# DIS Electron Finding in epi TDR Readiness

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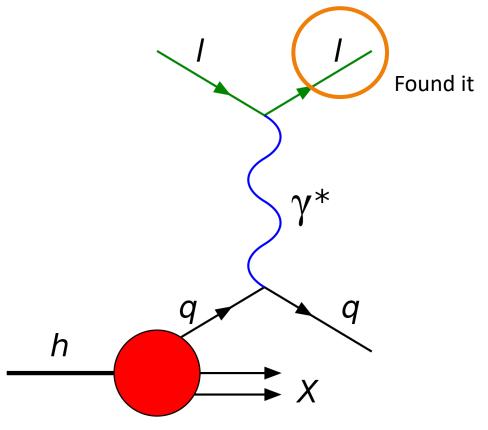
SPECIAL THANKS: Tyler Kutz, Brian Paige, Tristan Protzman, Temple U, Kong Tu, Andrii V, Markus D., Wouter D. Dmitry K. ....



#### **DIS Electron Finder**



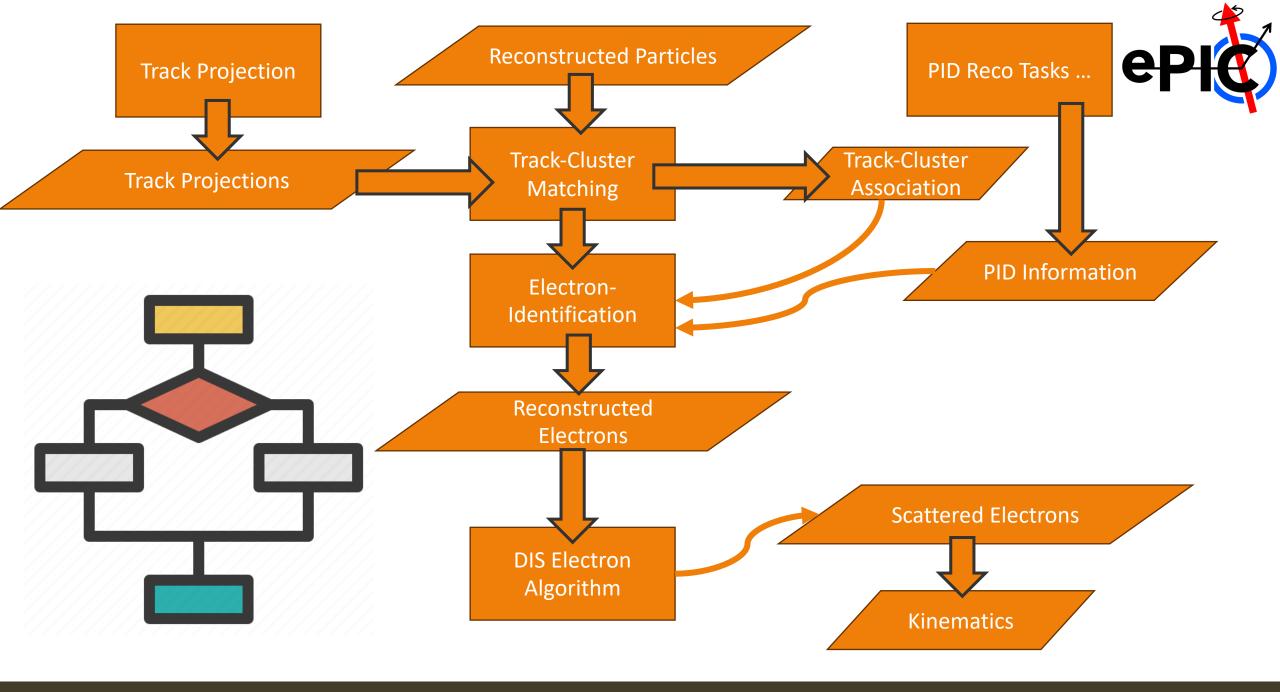
 Charge: Developing an efficient and accurate algorithm for identifying electrons and identifying the scattered electron of the DIS process

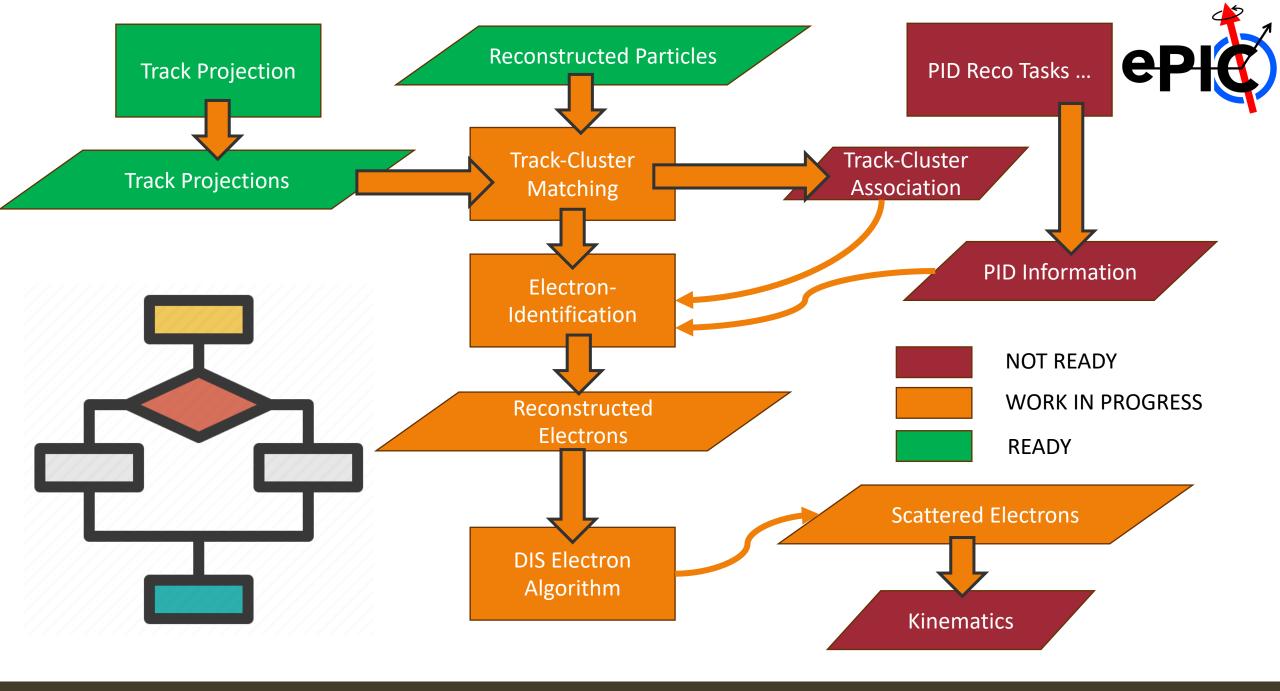


#### Major Goals:

- Develop unified electron identification
- Implement DIS lepton finder algorithms

Realistic DIS lepton finding is crucial for many benchmarks and analyses needed to inform detector design

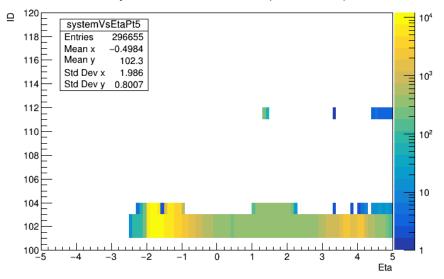




## Track Projections - Resolved



Subsystem ID Vs Track Eta (Pt > 5 GeV)



Track projections show ID (detector id) at eta values that should not be possible

Thanks to Tyler K., Brian P. and OSU students for working on this.

Thanks to software experts for help

 TrackPropagation\_factory: resolve an indexing bug by using ACTS surface ID fields #1257

Fixed and merged into eicrecon

## **Track Cluster Matching**



- Initial Track-Cluster Matching algorithm in development
- Association between Reco Cluster and ReconstructedParticle #52

#### Blocking Issue:

- Tracks are not filled
- Skip directly to Reconstructed Particles
- No relationship for associating projection<->track<-</li>>reconstructed particleSolution:

Tracks as first-class citizen

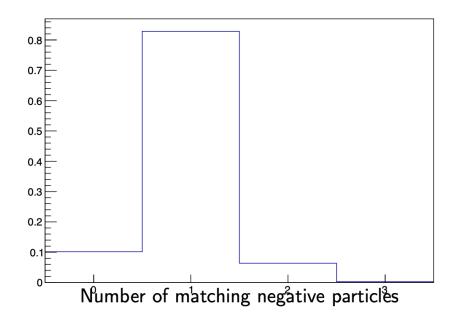
#### Update to data-model

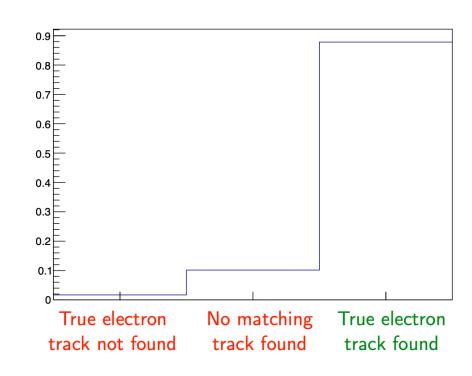
```
495 +
         edm4eic::TrackClusterMatch:
497 +
           Description: "Association between a Cluster and a ReconstructedParticle"
498
           Author: "D. Brandenburg"
499
           Members:
             - uint32_t
                                                  // Index of corresponding cluster (position
                                 cluID
       in cluster array)
                                                  // Index of corresponding
501 +
             - uint32 t
                                 plcID
       ReconstructedParticle (position in reco particle array)
                                                  // weight of this association
502 +
             float
                                 weight
           OneToOneRelations:
             - edm4eic::Cluster clu
                                                   // reference to the cluster
             - edm4eic::ReconstructedParticle plc
                                                             // reference to the Reco
       particle
```

#### **Electron Identification**



- Without Track-Cluster matching
- Partial truth information used (association with cluster)
- No clustering only leading hit probably leads to worse E/p performance
- Initial selection 0.9 < E/p < 1.2 needs optimization





**Tyler Kutz** 

Monday, February 5, 2024 eID | Work Meeting 7

#### **Electron Identification**



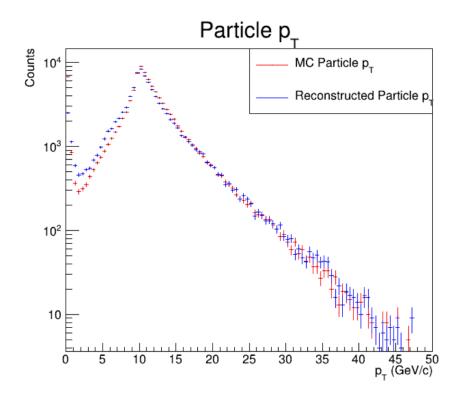
#### **Excellent work by Tristan Protzman (Lehigh University)**

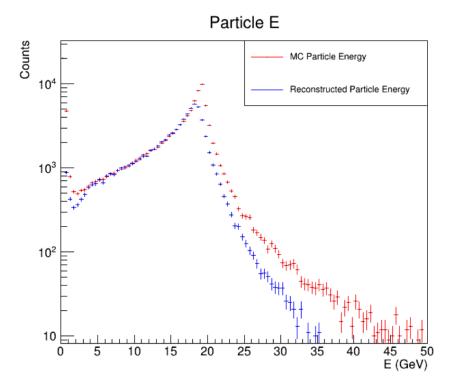
- Study E/p in electron endcap, barrel, hadronic endcap
- Use the neutral current DIS productions from the 23.12.0 simulation campaign
- Study done at 5x41, 10x100, 18x275 collision energy
  - Focus on 18x275, remainder in backup
  - Use min Q^2 of 1, 10, 100, and 1000

## **Electron Spectra**



- $18 \times 275$ ,  $Q_{min}^2 = 100$
- Electron only





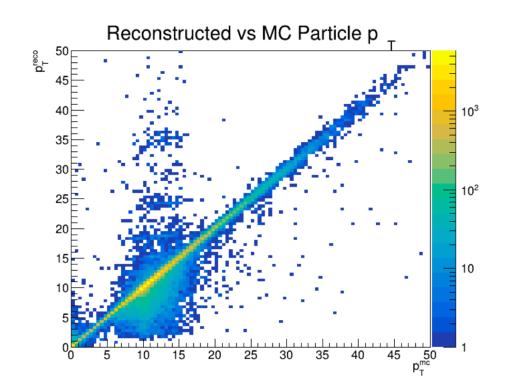
**Tristan Protzman (Lehigh University)** 

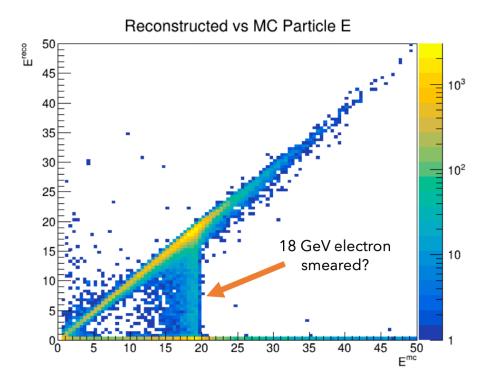
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## **Electron Energy Correlation**



- $18 \times 275$ ,  $Q_{min}^2 = 100$
- Electrons only

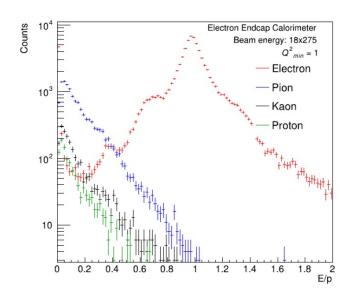




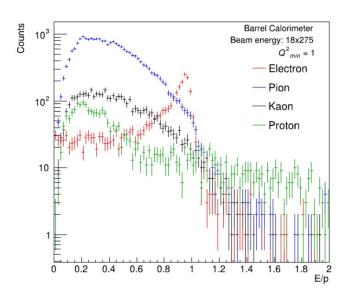
Iristan Protzman (Lenign University)

## Electron E/p - low Q^2





 $0.00 \le E/p \le 2.00$  Purity: 0.84 Efficiency: 1.00  $0.60 \le E/p \le 2.00$  Purity: 0.99 Efficiency: 0.90  $0.80 \le E/p \le 1.20$  Purity: 1.00 Efficiency: 0.69  $0.80 \le E/p \le 2.00$  Purity: 1.00 Efficiency: 0.81  $0.88 \le E/p \le 1.10$  Purity: 1.00 Efficiency: 0.56  $0.90 \le E/p \le 1.20$  Purity: 1.00 Efficiency: 0.59  $0.90 \le E/p \le 1.10$  Purity: 1.00 Efficiency: 0.59  $0.90 \le E/p \le 1.10$  Purity: 1.00 Efficiency: 0.53



 $0.00 \le E/p \le 2.00$  Purity: 0.09 Efficiency: 1.00  $0.60 \le E/p \le 2.00$  Purity: 0.26 Efficiency: 0.72  $0.80 \le E/p \le 1.20$  Purity: 0.45 Efficiency: 0.54  $0.80 \le E/p \le 2.00$  Purity: 0.42 Efficiency: 0.56  $0.88 \le E/p \le 1.10$  Purity: 0.54 Efficiency: 0.41  $0.90 \le E/p \le 1.20$  Purity: 0.52 Efficiency: 0.38  $0.90 \le E/p \le 1.10$  Purity: 0.55 Efficiency: 0.37

Very good separation in backwards direction

0.8 < E/p < 2 (and probably higher) captures most electrons with good purity

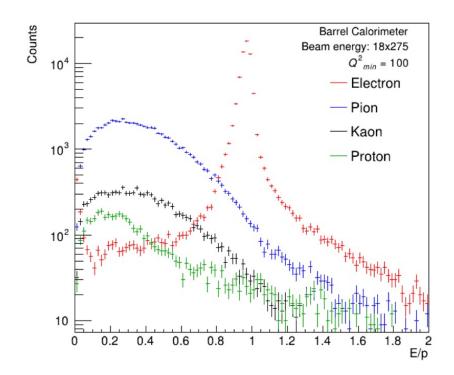
Smaller peak in barrel, but much fewer electrons here

Similar results with  $Q_{min}^2=10$ , with larger peak in barrel

**Iristan Protzman (Lehigh University)** 

## Electron E/p - Mid Q^2





 $0.00 \le E/p \le 2.00$  Purity: 0.51 Efficiency: 1.00  $0.60 \le E/p \le 2.00$  Purity: 0.83 Efficiency: 0.96  $0.80 \le E/p \le 1.20$  Purity: 0.94 Efficiency: 0.90  $0.80 \le E/p \le 2.00$  Purity: 0.92 Efficiency: 0.94  $0.88 \le E/p \le 1.10$  Purity: 0.96 Efficiency: 0.85  $0.90 \le E/p \le 1.20$  Purity: 0.96 Efficiency: 0.84  $0.90 \le E/p \le 1.10$  Purity: 0.97 Efficiency: 0.82

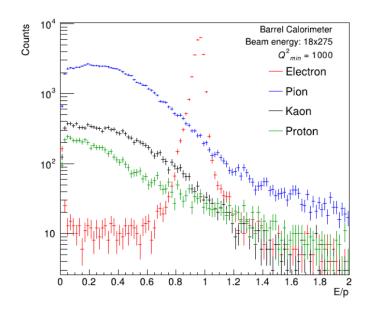
Almost all electrons in barrel

0.8 < E/p < 1.2 seems to be good balance between efficiency and purity

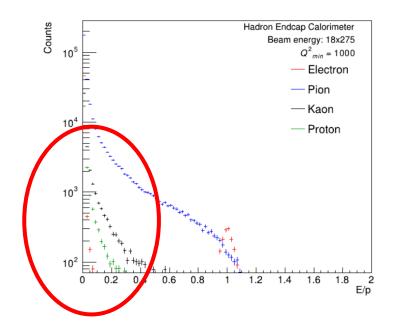
**Tristan Protzman (Lehigh University)** 

## Electron E/p - High Q^2





 $0.00 \le E/p \le 2.00$  Purity: 0.21 Efficiency: 1.00  $0.60 \le E/p \le 2.00$  Purity: 0.57 Efficiency: 0.98  $0.80 \le E/p \le 1.20$  Purity: 0.80 Efficiency: 0.95  $0.80 \le E/p \le 2.00$  Purity: 0.73 Efficiency: 0.97  $0.88 \le E/p \le 1.10$  Purity: 0.88 Efficiency: 0.90  $0.90 \le E/p \le 1.20$  Purity: 0.87 Efficiency: 0.88  $0.90 \le E/p \le 1.10$  Purity: 0.89 Efficiency: 0.87



 $0.00 \le E/p \le 2.00$  Purity: 0.11 Efficiency: 1.00  $0.60 \le E/p \le 2.00$  Purity: 0.15 Efficiency: 0.04  $0.80 \le E/p \le 1.20$  Purity: 0.32 Efficiency: 0.04  $0.80 \le E/p \le 2.00$  Purity: 0.27 Efficiency: 0.04  $0.88 \le E/p \le 1.10$  Purity: 0.44 Efficiency: 0.03  $0.90 \le E/p \le 1.20$  Purity: 0.45 Efficiency: 0.03  $0.90 \le E/p \le 1.10$  Purity: 0.48 Efficiency: 0.03

Most electrons again in barrel

0.8 < E/p < 1.2 still seems to be a good balance, though purity can be improved with 0.88 < E/p < 1.10

Few electrons make it the the hadronic endcap, poor separation here

Many electrons with  $E/p \sim 0$ , not fully understood yet

**Tristan Protzman (Lehigh University)** 

## **DIS Lepton Finder Algorithms**



Goal: identify the DIS lepton using only final state information

In case of one electron anything will work. In case of multiple electrons even a simple rule, such as take electron with "maximal" energy/pz /etc. will work for 50+% cases if only 2 electrons are present. Basically, implemented in Rivet.

#### **Challenges:**

- How to select the DIS electron when there are multiple candidates? Essentially check the hadronic final state kinematics is consistent with the kinematics of each electron, i.e. combine multiple measurements [1].
- How to associate the semi-hard radiation from the electron with the electron? Try to cluster some energy from the calorimeter deposits with the electron. Cluster the photons from the interaction point if they are angularly close to the electron?

Credit: Andrii Verbytskyi

## **DIS Lepton Finder Algorithms**

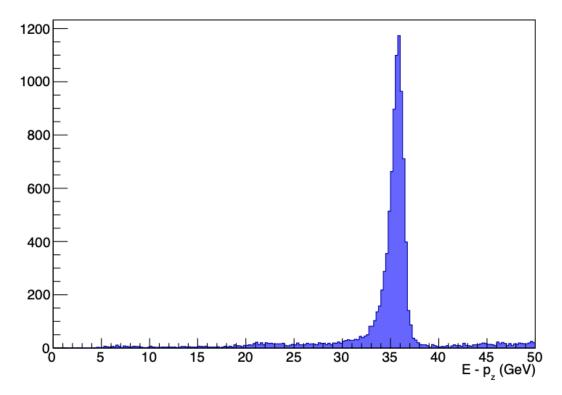


- So far two variants:
- Algorithms and factories for scattered electrons #1277
- ScatteredElectronsTruth finds MC scattered Electron and get associated reconstructed particle – collection of length 1, always the true\* electron
- ScatteredElectronsEMinusPz computes E-Pz for electron candidates and corresponding hadronic final state
- \*seems there is some issue in Pythia with the identification of the scattered electron from MC info...

## E-Pz algorithm



- Track matching next milestone for fully reconstruction-based E/p cut
- Further ePID refinement with addition of  $E-p_z$  cut (thanks to K. Tu for help on implementation)
- Implementation of EICrecon algorithm/ factory underway
- Still considering best output format, how to handle multiple DIS electron candidates, etc.



Thank you to Kong Tu and Tyler Kutz

#### ScatteredElectronsTruth



```
inline auto find_first_scattered_electron(const edm4hep::MCParticleCollection* mcparts) {
   return find_first_with_status_pdg(mcparts, {1}, {11});
}
```

```
template<class T>
auto find_first_with_status_pdg(
    const T* parts,
    const std::set<int32_t>& status,
    const std::set<int32_t>& pdg) {
 T c;
  c.setSubsetCollection();
  for (const auto& p: *parts) {
    if (status.count(p.getGeneratorStatus()) > 0 &&
        pdg.count(p.getPDG()) > 0) {
      c.push back(p);
      break:
  return c;
```

#### ScatteredElectronsTruth



```
// Get first scattered electron
const auto ef_coll = find_first_scattered_electron(mcparts);
if (ef coll.size() = \emptyset) {
  m_log→trace("No truth scattered electron found");
 return;
// Associate first scattered electron
// with reconstructed electron
auto ef_assoc = rcassoc→begin();
for (; ef_assoc \neq rcassoc\rightarrowend(); ++ef_assoc) {
  if (ef_assoc→getSimID() = (unsigned) ef_coll[0].getObjectID().index) {
    break;
```

### ScatteredElectronsEMinusPz



```
for (const auto& p: *rcparts) {
 // this is a hack - getObjectID() only works within
 // a collections, not unique across all collections.
 // What we want is to add all reconstructed particles
 // except the one we are currently considering as the
  // (scattered) electron candidate.
 // This does work though and in general it has only
 // one match as I would hope (tested on pythia events)
 if ( edm4hep::utils::magnitude(p.getMomentum())
    ≠ edm4hep::utils::magnitude(e.getMomentum()) ) {
   vHadron.SetCoordinates(
        p.getMomentum().x,
        p.getMomentum().y,
       p.getMomentum().z,
       m pion // Assume pion for hadronic state
      );
   // Sum hadronic final state
   vHadronicFinalState += vHadron;
 } else {
   m_log→trace( "Skipping electron in hadronic final state" );
```

Issue: getObjectID() only works within same collection

Solution: use Subset Collection (PR in progress)

#### **ScatteredElectronsEMinusPz**



Compute E-Pz for all (negative charge) electron candidates identified in ReconstructedElectron collection

#### **ScatteredElectronsEMinusPz**



```
// map sorts in ascending order by default
// sort by descending
bool first = true;
// for (auto kv : scatteredElectronsMap) {
for (auto kv = scatteredElectronsMap.rbegin(); kv \neq scatteredElectronsMap.rend(); ++kv) {
 double EMinusPz = kv→first;
 // Do not save electron candidates that
 // are not within range
 if ( EMinusPz > m cfg.maxEMinusPz
    EMinusPz < m_cfg.minEMinusPz ){</pre>
   continue:
```

Returns 'ScatteredElectronsEMinusPz` collection – ranked with the highest E-Pz first, then in descending order

Even without strict electron PID – right "most" of the time

## Checking the Scattered Electron



 Independent use of ScatteredElectronsTruth\_factory and ScatteredElectronsEMinusPz\_factory allows us to 'check' the algorithm's performance

```
[reco:ScatteredElectronsEMinusPz] [trace] We have 3 candidate electrons
[reco:ScatteredElectronsEMinusPz] [trace] Skipping electron in hadronic final state
[reco:ScatteredElectronsEMinusPz] [trace] E-Pz=17.044048885959747
[reco:ScatteredElectronsEMinusPz] [trace] ScatteredElectron has Pxyz=( 1.0658381, 2.5452766, -8.107633 )
[reco:ScatteredElectronsEMinusPz] [trace] skipping positron
[reco:ScatteredElectronsEMinusPz] [trace] skipping positron
[reco:ScatteredElectronsEMinusPz] [trace] Selecting candidates with 0 < E-Pz < 100000000
[reco:ScatteredElectronsEMinusPz] [trace] Max E-Pz Candidate:
[reco:ScatteredElectronsEMinusPz] [trace] ScatteredElectron has Pxyz=( 1.0658381, 2.5452766, -8.107633 )
[reco:ScatteredElectronsTruth] [trace] We found 1 scattered electron has Pxyz=( 1.0658381, 2.5452766, -8.107633 )
[reco:ScatteredElectronsTruth] [trace] TRUTH scattered electron has E-Pz = 17.044048885959747
[reco:ScatteredElectronsTruth] [trace] TRUTH scattered electron has Pxyz=( 1.065838098526001, 2.545276641845703, -8.107632637023926 )

TRUTH scattered electron has Pxyz=( 1.065838098526001, 2.545276641845703, -8.107632637023926 )
```

These allow 'baseline' algorithms to compare against as we develop more sophisticated (ML) algorithms

## Summary



- Recent Progress:
  - Resolve elusive Track Projection issues
  - Optimization studies on E/p selection
  - Updates to Electron Identification
  - Baseline algorithms for DIS electron finding
- Next Major Milestones
  - Proper use of `Tracks`
  - Fully integrate Track-Cluster Matching
  - Incorporate PID information
  - Setup for advanced lepton finder algorithms
  - Data format updates

SPECIAL THANKS: Tyler Kutz, Brian Paige, Tristan Protzman, Temple U, Kong Tu, Andrii V, Markus D., Wouter D. Dmitry K. ....

## Data format updates



- Track-Cluster Match
- Collection for ranked scattered electrons
  - Something like an object + a likelihood
  - Would be nice to also somehow associate an "algorithm data" object, e.g. E-Pz info for the ScatteredElectronsEMinusPz
- Kinematics formats for:
  - Inclusive
  - Exclusive
  - Etc.

#### Electron Finder 07-10-2023



- Progress in June (for July Sim Campaign)
  - PR #666: Provides association containers + truth associations merged into main on June 24
  - PR #751: Implement basic electron finding with truth cluster matching
  - Work in parallel with the RECO approach (see tasks below)
- Major Tasks (July):
  - Utilize the July sim campaign output:
    - Implement a processor to test DIS lepton finder
    - Check purity of selected electrons
  - Track Projection Factory: provide track projections at relevant detectors (Tyler Kutz)
    - ✓ Needed data structure identified
    - ✓ Prototype factory in progress
  - Track Match Factory: Matching of projecting tracks to clusters (volunteer?)
    - Nicholas Schmidt already has some code (processor) to study track matching
    - Provides a starting point for factory
  - Study of E/p cuts to implement (volunteer potentially identified, discussing next steps)
  - Study HCAL info for hadron rejection / electron id
- Plans for July sim campaign
  - Utilize "ReconstructedElectrons" to test-drive DIS lepton finder (should be in EICRecon for Aug)
  - Continue work towards towards fully RECO level (complete track matching / compare to truth level matching)

## Truth approach

- PR #751 Add reconstructed electron factory, algorithm utilizing E/p cut
  - https://github.com/eic/EICrecon/pull/751
  - ReconstructedElectrons Factory
  - Input:

```
{"MCParticles", "ReconstructedChargedParticles", "ReconstructedChargedParticleAssociations",

"EcalBarrelScFiClusterAssociations",

"EcalEndcapNClusterAssociations",

"EcalEndcapPClusterAssociations",

"EcalEndcapPInsertClusterAssociations",

"EcalLumiSpecClusterAssociations",

"EcalLumiSpecClusterAssociations",
```

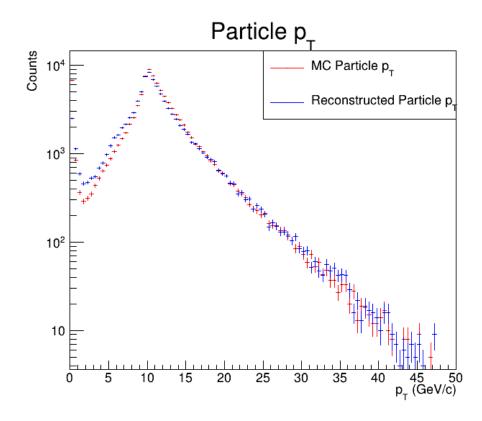
- Output: "ReconstructedElectrons"
- Utilizes the ElectronReconstruction Algorithm
  - Any track with an ECAL match
  - Accept if 0.9 < E/p < 1.2 (needs to be studied and optimized)</li>
  - TODO: use HCAL
  - TODO: handle multiple matches
- This is meant to be initial skeleton keep same structure for RECO approach

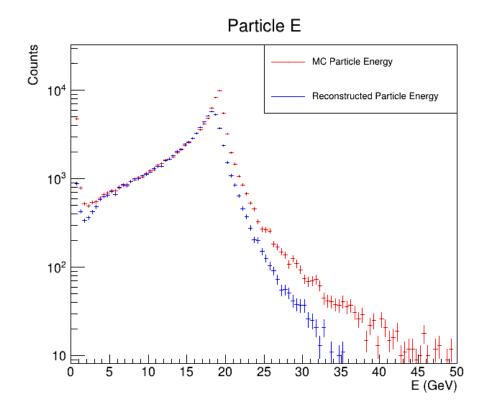
#### Overview

- Study E/p in electron endcap, barrel, hadronic endcap
- Use the neutral current DIS productions from the 23.12.0 simulation campaign
- Study done at 5x41, 10x100, 18x275 collision energy
  - Focus on 18x275, remainder in backup
- Use min  $Q^2$  of 1, 10, 100, and 1000

## Spectra

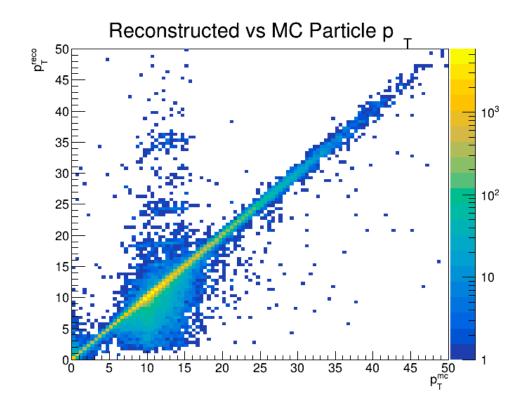
- 18x275,  $Q_{min}^2 = 100$
- Electron only

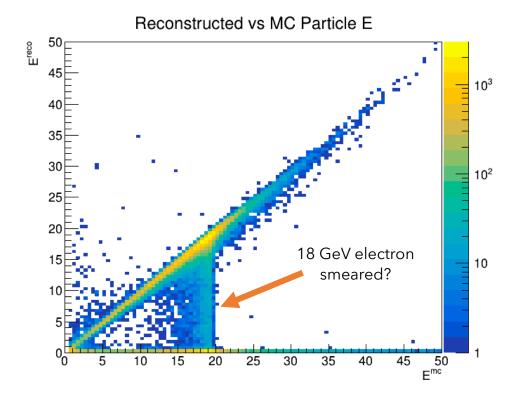




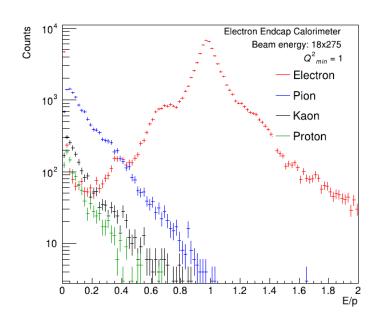
## Correlation

- 18x275,  $Q_{min}^2 = 100$
- Electrons only

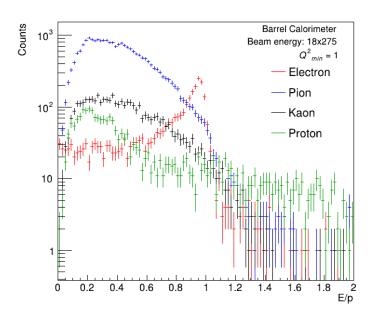




## Low $Q^2$



 $0.00 \le E/p \le 2.00$  Purity: 0.84 Efficiency: 1.00  $0.60 \le E/p \le 2.00$  Purity: 0.99 Efficiency: 0.90  $0.80 \le E/p \le 1.20$  Purity: 1.00 Efficiency: 0.69  $0.80 \le E/p \le 2.00$  Purity: 1.00 Efficiency: 0.81  $0.88 \le E/p \le 1.10$  Purity: 1.00 Efficiency: 0.56  $0.90 \le E/p \le 1.20$  Purity: 1.00 Efficiency: 0.59  $0.90 \le E/p \le 1.10$  Purity: 1.00 Efficiency: 0.59  $0.90 \le E/p \le 1.10$  Purity: 1.00 Efficiency: 0.53



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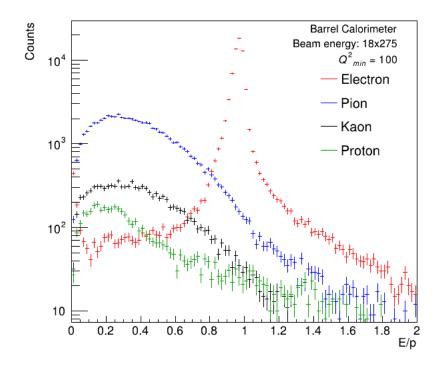
Very good separation in backwards direction

0.8 < E/p < 2 (and probably higher) captures most electrons with good purity

Smaller peak in barrel, but much fewer electrons here

Similar results with  $Q_{min}^2 = 10$ , with larger peak in barrel

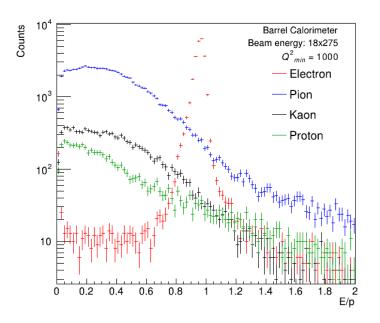
# $Mid Q^2$



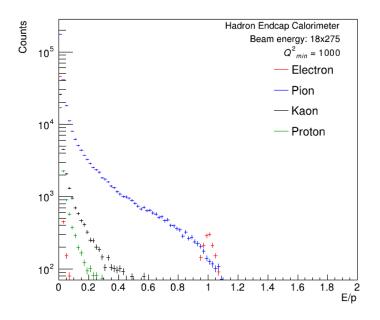
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0.8 < E/p < 1.2 seems to be good balance between efficiency and purity

# High $Q^2$



 $0.00 \le E/p \le 2.00$  Purity: 0.21 Efficiency: 1.00  $0.60 \le E/p \le 2.00$  Purity: 0.57 Efficiency: 0.98  $0.80 \le E/p \le 1.20$  Purity: 0.80 Efficiency: 0.95  $0.80 \le E/p \le 2.00$  Purity: 0.73 Efficiency: 0.97  $0.88 \le E/p \le 1.10$  Purity: 0.88 Efficiency: 0.90  $0.90 \le E/p \le 1.20$  Purity: 0.87 Efficiency: 0.88  $0.90 \le E/p \le 1.10$  Purity: 0.89 Efficiency: 0.87



 $0.00 \le E/p \le 2.00$  Purity: 0.11 Efficiency: 1.00  $0.60 \le E/p \le 2.00$  Purity: 0.15 Efficiency: 0.04  $0.80 \le E/p \le 1.20$  Purity: 0.32 Efficiency: 0.04  $0.80 \le E/p \le 2.00$  Purity: 0.27 Efficiency: 0.04  $0.88 \le E/p \le 1.10$  Purity: 0.44 Efficiency: 0.03  $0.90 \le E/p \le 1.20$  Purity: 0.45 Efficiency: 0.03  $0.90 \le E/p \le 1.10$  Purity: 0.48 Efficiency: 0.03  $0.90 \le E/p \le 1.10$  Purity: 0.48 Efficiency: 0.03

Most electrons again in barrel

0.8 < E/p < 1.2 still seems to be a good balance, though purity can be improved with 0.88 < E/p < 1.10

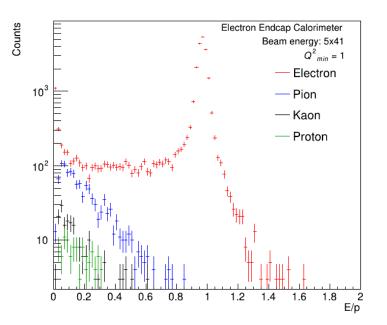
Few electrons make it the the hadronic endcap, poor separation here

Many electrons with  $E/p \sim 0$ , not fully understood yet

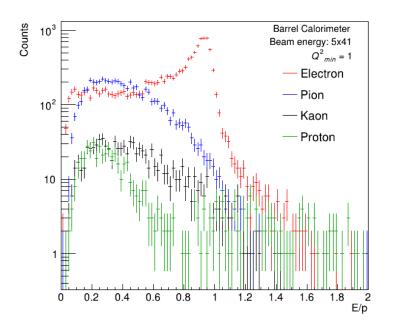
# Backup

# 5x41

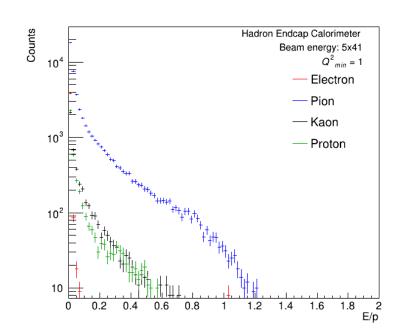
# $5x41 \ Q_{min}^2 = 1$



 $0.00 \le E/p \le 2.00$  Purity: 0.95 Efficiency: 1.00  $0.60 \le E/p \le 2.00$  Purity: 1.00 Efficiency: 0.83  $0.80 \le E/p \le 1.20$  Purity: 1.00 Efficiency: 0.78  $0.80 \le E/p \le 2.00$  Purity: 1.00 Efficiency: 0.79  $0.88 \le E/p \le 1.10$  Purity: 1.00 Efficiency: 0.75  $0.90 \le E/p \le 1.20$  Purity: 1.00 Efficiency: 0.74  $0.90 \le E/p \le 1.10$  Purity: 1.00 Efficiency: 0.74

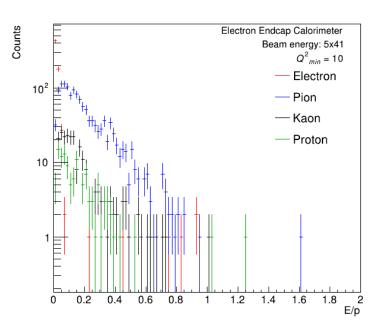




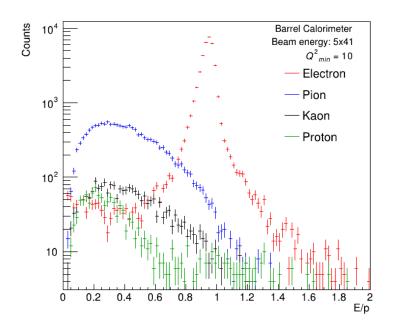


 $0.00 \le E/p \le 2.00$  Purity: 0.07 Efficiency: 1.00  $0.60 \le E/p \le 2.00$  Purity: 0.05 Efficiency: 0.03  $0.80 \le E/p \le 1.20$  Purity: 0.09 Efficiency: 0.02  $0.80 \le E/p \le 2.00$  Purity: 0.11 Efficiency: 0.03  $0.88 \le E/p \le 1.10$  Purity: 0.12 Efficiency: 0.01  $0.90 \le E/p \le 1.20$  Purity: 0.14 Efficiency: 0.02  $0.90 \le E/p \le 1.10$  Purity: 0.13 Efficiency: 0.01

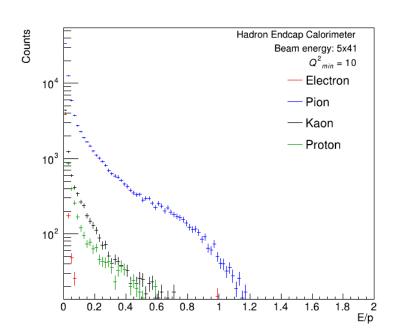
# $5x41 \ Q_{min}^2 = 10$



 $0.00 \le E/p \le 2.00$  Purity: 0.28 Efficiency: 1.00  $0.60 \le E/p \le 2.00$  Purity: 0.09 Efficiency: 0.01  $0.80 \le E/p \le 1.20$  Purity: 0.30 Efficiency: 0.00  $0.80 \le E/p \le 2.00$  Purity: 0.25 Efficiency: 0.00  $0.88 \le E/p \le 1.10$  Purity: 0.40 Efficiency: 0.00  $0.90 \le E/p \le 1.20$  Purity: 0.40 Efficiency: 0.00  $0.90 \le E/p \le 1.10$  Purity: 0.40 Efficiency: 0.00  $0.90 \le E/p \le 1.10$  Purity: 0.40 Efficiency: 0.00



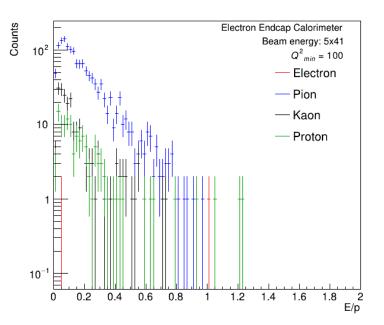
 $0.00 \le E/p \le 2.00$  Purity: 0.70 Efficiency: 1.00  $0.60 \le E/p \le 2.00$  Purity: 0.91 Efficiency: 0.97  $0.80 \le E/p \le 1.20$  Purity: 0.97 Efficiency: 0.92  $0.80 \le E/p \le 2.00$  Purity: 0.97 Efficiency: 0.94  $0.88 \le E/p \le 1.10$  Purity: 0.98 Efficiency: 0.82  $0.90 \le E/p \le 1.20$  Purity: 0.98 Efficiency: 0.76  $0.90 \le E/p \le 1.10$  Purity: 0.98 Efficiency: 0.75



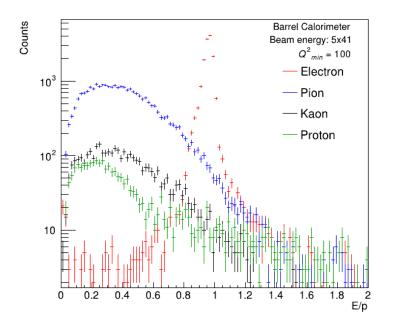
 $0.00 \le E/p \le 2.00$  Purity: 0.04 Efficiency: 1.00  $0.60 \le E/p \le 2.00$  Purity: 0.05 Efficiency: 0.04  $0.80 \le E/p \le 1.20$  Purity: 0.10 Efficiency: 0.03  $0.80 \le E/p \le 2.00$  Purity: 0.10 Efficiency: 0.04  $0.88 \le E/p \le 1.10$  Purity: 0.12 Efficiency: 0.02  $0.90 \le E/p \le 1.20$  Purity: 0.15 Efficiency: 0.03  $0.90 \le E/p \le 1.10$  Purity: 0.13 Efficiency: 0.02

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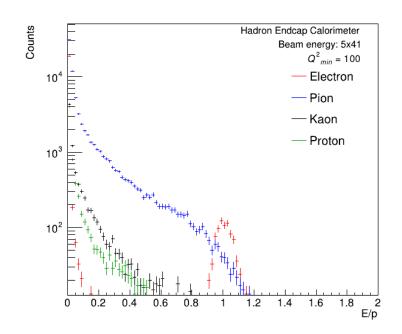
# $5x41 Q_{min}^2 = 100$



 $0.00 \le E/p \le 2.00$  Purity: 0.00 Efficiency: 1.00  $0.60 \le E/p \le 2.00$  Purity: 0.04 Efficiency: 0.67  $0.80 \le E/p \le 1.20$  Purity: 0.11 Efficiency: 0.33  $0.80 \le E/p \le 2.00$  Purity: 0.10 Efficiency: 0.33  $0.88 \le E/p \le 1.10$  Purity: 0.20 Efficiency: 0.33  $0.90 \le E/p \le 1.20$  Purity: 0.17 Efficiency: 0.33  $0.90 \le E/p \le 1.10$  Purity: 0.20 Efficiency: 0.33

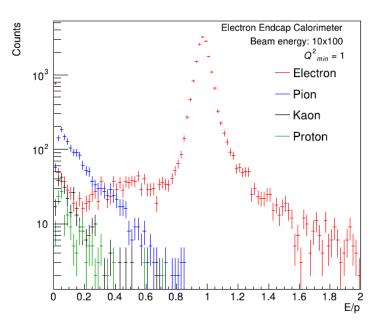


 $0.00 \le E/p \le 2.00$  Purity: 0.34 Efficiency: 1.00  $0.60 \le E/p \le 2.00$  Purity: 0.72 Efficiency: 0.99  $0.80 \le E/p \le 1.20$  Purity: 0.89 Efficiency: 0.97  $0.80 \le E/p \le 2.00$  Purity: 0.87 Efficiency: 0.98  $0.88 \le E/p \le 1.10$  Purity: 0.94 Efficiency: 0.92  $0.90 \le E/p \le 1.20$  Purity: 0.93 Efficiency: 0.90  $0.90 \le E/p \le 1.10$  Purity: 0.94 Efficiency: 0.89

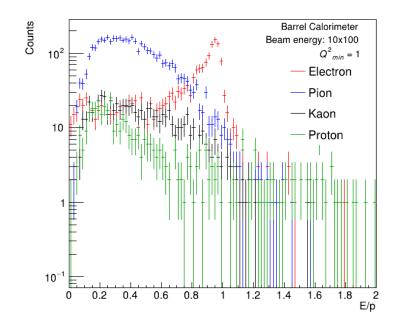


$$\begin{split} 0.00 &\leq E/p \leq 2.00 \text{ Purity: } 0.19 \text{ Efficiency: } 1.00 \\ 0.60 &\leq E/p \leq 2.00 \text{ Purity: } 0.24 \text{ Efficiency: } 0.05 \\ 0.80 &\leq E/p \leq 1.20 \text{ Purity: } 0.45 \text{ Efficiency: } 0.04 \\ 0.80 &\leq E/p \leq 2.00 \text{ Purity: } 0.44 \text{ Efficiency: } 0.04 \\ 0.88 &\leq E/p \leq 1.10 \text{ Purity: } 0.58 \text{ Efficiency: } 0.04 \\ 0.90 &\leq E/p \leq 1.20 \text{ Purity: } 0.60 \text{ Efficiency: } 0.04 \\ 0.90 &\leq E/p \leq 1.10 \text{ Purity: } 0.62 \text{ Efficiency: } 0.04 \\ \end{split}$$

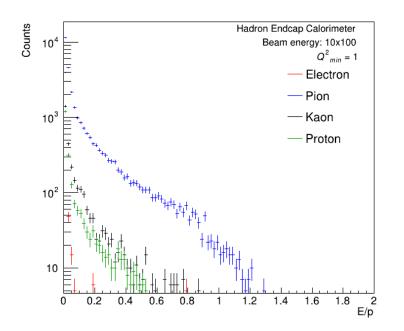
# 10x100



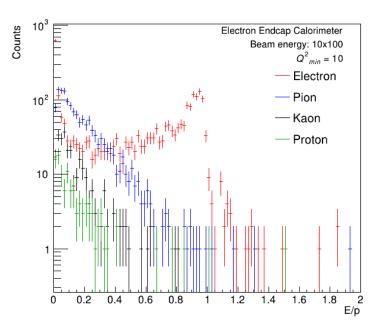
 $0.00 \le E/p \le 2.00$  Purity: 0.91 Efficiency: 1.00  $0.60 \le E/p \le 2.00$  Purity: 1.00 Efficiency: 0.92  $0.80 \le E/p \le 1.20$  Purity: 1.00 Efficiency: 0.86  $0.80 \le E/p \le 2.00$  Purity: 1.00 Efficiency: 0.90  $0.88 \le E/p \le 1.10$  Purity: 1.00 Efficiency: 0.81  $0.90 \le E/p \le 1.20$  Purity: 1.00 Efficiency: 0.81  $0.90 \le E/p \le 1.10$  Purity: 1.00 Efficiency: 0.79



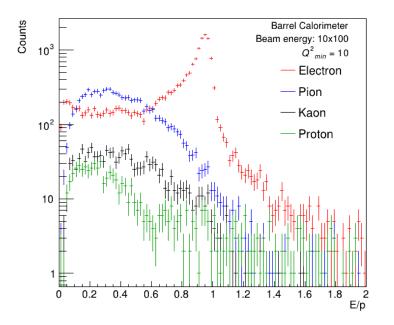
 $0.00 \le E/p \le 2.00$  Purity: 0.26 Efficiency: 1.00  $0.60 \le E/p \le 2.00$  Purity: 0.53 Efficiency: 0.70  $0.80 \le E/p \le 1.20$  Purity: 0.74 Efficiency: 0.54  $0.80 \le E/p \le 2.00$  Purity: 0.70 Efficiency: 0.55  $0.88 \le E/p \le 1.10$  Purity: 0.80 Efficiency: 0.40  $0.90 \le E/p \le 1.20$  Purity: 0.80 Efficiency: 0.37  $0.90 \le E/p \le 1.10$  Purity: 0.83 Efficiency: 0.36



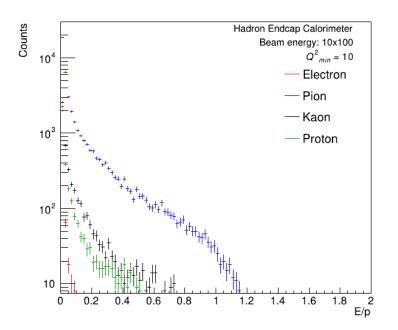
 $0.00 \le E/p \le 2.00$  Purity: 0.11 Efficiency: 1.00  $0.60 \le E/p \le 2.00$  Purity: 0.04 Efficiency: 0.01  $0.80 \le E/p \le 1.20$  Purity: 0.06 Efficiency: 0.01  $0.80 \le E/p \le 2.00$  Purity: 0.06 Efficiency: 0.01  $0.88 \le E/p \le 1.10$  Purity: 0.08 Efficiency: 0.01  $0.90 \le E/p \le 1.20$  Purity: 0.09 Efficiency: 0.01  $0.90 \le E/p \le 1.10$  Purity: 0.08 Efficiency: 0.01  $0.90 \le E/p \le 1.10$  Purity: 0.08 Efficiency: 0.01



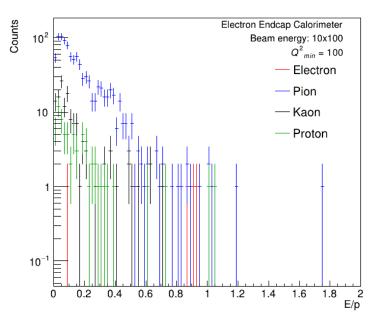
 $0.00 \le E/p \le 2.00$  Purity: 0.61 Efficiency: 1.00  $0.60 \le E/p \le 2.00$  Purity: 0.96 Efficiency: 0.45  $0.80 \le E/p \le 1.20$  Purity: 0.98 Efficiency: 0.31  $0.80 \le E/p \le 2.00$  Purity: 0.98 Efficiency: 0.32  $0.88 \le E/p \le 1.10$  Purity: 0.99 Efficiency: 0.23  $0.90 \le E/p \le 1.20$  Purity: 0.99 Efficiency: 0.20  $0.90 \le E/p \le 1.10$  Purity: 0.99 Efficiency: 0.20



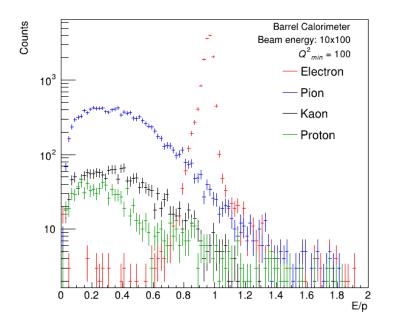
 $0.00 \le E/p \le 2.00$  Purity: 0.64 Efficiency: 1.00  $0.60 \le E/p \le 2.00$  Purity: 0.86 Efficiency: 0.74  $0.80 \le E/p \le 1.20$  Purity: 0.94 Efficiency: 0.57  $0.80 \le E/p \le 2.00$  Purity: 0.93 Efficiency: 0.59  $0.88 \le E/p \le 1.10$  Purity: 0.96 Efficiency: 0.45  $0.90 \le E/p \le 1.20$  Purity: 0.96 Efficiency: 0.41  $0.90 \le E/p \le 1.10$  Purity: 0.96 Efficiency: 0.40



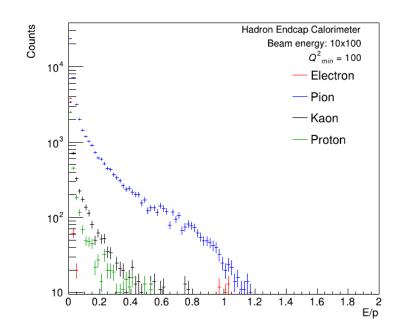
 $0.00 \le E/p \le 2.00$  Purity: 0.05 Efficiency: 1.00  $0.60 \le E/p \le 2.00$  Purity: 0.05 Efficiency: 0.03  $0.80 \le E/p \le 1.20$  Purity: 0.09 Efficiency: 0.03  $0.80 \le E/p \le 2.00$  Purity: 0.09 Efficiency: 0.03  $0.88 \le E/p \le 1.10$  Purity: 0.11 Efficiency: 0.02  $0.90 \le E/p \le 1.20$  Purity: 0.13 Efficiency: 0.02  $0.90 \le E/p \le 1.10$  Purity: 0.11 Efficiency: 0.02



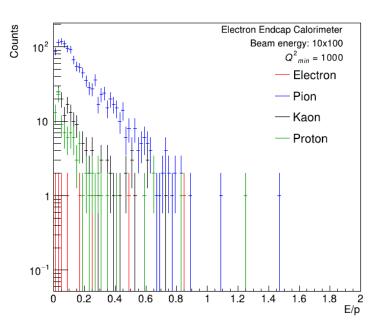
 $0.00 \le E/p \le 2.00$  Purity: 0.01 Efficiency: 1.00  $0.60 \le E/p \le 2.00$  Purity: 0.14 Efficiency: 0.42  $0.80 \le E/p \le 1.20$  Purity: 0.29 Efficiency: 0.42  $0.80 \le E/p \le 2.00$  Purity: 0.26 Efficiency: 0.42  $0.88 \le E/p \le 1.10$  Purity: 0.30 Efficiency: 0.25  $0.90 \le E/p \le 1.20$  Purity: 0.22 Efficiency: 0.17  $0.90 \le E/p \le 1.10$  Purity: 0.25 Efficiency: 0.17



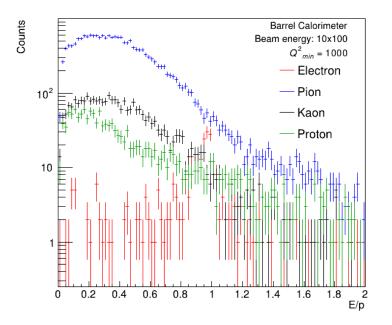




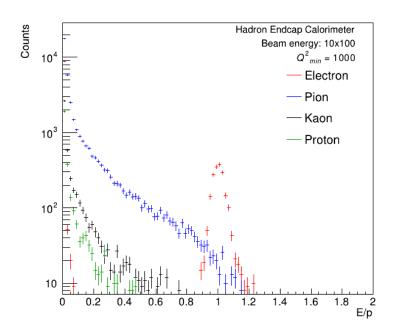
$$\begin{split} 0.00 &\leq E/p \leq 2.00 \text{ Purity: } 0.07 \text{ Efficiency: } 1.00 \\ 0.60 &\leq E/p \leq 2.00 \text{ Purity: } 0.06 \text{ Efficiency: } 0.03 \\ 0.80 &\leq E/p \leq 1.20 \text{ Purity: } 0.13 \text{ Efficiency: } 0.03 \\ 0.80 &\leq E/p \leq 2.00 \text{ Purity: } 0.12 \text{ Efficiency: } 0.03 \\ 0.88 &\leq E/p \leq 1.10 \text{ Purity: } 0.18 \text{ Efficiency: } 0.02 \\ 0.90 &\leq E/p \leq 1.20 \text{ Purity: } 0.20 \text{ Efficiency: } 0.02 \\ 0.90 &\leq E/p \leq 1.10 \text{ Purity: } 0.20 \text{ Efficiency: } 0.02 \\ \end{split}$$



 $0.00 \le E/p \le 2.00$  Purity: 0.01 Efficiency: 1.00  $0.60 \le E/p \le 2.00$  Purity: 0.03 Efficiency: 0.12  $0.80 \le E/p \le 1.20$  Purity: 0.17 Efficiency: 0.12  $0.80 \le E/p \le 2.00$  Purity: 0.10 Efficiency: 0.12  $0.88 \le E/p \le 1.10$  Purity: 0.00 Efficiency: 0.00  $0.90 \le E/p \le 1.20$  Purity: 0.00 Efficiency: 0.00  $0.90 \le E/p \le 1.10$  Purity: 0.00 Efficiency: 0.00  $0.90 \le E/p \le 1.10$  Purity: 0.00 Efficiency: 0.00

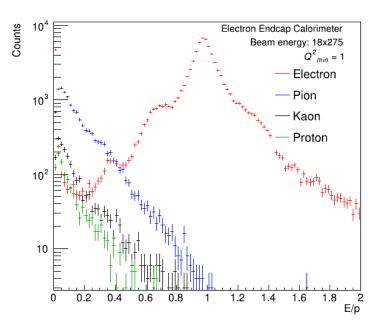


 $0.00 \le E/p \le 2.00$  Purity: 0.01 Efficiency: 1.00  $0.60 \le E/p \le 2.00$  Purity: 0.05 Efficiency: 0.85  $0.80 \le E/p \le 1.20$  Purity: 0.11 Efficiency: 0.63  $0.80 \le E/p \le 1.20$  Purity: 0.10 Efficiency: 0.76  $0.88 \le E/p \le 1.10$  Purity: 0.16 Efficiency: 0.50  $0.90 \le E/p \le 1.20$  Purity: 0.15 Efficiency: 0.51  $0.90 \le E/p \le 1.10$  Purity: 0.17 Efficiency: 0.47

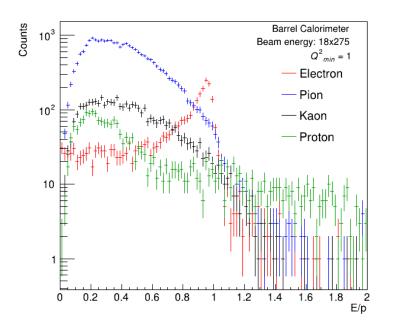


 $0.00 \le E/p \le 2.00$  Purity: 0.20 Efficiency: 1.00  $0.60 \le E/p \le 2.00$  Purity: 0.61 Efficiency: 0.18  $0.80 \le E/p \le 1.20$  Purity: 0.79 Efficiency: 0.17  $0.80 \le E/p \le 2.00$  Purity: 0.78 Efficiency: 0.18  $0.88 \le E/p \le 1.10$  Purity: 0.87 Efficiency: 0.17  $0.90 \le E/p \le 1.20$  Purity: 0.88 Efficiency: 0.17  $0.90 \le E/p \le 1.10$  Purity: 0.88 Efficiency: 0.17

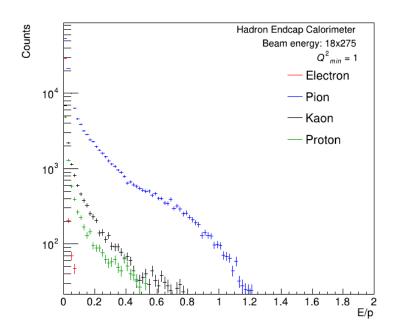
# 18x275



 $0.00 \le E/p \le 2.00$  Purity: 0.84 Efficiency: 1.00  $0.60 \le E/p \le 2.00$  Purity: 0.99 Efficiency: 0.90  $0.80 \le E/p \le 1.20$  Purity: 1.00 Efficiency: 0.69  $0.80 \le E/p \le 2.00$  Purity: 1.00 Efficiency: 0.81  $0.88 \le E/p \le 1.10$  Purity: 1.00 Efficiency: 0.56  $0.90 \le E/p \le 1.20$  Purity: 1.00 Efficiency: 0.59  $0.90 \le E/p \le 1.10$  Purity: 1.00 Efficiency: 0.53

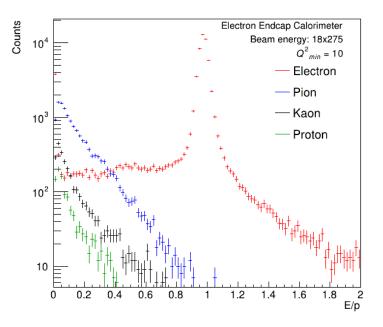




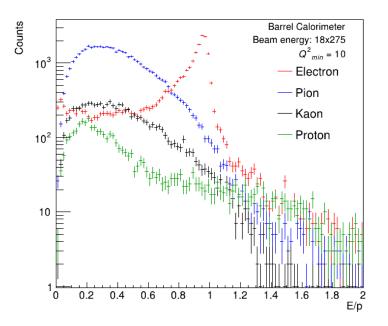


$$\begin{split} 0.00 &\leq E/p \leq 2.00 \text{ Purity: } 0.16 \text{ Efficiency: } 1.00 \\ 0.60 &\leq E/p \leq 2.00 \text{ Purity: } 0.04 \text{ Efficiency: } 0.01 \\ 0.80 &\leq E/p \leq 1.20 \text{ Purity: } 0.09 \text{ Efficiency: } 0.01 \\ 0.80 &\leq E/p \leq 2.00 \text{ Purity: } 0.09 \text{ Efficiency: } 0.01 \\ 0.88 &\leq E/p \leq 1.10 \text{ Purity: } 0.11 \text{ Efficiency: } 0.01 \\ 0.90 &\leq E/p \leq 1.20 \text{ Purity: } 0.13 \text{ Efficiency: } 0.01 \\ 0.90 &\leq E/p \leq 1.10 \text{ Purity: } 0.11 \text{ Efficiency: } 0.00 \\ \end{split}$$

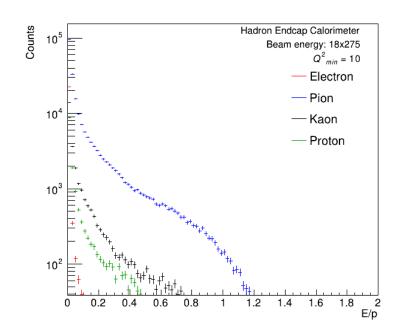
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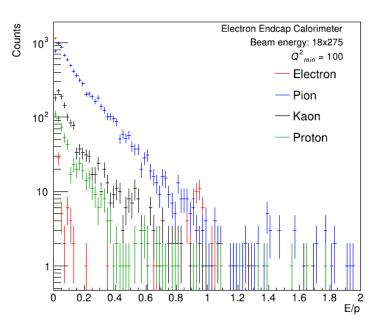
 $0.00 \le E/p \le 2.00$  Purity: 0.78 Efficiency: 1.00  $0.60 \le E/p \le 2.00$  Purity: 0.99 Efficiency: 0.85  $0.80 \le E/p \le 1.20$  Purity: 1.00 Efficiency: 0.79  $0.80 \le E/p \le 2.00$  Purity: 1.00 Efficiency: 0.82  $0.88 \le E/p \le 1.10$  Purity: 1.00 Efficiency: 0.76  $0.90 \le E/p \le 1.20$  Purity: 1.00 Efficiency: 0.76  $0.90 \le E/p \le 1.10$  Purity: 1.00 Efficiency: 0.75



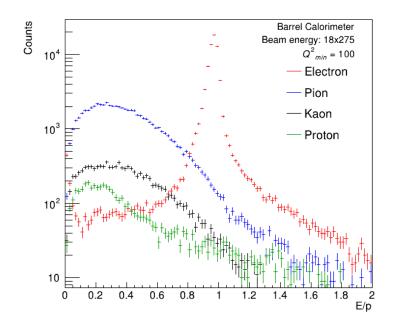




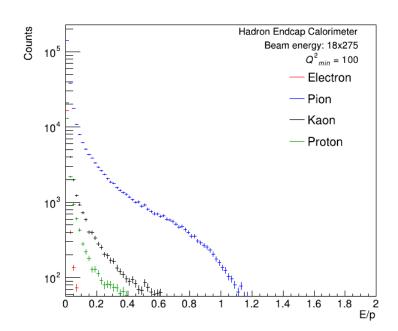
$$\begin{split} 0.00 &\leq E/p \leq 2.00 \text{ Purity: } 0.08 \text{ Efficiency: } 1.00 \\ 0.60 &\leq E/p \leq 2.00 \text{ Purity: } 0.05 \text{ Efficiency: } 0.02 \\ 0.80 &\leq E/p \leq 1.20 \text{ Purity: } 0.10 \text{ Efficiency: } 0.02 \\ 0.80 &\leq E/p \leq 2.00 \text{ Purity: } 0.10 \text{ Efficiency: } 0.02 \\ 0.88 &\leq E/p \leq 1.10 \text{ Purity: } 0.13 \text{ Efficiency: } 0.01 \\ 0.90 &\leq E/p \leq 1.20 \text{ Purity: } 0.14 \text{ Efficiency: } 0.01 \\ 0.90 &\leq E/p \leq 1.10 \text{ Purity: } 0.14 \text{ Efficiency: } 0.01 \\ \end{split}$$



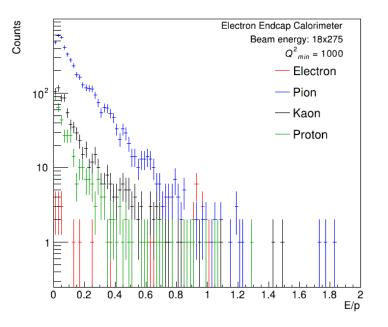
 $0.00 \le E/p \le 2.00$  Purity: 0.11 Efficiency: 1.00  $0.60 \le E/p \le 2.00$  Purity: 0.15 Efficiency: 0.05  $0.80 \le E/p \le 1.20$  Purity: 0.37 Efficiency: 0.04  $0.80 \le E/p \le 2.00$  Purity: 0.25 Efficiency: 0.04  $0.88 \le E/p \le 1.10$  Purity: 0.52 Efficiency: 0.04  $0.90 \le E/p \le 1.20$  Purity: 0.51 Efficiency: 0.04  $0.90 \le E/p \le 1.10$  Purity: 0.55 Efficiency: 0.04



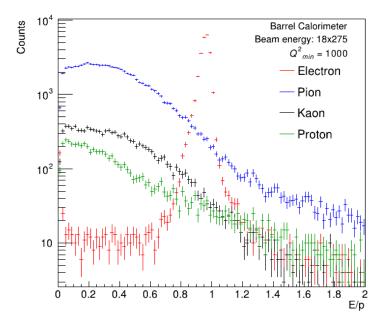


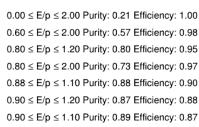


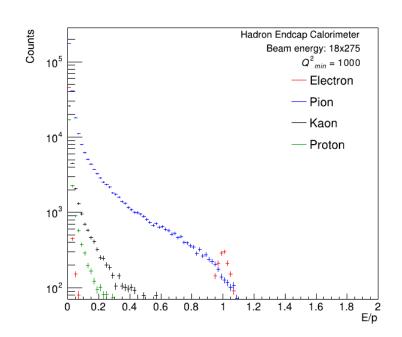
$$\begin{split} 0.00 &\leq E/p \leq 2.00 \text{ Purity: } 0.05 \text{ Efficiency: } 1.00 \\ 0.60 &\leq E/p \leq 2.00 \text{ Purity: } 0.05 \text{ Efficiency: } 0.03 \\ 0.80 &\leq E/p \leq 1.20 \text{ Purity: } 0.11 \text{ Efficiency: } 0.03 \\ 0.80 &\leq E/p \leq 2.00 \text{ Purity: } 0.09 \text{ Efficiency: } 0.03 \\ 0.88 &\leq E/p \leq 1.10 \text{ Purity: } 0.14 \text{ Efficiency: } 0.02 \\ 0.90 &\leq E/p \leq 1.20 \text{ Purity: } 0.15 \text{ Efficiency: } 0.02 \\ 0.90 &\leq E/p \leq 1.10 \text{ Purity: } 0.15 \text{ Efficiency: } 0.02 \\ \end{split}$$



 $0.00 \le E/p \le 2.00$  Purity: 0.01 Efficiency: 1.00  $0.60 \le E/p \le 2.00$  Purity: 0.10 Efficiency: 0.57  $0.80 \le E/p \le 1.20$  Purity: 0.26 Efficiency: 0.50  $0.80 \le E/p \le 2.00$  Purity: 0.20 Efficiency: 0.50  $0.88 \le E/p \le 1.10$  Purity: 0.39 Efficiency: 0.50  $0.90 \le E/p \le 1.20$  Purity: 0.36 Efficiency: 0.47  $0.90 \le E/p \le 1.10$  Purity: 0.41 Efficiency: 0.47







$$\begin{split} 0.00 & \leq E/p \leq 2.00 \text{ Purity: } 0.11 \text{ Efficiency: } 1.00 \\ 0.60 & \leq E/p \leq 2.00 \text{ Purity: } 0.15 \text{ Efficiency: } 0.04 \\ 0.80 & \leq E/p \leq 1.20 \text{ Purity: } 0.32 \text{ Efficiency: } 0.04 \\ 0.80 & \leq E/p \leq 2.00 \text{ Purity: } 0.27 \text{ Efficiency: } 0.04 \\ 0.88 & \leq E/p \leq 1.10 \text{ Purity: } 0.44 \text{ Efficiency: } 0.03 \\ 0.90 & \leq E/p \leq 1.20 \text{ Purity: } 0.45 \text{ Efficiency: } 0.03 \\ 0.90 & \leq E/p \leq 1.10 \text{ Purity: } 0.48 \text{ Efficiency: } 0.03 \\ \end{split}$$