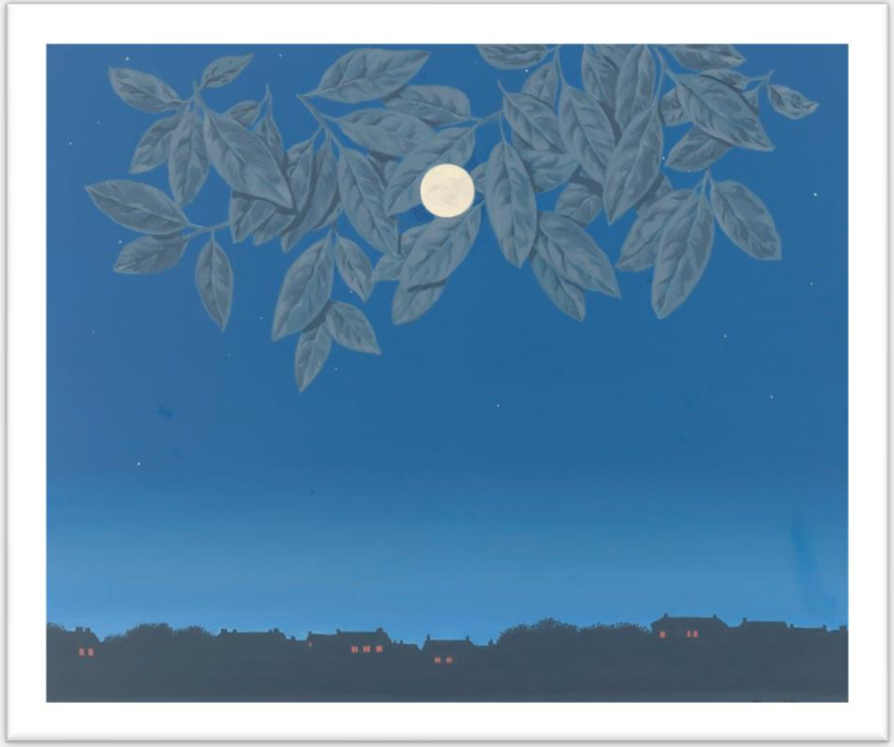

René Magritte, *La page blanche* (1967)

Logarithmic soft graviton theorem from asymptotic symmetries

based on 2309.11220 w/

Shreyansh Agrawal, Kevin Nguyen & Romain Ruzziconi

European Research Council
Established by the European Commission

Laura DONNAY
SISSA

Brookhaven National Laboratory (BNL)
18 Jan 2024

Intro and motivations

Quantum gravity in 4d asymptotically flat spacetimes → vanishing cosmological constant

$$\Lambda = 0$$

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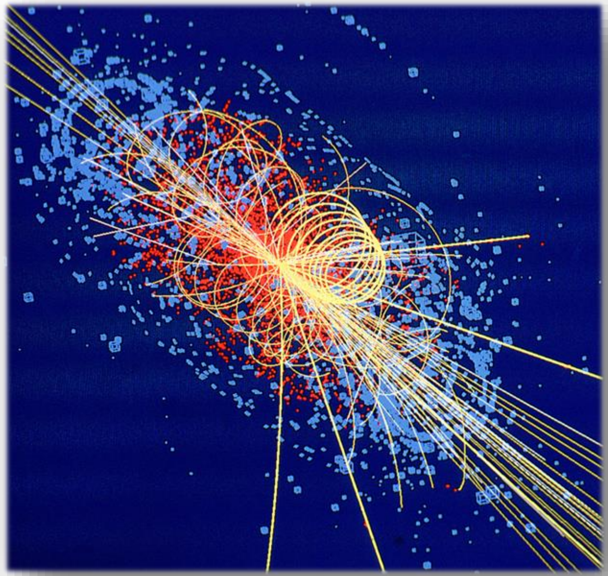
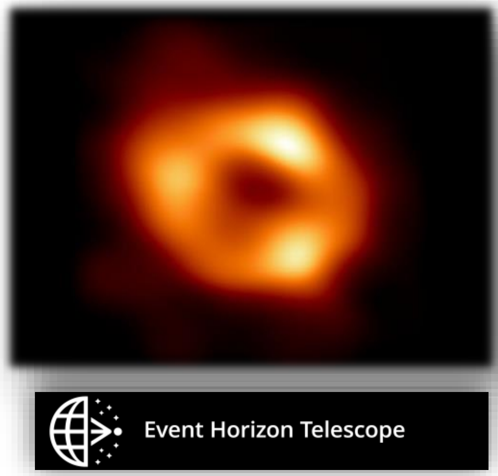
$$\Lambda = 0$$

These spacetimes are relevant



from collider physics …



… to astrophysics
(< cosmological scales)

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Recent developments in quantum gravity for flat spacetimes

- Universal connections between fundamental results (from the 1960’s) in
  General Relativity and **Quantum Field Theory**

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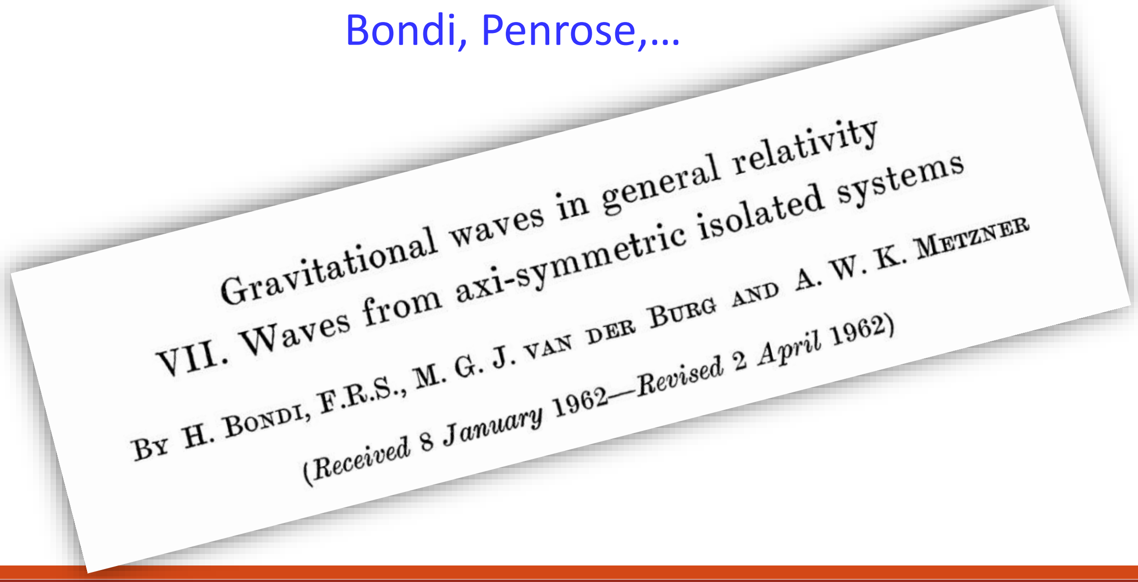
Gravitational waves in general relativity
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VOLUME 140, NUMBER 2B 25 OCTOBER 1965

Infrared Photons and Gravitons*
Steven Weinberg†
Department of Physics, University of California, Berkeley, California
(Received 1 June 1965)

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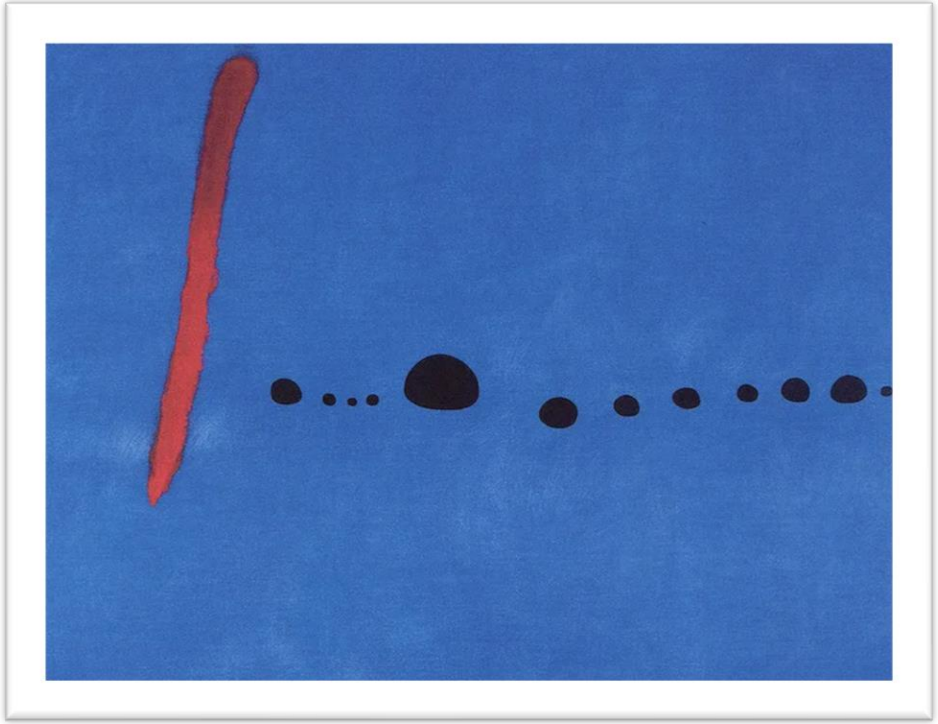
- General lesson: the infrared structure is much richer than we thought!

  new observables, new patterns in scattering amplitudes,…

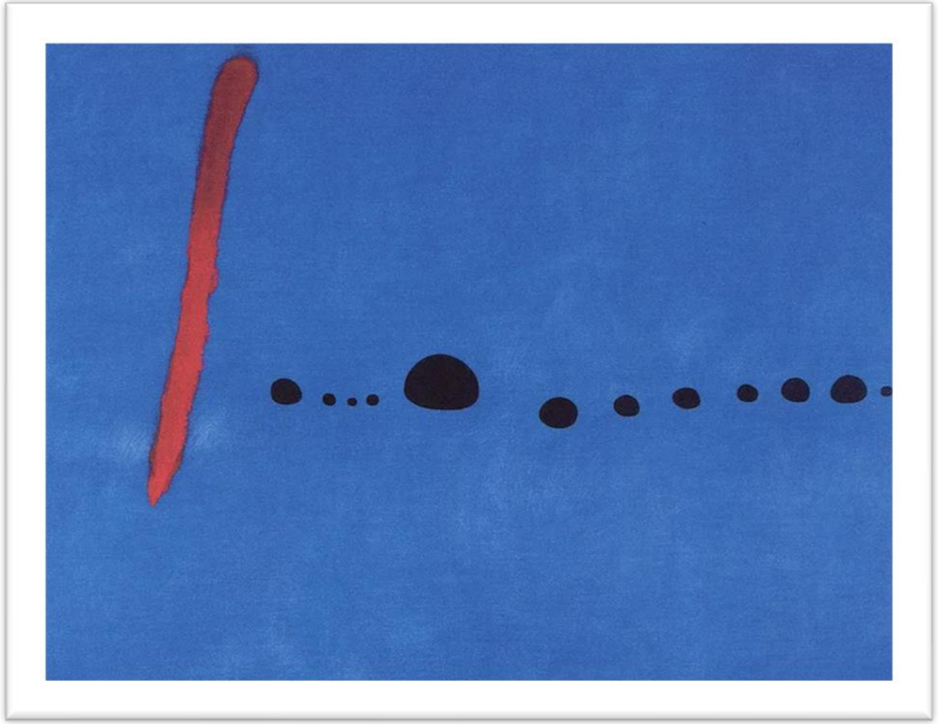
  → towards a ´holographic description'



Joan Miró, *Bleu II* (1961)

Outline

1. Asymptotically flat spacetimes
2. Soft theorems ⟷ asymptotic symmetries
3. New results for the *logarithmic* soft graviton theorem



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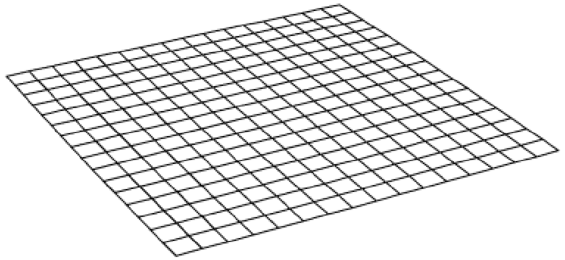
A surprise in flat spacetimes

The BMS symmetries [Bondi-Metzner-van der Burg; Sachs, '62]

Minkowski metric (flat spacetime) in 4D



The geometry is described by the line element

$$ds^2 = -\mathrm{d}t^2 + \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}x_3^2$$

(measure of distance in flat spacetime)

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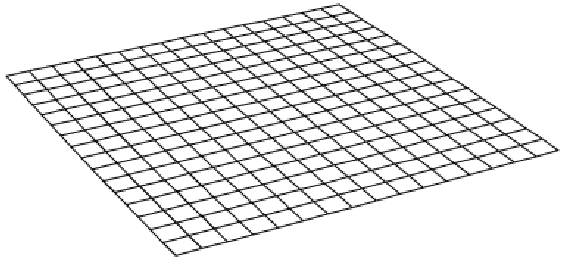

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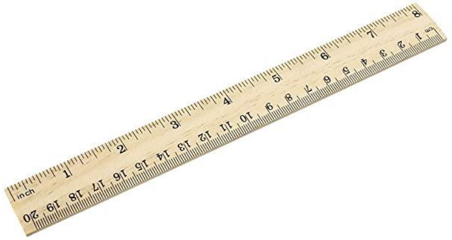

Change to *Bondi coordinates* $(u, r, z, \bar{z})$

$$\boxed{t = u + r, \quad x_1 = \frac{r(z+\bar{z})}{1+z\bar{z}} \\ x_2 = \frac{-ir(z-\bar{z})}{1+z\bar{z}}, \quad x_3 = \frac{r(1-z\bar{z})}{1+z\bar{z}}}$$

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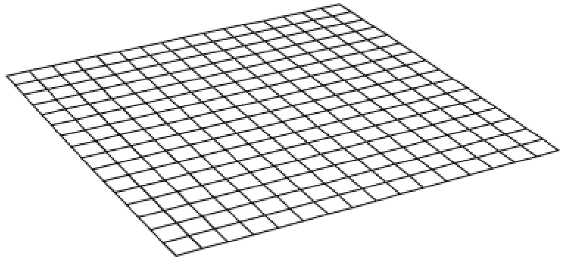

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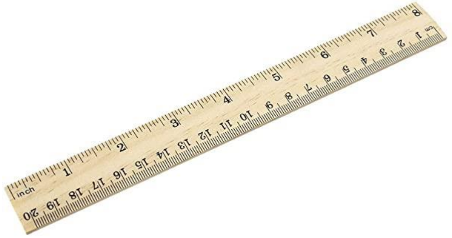

$$z = e^{i\phi}\cot\frac{\theta}{2}$$

$$\bar{z} = e^{-i\phi}\cot\frac{\theta}{2}$$

$$\gamma_{z\bar{z}} = \frac{2}{(1+z\bar{z})^2}$$

sphere angles

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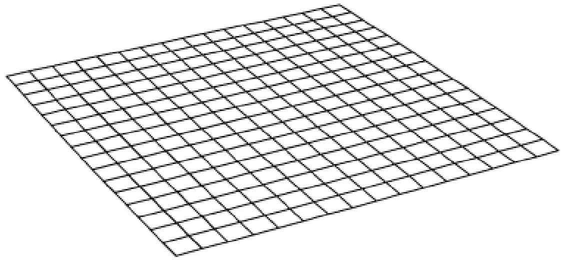

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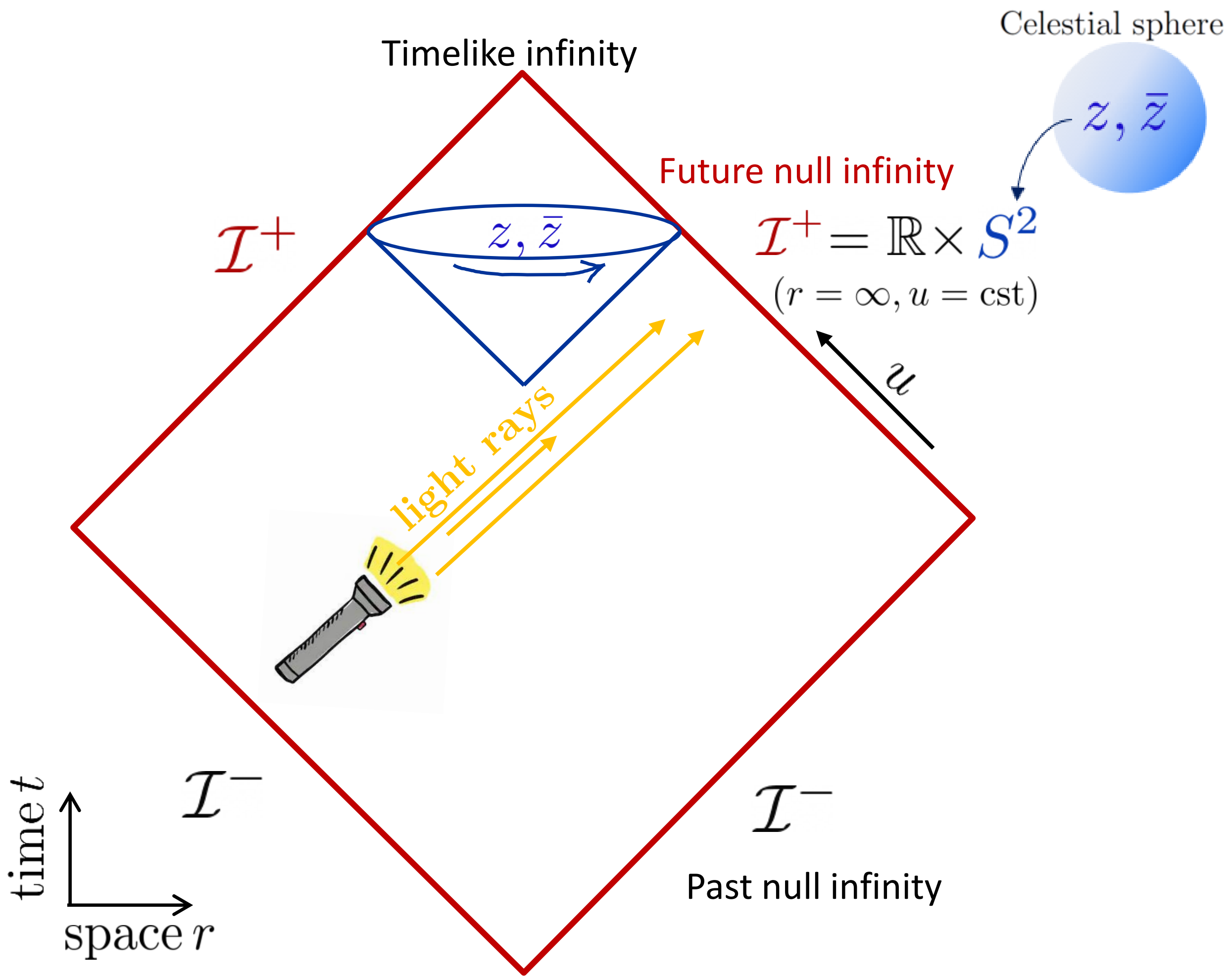
$u = t - r$: 'retarded' null time



Penrose diagram of Minkowski

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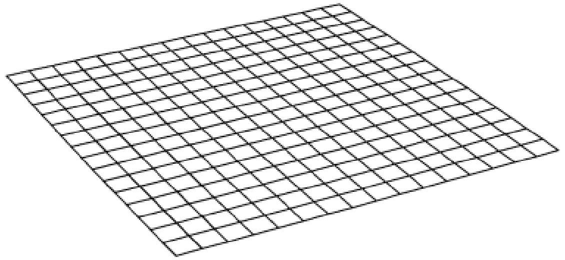

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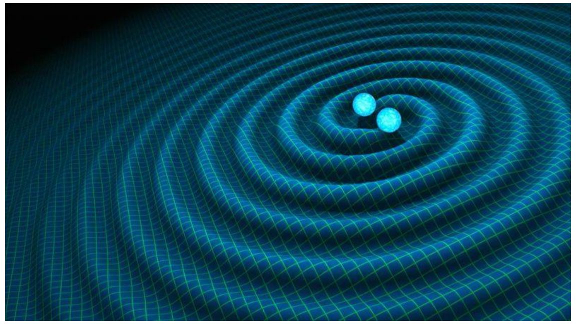
***Asymptotically* flat spacetime** (as $r \to \infty$)



Curved spacetime that looks flat seen from a far distance. The deviation from Minkowski is dictated by boundary conditions for the metric.

$$ds^2 = -\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}r + 2r^2\gamma_{z\bar{z}}\,\mathrm{d}z\mathrm{d}\bar{z} \quad + \frac{2M}{r}\mathrm{d}u^2 + rC_{zz}\mathrm{d}z^2 + D^zC_{zz}\mathrm{d}u\mathrm{d}z$$

$$+\frac{1}{r}\left(\frac{4}{3}(N_z + u\partial_z m_B) - \frac{1}{4}\partial_z(C_{zz}C^{zz})\right)\mathrm{d}u\mathrm{d}z + c.c. + \cdots$$

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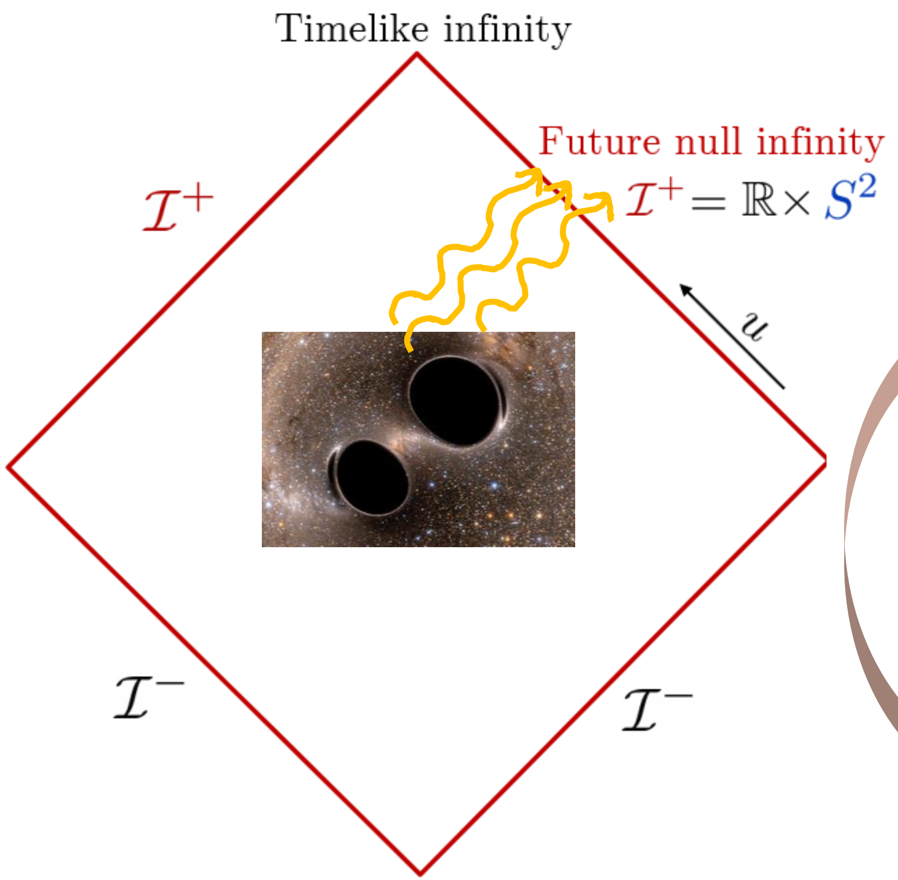

$$
\begin{aligned}
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\end{aligned}
$$

$M(u, z, \bar{z})$ gives the *energy (e.g. black hole mass)*

$N_z(u, z, \bar{z})$ gives the *angular momentum*

$C_{zz}(u, z, \bar{z})$ indicates the presence of **gravitational waves!**

$$\partial_u C_{zz} \neq 0$$

Mathematical description of a **radiating spacetime**

What are the symmetries of asymptotically flat spacetimes?

What are the symmetries of asymptotically flat spacetimes?

what was expected

Poincaré

4 spacetime translations
6 Lorentz transformations

what was found

Bondi-Metzner-Sachs (BMS) (‘62)

Infinite-dimensional extension!

Supertranslations

[Bondi, van der Burg, Metzner ’62] [Sachs ’62]
[Barnich, Troessaert ’10]

Asymptotically flat spacetimes in Bondi gauge:

$$r \to \infty \qquad\qquad (u, r, x^A)\,, \quad x^A = (z, \bar{z})$$

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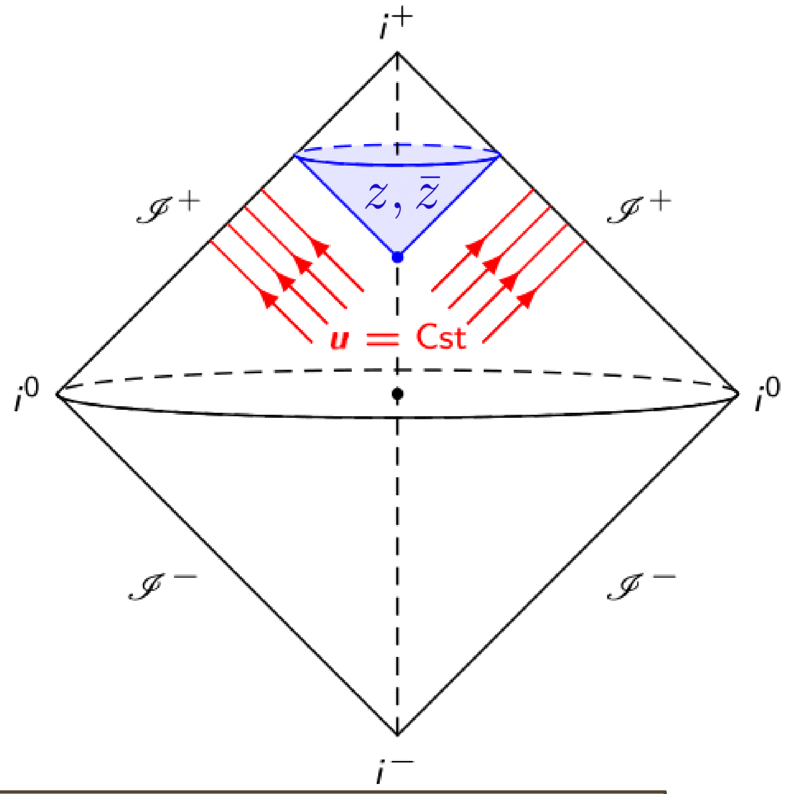

BMS supertranslation symmetries:

$$\xi = \boxed{\mathcal{T}(z, \bar{z})}\partial_u + \cdots$$

arbitrary function
on the celestial sphere

4 Poincaré translations

↓ *Symmetry enhancement*

∞ BMS *super*translations

Supertranslations

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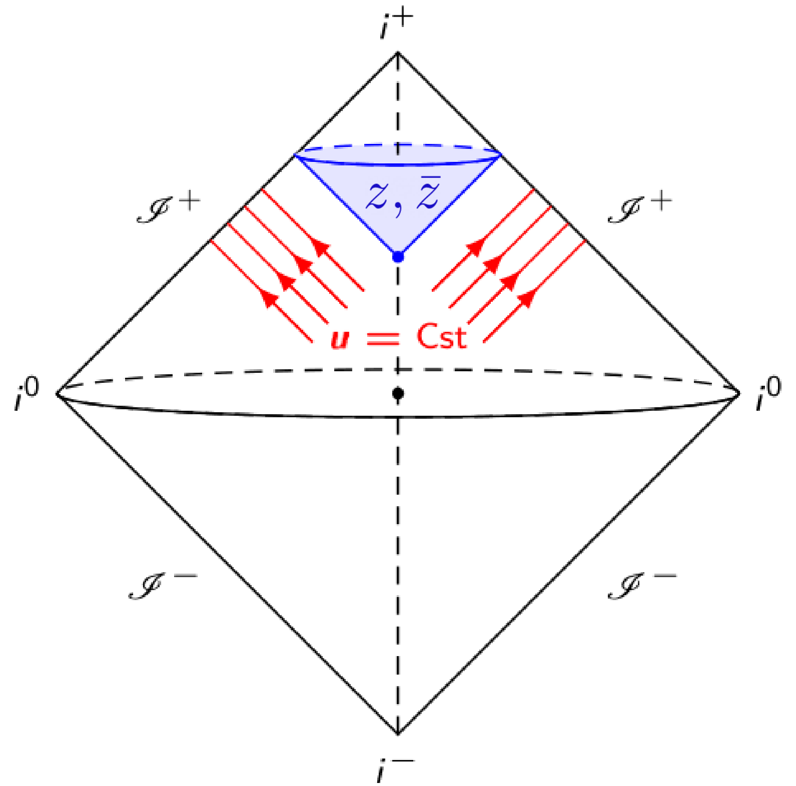

BMS supertranslation symmetries:

$$\xi = \mathcal{T}(z, \bar{z})\partial_u + \cdots$$

act non-trivially on the gravitational solution space

$$\delta_\xi M = \cdots \qquad \delta_\xi C_{zz} = (\cdots)C_{zz} - 2D_z^2\mathcal{T}$$

4 Poincaré translations

↓ *Symmetry enhancement*

∞ BMS *super*translations

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The fact that α is an arbitrary function of its arguments is quite annoying in some respects.

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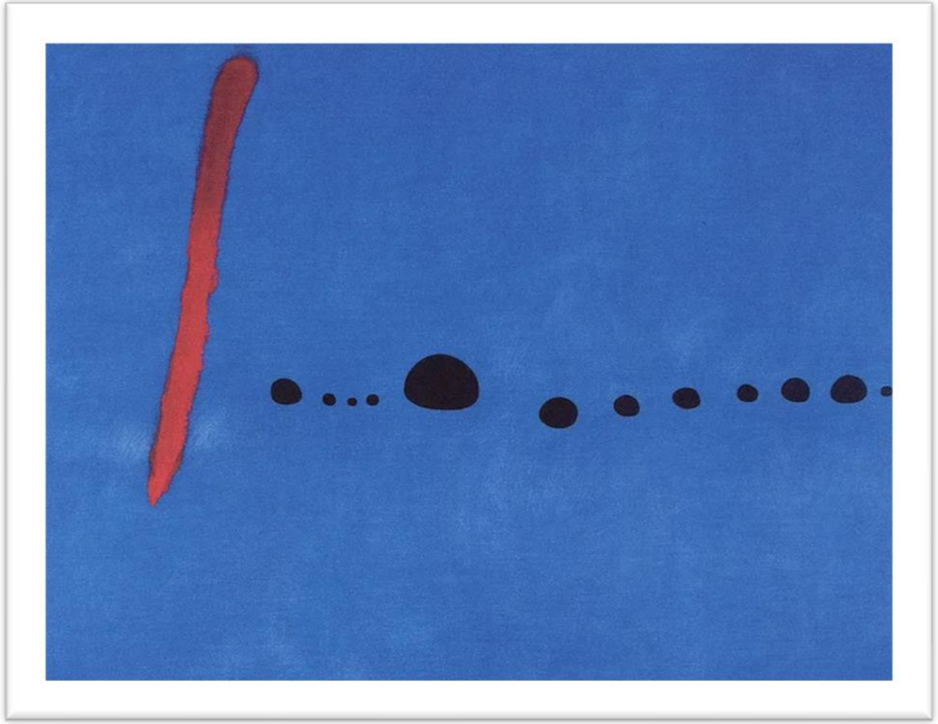
[Sachs '62]

But…

Conceivably α is a blessing in disguise and the representations of the group (3·12) are more interesting than the representations of the Lorentz group.



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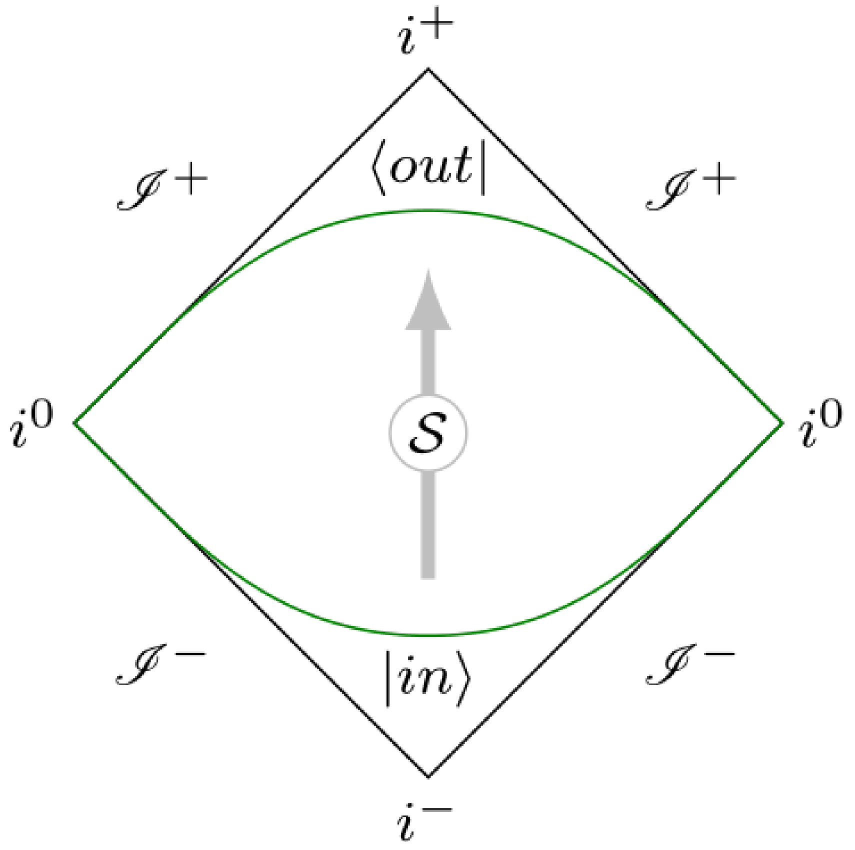
BMS and the scattering problem

Seminal observation: BMS symmetries constrain the gravitational scattering problem! [Strominger '14]

$\langle out | S | in \rangle$



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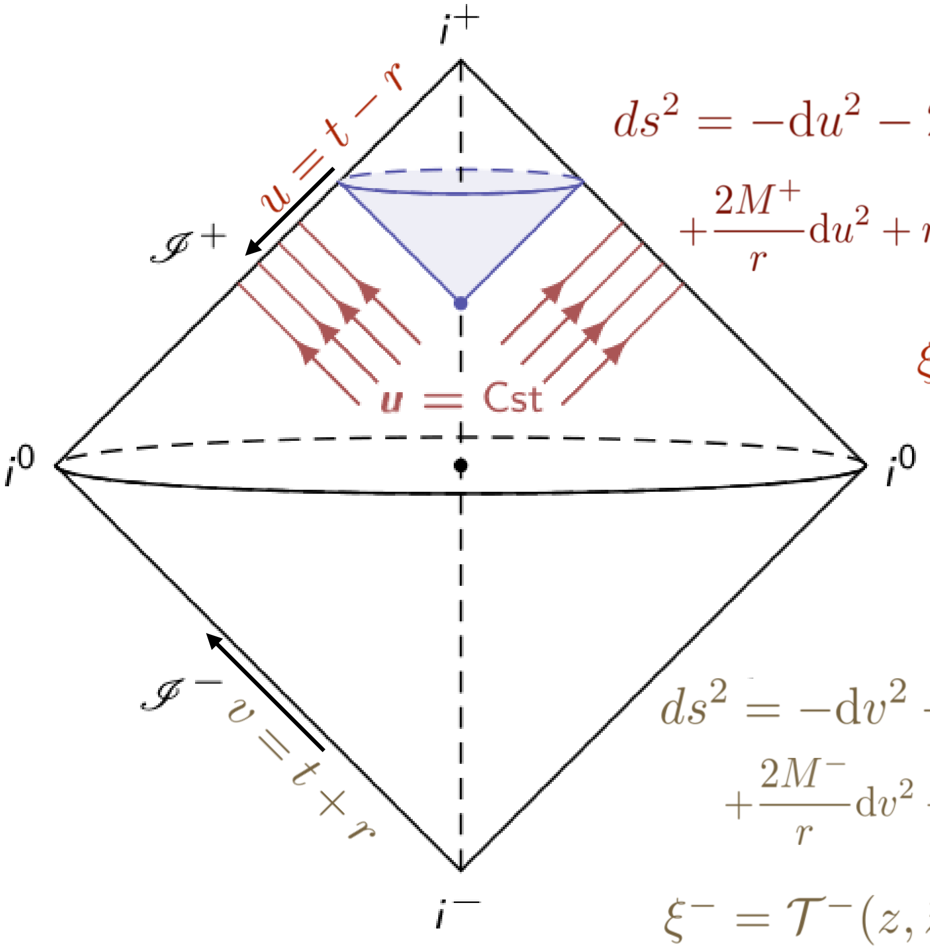

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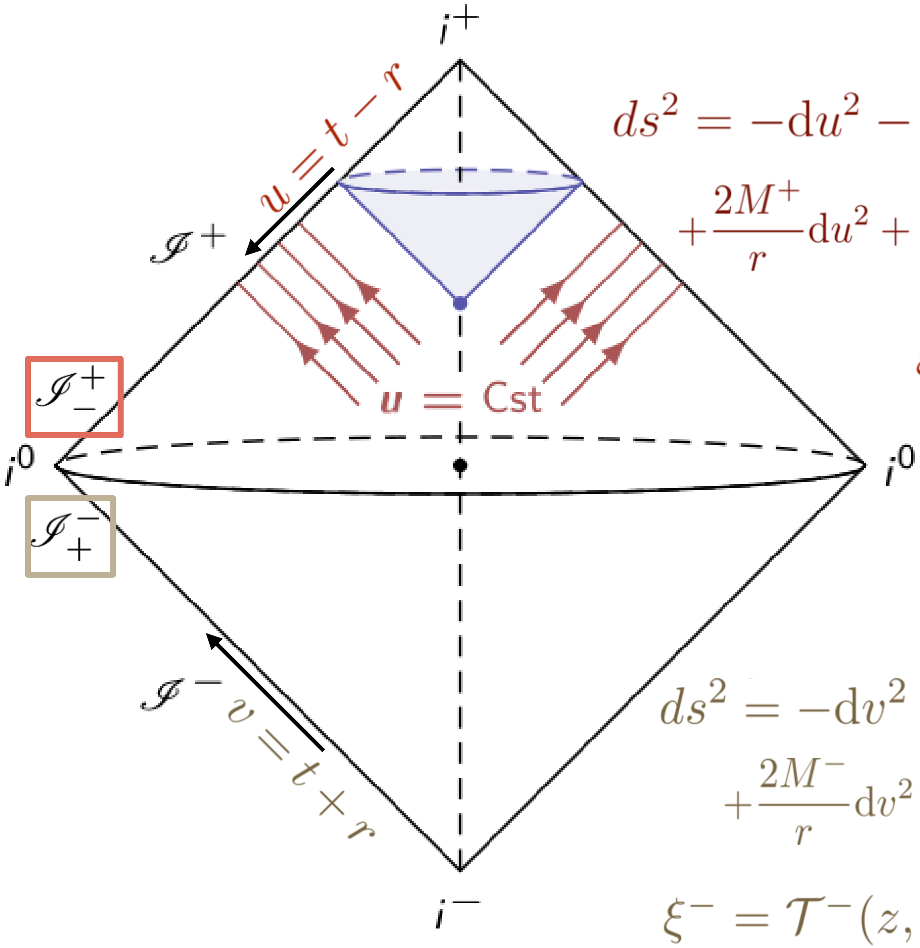

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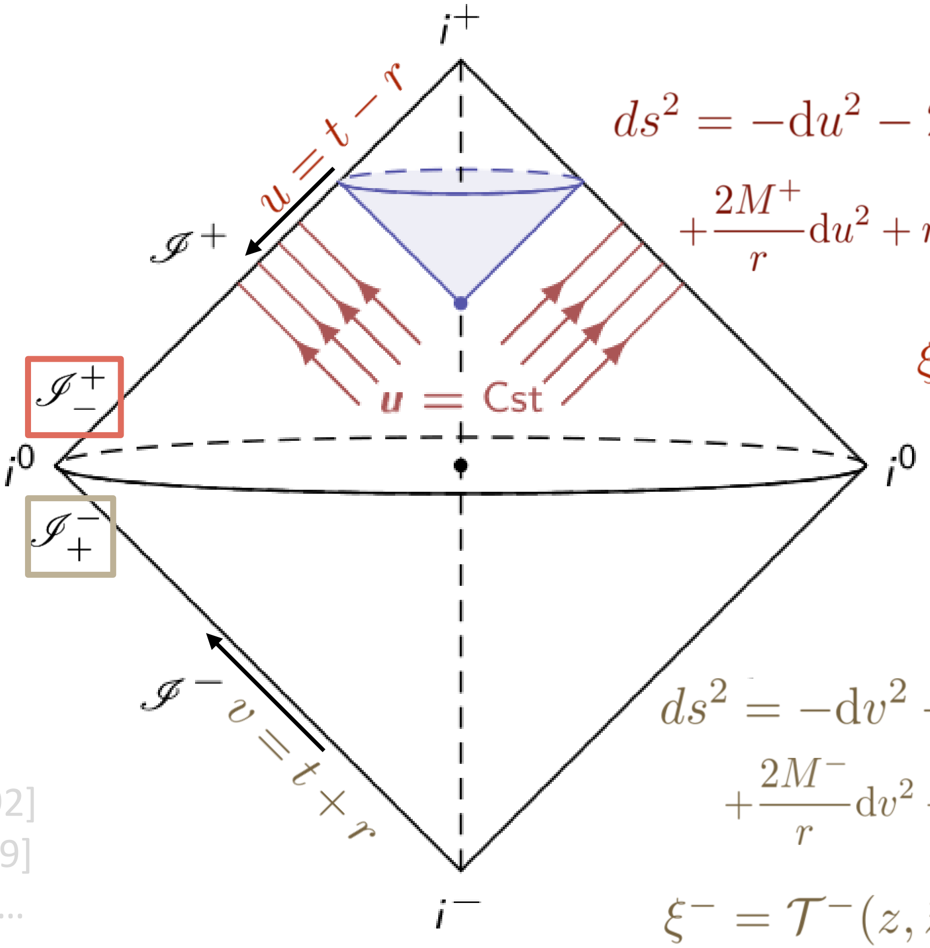
$$M^-(v,z,\bar{z})|_{\mathscr{I}^-_+} = M^+(u,z,\bar{z})|_{\mathscr{I}^+_-}$$

$$\mathcal{T}^-(z,\bar{z})|_{\mathscr{I}^-_+} = \mathcal{T}^+(z,\bar{z})|_{\mathscr{I}^+_-}$$

[Strominger '14]; see also [Herberthson, Ludvigsen '92] [Troessaert '18][Henneaux, Troessaert '18][Prabhu '19] [Kroon, Mohamed '21][Capone, Nguyen, Parisini '22]...



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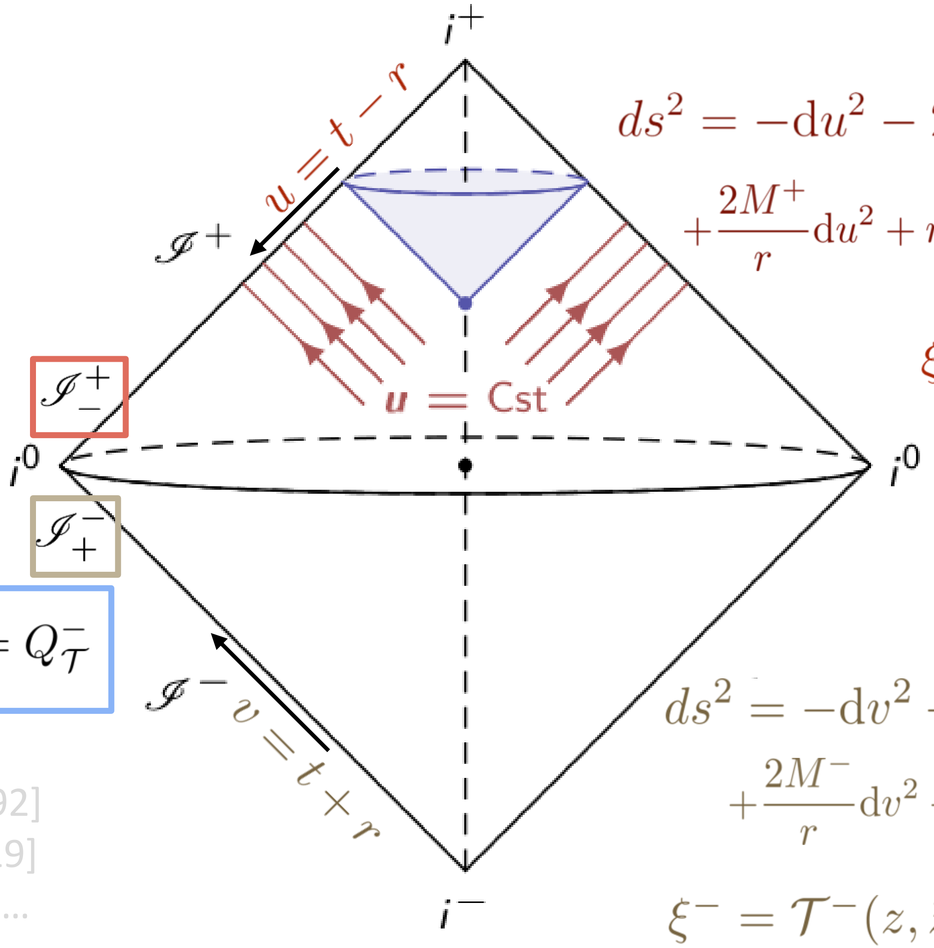

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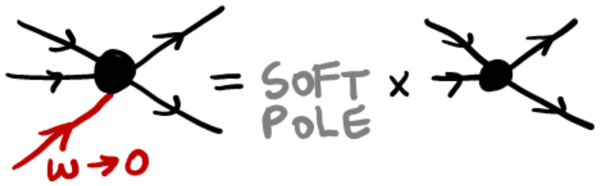
BMS and the scattering problem

Prime example:

The **leading soft graviton theorem** [Weinberg '65]

$\mathcal{A}_n = \langle out | S | in \rangle$

+soft particle (energy $\omega \to 0$)



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$$n \text{ hard particles } (p_k) + \text{external graviton } (q)$$

$$\lim_{\omega\to 0} \mathcal{A}_{n+1}(q) = S^{(0)}\mathcal{A}_n + \mathcal{O}(q^0)$$

$$S^{(0)} = \sum_{k=1}^{n} \frac{p_k^\mu p_k^\nu \varepsilon_{\mu\nu}(q)}{p_k \cdot q}$$

$\mathcal{A}_n = \langle \mathrm{out}|S|\mathrm{in}\rangle$

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BMS and the scattering problem

Prime example:

The **leading soft graviton theorem** [Weinberg '65]

$$n \text{ hard particles } (p_k) + \text{external graviton } (q)$$

$$\lim_{\omega\to 0} \mathcal{A}_{n+1}(q) = S^{(0)}\mathcal{A}_n + \mathcal{O}(q^0)$$

$$S^{(0)} = \sum_{k=1}^{n} \frac{p_k^\mu p_k^\nu \varepsilon_{\mu\nu}(q)}{p_k \cdot q}$$

$\mathcal{A}_n = \langle out|S|in\rangle$

+soft particle (energy $\omega \to 0$)

is nothing but the **Ward identity** associated to **supertranslation** symmetry [He, Lysov, Mitra, Strominger '15]

$$\langle out|Q_{\mathcal{T}}^{+}\mathcal{S} - \mathcal{S}Q_{\mathcal{T}}^{-}|in\rangle = 0$$

supertranslation charge

$$Q_{\mathcal{T}} = \frac{1}{4\pi G}\int d^2z\sqrt{\gamma}\,\mathcal{T}M$$

3 languages for the same IR physics

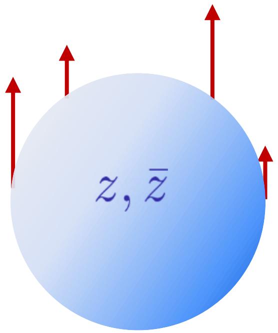
[Strominger '18]

Asymptotic symmetries

General Relativity

supertranslations

[Bondi-Metzner-Sachs '62]



$$\Delta C_{AB} \neq 0$$

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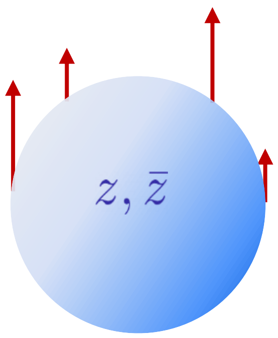


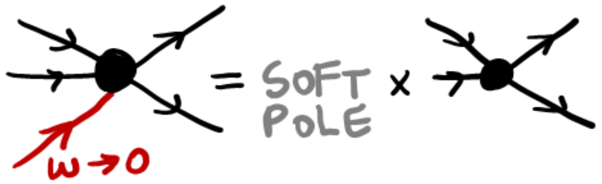
$$\Delta C_{AB} \neq 0$$

Soft theorems

Quantum Field Theory

leading soft graviton theorem

[Weinberg '65]



3 languages for the same IR physics

[Strominger '18]

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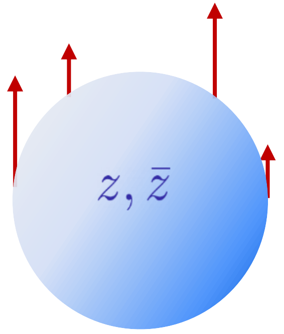

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Soft theorems

Quantum Field Theory

leading soft graviton theorem

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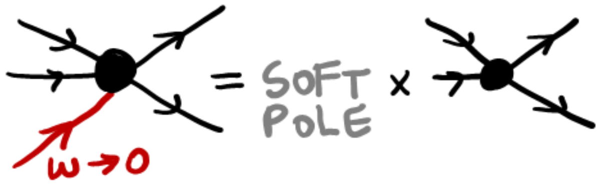
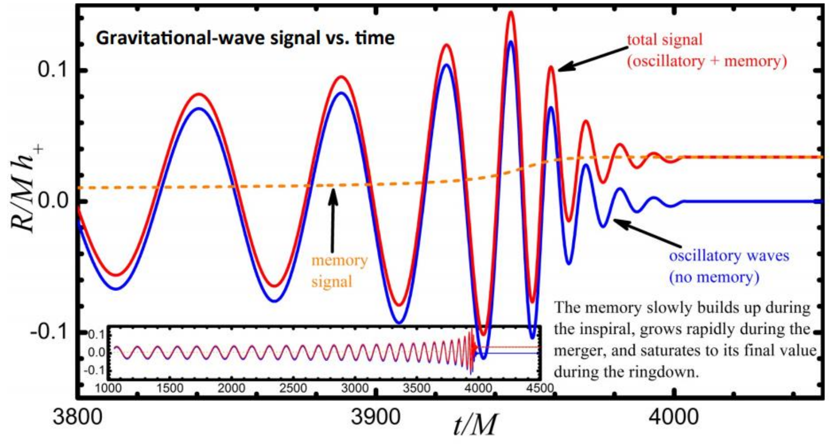


Memory effects

GW observation

displacement memory

[Zel'dovich, Polnarev, Braginskii, Thorne, Christodoulou] ... 70s – 90s

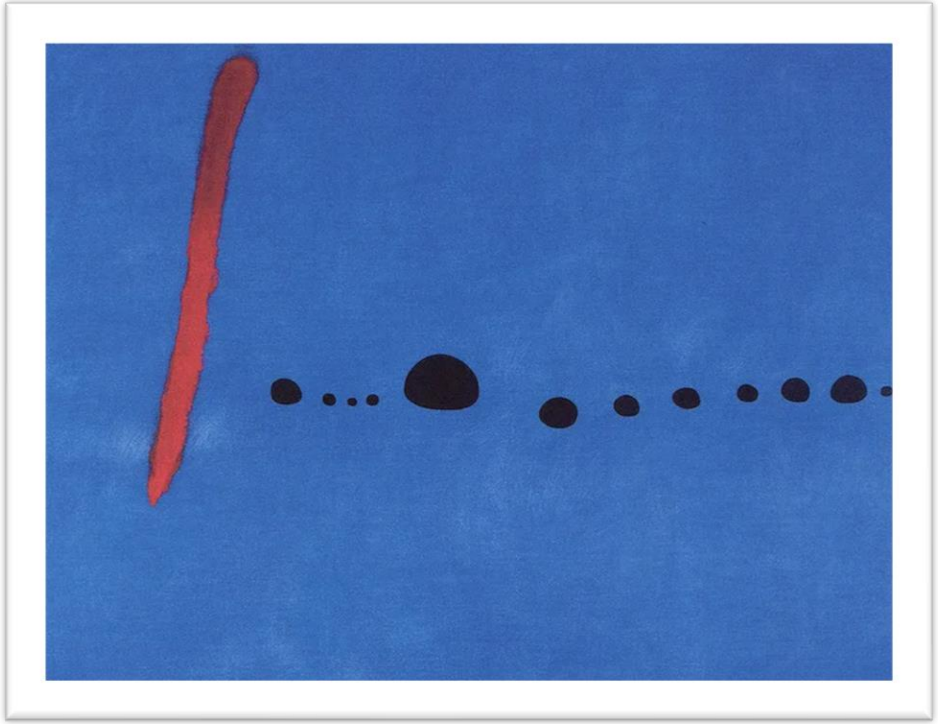


[Favata, '10]

Conclusions so far

Physics in the deep infrared is **much richer**, more subtle and **much less understood** than we previously thought.

The boundary of **flat space** exhibits an **infinite** amount of **symmetries** which constrain the scattering problem.



Joan Miró, *Bleu II* (1961)

Outline

1. Asymptotically flat spacetimes
2. Soft theorems $\longleftrightarrow$ asymptotic symmetries
3. New results for the logarithmic soft graviton theorem

~~Logarithmic~~ soft theorems

- Tree-level soft graviton theorem
(assuming a power series expansion in the soft momentum $q = \omega \hat{q}$)

[Weinberg '65]
[Cachazo, Strominger '14]

$$\mathcal{M}_{n+1} \overset{\omega \to 0}{=} \left[\omega^{-1} S_n^{(0)} + \omega^0 S_n^{(1)} \right] \mathcal{M}_n + \mathcal{O}(\omega)$$

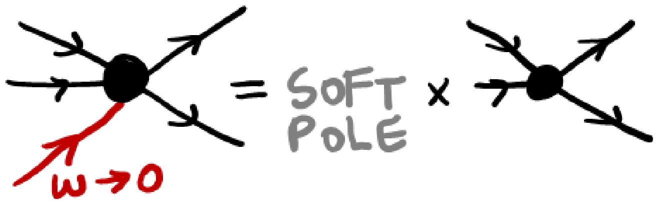
~~Logarithmic~~ soft theorems

- Tree-level soft graviton theorem
  (assuming a power series expansion in the soft momentum $q = \omega \hat{q}$)

[Weinberg '65]
[Cachazo, Strominger '14]

$$\mathcal{M}_{n+1} \overset{\omega \to 0}{=} \Big[\underbrace{\omega^{-1} S_n^{(0)}}_{\text{leading}} + \underbrace{\omega^{0} S_n^{(1)}}_{\text{subleading}}\Big] \mathcal{M}_n + \mathcal{O}(\omega)$$

$$S_n^{(0)} = \frac{\kappa}{2} \sum_{i=1}^{n} \frac{p_i^\mu p_i^\nu \varepsilon_{\mu\nu}(\hat{q})}{p_i \cdot \hat{q}}$$



$$S_n^{(1)} = -\frac{i\kappa}{2} \sum_{i=1}^{n} \frac{p_i^\mu \, \varepsilon_{\mu\nu}(\hat{q}) \, q_\lambda}{p_i \cdot q} \left(J_i^{\lambda\nu} + S_i^{\lambda\nu}\right)$$

$$\kappa = \sqrt{32\pi G}$$

Logarithmic soft theorems

- One-loop corrections generate logarithmic corrections!

$$\mathcal{M}_{n+1} \overset{\omega\to 0}{=} \left[\omega^{-1}\, S_n^{(0)} - \frac{\kappa^2}{4} \ln\omega\, S_n^{(\ln)}\right] \mathcal{M}_n + \mathcal{O}(\omega^0)$$

dominate over the subleading term

[Laddha, Sen ’18 ‘19] [Sahoo, Sen ‘18] [Saha, Sahoo, Sen ‘19][Krishna, Sahoo ‘23]
[Ciafaloni, Colferai, Veneziano ‘18] [Addazi, Bianchi, Veneziano ’19]
[di Vecchia, Heissenberg, Russo, Veneziano ‘23]

Logarithmic soft theorems

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[Sahoo, Sen '18][…]

$$\begin{aligned} S_n^{(\ln)} =& \frac{i\kappa}{8\pi} \sum_i \frac{\varepsilon_{\mu\nu} p_i^\mu p_i^\nu}{p_i \cdot q} \sum_j \delta_{\eta,\eta_j}\, q\cdot p_j \\ &+ \frac{i\kappa}{16\pi} \sum_i \frac{\varepsilon_{\mu\nu} p_i^\nu q_\rho}{p_i \cdot q} \sum_j \delta_{\eta_i,\eta_j} (p_i\cdot p_j)\,(p_i^\mu p_j^\rho - p_j^\mu p_i^\rho) \frac{2(p_i\cdot p_j)^2 - 3p_i^2 p_j^2}{\left[(p_i\cdot p_j)^2 - p_i^2 p_j^2\right]^{3/2}} \\ &- \frac{\kappa}{8\pi^2} \sum_i \frac{\varepsilon_{\mu\nu} p_i^\nu p_i^\nu}{p_i\cdot q} \sum_j q\cdot p_j \ln|\hat{q}\cdot\hat{p}_j| \\ &- \frac{\kappa}{32\pi^2} \sum_i \frac{p_i^\mu \varepsilon_{\mu\nu} q_\lambda}{p_i\cdot q} \left(p_i^\lambda \frac{\partial}{\partial p_{i\nu}} - p_i^\nu \frac{\partial}{\partial p_{i\lambda}}\right) \sum_j \frac{2(p_i\cdot p_j)^2 - p_i^2 p_j^2}{\left[(p_i\cdot p_j)^2 - p_i^2 p_j^2\right]^{1/2}} \ln\left(\frac{p_i\cdot p_j + \sqrt{(p_i\cdot p_j)^2 - p_i^2 p_j^2}}{p_i\cdot p_j - \sqrt{(p_i\cdot p_j)^2 - p_i^2 p_j^2}}\right) \end{aligned}$$

Logarithmic soft theorems

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$$\mathcal{M}_{n+1} \overset{\omega\to 0}{=} \left[\omega^{-1}\, S_n^{(0)} - \frac{\kappa^2}{4} \ln\omega\, S_n^{(\ln)}\right] \mathcal{M}_n + \mathcal{O}(\omega^0)$$

[Sahoo, Sen '18][…]

$$\begin{aligned}
S_n^{(\ln)} = & \boxed{\frac{i\kappa}{8\pi} \sum_i \frac{\varepsilon_{\mu\nu} p_i^\mu p_i^\nu}{p_i\cdot q} \sum_j \delta_{\eta,\eta_j}\, q\cdot p_j \\
& + \frac{i\kappa}{16\pi} \sum_i \frac{\varepsilon_{\mu\nu} p_i^\nu q_\rho}{p_i\cdot q} \sum_j \delta_{\eta_i,\eta_j} (p_i\cdot p_j)\, (p_i^\mu p_j^\rho - p_j^\mu p_i^\rho) \frac{2(p_i\cdot p_j)^2 - 3p_i^2 p_j^2}{\left[(p_i\cdot p_j)^2 - p_i^2 p_j^2\right]^{3/2}}} \\
& - \frac{\kappa}{8\pi^2} \sum_i \frac{\varepsilon_{\mu\nu} p_i^\nu p_i^\nu}{p_i\cdot q} \sum_j q\cdot p_j \ln|\hat{q}\cdot\hat{p}_j| \\
& - \frac{\kappa}{32\pi^2} \sum_i \frac{p_i^\mu \varepsilon_{\mu\nu} q_\lambda}{p_i\cdot q} \left(p_i^\lambda \frac{\partial}{\partial p_{i\nu}} - p_i^\nu \frac{\partial}{\partial p_{i\lambda}}\right) \sum_j \frac{2(p_i\cdot p_j)^2 - p_i^2 p_j^2}{\left[(p_i\cdot p_j)^2 - p_i^2 p_j^2\right]^{1/2}} \ln\left(\frac{p_i\cdot p_j + \sqrt{(p_i\cdot p_j)^2 - p_i^2 p_j^2}}{p_i\cdot p_j - \sqrt{(p_i\cdot p_j)^2 - p_i^2 p_j^2}}\right)
\end{aligned}$$

"classical"

Logarithmic soft theorems

- One-loop corrections generate logarithmic corrections!

$$\mathcal{M}_{n+1} \overset{\omega\to 0}{=} \left[\omega^{-1}\, S_n^{(0)} - \frac{\kappa^2}{4}\ln\omega\, S_n^{(\ln)}\right]\mathcal{M}_n + \mathcal{O}(\omega^0)$$

[Sahoo, Sen '18][...]

"classical"

$$S_n^{(\ln)} = \frac{i\kappa}{8\pi}\sum_i \frac{\varepsilon_{\mu\nu}p_i^\mu p_i^\nu}{p_i\cdot q}\sum_j \delta_{\eta,\eta_j}\, q\cdot p_j$$

$$+ \frac{i\kappa}{16\pi}\sum_i \frac{\varepsilon_{\mu\nu}p_i^\nu q_\rho}{p_i\cdot q}\sum_j \delta_{\eta_i,\eta_j}(p_i\cdot p_j)\left(p_i^\mu p_j^\rho - p_j^\mu p_i^\rho\right)\frac{2(p_i\cdot p_j)^2 - 3p_i^2p_j^2}{\left[(p_i\cdot p_j)^2 - p_i^2p_j^2\right]^{3/2}}$$

"quantum"

$$- \frac{\kappa}{8\pi^2}\sum_i \frac{\varepsilon_{\mu\nu}p_i^\nu p_i^\nu}{p_i\cdot q}\sum_j q\cdot p_j \ln|\hat{q}\cdot\hat{p}_j|$$

$$- \frac{\kappa}{32\pi^2}\sum_i \frac{p_i^\mu \varepsilon_{\mu\nu}q_\lambda}{p_i\cdot q}\left(p_i^\lambda\frac{\partial}{\partial p_{i\nu}} - p_i^\nu\frac{\partial}{\partial p_{i\lambda}}\right)\sum_j \frac{2(p_i\cdot p_j)^2 - p_i^2p_j^2}{[(p_i\cdot p_j)^2 - p_i^2p_j^2]^{1/2}}\ln\left(\frac{p_i\cdot p_j + \sqrt{(p_i\cdot p_j)^2 - p_i^2p_j^2}}{p_i\cdot p_j - \sqrt{(p_i\cdot p_j)^2 - p_i^2p_j^2}}\right)$$

Classical log $\longleftrightarrow$ tails

- $$\boxed{S_n^{(\ln)} = \frac{i\kappa}{8\pi}\sum_i \frac{\varepsilon_{\mu\nu}p_i^\mu p_i^\nu}{p_i\cdot q}\sum_j \delta_{\eta,\eta_j}\, q\cdot p_j + \frac{i\kappa}{16\pi}\sum_i \frac{\varepsilon_{\mu\nu}p_i^\nu q_\rho}{p_i\cdot q}\sum_j \delta_{\eta_i,\eta_j}(p_i\cdot p_j)\left(p_i^\mu p_j^\rho - p_j^\mu p_i^\rho\right)\frac{2(p_i\cdot p_j)^2 - 3p_i^2p_j^2}{\left[(p_i\cdot p_j)^2 - p_i^2p_j^2\right]^{3/2}}}$$

"classical"

effect of gravitational **drag** on the soft graviton

\+ effect of **late time** gravitational radiation (due to the late time acceleration of the particles via long range gravitational interaction)

Quantum log corrections

- $$S_n^{(\ln)} = \frac{i\kappa}{8\pi} \sum_i \frac{\varepsilon_{\mu\nu} p_i^\mu p_i^\nu}{p_i \cdot q} \sum_j \delta_{\eta,\eta_j}\, q \cdot p_j$$
$$+ \frac{i\kappa}{16\pi} \sum_i \frac{\varepsilon_{\mu\nu} p_i^\nu q_\rho}{p_i \cdot q} \sum_j \delta_{\eta_i,\eta_j} (p_i \cdot p_j) \left(p_i^\mu p_j^\rho - p_j^\mu p_i^\rho\right) \frac{2(p_i \cdot p_j)^2 - 3 p_i^2 p_j^2}{\left[(p_i \cdot p_j)^2 - p_i^2 p_j^2\right]^{3/2}}$$

"quantum"

$$- \frac{\kappa}{8\pi^2} \sum_i \frac{\varepsilon_{\mu\nu} p_i^\nu p_i^\nu}{p_i \cdot q} \sum_j q \cdot p_j \ln|\hat{q} \cdot \hat{p}_j| \qquad \omega \gg \text{loop momentum}$$

$$- \frac{\kappa}{32\pi^2} \sum_i \frac{p_i^\mu \varepsilon_{\mu\nu} q_\lambda}{p_i \cdot q} \left(p_i^\lambda \frac{\partial}{\partial p_{i\nu}} - p_i^\nu \frac{\partial}{\partial p_{i\lambda}}\right) \sum_j \frac{2(p_i \cdot p_j)^2 - p_i^2 p_j^2}{\left[(p_i \cdot p_j)^2 - p_i^2 p_j^2\right]^{1/2}} \ln\left(\frac{p_i \cdot p_j + \sqrt{(p_i \cdot p_j)^2 - p_i^2 p_j^2}}{p_i \cdot p_j - \sqrt{(p_i \cdot p_j)^2 - p_i^2 p_j^2}}\right)$$

$\omega \ll$ loop momentum

- computed via one-loop diagrams in a theory of minimally coupled scalars

  [Laddha, Sen '18] [Sahoo, Sen '18] [Saha, Sahoo, Sen '19]

- recently generalized to scattering of particles of arbitrary spin ⟶ universal results!

  [Krishna, Sahoo '23]

Question

Do log soft theorems arise from symmetry conservation?

$$S_n^{(\ln)} = \frac{i\kappa}{8\pi}\sum_i \frac{\varepsilon_{\mu\nu}p_i^\mu p_i^\nu}{p_i\cdot q}\sum_j \delta_{\eta,\eta_j}\, q\cdot p_j$$
$$+\frac{i\kappa}{16\pi}\sum_i \frac{\varepsilon_{\mu\nu}p_i^\nu q_\rho}{p_i\cdot q}\sum_j \delta_{\eta_i,\eta_j}(p_i\cdot p_j)\left(p_i^\mu p_j^\rho - p_j^\mu p_i^\rho\right)\frac{2(p_i\cdot p_j)^2-3p_i^2p_j^2}{\left[(p_i\cdot p_j)^2-p_i^2p_j^2\right]^{3/2}}$$
$$-\frac{\kappa}{8\pi^2}\sum_i \frac{\varepsilon_{\mu\nu}p_i^\nu p_i^\nu}{p_i\cdot q}\sum_j q\cdot p_j \ln|\hat{q}\cdot\hat{p}_j|$$
$$-\frac{\kappa}{32\pi^2}\sum_i \frac{p_i^\mu\varepsilon_{\mu\nu}q_\lambda}{p_i\cdot q}\left(p_i^\lambda\frac{\partial}{\partial p_{i\nu}} - p_i^\nu\frac{\partial}{\partial p_{i\lambda}}\right)\sum_j \frac{2(p_i\cdot p_j)^2-p_i^2p_j^2}{\left[(p_i\cdot p_j)^2-p_i^2p_j^2\right]^{1/2}}\ln\left(\frac{p_i\cdot p_j+\sqrt{(p_i\cdot p_j)^2-p_i^2p_j^2}}{p_i\cdot p_j-\sqrt{(p_i\cdot p_j)^2-p_i^2p_j^2}}\right)$$

→ yes, from local conformal symmetries (aka ‘superrotations’)

[Agrawal, LD, Nguyen, Ruzziconi ‘23]

Superrotations

- Supertranslation symmetries:

$$\xi = \boxed{\mathcal{T}(z,\bar{z})}\partial_u + \cdots$$

arbitrary function
on the celestial sphere

- Superrotation symmetries:

$$\xi = \boxed{\mathcal{Y}(z)}\partial_z + \tfrac{u}{2} D_z \mathcal{Y}(z) + \cdots$$

local conformal Killing vector
(>< 6 Lorentz transformations which are *globally* well-defined)

[Barnich, Troessaert '10]

Superrotations

- Supertranslation symmetries:

$$\xi = \boxed{\mathcal{T}(z,\bar{z})}\partial_u + \cdots$$

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- Superrotation symmetries:

$$\xi = \boxed{\mathcal{Y}(z)}\partial_z + \tfrac{u}{2} D_z \mathcal{Y}(z) + \cdots$$

local conformal Killing vector
(>< 6 Lorentz transformations which are *globally* well-defined)

Poincaré group**: 4 translations + 6 Lorentz transfo.**

↓ *Symmetry enhancement* ↓ *Symmetry enhancement*

BMS group**:** ∞ ***super*translations** + ∞ ***super*rotations**

[Barnich, Troessaert '10]

Logarithmic soft theorems

$$
\begin{aligned}
S_n^{(\ln)} &= \frac{i\kappa}{8\pi}\sum_i \frac{\varepsilon_{\mu\nu}p_i^\mu p_i^\nu}{p_i\cdot q}\sum_j \delta_{\eta,\eta_j}\, q\cdot p_j \\
&+ \frac{i\kappa}{16\pi}\sum_i \frac{\varepsilon_{\mu\nu}p_i^\nu q_\rho}{p_i\cdot q}\sum_j \delta_{\eta_i,\eta_j}(p_i\cdot p_j)\left(p_i^\mu p_j^\rho - p_j^\mu p_i^\rho\right)\frac{2(p_i\cdot p_j)^2-3p_i^2p_j^2}{\left[(p_i\cdot p_j)^2-p_i^2p_j^2\right]^{3/2}} \\
&- \frac{\kappa}{8\pi^2}\sum_i \frac{\varepsilon_{\mu\nu}p_i^\nu p_i^\nu}{p_i\cdot q}\sum_j q\cdot p_j \ln|\hat{q}\cdot\hat{p}_j| \\
&- \frac{\kappa}{32\pi^2}\sum_i \frac{p_i^\mu\varepsilon_{\mu\nu}q_\lambda}{p_i\cdot q}\left(p_i^\lambda\frac{\partial}{\partial p_{i\nu}} - p_i^\nu\frac{\partial}{\partial p_{i\lambda}}\right)\sum_j \frac{2(p_i\cdot p_j)^2-p_i^2p_j^2}{\left[(p_i\cdot p_j)^2-p_i^2p_j^2\right]^{1/2}}\ln\left(\frac{p_i\cdot p_j+\sqrt{(p_i\cdot p_j)^2-p_i^2p_j^2}}{p_i\cdot p_j-\sqrt{(p_i\cdot p_j)^2-p_i^2p_j^2}}\right)
\end{aligned}
$$

- The log corrections can be written in a much simpler form

$$
\boxed{S_n^{(\ln)} = \hat{\sigma}'_{n+1}(\hat{q})\, S_n^{(0)} - S_n^{(1)J}\,\sigma_n}
$$

[Sahoo, Sen ‘18]

leading
soft factor

subleading
soft factor

Logarithmic soft theorems

$$
\begin{aligned}
S_n^{(\ln)} = &\frac{i\kappa}{8\pi} \sum_i \frac{\varepsilon_{\mu\nu} p_i^\mu p_i^\nu}{p_i \cdot q} \sum_j \delta_{\eta,\eta_j}\, q \cdot p_j \\
&+ \frac{i\kappa}{16\pi} \sum_i \frac{\varepsilon_{\mu\nu} p_i^\nu q_\rho}{p_i \cdot q} \sum_j \delta_{\eta_i,\eta_j} (p_i \cdot p_j)\, (p_i^\mu p_j^\rho - p_j^\mu p_i^\rho) \frac{2(p_i \cdot p_j)^2 - 3p_i^2 p_j^2}{\left[(p_i \cdot p_j)^2 - p_i^2 p_j^2\right]^{3/2}} \\
&- \frac{\kappa}{8\pi^2} \sum_i \frac{\varepsilon_{\mu\nu} p_i^\nu p_i^\nu}{p_i \cdot q} \sum_j q \cdot p_j \ln|\hat{q} \cdot \hat{p}_j| \\
&- \frac{\kappa}{32\pi^2} \sum_i \frac{p_i^\mu \varepsilon_{\mu\nu} q_\lambda}{p_i \cdot q} \left(p_i^\lambda \frac{\partial}{\partial p_{i\nu}} - p_i^\nu \frac{\partial}{\partial p_{i\lambda}} \right) \sum_j \frac{2(p_i \cdot p_j)^2 - p_i^2 p_j^2}{[(p_i \cdot p_j)^2 - p_i^2 p_j^2]^{1/2}} \ln \left(\frac{p_i \cdot p_j + \sqrt{(p_i \cdot p_j)^2 - p_i^2 p_j^2}}{p_i \cdot p_j - \sqrt{(p_i \cdot p_j)^2 - p_i^2 p_j^2}} \right)
\end{aligned}
$$

- The log corrections can be written in a much simpler form

* I drop here some numerical factors

$$
\boxed{S_n^{(\ln)} = \hat{\sigma}'_{n+1}(\hat{q})\, S_n^{(0)} - S_n^{(1)J}\, \sigma_n}
$$

[Sahoo, Sen ‘18]

leading soft factor

subleading soft factor

$$
\beta_{ij} = \sqrt{1 - (\hat{p}_i \cdot \hat{p}_j)^{-2}}
$$

relative velocity of particles *i* and *j*

where
$$
\hat{\sigma}'_{n+1}(\hat{q}) = \sum_{i=1}^{n} (p_i \cdot \hat{q}) \ln(\hat{p}_i \cdot \hat{q})
$$

$$
\sigma_n = \sum_{ij} \eta_i \eta_j\, m_i m_j \frac{1 + \beta_{ij}^2}{\beta_{ij}\sqrt{1 - \beta_{ij}^2}} \left(i\pi \delta_{\eta_i,\eta_j} - \frac{1}{2} \ln \frac{1 + \beta_{ij}}{1 - \beta_{ij}} \right)
$$

$\eta_i = \pm 1$ (in or out)

Logarithmic soft theorems

- $$S_n^{(\ln)} = \boxed{\hat{\sigma}'_{n+1}(\hat{q})\, S_n^{(0)}} - \boxed{S_n^{(1)J}\ \sigma_n}$$ [Sahoo, Sen '18]

1 2

where $$\hat{\sigma}'_{n+1}(\hat{q}) = \sum_{i=1}^{n} (p_i \cdot \hat{q}) \ln(\hat{p}_i \cdot \hat{q})$$

$$\sigma_n = \sum_{ij} \eta_i \eta_j\, m_i m_j \frac{1+\beta_{ij}^2}{\beta_{ij}\sqrt{1-\beta_{ij}^2}} \left(i\pi\delta_{\eta_i,\eta_j} - \frac{1}{2} \ln \frac{1+\beta_{ij}}{1-\beta_{ij}} \right)$$

Logarithmic soft theorems

- $$S_n^{(\ln)} = \boxed{\hat{\sigma}'_{n+1}(\hat{q})\, S_n^{(0)}} - S_n^{(1)J}\,\sigma_n$$ [Sahoo, Sen '18]

  1

  where $$\hat{\sigma}'_{n+1}(\hat{q}) = \sum_{i=1}^{n} (p_i \cdot \hat{q}) \ln(\hat{p}_i \cdot \hat{q})$$

- The first contribution 1 is reproduced by the superrotation Ward identity

$$F_{\mathcal{Y}}^{soft,new} = \frac{2}{\kappa^2} \int d^2 z\, \mathcal{Y} \left[-\partial^3 (C^{(0)} \mathcal{N}_{\bar{z}\bar{z}}^{(0)}) + 3\bar{\partial}^2 \mathcal{N}_{zz}^{(0)} \partial C^{(0)} + C^{(0)} \partial \bar{\partial}^2 \mathcal{N}_{zz}^{(0)} \right]$$

Noether charge for superrotations

i.e. $$\boxed{\langle \text{out} | F_{\mathcal{Y}}^{soft,new}(\mathscr{I}^+)\, \mathcal{S} + \mathcal{S}\, F_{\mathcal{Y}}^{soft,new}(\mathscr{I}^-) | \text{in} \rangle = -\frac{i\kappa}{16\pi\epsilon} \int d^2 z\, \mathcal{Y}\, \partial^3 (\hat{\sigma}'_{n+1}\, S_n^{(0)-}) \langle \text{out} | \mathcal{S} | \text{in} \rangle}$$

[Agrawal, LD, Nguyen, Ruzziconi '23]; see also [LD, Nguyen, Ruzziconi '22][Pasterski '22]

Logarithmic soft theorems

- $$S_n^{(\ln)} = \boxed{\hat{\sigma}'_{n+1}(\hat{q})\, S_n^{(0)}} - \boxed{S_n^{(1)J}\ \sigma_n}$$ [Sahoo, Sen '18]

  1 2

- The first contribution 1 is reproduced by the superrotation Ward identity

- The second contribution 2 is reproduced by the `**dressed**' superrotation charge at timelike infinity; see details in [Agrawal, LD, Nguyen, Ruzziconi '23]

Conclusions

Conclusions

- **Logarithmic** corrections to the soft graviton theorems can be obtained from superrotation Ward identities. [Agrawal, LD, Nguyen, Ruzziconi '23] **QED**: see [Campiglia, Laddha '19]

Conclusions

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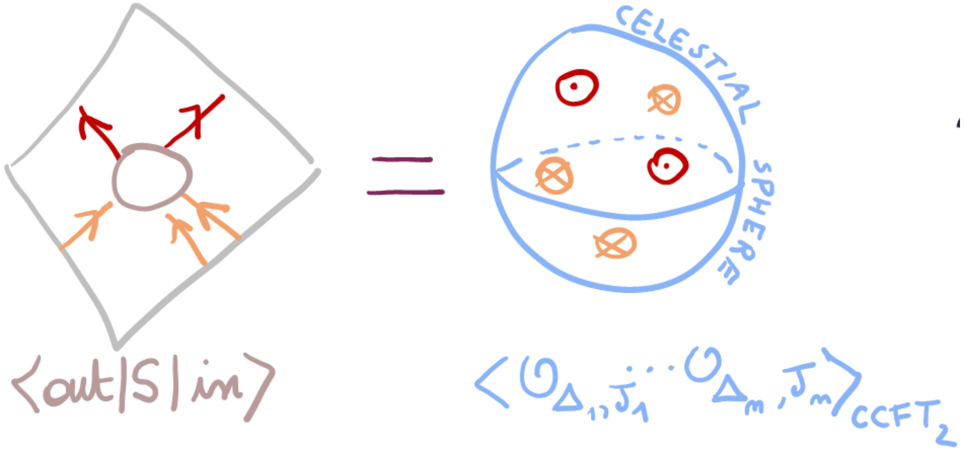
    'Celestial Holography'



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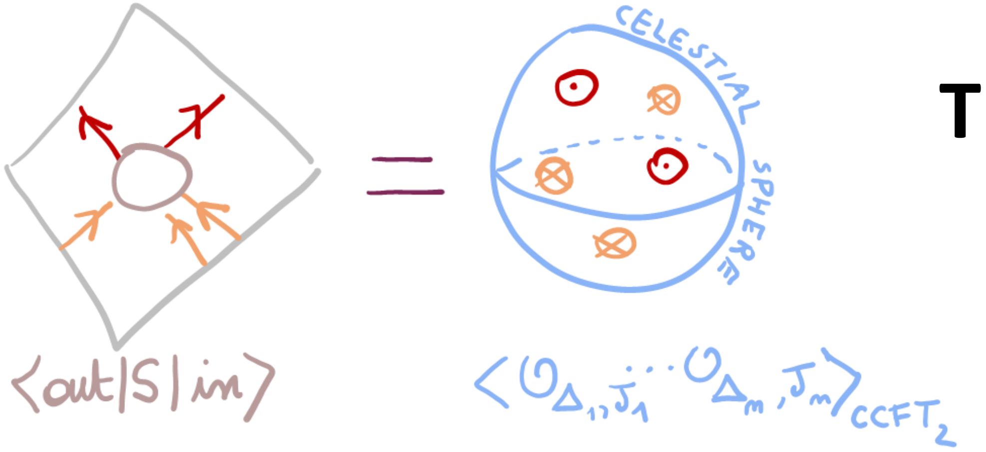

Thank you!