

Effect of initial covariance matrix for real-seeded tracking

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with help from Jeetendra Gupta and Shujie Li

How does initial error matrix affect our tracking?

```
trackparam.setLocError({0.1,0.1}); //covariance of location  
trackparam.setMomentumError({0.05,0.05,0.05}); // covariance on theta/phi/q/p  
trackparam.setTimeError(0.1); // error on time
```

<https://github.com/eic/ElCrecon/blob/main/src/algorithms/tracking/TrackSeeding.cc>

These values seem to have been set somewhat arbitrarily.

How does initial error matrix affect our tracking?

Parameters Covariance matrix

$$C = \begin{bmatrix} \sigma^2(l_0) & \text{cov}(l_0, l_1) & \text{cov}(l_0, \phi) & \text{cov}(l_0, \theta) & \text{cov}(l_0, q/p) \\ \cdot & \sigma^2(l_1) & \text{cov}(l_1, \phi) & \text{cov}(l_1, \theta) & \text{cov}(l_1, q/p) \\ \cdot & \cdot & \sigma^2(\phi) & \text{cov}(\phi, \theta) & \text{cov}(\phi, q/p) \\ \cdot & \cdot & \cdot & \sigma^2(\theta) & \text{cov}(\theta, q/p) \\ \cdot & \cdot & \cdot & \cdot & \sigma^2(q/p) \end{bmatrix}$$

Evolution of parameters covariance

$$C^f = J \cdot C^i \cdot J^T,$$

$$J = \begin{bmatrix} \frac{\partial l_0^f}{\partial l_0^i} & \cdots & \frac{\partial l_0^f}{\partial (q/p)^i} \\ \vdots & \ddots & \vdots \\ \frac{\partial (q/p)^f}{\partial l_0^i} & \cdots & \frac{\partial (q/p)^f}{\partial (q/p)^i} \end{bmatrix},$$

Hit residual and chi-square

$$\text{Res} = \vec{x}_{\text{calibrated}} - \mathbf{H}\vec{x}_{\text{predicted}}$$

$$\chi^2 = \text{Res}^T (\mathbf{C}_{\text{calibrated}} + \mathbf{H}\mathbf{C}_{\text{predicted}}\mathbf{H}^T)^{-1} \text{Res}$$

H is observation matrix

$\mathbf{C}_{\text{calibrated}}$ is measurement covariance

$\mathbf{C}_{\text{predicted}}$ is predicted estimate covariance

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Hit residual and chi-square

$$\text{Res} = \vec{x}_{\text{calibrated}} - \mathbf{H}\vec{x}_{\text{predicted}}$$

$$\chi^2 = \text{Res}^T \left(\mathbf{C}_{\text{calibrated}} + \mathbf{H}\mathbf{C}_{\text{predicted}}\mathbf{H}^T \right)^{-1} \text{Res}$$

For Silicon endcap hits, for example, we have

Evolution of parameters covariance

$$C^f = J \cdot C^i \cdot J^T,$$

$$J = \begin{bmatrix} \frac{\partial l_0^f}{\partial l_0^i} & \cdots & \frac{\partial l_0^f}{\partial (q/p)^i} \\ \vdots & \ddots & \vdots \\ \frac{\partial (q/p)^f}{\partial l_0^i} & \cdots & \frac{\partial (q/p)^f}{\partial (q/p)^i} \end{bmatrix},$$

$$\sigma_{xx} = \left(\frac{20 \text{ um}}{\sqrt{12}} \right)^2 = 0.000033 \text{ mm}^2$$

```
SiEndcapTrackerRecHits.position.x = 207.386734, 108.986137, 1
SiEndcapTrackerRecHits.position.y = -325.804352, -168.049530, 1
SiEndcapTrackerRecHits.position.z = 1349.864990, 699.864990, 1
SiEndcapTrackerRecHits.positionError.xx = 0.000033, 0.000033, 1
SiEndcapTrackerRecHits.positionError.yy = 0.000033, 0.000033, 1
SiEndcapTrackerRecHits.positionError.zz = 0.000000, 0.000000, 1
```

Hits in tracking detectors for 10k single-muon events

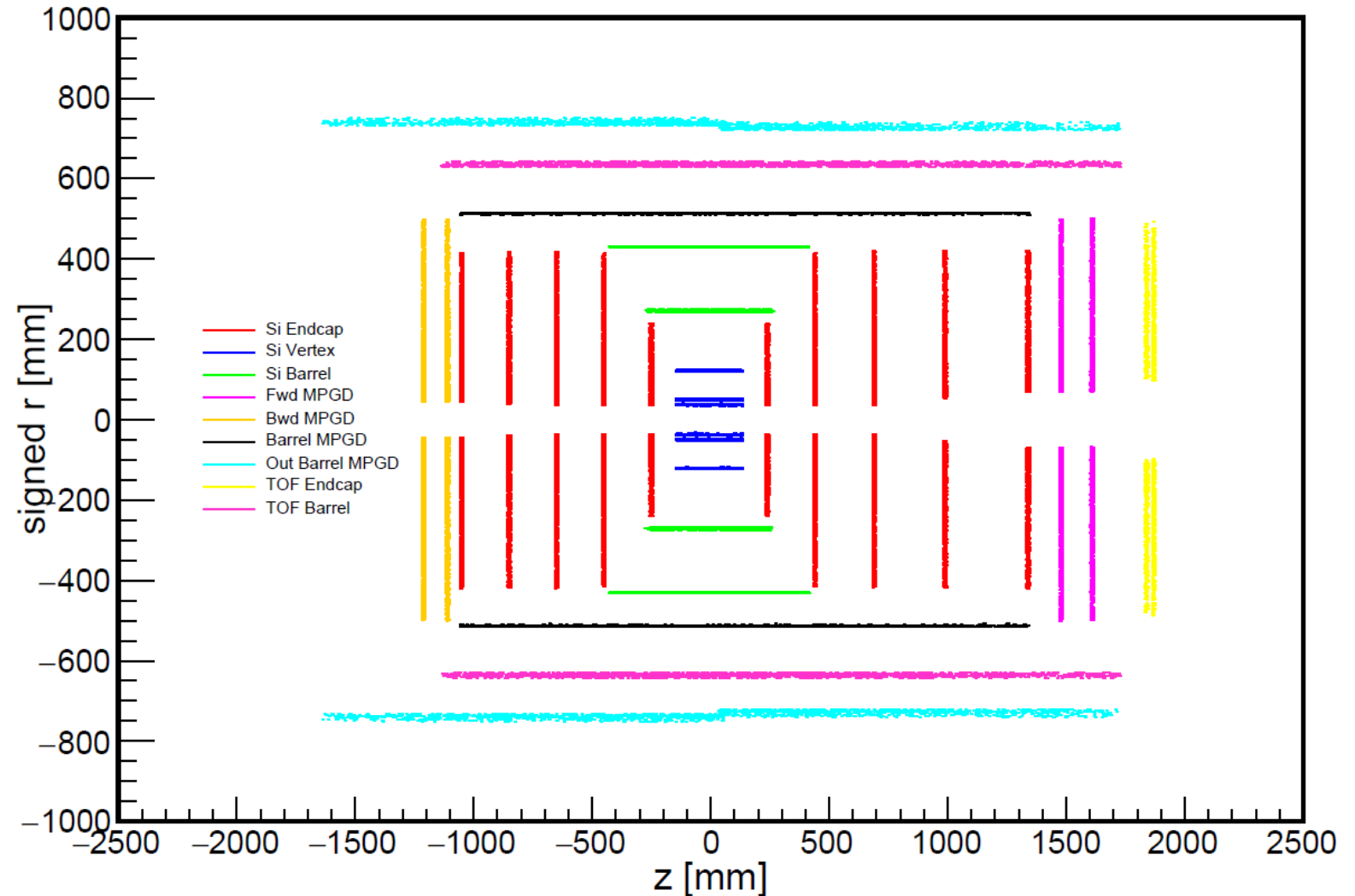
Geant-level tracker hits

Single μ^- generated:

$0.5 \text{ GeV}/c < P < 20 \text{ GeV}/c$

$-4 < \eta < 4$

Generated vertex: (0,0,0) mm

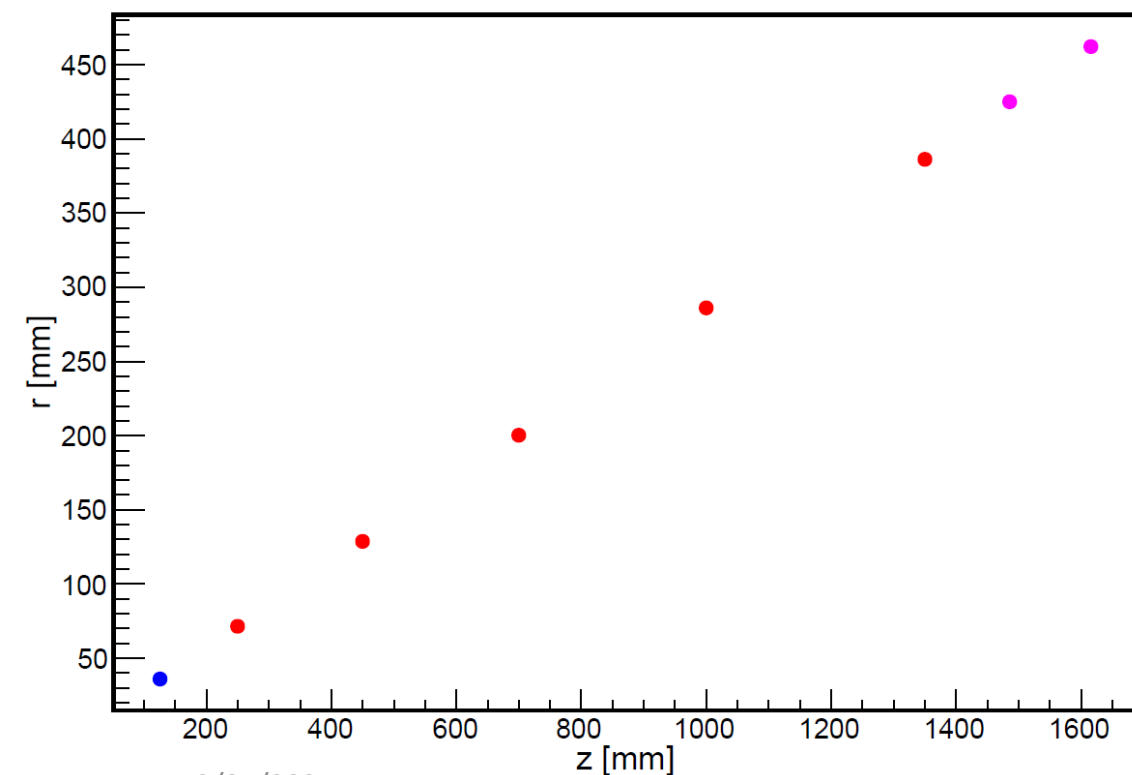


Tracker hits associated with primary particle in event #1

Muon (px,py,pz) = (2.53,-3.82,16.03) GeV/c

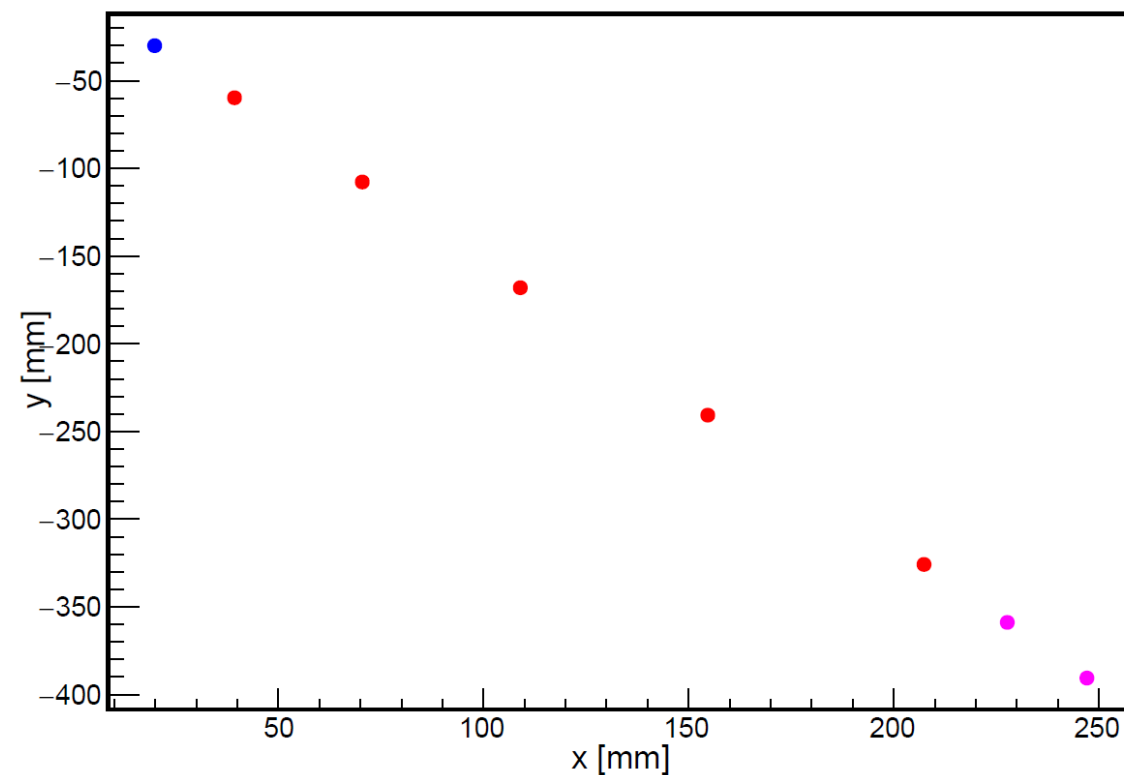
- Si Endcap
- Si Vertex
- Si Barrel
- Fwd MPGD
- Bwd MPGD
- Barrel MPGD
- Out Barrel MPGD
- TOF Endcap
- TOF Barrel

Tracker hits from primary particle for event 1



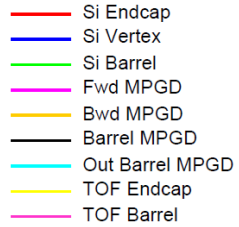
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Tracker hits from primary particle for event 1



6

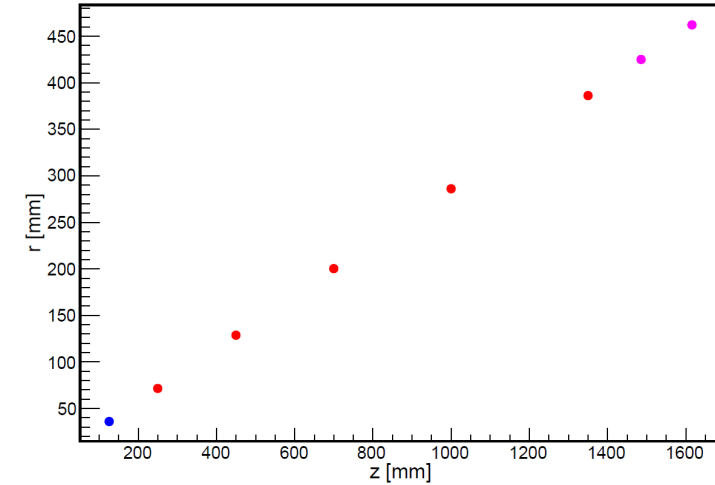
Reconstructed tracks in event #1



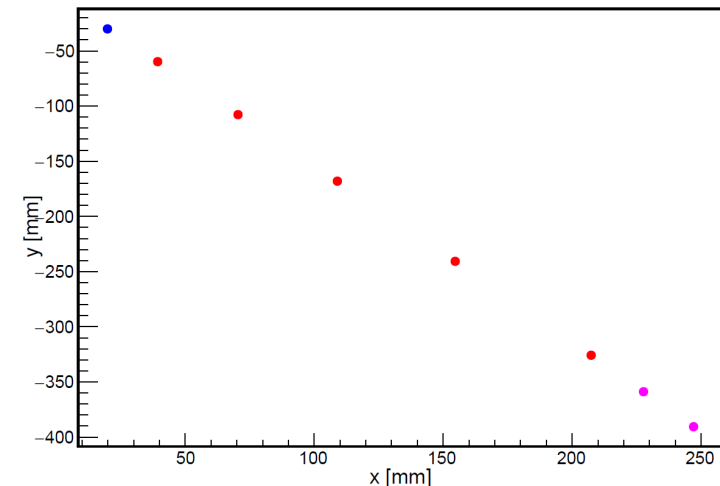
Muon (px,py,pz) = (2.53,-3.82,16.03) GeV/c

Event #	Track #	# Meas.	# Outliers	Total Chi2	Track Total Momentum [GeV/c]
* 1 *	0 *	6 *	2 *	9.7384395 *	16.736422 *
* 1 *	1 *	6 *	2 *	20.329893 *	16.844126 *
* 1 *	2 *	6 *	2 *	3.8973052 *	16.771324 *
* 1 *	3 *	6 *	2 *	3.9100885 *	16.763809 *
* 1 *	4 *	6 *	2 *	3.7853021 *	16.765380 *

Tracker hits from primary particle for event 1



Tracker hits from primary particle for event 1



Acts hits selection

For given track state, calculate chi2 of all hits on surface and rank, find chi2_{\min}

- If no hit on surface → Hole
- If $\text{chi2}_{\min} > \text{chi2}_{\text{cutoff}}$ (default = 15) → Outlier
- If $\text{chi2}_{\min} < \text{chi2}_{\text{cutoff}}$ → Measurement (up to $\text{numMeasurements}_{\text{CutOff}}=10$ default)

Reconstructed tracks in event #1

- Si Endcap
- Si Vertex
- Si Barrel
- Fwd MPGD
- Bwd MPGD
- Barrel MPGD
- Out Barrel MPGD
- TOF Endcap
- TOF Barrel

Muon (px,py,pz) = (2.53,-3.82,16.03) GeV/c

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* 1 *	3 *	6 *	2 *	3.9100885 *	16.763809 *
* 1 *	4 *	6 *	2 *	3.7853021 *	16.765380 *

All tracks have same set of hits as measurements and outliers, but very different chi-square values

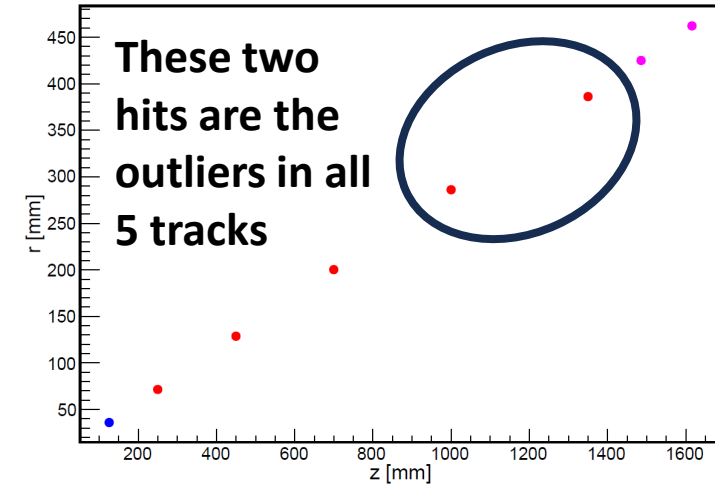
CentralCKFSeededTrajectories.outlierHits_begin = 0, 2, 4, 6, 8

CentralCKFSeededTrajectories.outlierHits_end = 2, 4, 6, 8, 10

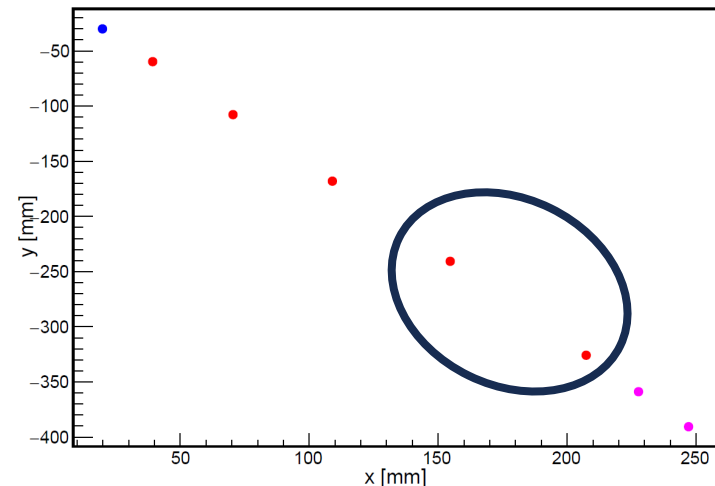
_CentralCKFSeededTrajectories_outlierHits.index = 1, 3, 1, 3, 1, 3, 1, 3, 1, 3

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Tracker hits from primary particle for event 1



Tracker hits from primary particle for event 1



What if we increase these initial errors by a factor of 10?

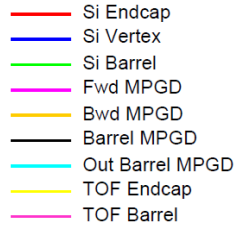
x10 for all 6 numbers
(time is not used)

```
trackparam.setLocError({0.1,0.1}); //covariance of location  
trackparam.setMomentumError({0.05,0.05,0.05}); // covariance on theta/phi/q/p  
trackparam.setTimeError(0.1); // error on time
```

<https://github.com/eic/ElCrecon/blob/main/src/algorithms/tracking/TrackSeeding.cc>

These values seem to have been set somewhat arbitrarily.

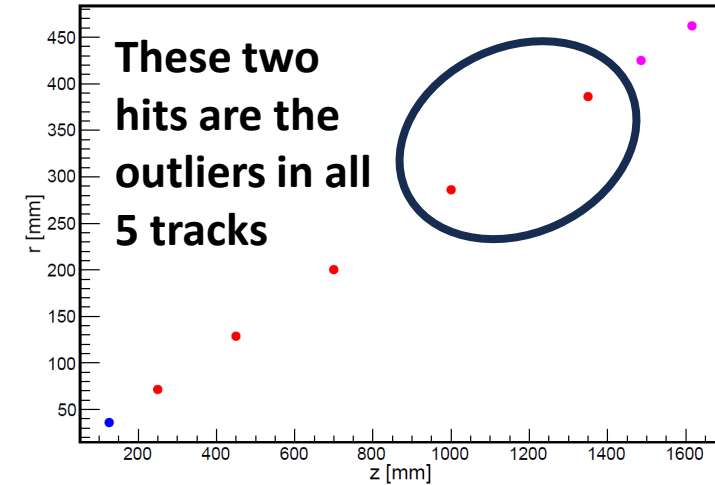
Reconstructed tracks in event #1



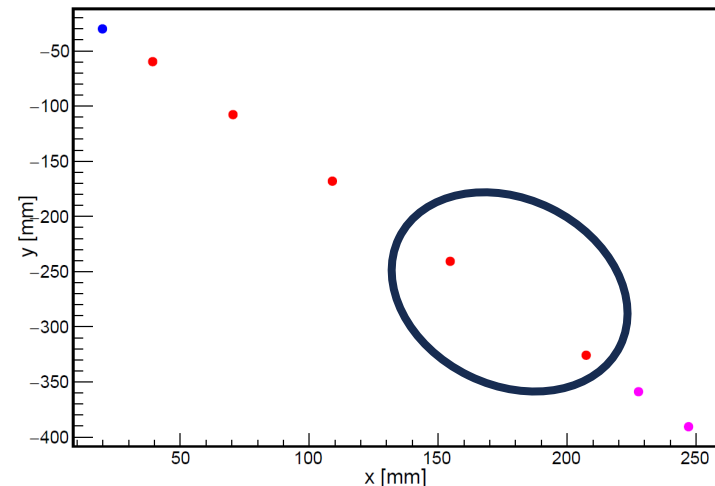
Muon (px,py,pz) = (2.53,-3.82,16.03) GeV/c

Event #	Track #	# Meas.	# Outliers	Total Chi2	Track Total Momentum [GeV/c]	
* 1 *	0 *	6 *	2 *	9.7384395 *	16.736422 *	with default initial error matrix values
* 1 *	1 *	6 *	2 *	20.329893 *	16.844126 *	
* 1 *	2 *	6 *	2 *	3.8973052 *	16.771324 *	
* 1 *	3 *	6 *	2 *	3.9100885 *	16.763809 *	
* 1 *	4 *	6 *	2 *	3.7853021 *	16.765380 *	
* 1 *	0 *	6 *	2 *	3.8872442 *	16.766671 *	With x10 initial error matrix values
* 1 *	1 *	6 *	2 *	4.0938601 *	16.758594 *	
* 1 *	2 *	6 *	2 *	3.7222030 *	16.762185 *	
* 1 *	3 *	6 *	2 *	3.7234082 *	16.761795 *	
* 1 *	4 *	6 *	2 *	3.7219612 *	16.761828 *	

Tracker hits from primary particle for event 1



Tracker hits from primary particle for event 1



Per-layer chi-square values for **measurements** (sum is equal to total track chi-square)

Default initial error values

1.6551995
0.7588014
2.2014386
9.8320589
5.8656659
0.0167279

x10 initial error values

1.7308549
0.5881769
0.8303119
0.8844062
0.0599426
0.0001672

Per-layer chi-square values for **outliers**

Default initial error values

46.156986
23.003814

x10 initial error values

46.684082
23.178184

Track Total Momentum [GeV/c]

16.736422 *

with default initial error matrix values

* 1 * 1 * 6 * 2 * 20.329893 * 16.844126 *

* 1 * 2 * 6 * 2 * 3.8973052 * 16.771324 *

* 1 * 3 * 6 * 2 * 3.9100885 * 16.763809 *

* 1 * 4 * 6 * 2 * 3.7853021 * 16.765380 *

* 1 * 0 * 6 * 2 * 3.8872442 * 16.766671 *

* 1 * 1 * 6 * 2 * 4.0938601 * 16.758594 *

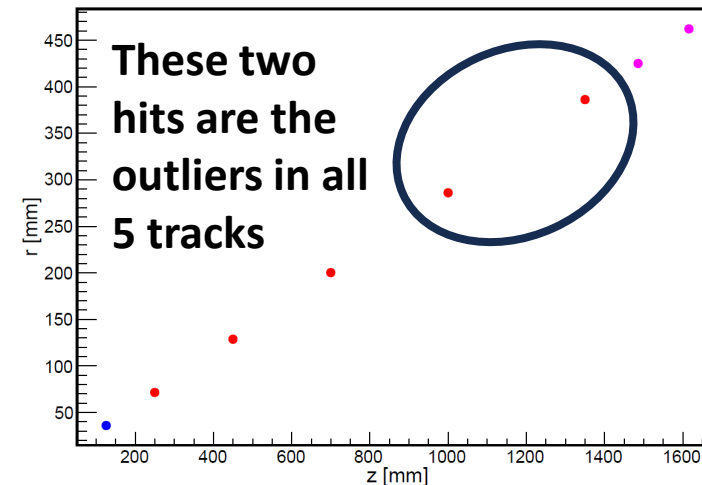
* 1 * 2 * 6 * 2 * 3.7222030 * 16.762185 *

* 1 * 3 * 6 * 2 * 3.7234082 * 16.761795 *

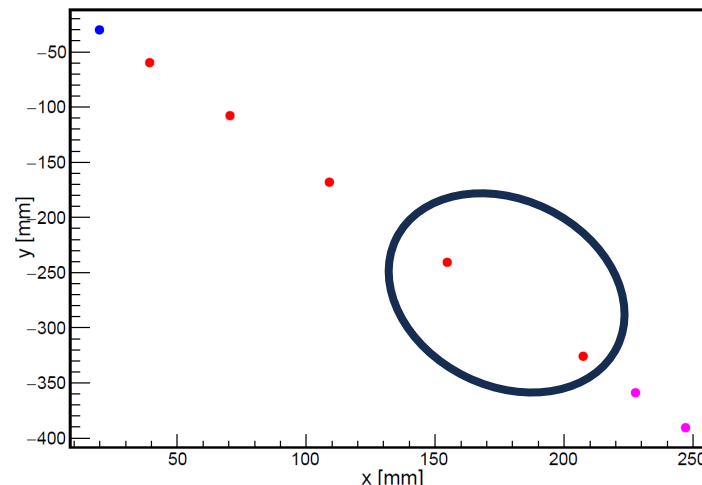
* 1 * 4 * 6 * 2 * 3.7219612 * 16.761828 *

With x10 initial error matrix values

Tracker hits from primary particle for event 1



Tracker hits from primary particle for event 1



What about truth-seeded tracking for event #1?

- Si Endcap
- Si Vertex
- Si Barrel
- Fwd MPGD
- Bwd MPGD
- Barrel MPGD
- Out Barrel MPGD
- TOF Endcap
- TOF Barrel

Muon (px,py,pz) = (2.53,-3.82,16.03) GeV/c

Event #	Track #	# Meas.	# Outliers	Total Chi2	Track Total Momentum [GeV/c]
1 *	0 *	6 *	2 *	3.7294552 *	16.762197

Measurement chi-square per-layer

1.8605283
 0.5367326
 0.5832352
 0.7475767
 0.0013756
 6.546e-06

Outlier chi-square per layer

53.284168
 26.377220

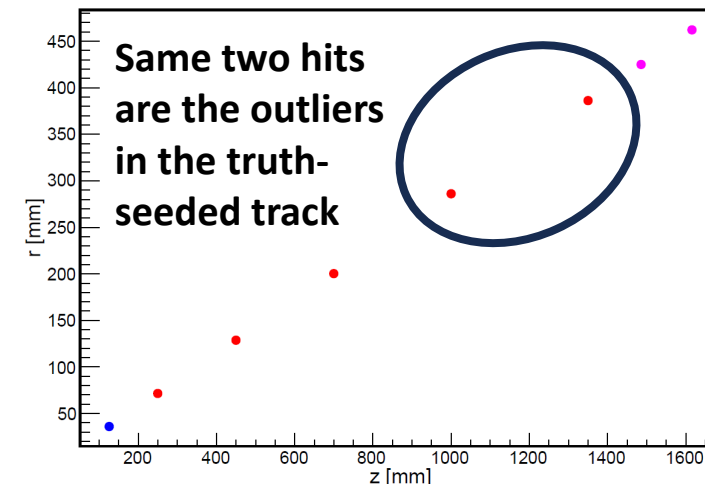
Initial track error matrix values for truth-seeded tracking

```
track_parameter.setLocError({1.0, 1.0}); // sqrt(variance) of location [mm]
track_parameter.setMomentumError({0.01, 0.05, 0.1}); // sqrt(variance) on theta, phi, q/p [rad, rad, e/GeV]
track_parameter.setTimeError(10e9); // error on time [ns]
```

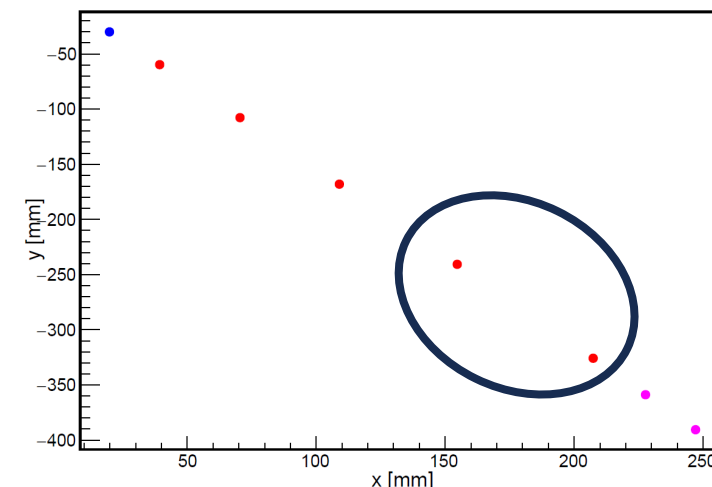
<https://github.com/eic/EICrecon/blob/main/src/algorithms/tracking/TrackParamTruthInit.cc>

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Tracker hits from primary particle for event 1



Tracker hits from primary particle for event 1



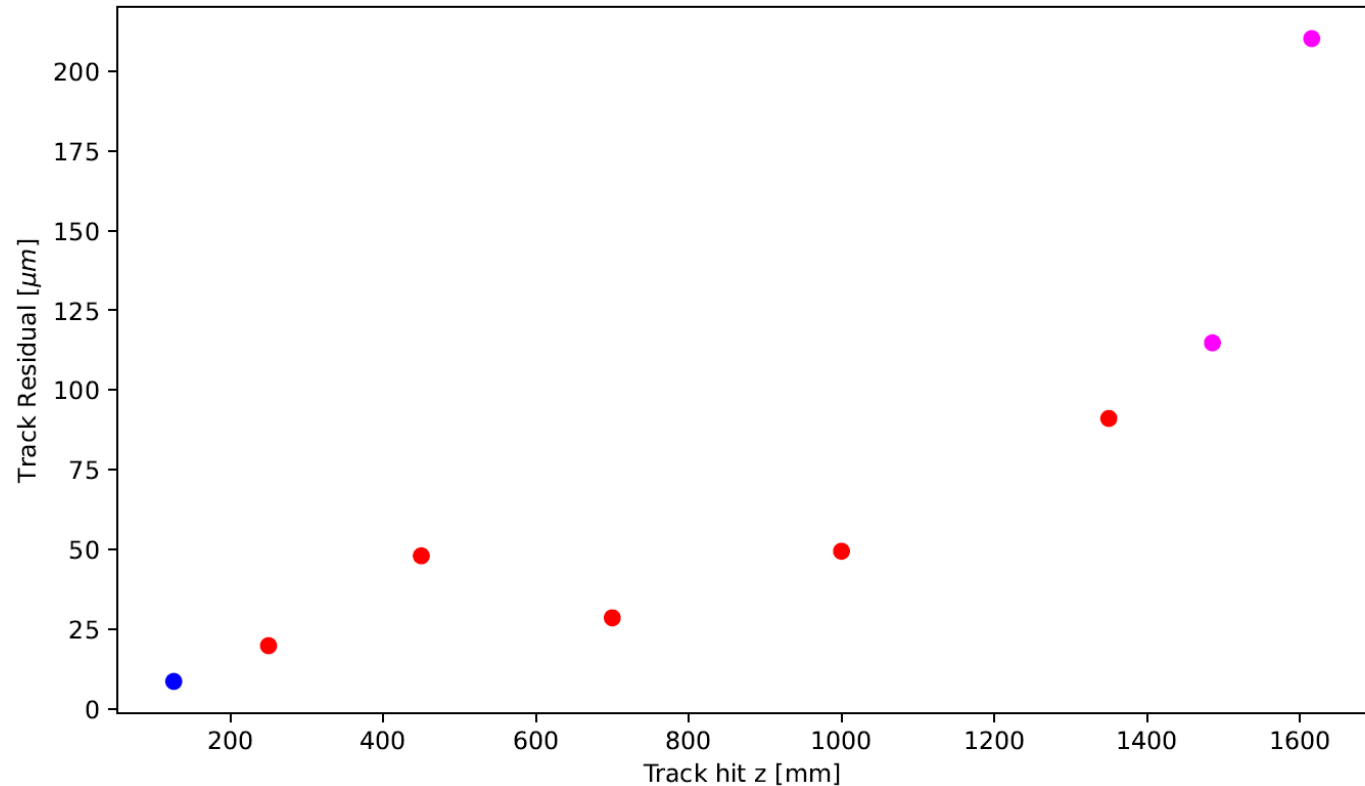
Track residuals for truth-seeded tracking in event #1

- Si Endcap
- Si Vertex
- Si Barrel
- Fwd MPGD
- Bwd MPGD
- Barrel MPGD
- Out Barrel MPGD
- TOF Endcap
- TOF Barrel

For truth-seeded tracking, we save the track states (x,y,z) at the various tracking layers. So, we can calculate residuals.

Vertex layer residual: $\sqrt{r\Delta\phi^2 + \Delta z^2}$

Si endcap and Fwd MPGD residuals: $\sqrt{\Delta x^2 + \Delta y^2}$



Track residuals for truth-seeded tracking in event #1

- Si Endcap
- Si Vertex
- Si Barrel
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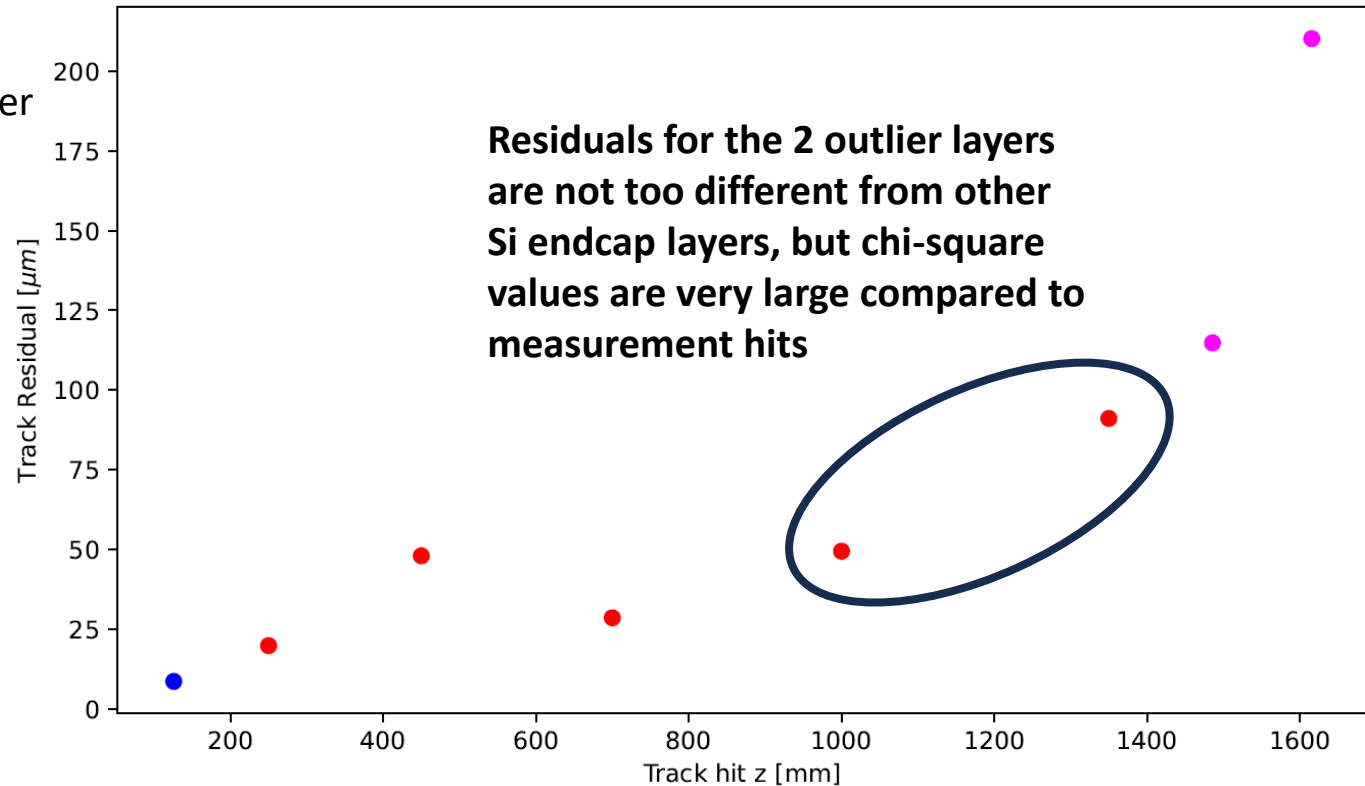
Si endcap and Fwd MPGD residuals: $\sqrt{\Delta x^2 + \Delta y^2}$

Measurement chi-square per-layer

1.8605283
0.5367326
0.5832352
0.7475767
0.0013756
6.546e-06

Outlier chi-square per layer

53.284168
26.377220



Track residuals for truth-seeded tracking in event #1

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- Fwd MPGD
- Bwd MPGD
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- Out Barrel MPGD
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- TOF Barrel

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Vertex layer residual: $\sqrt{r\Delta\phi^2 + \Delta z^2}$

Si endcap and Fwd MPGD residuals: $\sqrt{\Delta x^2 + \Delta y^2}$

Measurement chi-square per-layer

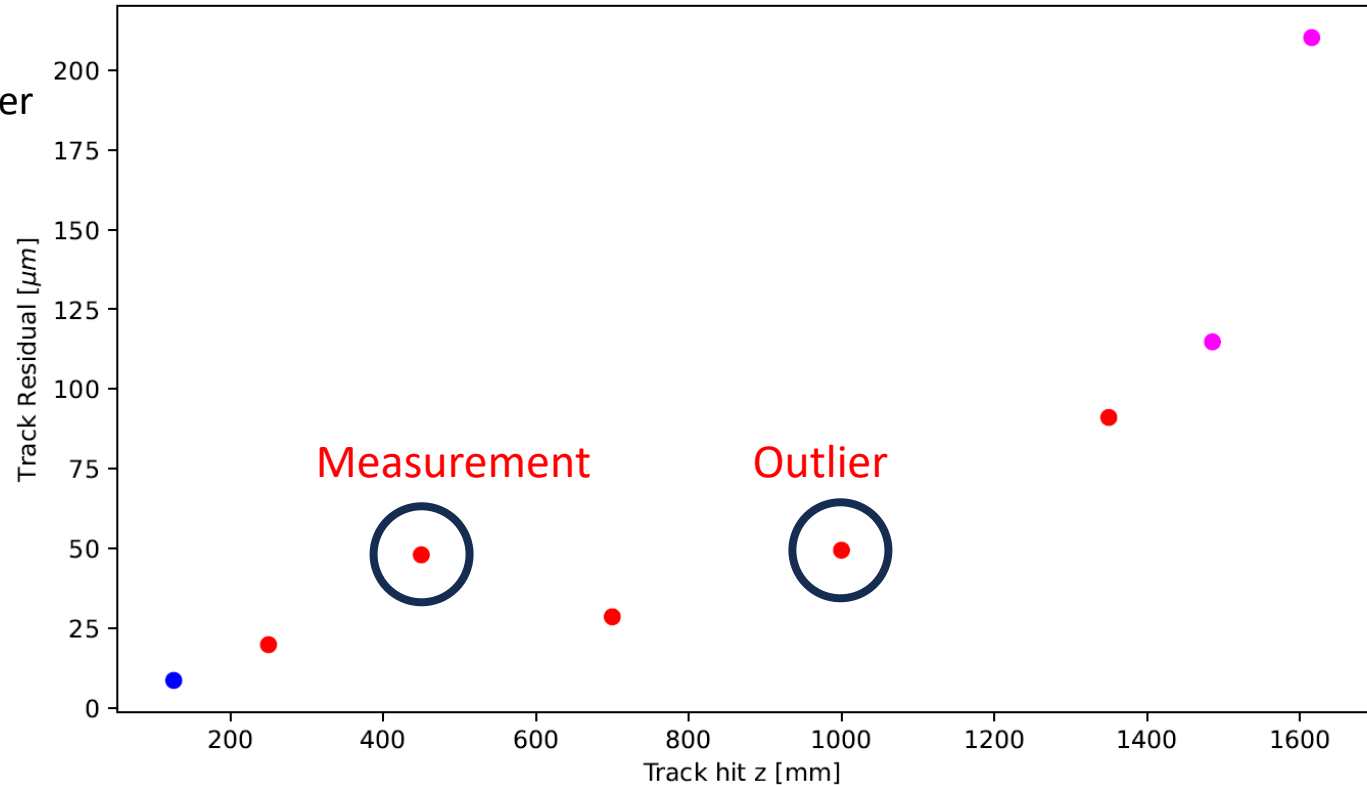
Outlier chi-square per layer

- 1.8605283
- 0.5367326
- 0.5832352
- 0.7475767
- 0.0013756
- 6.546e-06

- 53.284168
- 26.377220

Why is there such a large chi-square difference if the residuals are comparable? $C_{\text{calibrated}}$ is same for all Silicon endcap hits.

$$\chi^2 = \text{Res}^T (C_{\text{calibrated}} + \mathbf{H}C_{\text{predicted}}\mathbf{H}^T)^{-1} \text{Res}$$



Results for a few events – real-seeded tracking

Event # Track # # Meas. # Outliers Total Chi2 Track Total Momentum [GeV/c]

* 0 *	0 *	5 *	0 *	11.943484 *	13.88873 *	* 0 *	0 *	5 *	0 *	10.844925 *	13.883998 *
* 0 *	1 *	5 *	0 *	11.479335 *	13.898334 *	* 0 *	1 *	5 *	0 *	10.843421 *	13.884253 *
* 0 *	2 *	5 *	0 *	10.930010 *	13.887604 *	* 0 *	2 *	5 *	0 *	10.828397 *	13.884164 *
* 1 *	0 *	6 *	2 *	9.7384395 *	16.736422 *	* 1 *	0 *	6 *	2 *	3.8872442 *	16.766671 *
* 1 *	1 *	6 *	2 *	20.329893 *	16.844126 *	* 1 *	1 *	6 *	2 *	4.0938601 *	16.758594 *
* 1 *	2 *	6 *	2 *	3.8973052 *	16.771324 *	* 1 *	2 *	6 *	2 *	3.7222030 *	16.762185 *
* 1 *	3 *	6 *	2 *	3.9100885 *	16.763809 *	* 1 *	3 *	6 *	2 *	3.7234082 *	16.761795 *
* 1 *	4 *	6 *	2 *	3.7853021 *	16.765380 *	* 1 *	4 *	6 *	2 *	3.7219612 *	16.761828 *
* 2 *	0 *	3 *	4 *	20.717800 *	2.0906528 *	* 2 *	0 *	4 *	3 *	4.7161579 *	2.0912679 *
* 2 *	1 *	4 *	3 *	4.6227774 *	2.0913856 *	* 2 *	1 *	4 *	3 *	4.5198316 *	2.0912179 *
* 2 *	2 *	4 *	3 *	5.3654336 *	2.0905943 *	* 2 *	2 *	4 *	3 *	4.5265750 *	2.0912083 *
* 3 *	0 *	2 *	0 *	9.0088758 *	5.2862491 *	* 3 *	0 *	7 *	0 *	15.135988 *	19.400565 *
* 3 *	1 *	7 *	0 *	25.266552 *	19.254458 *	* 3 *	1 *	7 *	0 *	13.530672 *	19.385273 *
* 3 *	2 *	7 *	0 *	13.754158 *	19.407079 *	* 3 *	2 *	7 *	0 *	13.315017 *	19.385572 *
* 3 *	3 *	7 *	0 *	13.493220 *	19.401356 *	* 3 *	3 *	7 *	0 *	13.309839 *	19.385123 *
* 3 *	4 *	7 *	0 *	13.765888 *	19.399163 *	* 3 *	4 *	7 *	0 *	13.313103 *	19.385551 *
* 4 *	0 *	6 *	0 *	22.044910 *	2.5025064 *	* 4 *	0 *	6 *	0 *	21.834493 *	2.5023918 *
* 4 *	1 *	6 *	0 *	23.182231 *	2.502683 *	* 4 *	1 *	6 *	0 *	21.850294 *	2.5023879 *
* 5 *	0 *	6 *	0 *	23.770883 *	19.333478 *	* 5 *	0 *	6 *	0 *	23.249582 *	19.329668 *
* 5 *	1 *	6 *	0 *	23.226663 *	19.329731 *	* 5 *	1 *	6 *	0 *	23.215953 *	19.329855 *
* 5 *	2 *	6 *	0 *	23.250181 *	19.330913 *	* 5 *	2 *	6 *	0 *	23.221796 *	19.329798 *
* 5 *	3 *	6 *	0 *	23.263805 *	19.329128 *	* 5 *	3 *	6 *	0 *	23.213615 *	19.329921 *

with default
initial error
matrix values

With x10 initial
error matrix
values

Conclusions

- Real-seeded tracking results are sensitive to initial values used in the covariance matrix.
- If these values were set arbitrarily, we should think about how best to set them going forward:
 1. If we have tracks that have the same set of measurement hits and outlier hit, they should have the about the same track parameters and chi-squared. So, we should not be sensitive to which triplet is used for seeding.
 2. We can look at the seed resolutions and set the initial error values to be some factor larger than the resolution.
- Tried to understand hit residuals and chi-squared values.