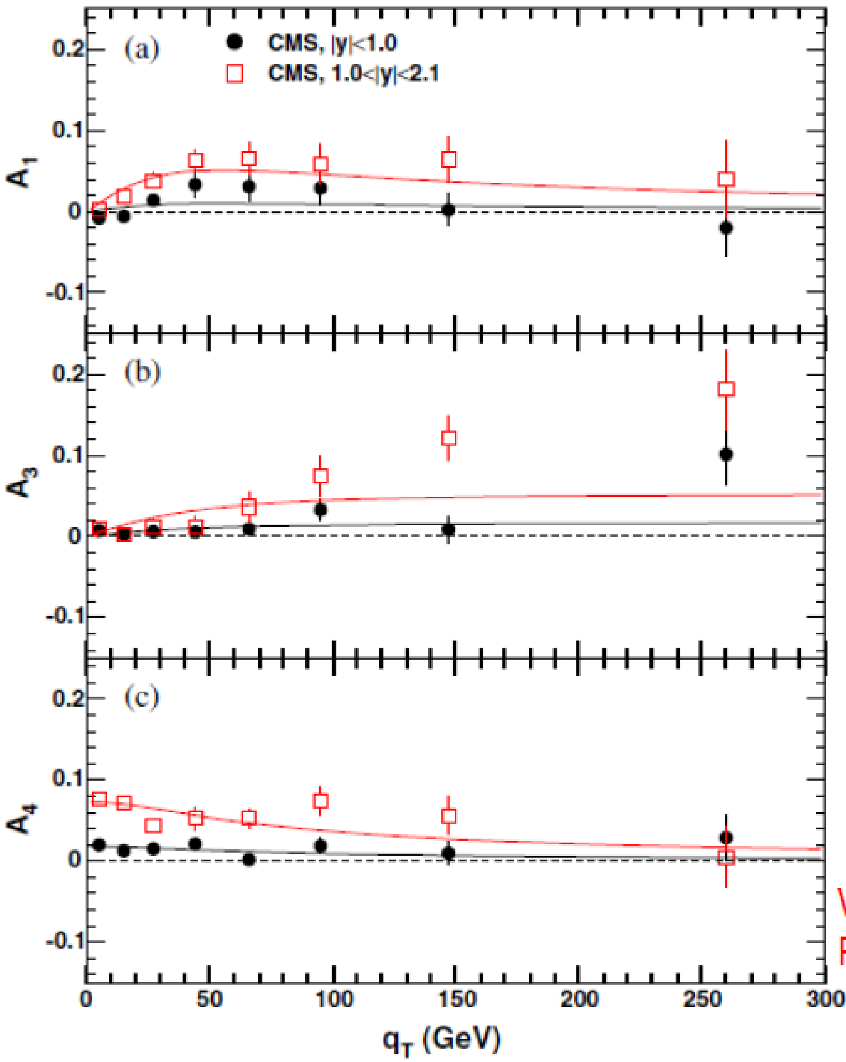
(Z production in $p + \bar{p}$ collision at 1.96 TeV)



Solid curves correspond to a mixture of 27.5% qG and 72.5% $q\bar{q}$ processes, and $\left\langle \sin^2\theta_1 \cos 2\phi_1 \right\rangle / \left\langle \sin^2\theta_1 \right\rangle = 0.85$

Violation of Lam-Tung relation is not ruled out

Compare CMS data on A_1, A_3 and A_4 with calculations



$$A_1 = r_1\left[f\frac{q_T Q}{Q^2+q_T^2} + (1-f)\frac{\sqrt{5}q_T Q}{Q^2+5q_T^2}\right]$$

$$A_3 = r_3\left[f\frac{q_T}{\sqrt{Q^2+q_T^2}} + (1-f)\frac{\sqrt{5}q_T}{\sqrt{Q^2+5q_T^2}}\right]$$

$$A_4 = r_4\left[f\frac{Q}{\sqrt{Q^2+q_T^2}} + (1-f)\frac{Q}{\sqrt{Q^2+5q_T^2}}\right]$$

Rapidity of A_1, A_3 and A_4 are well described

W.C. Chang, R.E. McClellan, J.C. Peng, O. Teryaev
Phys. Rev. D 96, 054020 (2017)

Other implications of the "geometric model"

- Extend this study to semi-inclusive DIS at high p_T (involving two hadrons and two leptons)
  - Relevant for EIC measurements
- Rotational invariance, equality, and inequality relations formed by various angular distribution coefficients
  - See preprint arXiv: 1808.04398 (Phys Lett B789 (2019) 352)
- Comparison with pQCD calculations
  - See preprint arXiv: 1811.03256 (PRD 99 (2019) 014032)
  - Lambertson and Vogelsang, PRD 93 (2016) 114013

Other implications

On the Rotational Invariance and Non-Invariance of Lepton Angular Distributions in Drell-Yan and Quarkonium Production

Jen-Chieh Peng[a], Daniël Boer[b], Wen-Chen Chang[c], Randall Evan McClellan[a,d], Oleg Teryaev[e]

arXiv:1808.04398 (Phys Lett B789 (2019) 352)

Quantities invariant under rotations along the y-axis (Faccioli et al.)

$$\mathcal{F} = \frac{1+\lambda+\nu}{3+\lambda}$$

$$\mathcal{F} = \frac{1+\lambda_0 - 2\lambda_0 \sin^2\theta_1 \sin^2\phi_1}{3+\lambda_0} = \frac{1+\lambda_0 - 2\lambda_0 y_1^2}{3+\lambda_0}$$

$$\tilde{\lambda} = \frac{2\lambda+3\nu}{2-\nu}$$

$$\tilde{\lambda} = \frac{\lambda_0 - 3\lambda_0 \sin^2\theta_1 \sin^2\phi_1}{1+\lambda_0 \sin^2\theta_1 \sin^2\phi_1} = \frac{\lambda_0 - 3\lambda_0 y_1^2}{1+\lambda_0 y_1^2}$$

$$\tilde{\lambda}' = \frac{(\lambda-\nu/2)^2 + 4\mu^2}{(3+\lambda)^2}$$

$$\tilde{\lambda}' = \frac{\lambda_0^2 (z_1^2 + x_1^2)^2}{(3+\lambda_0)^2} = \frac{\lambda_0^2 (1-y_1^2)^2}{(3+\lambda_0)^2}$$

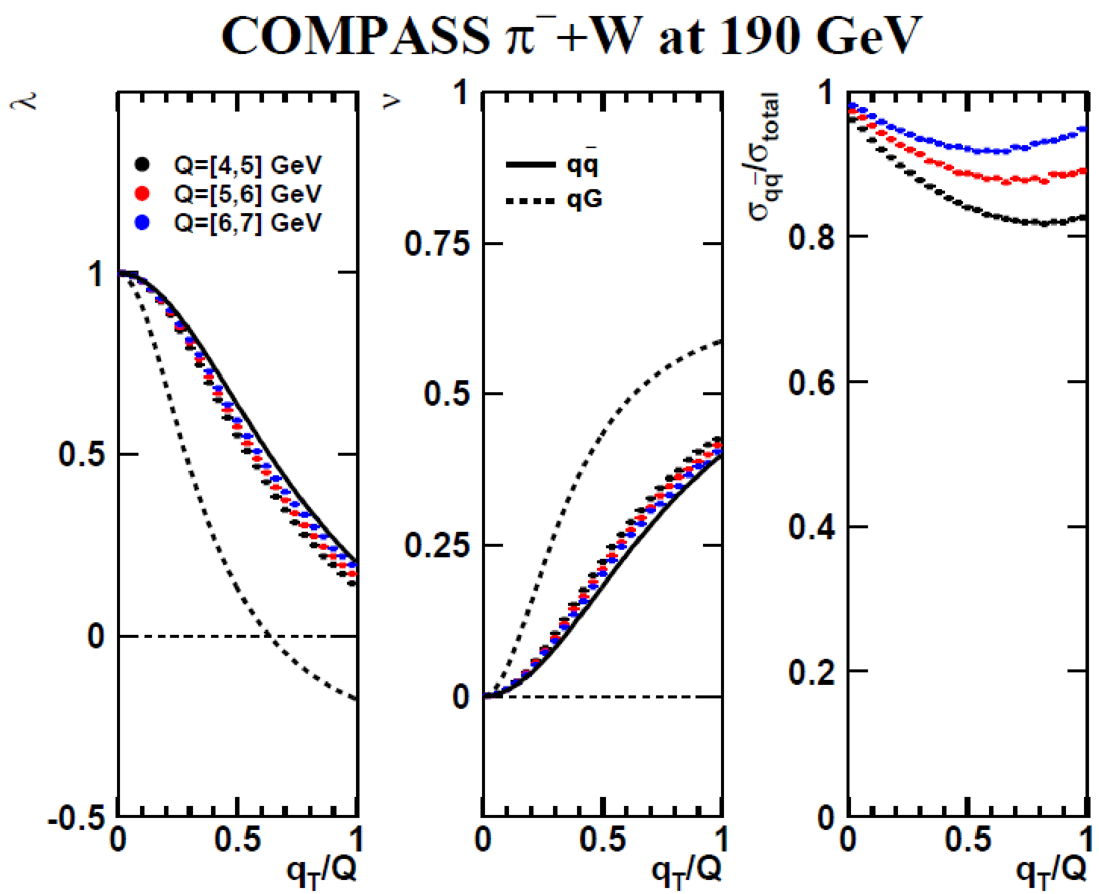
$y_1 = \sin\theta_1 \sin\phi_1$ is the component of $\hat{z}'$ along the y-axis in the dilepton rest frame; invariant under rotation along y-axis

Other implications

- Extend this study to fixed-target Drell-Yan

Lepton Angular Distributions of Fixed-target Drell-Yan Experiments in Perturbative QCD and a Geometric Approach

Wen-Chen Chang,[1] Randall Evan McClellan,[2,3] Jen-Chieh Peng,[3] and Oleg Teryaev[4]



arXiv:1811.03256v1

PRD 99 (2019) 014032

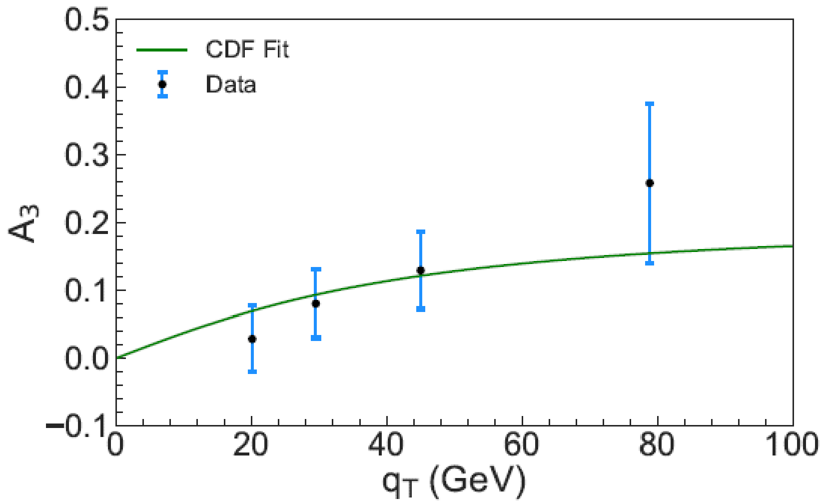
Other implications

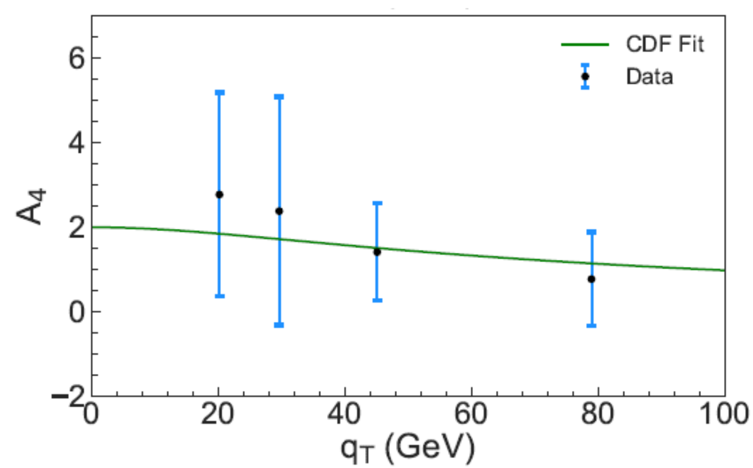
Extend this study to W-boson production at CDF

PHYSICAL REVIEW D **103**, 034011 (2021)

Lepton angular distribution of *W* boson productions

Yang Lyu,[1,2] Wen-Chen Chang,[3] Randall Evan McClellan,[1,4] Jen-Chieh Peng,[1] and Oleg Teryaev[5]



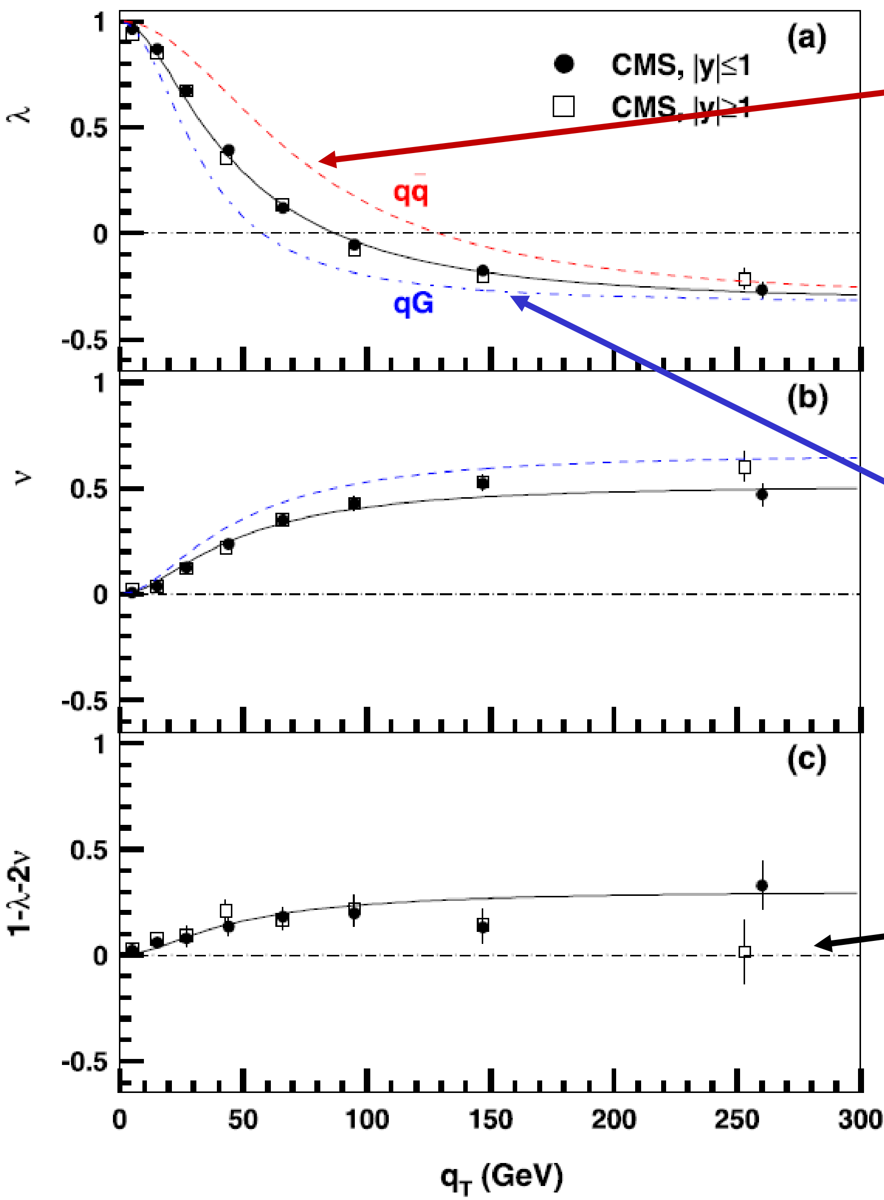


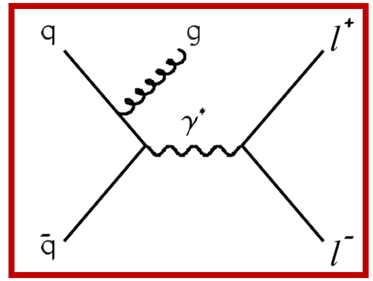
W-boson production in p-pbar collision from CDF

Other implications

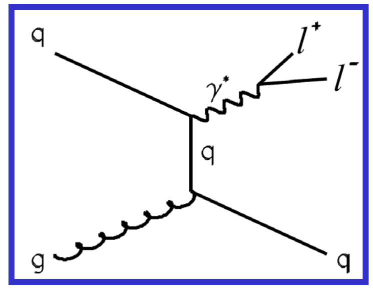
- Extend this study to Z plus jets data at LHC
  - The angular distribution coefficients are expected to be different, in general, for Z plus single jet and Z plus multi-jets events
  - Lam-Tung relation is expected to be satisfied by Z plus single jet events, but badly violated by Z plus two or more jets.
  - The q_T dependence of A_0 would be different for Z plus a single quark jet events and Z plus a single gluon jet events (can lead to the validation of various algorithms for quark/gluon jets separation)
  - Would be great to have these data from LHC!

Expected Z plus jets results





Z plus single gluon jet



Z plus single quark jet

Z plus single jet

Phys. Lett. B797 (2019) 134895

Summary

- A "geometric model" is developed to understand many features of the lepton angular distribution in Drell-Yan and quarkonium productions in hadron collisions
- The lepton angular distribution coefficients $A_0 - A_7$ can be described in terms of the polar and azimuthal angles of the $q - \overline{q}$ axis (natural axis)
- Violation of the Lam-Tung relation is due to the acoplanarity of the $q - \overline{q}$ axis and the hadron plane. This can come from order α_s^2 or higher processes or from intrinsic k_T
- This approach can be extended to Drell-Yan and quarkonium productions $(J/\Psi, \Psi', \Upsilon(1S), \Upsilon(3S), \Upsilon(3S))$ which could be probed at sPHENIX

Some other possible physics topics at sPHENIX

1) In transversely polarized p - p collision, one could measur A_N (left-right asymmetry) for $J/\psi, \psi'$, $\Upsilon(1S, 2S, 3S), \Lambda, K^*, D$ production.

2) If one could identify Drell-Yan signals, it would be interesting to measure Sivers asymmetry (in A_N), as well as other TMDs.

3) One could study intrinsic charm in the proton using forward J/ψ production or D-meson production, or intrinsic strange using forward Λ production.

Some other possible physics topics at sPHENIX

4) If the vertex detectors work well, it might be possible to observe B-meson production through the $b \rightarrow J/\psi X$ decay.

5) If one could observe Drell-Yan events, then it would be very interesting to measure (p+A) / (p+p) ratios, which can probe antiquark saturation (or shadowing) in nucelus at small-x, complementary to gluon saturation.

6) For any of the above topics, one could use the A-A collision data as a warm-up to optimize analysis algorithms.