BUILDING BLOCKS OF FLAVOUR IN SMEFT

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(based on *arXiv:2210.09316* and WIP w/ C. Machado, S. Renner, B. Smith)

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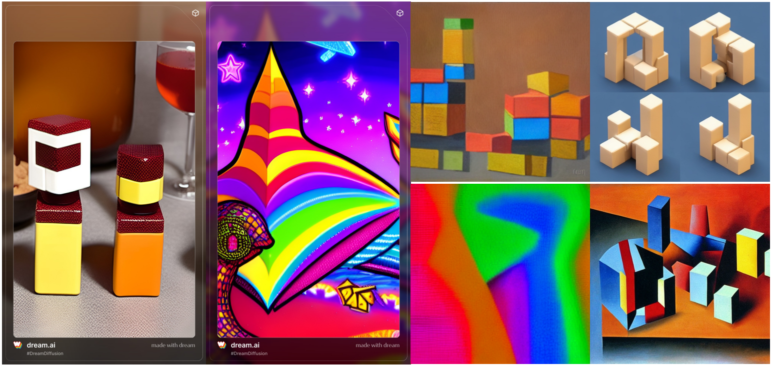
WHAT'S A BUILDING BLOCK?



(Various AI text-to-image interpretations of the paper title)

Key point: $SU(3)^5$ decomposition usefully organises the flavour space of a *completely generic* SMEFT, and simplifies the running.

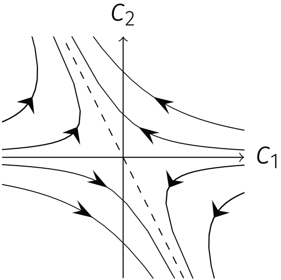
HEAVY NP HAS MANY FLAVOURFUL PARAMETERS

Effects of heavy NP described by contact interactions, e.g.,

$$
\begin{aligned}
c_{H\Box}\,\left(H^\dagger H\right)\Box\left(H^\dagger H\right) \\
c_{ij}\left(H^\dagger i\overleftrightarrow{D} H\right)\left(\overline{Q}^i\gamma Q^j\right) \\
c_{ijkl}\left(\overline{Q}^i\gamma Q^j\right)\left(\overline{Q}^k\gamma Q^l\right)
\end{aligned}
$$

Most are flavourful: compare 2499 (3 generation) versus 76 (1 generation) real parameters in the dim. 6 B-conserving SMEFT.

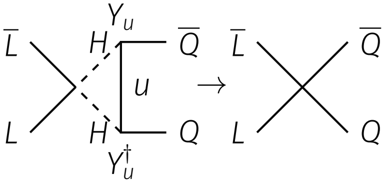
We want to analyse physically meaningful subsets of these parameters.



$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}C_i = \frac{1}{16\pi^2}\gamma_{ij}C_j$$

Parameters mix via γ, populated by SM couplings. (Alonso, Jenkins, Manohar, and Trott 2014)

Much of it is flavourful, e.g.



Are all physically meaningful subsets mixed by running?

THE PLAN

- $SU(3)^5$ decomposition as a generic organising principle (Machado, Renner, and Sutherland 2023)
- Structures in the RG due to helicity (Cheung and Shen 2015)
- Structures in the RG due to flavour (Machado, Renner, and Sutherland 2023)
- Relevant directions in the IR (preliminary work)

THE SM(EFT) HAS A BROKEN $SU(3)^5$ SYMMETRY

The SM(EFT) has a hierarchically broken $SU(3)_Q \times SU(3)_u \times SU(3)_d \times SU(3)_L \times SU(3)_e$ symmetry.

$$\begin{aligned}\mathcal{L} =& i\overline{Q}^i \not{D} Q^i + i\overline{u}^i \not{D} u^i + [\text{sim. for } d, L, e] \\ & - [Y^U]_{ij} \overline{Q}^i \tilde{H} u^j + \text{h.c.} + [\text{sim. for } Y^D, Y^E] \\ & + c_{ijkl} \left(\overline{Q}^i \gamma Q^j\right) \left(\overline{Q}^k \gamma Q^l\right) + [\text{other ops}]\end{aligned}$$

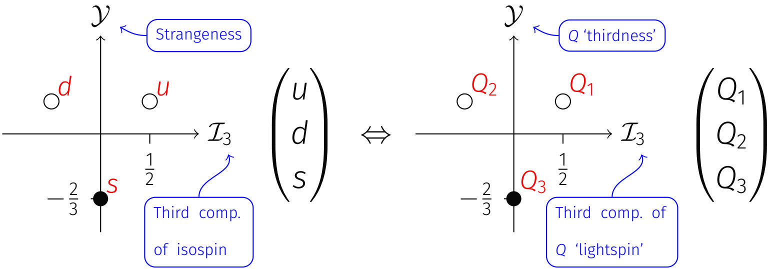
The kinetic terms are invariant under $Q^i \to U_Q^{ij} Q^j$, $u^i \to U_u^{ij} u^j$, ...

The Yukawas break different parts of these symmetries — their components are *charged* under $SU(3)^5$.

The SMEFT operators are also charged under $SU(3)^5$.

FLAVOUR DECOMPOSITION

Cf. $SU(3)$ of uds: The SM Yukawas hierarchically break its $SU(3)_Q \times SU(3)_u \times SU(3)_d \times SU(3)_L \times SU(3)_e$ symmetry.

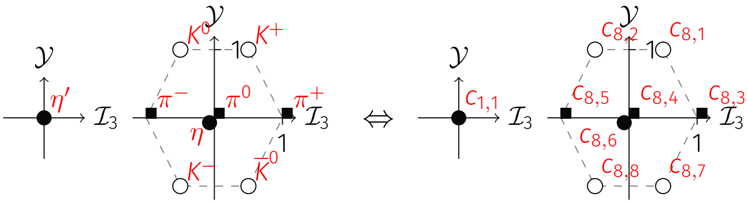


There are 20 flavour quantum numbers in total

$$\{d, \mathcal{I}, \mathcal{I}_3, \mathcal{Y}\}_F \qquad \forall F \in \{Q, u, d, L, e\}$$

FLAVOUR DECOMPOSITION: ONE FERMION CURRENT

For $\left(H^\dagger i\overleftrightarrow{D}H\right)\left(\overline{Q}^i\gamma Q^j\right)$: $\mathbf{1}_Q \oplus \mathbf{8}_Q$



Total lightspin ($\mathcal{I}$) key: ● $=0$, ○ $=\frac{1}{2}$, ■ $=1$.

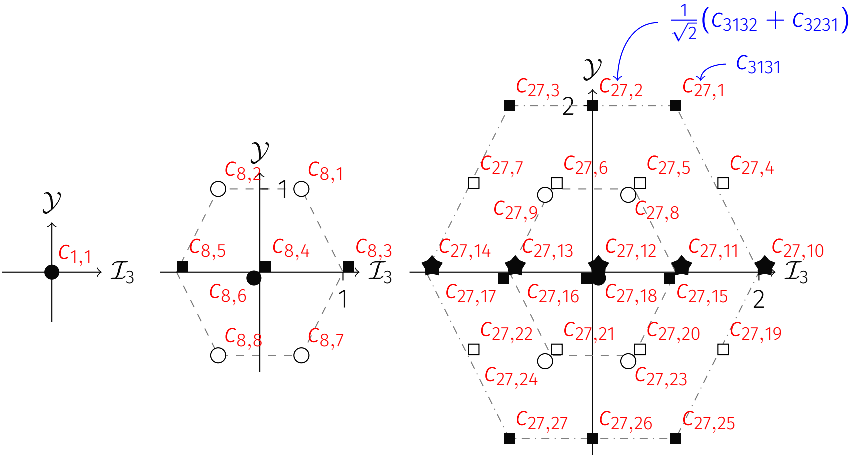
Compare

$$\eta \sim \left(u\overline{u} + d\overline{d} - 2s\overline{s}\right)$$

and

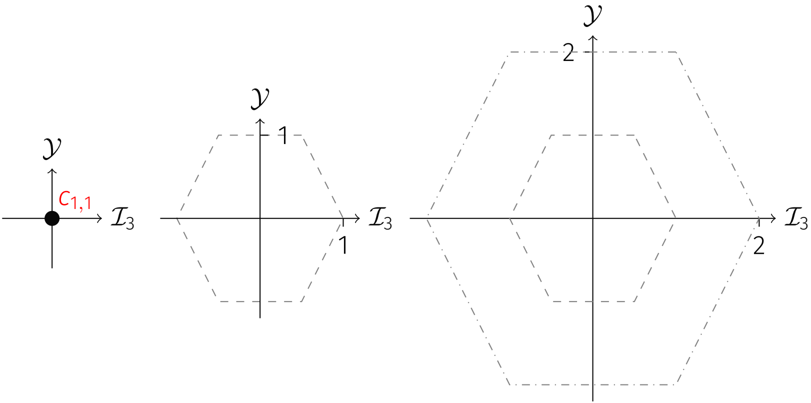
$$c_{8,6}\left(H^\dagger i\overleftrightarrow{D}H\right)\left(\overline{u}_L^i\gamma u_L^j + \overline{c}_L^i\gamma c_L^j - 2\overline{t}_L^i\gamma t_L^j\right)$$

For $\left(\overline{Q}^i \gamma Q^j\right)\left(\overline{Q}^k \gamma Q^l\right)$: $\mathbf{1}_Q \oplus \mathbf{1}_Q \oplus \mathbf{8}_Q \oplus \mathbf{8}_Q \oplus \mathbf{27}_Q$



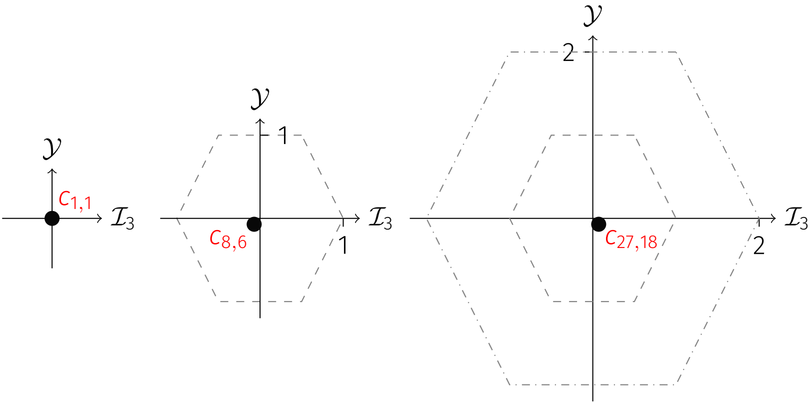
Total lightspin ($\mathcal{I}$) key: ● $= 0$, ○ $= \frac{1}{2}$, ■ $= 1$, □ $= \frac{3}{2}$, ★ $= 2$.

Flavour universal ($SU(3)^5$ symmetric)



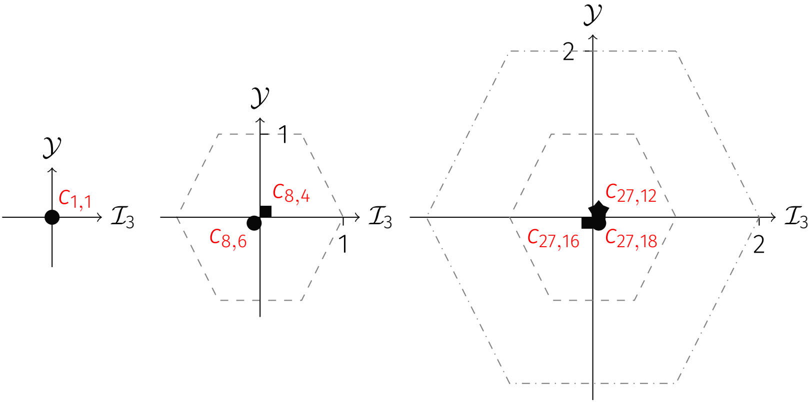
Total lightspin ($\mathcal{I}$) key: ● $= 0$, ○ $= \frac{1}{2}$, ■ $= 1$, □ $= \frac{3}{2}$, ⬟ $= 2$.

$SU(2)^5$ symmetric



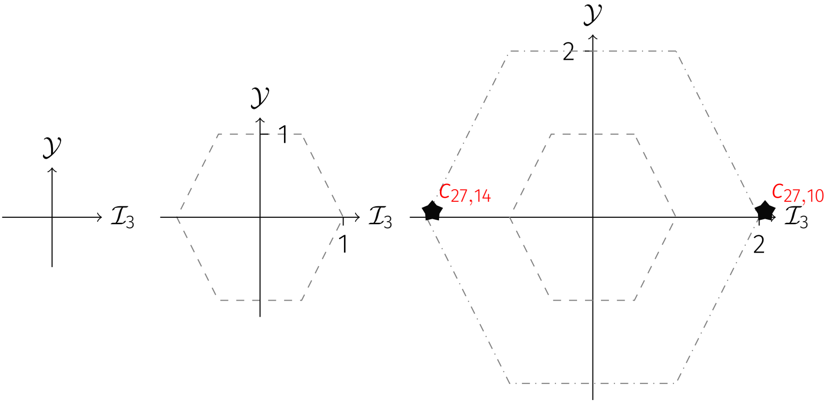
Total lightspin ($\mathcal{I}$) key: ● = 0, ○ = $\frac{1}{2}$, ■ = 1, □ = $\frac{3}{2}$, ⬟ = 2.

Flavour conserving



Total lightspin ($\mathcal{I}$) key: ● = 0, ○ = $\frac{1}{2}$, ■ = 1, □ = $\frac{3}{2}$, ★ = 2.

$\Delta F = 2$, e.g. Kaon mixing



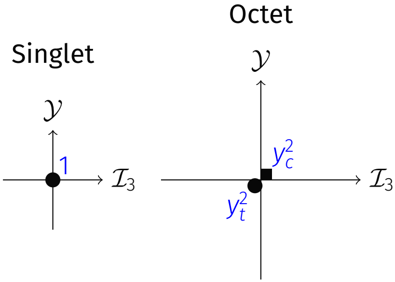
Total lightspin ($\mathcal{I}$) key: ● $= 0$, ○ $= \frac{1}{2}$, ■ $= 1$, □ $= \frac{3}{2}$, ★ $= 2$.

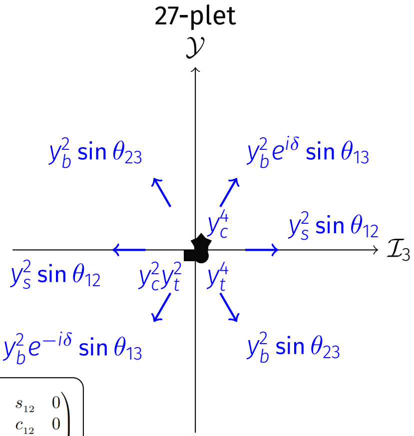
$su(3)^5$ NATURALLY CLASSIFIES COEFFICIENTS

Lepton \ Quark	① $d_{\{Q,u,d\}} = 1$	② $d_{\{Q,u\}} > 1$, $\{\mathcal{I}_3, \mathcal{Y}\}_{\{Q,u\}} = 0$	③ $(\mathcal{I}^2 + \frac{3}{4}\mathcal{Y}^2)_{\{Q,u,d\}} = 1$	④ $(\mathcal{I}^2 + \frac{3}{4}\mathcal{Y}^2)_{\{Q,u,d\}} > 1$
Ⓐ $d_{\{L,e\}} = 1$	Higgs, EW, ...	top, MFV FCNCs	non-MFV FCNCs	e.g. meson mixing
Ⓑ $d_{\{L,e\}} > 1$, $\{\mathcal{I}_3, \mathcal{Y}\}_{\{L,e\}} = 0$	LFUV (quark flavour conserved) e.g. LFUV in Z decays	LFUV in MFV FCNCs	LFUV in non-MFV FCNCs e.g. R_K	-
Ⓒ $(\mathcal{I}^2 + \frac{3}{4}\mathcal{Y}^2)_{\{L,e\}} = 1$	LFV (quark flavour conserved) e.g. $\mu \to 3e$, $H \to \tau\mu$	LFV in MFV FCNCs	LFV in non-MFV FCNCs e.g. $B \to K\mu e$	-
Ⓓ $(\mathcal{I}^2 + \frac{3}{4}\mathcal{Y}^2)_{\{L,e\}} > 1$	e.g. muonium oscillations, $\tau^+ \to \mu^- e^+ e^+$	-	-	-

(Machado, Renner, and Sutherland 2023)

For Q charged Wilson coeffs



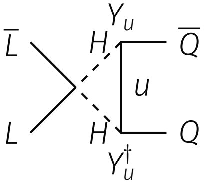


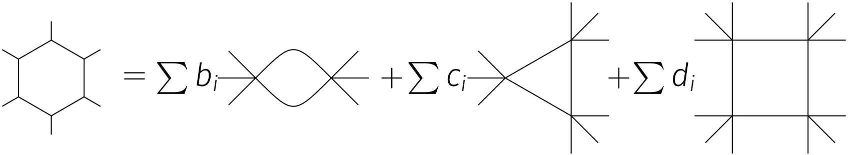
$$V_{\rm CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(Working in up basis.)

BLOCKS FROM HELICITY

Consider the Passarino-Veltman decomposition of a one loop diagram, e.g.

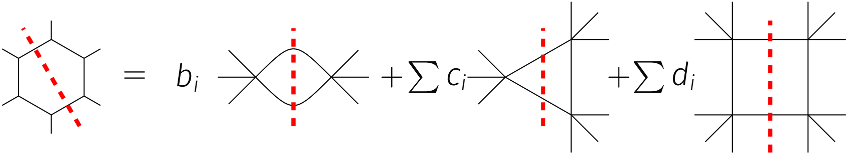




It contains UV and IR divergences. Anomalous dimensions are encoded in the *b*s.

CUTTING THROUGH THE NOISE

“Cut” both sides by placing two propagators on-shell



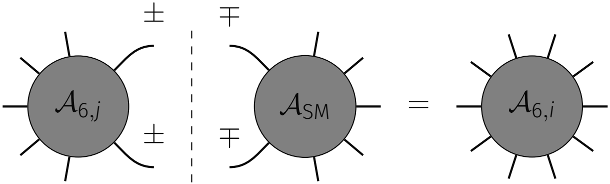
Take all possible cuts, obtain a set of linear equations for $\{b_i, c_i, d_i\}$.

For many SMEFT amplitudes, the LHS vanishes for all cuts, and therefore $b_i = c_i = d_i = 0$.

For how to calculate EFT RGs onshell, see (Caron-Huot and Wilhelm 2016), (Jiang, Ma, and Shu 2021), (Baratella, Fernandez, and Pomarol 2020), (Elias Miró, Ingoldby, and Riembau 2020), ...

HELICITY SELECTION RULES

LHSs vanish because tree-level SM ultra-helicity violating amplitudes vanish, e.g., $\mathcal{A}_{\mathrm{SM}}(g^+g^+g^+g^-)$. (Cheung and Shen 2015)

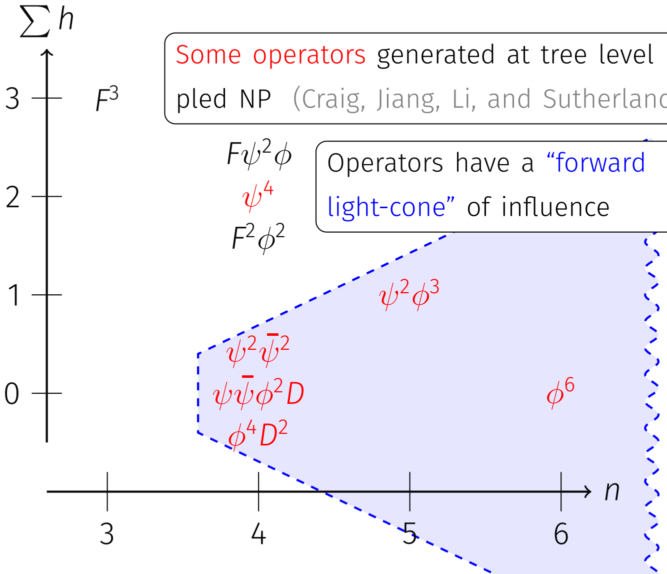


$$\begin{pmatrix} n_j \\ \sum h_j \end{pmatrix} + \begin{pmatrix} -4 \\ 0 \end{pmatrix} + \begin{pmatrix} n_{\mathrm{SM}} \\ \sum h_{\mathrm{SM}} \end{pmatrix} = \begin{pmatrix} n_i \\ \sum h_i \end{pmatrix} \quad \begin{matrix} \text{[\# legs]} \\ \text{[Tot. helicity]} \end{matrix}$$

$|\sum h_{\mathrm{SM}}| \leq n_{\mathrm{SM}} - 4$ at tree level with the exception of $\mathcal{A}(Q^+u^+Q^+d^+) \propto Y_u \times Y_d$

A "CAUSAL" RG

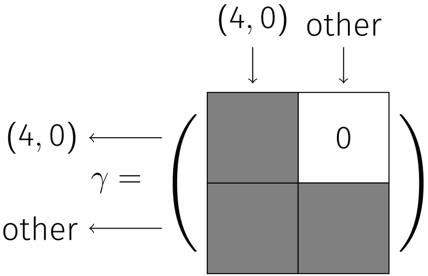
(Alonso, Jenkins, and Manohar 2014; Cheung and Shen 2015)



Some operators generated at tree level by weakly coupled NP (Craig, Jiang, Li, and Sutherland 2020)

Operators have a "forward light-cone" of influence

FOCUS ON THE $(4, 0)$ BLOCK

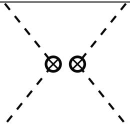
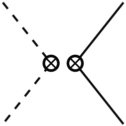
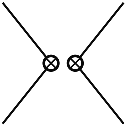


Nothing runs into it, except a few $(4, 2)$ operators, but that's suppressed by $Y_u \times Y_d$. By the same token, drop $\mathcal{O}_{Hud}$ and $\mathcal{O}_{LedQ}$.

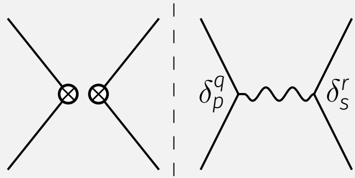
The block contains 1460 of 2499 parameters, all tree-level generated.

BLOCKS FROM FLAVOUR

All considered operators are the product of two currents

Class	Example	Notation
$\phi^4 D^2$	$(H^\dagger i\overleftrightarrow{D}H)^2, (H^\dagger i\overleftrightarrow{D}\sigma^I H)^2$	
$\psi\bar{\psi}\phi^2 D$	$(H^\dagger i\overleftrightarrow{D}H)(\overline{u}_R\gamma u_R), (H^\dagger i\overleftrightarrow{D}\sigma^I H)(\overline{L}_L\gamma\sigma^I L_L)$	
$\psi^2\bar{\psi}^2$	$(\overline{Q}_L\gamma\lambda^A Q_L)(\overline{u}_R\gamma\lambda^A u_R), (\overline{L}_L\gamma L_L)^2$	

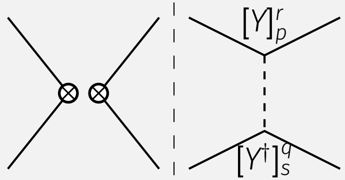
THE FOUR TYPES OF RUNNING: IR FINITE GAUGE



It is *flavourful* — it lifts flavour universal pieces relative to non-universal ones.

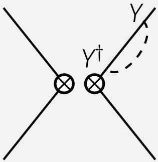
It can change operator type.

THE FOUR TYPES OF RUNNING: IR FINITE YUKAWA



It is *flavourful* — it affects the third generation more than others.

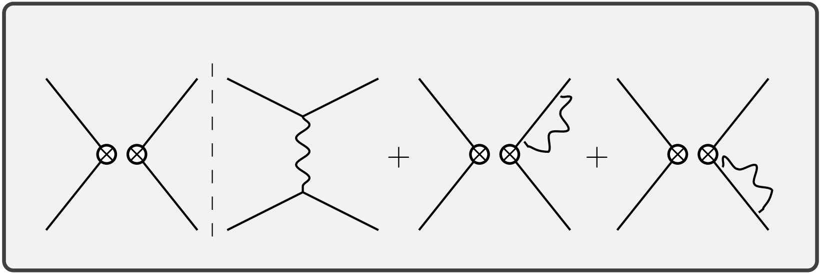
It can change operator type.



It is *flavourful* — it affects third generation more than others.

It *cannot* change operator type.

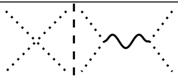
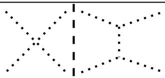
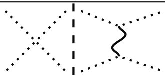
THE FOUR TYPES OF RUNNING: IR DIVERGENT GAUGE



It is *flavourless*. It often vanishes due to non-renormalisation of number current.

It *cannot* change operator type (other than mixing different gauge structures, e.g. $\mathcal{O}_{ud}^{(1)} \leftrightarrow \mathcal{O}_{ud}^{(8)}$).

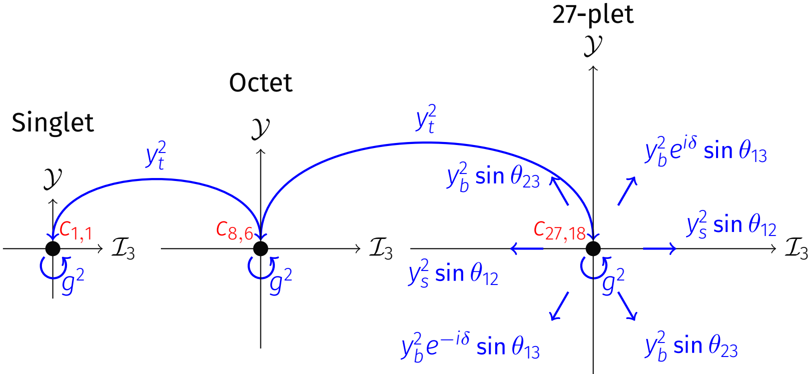
THE FOUR TYPES OF RUNNING: SUMMARY

γ contribution	Cut topology	Flavour action
IR-finite gauge		singlets $\leftrightarrow$ singlets
IR-finite Yukawa		mixes irreps
IR-divergent gauge	 and collinear	blind
IR-divergent Yukawa	collinear	mixes irreps

(Also a couple flavourless Higgs quartic interactions.)

Yukawa charges dictate running in different directions.

For Q charged Wilson coeffs



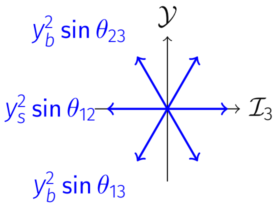
$$V_{\text{CKM}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(Working in up basis.)

WHY GAUGE+y_t IS A GOOD APPROXIMATION

In the up basis, the Yukawas' flavour violation is small.

Instead, flavour violation is through 'diagonal' y_t running + matching.



$$\overline{Q}^i \gamma \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}^{ij} Q^j \overset{\text{Run}}{\rightarrow} \overline{Q}^i \gamma \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & b \end{pmatrix}^{ij} Q^j \underset{Q=\begin{pmatrix} u \\ Vd \end{pmatrix}}{\overset{\text{Match}}{\longrightarrow}} (b-a)(V^{\text{CKM}}_{3i})^* V^{\text{CKM}}_{3j} \overline{d}^i \gamma d^j$$

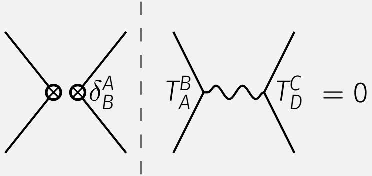
y_t important: comparable to g_s and appears frequently

$$\alpha_t(m_Z) = \frac{y_t^2}{4\pi} \approx 0.08 \,; \qquad \alpha_s(m_Z) = \frac{g_s^2}{4\pi} \approx 0.12 \,.$$

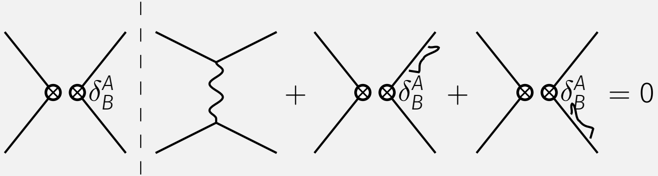
g_s IS LESS IMPORTANT THAN y_t

g_s appears less frequently than y_t as it requires non-trivial colour structures (other than δ^A_B)

Because $Tr[T] = 0$



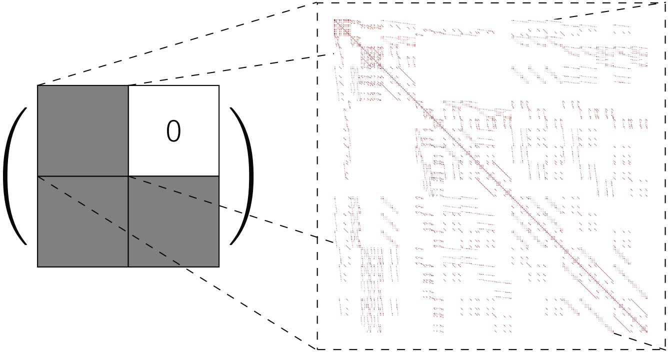
Because number current is conserved



FLAVOUR BLOCKS: GAUGE AND y_t APPROX.

In ***DSixTools***'s basis, the matrix is sparse and messy.

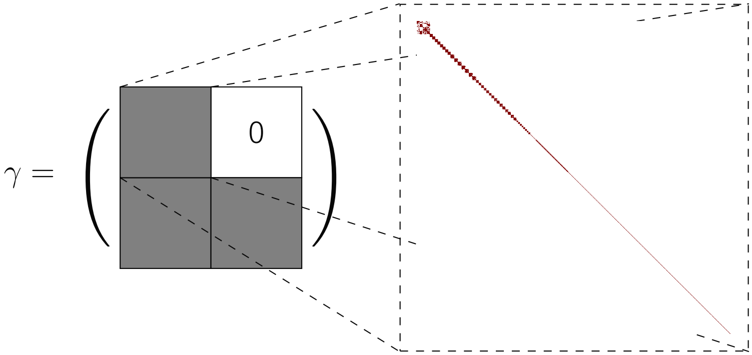
$\gamma =$



This is not a complaint about ***DSixTools***, which has been very useful for this study! (Fuentes-Martin, Ruiz-Femenia, Vicente, and Virto 2021)

FLAVOUR BLOCKS: GAUGE AND y_t APPROX.

$SU(3)^5$ decomposition block diagonalises sparse 1460 by 1460 anomalous dim. matrix of current-current operators.

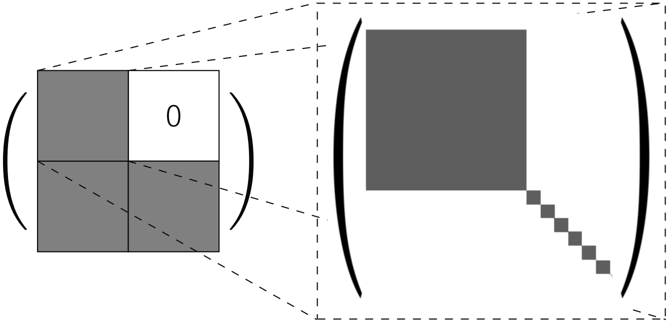


(Blocks from conserved charges: $\{\mathcal{I}, \mathcal{I}_3, \mathcal{Y}\}_{\{Q,u\}}$, $\{d, \mathcal{I}, \mathcal{I}_3, \mathcal{Y}\}_{\{d,L,e\}}$)

FLAVOUR BLOCKS: NO APPROX.

$SU(3)^5$ decomposition block diagonalises sparse 1460 by 1460 anomalous dim. matrix of current-current operators.

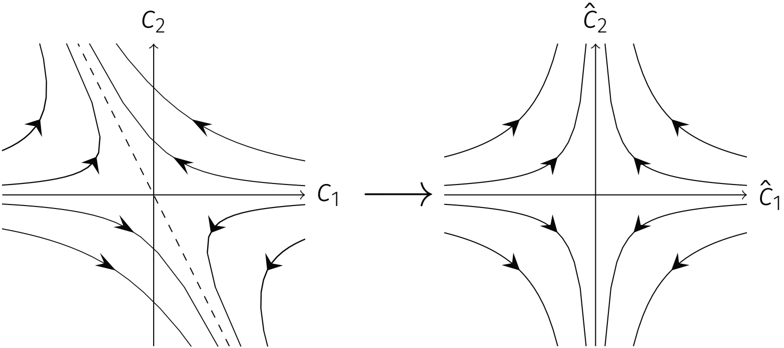
$\gamma =$



(Blocks from conserved charges: $\mathcal{I}_{3,L} + \mathcal{I}_{3,e},\ \mathcal{Y}_L + \mathcal{Y}_e$)

IR RELEVANT DIRECTIONS

MIXING IS A BASIS ARTEFACT

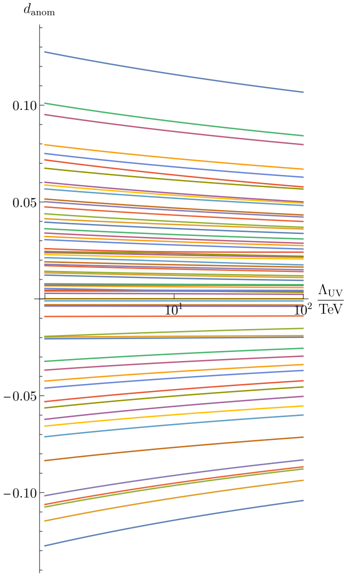


Diagonalise the anomalous dimension matrix

$$\frac{\mathrm{d}\hat{c}_i}{\mathrm{d}\ln\mu} = \frac{\hat{\gamma}_i \hat{c}_i}{16\pi^2} \implies \mathcal{A}_{4\text{-pt}} \sim \hat{c}_i^{(6)}(E)\left(\frac{E}{M}\right)^2 = \hat{c}_i^{(6)}(M)\left(\frac{E}{M}\right)^{2+\frac{\hat{\gamma}_i}{16\pi^2}}$$

(To account for running of SM coeffs, $\gamma \to \frac{\int \gamma(\mu)\mathrm{d}\ln\mu}{\int \mathrm{d}\ln\mu}$.)

OPERATOR SPECTRUM IN GAUGE+y_t APPROX. (PRELIMINARY)



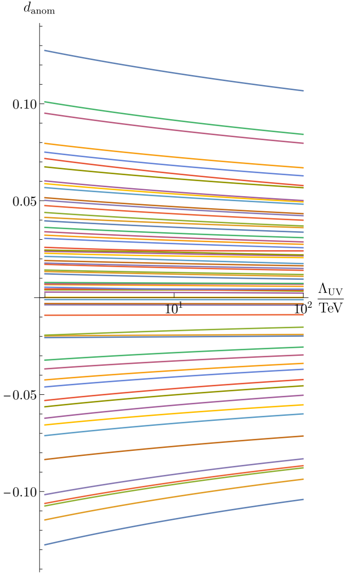
Diagonalise the 61×61 block. (The biggest block, mixing flavour universal and 3rd gen. operators.)

Contains 533 y_t^2 entries, 138 g_s^2 entries.

Individual entries $\frac{g_s^2(m_Z)}{16\pi^2} = 0.01$ add up to $\pm\mathcal{O}(0.1)$ eigenvalues.

Directions double/halve from 50 TeV to 174 GeV.

OPERATOR SPECTRUM IN GAUGE+y_t APPROX. (PRELIMINARY)



Most IR irrelevant: Dimension 6.12

$\mathcal{O} \approx 0.94\mathcal{O}_{HD} + 0.23\mathcal{O}_{H\Box} - 0.19(\mathcal{O}_{Hu})_{8,6} + \ldots$

Most IR relevant: Dimension 5.88

$\mathcal{O} \approx 0.63(\mathcal{O}^{(8)}_{Qu})_{8,6,1,1} - 0.59(\mathcal{O}^{(8)}_{Qd})_{8,6,1,1} - 0.46(\mathcal{O}^{(8)}_{Qd})_{8,6,8,6} + \ldots$

SUMMARY

An $SU(3)^5$ decomposition usefully organises the flavour parameter space of a *completely generic* SMEFT.

It contains flavour Ansätze within identifable subsets.

It simplifies RG effects, to the point that they are (semi)analytically soluble.

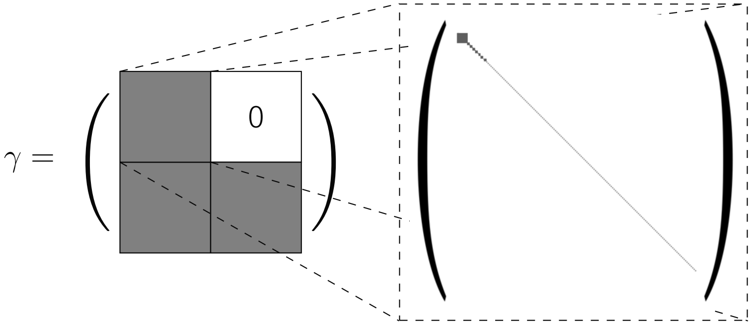
The RG of the SMEFT is not a black box, but a beautifully simple and flavourful machine!

Thank you

Backup

FLAVOUR BLOCKS: GAUGE AND y_t APPROX.

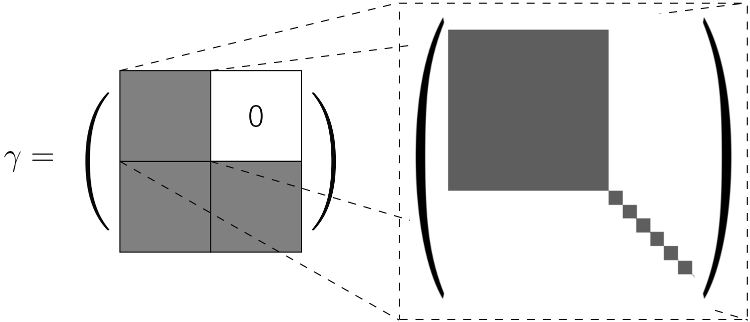
Conserved: $\{\mathcal{I}, \mathcal{I}_3, \mathcal{Y}\}_{\{Q,u\}}$, $\{d, \mathcal{I}, \mathcal{I}_3, \mathcal{Y}\}_{\{d,L,e\}}$



Block size	61	17	13	12	8	2	1
Multiplicity	1	7	8	15	8	217	498

FLAVOUR BLOCKS: NO APPROX.

Lepton number conserved: $\mathcal{I}_{3,L} + \mathcal{I}_{3,e},\ \mathcal{Y}_L + \mathcal{Y}_e$



Block size	932	81	4	3
Multiplicity	1	6	6	6