THE HIGHS AND LOWS OF LOW-x PHYSICS

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OUTLINE

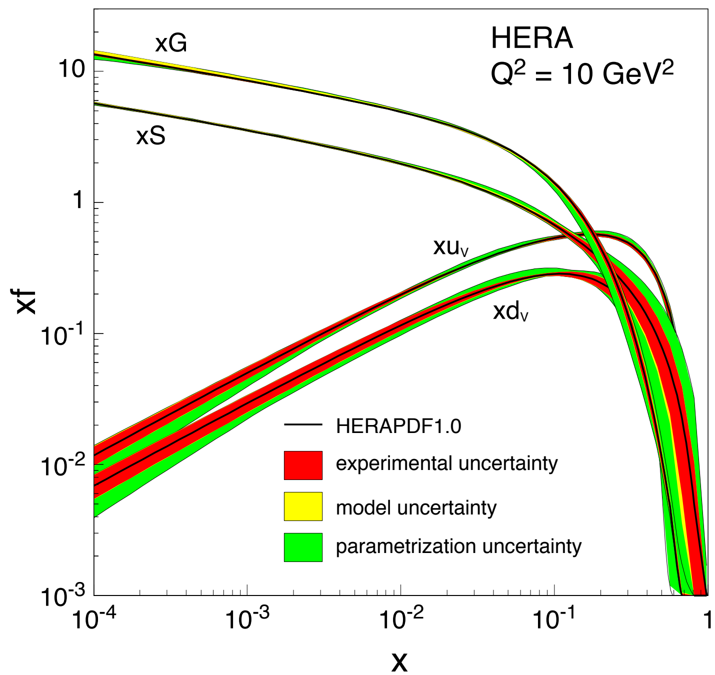
- DIS at small x: nonlinear evolution, saturation, saturation scale and its dependence on x and A.

- Saturation searches at EIC:
  - Structure functions;
  - Correlations;
  - Diffraction.

DIS at Small x

Gluons at Small-x

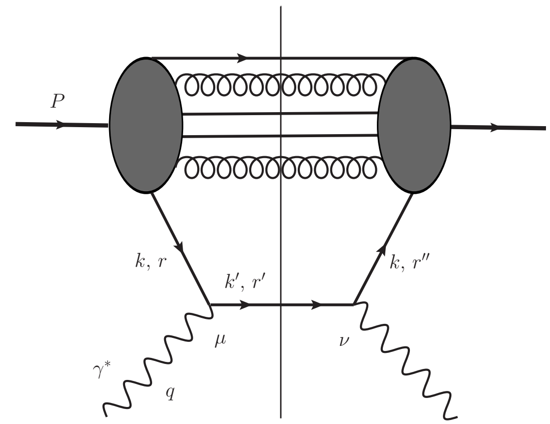
- There is a large number of small-x gluons (and quarks) in a proton:



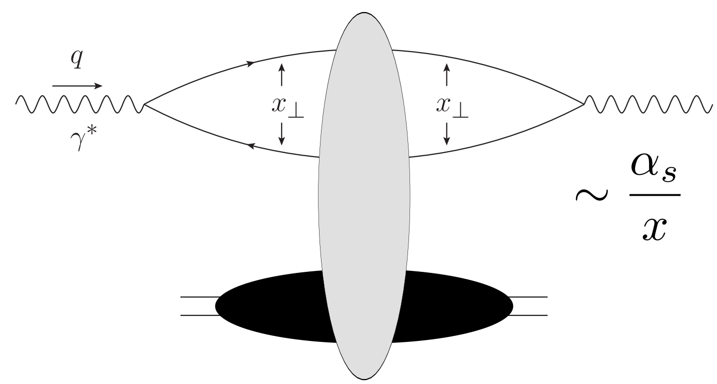
- $G(x, Q^2)$, $q(x, Q^2)$ = gluon and quark number densities (q=u,d, or S for sea).

Dipole picture of DIS

- At small x, the dominant contribution to DIS structure functions does not come from the handbag diagram.



- Instead, the dominant term comes from the dipole picture of DIS, where the virtual photon splits into a quark-antiquark pair, which then interacts with the target.



$$\sim \frac{\alpha_s}{x}$$

Dipole picture of DIS

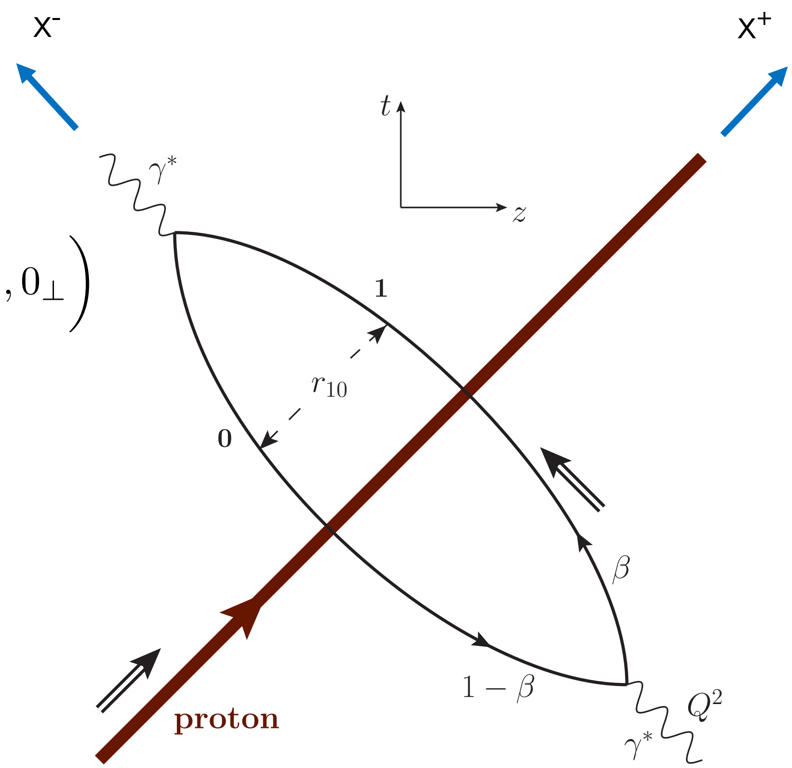
$$W^{\mu\nu} = \frac{1}{4\pi M_p} \int d^4x \, e^{iq\cdot x} \, \langle P| j^\mu(x) \, j^\nu(0) |P\rangle$$

$$q^\mu = \left(\frac{Q^2}{2q^-}, q^-, 0_\perp \right)$$

Large q^- → large x^- separation

$$e^{iq\cdot x} = e^{i\frac{Q^2}{2q^-}x^- + iq^- x^+}$$

$$x^\pm = \frac{t \pm z}{\sqrt{2}}$$

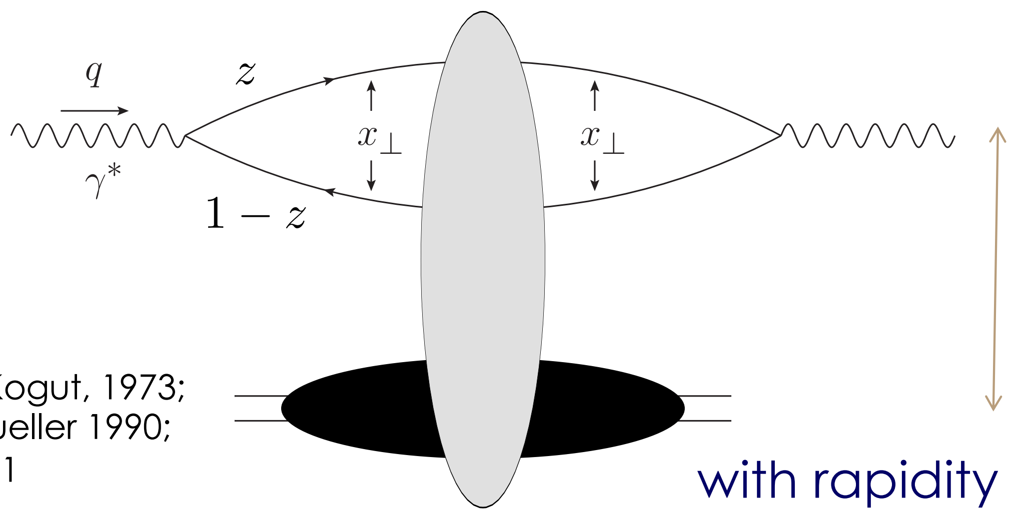


aka the "shock wave"

Dipole Amplitude

- The total DIS cross section is expressed in terms of the (Im part of the) forward quark dipole amplitude N:

$$\sigma_{tot}^{\gamma^* A} = \int \frac{d^2 x_\perp}{2\,\pi}\, d^2 b_\perp \int\limits_0^1 \frac{dz}{z\,(1-z)}\; |\Psi^{\gamma^* \to q \bar{q}}(\vec{x}_\perp, z)|^2 \; N(\vec{x}_\perp, \vec{b}_\perp, Y)$$



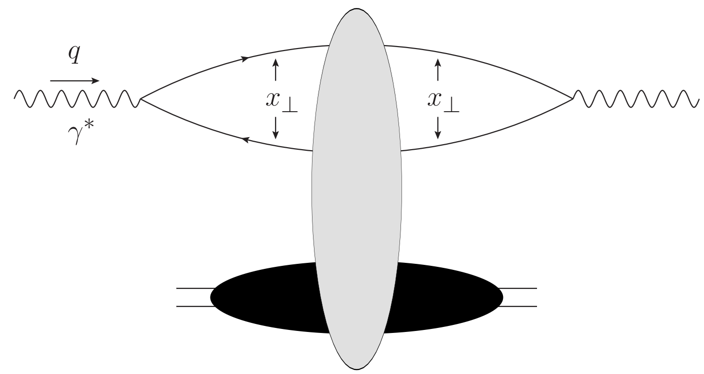
b is the Fourier conjugate to q with $t = -\,q^2$, making the dipole amplitude N similar to the GPDs at zero skewedness.

$b_\perp,\ Y$

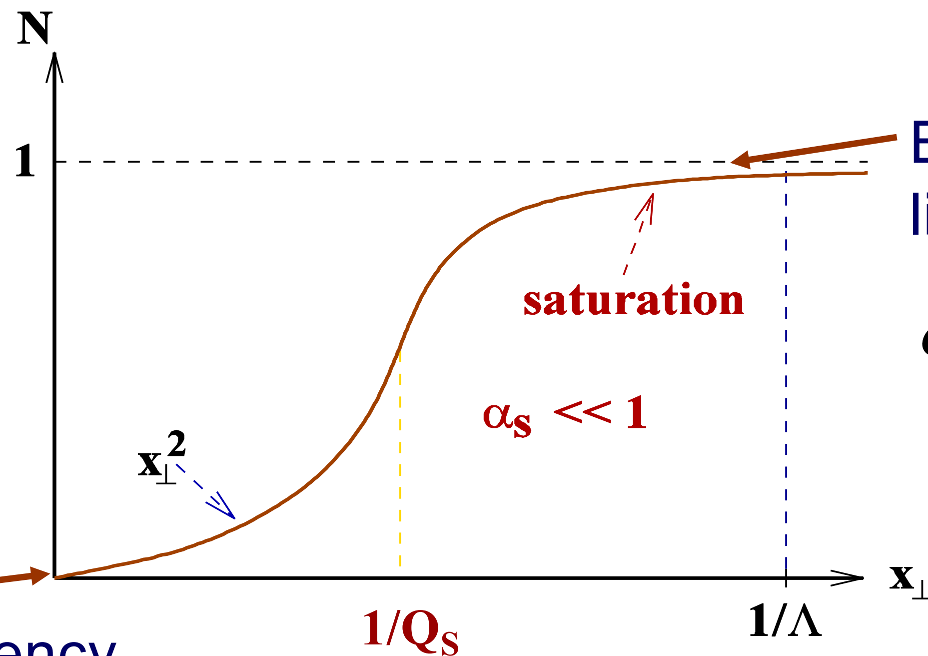
Gribov, 1970; Bjorken and Kogut, 1973; Frankfurt, Strikman 1988; Mueller 1990; Nikolaev and Zakharov 1991

with rapidity Y=ln(1/x)

Dipole Amplitude



The dipole-nucleus amplitude
as a function of the dipole size is



Color
transparency

Black disk
limit,

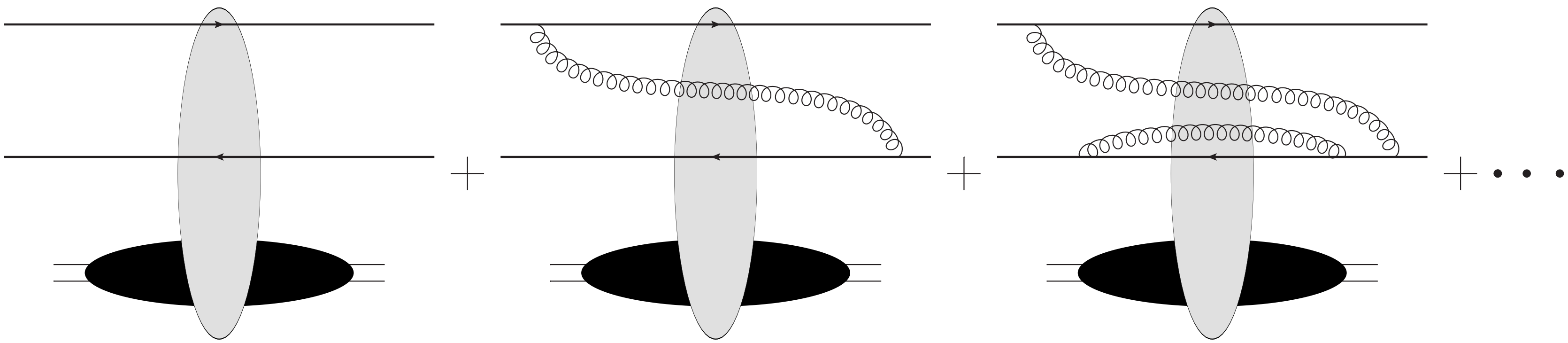
$$\sigma_{tot} < 2\pi R^2$$

$$\sigma^{q\bar{q}A} = 2\int d^2b\, N(x_\perp, b_\perp, Y)$$

Small-x Evolution

- Energy dependence comes in through the long-lived *s*-channel gluon corrections (higher Fock states):

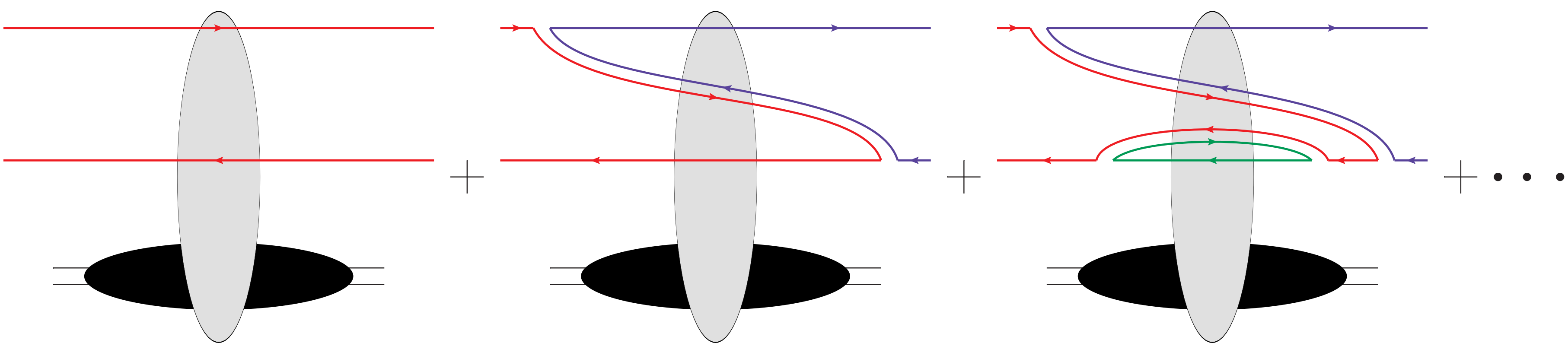
$$\alpha_s \, \ln s \sim \alpha_s \, \ln \frac{1}{x} \sim 1$$



These extra gluons bring in powers of $\alpha_S \ln s$, such that when $\alpha_S << 1$ and $\ln s >> 1$ this parameter is $\alpha_S \ln s \sim 1$ (leading logarithmic approximation, LLA).

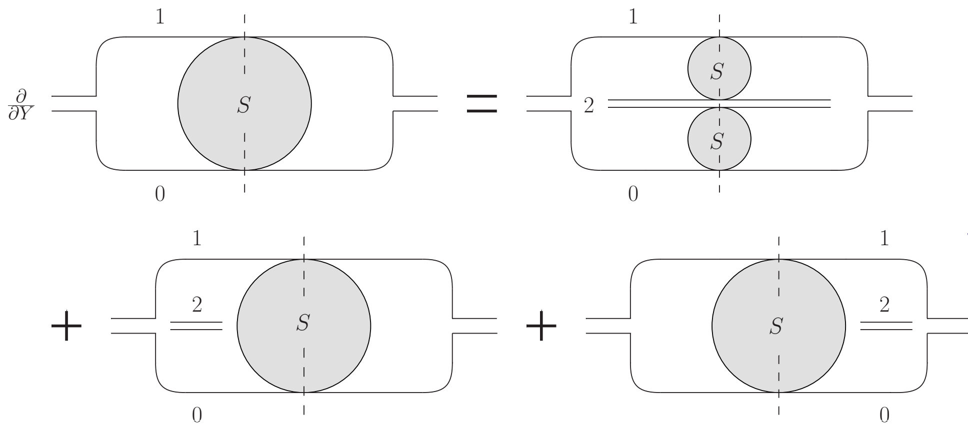
Small-x Evolution: Large N_c Limit

- How do we resum this cascade of gluons?
- The simplification comes from the large-N_c limit, where each gluon becomes a quark-antiquark pair: $3 \otimes \bar{3} = 1 \oplus 8 \quad \Rightarrow \quad N_c \otimes \bar{N}_c = 1 \oplus (N_c^2 - 1) \approx N_c^2 - 1$
- Gluon cascade becomes a dipole cascade (each color outlines a dipole):



Nonlinear Evolution

To sum up the gluon cascade at large-N_C we write the following equation for the dipole S-matrix:



dashed line = all interactions with the target

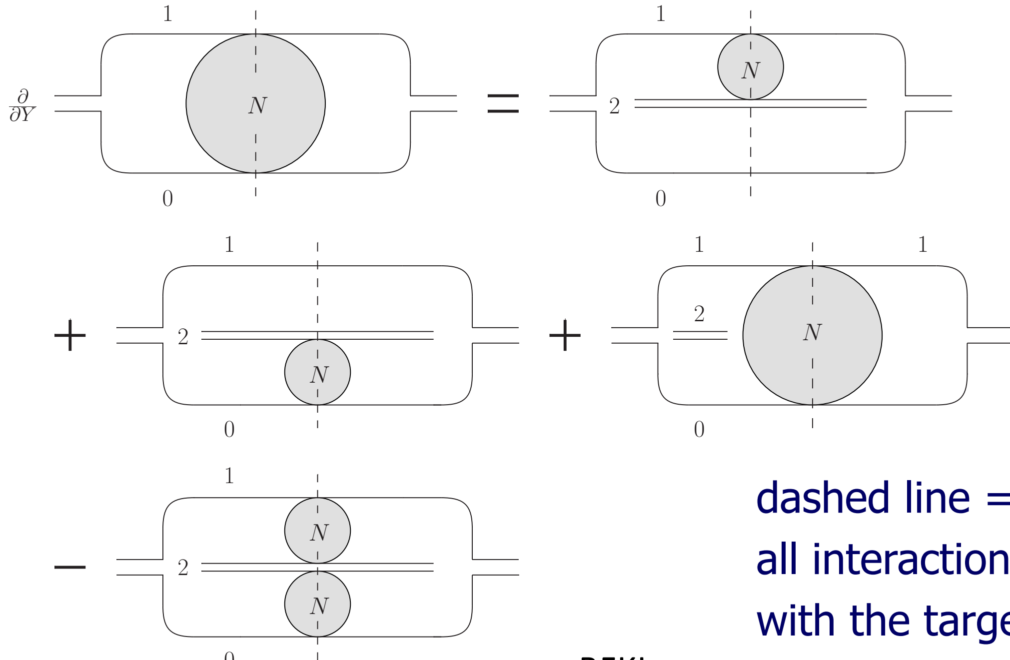
$$Y = \ln \frac{1}{x} \sim \ln s$$

$$\partial_Y S_{\mathbf{x}_0,\mathbf{x}_1}(Y) = \frac{\alpha_s N_c}{2\pi^2} \int d^2 x_2 \frac{x_{01}^2}{x_{02}^2 \, x_{21}^2} \left[S_{\mathbf{x}_0,\mathbf{x}_2}(Y) \, S_{\mathbf{x}_2,\mathbf{x}_1}(Y) - S_{\mathbf{x}_0,\mathbf{x}_1}(Y) \right]$$

Remembering that S=1 + i T = 1 - N where N = Im(T) we can rewrite this equation in terms of the dipole scattering amplitude N.

Nonlinear evolution at large N_c

As N=1-S we write



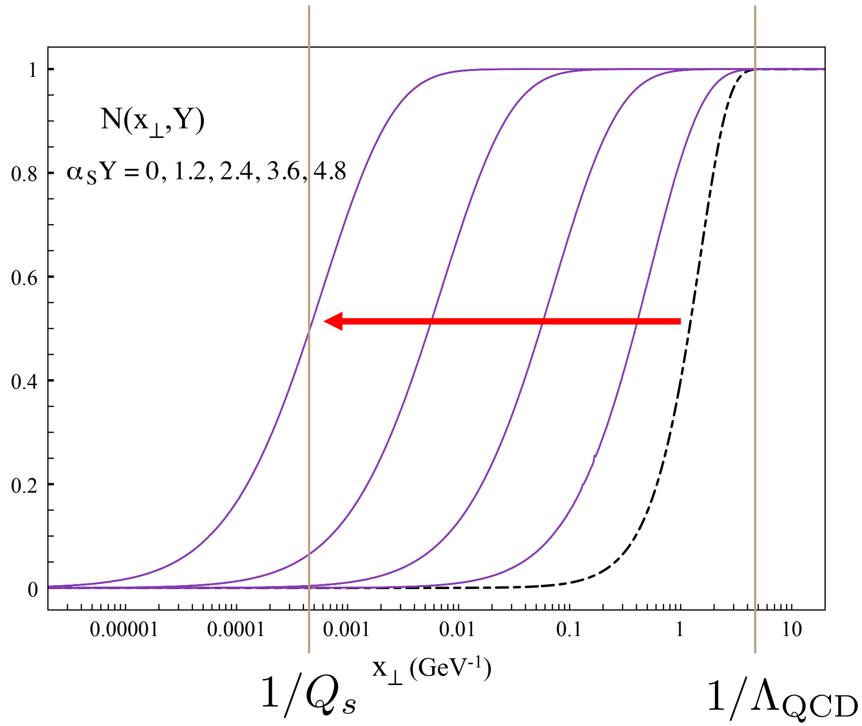
dashed line =
all interactions
with the target

BFKL

$$\partial_Y N_{\mathbf{x}_0,\mathbf{x}_1}(Y) = \frac{\alpha_s N_c}{2\pi^2} \int d^2x_2 \, \frac{x_{01}^2}{x_{02}^2 \, x_{21}^2} \, \left[N_{\mathbf{x}_0,\mathbf{x}_2}(Y) + N_{\mathbf{x}_2,\mathbf{x}_1}(Y) - N_{\mathbf{x}_0,\mathbf{x}_1}(Y) - N_{\mathbf{x}_0,\mathbf{x}_2}(Y) \, N_{\mathbf{x}_2,\mathbf{x}_1}(Y) \right]$$

Balitsky '96, Yu.K. '99; beyond large N_c, JIMWLK evolution, 0.1% correction for the dipole amplitude

Solution of BK equation



numerical solution
by J. Albacete '03

Energy increases → Q_s increases moving further away from Λ_{QCD}

We conclude that

$$Q_s^2 \sim \left(\frac{1}{x}\right)^{\lambda}$$

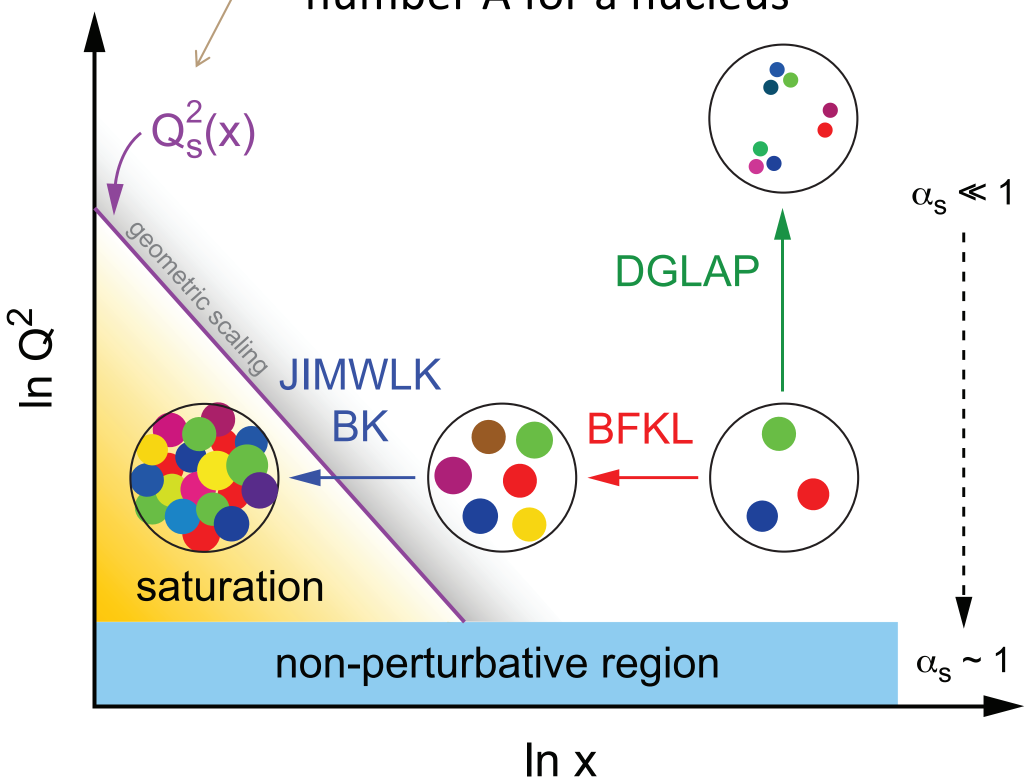
BK solution preserves the black disk limit, N<1 always (unlike the linear BFKL equation)

$$\sigma^{q\bar{q}A} = 2\int d^2b\, N(x_\perp, b_\perp, Y)$$

Map of High Energy QCD

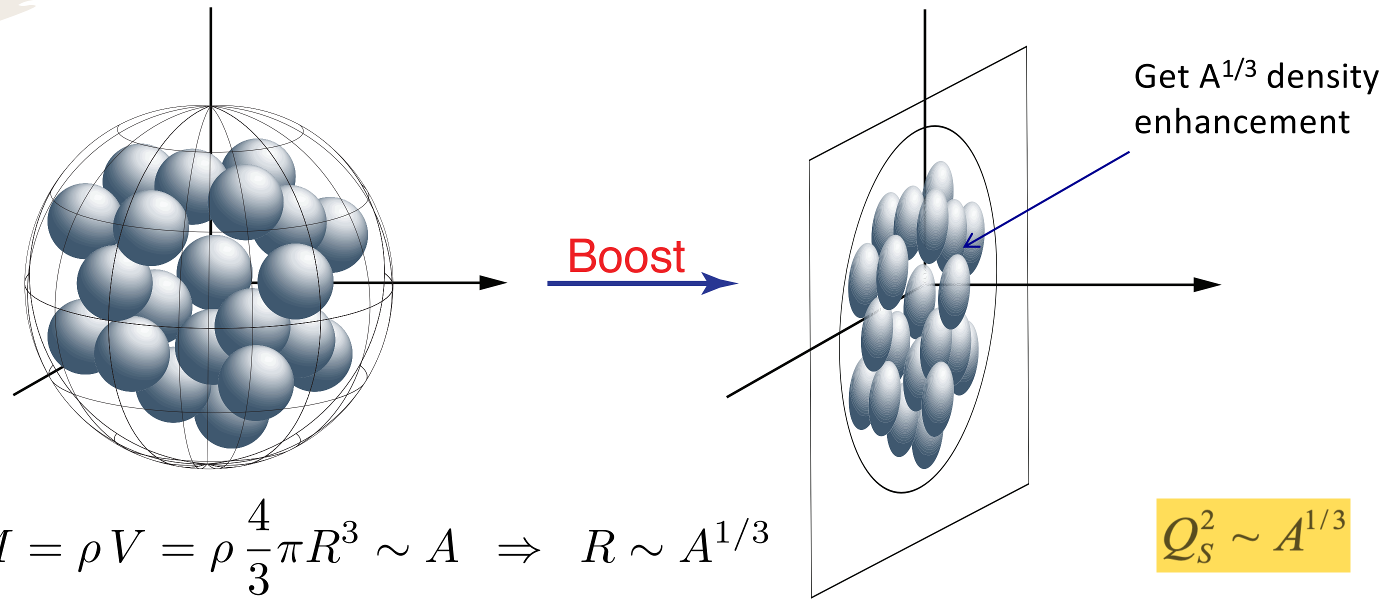
Saturation Scale
grows with energy and atomic number A for a nucleus

$$Q_s^2 \sim \left(\frac{A}{x}\right)^{1/3}$$



BFKL, DGLAP – linear equations
BK/JIMWLK – nonlinear

McLerran-Venugopalan Model



$$M = \rho V = \rho \frac{4}{3}\pi R^3 \sim A \quad \Rightarrow \quad R \sim A^{1/3}$$

$$Q_S^2 \sim A^{1/3}$$

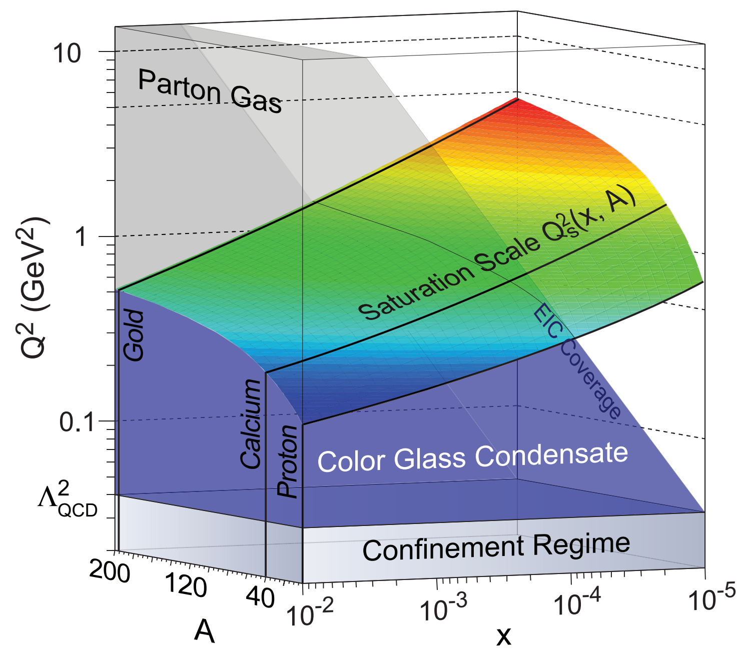
- Large gluon density gives a large momentum scale Q_s (the saturation scale): Q_s^2 ~ # gluons per unit transverse area ~ $A^{1/3}$ (nuclear oomph).
- For $Q_s \gg \Lambda_{QCD}$, get a theory at weak coupling $\alpha_s(Q_s^2) \ll 1$.

Saturation Scale

To summarize, saturation scale is an increasing function of both energy (1/x) and A:

$$Q_s^2 \sim \left(\frac{A}{x}\right)^{1/3}$$

Gold nucleus provides an enhancement by $197^{1/3}$, which is equivalent to doing scattering on a proton at 197 times smaller *x* / higher *s*!



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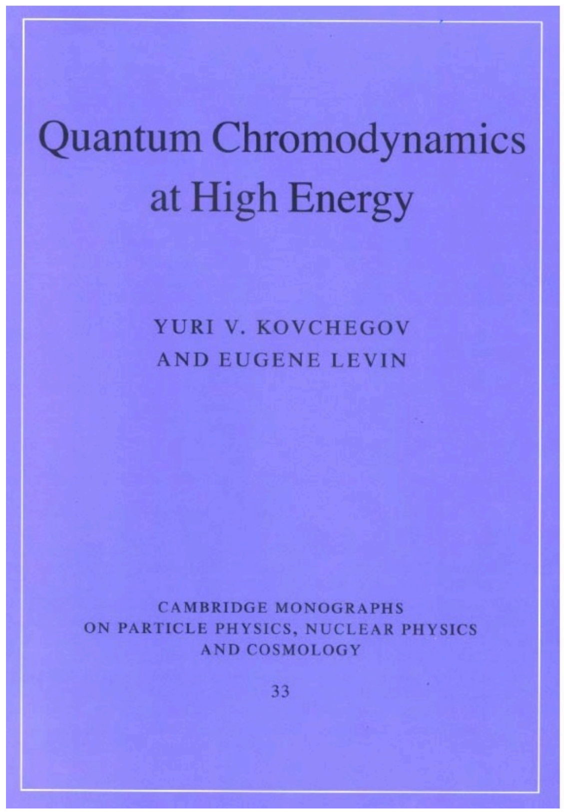
- H.Weigert, hep-ph/0501087
- J.Jalilian-Marian, Yu.K., hep-ph/0505052
- F. Gelis et al, arXiv:1002.0333 [hep-ph]
- J.L. Albacete, C. Marquet, arXiv:1401.4866 [hep-ph]
- A. Morreale, F. Salazar, arXiv:2108.08254 [hep-ph]
- and…

References

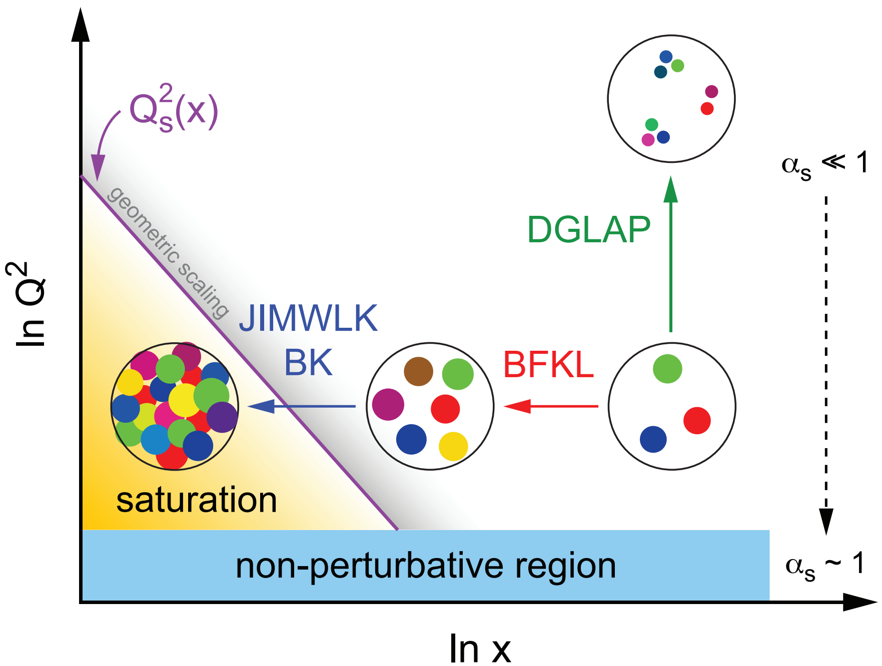


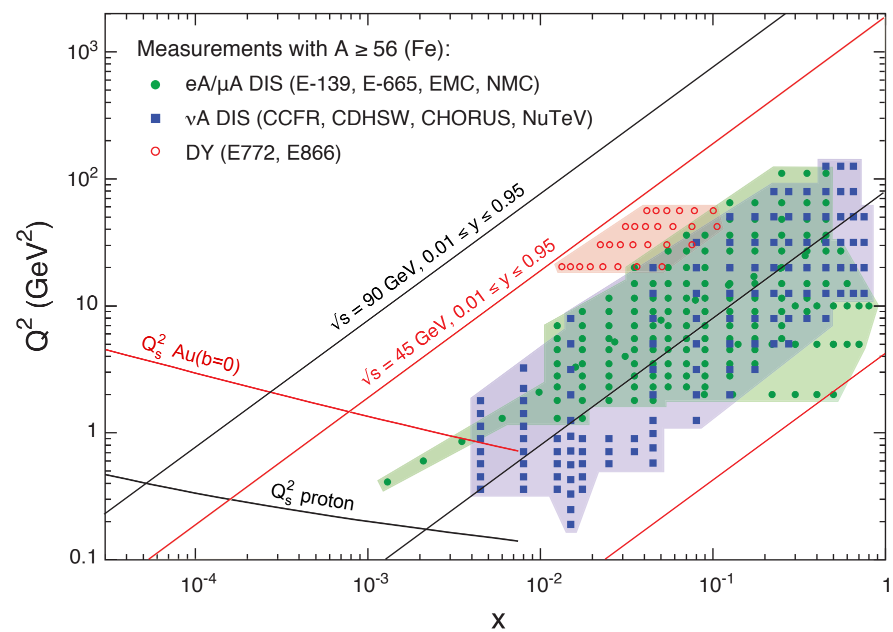
Published in September 2012
by Cambridge U Press

Saturation Physics at EIC

Can Saturation be Discovered at EIC?

EIC will have an unprecedented small-x reach for DIS on large nuclear targets, enabling decisive tests of saturation and non-linear evolution:

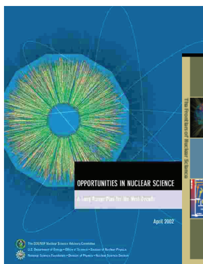




Plots from the EIC White Paper, '12, '14 (2nd ed).

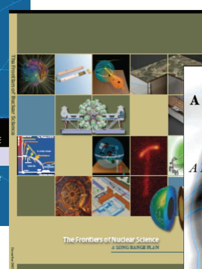
EIC Literature

2002



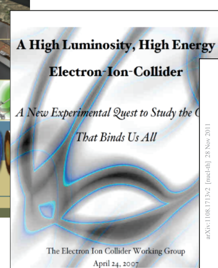
"…essential accelerator and detector R&D [for EIC] should be given very high priority in the short term."

2007



"We recommend the allocation of resources …to lay the foundation for a polarized Eletron-Ion Collider…"

2009

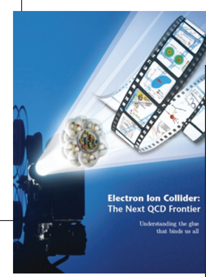


"..a new dedicated facility will be essential for answering some of the most central questions."

2010

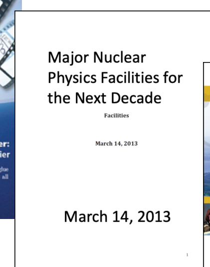
"The quantitative study of matter in this new regime [where abundant gluons dominate] requires a new experimental facility: an Electron Ion Collider.."

2012



"…essential accelerator and detector R&D [for EIC] should be given very high priority in the short term."

2013



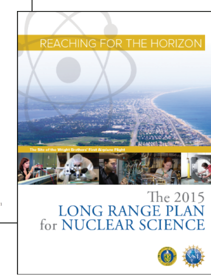
Major Nuclear Physics Facilities for the Next Decade

March 14, 2013

"Electron-Ion Collider..absolutely central to the nuclear science program of the next decade."

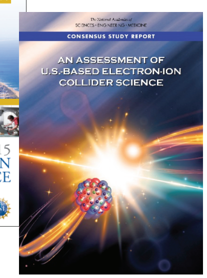
"a high-energy high-luminosity polarized EIC [is] the highest priority for new facility construction following the completion of FRIB."

2015



The science questions that an EIC will answer are central to completing an understanding of atoms as well as being integral to the agenda of nuclear physics today."

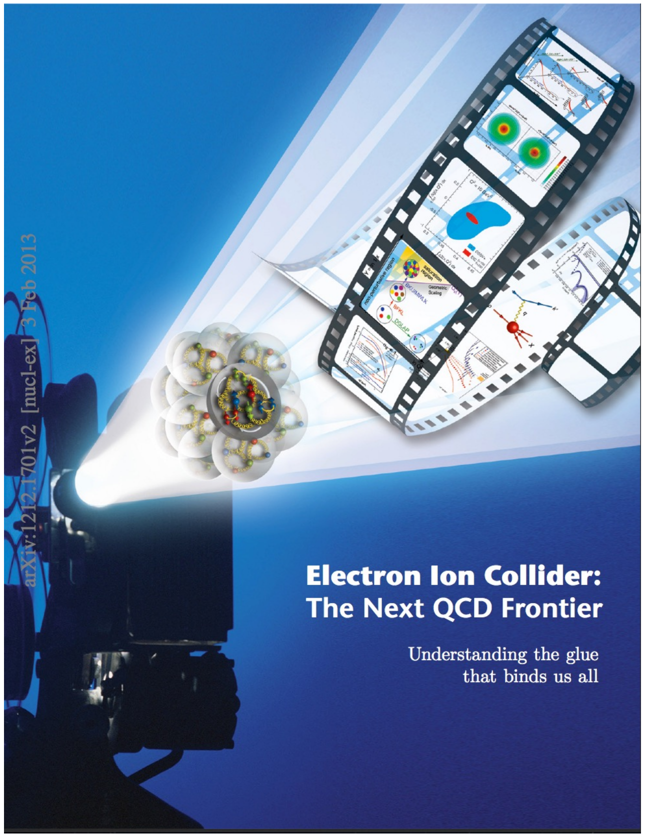
2018



+ EIC Yellow Report, 2021

Electron-Ion Collider (EIC) White Paper

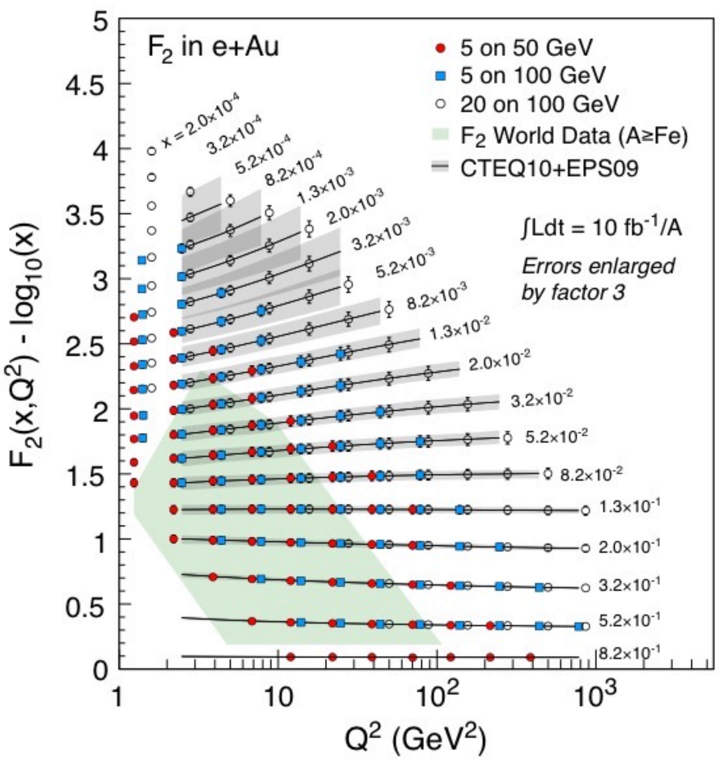
- EIC WP was finished in late 2012 + 2nd edition in 2014
- A several-year effort by a 19-member committee + 58 co-authors
- arXiv:1212.1701 [nucl-ex]
- We will follow the physics discussion and use the plots from this WP.

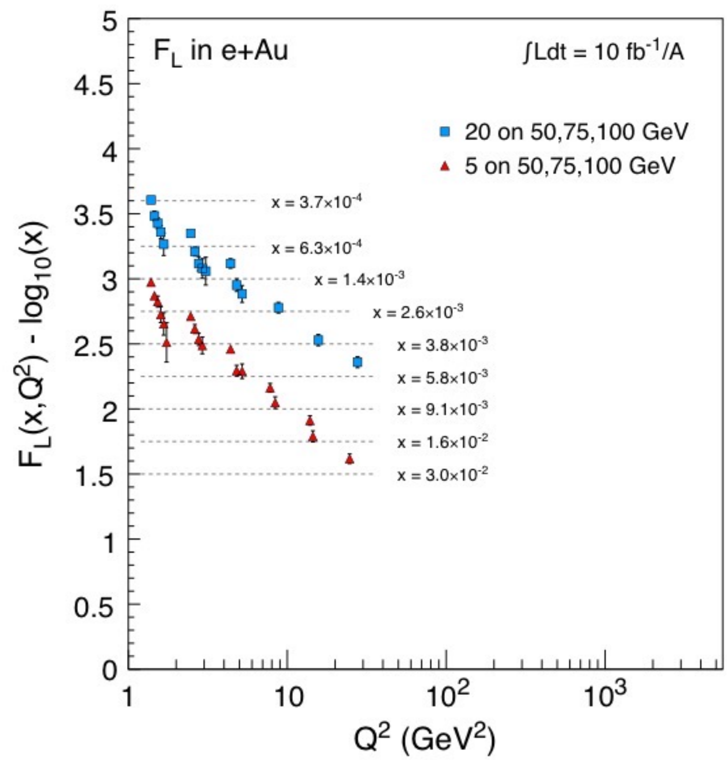


(i) Nuclear Structure Functions

Structure Functions at EIC

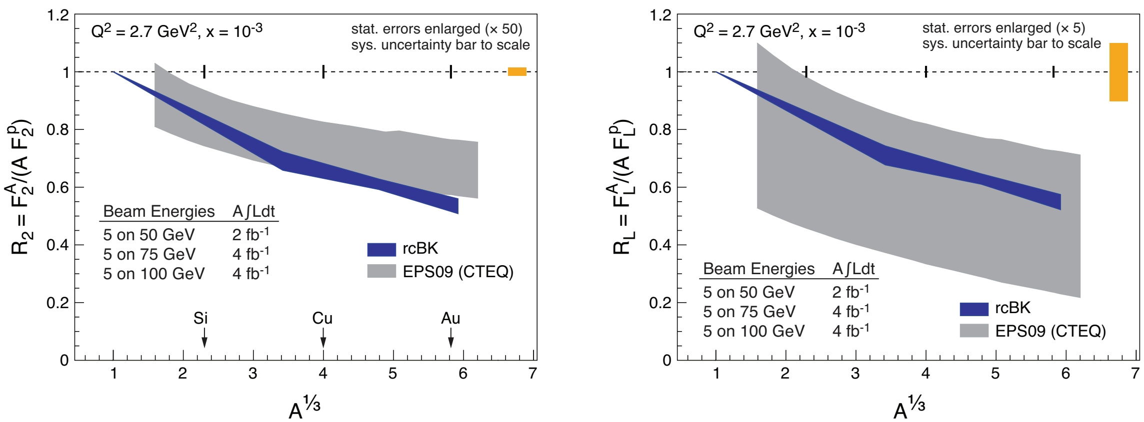
Nuclear structure functions F_2 and F_L (parts of σ^{e+A} cross section) which will be measured at EIC (values = EPS09+PYTHIA). Shaded area = (x, Q^2) range of the world e+A data.

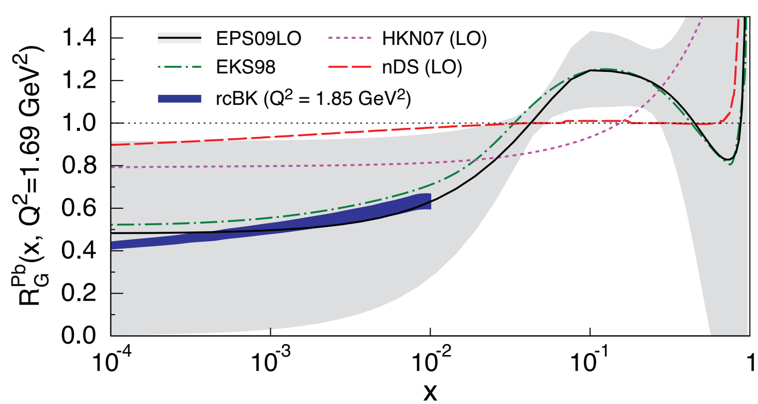




Nuclear Shadowing

- Saturation effects may explain nuclear shadowing: reduction of the number of gluons per nucleon with decreasing x and/or increasing A:

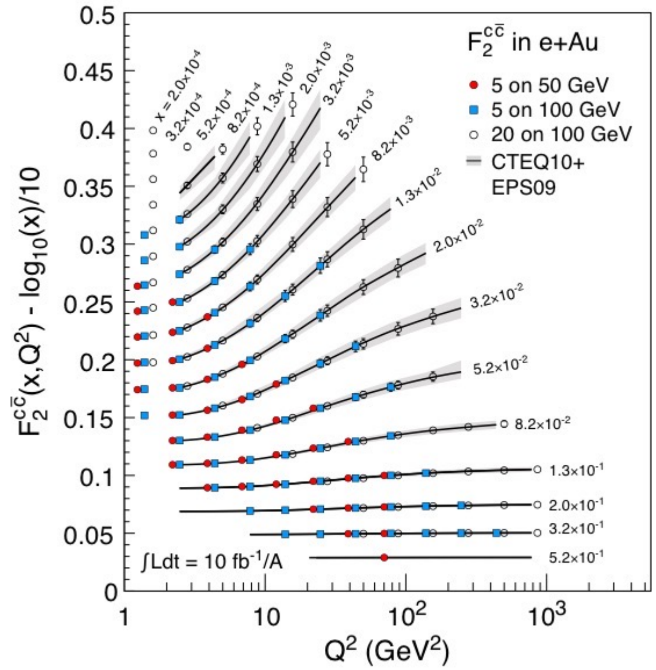


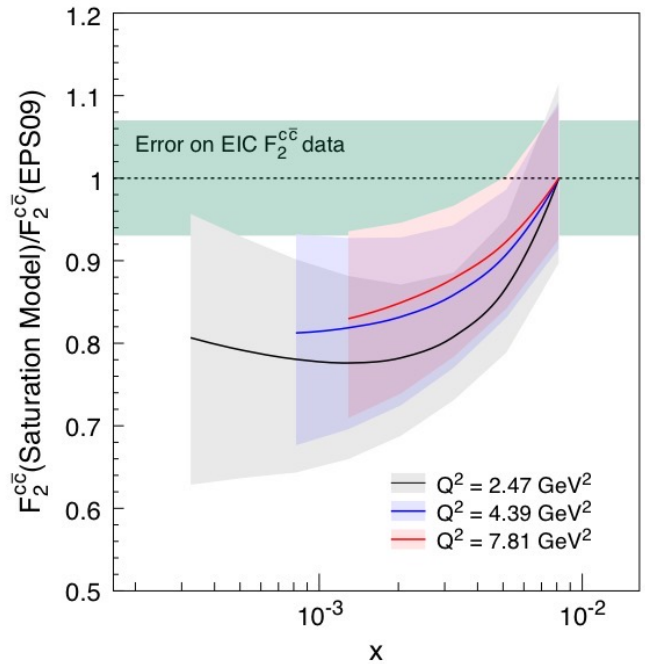


But: as DGLAP does not predict the x- and A-dependences, it needs to be constrained by the data.

Note that including heavy flavors (charm) for F_2 and F_L should help distinguish between the saturation versus non-saturation predictions.

Nuclear Shadowing for Charm



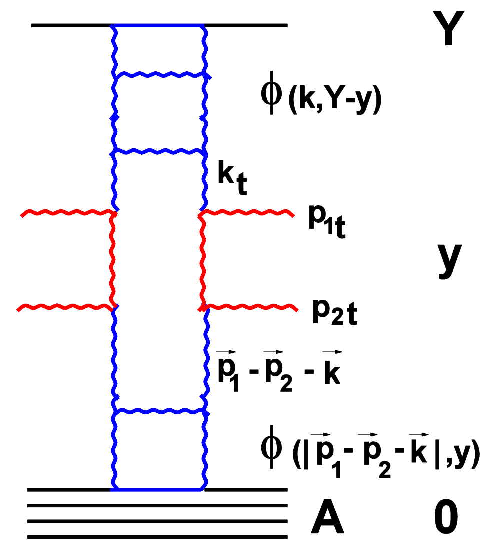


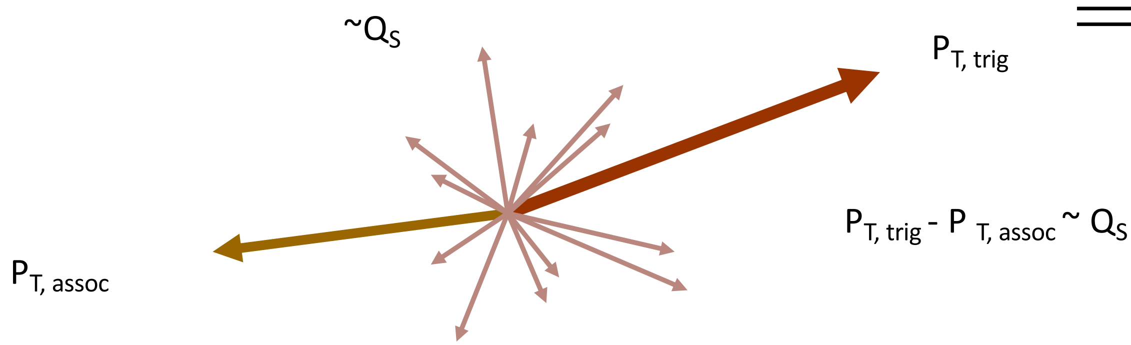
may help distinguish saturation vs DGLAP-based prediction

(ii) Di-Hadron Correlations

De-correlation

- Small-x evolution ↔ multiple emissions
- Multiple emissions → de-correlation.

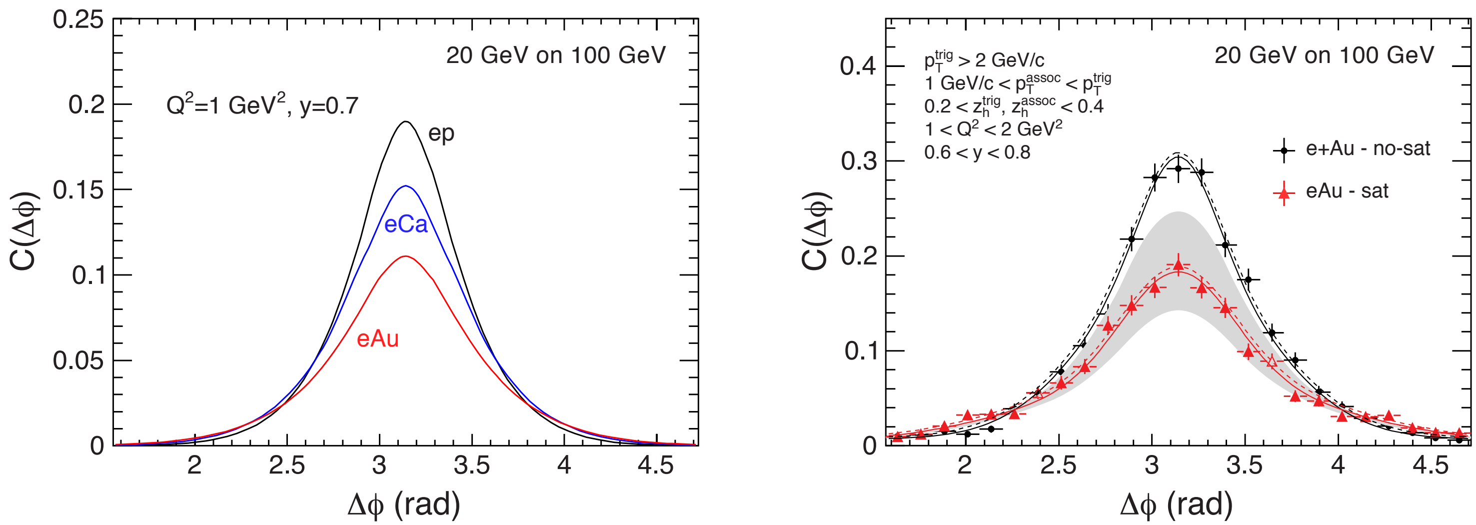




- B2B jets may get de-correlated in p_T with the spread of the order of Q_S

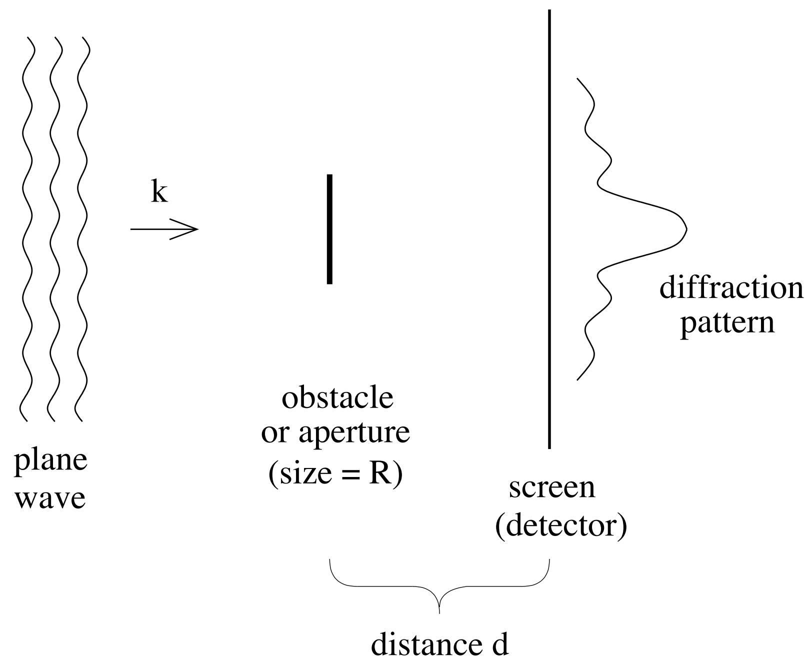
Di-hadron Correlations

Depletion of di-hadron correlations is predicted for e+A as compared to e+p. (Dominguez et al ‘11; Zheng et al ‘14). This is a signal of saturation.



(iii) Diffraction

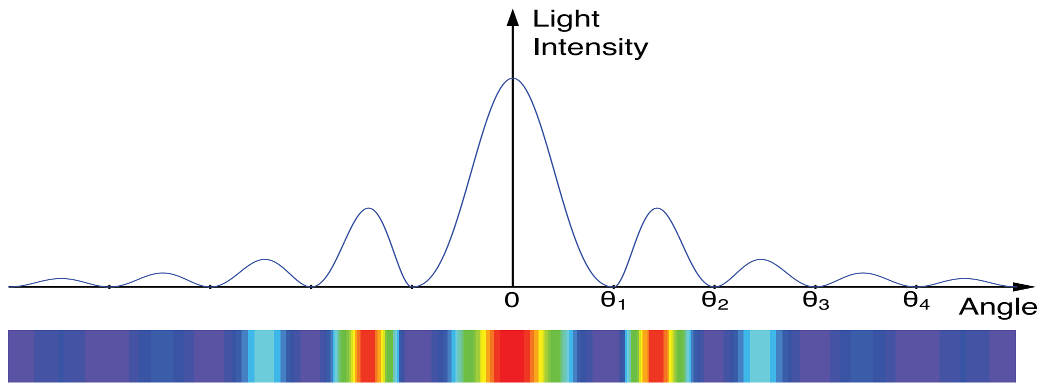
Diffraction in optics

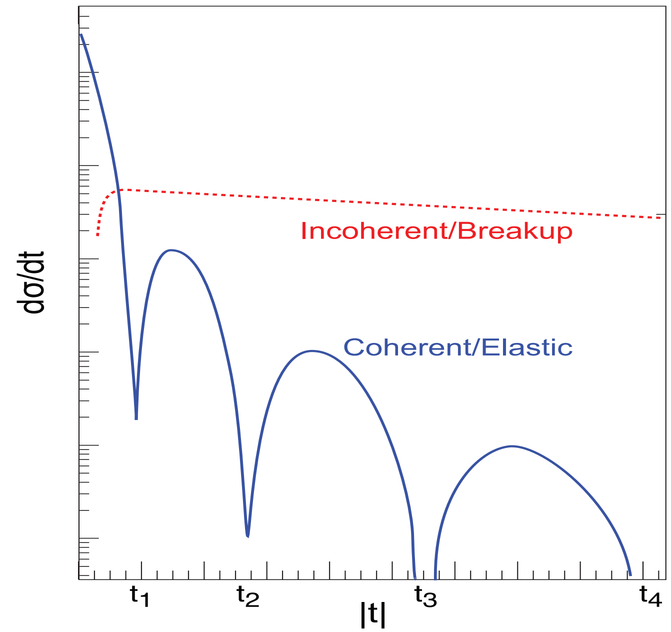


Diffraction pattern contains information about the size *R* of the obstacle and about the optical “blackness” of the obstacle.
In optics, diffraction pattern is studied as a function of the angle θ. In high energy scattering the diffractive cross sections are plotted as a function of the Mandelstam variable *t* with $\sqrt{|t|} = k \sin\theta$.

Optical Analogy

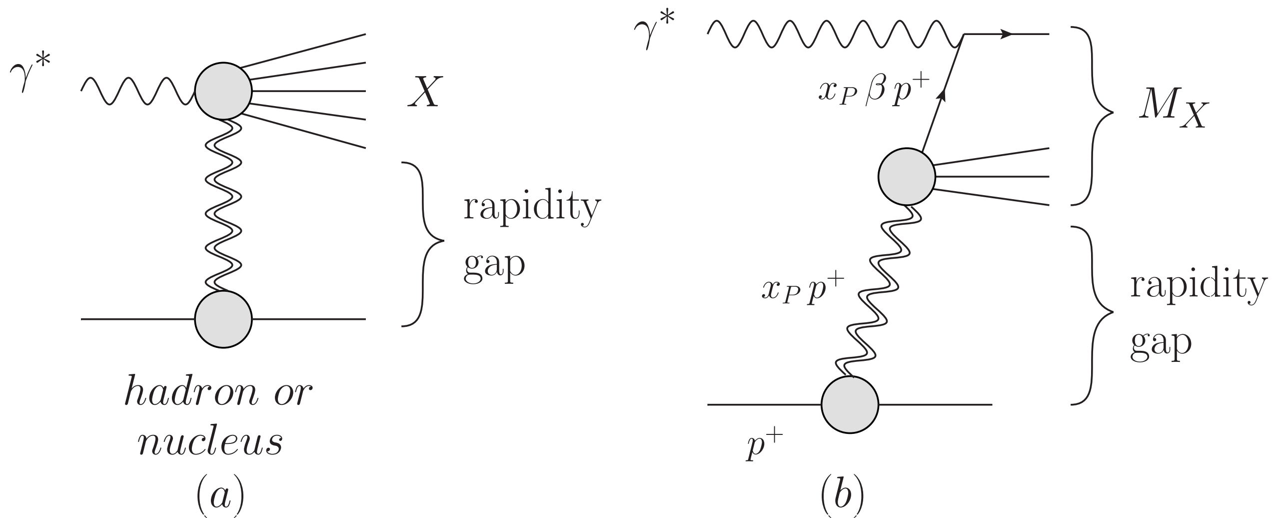
Diffraction in high energy scattering is not very different from diffraction in optics: both have diffractive maxima and minima:





Coherent: target stays intact;
Incoherent: target nucleus breaks up, but nucleons are intact.

Diffraction terminology

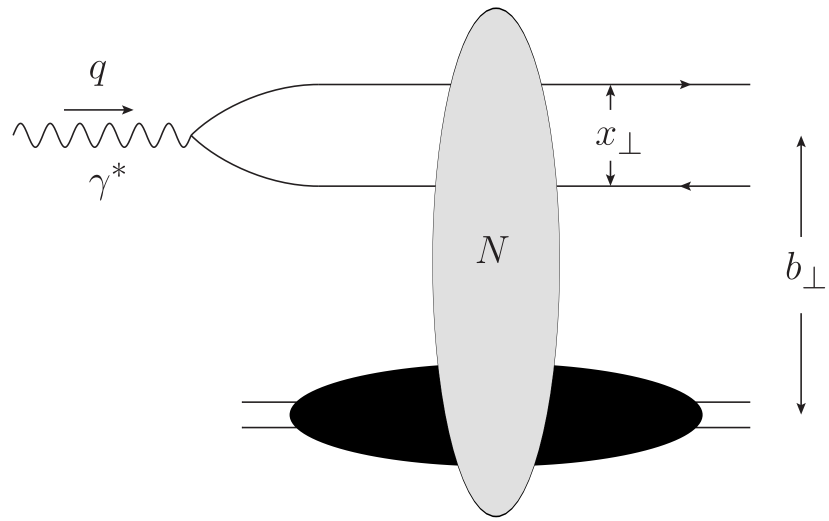


W^2 = cms energy squared for the photon+proton/nucleus system

$$x_P = \frac{Q^2 + M_X^2}{Q^2 + W^2} \approx \frac{M_X^2}{W^2} \qquad \beta = \frac{x_{Bj}}{x_P} = \frac{Q^2}{Q^2 + M_X^2} \approx \frac{Q^2}{M_X^2}$$

Quasi-elastic DIS

Consider the case when nothing but the quark-antiquark pair, i.e., two jets, are produced:



The quasi-elastic cross section is then proportional to the square of the dipole amplitude N:

$$\sigma_{el}^{\gamma^* A} = \int \frac{d^2 x_\perp}{4\,\pi}\, d^2 b_\perp \int\limits_0^1 \frac{dz}{z\,(1-z)}\, |\Psi^{\gamma^* \to q\bar{q}}(\vec{x}_\perp, z)|^2\ N^2(\vec{x}_\perp, \vec{b}_\perp, Y)$$

Buchmuller et al '97, McLerran and Yu.K. '99

Diffraction on a black disk

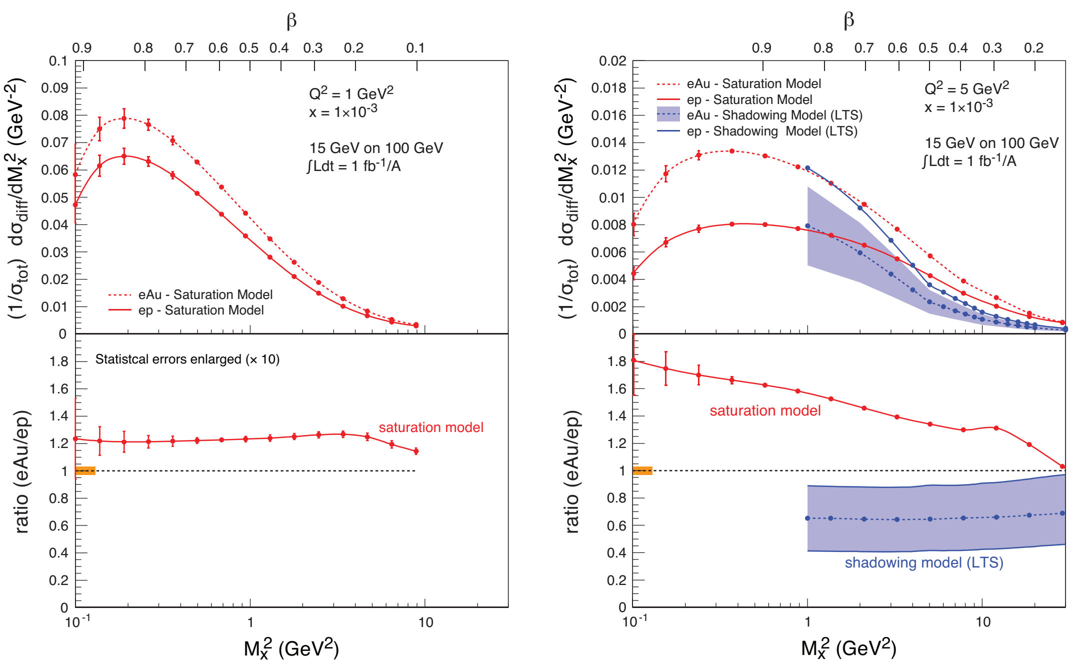
- For low Q^2 (large dipole sizes) the black disk limit is reached with N=1
- Diffraction (elastic scattering) becomes a half of the total cross section

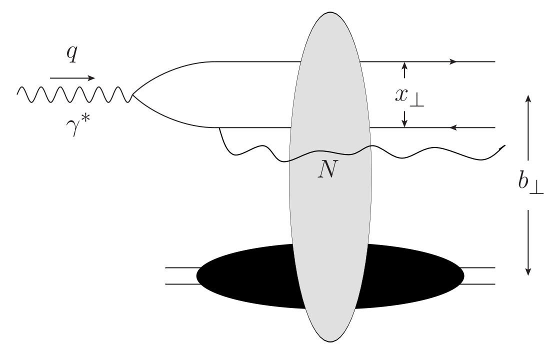
$$\frac{\sigma_{el}^{q\bar{q}A}}{\sigma_{tot}^{q\bar{q}A}} = \frac{\int d^2b\, N^2}{2\int d^2b\, N} \longrightarrow \frac{1}{2}$$

- Large fraction of diffractive events in DIS is a signature of reaching the black disk limit!
- HERA: ~15% (unexpected!) ; EIC: ~25% expected from saturation

Diffractive over total cross sections

- Here's an EIC measurement which may distinguish saturation from non-saturation approaches (from the 2012 EIC White Paper), using **diffractive to total double ratio**:



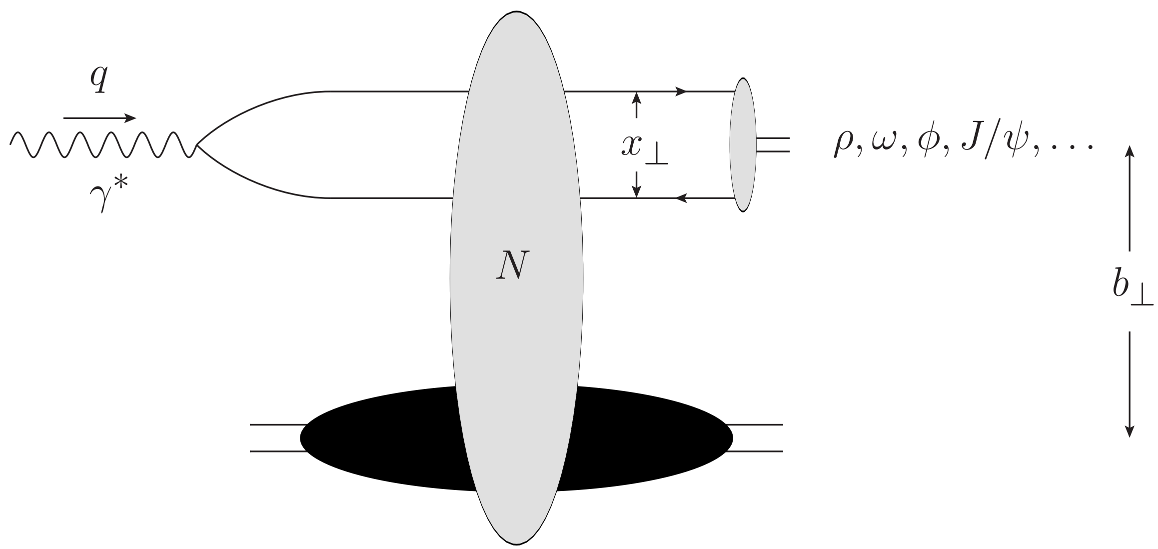


sat = Kowalski et al '08, plots generated by Marquet

no-sat = Leading Twist Shadowing (LTS), Kopeliovich, Tarasov, '02; Frankfurt, Guzey, Strikman '04, plots by Guzey

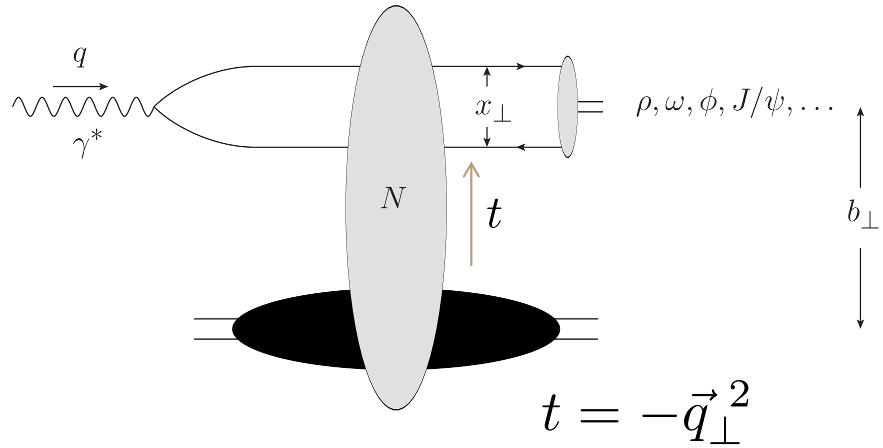
Exclusive Vector Meson Production

- An important diffractive process which can be measured at EIC is exclusive vector meson production (cf. UPCs):



Exclusive VM Production: Probe of Spatial Gluon Distribution

- Differential exclusive VM production cross section is



$$\frac{d\sigma^{\gamma^*+A\to V+A}}{dt} = \frac{1}{4\pi}\left|\int d^2b\, e^{-i\,\vec{q}_\perp\cdot\vec{b}_\perp}\, T^{q\bar{q}A}(\hat{s},\vec{b}_\perp)\right|^2$$

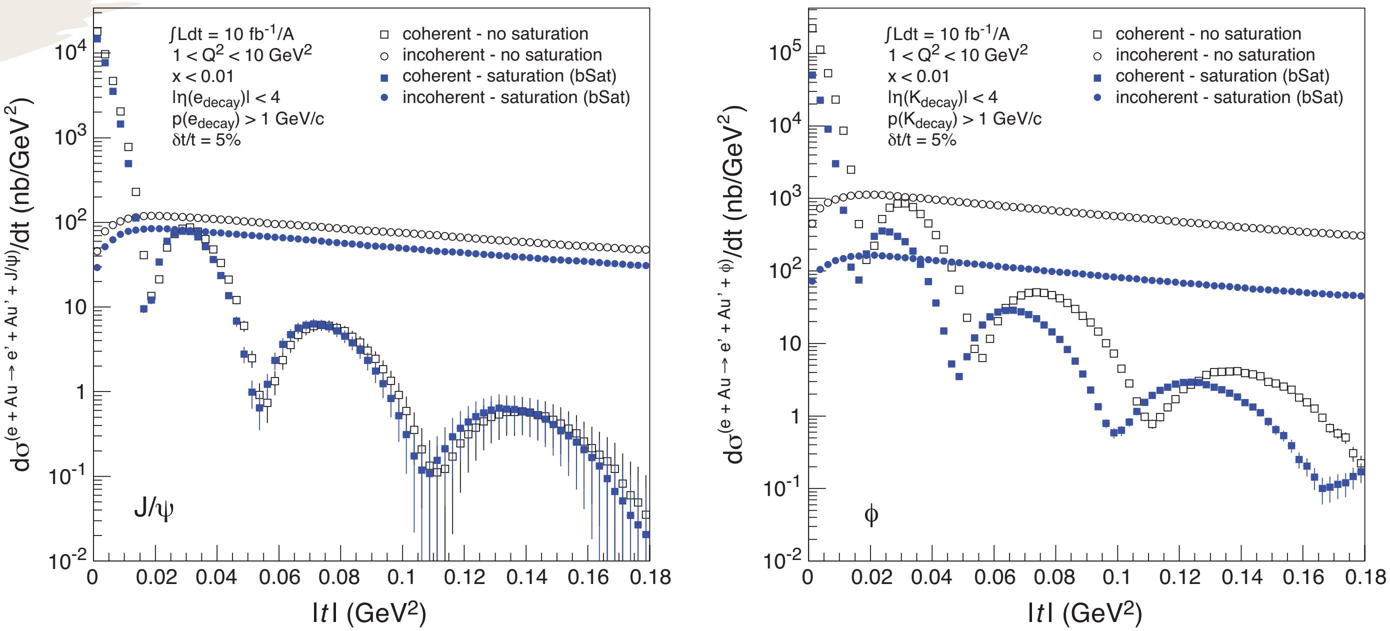
- the T-matrix is related to the dipole amplitude N:

$$T^{q\bar{q}A}(\hat{s},\vec{b}_\perp) = i\int \frac{d^2x_\perp}{4\,\pi}\int_0^1 \frac{dz}{z\,(1-z)}\;\Psi^{\gamma^*\to q\bar{q}}(\vec{x}_\perp,z)\;N(\vec{x}_\perp,\vec{b}_\perp,Y)\,\Psi^V(\vec{x}_\perp,z)^*$$

Brodsky et al '94, Ryskin '93

- Can study *t*-dependence of the *dσ/dt* and look at different mesons to find the dipole amplitude *N(x,b,Y)* (Munier, Stasto, Mueller '01).
- Learn about the gluon distribution in space. This is similar to GPDs.

Exclusive VM Production as a Probe of Saturation



Plots by T. Toll and T. Ullrich using the Sartre event generator
(b-Sat (=GBW+b-dep+DGLAP) + WS + MC, from the 2012 EIC White Paper).

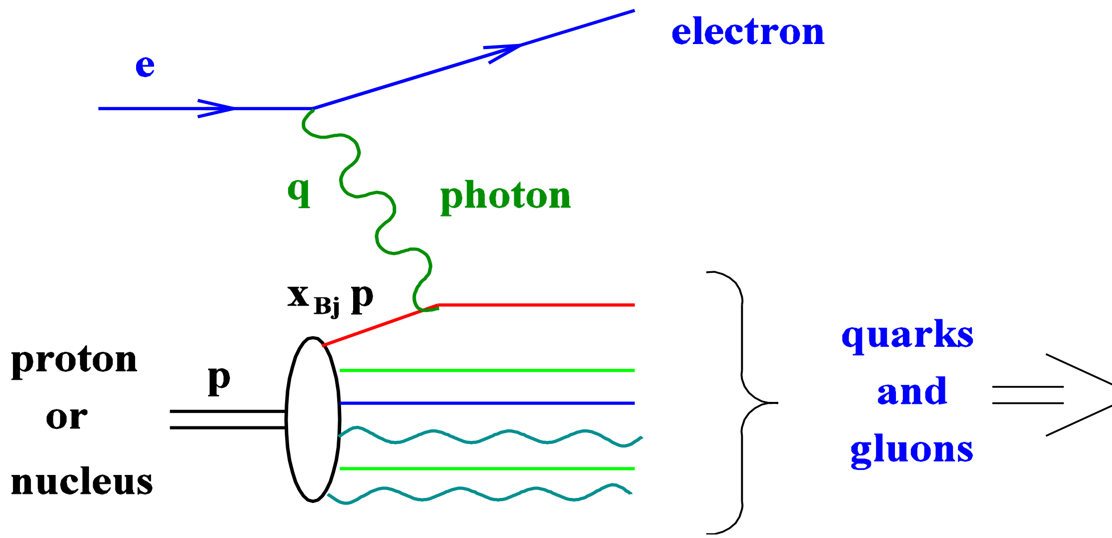
- J/psi is smaller, less sensitive to saturation effects
- Phi meson is larger, more sensitive to saturation effects

Conclusions

- In recent decades we have seen a lot of progress in our understanding of QCD in high energy collisions.
- Non-linear small-x evolution equations have been derived, unitarizing the BFKL evolution, while predicting and quantifying gluon saturation. They led to a lot of successful phenomenology at HERA, RHIC, and LHC, providing possible evidence for gluon saturation (not discussed today).
- EIC will be able to test the predictions of these nonlinear small-x evolution equations and may complete the discovery of parton saturation.

Backup Slides

Kinematics of DIS



- Photon carries 4-momentum q_μ, its virtuality is

$$Q^2 = -q_\mu q^\mu$$

- Photon hits a quark in the proton carrying momentum $x_{Bj}p$ with p being the proton's momentum. Parameter x_{Bj} is the Bjorken *x* variable.

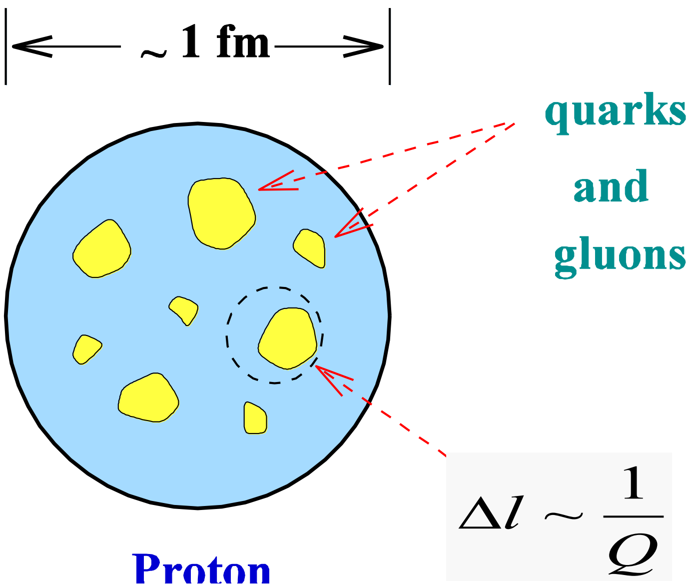
Physical Meaning of Q

Uncertainty principle teaches us that

$$\Delta p \Delta l \approx \hbar$$

which means that the photon probes the proton at the distances of the order (ħ=1)

$$\Delta l \sim \frac{1}{Q}$$

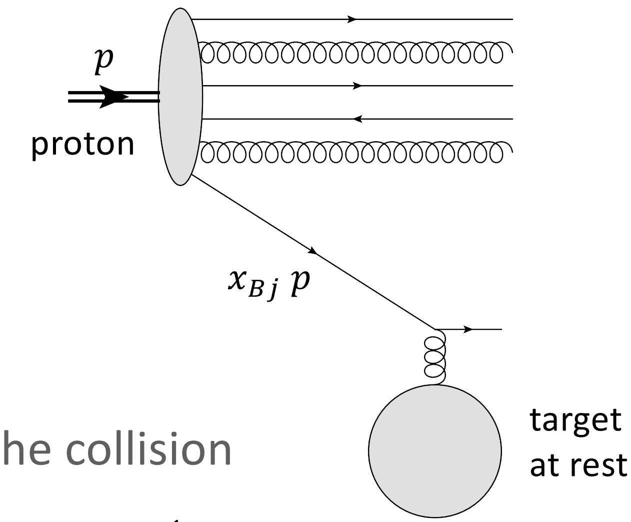


Large Momentum Q = Short Distances Probed

Physical Meaning of Bjorken x

The quarks and gluons that interact with the target have their typical momenta on the order of the typical momentum in the target,

$$x_{Bj}\, p \ \approx q \approx m.$$



Then the energy of the collision

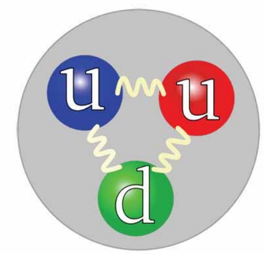
$$E \sim p \sim \frac{1}{x_{Bj}}$$

High Energy = Small x

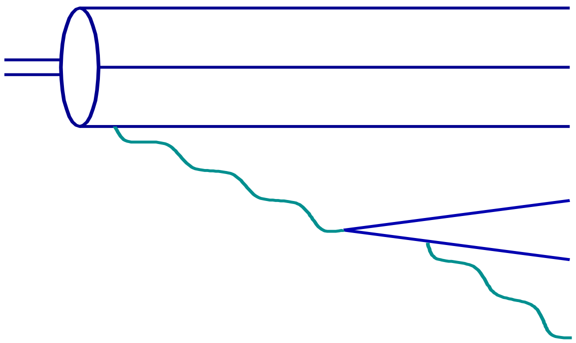
Gluons and Quarks in the Proton

⇨ There is a huge number of quarks, anti-quarks and gluons at small-x !

⇨ How do we reconcile this result with the picture of the proton made up of three valence quarks?



⇨ Qualitatively we understand that these extra quarks and gluons are emitted by the original three valence quarks in the proton.



Running of QCD Coupling Constant

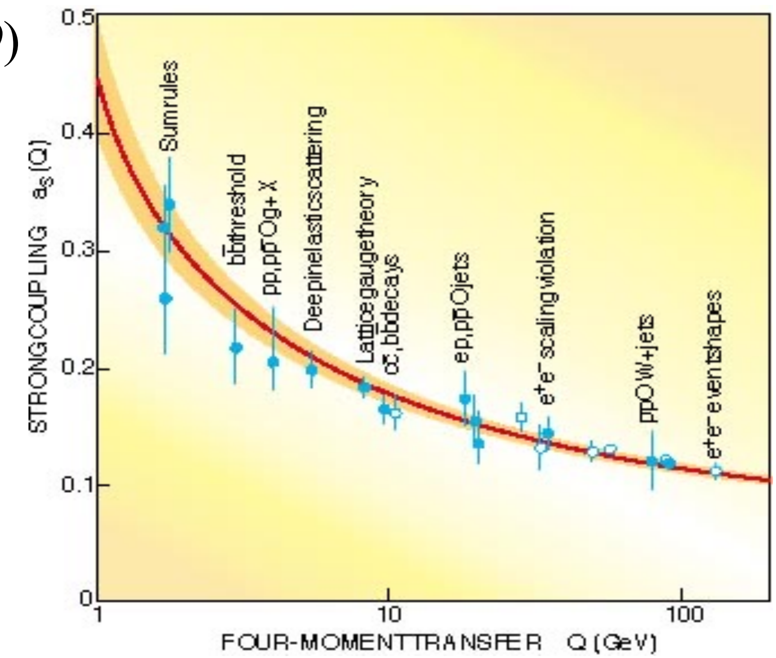
⇨QCD coupling constant $\alpha_S = \frac{g^2}{4\pi}$ changes with the momentum scale involved in the interaction

$$\alpha_S = \alpha_S(Q)$$

Asymptotic Freedom!

Gross and Wilczek, Politzer, ca '73

Physics Nobel Prize 2004!



For short distances x < 0.2 fm, or, equivalently, large momenta k > 1 GeV the QCD coupling is small $\alpha_S << 1$ and interactions are weak.

The main principle

- Saturation physics is based on the existence of a large internal transverse momentum scale Q_S which grows with both decreasing Bjorken *x* and with increasing nuclear atomic number A

$$Q_s^2 \sim A^{1/3} \left(\frac{1}{x}\right)^{\lambda}$$

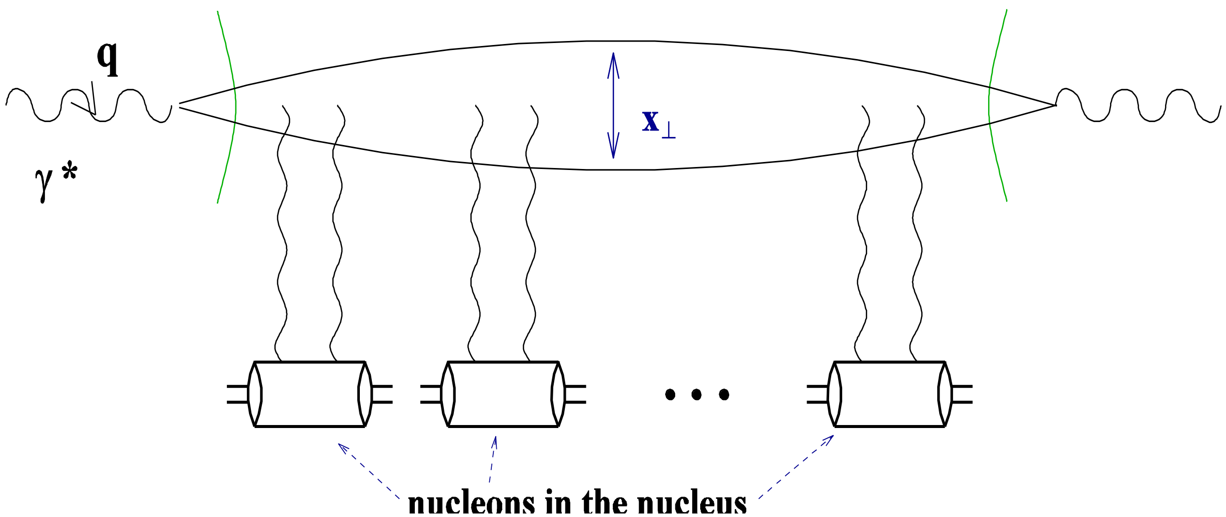
such that

$$\alpha_S = \alpha_S(Q_S) << 1$$

and we can use perturbation theory to calculate total cross sections, particle spectra and multiplicities, correlations, etc, from first principles.

DIS in the Classical Approximation

The DIS process in the rest frame of the target nucleus is shown below.



$$\sigma_{tot}^{\gamma * A}(x_{Bj}, Q^2) = |\Psi^{\gamma * \rightarrow q\bar{q}}|^2 \otimes N(x_\perp, Y = \ln 1/x_{Bj})$$

with rapidity Y=ln(1/x)

Dipole Amplitude

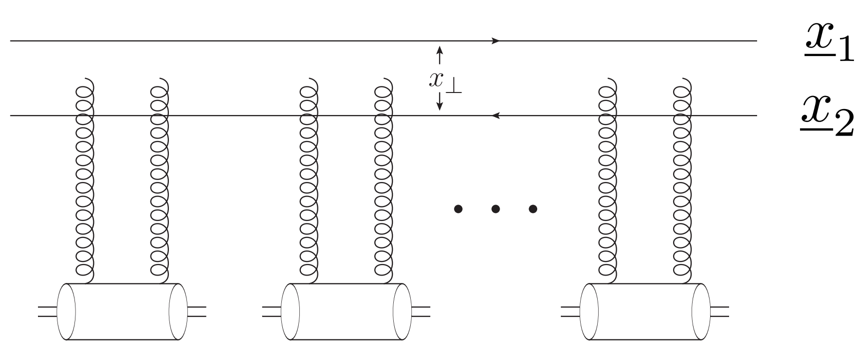
- The quark dipole amplitude is defined by

$$N(\underline{x}_1, \underline{x}_2) = 1 - \frac{1}{N_c} \left\langle \mathrm{tr} \left[V(\underline{x}_1) \, V^\dagger(\underline{x}_2) \right] \right\rangle$$

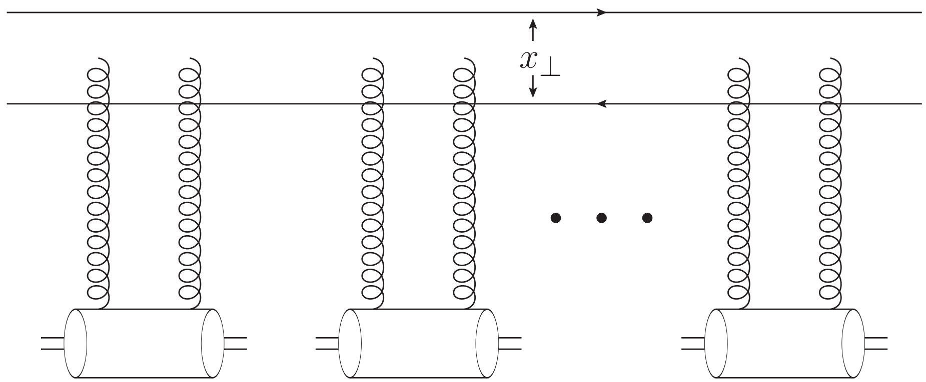
- Here we use the Wilson lines along the light-cone direction

$$V_{\underline{x}} = \mathcal{P} \exp \left[ig \int\limits_{-\infty}^{\infty} dx^- A^+(0^+, x^-, \underline{x}) \right]$$

- In the classical Glauber-Mueller/McLerran-Venugopalan approach the dipole amplitude resums multiple rescatterings:



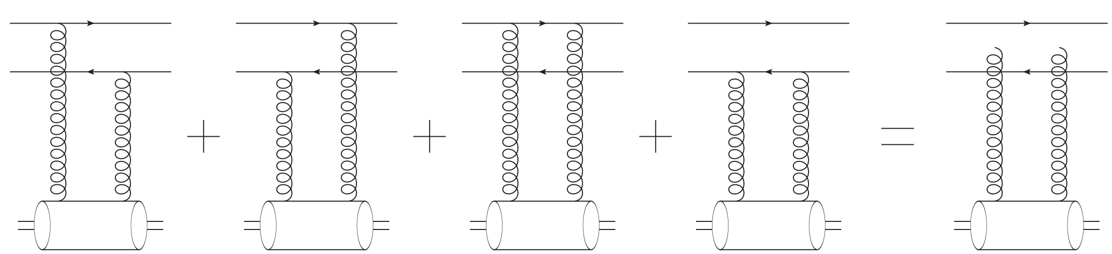
Quasi-classical dipole amplitude



A.H. Mueller, ‘90

Lowest-order interaction with each nucleon – two gluon exchange – lead to the following resummation parameter:

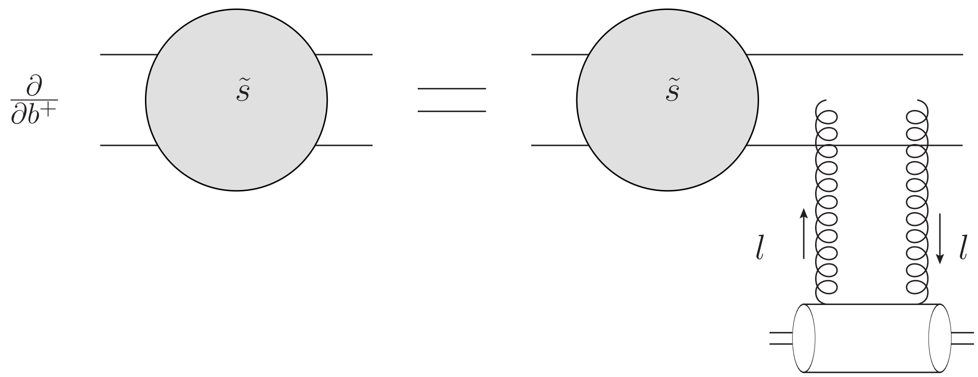
$$\alpha_s^2 \, A^{1/3}$$



Quasi-classical dipole amplitude

- To resum multiple rescatterings, note that the nucleons are independent of each other and rescatterings on the nucleons are also independent.

- One then writes an equation (Mueller '90)



Each scattering!

$$N(x_\perp, Y) = 1 - \exp\left[-\frac{x_\perp^2 Q_s^2}{4} \ln \frac{1}{x_\perp \Lambda}\right]$$

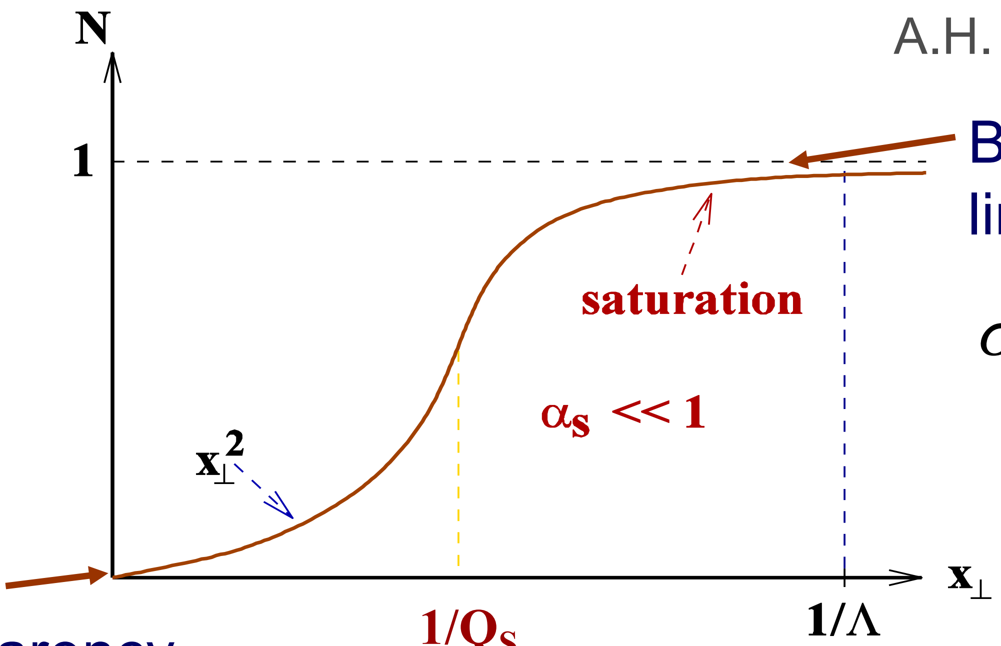
DIS in the Classical Approximation

The dipole-nucleus amplitude in the classical approximation is

$$\sigma^{q\bar{q}A} = 2 \int d^2b\, N(x_\perp, b_\perp, Y)$$

$$N(x_\perp, Y) = 1 - \exp\left[-\frac{x_\perp^2\, Q_s^2}{4}\, \ln\frac{1}{x_\perp\, \Lambda}\right]$$

A.H. Mueller, ‘90



Black disk limit,

$$\sigma_{tot} < 2\pi R^2$$

Color transparency

$Q_S^2 \sim A^{1/3}$

Black Disk Limit

- Start with basic scattering theory: the final and initial states are related by the S-matrix operator,

$$|\psi_f\rangle = \hat{S}\,|\psi_i\rangle$$

- Write it as $$|\psi_f\rangle = |\psi_i\rangle + \left[\hat{S} - 1\right]\,|\psi_i\rangle$$

- The total cross section is

$$\sigma_{tot} \propto \left|\left[\hat{S} - 1\right]\,|\psi_i\rangle\right|^2 = 2 - S - S^*$$

where the forward matrix element of the S-matrix operator is

$$S = \langle\psi_i|\,\hat{S}\,|\psi_i\rangle$$

and we have used unitarity of the S-matrix

$$\hat{S}\,\hat{S}^\dagger = 1$$

Black Disk Limit

- Now, since $|\psi_f\rangle = |\psi_i\rangle + \left[\hat{S} - 1\right] |\psi_i\rangle$

  the elastic cross section is

$$\sigma_{el} \propto \left| \langle\psi_i| \left[\hat{S} - 1\right] |\psi_i\rangle \right|^2 = |1 - S|^2$$

- The inelastic cross section can be found via

$$\sigma_{tot} = \sigma_{inel} + \sigma_{el}$$

- In the end, for scattering with impact parameter b we write

$$\sigma_{tot} = 2 \int d^2b \, [1 - \mathrm{Re}\, S(b)]$$

$$\sigma_{el} = \int d^2b \, |1 - S(b)|^2$$

$$\sigma_{inel} = \int d^2b \left[1 - |S(b)|^2\right]$$

Unitarity Limit

- Unitarity implies that

$$1 = \langle \psi_i | \, \hat{S} \, \hat{S}^\dagger \, | \psi_i \rangle = \sum_X \langle \psi_i | \, \hat{S} | X \rangle \, \langle X | \hat{S}^\dagger \, | \psi_i \rangle \geq |S|^2$$

- Therefore

$$|S| \leq 1$$

leading to the unitarity bound on the total cross section

$$\sigma_{tot} = 2 \int d^2 b \, [1 - \mathrm{Re}\, S(b)] \leq 4 \int d^2 b = 4 \pi R^2$$

- Notice that when S=-1 the inelastic cross section is zero and

$$\sigma_{tot} = 2 \int d^2 b \, [1 - \mathrm{Re}\, S(b)]$$

$$\sigma_{tot} = 4 \pi R^2 = \sigma_{el}$$

$$\sigma_{el} = \int d^2 b \, |1 - S(b)|^2$$

This limit is realized in low-energy scattering!

$$\sigma_{inel} = \int d^2 b \left[1 - |S(b)|^2 \right]$$

Black Disk Limit

- At high energy inelastic processes dominate over elastic. Imposing

$$\sigma_{inel} \geq \sigma_{el}$$

we get

$$\mathrm{Re}\, S \geq 0$$

- The bound on the total cross section is (aka the **black disk limit**)

$$\sigma_{tot} = 2 \int d^2b \,[1 - \mathrm{Re}\, S] \leq 2 \int d^2b = 2\pi R^2$$

- The inelastic and elastic cross sections at the black disk limit are

$$\sigma_{inel} = \sigma_{el} = \pi R^2$$

$$\sigma_{tot} = 2 \int d^2b \,[1 - \mathrm{Re}\, S(b)]$$

$$\sigma_{el} = \int d^2b \,|1 - S(b)|^2$$

$$\sigma_{inel} = \int d^2b \left[1 - |S(b)|^2\right]$$

Notation

- At high energies $\mathrm{Im}\, S \approx 0$

  Define the dipole amplitude N as the imaginary part of the dipole T-matrix (S=1+iT), such that
  $$\mathrm{Re}\, S = 1 - N$$

- The cross sections are
$$\sigma_{tot} = 2\int d^2b\, N(x_\perp, b_\perp)$$
$$\sigma_{el} = \int d^2b\, N^2(x_\perp, b_\perp)$$
$$\sigma_{inel} = \int d^2b\, \left[2\, N(x_\perp, b_\perp) - N^2(x_\perp, b_\perp)\right]$$

- We see that N=1 is the black disk limit. Hence $N \leq 1$ as we saw above.