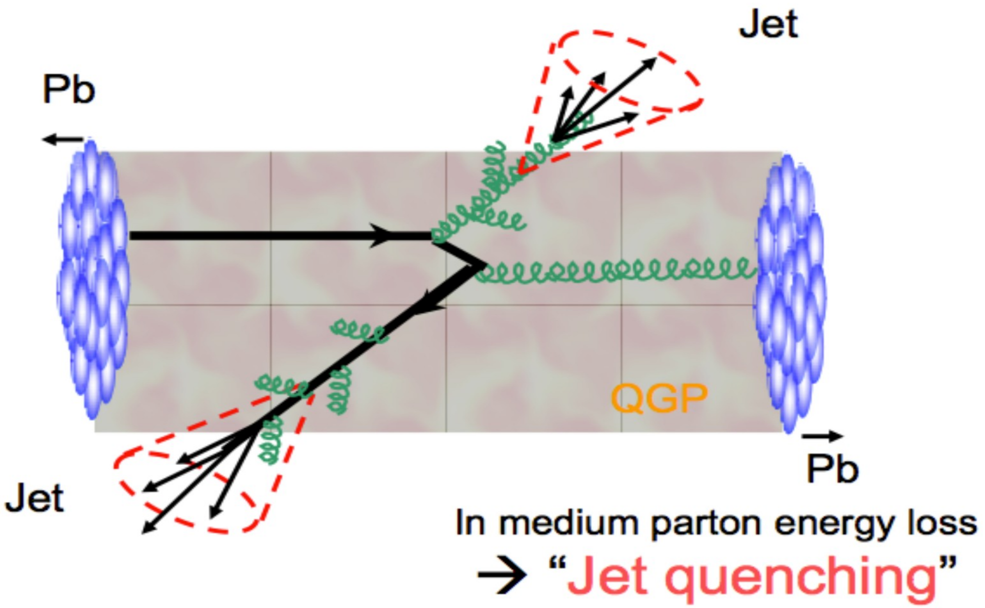
A jet quench and a theorist walk into a bar...

Peter Arnold

University of Virginia

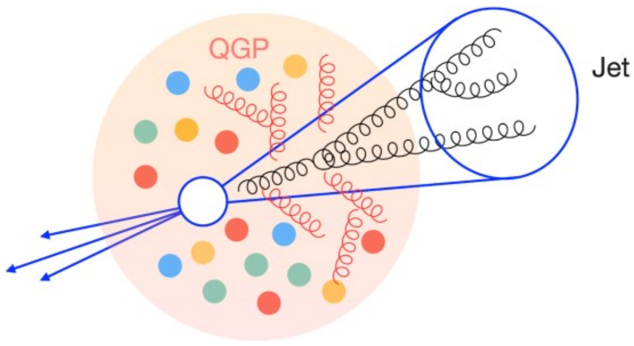
Cartoon depictions of jet quenching:

side view

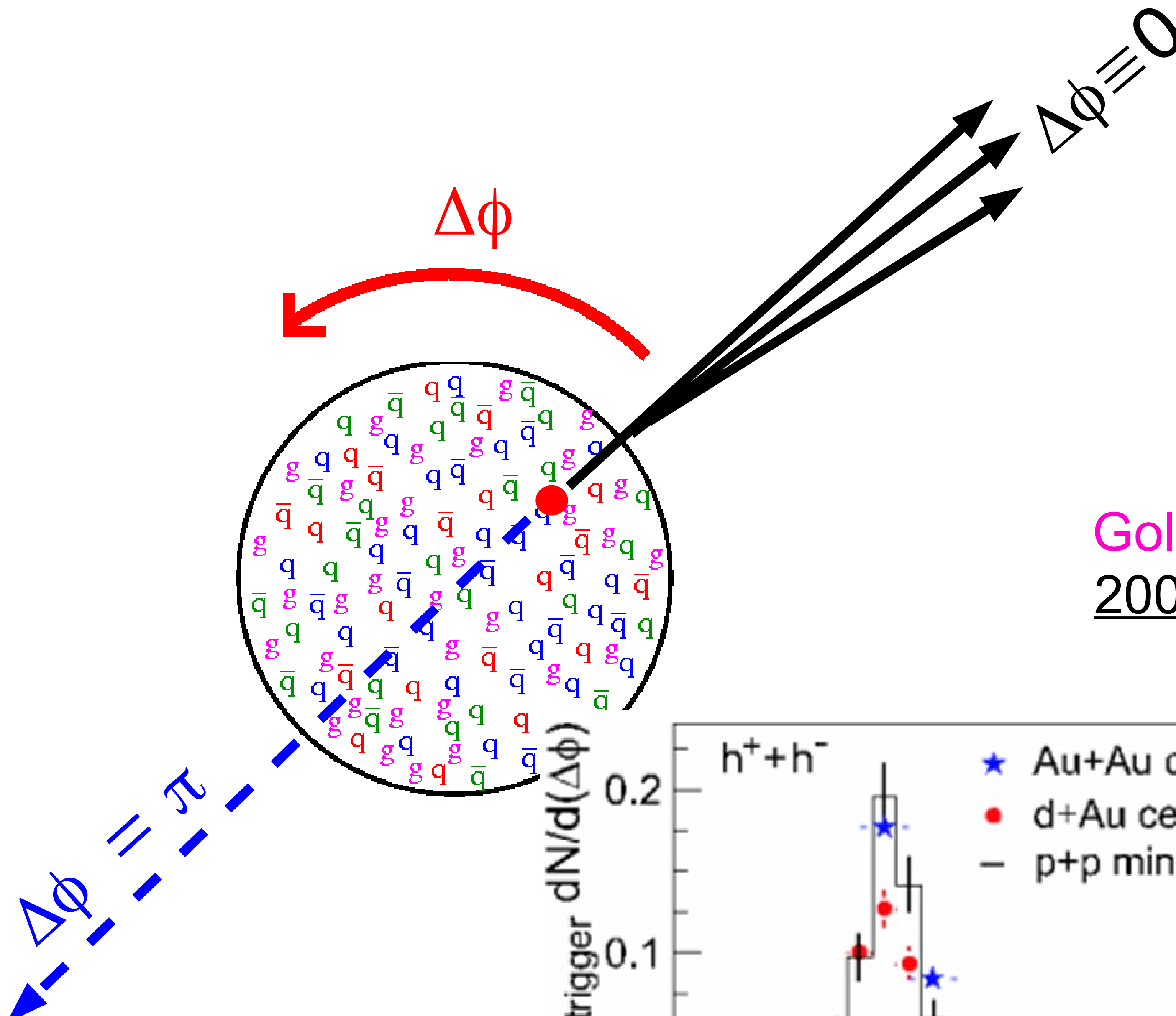


[source: 2015 talk by K.E. Raghav for CMS]

end view



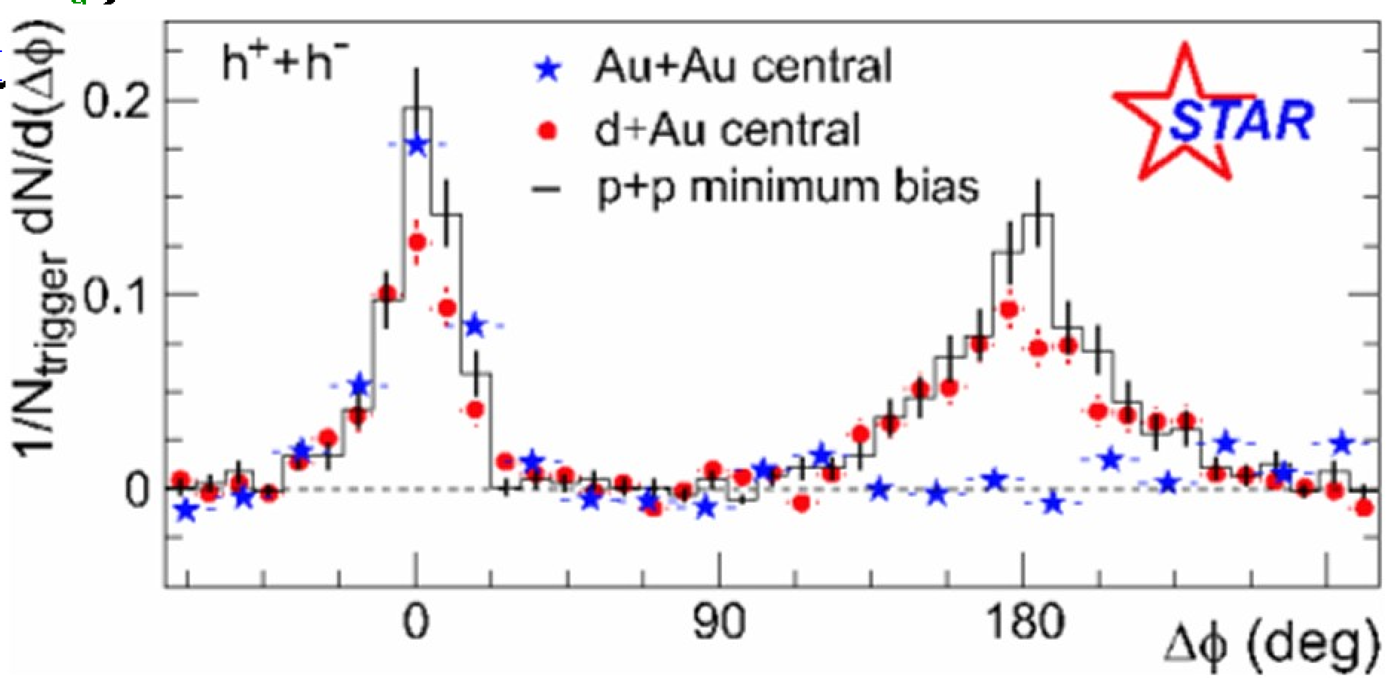
[source: logo of INT-21-2B program]



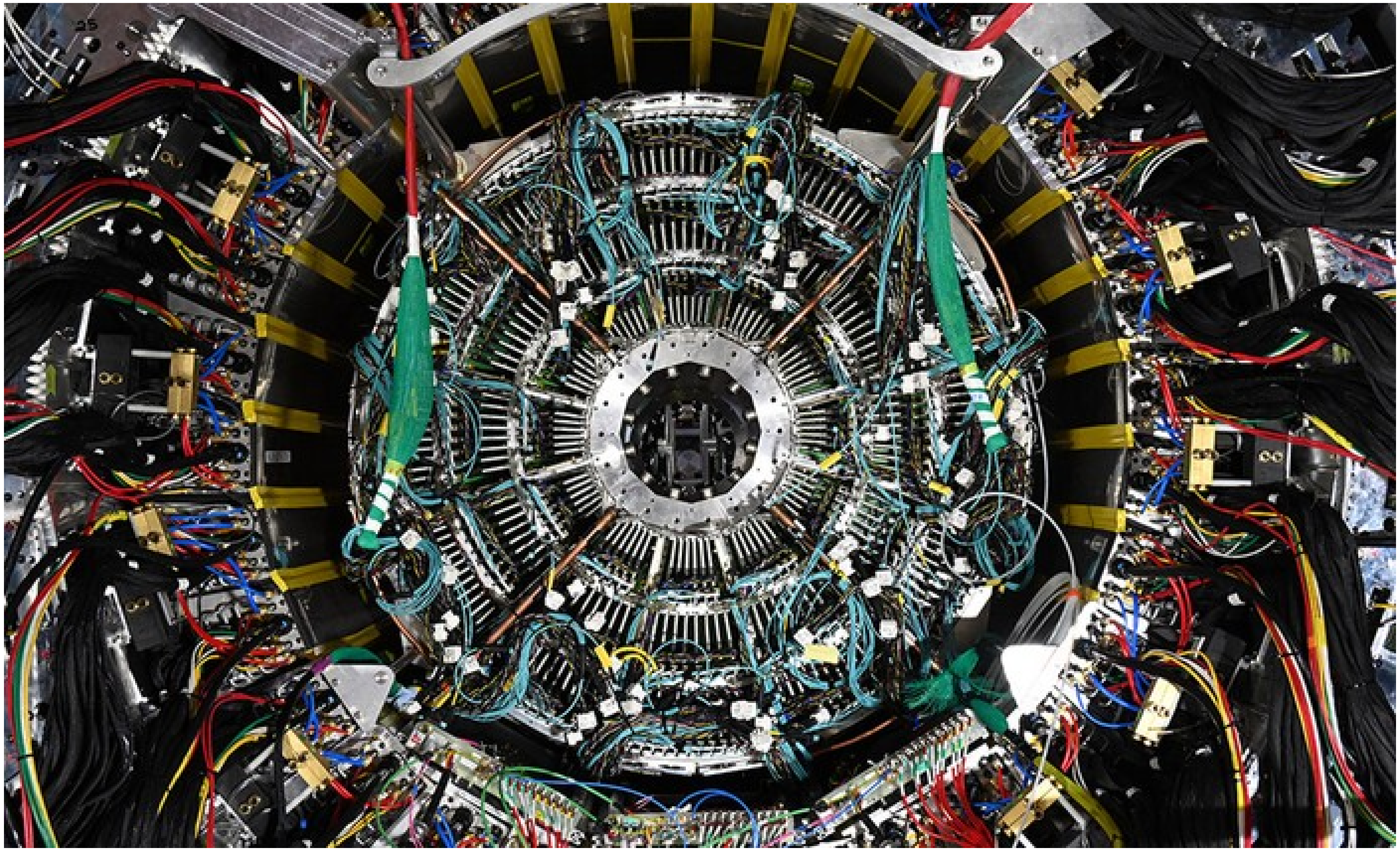
Golden oldie!

2003 result from STAR

PRL 91 (2003) 072304



20 years later… sPHENIX !



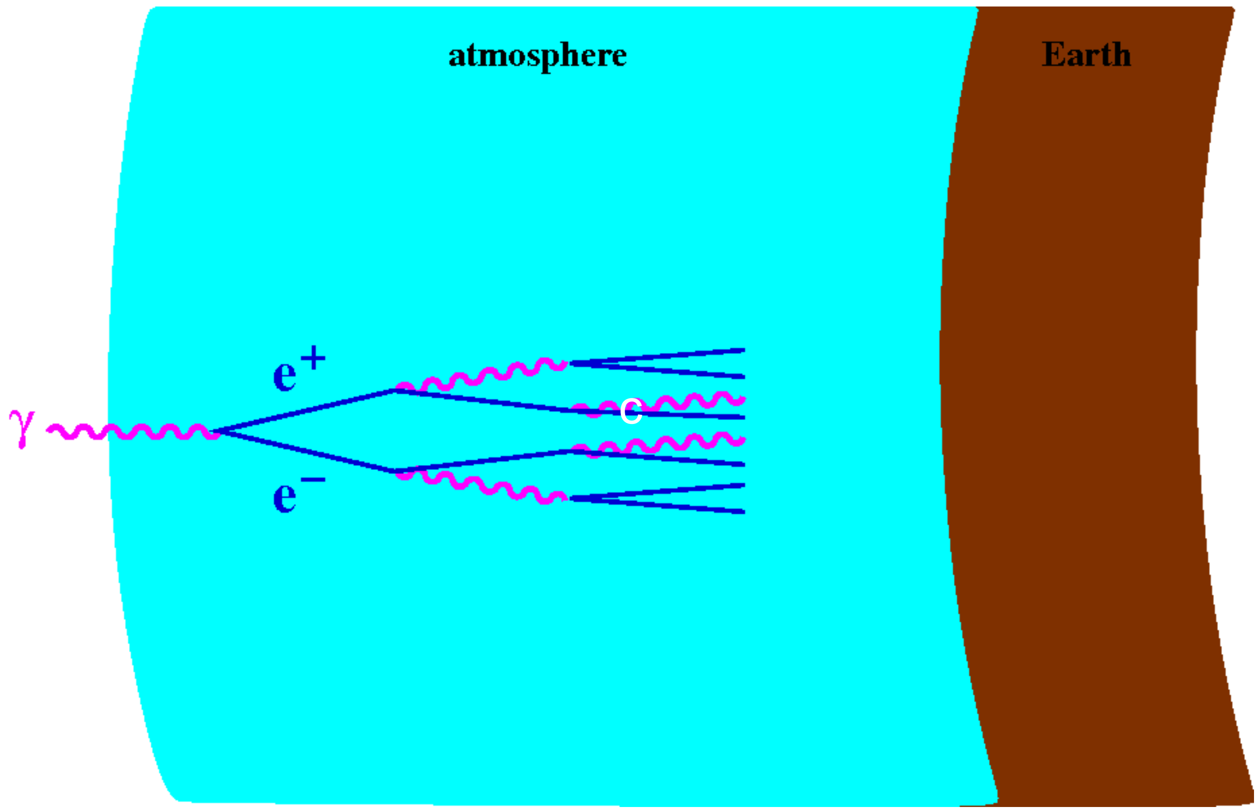
[source: BNL Newsroom]

The story begins with

The LPM Effect

(Landau, Pomeranchuk, Migdal)

Think about QED showers in a medium, e.g.



[Oversimplification: Only electromagnetic shower shown.]

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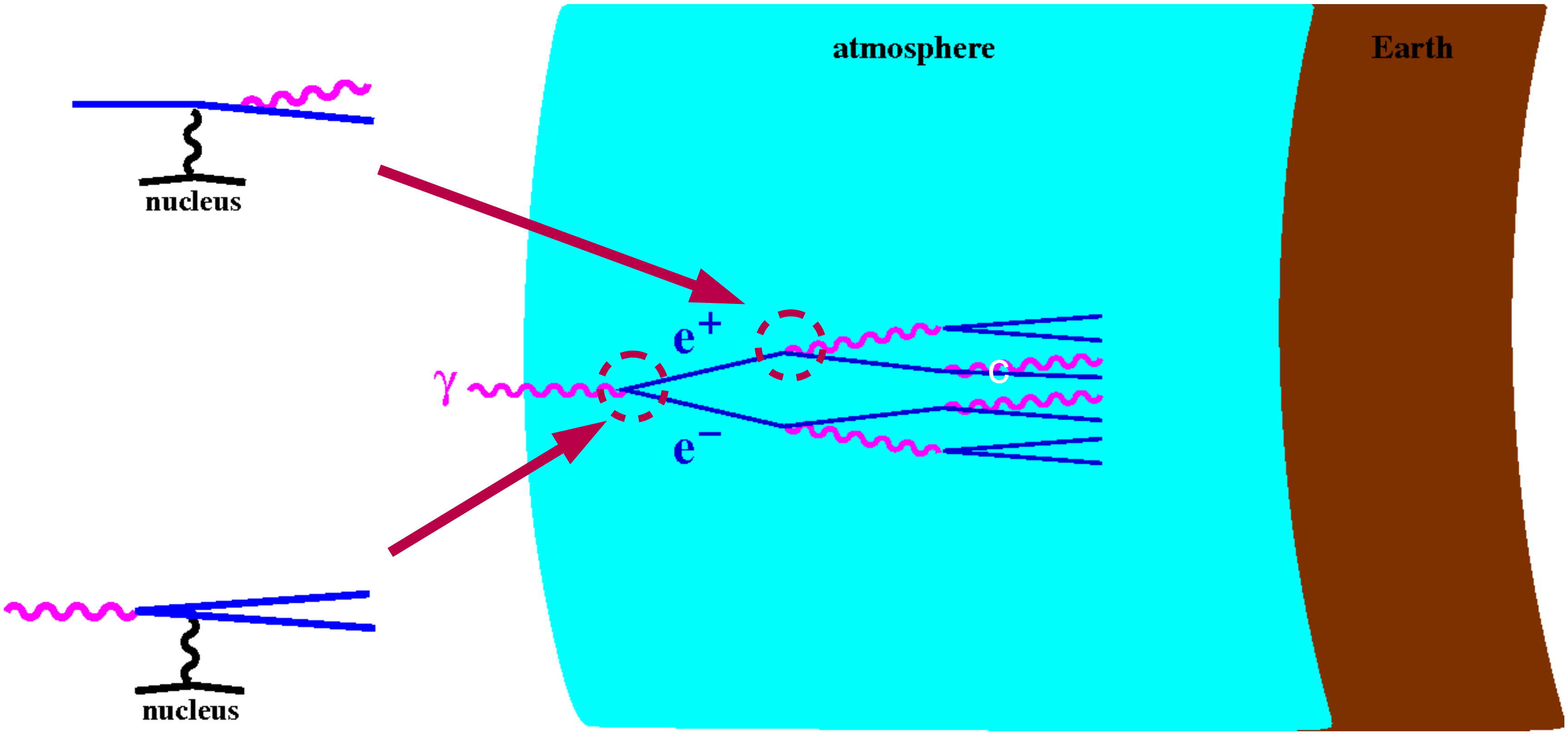


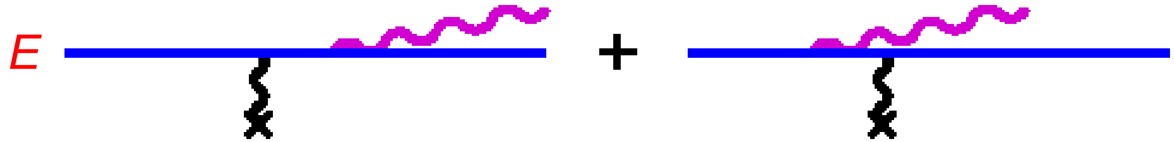
[Oversimplification: Only electromagnetic shower shown.]

A subtlety arises in the rate for those splitting processes!

Hard bremsstrahlung rate

(LPM effect is similar for pair production but harder to motivate with hand-waving.)

Naively, bremsstrahlung involves computing



Prob. of brem ~ α per collision with medium
(up to logs)

Hard bremsstrahlung rate

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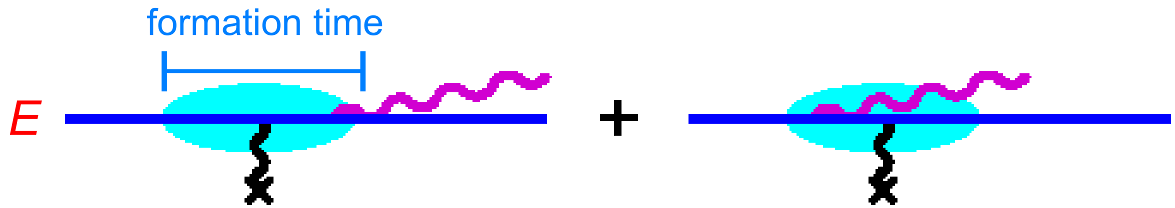

Formation time means quantum duration of splitting process.

Formation time grows with energy *E*.

Prob. of brem ~ α per collision with medium
(up to logs)

Hard bremsstrahlung rate

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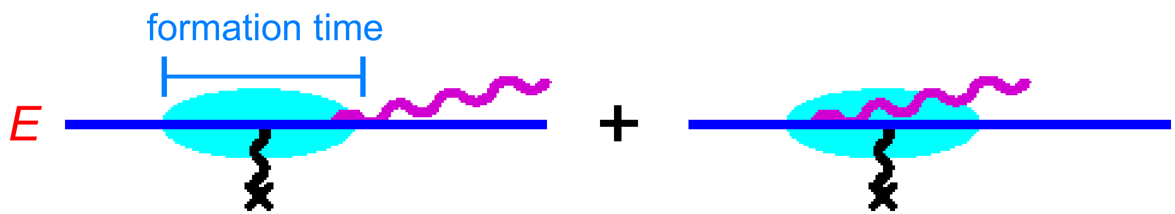
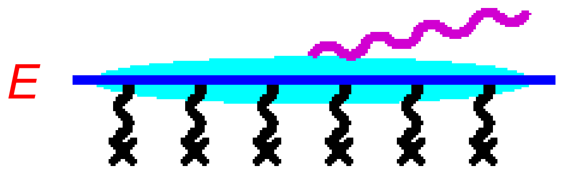


Formation time means quantum duration of splitting process.

Formation time grows with energy E.

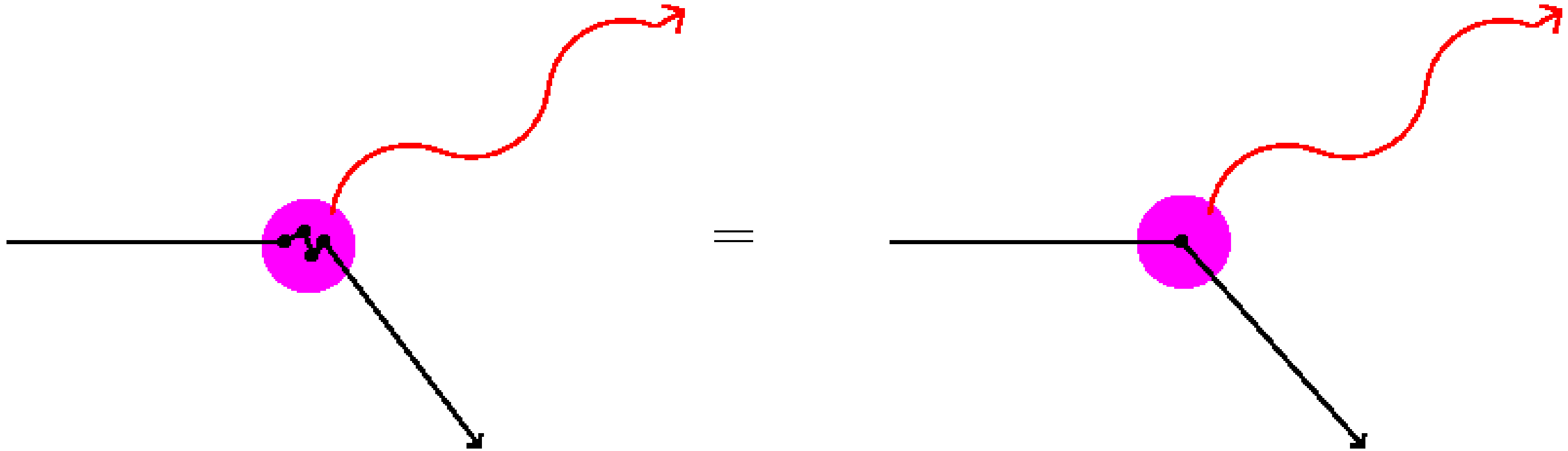
Landau and Pomeranchuk wondered:

What happens when formation time $\gg$ mean free time between collisions w/ medium?



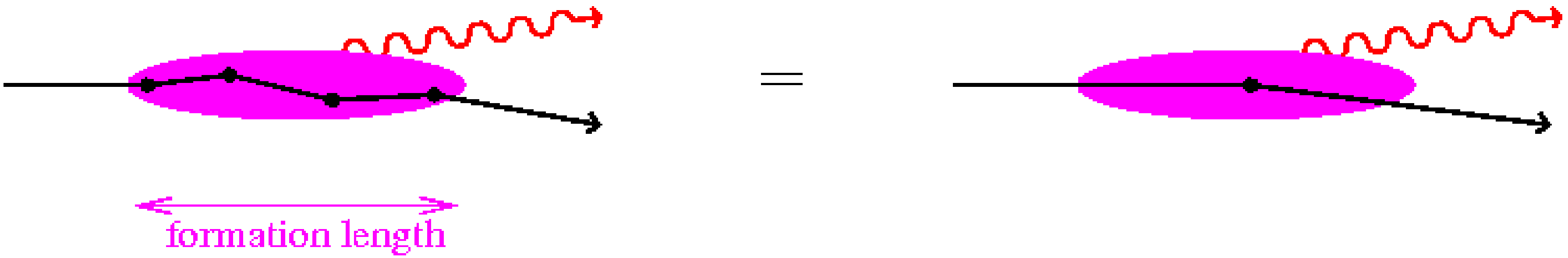
Why does formation time grow with energy?

Warm-up: Recall that light cannot resolve details smaller than its wavelength.



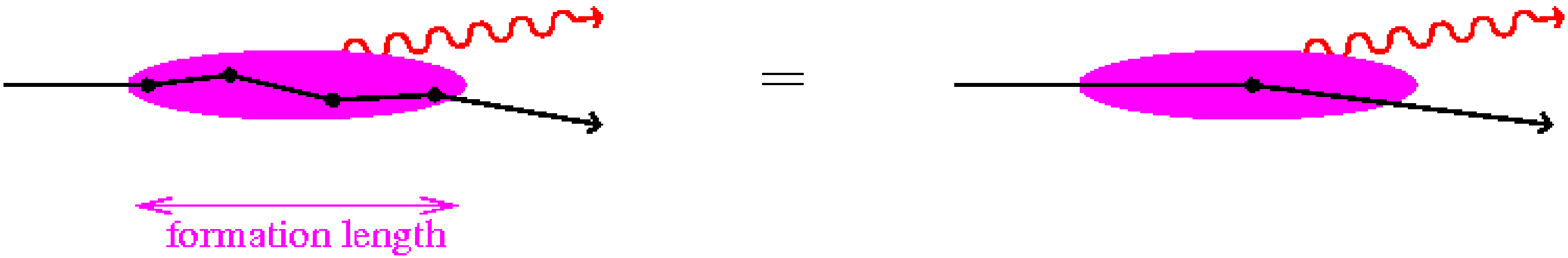
Now: Just Lorentz boost above picture by a lot!

Why does formation time grow with energy?



(1) **bigger *E*** requires bigger boost → more time dilation → **longer formation length**
(2) big boost → this process is **very collinear.**

Why does formation time grow with energy?



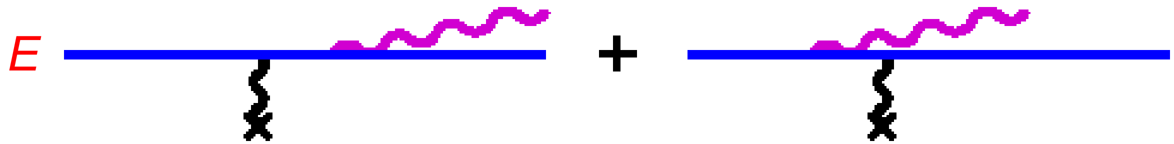
(1) **bigger *E*** requires bigger boost → more time dilation → **longer formation length**
(2) big boost → this process is **very collinear.**

This argument can also be run backward:
Any physics which makes splitting **less collinear** → **shorter formation length**.

Take-away: multiple elastic collisions within a formation time do **not** provide additional chances for bremsstrahlung.

Consequence

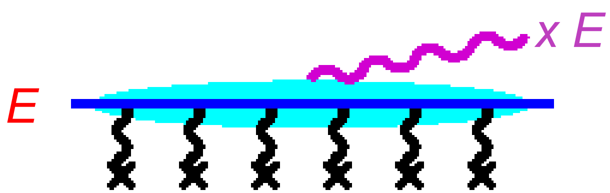
Naively:



Prob. of brem ~ α per collision

LPM Effect:

What happens when formation time $\gg$ mean free time between collisions w/ medium?



Prob. of brem ~ α per formation time

QED (1950s): LPM [Landau-Pomeranchuk & Migdal]
QCD (1990s): BDMPS-Z + many later variations
[BDMPS=Baier,Dokshitzer,Mueller,Schiff; Z=Zakharov]

⇒ calculation of splitting rates $\dfrac{d\Gamma^{\rm split}}{dx}$

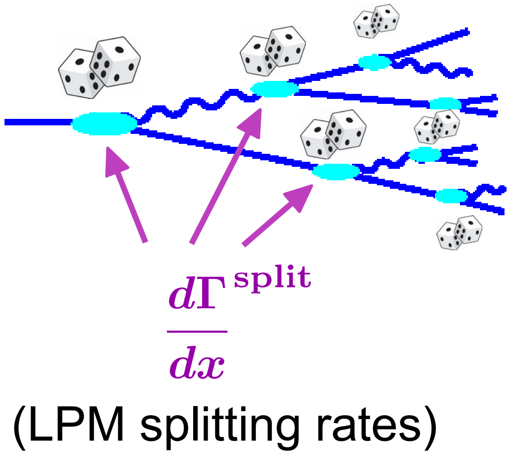
QED LPM effect well tested with thin foil target by SLAC E-146 (1995)

---

various QCD variations/specializations/generalizations/alternatives include
ASW=Armesto,Salgado,Wiedemann; GLV=Gyulassy,Levai,Vitaev;
AMY=Arnold,Moore,Yaffe; HT = HIgher Twist approach (Wang, Guo + Majumder)

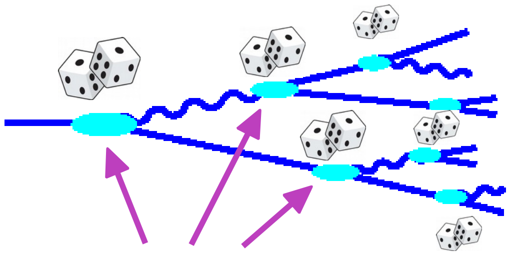
A new concern

Can we then describe in-medium shower development by



?

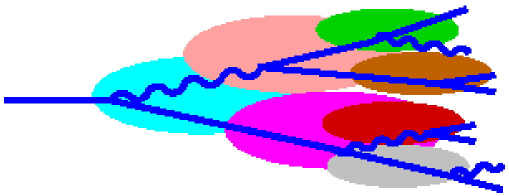
Or can splittings overlap?



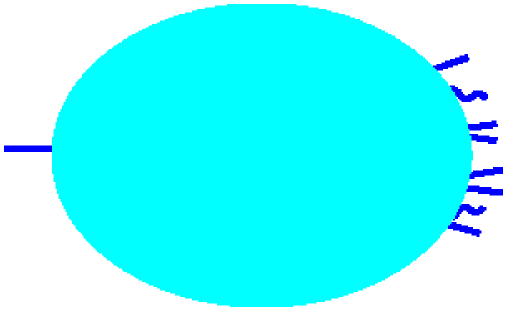
$$\frac{d\Gamma^{\text{split}}}{dx}$$

(LPM splitting rates)

vs.



vs.



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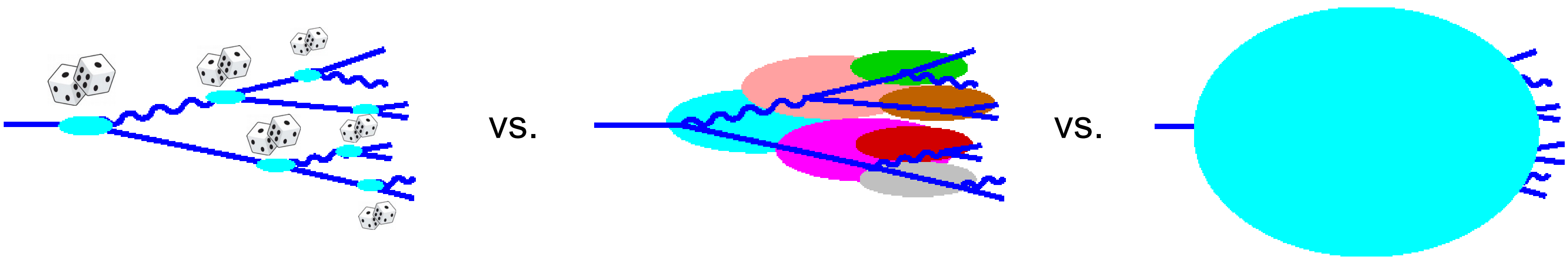
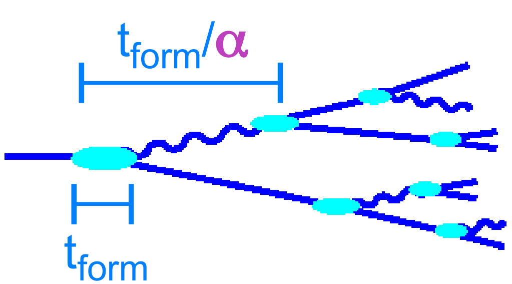


Prob. of brem ~ α per formation time

→ Prob. two consecutive splittings overlap ~ α

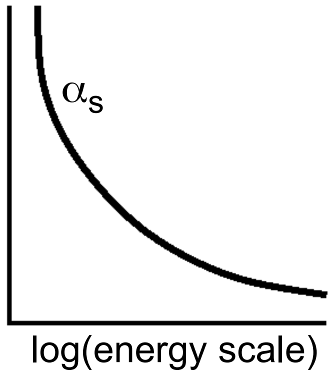
All depends on how big α is!

For small α, there is a hierarchy of scales that (typically) separates the splittings:



How big is α_s?

Answer depends on scale:



and also on context:

- Higgs production (relevant scale for coupling ~ 125 GeV)

  Perturbation theory works great! (provided you factorize out parton PDFs)

- QGP properties at the *unacheivably*(!) large temperature T = 125 GeV

  Convergence of a straight-up small coupling expansion more or less sucks

  … and it's an expansion in $\alpha^{1/2}$ instead of α.

  e.g. free energy = T^4 (# + #α + #$\alpha^{3/2}$ + …)

How big is α_s?

And for QGP properties at achievable temperatures:

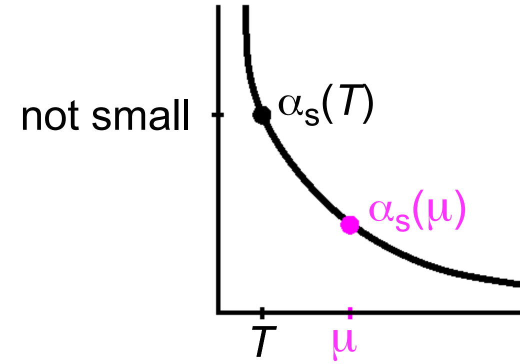
RHIC Scientists Serve Up 'Perfect' Liquid

New state of matter more remarkable than predicted — raising many new questions

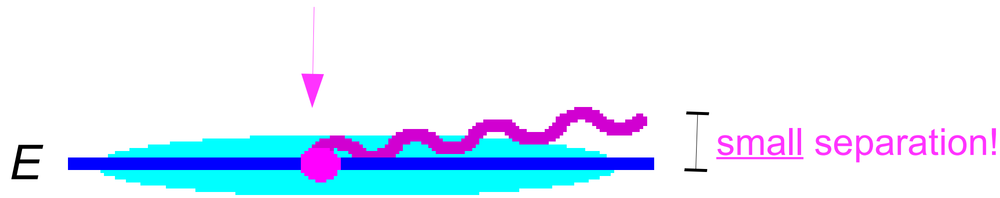
April 18, 2005

TAMPA, FL — The four detector groups conducting research at the Relativistic Heavy Ion Collider (RHIC) — a giant atom "smasher" located at the U.S. Department of Energy's Brookhaven National Laboratory — say they've created a new state of hot, dense matter out of the quarks and gluons that are the basic particles of atomic nuclei, but it is a state quite different and even more remarkable than had been predicted. In peer-reviewed papers summarizing the first three years of RHIC findings, the scientists say that instead of behaving like a gas of free quarks and gluons, as was expected, the matter created in RHIC's heavy ion collisions appears to be more like a *liquid*.

How big is α_s?



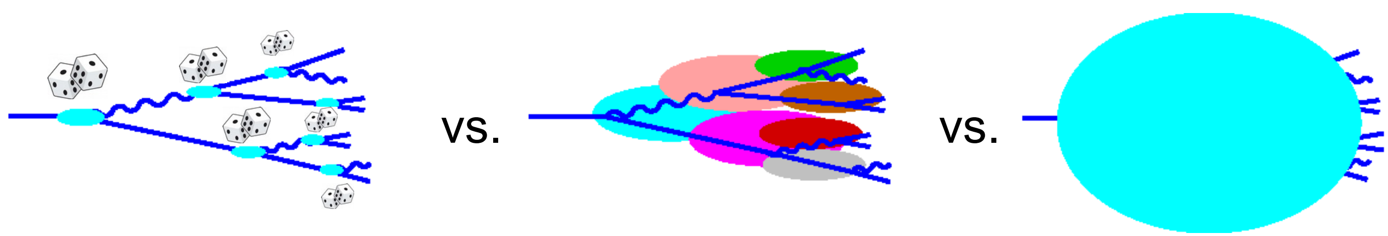
Scale dependence: $\alpha_s(T)$ or $\alpha_s(E)$ or $\alpha_s(?)$



In the situation I will consider later, it's

Prob. of brem ~ $\alpha_s(\mu)$ per formation time with $\boldsymbol{\mu \propto E^{1/4}}$

That is the α_s that will determine



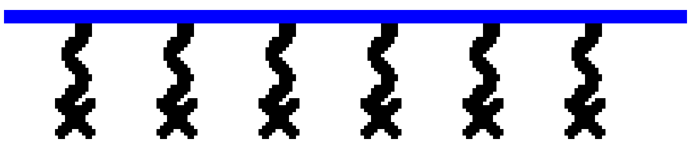
Context: How good is the small coupling expansion for a given size of $\alpha_s(\mu)$?

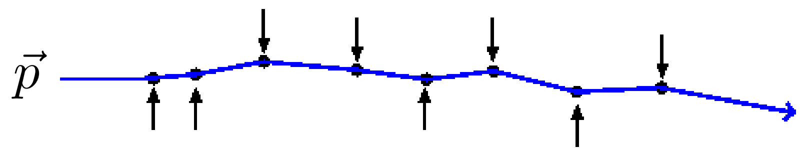
- I should do an overlapping formation-time calculation to find out!
- Also, can I "factorize out" the complicated $\alpha_s(T)$ physics of the QGP? →

What do we need from the QGP?

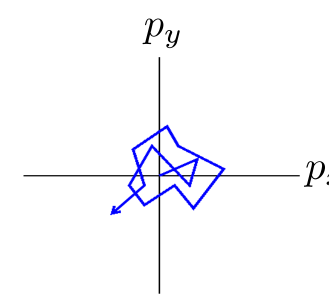
Bremsstrahlung arises b/c high-energy partons deflected by small random kicks from the medium.

Start with a cartoon for a weakly-coupled plasma:





Random kicks from medium change p_T by tiny amounts << *E*
→ Random walk in transverse momentum plane:
→ some prob. distribution P(p_T) for *net* kick during formation time.



For a large enough number of kicks, P(p_T) will become (with important caveats) a Gaussian distribution, characterized completely by a single number: its width

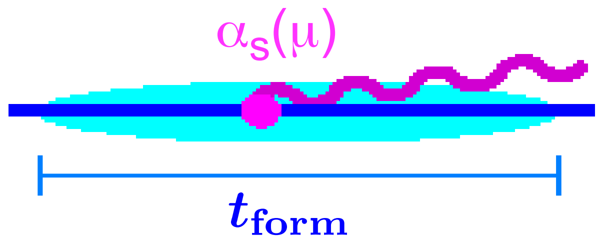
$$(p_\perp)_{\rm rms} \propto \sqrt{N_{\rm kicks}} \propto \sqrt{\text{distance travelled}} \quad \rightarrow \quad \langle p_\perp^2 \rangle = \hat{q} \times (\text{distance travelled})$$

$\hat{q}$ defined as this proportionality constant

A strongly-coupled plasma:

Same argument works as long as formation time >> correlation length in plasma ~ 1/T

Scales for E

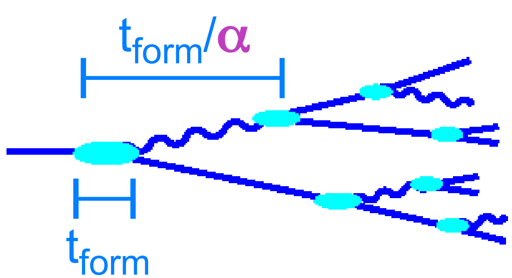


(ignoring possible overlapping splittings)

for medium-induced hard bremsstrahlung

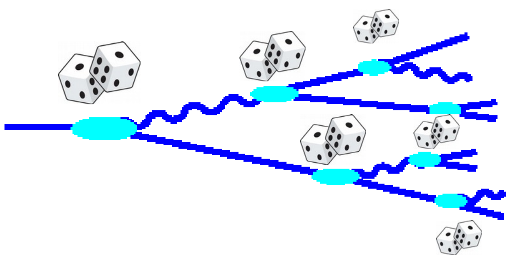
$$t_{\text{form}} \sim \sqrt{\frac{E}{\hat{q}}}$$

$$\mu \sim (\hat{q}E)^{1/4}$$



typical distance between hard splittings $\sim \dfrac{1}{\alpha_s(\mu)}\sqrt{\dfrac{E}{\hat{q}}}$

Summary so far



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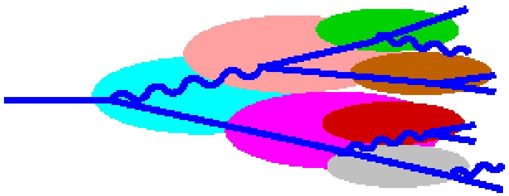

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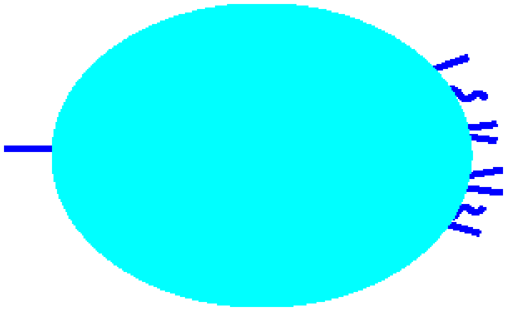


$\alpha_s(\mu)$ **small**

a “standard” picture
of a shower

$\alpha_s(\mu)$ **big**

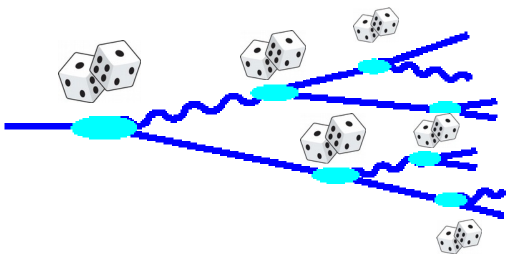
HELP!

Turn to AdS/CFT for
qualitative insight?

The stakes

Should we believe anyone using Feynman diagrams to describe medium-induced showering?

And how do we tell if



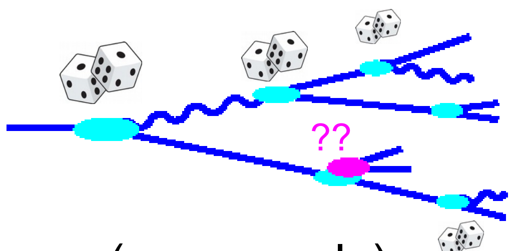
is a good or bad picture for reasonable values of $\alpha_s(\mu)$?

Two approaches

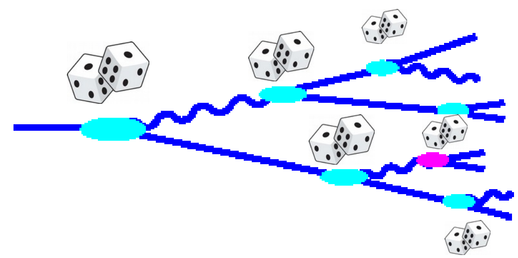
(1) EXTERNAL VALIDATION: Confront w/ experiment.
But…. many confounding factors.

(2) INTERNAL CONSISTENCY: Test with theory!

Question:
Are the first corrections



to



(an example)

small for reasonable values of $\alpha_s(\mu)$?

Perks for theorists:

- May avoid confounding factors by testing in simplified situations.
- Can test on simple shower characteristics not accessible to experiment.

So…

A theorist thought experiment

work with

Shahin
Iqbal

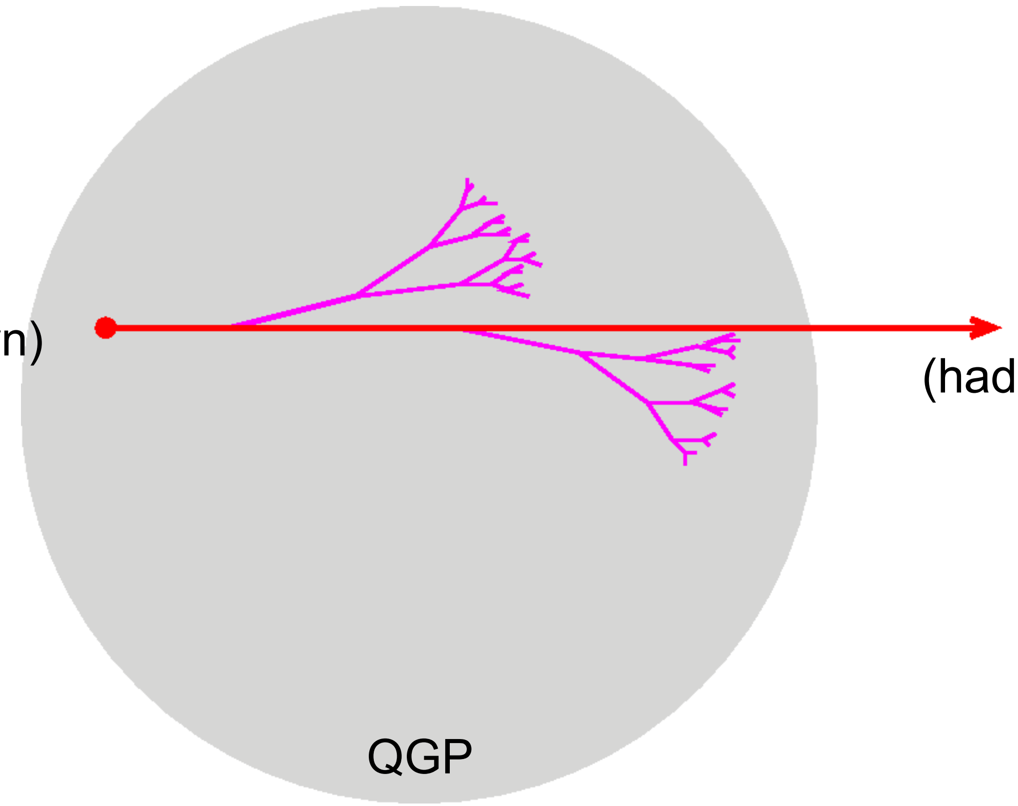
Omar
Elgedawy

Simplifying assumptions

- A static, homogeneous, “infinite”-size QGP

  “infinite” will mean so large that the shower deposits ***all*** its energy in the medium

For the purpose of this discussion, think not of



but instead just ...

Cascades that stop in-medium

A theorist thought experiment

work with

Shahin Iqbal

Omar Elgedawy

Simplifying assumptions

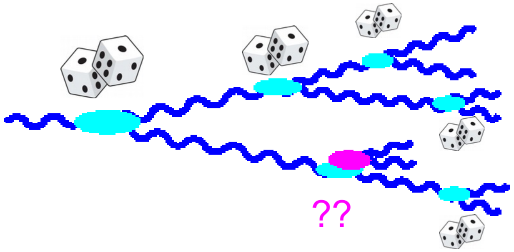
- A static, homogeneous, “infinite”-size QGP

  “infinite” will mean so large that the shower deposits *all* its energy in the medium

- Start with a parton that is (approx.) on-shell.

- Study gluon-initiated showers in large-N_c limit (w/ N_f fixed)

⇨ Only g→gg splittings consider (so far!)



A theorist thought experiment

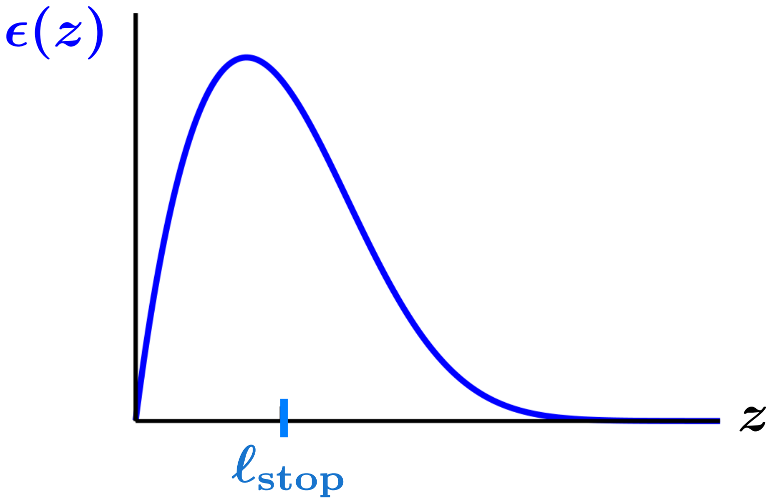
work with

Shahin Iqbal

Omar Elgedawy

Something theorists could "observe":

(statistically averaged) distribution of energy deposited by shower as a function of distance *z*



$\ell_{\rm stop} \equiv \langle z \rangle$ (1st moment of energy deposition distribution)

$$\ell_{\rm stop} \sim \frac{t_{\rm form}}{\alpha} \sim \frac{1}{\alpha}\sqrt{\frac{E}{\hat{q}}}$$

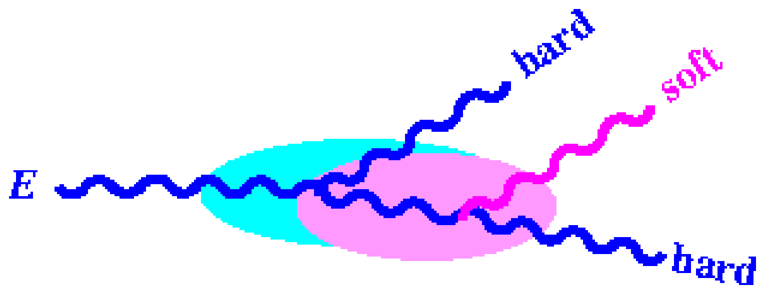
Note: $\ell_{\rm stop}$ depends on $\hat{q}$

How big are the overlap corrections to ε(z)?

Answer: **BIG !** … which has been know since Blaizot and Mehtar-Tani (2014)
Iancu (2014)
Wu (2014)

[building on radiative corrections to $\hat{q}$ found by Liou, Mueller, Wu (2013)]

(1) BIG because there is a double-log enhancement coming from SOFT radiation:

Prob. of overlap 

suppressed by (in my application)

$$\alpha_s(\mu)\,\ln^2\!\left(\frac{E}{T}\right)$$ ⇒ BIG result for large *E*

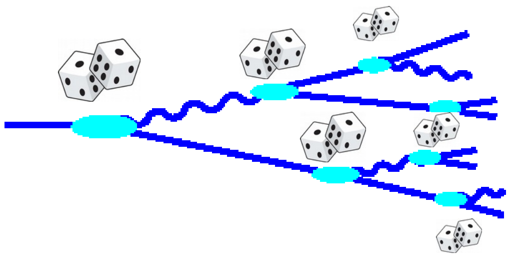
(2) But these BIG soft-radiation effects can be absorbed into an effective value of $\hat{q}$:

$$\hat{q} \longrightarrow \hat{q}_{\rm eff}(E) = \hat{q}\left[1 + \#\alpha_s \ln^2\!\left(\frac{E}{T}\right)\right]$$

Can even be re-summed at leading log to all orders in α_s

AN ASIDE

That analysis confirmed a qualitative lesson learned from gauge-gravity duality.
For N=4 supersymmetric QCD plasma:



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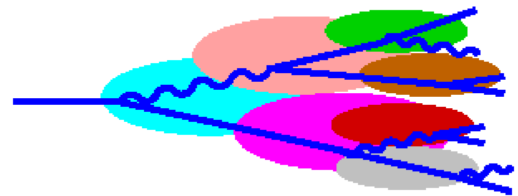

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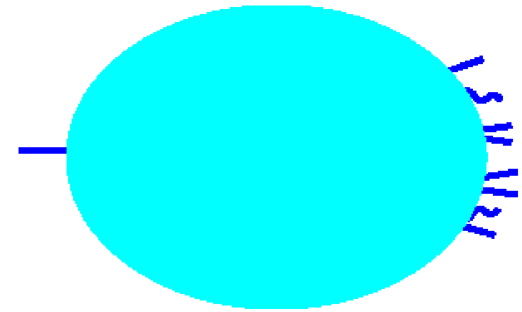

α_s small

α_s big

usual LPM/BDMPS-Z analysis

$$\ell_{\rm stop} \sim \frac{1}{\alpha}\sqrt{\frac{E}{\hat q}} \propto E^{1/2}$$

gauge-gravity duality

$$\ell_{\rm stop} \propto E^{1/3}$$

Gubser, Gulotta, Pufu, Rocha (2008)
Hatta, Iancu, Mueller (2008)
Chesler, Jensen, Karch, Yaffe (2009)

> Lesson for real QCD?: the exponent should depend on α_s

By *resumming* their large soft-emission logarithms to all orders in α_s, the previous authors obtained an explicit result for real QCD that verified this for small α_s:

$$\hat q \longrightarrow \hat q_{\rm eff}(E) \propto E^{\#\sqrt{\alpha_s}}$$

which gives

$$\ell_{\rm stop} \sim \frac{1}{\alpha}\sqrt{\frac{E}{\hat q_{\rm eff}(E)}} \propto E^{\frac{1}{2}(1-\#\sqrt{\alpha_s})}$$

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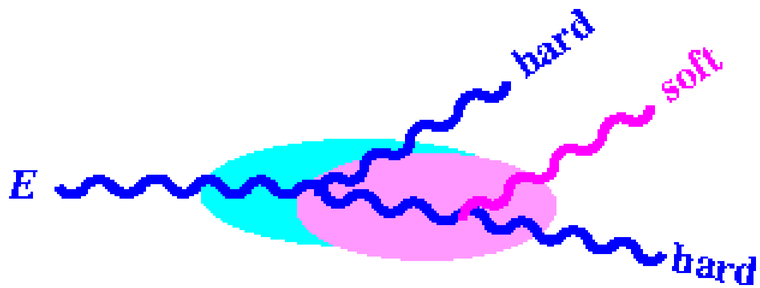

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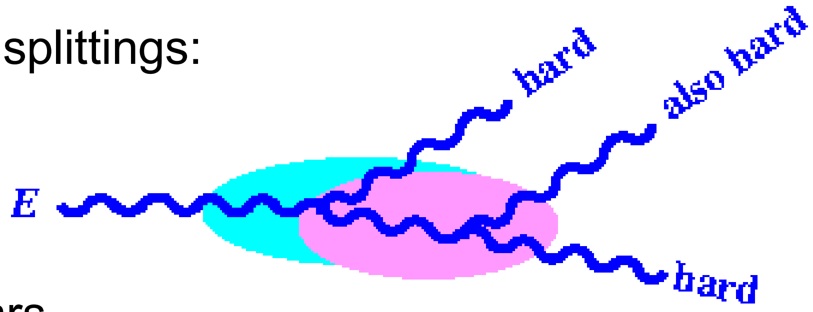
Can even be re-summed at leading log to all orders in α_s

A REFINED QUESTION

How big are overlap effects that cannot be absorbed in $\hat{q}$?

(1) Need to calculate overlap of two hard splittings:



Extremely difficult calculation.

After lots of QFT and many (!!) years ...

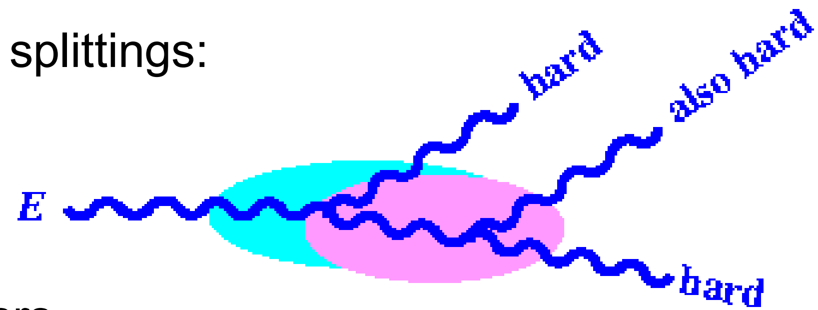
Completed (for gluons) in 2022 with S. Iqbal and

Tyler Gorda

A REFINED QUESTION

How big are overlap effects that <u>cannot</u> be absorbed in $\hat{q}$?

(1) Need to calculate overlap of two <u>hard</u> splittings:



Extremely difficult calculation.

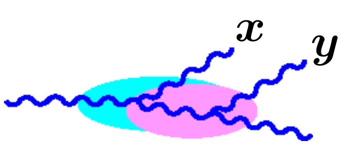
After lots of QFT and many (!!) years ...

Completed (for gluons) in 2022 with S. Iqbal and

Tyler Gorda

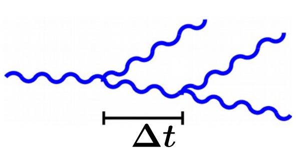
<u>Technical note</u>

The drawing above is short-hand for what we call



$\Delta \dfrac{d\Gamma}{dx\,dy} \equiv$ the overlap <u>correction</u> to two independent splittings

$$= \left[\begin{array}{c} \text{full calculation of double splitting rate} \\ \left\langle \left| \int_0^\infty d(\Delta t) \right.\right. \end{array} \right.$$



$$\left. \left. + \cdots \right|^2 \right\rangle_{\substack{\text{medium}\\\text{avg}}} \Bigg] - \left[\begin{array}{c} \text{pretending the two splittings} \\ \text{are independent dice roles} \\ \dfrac{d\Gamma^{\text{split}}}{dx} \text{ and } \dfrac{d\Gamma^{\text{split}}}{dy} \end{array} \right]$$

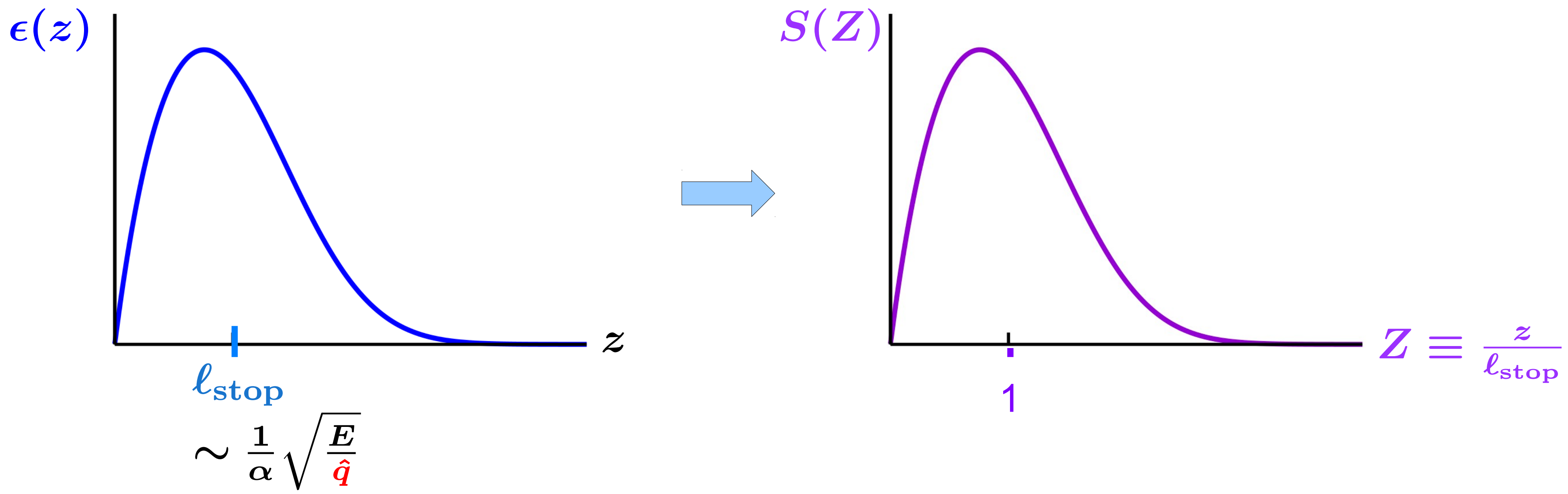
which cancels except for contributions from splittings separated by $\Delta t \lesssim t_{\text{form}}$

A REFINED QUESTION

How big are overlap effects that cannot be absorbed in $\hat{q}$?

(2) Choose a theorist observable that is insensitive to $\hat{q}$:

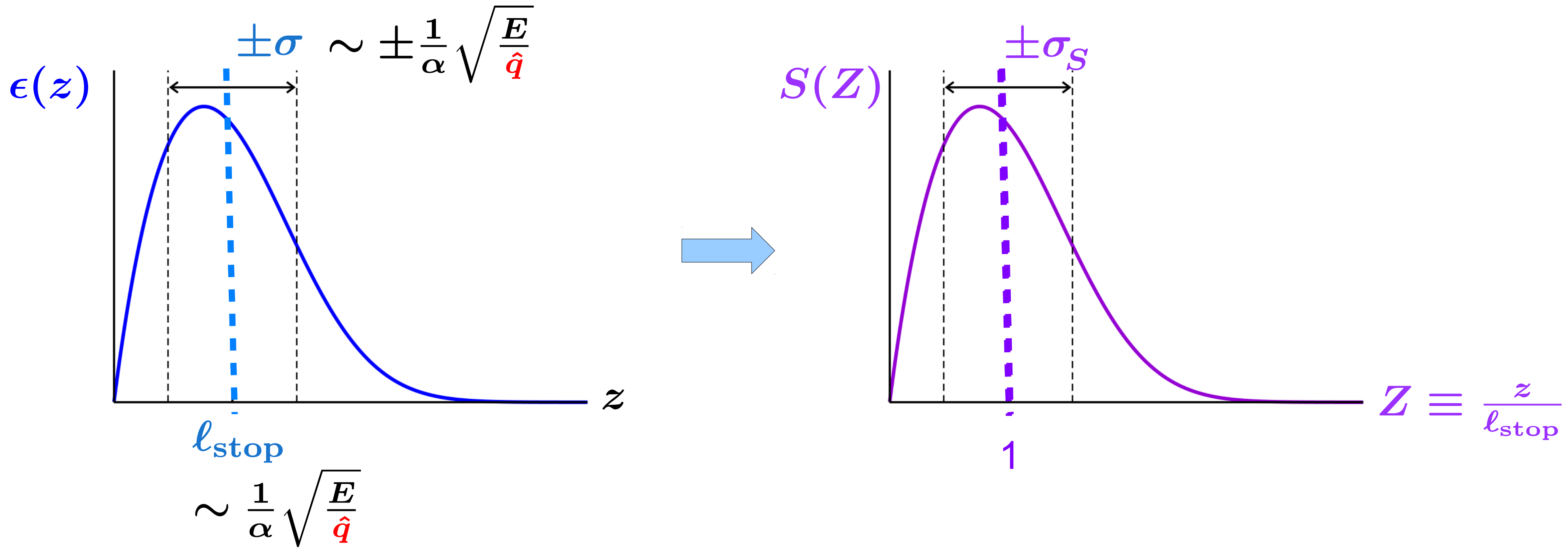
consider the shape $S(Z)$ of the energy deposition distribution:



A REFINED QUESTION

How big are overlap effects that <u>cannot</u> be absorbed in $\hat{q}$?

<u>Example</u>

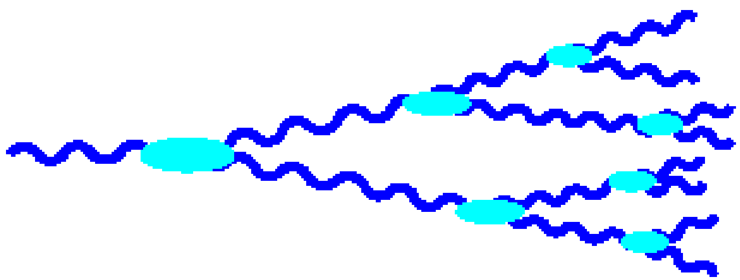


$\sigma_S = \frac{\sigma}{\ell_{\rm stop}}$ is independent of $\hat{q}$ *

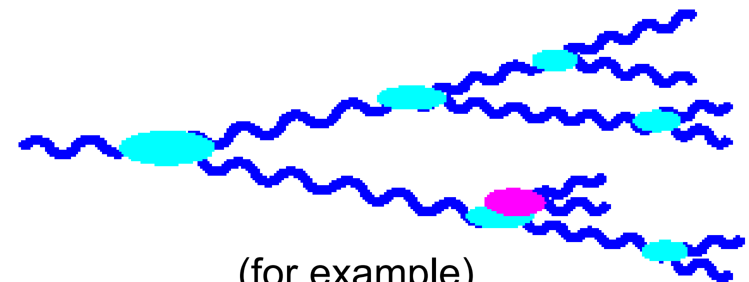
* Important, interesting, and resolvable caveats that I may not have time to explain.

How to account for overlaps in showers

Think of

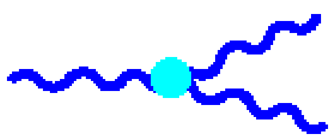


+



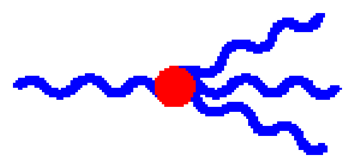
(for example)

as “standard” shower development with independent splittings but two types of localized, independent vertices:



1→2 (normal LPM)

$$\frac{d\Gamma^{\rm split}}{dx}$$



1→3 (overlap correction)

$$\Delta\frac{d\Gamma}{dx\,dy}$$

Then treat these “splitting” probabilities as purely classical.

RESULTS

To start: the width of the shape *S*(*Z*) of energy deposition

Large-N_f QED [2018 w/ S. Iqbal]:

"LO" means "ignoring overlaps"

charge deposition

$$\sigma_S = \frac{\sigma}{\ell_{\text{stop}}} = \left(\frac{\sigma}{\ell_{\text{stop}}}\right)_{\text{LO}} \left[1 - 0.87\, N_f \alpha(\mu)\right]$$

Large-N_c QCD (gluons only) [2022 w/ S. Iqbal and O. Elgedawy]:

energy deposition

$$\sigma_S = \frac{\sigma}{\ell_{\text{stop}}} = \left(\frac{\sigma}{\ell_{\text{stop}}}\right)_{\text{LO}} \left[1 + \text{???}\, N_c \alpha_s(\mu)\right]$$

DRUM ROLL PLEASE

RESULTS

To start: the width of the shape S(Z) of energy deposition

Large-N_f QED [2018 w/ S. Iqbal]:

"LO" means "ignoring overlaps"

charge deposition

$$\sigma_S = \frac{\sigma}{\ell_{\rm stop}} = \left(\frac{\sigma}{\ell_{\rm stop}}\right)_{\rm LO} \left[1 - 0.87\, N_{\rm f}\alpha(\mu)\right]$$

Large-N_c QCD (gluons only) [2022 w/ S. Iqbal and O. Elgedawy]:

energy deposition

$$\sigma_S = \frac{\sigma}{\ell_{\rm stop}} = \left(\frac{\sigma}{\ell_{\rm stop}}\right)_{\rm LO} \left[1 - 0.02\, N_{\rm c}\alpha_{\rm s}(\mu)\right]$$

RESULTS

To start: the width of the shape S(Z) of energy deposition

Large-N_f QED [2018 w/ S. Iqbal]:

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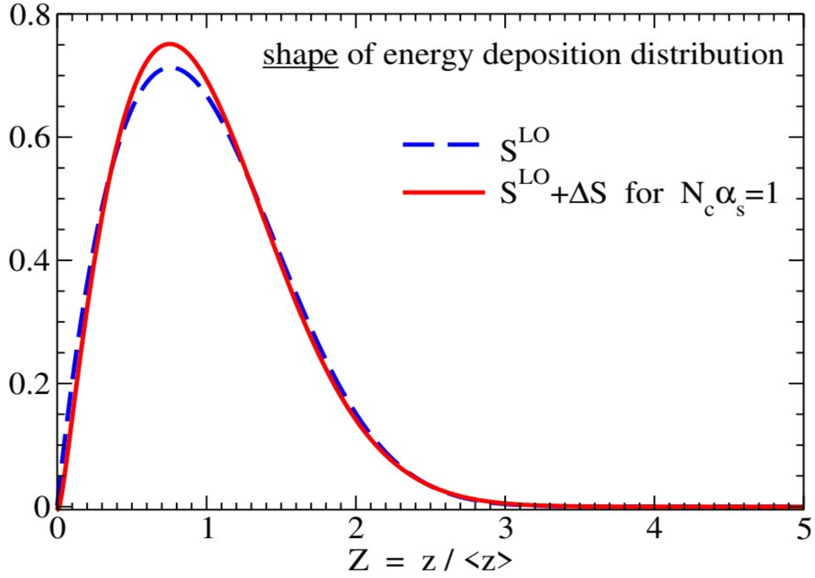

Conclusion for this test

Overlap corrections that cannot be absorbed into $\hat{q}$ are negligible.

The QED and gluon results are very different: Discuss!

Large-N_f QED $$\sigma_S = \frac{\sigma}{\ell_{\rm stop}} = \left(\frac{\sigma}{\ell_{\rm stop}}\right)_{\rm LO} \left[1 - 0.87\, N_{\rm f} \alpha(\mu)\right]$$

Large-N_c gluons $$\sigma_S = \frac{\sigma}{\ell_{\rm stop}} = \left(\frac{\sigma}{\ell_{\rm stop}}\right)_{\rm LO} \left[1 - 0.02\, N_{\rm c} \alpha_{\rm s}(\mu)\right]$$

(i) Is the minuteness of the QCD result robust? e.g.

- Could it be just an *accidental* cancellation specific to gluons and energy stopping?
- What happens if we include quarks in the calculations?
- Is there a qualitative difference between charge deposition and energy deposition?

(ii) Can we understand why the QED and QCD results are so different?

The QED and gluon results are very different: Discuss!

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Preliminary results from work in progress: (i) robust and (ii) understandable.

The QED and gluon results are very different: Discuss!

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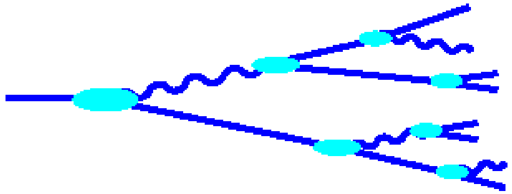
Preliminary results from work in progress: (i) robust and (ii) understandable.

- Is there something special about the large-N_c limit?
- What about non-"theory thought experiment" situations that experimentalists care about?

Working Conclusion

Weak-coupling analysis (i.e. "Feynman" diagrams) for hard medium-induced splittings



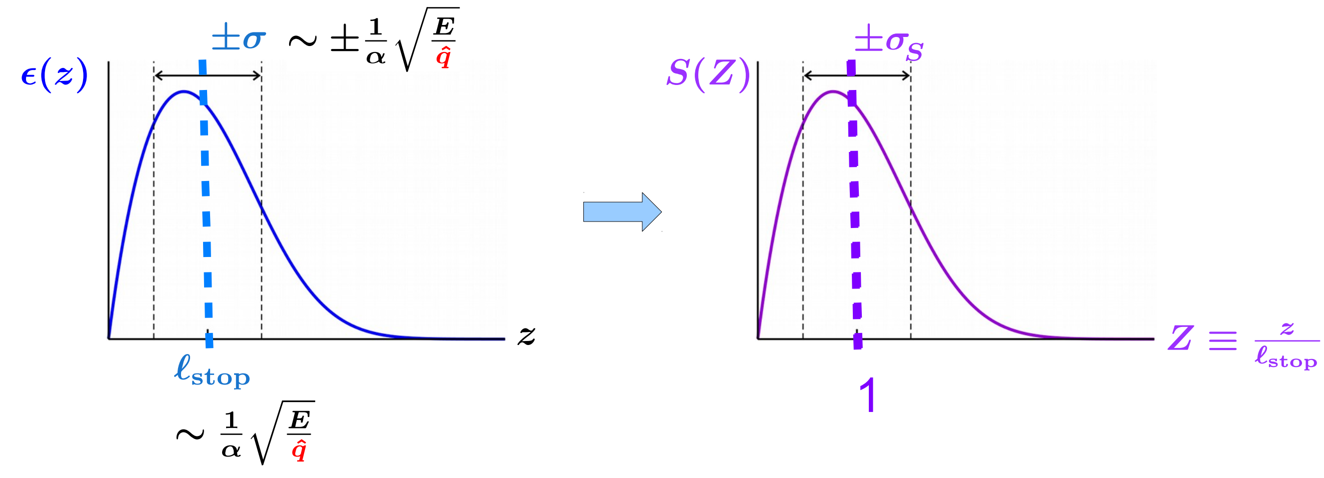
okay (at least in the situation analyzed) provided all the complicated QGP stuff is absorbed into $\hat{q}$ and then $\hat{q}_{\rm eff}(E)$ is run with energy.

If you want more info, and to find out about issues oversimplified in this presentation, see PRL 131 (2023) 162302 [arXiv:2212.08086] & PRD 108 (2023) 074015 [arXiv:2302.10215]

Shrouded from view in this presentation ...

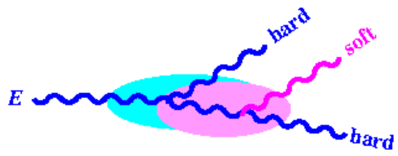
I half-lied about something

Remember



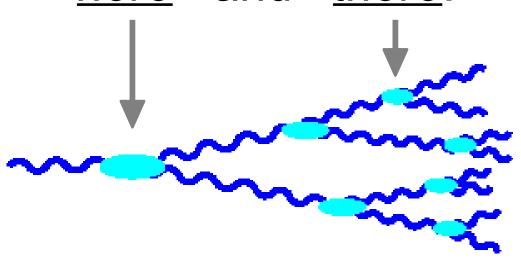
$\sigma_S = \frac{\sigma}{\ell_{\rm stop}}$ is independent of $\hat{q}$ *

and why we did that:



$$\hat{q} \longrightarrow \hat{q}_{\rm eff}(E) = \hat{q}\left[1 + \#\alpha_{\rm s} \ln^2\left(\frac{E}{T}\right)\right]$$

But then $\hat{q}_{\rm eff}(E)$ is different *here* and *there*.



Those difference don't quite cancel in $\sigma_S = \sigma/\ell_{\rm stop}$ and $S(Z)$.

They cancel at leading log but leave behind BIG single-log corrections to σ_S and $S(Z)$:

overlap corrections $\sim \alpha_{\rm s}(\mu) \ln\left(\frac{E}{T}\right)$

Factorization

Remember that soft radiation can be absorbed into $\hat{q}$.

When factorizing away some IR or UV physics in QFT, we must introduce a *factorization scale* to do NLO calculations.

Examples

UV divergences absorbed into couplings: renormalization scale μ

Collinear divergences absorbed into PDFs: factorization scale $M_{\rm fac}$

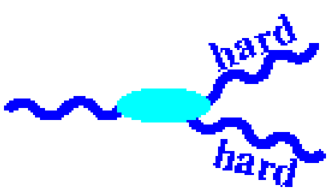
Such factorization scales appear explicitly inside logarithms in NLO results.

- Set them to the appropriate physics scale for the process.
- Check sensitivity to the precise choice of scale.

Our problem

To factorize *all* the soft radiation effects into $\hat{q}_{\rm eff}$,
we introduce an energy factorization scale

$$\Lambda_{\bf fac} = \# \left(\text{min energy of daughters of} \right)$$



where # = any reasonable O(1) number.

The overlap result shown earlier was the result for # = 1.

Now showing dependence on the normalization # of the factorization scale:

$$\boldsymbol{\sigma_S} = \frac{\boldsymbol{\sigma}}{\ell_{\mathbf{stop}}} = \left(\frac{\boldsymbol{\sigma}}{\ell_{\mathbf{stop}}}\right)_{\mathbf{LO}} \left[\mathbf{1} - (\mathbf{0.02} + \mathbf{0.001\,ln\,\#})\boldsymbol{N_{\mathrm{c}}\alpha_{\mathrm{s}}(\mu)}\right]$$

Extremely weak dependence on factorization scale.