

2 **Reconstructing the kinematics of semi-inclusive deep** 3 **inelastic scattering at the EIC utilizing the hadronic final** 4 **state and machine learning**

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8 **ABSTRACT:** Semi-inclusive deep inelastic scattering, $e(k) + P(p) \rightarrow e'(k') + h(P_h) + X$, is critical
9 to mapping the three-dimensional momentum structure of the nucleon and is a major focus of
10 the Electron-Ion Collider experimental project [1]. The Electron-Ion Collider will feature high
11 luminosity, precise charged hadron tracking and particle identification, and polarized electron and
12 ion beams providing the ability to study many semi-inclusive deep inelastic scattering observables
13 across the wide kinematic space available at an e-A collider. In this study, full simulations of the
14 EIC project detector are used to demonstrate methods utilizing the hadronic final state (particles
15 produced from hadronization of struck quark) to determine the four-momentum of the exchanged
16 virtual photon in deep-inelastic scattering events, around which the relevant angles and transverse
17 momenta of semi-inclusive DIS are defined. These approaches to reconstruction include a full
18 summation of the hadronic final state, extending on methods developed at HERA for inclusive-DIS
19 kinematic reconstruction, as well as the first machine learning approach to semi-inclusive DIS
20 kinematic reconstruction. The performance of these methods are compared to the reconstruction
21 using only the scattered lepton, with improved kinematic resolution demonstrated in much of the
22 kinematic space at the Electron-Ion Collider.

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33 1 Introduction

Deep inelastic scattering (DIS) is used to probe the partonic substructure of nucleons and is a foundation of the experimental program at the future Electron-Ion Collider (EIC) [1]. In this electroweak process, a lepton scatters with a single parton inside the nucleon through the exchange of a virtual photon or W/Z-boson. The differential cross-section of inclusive-DIS, in which the scattered lepton is measured, is proportional to Parton Distribution Functions (PDFs), giving the probability of finding a parton with fractional momentum x . The point-like lepton probe makes this a clean process, with inclusive-DIS, $e(k) + P(p) \rightarrow e'(k') + X$, kinematics defined as

$$\begin{aligned} q &= k - k', \quad Q^2 = -q^2, \\ x &= \frac{Q^2}{2P \cdot q}, \quad y = \frac{P \cdot q}{P \cdot k}, \end{aligned} \tag{1.1}$$

34 where q is the four-momentum of the exchanged virtual photon. Q^2 is interpreted as the resolution
35 of the probe, and x is the fraction of the longitudinal momentum of the nucleon carried by the struck
36 parton. y can be interpreted as the "inelasticity" of the hard scattering. With $s = (P + k)^2$, these
37 quantities can be related by $Q^2 = xys$.

38 Final state hadrons resulting from the hadronization of the struck parton can be measured in
39 addition to the scattered lepton, $e(k) + P(p) \rightarrow e'(k') + h(P_h) + X$, known as semi-inclusive
40 DIS (SIDIS). The cross-section is then proportional to combinations of PDFs with fragmentation
41 functions (FFs) [2], with FFs describing the probability of a struck parton forming a specific
42 hadron [3]. The additional scale provided by the transverse momentum of the hadron with respect
43 to the virtual photon provides sensitivity to the transverse momentum dependence of PDFs and
44 FFs [2], making semi-inclusive measurements a valuable tool for probing the three-dimensional

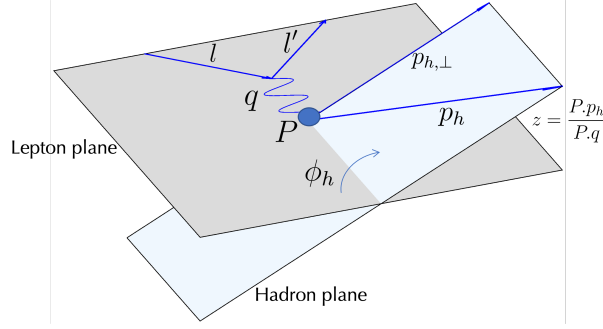


Figure 1. SIDIS kinematics in nucleon target rest frame.

45 structure of nucleons in momentum space. Additionally, measurements of the semi-inclusive
 46 production of different hadrons such as pions and kaons can give access to the flavor makeup of sea
 47 quarks in the nucleon [4]. SIDIS spin-asymmetry measurements at experiments such as HERMES
 48 [5], COMPASS [6], and CLAS [7] have provided critical experimental information on the spin
 49 structure of the nucleon, and SIDIS measurements have been historically carried out at the first e-p
 50 collider HERA [8, 9].

51 **1.1 Semi-inclusive DIS kinematics**

52 The mapping of TMD-PDFs and TMD-FFs through SIDIS measurements requires a precise deter-
 53 mination of both the inclusive DIS kinematics as well as kinematic variables related to the selected
 54 final state hadron. These kinematics are defined in the rest frame of the nucleon target [10] (figure
 55 1). In single hadron SIDIS, the transverse momentum scale of the process is defined as the mo-
 56 mentum of the hadron transverse to the virtual photon axis, $p_{h,\perp}$, which enters into both the PDFs
 57 and FFs. The relevant azimuthal angle of the final state hadron is taken between the hadron plane,
 58 spanning \vec{q} and \vec{p}_h , and the lepton plane, spanning \vec{q} and \vec{k} . These kinematic quantities are depicted
 59 in figure 1, with the convnetions defined in reference [10]. Finally, an additional scaling variable z
 60 enters into the fragmentation functions, defined as $z = P \cdot p_h / P \cdot q$, the fraction of the energy of the
 61 struck quark carried by the measured hadron.

62 **2 SIDIS reconstruction methods**

63 As the SIDIS kinematic quantities are defined around the four-momentum of the virtual photon, q
 64 must be well-constrained throughout the entire kinematic space available at the EIC. In past SIDIS
 65 studies carried out at fixed-target experiments and HERA, q is determined using only the scattered
 66 lepton, $q = k - k'$. Studies of SIDIS kinematic reconstruction in EIC simulation for the EIC yellow
 67 report [1] found the kinematic resolution achieved using this method to be reliable at large values of
 68 y , but with performance degrading rapidly for values of $y < 0.1$, as y is proportional to the energy
 69 loss of the lepton.

70 The low- y kinematic space is critical for the measurement of a variety of SIDIS observables at
 71 the EIC [11]. Due to depolarization factors which are dependent on y , the low- y region provides the
 72 greatest sensitivity to observables with an unpolarized lepton beam, such as target spin asymme-

73 tries [11]. Additionally, the large- x and low- Q^2 region accessible at low- y provides critical overlap
 74 with the kinematic space covered by fixed-target SIDIS measurements.

75 In this study, we demonstrate two methods which utilize information from the hadronic final
 76 state (all final state particles produced from the struck parton) in addition to the scattered electron
 77 to determine the four-momentum of the virtual photon, and thus improve the resolution of SIDIS
 78 kinematics at low- y .

79 2.1 Hadronic final state methods

80 Due to conservation of momentum and energy, the hadronic final state (HFS) of the DIS interaction,
 81 or all particles in the hadronization jet resulting from the struck quark, will also contain enough
 82 information to fully constrain q . Methods to reconstruct the inclusive DIS kinematics x , Q^2 , and
 83 y using the HFS were developed for the analysis of HERA data [12]. These included methods
 84 utilizing only the HFS which are required for the analysis of charged-current DIS, such as the
 85 Jacquet-Blondel method, or hybrid HFS-scattered electron methods, such as the double angle
 86 method [12]. HERA simulation studies found these additional methods to provide advantages in
 87 precision in some regions of the kinematic space, and in some cases benefits when considering
 88 effects of QED radiation, such as the Σ -method [13]. In this section, we introduce two methods
 89 further utilizing the HFS to improve the reconstruction of SIDIS kinematics, making these methods
 90 the first use of the full HFS for kinematic reconstruction in SIDIS.

91 2.1.1 Constraining q with transverse recoil, y , and Q^2

92 As defined in equation 1.1, the virtual photon four-momentum is used to define the inclusive
 93 DIS variables x, y, Q^2 in their Lorentz invariant form. Given that the alternative methods of
 94 reconstructing DIS variables developed at HERA [12] outperform the electron method in some
 95 regions of the kinematic space, the determination of the inclusive-DIS variables can be leveraged
 96 to place constraints on the reconstruction of the virtual photon four-momentum q .

97 Using the inclusive DIS variables as computed from a hybrid HFS-electron method, such as the
 98 double-angle method [12], and with q_x, q_y taken from the transverse recoil of either the scattered
 99 electron or the hadronic final state, the remaining two components of q can be constrained from the
 100 following system of equations:

$$\left. \begin{aligned} (q_x, q_y) &= \text{HFS } \vec{p}_T \parallel \text{electron } \vec{p}_T, \\ Q^2 &= -q^2, \\ y &= \frac{P \cdot q}{P \cdot k} \end{aligned} \right\} (q_x, q_y, q_z, q_t). \quad (2.1)$$

102 In a kinematic region in which the hybrid DIS method has better overall accuracy than the
 103 electron method, this can result in an improved determination of q_z, q_E , as demonstrated by us for
 104 the first time in simulation studies for the EIC yellow report [1].

105 2.1.2 Machine learning approach

106 To produce a further optimized combination of the information from the scattered electron and HFS
 107 for the reconstruction of q , a machine learning (ML) approach was developed using the full final

108 state information of simulated DIS events. Studies have been conducted on the use of machine
 109 learning for directly reconstructing inclusive DIS kinematics in the context of e-p colliders, using
 110 simulated HERA data to demonstrate the ability of machine learning approaches to outperform
 111 traditional reconstruction methods as well as minimize the impact of radiative corrections [14, 15].
 112 Additionally, recent work utilizing Bayesian Neural Networks [16] and kinematic fitting based on
 113 Bayesian inference [17] have demonstrated methods which both outperform traditional methods of
 114 reconstruction inclusive DIS kinematics and providing event-level uncertainty quantification. Our
 115 application of machine learning to reconstructing q was first demonstrated on the full simulation
 116 developed for the ATHENA detector proposal [18, 19].

117 In this study, a neural network is trained to determine the four-momentum of the virtual photon
 118 q , rather than directly reconstructing the SIDIS kinematics. The network architecture utilized for
 119 this application is Particle Flow Networks (PFN) [20]. This architecture takes as input the features
 120 of an unordered and variable size set of particles. The features p_i of each particle are passed
 121 individually through a first set of fully connected layers Φ , after which they are summed over to
 122 form a latent space which is passed to a final set of fully connected layers F ,

$$123 \quad \text{PFN} = F\left(\sum_{i=1}^M \Phi(p_i)\right)$$

124 [20]. Global features of the event, not associated with any particular particle, can be concatenated
 125 with the latent space variables formed after summing over the outputs of Φ .

126 The values of p_x, p_y, p_z, E from the set of all HFS particles was input to the layers Φ , with
 127 the size of the HFS varying. The global features of the event were taken as the components of the
 128 virtual photon four momentum as determined using the electron method, $q = k - k'$, as well as the
 129 value of $-\log_{10}(x)$ and $\log_{10}(Q^2)$ as reconstructed from the electron, double-angle, and Σ methods
 130 (methods defined in [12]).

131 The Particle Flow Network architecture was used through the Energyflow python package
 132 [20]. Each hidden layer was followed by the 'relu' activation function, with linear final output. A
 133 mean-squared-error loss function was chosen and two models were trained: one to reconstruct the
 134 lab-frame q_x and q_y , and one to reconstruct q_z, q_E . The number of hidden layers and units per
 135 layer for each network was optimized using a grid search by selecting the parameters resulting in
 136 the minimal validation loss after 50 epochs. The network trained to reconstruct q_x and q_y had 3 Φ
 137 layers of 350 units and 3 F layers of 350 units. The network trained to reconstruct q_z and q_E had 3
 138 Φ layers of 350 units and 3 F layers of 200 units.

139 3 ePIC Simulation Dataset

140 In this analysis, the above described reconstruction methods are demonstrated again on the further
 141 developed full simulation of the ePIC detector. The ePIC detector will be the first EIC detector [21],
 142 located at interaction point-6 with data taking expected to begin in the early 2030s. ePIC is planned
 143 to utilize a 1.7T solenoidal magnet and Si MAPS tracking system, providing precise momentum
 144 reconstruction of charged particles with a minimum transverse momentum of 0.1 GeV/c. Precise
 145 electromagnetic calorimetry will be carried out with a PbWO4 backwards ECal, and an imaging

146 barrel ECal. Wide particle ID coverage will be provided by various DIRC and RICH detectors in
147 each region of the detector.

148 The dataset used in this analysis is the July 2023 full simulation of the ePIC detector. The
149 July 2023 ePIC simulation campaign contains full simulation and reconstruction of the ePIC
150 calorimetry and tracking systems. The detector geometry [22] is implemented in dd4hep [23] with
151 reconstruction carried out with EICrecon [24].

152 The events used for this analysis were neutral-current DIS events generated using pythia-8
153 [25] including beam momentum smearing and a beam crossing-angle of 25 mrad. The energy
154 configuration used in this study was an electron beam energy of 18GeV and proton beam energy of
155 275GeV. QED radiative corrections are not applied in this dataset. Standard event-level DIS cuts
156 were placed of $Q^2 > 1\text{GeV}^2$ and HFS invariant mass $W > 3\text{ GeV}$, and it was required that more than
157 one particle be reconstructed in the HFS.

158 The reconstructed particle information used for this analysis of kinematic reconstruction was
159 primarily sourced from the ePIC tracking systems. The scattered electron was determined by taking
160 the matching reconstructed charged track to the MC-truth scattered electron ID, as a more realistic
161 DIS-electron finder is still under development. The HFS four-momenta were taken from all other
162 reconstructed charged tracks with a minimum lab frame transverse momentum of 0.1 GeV/c, with
163 any calorimeter clusters not associated with a track taken as additional neutral HFS particles.

164 The two PFN models were each trained on 1.6 million events with 1 million events set aside
165 for final validation.

166 4 Kinematic reconstruction results

167 On the 1 million event validation dataset, SIDIS kinematics for positive pions were computed using
168 q as reconstructed from the electron, hybrid HFS-electron, and the ML method. The HFS-electron
169 hybrid method in this study used the transverse recoil as measured from the scattered electron and
170 the double-angle method to compute y and Q^2 . Typical cuts on the true SIDIS kinematics were
171 placed requiring $p_{h,\perp} > 0.1\text{ GeV}/c$ and $z > 0.2$.

172 In figure 2, the mean and RMS of the error or relative error in $z, p_{h,\perp}, \phi_h$ is plotted for
173 values of y_{true} , which best captures the behavior of the electron reconstruction method. At large-
174 y , the electron energy loss is larger and the electron method performs well, as expected. The
175 PFN reconstruction at large- y successfully matches the performance of the electron method for
176 reconstruction of z and $p_{h,\perp}$, with slightly worse resolution of ϕ_h .

177 For decreasing y , the electron method performance degrades rapidly for all SIDIS kinematic
178 quantities. In contrast, the methods utilizing the HFS maintain a narrow distribution better centered
179 around zero even at very low y -values of 0.005. The ML method in-particular outperforms or
180 equals the electron method for z and $p_{h,\perp}$ in all y -bins, and for ϕ_h in all but the highest y -bin. The
181 decrease in performance as a function of y is significantly more gradual with the ML approach. As
182 the network is directly given information on q as reconstructed from the electron method, it appears
183 that the network is able to learn additional corrections to the electron method through correlations
184 with the full hadronic final state.

SIDIS kinematic resolutions, $Q^2 > 1\text{GeV}^2$, π^+ , $z > 0.2$ ePIC 23.07.1

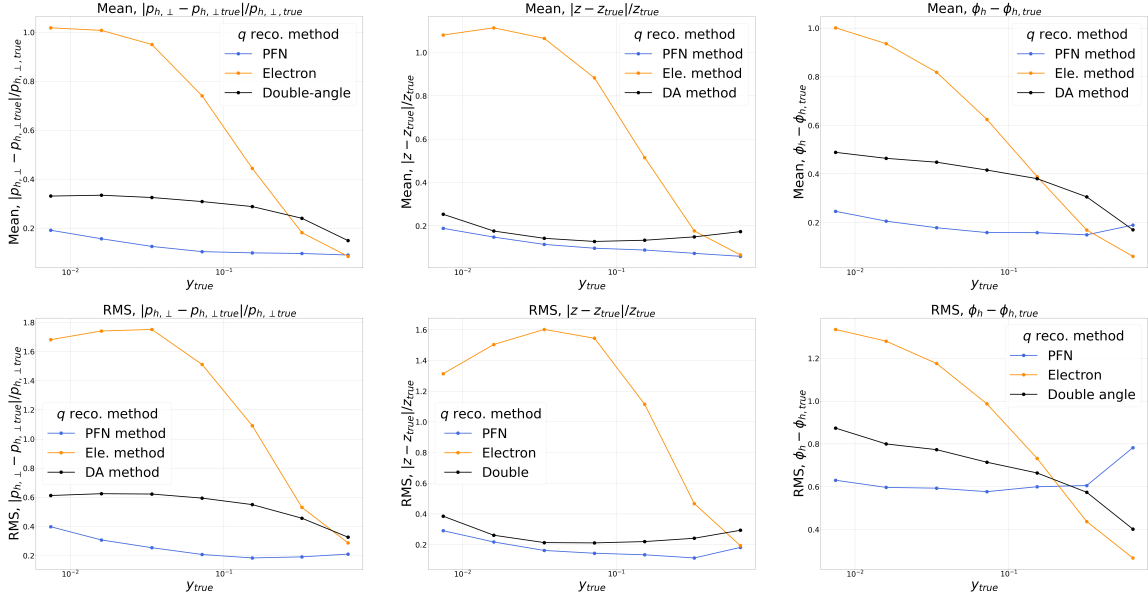


Figure 2. Absolute relative error in $p_{h,\perp}$ and z and error in ϕ_h with validation dataset in bins of y_{true} ranging from $y=0.005$ to $y=1$. A cut is placed requiring the absolute relative error be less than 1000% to understand the core of the distribution, which removes about 0.5% of events for the electron method. The results using each of the electron method, DA+HFS method, and PFN trained as described are shown. Top row displays the mean of these distributions as a function of y , and bottom row displays the RMS of these distributions.

185 5 Conclusion

186 Using the July 2023 full simulation of the ePIC detector, we have demonstrated two methods which
 187 improve on the reconstruction of semi-inclusive DIS kinematics by combining information from
 188 the hadronic final state and the scattered electron. Both HFS methods demonstrated maintain stable
 189 reconstruction at low- y where the electron method fails, and the use of Particle Flow Networks
 190 to reconstruct the virtual photon four-momentum outperforms the electron method for almost all
 191 values of y . This first use of a machine learning approach to the reconstruction of SIDIS kinematics
 192 will provide a better understanding of the ultimate kinematic resolution that will be achieved with
 193 the ePIC detector. As the ePIC simulation is developed further, the impacts of QED radiative
 194 corrections and a realistic DIS-electron finder must be investigated.

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