Challenges on the phenomenology of GPDs

Hervé Dutrieux (William & Mary)

Some collaborators
William & Mary and JLab: K. Orginos, J. Karpie, C. Monahan, ...
PARTONS @ Saclay and Warsaw: H. Moutarde, C. Mezrag, V. Bertone, P. Sznajder, ...
Marseille: S. Zafeiropoulos

October 26th, 2023 – ePIC@BNL Meeting – hldutrieux@wm.edu

WILLIAM & MARY

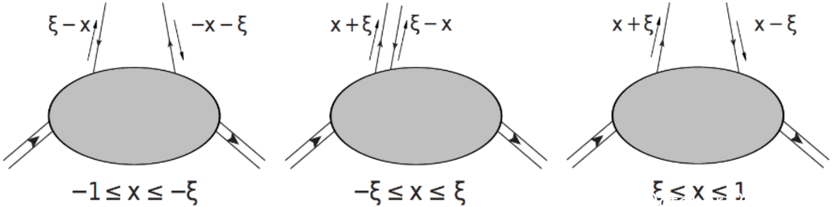
Generalized parton distributions

Spin-1/2 hadron, parton-helicity averaged quark GPDs H^q and E^q in the lightcone gauge
[Müller et al, 1994], **[Radyushkin, 1996]**, **[Ji, 1997]**

$$\frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi} e^{ixP^+z^-} \left\langle p_2 \middle| \bar{\psi}^q\left(-\frac{z}{2}\right)\gamma^+\psi^q\left(\frac{z}{2}\right) \middle| p_1 \right\rangle \Bigg|_{z_\perp=0,\ z^+=0}$$

$$= \frac{1}{2P^+}\left(H^q(x,\xi,t)\bar{u}(p_2)\gamma^+ u(p_1) + E^q(x,\xi,t)\bar{u}(p_2)\frac{i\sigma^{+\mu}\Delta_\mu}{2M}u(p_1)\right) \quad (1)$$

$$p_2 - p_1 = \Delta,\ t = \Delta^2,\ P = \frac{1}{2}(p_1+p_2),\ \xi = -\frac{\Delta^+}{2P^+}\,. \quad (2)$$



GPDs at small ξ – t dependence

- When $x \gg \xi$, negligible asymmetry between incoming $(x - \xi)$ and outgoing $(x + \xi)$ parton longitudinal momentum fraction $\rightarrow$ **smooth limit of GPDs**

$$H(x, \xi, t, \mu^2) \approx H(x, 0, t, \mu^2) \ \text{ for } x \gg \xi \, . \tag{3}$$

Impact parameter distribution (IPD) [Burkardt, 2000]

$$I_a(x, \mathbf{b}_\perp, \mu^2) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp \cdot \Delta_\perp} F^a(x, 0, t = -\Delta_\perp^2, \mu^2) \tag{4}$$

is the density of partons with plus-momentum x and transverse position $\mathbf{b}_\perp$ from the center of plus momentum in a hadron $\rightarrow$ **hadron tomography**

GPDs at small ξ – t dependence

- Extraction of the t-dependent PDF $H(x, 0, t, \mu^2)$?
  - Forward limit gives ordinary PDFs

$$H(x, 0, t = 0, \mu^2) = f(x, \mu^2) \, . \tag{5}$$

  - First Mellin moment gives elastic form factors

$$\int \mathrm{d}x \, H(x, 0, t) = F_1(t) \, . \tag{6}$$

- Better modelling of the t-dependent PDF requires more data, more difficult to obtain with larger systematic uncertainty
  - x-dependence at $\xi = 0$ computed on the lattice from the **non-local euclidean matrix elements** (LaMET [Ji, 2013], short-distance factorization [Radyushkin, 2017], ...)
  - Experimental data from exclusive processes: **most of these data have a particular sensitivity to the region $x \approx \xi$, so precisely not $x \gg \xi$!**
- How can one leverage the experimental data to constrain t-dependent PDFs?

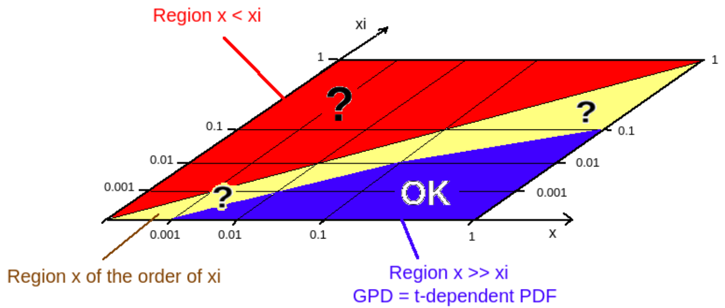
GPDs at small ξ – ξ dependence

- Why don't we just assume

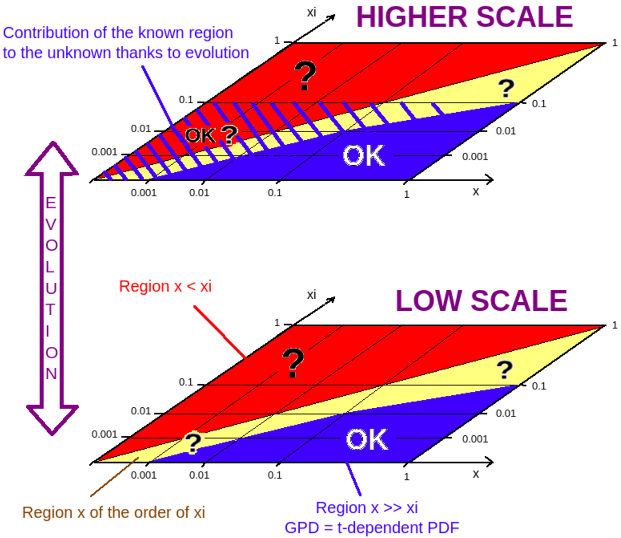
$$H(x,\xi,t,\mu^2) \approx H(x,0,t,\mu^2) \quad \text{for } \xi \ll 1 \text{ even if } x \approx \xi? \tag{7}$$

Because significant asymmetry between incoming and outgoing ($x + \xi \gg x - \xi$) parton momentum means very different dynamics, materialized *e.g.* by a very different behavior under evolution.

No reason for the ξ dependence to be negligible even at very small ξ. Skewness ratios $\frac{H(x,x)}{H(x,0)}$ as large as 1.6 have been advocated at small x. [Frankfurt et al, 1998] [Shuvaev et al, 1999]



GPDs at small ξ – ξ dependence

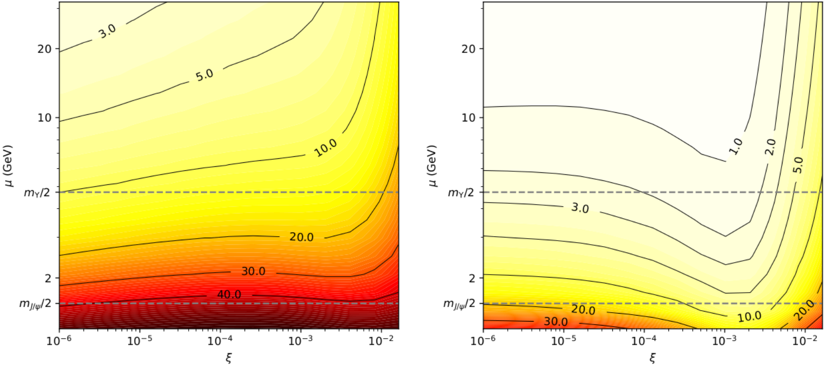


- Evolution displaces the GPD from the large x to the small x region
- Significant ξ dependence arises perturbatively in the small x and ξ region
- But how does it compare to the unknown ξ dependence at initial scale?

Obviously depends on the range of evolution, value of x and ξ, and profile of the known t-dependent PDF.

GPDs at small ξ – ξ dependence

Example: working at $t = 0$, with the MMHT2014 PDF **[Harland-Lang et al, 2015]** at 1 GeV (**prior knowledge of t-dependent PDF**). We want to assess the dominance of the region $x \gg \xi$ at initial scale in the value of the GPD on the diagonal as scale increases.
Pessimistic assumption on unknown ξ dependence at $x = \xi$ for 1 GeV: 60%.



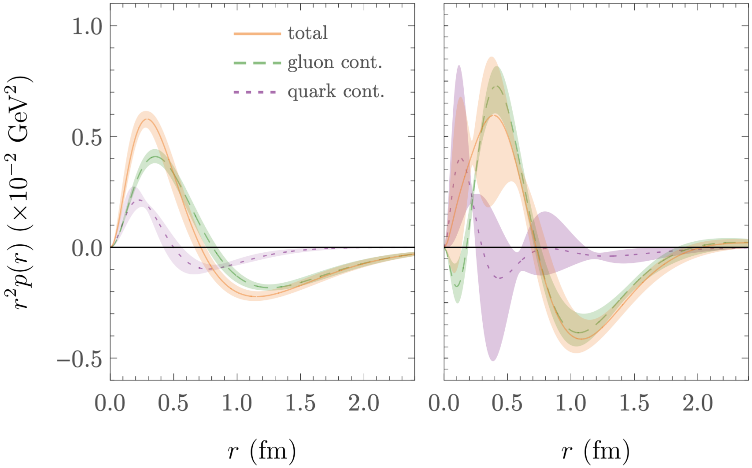
Uncertainty on the diagonal of the light sea quarks (left) and gluons (right) depending on $x = \xi$ and μ.
Stronger μ effect for gluons, divergence of PDFs at small x visible.

[HD, Winn, Bertone, 2023]

Perspectives

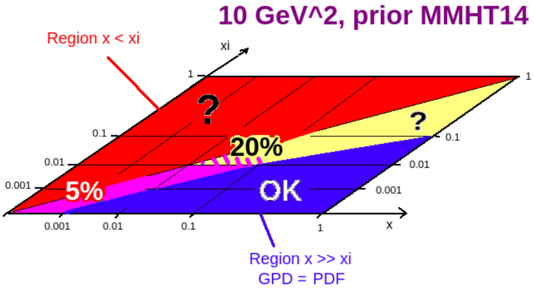
- Generating perturbatively the ξ dependence offers a well defined functional space for GPDs at small ξ which verifies the main theoretical constraints (polynomiality of Mellin moments, positivity, limits, ...)
- By subtracting the degree of freedom of the ξ dependence, we have regularized the deconvolution problem, and we have an evaluation of the uncertainty associated to this regularization.
- Limitations: higher order perturbative order, small x resummation of the ξ dependence unavailable.

Perspectives



[Shanahan, Detmold, 2018]

Deconvolution problem for $x < \xi$



- Summary of the situation for H^g at $t = 0$ with MMHT2014 PDFs as prior
- What is happening for $x < \xi$, and what is the **deconvolution problem**?
- GPDs satisfy a **polynomiality property** arising from Lorentz covariance: **[Ji, 1998]**, **[Radyushkin, 1999]**

$$\int_{-1}^{1} \mathrm{d}x\, x^n H^q(x,\xi,t,\mu^2) = \sum_{k=0\ even}^{n} A^q_{n,k}(t,\mu^2)\xi^k + \mathrm{mod}(n,2)\xi^{n+1}\, C^q_n(t,\mu^2)\,. \tag{8}$$

red contribution: if a function $D^q(\alpha,t,\mu)$ is odd in α, **[Polyakov, Weiss, 1999]**

$$\int_{-1}^{1} \mathrm{d}x\, x^n\, \Theta\left(1 - \frac{|x|}{|\xi|}\right) \mathrm{sgn}(\xi) D^q\left(\frac{x}{\xi}, t, \mu^2\right) = \mathrm{mod}(n,2)\xi^{n+1} \int_{-1}^{1} \mathrm{d}\alpha\, \alpha^n D^q(\alpha,t,\mu^2)\,. \tag{9}$$

Deconvolution problem for $x < \xi$

DVCS dispersion relation [Anikin, Teryaev, 2007], [Diehl, Ivanov, 2007]

$$\mathcal{C}_H(t, Q^2) = \mathrm{Re}\,\mathcal{H}(\xi, t, Q^2) - \frac{1}{\pi}\int_0^1 \mathrm{d}\xi' \,\mathrm{Im}\,\mathcal{H}(\xi', t, Q^2)\left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'}\right) \tag{10}$$

$$\overset{LO}{=} 2\sum_q e_q^2 \int_{-1}^{1} \mathrm{d}z\, \frac{D^q(z, t, Q^2)}{1 - z} \tag{11}$$

Since z is integrated out, only hope comes from the knowledge of the LO scale dependence of the D-term (ERBL equation). How effective is evolution to constrain it?

Shadow distributions

Find a distribution with reasonable shape such that it gives no experimental contribution at one scale, and check how big its contribution becomes as you move from the initial scale $\rightarrow$ measures worst case uncertainty propagation from experiment to fit

Deconvolution problem for $x < \xi$

Let's expand the D-term on a basis of Gegenbauer polynomials

$$D^q(z,t,\mu^2) = (1-z^2) \sum_{\text{odd } n} d_n^q(t,\mu^2)\, C_n^{3/2}(z) \tag{12}$$

Then

GFF C_a extraction

$$\int_{-1}^{1} \mathrm{d}z \, \frac{D^q(z,t,\mu^2)}{1-z} = 2 \sum_{\text{odd } n} d_n^q(t,\mu^2) \quad \text{and} \quad \int_{-1}^{1} \mathrm{d}z \, z D^q(z,t,\mu^2) = \frac{4}{5}\, d_1(t,\mu^2) \tag{13}$$

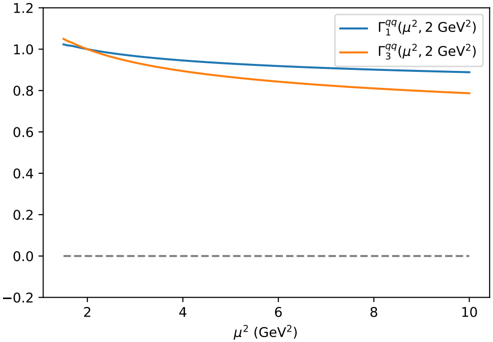
- There is a shadow D-term for

$$d_1(\mu_0^2) = -d_3(\mu_0^2)\,! \tag{14}$$

[HD, Lorcé, Moutarde, Sznajder, Trawinski, Wagner, 2021]: allowing two free parameters d_1 and d_3 results in an **inflation of uncertainty by a factor 20 with full correlation between fitted parameters** compared to just d_1 over a range of $Q^2 \in [1.5, 4]$ GeV^2

Extraction of GFFs

in preparation



Simplified evolution in the qq sector

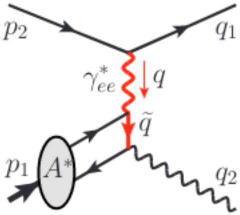
$$d_n^q(\mu^2) = \Gamma_n^{qq}(\mu^2, 2\ \mathrm{GeV}^2) d_n^q(2\ \mathrm{GeV}^2) \quad (15)$$

- current range of most DVCS data : [1.5, 4] GeV^2
- Over this range, Γ_1^{qq} and Γ_3^{qq} are numerically very close $\rightarrow$ little actual leverage in evolution to separate the two
- Estimate of the inflation on uncertainty when fitting jointly d_1 and d_3 compared to the sole d_1 :

$$\propto \left(1 - \frac{\Gamma_3^{qq}(Q^2_{\mathrm{max}}, Q^2_{\mathrm{min}})}{\Gamma_1^{qq}(Q^2_{\mathrm{max}}, Q^2_{\mathrm{min}})}\right)^{-1} \quad (16)$$

- **An increase thanks to EIC from [1.5, 4] GeV^2 to [1.5, 50] GeV^2 could yield a decrease by 3 times of the uncertainty on (d_1, d_3) due to the sole effect of increase in Q^2 range, without taking account a better experimental precision.**

Perspectives

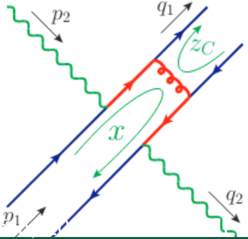


DVCS, TCS, DVMP: "moment-like" information on GPDs $\rightarrow x, \xi$ are not coupled directly to the hard scale [Qiu, Yu, 2022]

$$\tilde{q}^2 = \frac{Q^2 + q_2^2}{2\xi}\left[x - \xi\left(\frac{1 - q_2^2/Q^2}{1 + q_2^2/Q^2}\right)\right] + \mathcal{O}(t/Q^2) \qquad (17)$$

[Qiu, Yu, 2022]

Solution: entangle the flow of hard momentum with the x, ξ dependence: DDVCS [Guidal, Vanderhaeghen, 2003], [Belitsky, Müller, 2003], di-photon production [Pedrak et al, 2017], [Grocholski et al, 2020], photoproduction of photon-meson pair [Qiu, Yu, 2022] $\rightarrow$ avoids the single-photon channel!, ...



Conclusions

- Phenomenology of GPDs with lesser model dependence requires a global analysis program, over large kinematic range (EIC) and with many processes beyond the traditional DVCS, DVMP.
- Perturbative modelling of the ξ dependence of GPDs **in the region where** $|x| > |\xi|$ is an interesting avenue in EIC kinematics at small x and large range in Q^2
- Model independent access to GPDs in the moderate and large x region, as well as the D-term and pressure distribution requires exclusive processes with a **richer kinematic structure**.

Thank you for your attention!

Perspectives

- Other exclusive processes can be expressed in terms of GPDs. Close parent to DVCS is **time-like Compton scattering** (TCS) **[Berger et al, 2002]**. Although its measurement will reduce the uncertainty, especially on $\mathrm{Re}\,\mathcal{H}$ **[Jlab proposal PR12-12-001]**, and produce a valuable check of the universality of the GPD formalism, the similar nature of its convolution (see **[Müller et al, 2012]**) makes it subject to the same shadow GPDs.
- **Deeply virtual meson production** (DVMP) **[Collins et al, 1997]** is also an important source of knowledge on GPDs, with currently a larger lever arm in Q^2. The process involves form factors of the general form

$$\mathcal{F}(\xi, t) = \int_0^1 \mathrm{d}u \int_{-1}^{1} \frac{\mathrm{d}x}{\xi} \phi(u) T\left(\frac{x}{\xi}, u\right) F(x, \xi, t) \tag{18}$$

where $\phi(u)$ is the leading-twist meson distribution amplitude (DA).
- At LO, the GPD and DA parts of the integral factorize and shadow GPDs cancel the form factor.
- Situation at NLO remains to be clarified, it is foreseeable new shadow GPDs (dependent on the DA) could be generated also for this process.

Deeply virtual Compton scattering and the structure of hadrons

- Remarkably, GPDs allow access to gravitational form factors (GFFs) of the **energy-momentum tensor (EMT)** [Ji, 1997] defined for parton of type a

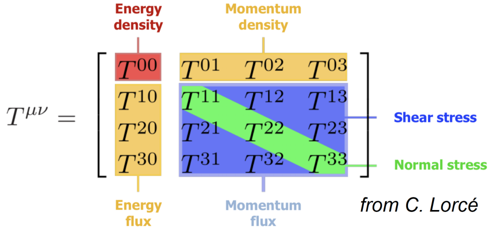
Gravitational form factors [Lorcé et al, 2017]

$$\langle p', s' | T_a^{\mu\nu} | p, s \rangle = \bar{u}(p', s') \Big\{ \frac{P^\mu P^\nu}{M} A_a(t, \mu^2) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} C_a(t, \mu^2) + M\eta^{\mu\nu} \bar{C}_a(t, \mu^2)$$
$$+ \frac{P^{\{\mu} i\sigma^{\nu\}\rho}\Delta_\rho}{4M} \left[A_a(t, \mu^2) + B_a(t, \mu^2) \right] + \frac{P^{[\mu} i\sigma^{\nu]\rho}\Delta_\rho}{4M} D_a(t, \mu^2) \Big\} u(p, s) \quad (19)$$

where

$$\Delta = p' - p, \;\; t = \Delta^2, \;\; P = \frac{p + p'}{2} \quad (20)$$

Deeply virtual Compton scattering and the structure of hadrons



from C. Lorcé

In the Breit frame ($\vec{P} = 0$, $t = -\vec{\Delta}^2$), radial distributions of energy and momentum in the proton are described by Fourier transforms of the **GFFs** w.r.t. variable $\vec{\Delta}$ **[Polyakov, 2003]**.

- Example of such distribution: radial pressure anisotropy profile

$$s_a(r, \mu^2) = -\frac{4M}{r^2} \int \frac{\mathrm{d}^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \frac{t^{-1/2}}{M^2} \frac{\mathrm{d}^2}{\mathrm{d}t^2} \left[t^{5/2} C_a(t, \mu^2) \right] \tag{21}$$

- This pressure profile can be extracted from **GPDs** thanks to *e.g.* for quarks

$$\int_{-1}^{1} \mathrm{d}x \, x \, H^q(x, \xi, t, \mu^2) = A_q(t, \mu^2) + 4\xi^2 C_q(t, \mu^2) \tag{22}$$

$$\int_{-1}^{1} \mathrm{d}x \, x \, E^q(x, \xi, t, \mu^2) = B_q(t, \mu^2) - 4\xi^2 C_q(t, \mu^2) \tag{23}$$

Extraction of GFFs

- At this stage, we don't need to fully extract the GPDs H or E to conveniently access the GFF $C_q(t, \mu^2)$. The **polynomiality property** gives that the GFF $C_q(t, \mu^2)$ only depends on the D-term via

$$\int_{-1}^{1} \mathrm{d}z\, z D^q(z, t, \mu^2) = 4 C_q(t, \mu^2) \tag{24}$$

- The experimental data is sensitive to the D-term through the **subtraction constant** defined by the **dispersion relation** (see *e.g.* [Diehl, Ivanov, 2007])

DVCS dispersion relation

$$\mathcal{C}_H(t, Q^2) = \mathrm{Re}\,\mathcal{H}(\xi, t, Q^2) - \frac{1}{\pi} \int_0^1 \mathrm{d}\xi' \,\mathrm{Im}\,\mathcal{H}(\xi', t, Q^2) \left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'} \right) \tag{25}$$

The subtraction constant $\mathcal{C}_H(t, Q^2)$ is a function of the D-term given at LO by

$$\mathcal{C}_H(t, Q^2) = 2 \sum_q e_q^2 \int_{-1}^{1} \mathrm{d}z \frac{D^q(z, t, Q^2)}{1 - z} \tag{26}$$

Extraction of GFFs

- How do we get from

$$\int_{-1}^{1} \mathrm{d}z \, \frac{D^q(z,t,\mu^2)}{1-z} \quad \text{to} \quad \int_{-1}^{1} \mathrm{d}z \, z D^q(z,t,\mu^2) \ ? \tag{27}$$

- This is a prototype of the more complicated GPD extraction problem we will face later on. The known solution is through evolution.
- Let's expand the D-term on a basis of Gegenbauer polynomials

$$D^q(z,t,\mu^2) = (1-z^2) \sum_{\text{odd } n} d_n^q(t,\mu^2) \, C_n^{3/2}(z) \tag{28}$$

Then

GFF C_a extraction

$$\int_{-1}^{1} \mathrm{d}z \, \frac{D^q(z,t,\mu^2)}{1-z} = 2 \sum_{\text{odd } n} d_n^q(t,\mu^2) \ \text{ and } \ \int_{-1}^{1} \mathrm{d}z \, z D^q(z,t,\mu^2) = \frac{4}{5} \, d_1(t,\mu^2) \tag{29}$$

Extraction of GFFs

- Since the LO subtraction constant reads

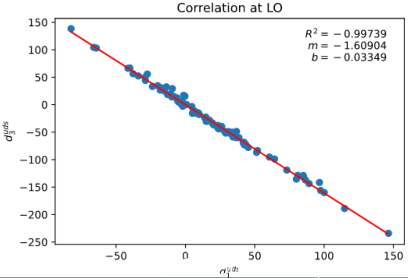
$$\int_{-1}^{1} \mathrm{d}z \, \frac{D^q(z,t,\mu^2)}{1-z} = 2 \sum_{\text{odd } n} d_n^q(t,\mu^2) \qquad (30)$$

if we allow d_3^q to be non-zero, at some scale μ_0^2, we can have $d_1^q(\mu_0^2) = -d_3^q(\mu_0^2)$, so a **vanishing subtraction constant, but non-zero GFF** $C_q(\mu_0^2)$. If the effect of evolution is not significant enough, these configurations are not ruled out and add a considerable uncertainty.

$d_1^{uds}(\mu_F^2)$	-0.5 ± 1.2

$\longrightarrow$

$d_1^{uds}(\mu_F^2)$	11 ± 25
$d_3^{uds}(\mu_F^2)$	-11 ± 26



Deconvolution problem at moderate x and ξ

General deconvolution problem: Compton form factors (CFFs) given by [Radyushkin, 1997], [Ji, Osborne, 1998], [Collins, Freund, 1999]

$$\mathcal{H}^q(\xi, t, Q^2) = \int_{-1}^{1} \frac{\mathrm{d}x}{\xi}\, T^q\left(\frac{x}{\xi}, \alpha_s, \frac{Q^2}{\mu^2}\right) H^q(x, \xi, t, \mu^2)\,. \tag{31}$$

- ambiguities in defining ξ from experimental quantities up to order $\mathcal{O}(t/Q^2)$, related issue of kinematic power corrections and higher twists [Braun et al, 2014], flavor decomposition [Cuic, Kumericki, Schäfer, 2020], ...
- $u, \bar{u}, d, \bar{d}, g \times$ 4 chiral-even GPDs = 20 GPDs $\times$ 3 dimensions = hundreds of parameters [Guo et al, 2022]

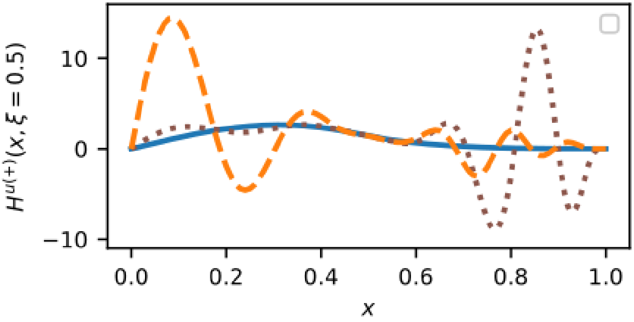
What is a reasonable shape for shadow GPD?

1. **Double distributions** [Radyushkin, 1997] as polynomials in their two variables (α, β)?
2. **Neural network model** of double distributions?

Deconvolution problem at moderate x and ξ

Double distributions as polynomials in their two variables (α, β) **[Bertone, HD, Mezrag, Moutarde, Sznajder, 2021]**

- Enforces polynomiality by construction
- Analytical computation of the CFF $\rightarrow$ exact cancellation possible at least up to NLO
- Precise test of the accuracy of evolution: at NLO, should vary as $\mathcal{O}(\alpha_s^2)$



- **Result**: the three models give CFFs that vary by $\approx 10^{-5}$ at moderate ξ over a range of $[1, 100]$ GeV^2 $\rightarrow$ **enormous inflation of uncertainty from experimental data at moderate ξ**
- **Limitation**: large fluctuations at large x unphysical, incompatible with positivity constraints

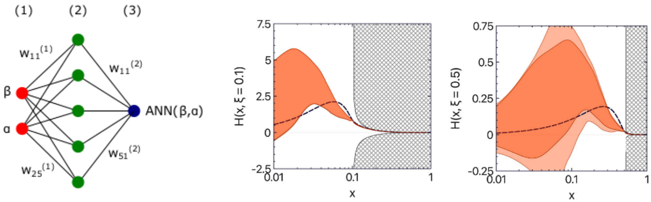
Deconvolution problem at moderate x and ξ

Neural network model of double distributions **[HD, Grocholski, Moutarde, Sznajder, 2022]**

- Enforces polynomiality by construction
- More flexible without the need of very large polynomial powers (precision issue for floating point computation)
- More flexible framework to implement positivity constraint: mock constraint

$$|H^q(x,\xi,t)| \leq \sqrt{f^q\left(\frac{x+\xi}{1+\xi}\right) f^q\left(\frac{x-\xi}{1-\xi}\right) \frac{1}{1-\xi^2}} \quad (32)$$

- Proof of concept – closure test :



Deconvoluting a Compton form factor

- Question was raised 20 years ago. Evolution was proposed as a crucial element in **[Freund, 1999]**, but the question has remained essentially open.
- We show that GPDs exist which bring contributions to the LO and NLO CFF of only subleading order even under evolution. We call them **LO and NLO shadow GPDs**.

Definition of an NLO shadow GPD

For a given scale μ_0^2,

$$\forall \xi, \forall t,\ T^q_{NLO}(Q^2, \mu_0^2) \otimes H^q(\mu_0^2) = 0 \quad \text{and} \quad H^q(x, \xi = 0, t = 0, \mu_0^2) = 0 \qquad (33)$$

$$\text{so for } Q^2 \text{ and } \mu^2 \text{ close enough to } \mu_0^2,\ T^q_{NLO}(Q^2, \mu^2) \otimes H^q(\mu^2) = \mathcal{O}(\alpha_s^2(\mu^2)) \qquad (34)$$

- Let H^q be an NLO shadow GPD, and G^q be any GPD. Then G^q and $G^q + H^q$ have the same forward limit, and the same NLO CFF up to a numerically small and theoretically subleading contribution.

Shadow GPDs at leading order

- Complete details in **[Bertone, HD, Mezrag, Moutarde, Sznajder, Phys.Rev.D 103 (2021) 11, 114019]**
- We search for our shadow GPDs as simple **double distributions (DD)** $F(\beta, \alpha, \mu^2)$ to respect polynomiality, with a zero D-term. Then, thanks to dispersion relations, we can restrict ourselves to the imaginary part only $\mathrm{Im}\ T^q(Q^2, \mu_0^2) \otimes H^q(\mu_0^2) = 0$.
- We search our DD as a polynomial of order N in (β, α), characterised by $\sim N^2$ coefficients c_{mn}:

$$F(\beta, \alpha, \mu_0^2) = \sum_{m+n \leq N} c_{mn}\, \alpha^m \beta^n \tag{35}$$

Shadow GPDs at next-to-leading order

- **First study beyond leading order:** Apart from the **LO** part, the NLO CFF is composed of a **collinear part** (compensating the α_s^1 term resulting from the convolution of the LO coefficient function and the evoluted GPD) and a genuine **1-loop NLO** part.

$$\mathcal{H}^q(\xi, Q^2) = C_0^q \otimes H^{q(+)}(\mu_0^2) + \alpha_s(\mu^2)\, C_1^q \otimes H^{q(+)}(\mu_0^2) + \alpha_s(\mu^2)\, C_{coll}^q \otimes H^{q(+)}(\mu_0^2) \log\left(\frac{\mu^2}{Q^2}\right) \quad (36)$$

An explicit calculation of each term for our polynomial double distribution gives that

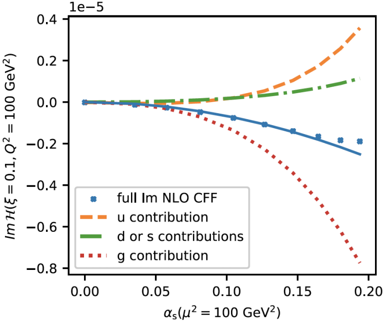
$$\mathrm{Im}\ T_{coll}^q(Q^2, \mu^2) \otimes H^q(\mu^2) \propto$$

$$\alpha_s(\mu^2) \log\left(\frac{\mu^2}{Q^2}\right) \left[\left(\frac{3}{2} + \log\left(\frac{1-\xi}{2\xi}\right)\right) \mathrm{Im}\ T_{LO}^q \otimes H^q(\mu^2) + \sum_{w=1}^{N+1} \frac{k_w^{(coll)}}{(1+\xi)^w} \right] \quad (37)$$

and assuming $\mathrm{Im}\ T_{LO}^q \otimes H^q(\mu^2) = 0$,

$$\mathrm{Im}\ T_1^q(Q^2, \mu^2) \otimes H^q(\mu^2) \propto \alpha_s(\mu^2) \left[\log\left(\frac{1-\xi}{2\xi}\right) \mathrm{Im}\ T_{coll}^q \otimes H^q(\mu^2) + \sum_{w=1}^{N-1} \frac{k_w^{(1)}}{(1+\xi)^w} \right]$$

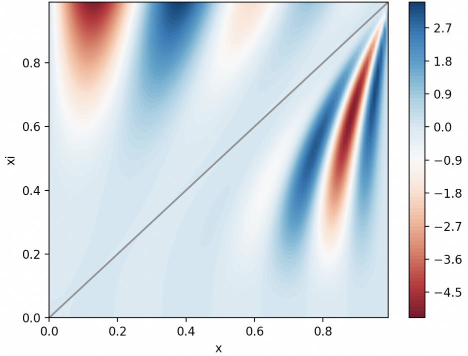
- By linearity of both the CFF convolution and the evolution equation, we can evaluate separately the contribution to the CFF of a quark shadow NLO GPD under evolution.
- We probe the prediction of evolution as $\mathcal{O}(\alpha_s^2(\mu^2))$ with our previous NLO shadow GPD on a lever-arm in Q^2 of $[1, 100]$ GeV^2 (typical collider kinematics) using APFEL++ code.

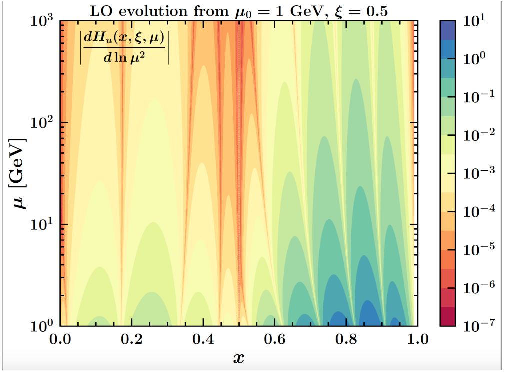


- The fit by $\alpha_s^2(\mu^2)$ is very good up to values of α_s of the order of its $\overline{MS}$ values. For larger values, large logs and higher orders slightly change the picture.
- The numerical effect of evolution remains very small. For a GPD of order 1, the NLO CFF is only of order 10^{-5}.

Shadow GPDs at next-to-leading order

- Cancelling both terms gives rise to two additional systems with a linear number of equations. The first NLO shadow GPD is found for $N = 21$, and adding the condition that the DD vanishes at the edges of its support gives a first solution for $N = 25$ (see below).





Color plot of an NLO shadow GPD at initial scale 1 GeV^2, and its evolution for $\xi = 0.5$ up to 10^6 GeV^2 via APFEL++ and PARTONS [Bertone].

Evolution of GPDs

GPD's dependence on scale is given by **renormalization group equations**. In the limit $\xi = 0$, usual DGLAP equation:

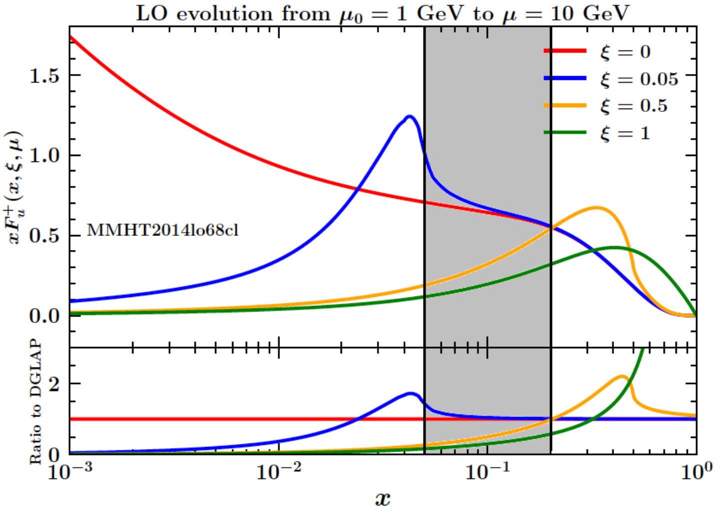
$$\frac{\mathrm{d}f^{q+}}{\mathrm{d}\mu}(x,\mu) = \frac{C_F \alpha_s(\mu)}{\pi\mu}\left\{ \int_x^1 \mathrm{d}y \, \frac{f^{q+}(y,\mu) - f^{q+}(x,\mu)}{y-x}\left[1 + \frac{x^2}{y^2}\right] \right.$$
$$\left. + f^{q+}(x,\mu)\left[\frac{1}{2} + x + \log\left(\frac{(1-x)^2}{x}\right)\right]\right\} \quad (39)$$

But in the limit $x = \xi$:

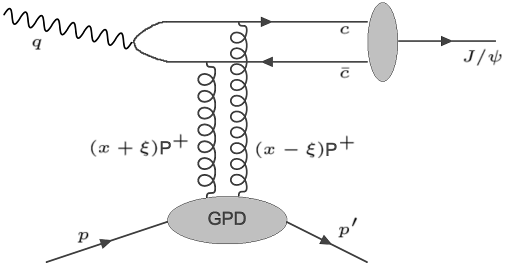
$$\frac{\mathrm{d}H^{q+}}{\mathrm{d}\mu}(x,x,\mu) = \frac{C_F \alpha_s(\mu)}{\pi\mu}\left\{ \int_x^1 \mathrm{d}y \, \frac{H^{q+}(y,x,\mu) - H^{q+}(x,x,\mu)}{y-x} \right.$$
$$\left. + H^{q+}(x,x,\mu)\left[\frac{3}{2} + \log\left(\frac{1-x}{2x}\right)\right]\right\} \quad (40)$$

Assuming that GPD = t-dependent PDF at small ξ and $x \approx \xi$ is incompatible with evolution, which generates an intrinsic ξ dependence!

Evolution of GPDs



Vector meson production



LO depiction of J/ψ photoproduction.

The region $x \sim \xi$ where significant perturbative ξ dependence occurs is crucial for the phenomenology of GPDs!
Transfer of four-momentum to the hadron $\rightarrow$ description in the framework of collinear factorization by **generalized parton distributions (GPDs)** and **non-relativistic QCD matrix element** for moderate or small photon virtuality $Q^2 = -q^2$. Hard scale provided by $m_V/2$ **[Jones et al, 2015]**.

$$\xi = \frac{p^+ - p'^+}{p^+ + p'^+} \approx \frac{x_B}{2}, \quad t = (p' - p)^2$$

Vector meson production

- Vector meson production amplitude up to NLO **[Ivanov et al, 2004]**:

$$\mathcal{F}(\xi, t) \propto \left(\frac{\langle O_1 \rangle_V}{m_V^3} \right)^{1/2} \sum_{a=q,g} \int_{-1}^{1} \mathrm{d}x \, T^a(x, \xi) \, F^a(x, \xi, t) \quad (41)$$

  where $\langle O_1 \rangle_V^{1/2}$ is the NR QCD matrix element, T a hard-scattering kernel and $F(x, \xi, t)$ is the GPD.
- The dominant region controlling the imaginary part of the amplitude is:

$$x \approx \xi \approx \frac{x_B}{2} \approx e^{-y} \frac{m_V}{2\sqrt{s}} \quad (42)$$

- At LHCb kinematics *e.g.*, typical values of x_B as low as $\sim 10^{-5}$.