Modular RICH Detector
Optics Design Optimization

for the second mRICH Prototype

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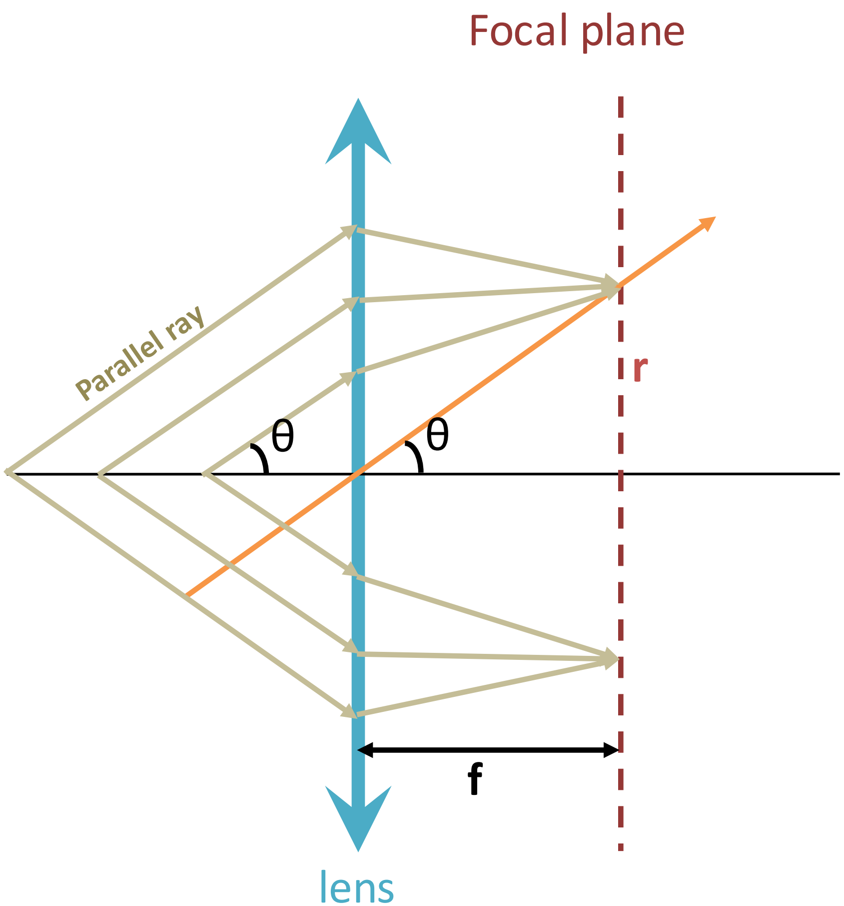
08-01-2016

New Focuses

- Study ring radius in terms of
  - Fresnel Lens focal length : f
  - Refractive index of aerogel : n
  - Momentum of incident particle (pi, kaon) : p
- Explore the possibility of optical design improvement for the second mRICH prototype

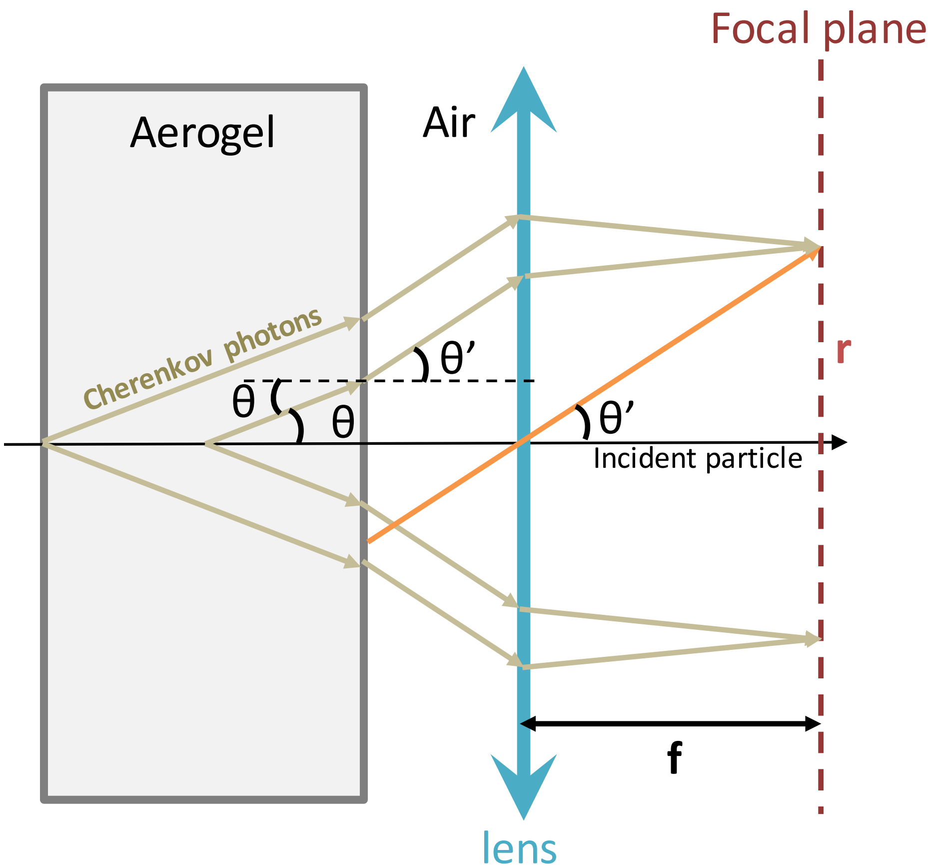
Lens Optics



Rule of ray diagram: An incident ray that **passes through the center (orange line)** of the lens will in effect **continue in the same direction** that it had when it entered the lens.

$$r = f \cdot \tan\theta$$

mRICH Optics



Proportional to focal length

$$r = f \cdot \tan\theta' \qquad \text{---- (1)}$$

Ring size is **independent** on thickness of the aerogel or air gap

From Snell’s Law,

$$\frac{\sin\theta'}{\sin\theta} = n$$

$$\sin\theta' = \mathrm{n} \cdot \sin\theta \qquad \text{---- (2)}$$

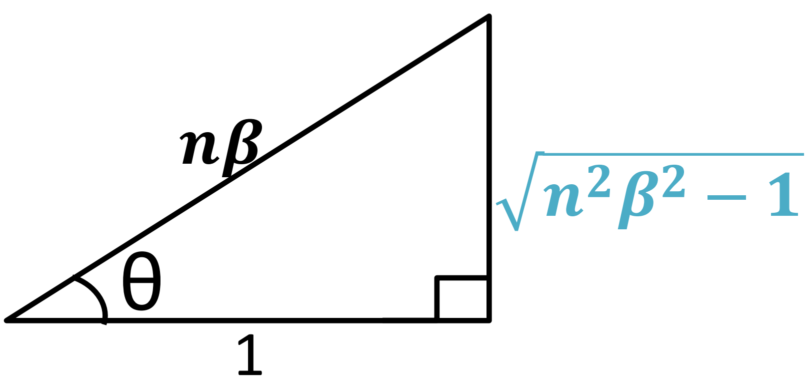
Cherenkov Angle

$$\cos\theta = \frac{1}{n\beta} \qquad \text{---- (3)}$$

Then, $\sin\theta = \frac{\sqrt{n^2\beta^2 - 1}}{n\beta}$

$$\sin\theta' = n \cdot \frac{\sqrt{n^2\beta^2 - 1}}{n\beta}$$

$$\sin\theta' = \frac{\sqrt{n^2\beta^2 - 1}}{\beta} \qquad \text{---- (4)}$$

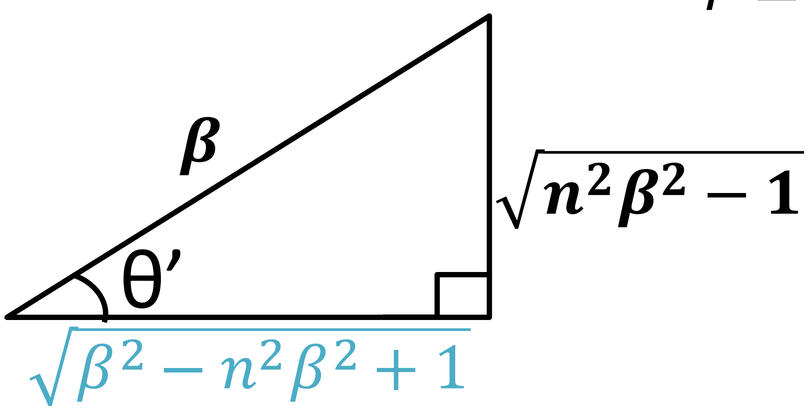


From $\sin\theta' = \frac{\sqrt{n^2\beta^2-1}}{\beta}$

$$\tan\theta' = \sqrt{\frac{n^2\beta^2-1}{\beta^2-n^2\beta^2+1}}$$

Thus, $$r = f \cdot \sqrt{\frac{n^2\beta^2-1}{\beta^2-n^2\beta^2+1}}$$

$$r = f \cdot \sqrt{\frac{n^2\beta^2-1}{(1-n^2)\beta^2+1}} \qquad \text{---- (5)}$$



Relativistic Momentum

$$p = \frac{m_0 v}{\sqrt{1-\beta^2}}$$

Since $v = \beta c$,
$$p = \frac{m_0 \beta}{\sqrt{1-\beta^2}} \cdot c$$

In natural unit,
$$p = \frac{m_0 \beta}{\sqrt{1-\beta^2}} \qquad \text{---- (6)}$$

Then,
$$\beta = \frac{p}{\sqrt{m_0^2 + p^2}} \qquad \text{---- (7)}$$

$$\beta^2 = \frac{p^2}{m_0^2 + p^2}$$

Let $\Delta = m_0^2 + p^2$,
$$\beta^2 = \frac{p^2}{\Delta} \qquad \text{---- (8)}$$

Cherenkov Ring Radius

Substitute (8) to (5)

$$r = f \cdot \sqrt{\frac{n^2 \cdot {p^2}/{\Delta} - 1}{(1-n^2) \cdot {p^2}/{\Delta} + 1}}$$

$$r = f \cdot \sqrt{\frac{n^2 p^2 - \Delta}{(1-n^2)p^2 + \Delta}}$$

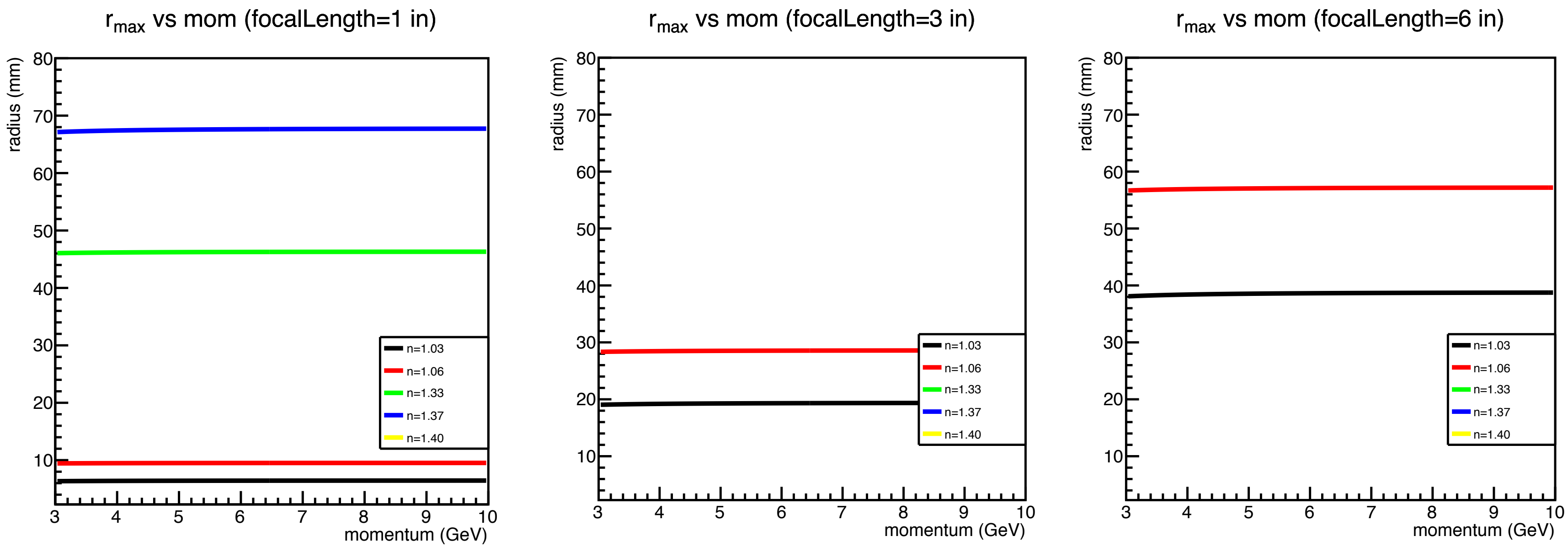
$$\mathrm{r} = f \cdot \sqrt{\frac{n^2 p^2 - \mathrm{m}_0^2 - \mathrm{p}^2}{(1-n^2)p^2 + \mathrm{m}_0^2 + \mathrm{p}^2}}$$

$$\boxed{r = f \cdot \sqrt{\frac{(n^2-1)p^2 - \mathrm{m}_0^2}{(2-n^2)p^2 + \mathrm{m}_0^2}}} \quad \text{---- (9)}$$

Thus,

$$\boxed{\Delta r = f \left[\sqrt{\frac{(n^2-1)p^2 - \mathrm{m}_\pi^2}{(2-n^2)p^2 + \mathrm{m}_\pi^2}} - \sqrt{\frac{(n^2-1)p^2 - \mathrm{m}_\kappa^2}{(2-n^2)p^2 + \mathrm{m}_\kappa^2}} \right]} \quad \text{--- (10)}$$

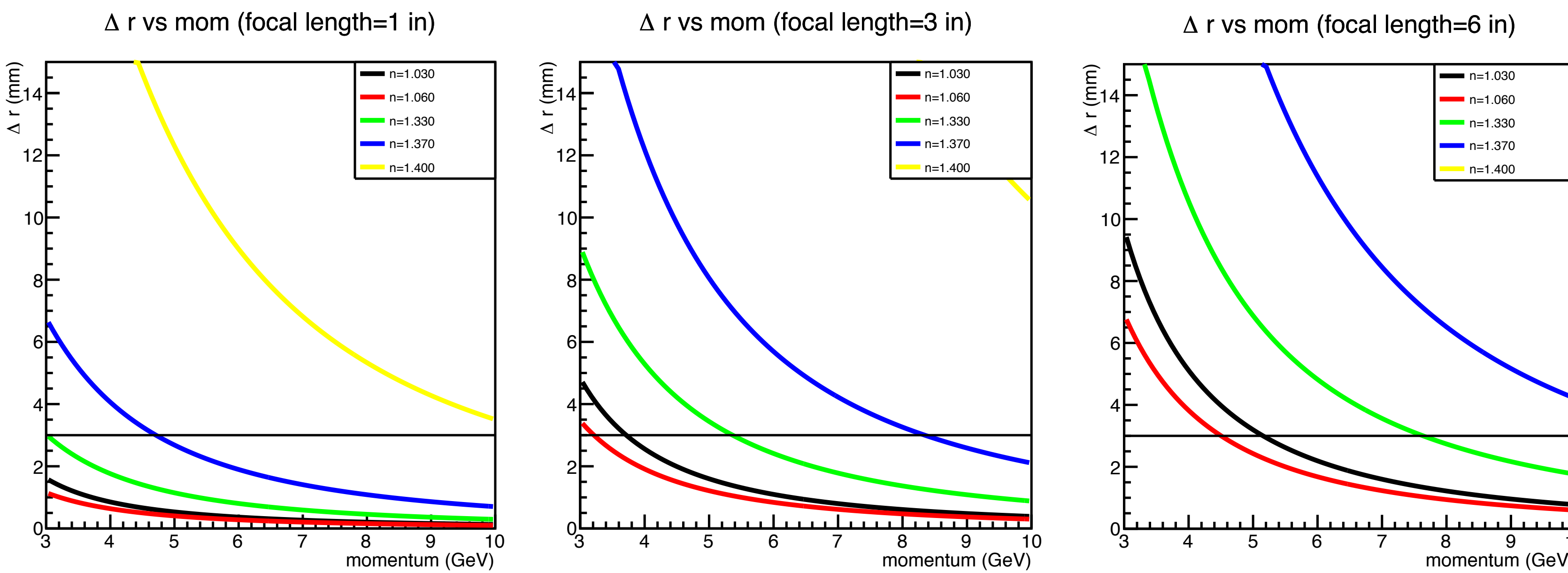
Maximum Radius vs Momentum



- Maximum radius with mass of pion
- From calculation but **not** simulation

$$r = f \cdot \sqrt{\frac{(n^2-1)p^2 - \mathrm{m}_0^2}{(2-n^2)p^2 + \mathrm{m}_0^2}}$$

Δr vs Momentum



$$\Delta r = f\left[\sqrt{\frac{(n^2-1)p^2 - \mathrm{m}_\pi^2}{(2-n^2)p^2 + \mathrm{m}_\pi^2}} - \sqrt{\frac{(n^2-1)p^2 - \mathrm{m}_\kappa^2}{(2-n^2)p^2 + \mathrm{m}_\kappa^2}}\right]$$

- Difference of Cherenkov ring radius from incident pion and kaon
- From calculation but **not** simulation
- Generally, Δr increases with n
- However, radiator with n=1.06 gives smaller Δr then radiator with n=1.03 !!!

Simulation Result with Different Focal Lengths

Detector Setup

- Aerogel gel
  - Thickness= 3.3cm
  - Refractive index=1.03
- Sensor
  - w/ glass window
  - 1mm gap between glass window
  - at the focal plane
  - Quantum eff. applied in the analysis code

- Fresnel lens (from Edmund Optics)

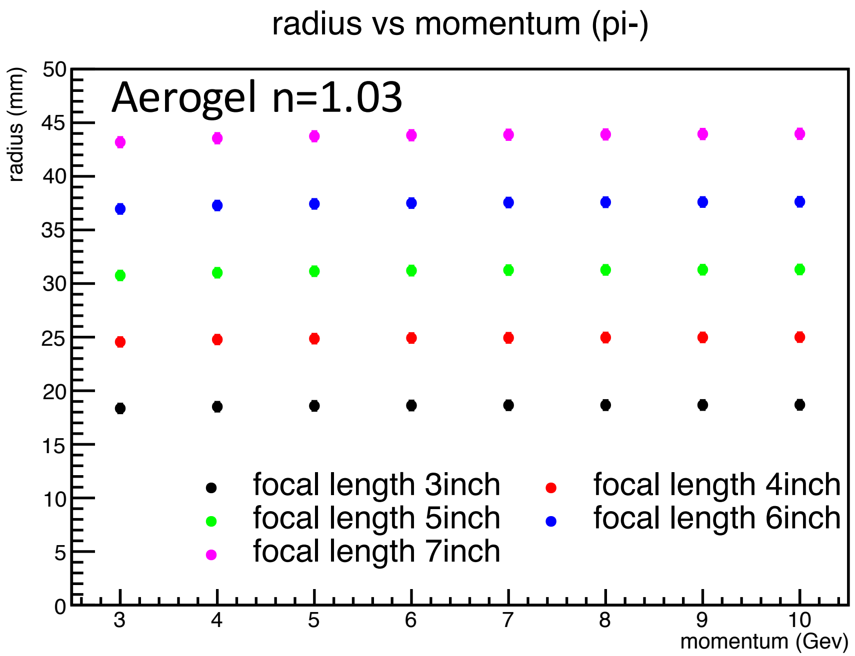
Focal Length (")	Groove density (groove/in)	Thickness (mm) (in simulation)	Refractive index
3"	100	2.04	1.49
4"	100	1.78	1.49
5"	125	1.77	1.49
6"	125	1.73	1.49
7"	50	2.30	1.49

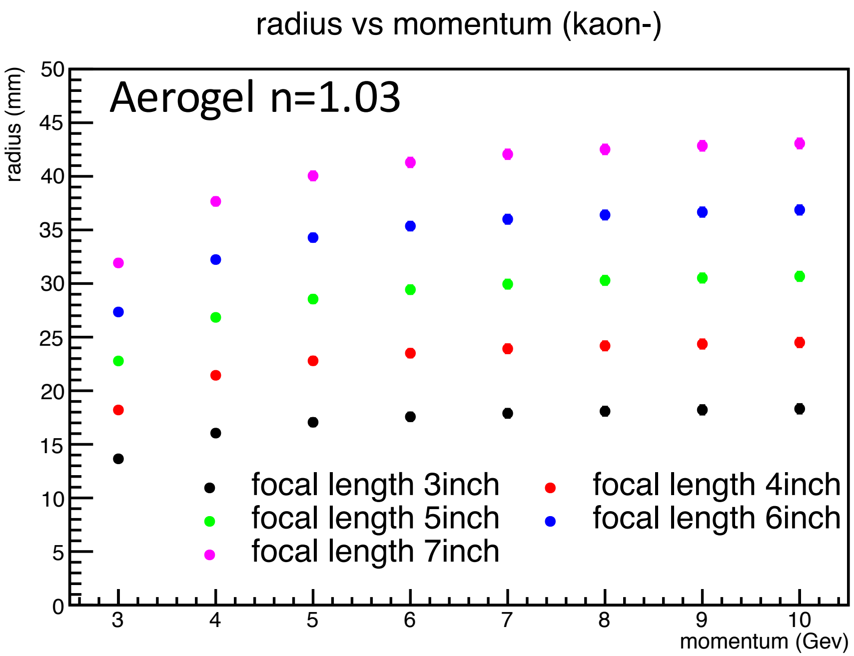
Lens absorption lengths need to be modified for more accurate simulation

Simulation Setup

- Particle : pion-, kaon-
- Momentum : 3-10 GeV, with 1GeV increment
- 1,000 event per run
- Toward center of detector
- Perpendicular to xy-plane

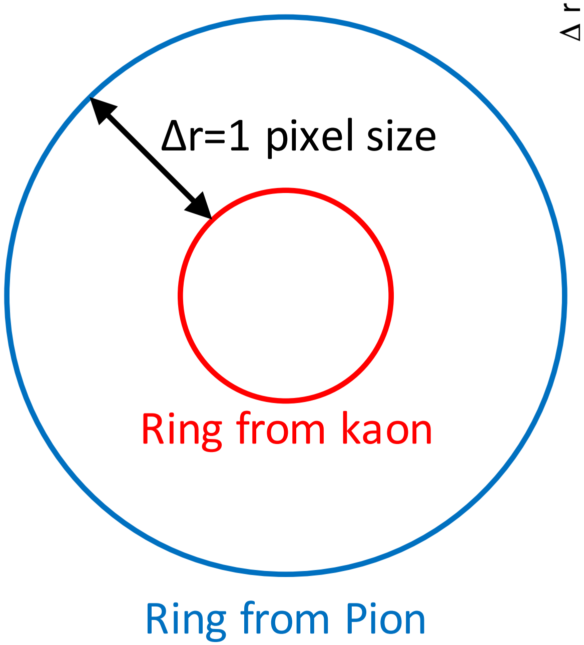
Radius vs Momentum from Simulation

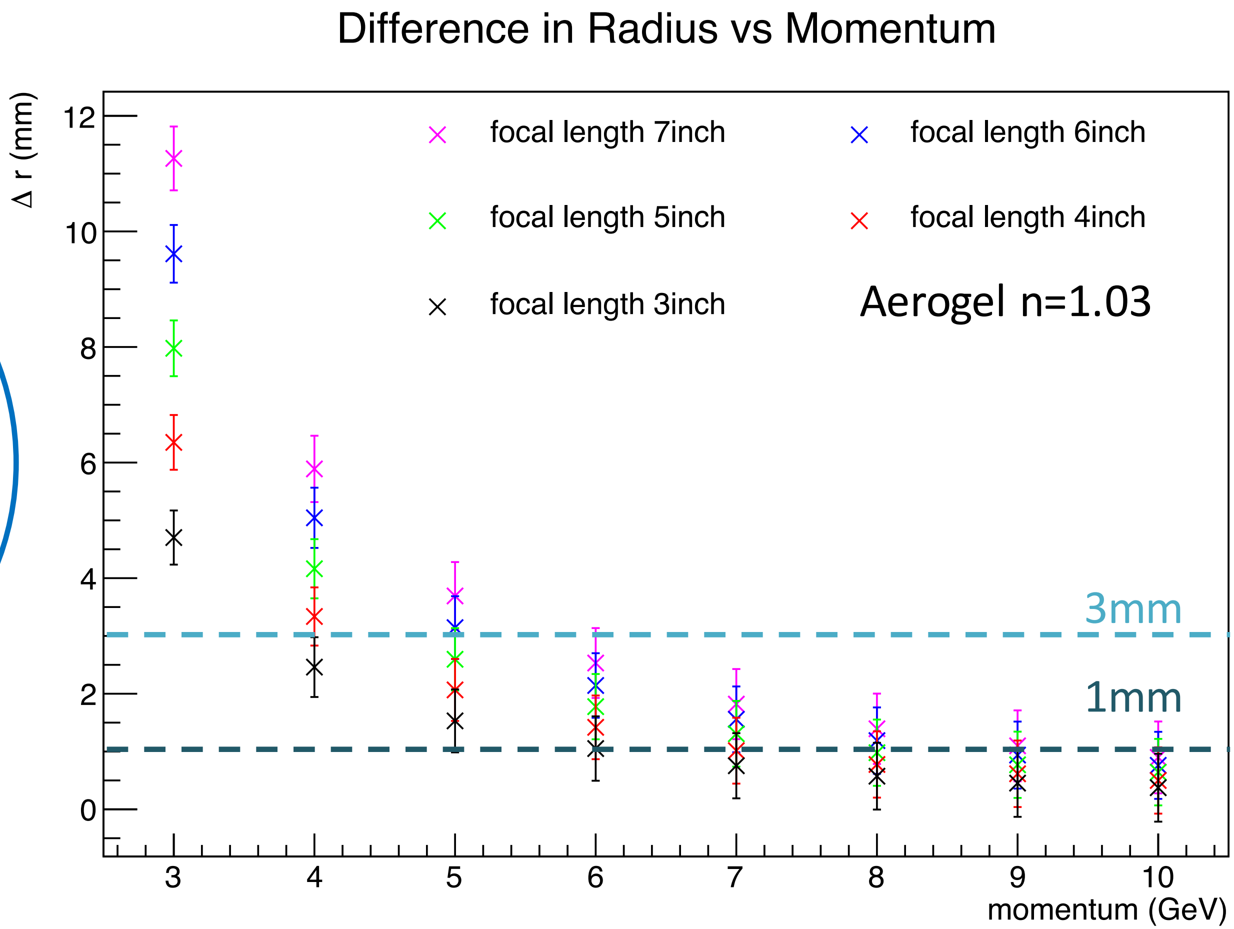




- Cherenkov ring radius increases with focal length
- Note that the error is too small compare to value of ring radius, thus error bars are hardly to seen in these plots
- Pion ring radius at 5GeV with 7” focal length is about 45mm. It can still be covered by the sensor plane ($53x53mm^2$, including gap between glass window)

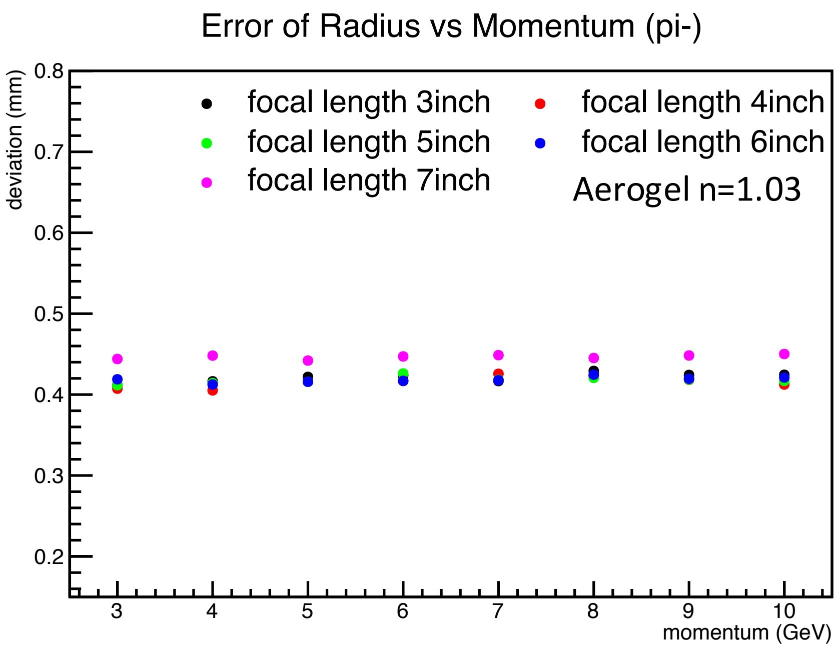
Required Pixel Size vs Momentum from Simulation

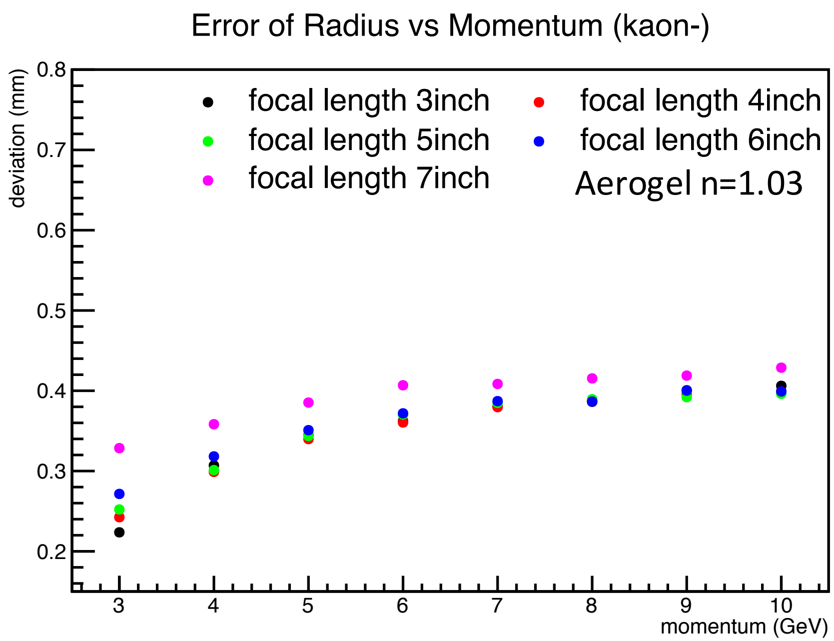




Required pixel size increases with focal length

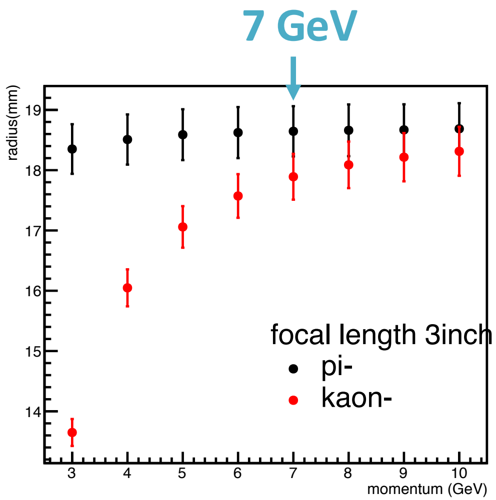
Deviation of Radius vs Momentum from Simulation

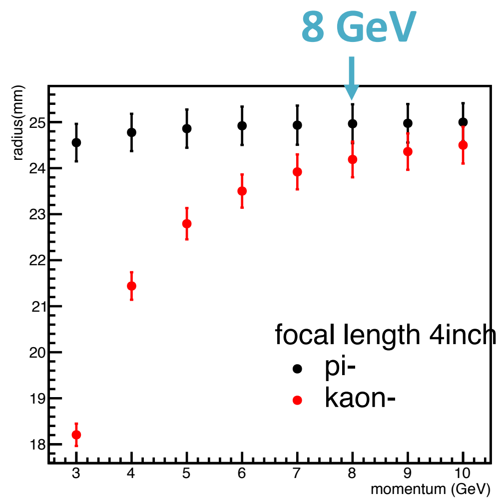


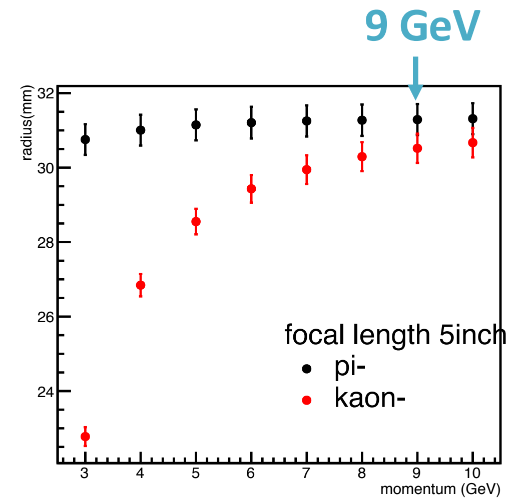


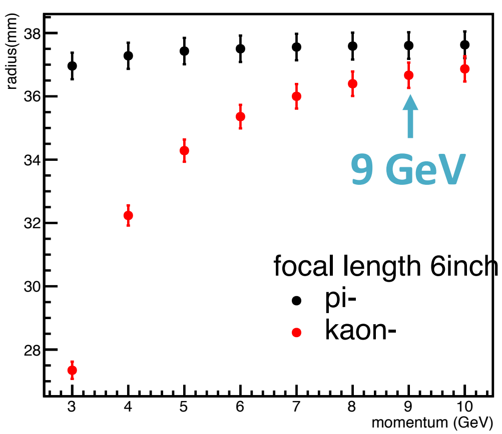
- Deviation of ring radius from pion increases by ~0.05mm when focal length increases from 3” to 7”
- Deviation of ring radius from kaon increases by ~0.1mm when focal length increases from 3” to 7”

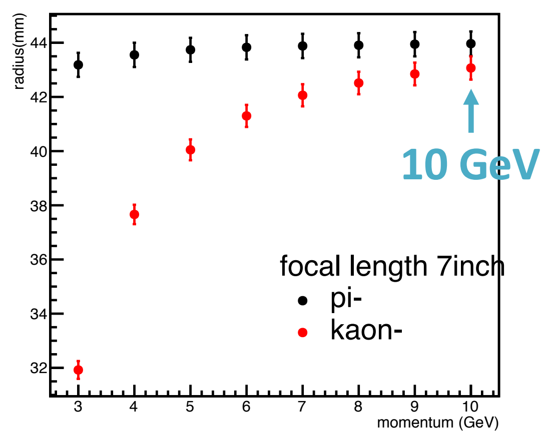
Maximum of Separation Limit (with infinite small pixel size) from Simulation











Since the changes of radius deviation is small (see slide 13), the upper (theoretical) momentum limit for PID increase with focal length

Total Internal Reflection

Internal reflection happen when $\theta = \theta_{cherenkov} \geq \theta_{critical}$

Cherenkov angle

$$\theta = \cos^{-1}\frac{1}{n\beta}$$

$$\cos\theta = \frac{1}{n\beta}$$

Critical angle of internal reflection

$$\theta = \sin\frac{1}{n}$$

$$\sin\theta = \frac{1}{n}$$

From

$$\cos^2\theta + \sin^2\theta = 1$$

$$\left(\frac{1}{n\beta}\right)^2 + \frac{1}{n^2} = 1$$

$$\frac{1}{\beta^2} = n^2 - 1$$

Upper limit of β:

$$\beta = \frac{1}{\sqrt{n^2 - 1}} \quad \text{---- (11)}$$

Total Internal Reflection

Case 1	$n^2 < 1$	No such material
Case 2	$n^2 = 1$	Same medium, no refraction or reflection
Case 3	$1 < n^2 < 2$ Or $1 < n < 1.414$	$\sqrt{n^2 - 1} < 1$ $\frac{1}{\sqrt{n^2-1}} > 1$ $\beta > 1$ Since β must be smaller than or equal to 1, Cherenkov angle always smaller then critical angle. →**Internal reflection will not happen**.
Case 4	$n^2 > 2$ or $n > 1.414$	$\text{Cherekov Threshold} < \beta < \frac{1}{\sqrt{n^2-1}}$ **(To avoid Internal reflection)** $\frac{1}{n} < \beta < \frac{1}{\sqrt{n^2-1}}$ Example: pyrex glass with n=1.47 $\beta_{max} = \frac{1}{\sqrt{1.47^2-1}} = 0.928$ and $\beta_{min} = \frac{1}{1.47} = 0.680$ From eq.(6), $0.130 \leq p_\pi \leq 0.349 \text{ GeV}$ $0.464 \leq p_\kappa \leq 1.245 \text{ GeV}$

Summary

- For the second mRICH prototype, based on this study, we plan to use a Fresnel lens with 6 inches focal length
- 3x3 mm^2 pixel size can separate pion and kaon up to 5 GeV/c
- Smaller pixel size (1x1 mm^2) sensor is required for pion and kaon seperation in high momentum region (≥8 GeV/c)

To do

- pi/e and K/proton separation

Back Up

Liquid Radiator

May be same material with coating to block reflected light
Ceramic? Acrylic? Metal?