QCD in space:
quark matter with loop calculations

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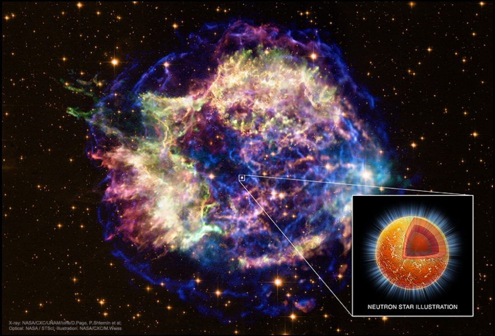
neutron stars with QCD

Quiescent neutron stars

neutron stars are celestial labs for dense (= lots of baryons in a small volume) strongly interacting matter



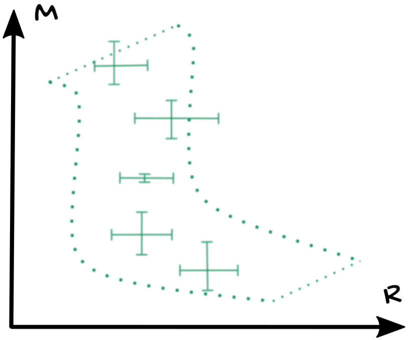
Cooling neutron star [Nasa, PD]

far denser than terrestrial labs and running all the time

classic observations like Mass, Radius from calm NSs

Path to new physics

Equation of State → Equations of Motion → Mass–Radius curve



Path to new physics

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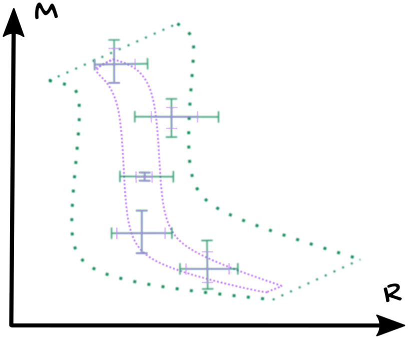


improve EoS with more accurate theory

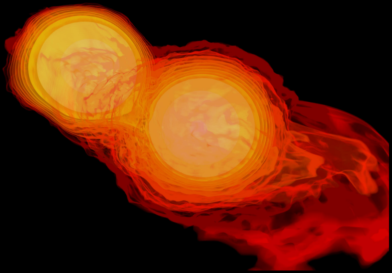
—or—

constrain points on the M, R-plane with better observations

One Day: Discrepancy! New Physics!!

Neutron star mergers

novel approach: neutron star mergers and gravitational waves



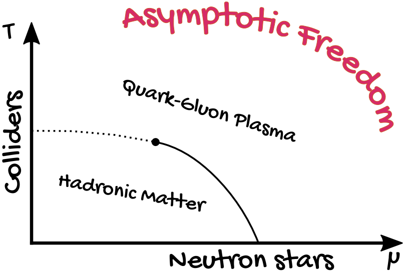
Merger simulation [Nasa, PD]

new observations: transport phenomena, tidal deformability, ...

Asymptotic Freedom

determining the EoS $\leftrightarrow$ determining the pressure

$$p \sim \ln Z \sim \ln \int \mathcal{D}x e^{-S}$$



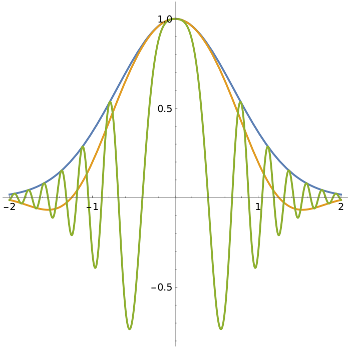
High densities
$\rightarrow$ high energies
$\rightarrow$ small couplings
$\rightarrow$ analytic control

...

... even neutron star cores aren't that far into deconfined phase: the coupling is never that small in real life

Lattice problems

in colliders, at finite temperature, simulating QFT on computers with lattice field theory works



finite density makes the action S complex $\rightarrow$ sign problem (e^{-S} is no longer a posdef probability distribution for Monte Carlo)

pQCD solutions (?)

goal: try to improve pQCD to get closer to NS cores to improve things on the denser side. requires very high precision, but gives unique and valuable insight to dense QCD:

- few other first-principles methods—at low densities effective nuclear / hadronic methods work great
- theoretically 'clean'—No fundamental obstacles, just difficult calculations
- and those calculations are fun—Lots of diagrams to draw and methods to use to compute loops: much more to understand vs vacuum (or even $T > 0$)

finite-density pQCD
(aka: I like diagrammatics)

Basic formalism

finite density $n \leftrightarrow$ finite chemical potential μ: 'Excess of stuff'

in momentum space: $p_0 \to p_0 - i\mu_f$ for every quark flavour f

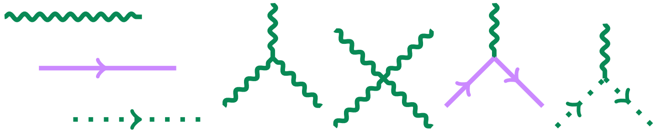
- complexifies integrands
- breaks Lorentz-invariance
- gives scales: μ_f an inherent property of the surrounding medium

(very similar to $T > 0$: $p_0 \to 2n\pi Tn$ or $(2n+1)\pi Tn$ for $n \in \mathbb{Z}$)

understanding diagrammatics helps understanding physics

QCD fields and pressure

three basic lines and four basic vertices in QCD:

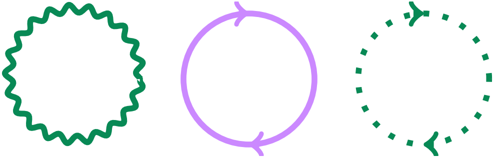


Feynman rules connect them to maths: more loops $\sim$ more integrals

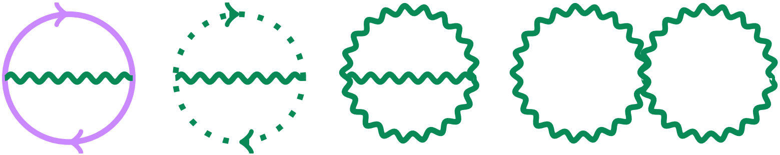
pressure (free energy, EoS): 'ln Z is the sum of all connected bubble diagrams', truncating to a given order gives a Taylor coefficients in coupling g_s

First two orders

first order is just simple loop (no interactions: free bosons and fermions)



at second order you also get vertices (QFT effects included now!)

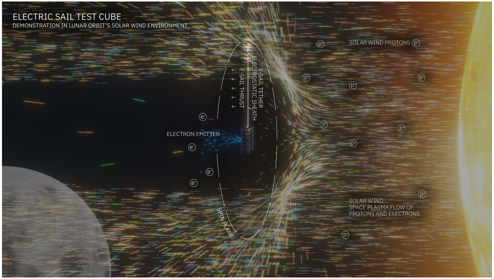


these are $\sim$ as easy to compute as they are to draw

integrals need a scale: only the graphs with quark lines carrying μ survive

Soft physics

particles interact with a thermal medium, especially low-energy (soft) gluons screened by all the dense stuff around them:

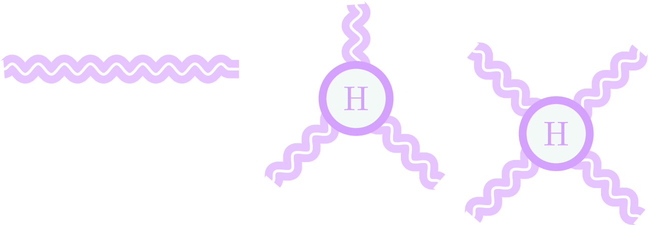


ESTCube / M. F. Palos, A. Maskava & R. M. Jansone [CC-BY 4.0]

diagrammatically: arbitrarily many loop insertions contribute at the same order for soft physics → combine them into a resummed line

Hard Thermal Loops

treat the soft gluons as extra lines and vertices with a screening mass $\propto \mu$: soft gluons and soft gluon vertices

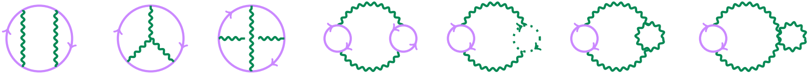


framework with ‘Hard Thermal Loops’ [Braaten–Pisarski NPB 337 (1990)], simplified description valid only for soft physics

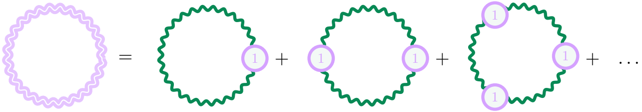
these ‘lag behind’ in a perturbative expansion: will not show up until the third order—but just keep drawing bubbles with these extra lines and keep track of when they show up

Third order and the ring sum

3-loop diagrams are manageable (only quark diagrams shown):



the first soft gluons show up: the soft gluon loop hides infinitely many diagrams inside it

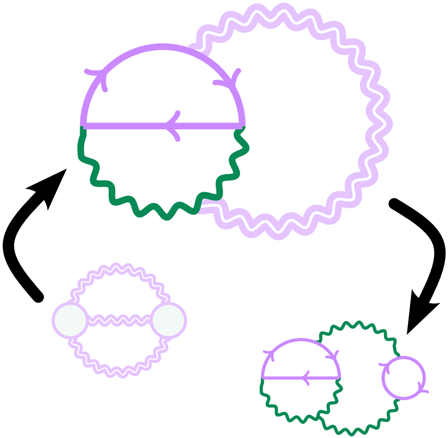


physically the pressure of a screened gluon

classic result [Freedman–McLerran, PRD 16 (1977)] with the rest of third order contributions

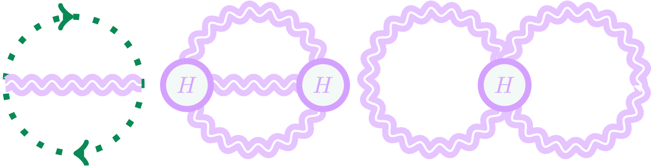
Fourth order and sectors

keep repeating the exercise—now three classes of diagrams: Soft , with just soft lines, Mixed , with soft and hard lines, Hard , with just hard lines



Soft sector

these are the same two-loop diagrams, but with soft lines (finite-density HTL is just for gluons, ghosts are needed for gauge invariance)

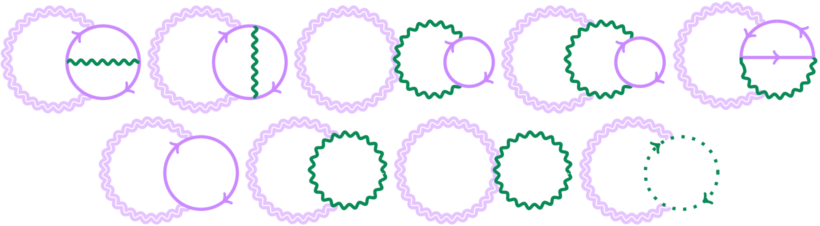


two-loop order screened pressure: how screened fields interact with other screened fields

[PRL 121 (2018) for one Taylor coefficient, PRL 104 & PRD 127 (both 2021) for the rest of the graphs, first in collab w/ Gorda, Kurkela, Romatschke, Vuorinen, last two -Romatschke +Paatelainen]

Mixed sector

one soft line connected to a hard graph



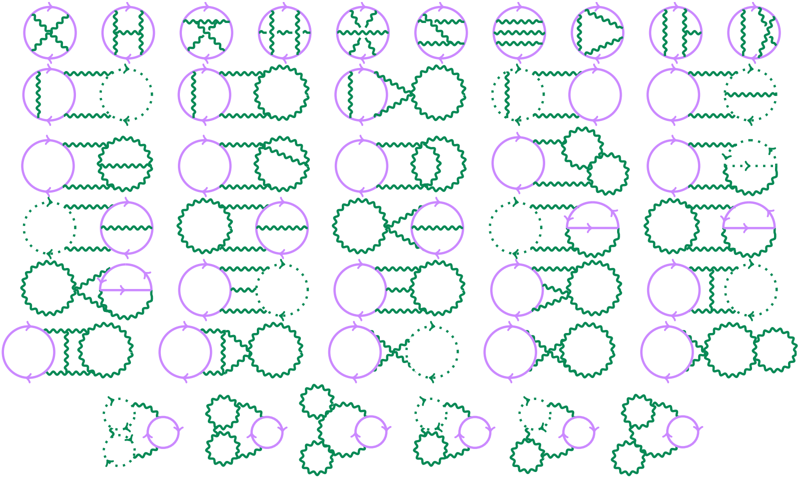
these are a bit unusual—there are contributions from all orders, and the ghost one already showed up before, just with soft momenta

they are corrections to the ring sum from the hard particles: how screened particles interact with hard particles

[2 × PRD 107 w/ Gorda, Kurkela, Österman, Paatelainen, Philipp Schicho, Seppänen, Vuorinen for QED , PRL 131 & JHEP 08 w/ Gorda, Paatelainen, Seppänen for QCD , all 2023]

Hard sector

easy to conceptualise, hard to calculate...



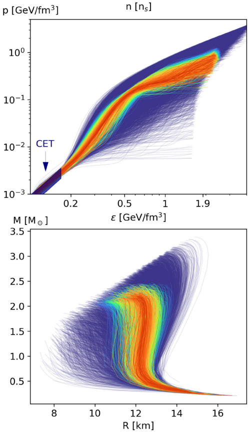
...and these are only the graphs without resummed counterparts!

Results

if you actually evaluate all of the above

$$
\begin{aligned}
p \approx \frac{3}{4\pi^2}\left(\frac{\mu_B}{3}\right)^4 \Bigg\{ 1 - 2\frac{\alpha_s}{\pi} \\
- 3\left(\frac{\alpha_s}{\pi}\right)^2 \left[\ln\left(3\frac{\alpha_s}{\pi}\right) + 3\ln X + 5.0 \right] \\
+ \left(\frac{\alpha_s}{\pi}\right)^3 \Bigg[\frac{11}{12}\ln^2\left(3\frac{\alpha_s}{\pi}\right) \\
- (-6.6 + 3\ln X)\ln\left(3\frac{\alpha_s}{\pi}\right) \\
+ 5.1 - 18.\ln X - \frac{9}{2} + \frac{2}{3}c_0 \Bigg] \Bigg\} + O(\alpha_s^4)
\end{aligned}
$$

this provides constraints to the behaviour of deconfined dense matter and NS cores, and tells us a part of the theory story about our stellar labs



[Gorda, Komoltsev, Kurkela, Astrophys.J. 950 (2023), cropped]

thanks for the attention,
have a space jellyfish.
(B0355 by Chandra [Nasa, PD])