Theoretical highlights on anisotropic flow calculations

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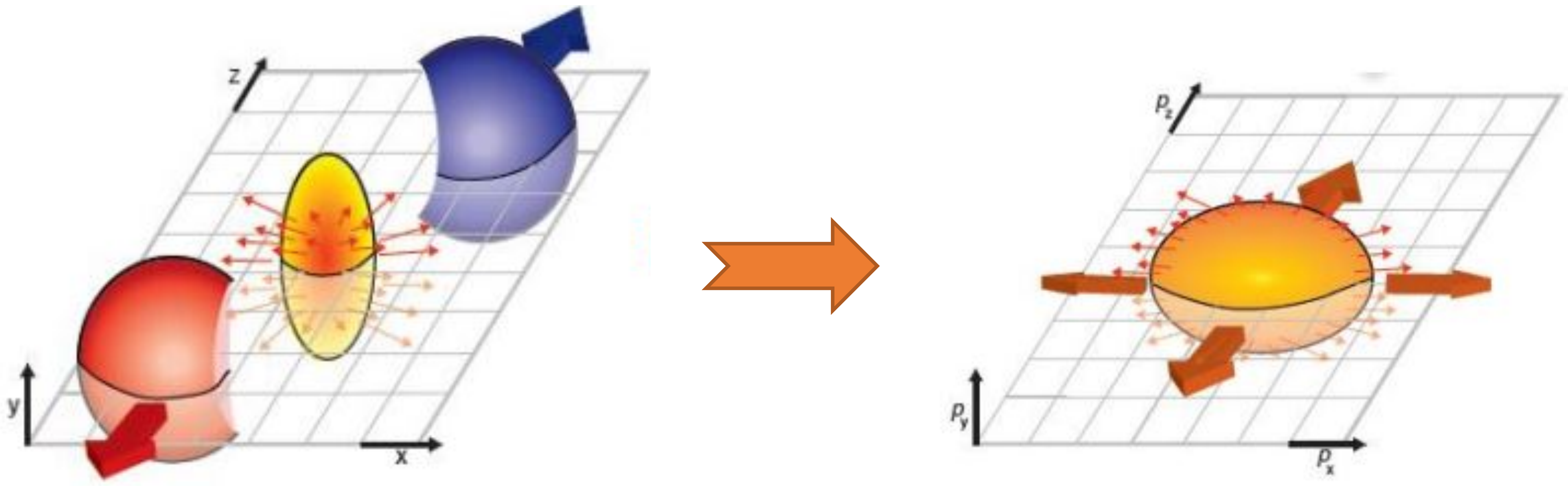
RHIC/AGS Annual Users' Meeting
June.11.2024

QGP signal (collective flow)

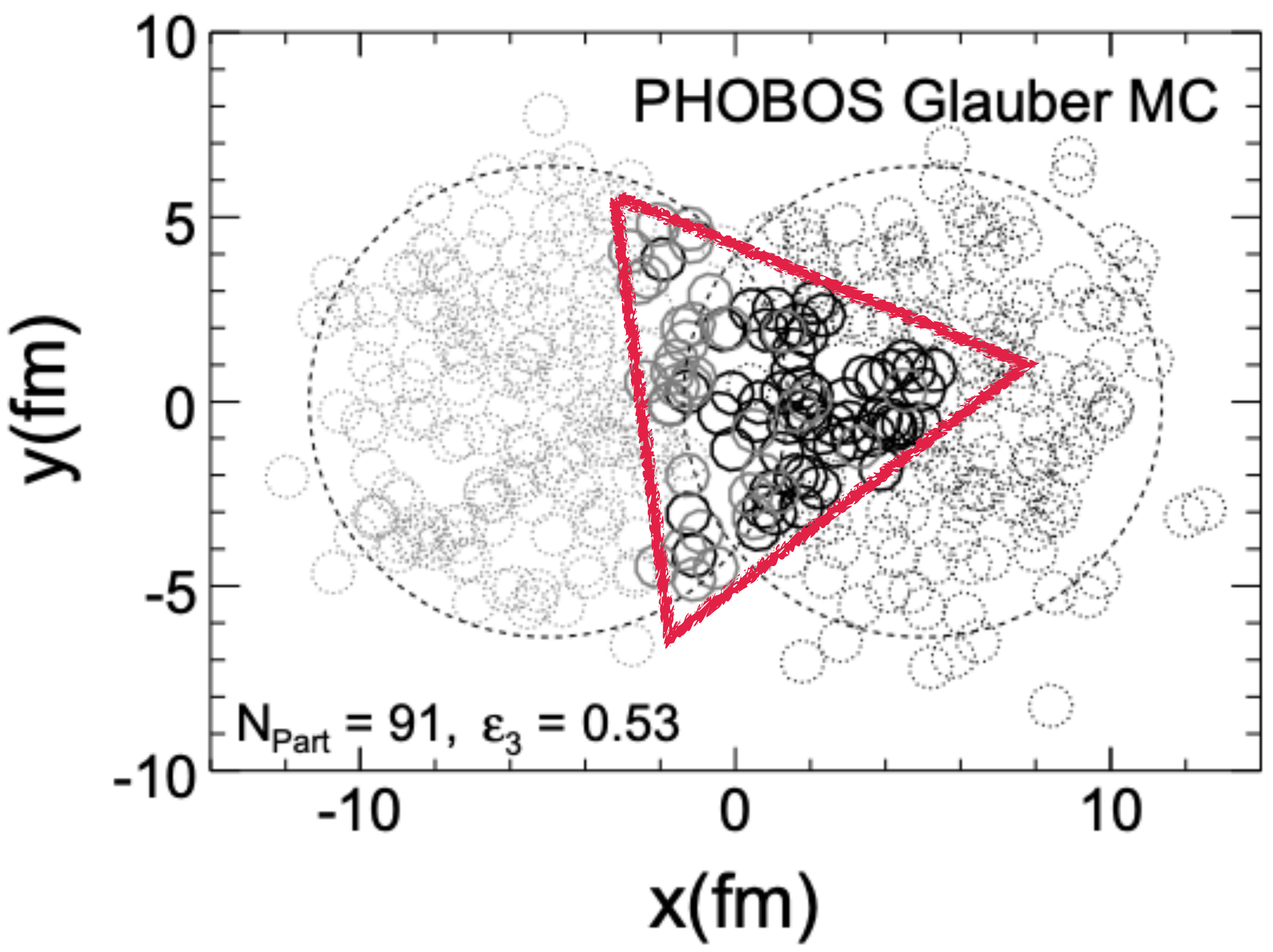


$$\frac{dN}{p_T dp_T dy d\phi} = \frac{dN}{2\pi p_T dp_T dy}\left[1 + 2\sum_n v_n\left(p_T, y\right)\cos\left(n\left(\phi - \Psi_n\left(p_T, y\right)\right)\right)\right]$$

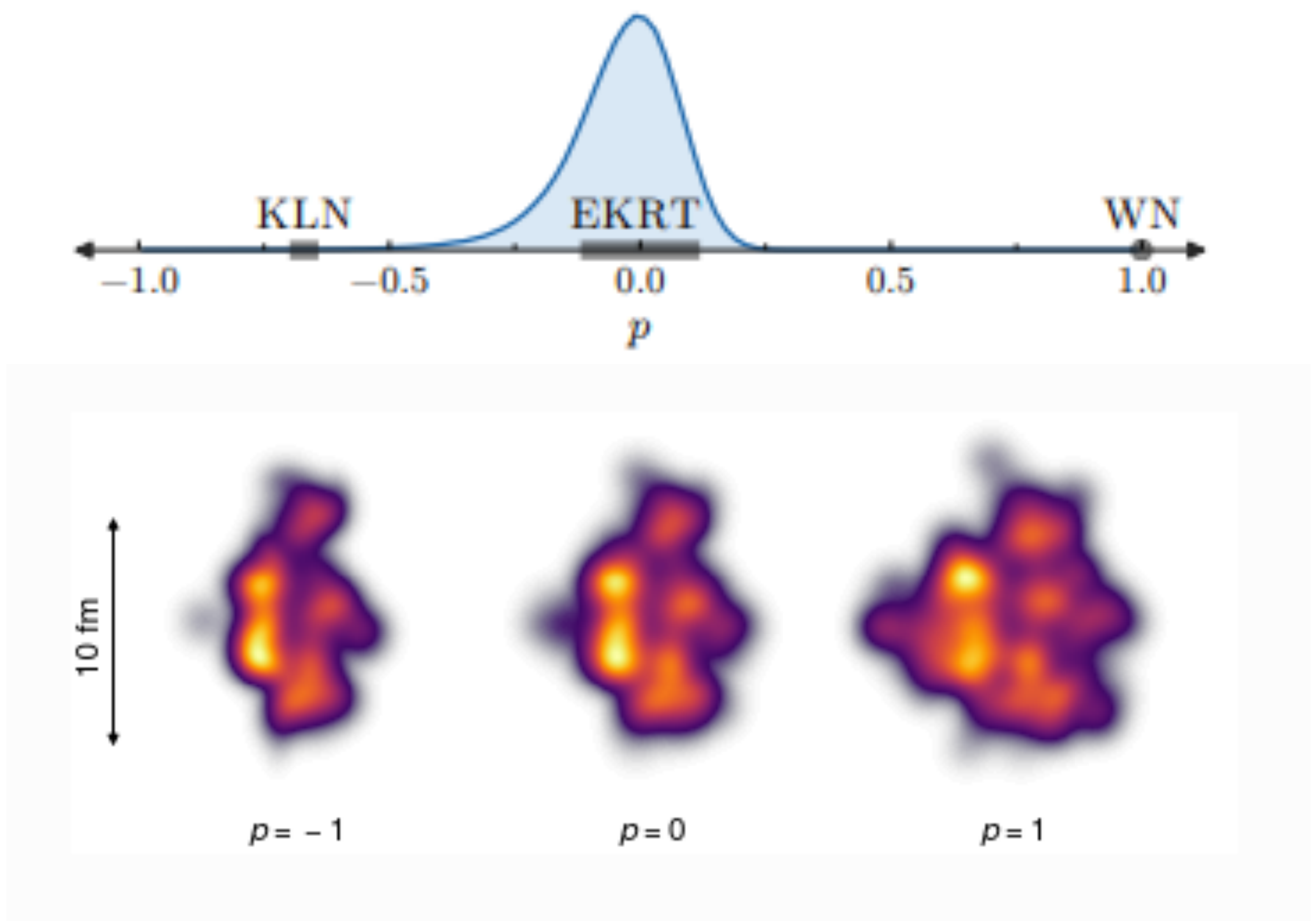
The collective flow of the QGP fireball converts the initial geometric anisotropy into final momentum anisotropy.

Why anisotropic flow/Flow fluctuations?

- Constrain the initial state
  - ✦ The initial spatial geometry $(v_2, v_3 \ldots)$



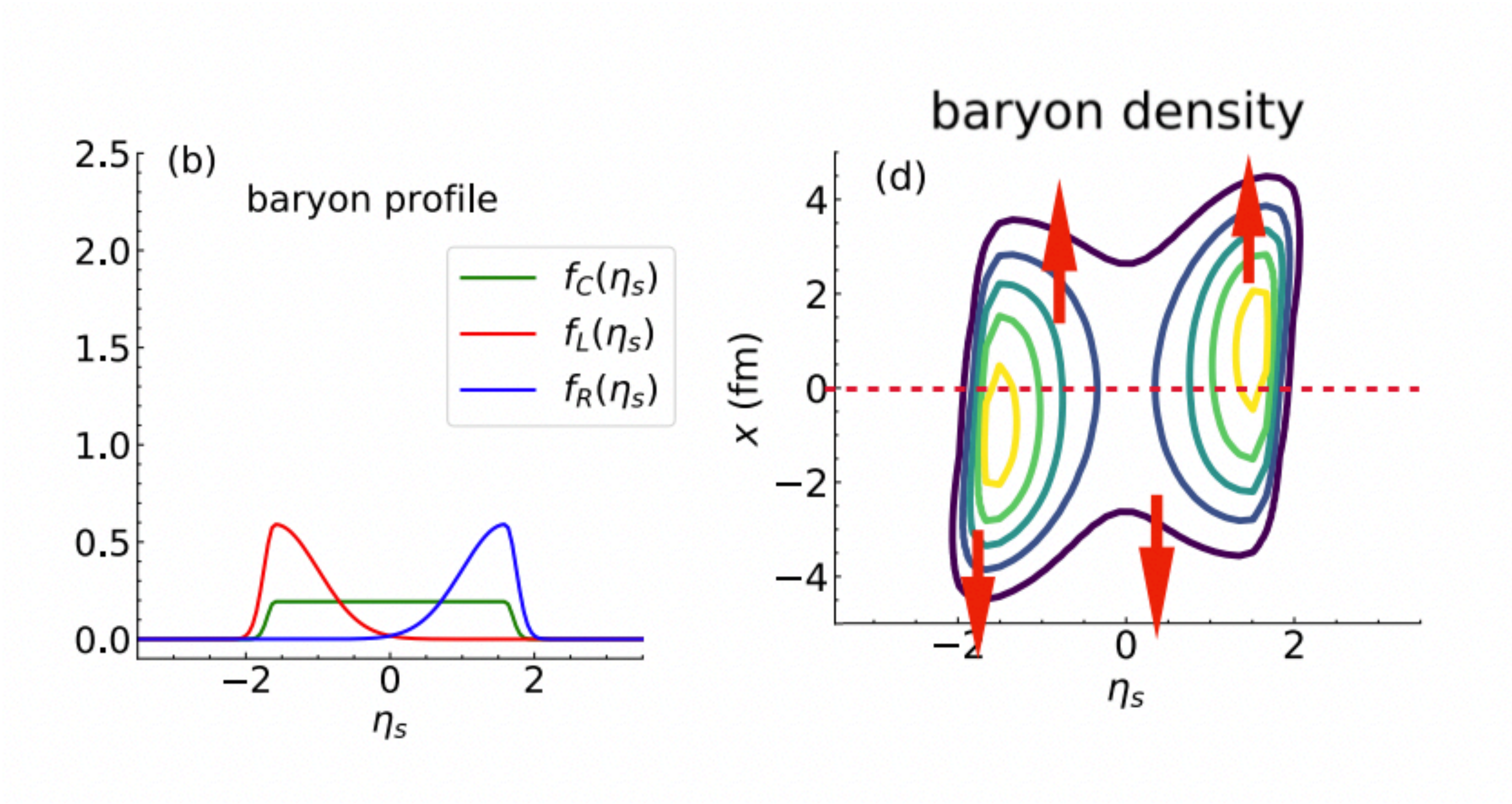
[Phys.Rev.C 81 (2010) 054905]

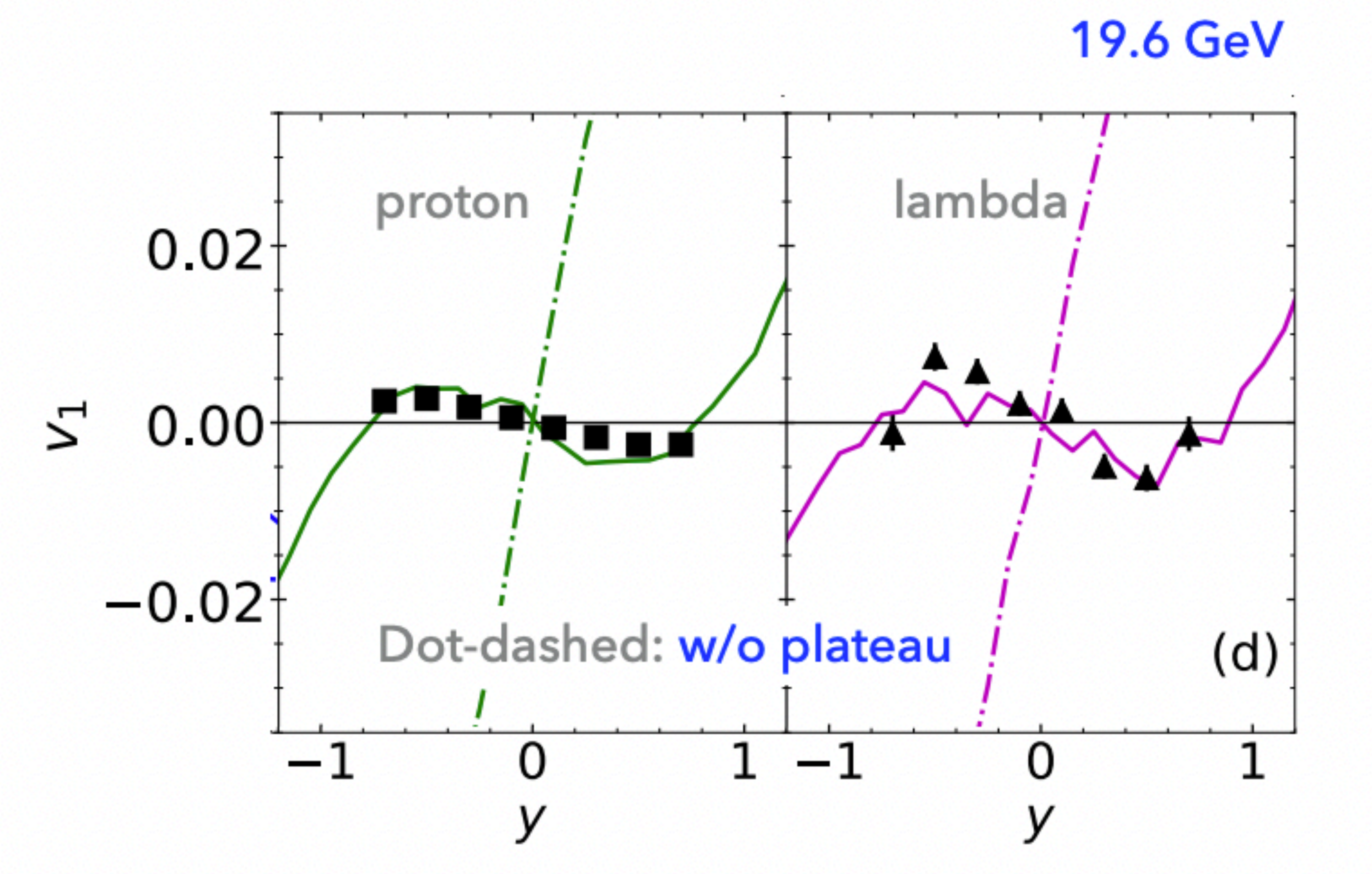


[Phys.Rev.C 94 (2016) 2, 024907]

Why anisotropic flow/Flow fluctuations?

- Constrain the initial state
  - ✦ The initial spatial geometry $(v_2, v_3 \ldots)$
  - ✦ The longitudinal structure

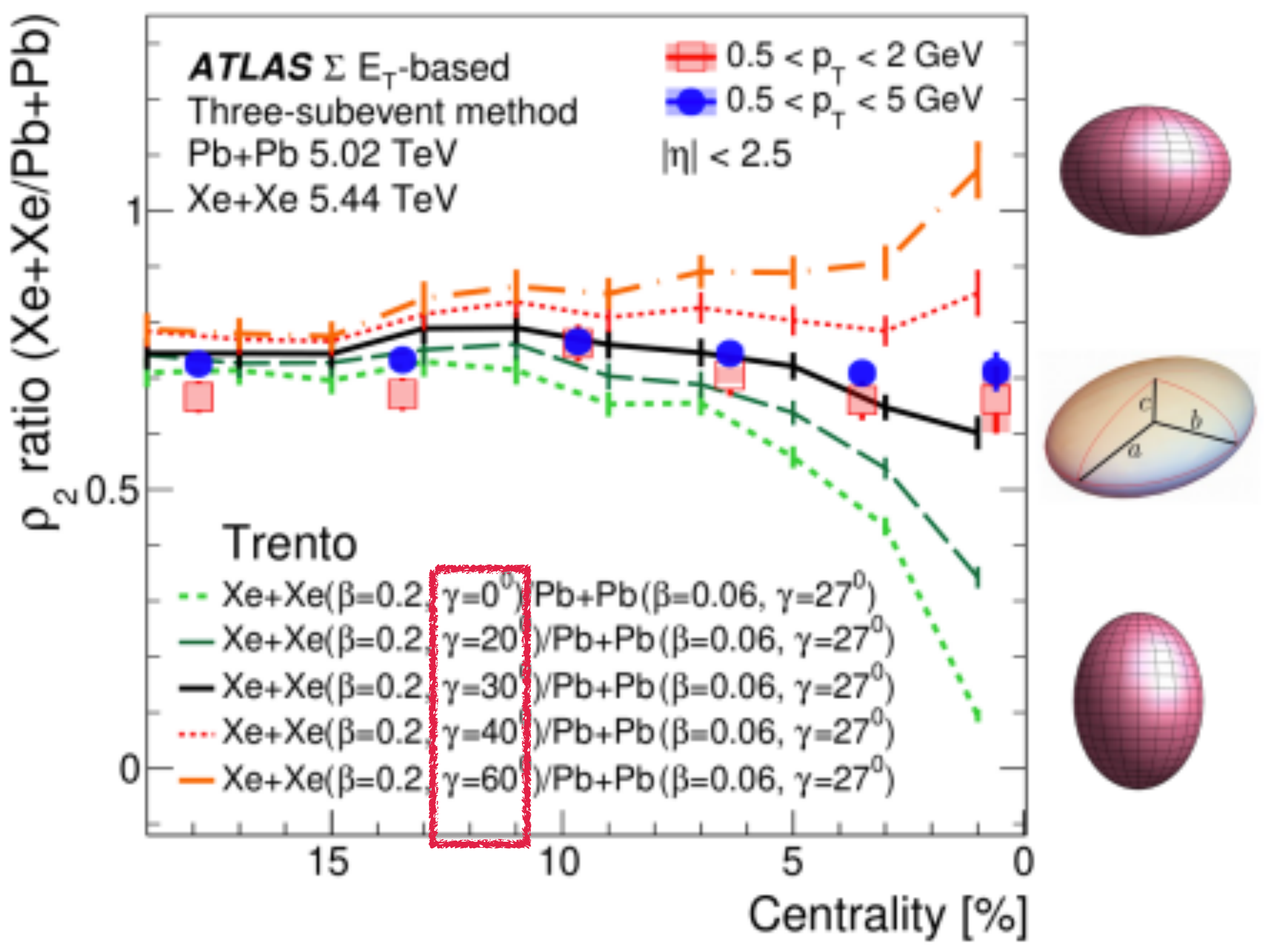




[Phys.Rev.C 108 (2023) 4, L041901]

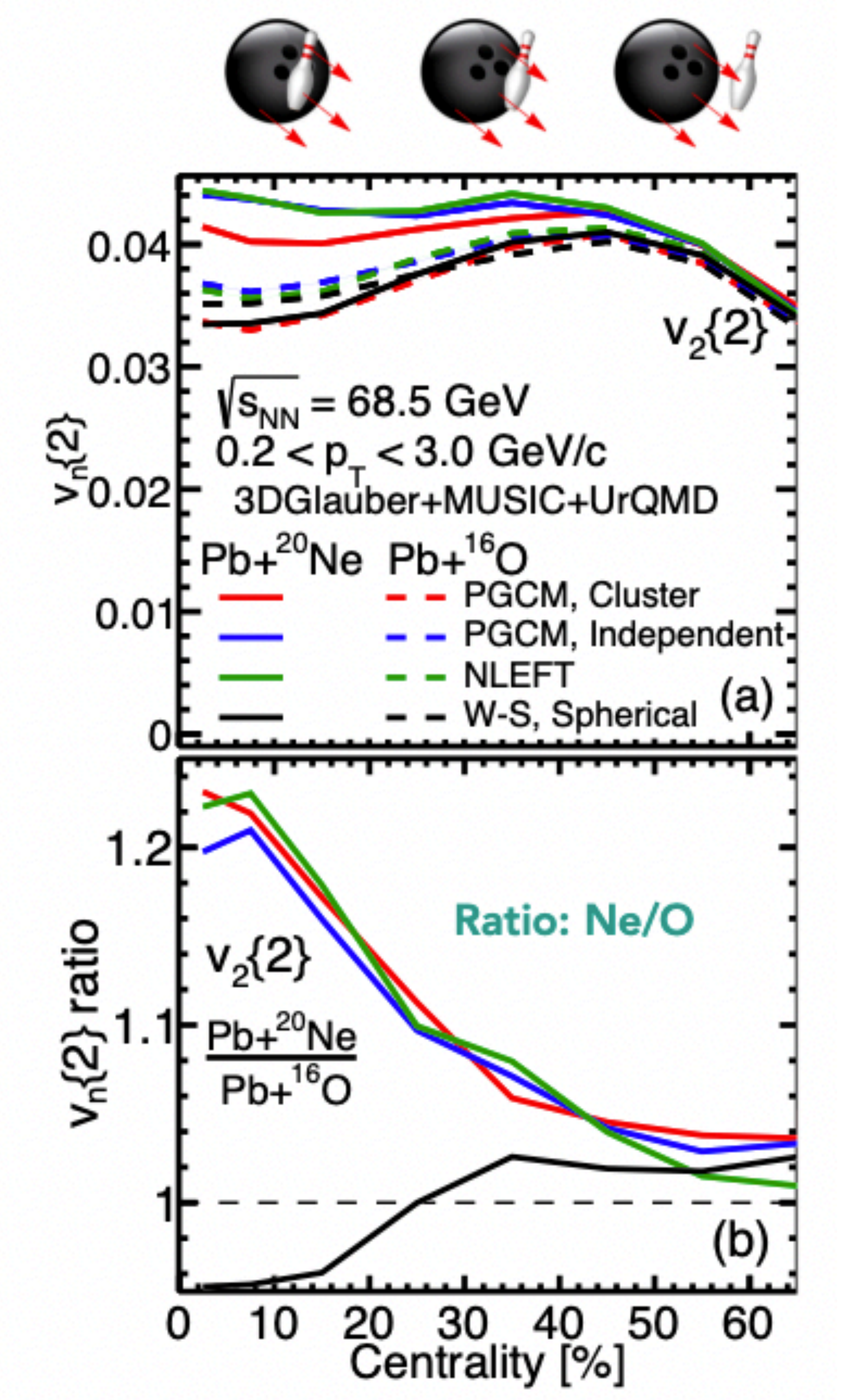
Why anisotropic flow/Flow fluctuations?

- Constrain the initial state
  - ✦ The initial spatial geometry $(v_2, v_3 \ldots)$
  - ✦ The longitudinal structure
  - ✦ The connection to nuclear structure at low energy



$$\rho(\vec{r}) = \left\{1 + \exp\left[r - R_0\left(1 + \beta_2\left[Y_2^0(\theta,\phi)\cos\gamma + Y_2^2(\theta,\phi)\sin\gamma\right]\right)\right]\right\}^{-1}$$

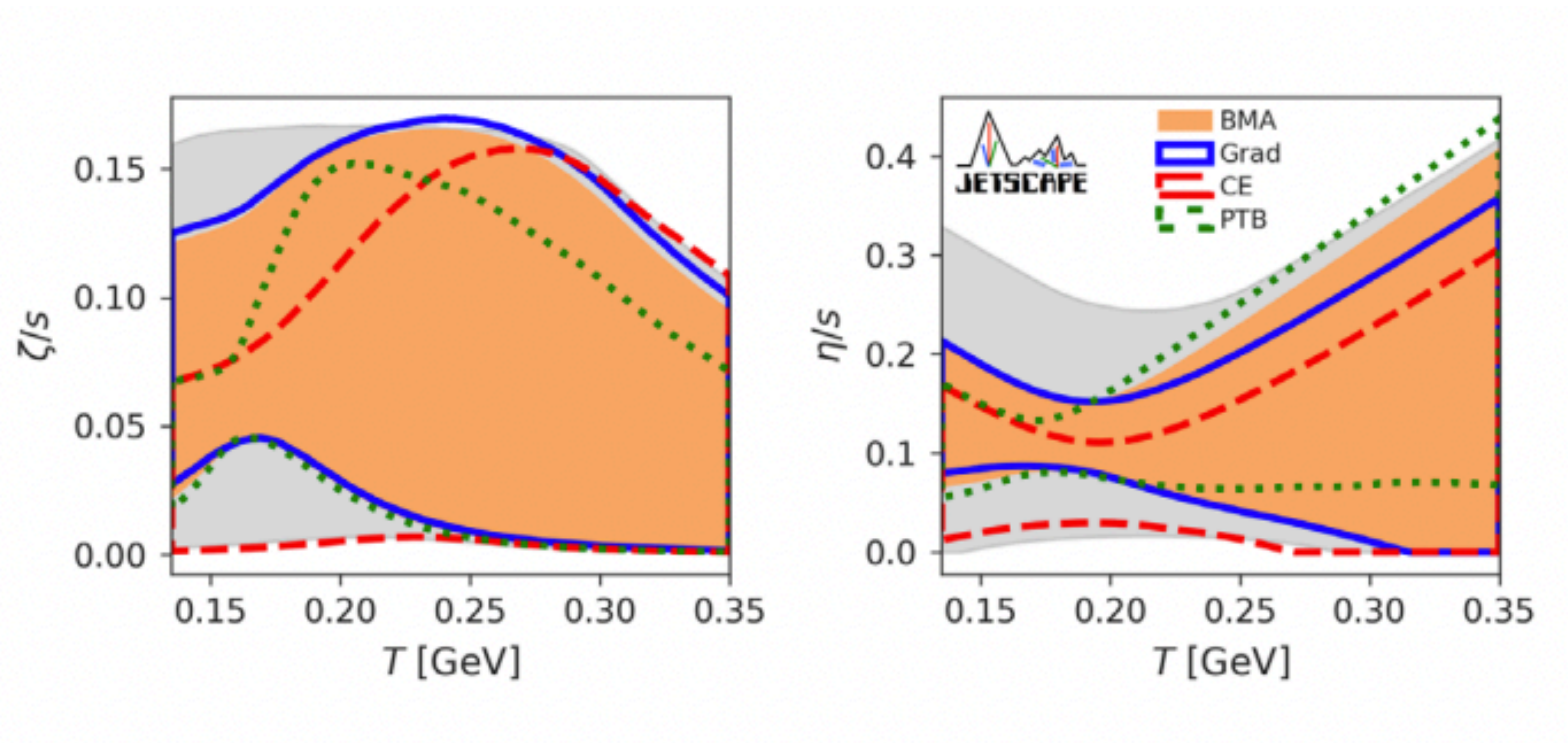
[Phys.Rev.C 107 (2023) 5, 054910]



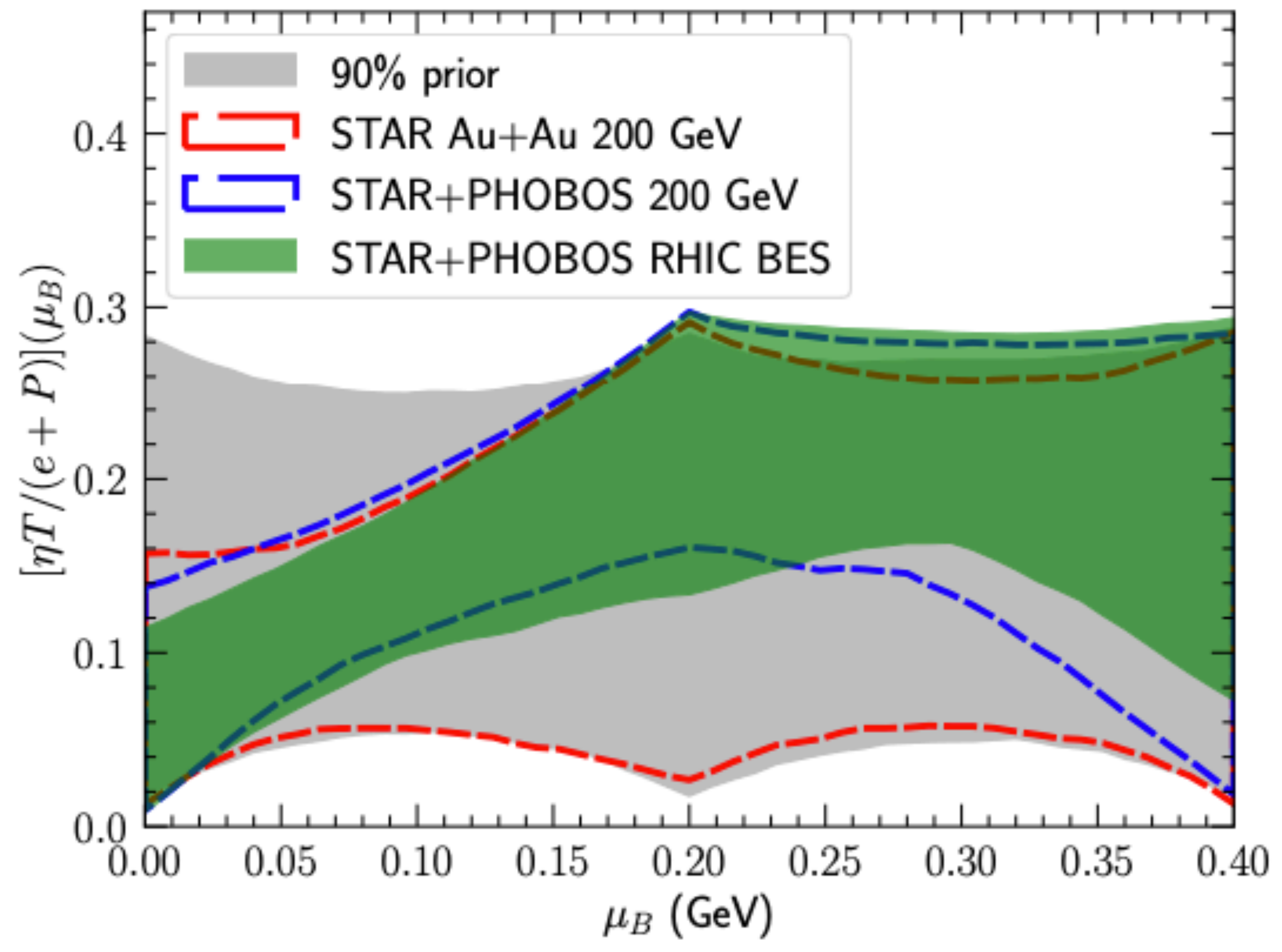
[e-Print: 2405.20210]

Why anisotropic flow/Flow fluctuations?

- Constrain the initial state
- Constrain transport properties of QGP $\frac{\eta}{s}(T,\mu_B)$ $\frac{\zeta}{s}(T,\mu_B)$

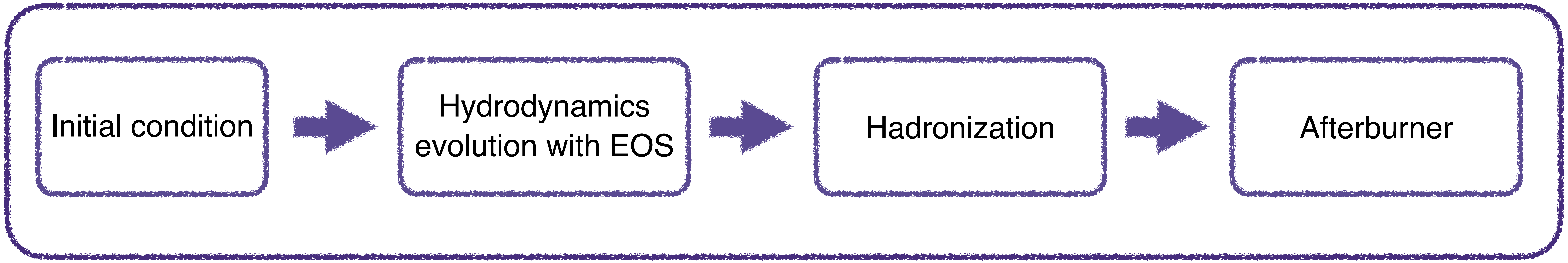


[Phys.Rev.Lett. 126 (2021) 24, 242301]



[Phys.Rev.Lett. 126 (2021) 24, 242301]

A standard hybrid framework



Initial condition: Glauber, Trento, AMPT, SMASH, IP-Glasma…

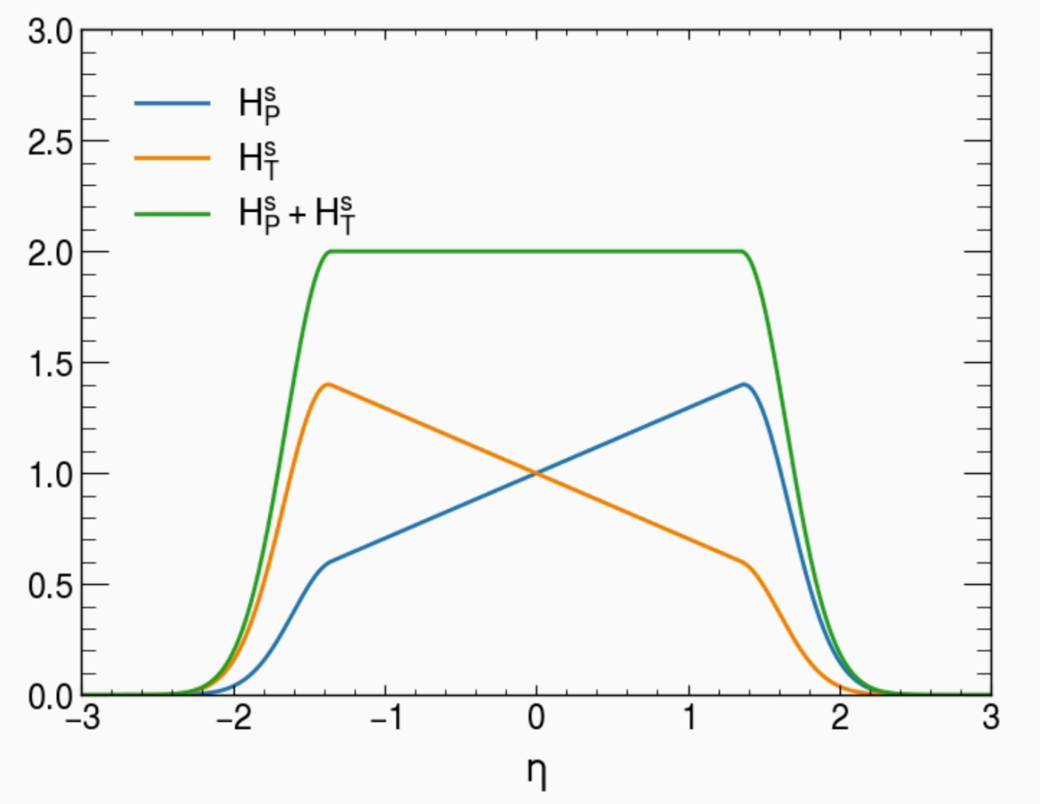
Hydrodynamics: Energy/baryon conservation+2nd Israel-Stewart-like equations including $\eta, \ \zeta, \ \kappa_B$

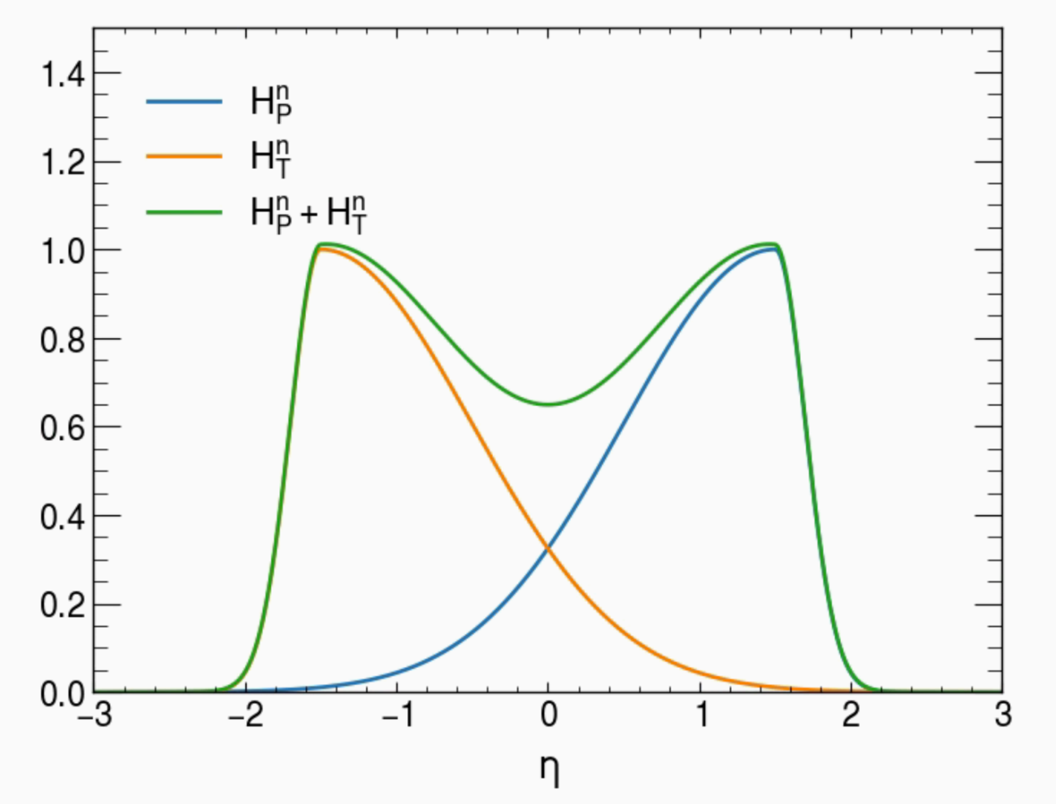
Hadronization: Cooper-Frye formula

Afterburner: URQMD/ SMSAH/JAM

[Moreland, Bernhard, and Bass, Phys. Rev. C 92, 011901 (2015)]
[Miller, Reygers, Sanders, and Steinberg, Ann. Rev. Nucl. Part. Sci. 57, 205 (2007)]
[Lin, Ko, Li, Zhang, and Pal, Phys. Rev. C72, 064901 (2005)]
[J. Weil et al., Phys. Rev. C 94, 054905 (2016), arXiv:1606.06642]
[Wu, Pang, Qin, and Wang, Nucl. Phys. A 1005, 121827 (2021)]
[Monnai, Schenke, and Shen, Phys. Rev. C 100, 024907 (2019)]
[Wu, Qin, Pang, and Wang, Phys. Rev. C 105, 034909 (2022)]
[Denicol, Gabriel S. et al. Phys.Rev. C98 (2018) no.3, 034916]
[S. A. Bass et al., Prog. Part. Nucl. Phys. 41, 255 (1998)]

CLVisc Model





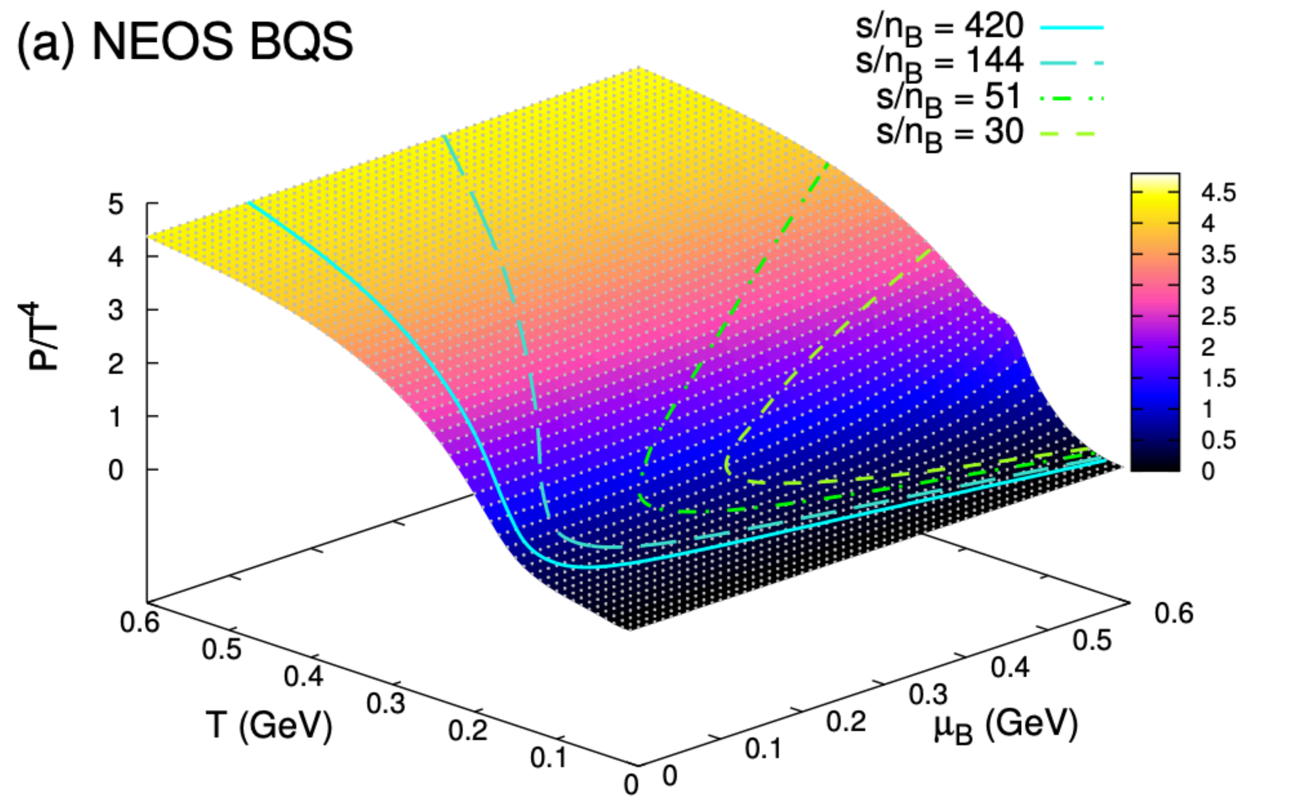
Initial condition: MCGlauber + envelope function

Hydro evolution:

$$\partial_\mu T^{\mu\nu} = 0 \qquad \Delta^{\mu\nu}_{\alpha\beta} D\pi^{\alpha\beta} = -\frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - \eta\sigma^{\mu\nu}\right) - \frac{4}{3}\pi^{\mu\nu}\theta - \frac{5}{7}\pi^{\alpha\langle\mu}\sigma^{\nu\rangle}_{\alpha} + \frac{9}{70}\frac{4}{e+P}\pi^{\langle\mu}_{\alpha}\pi^{\nu\rangle\alpha}$$

$$\partial_\mu J^\mu = 0 \qquad \Delta^{\mu\nu} DV_\mu = -\frac{1}{\tau_V}\left(V^\mu - \kappa_B \nabla^\mu \frac{\mu_B}{T}\right) - V^\mu\theta - \frac{3}{10}V_\nu\sigma^{\mu\nu}$$

Equation of state: NEOS-BQS (Taylor expansion, LQCD+hadron gas)



Particlization

$$\frac{dN}{dYp_T dp_T d\phi} = \frac{g_i}{(2\pi)^3}\int_\Sigma p^\mu d\Sigma_\mu f_{\rm eq}\left(1+\delta f_\pi + \delta f_V\right)$$

Afterburner: SMASH

$$p^\mu \partial_\mu f + mF^\mu \partial_{p_\mu}(f) = C[f]$$

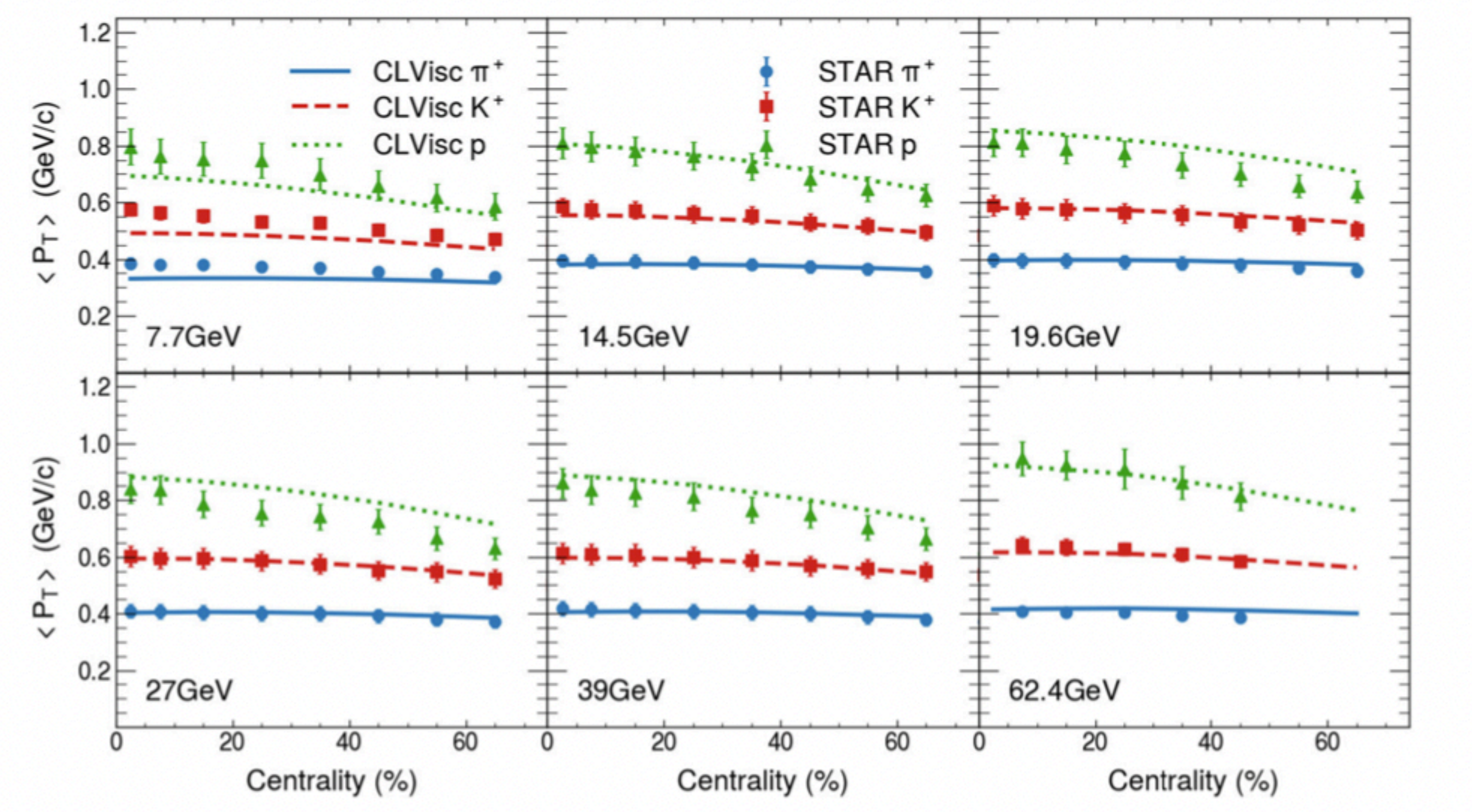
smash

[Monnai, Schenke and Shen, Phys. Rev. C 100, 024907 (2019)]

[Wu, Qin, Pang, and Wang, Phys. Rev. C 105, 034909 (2022)]

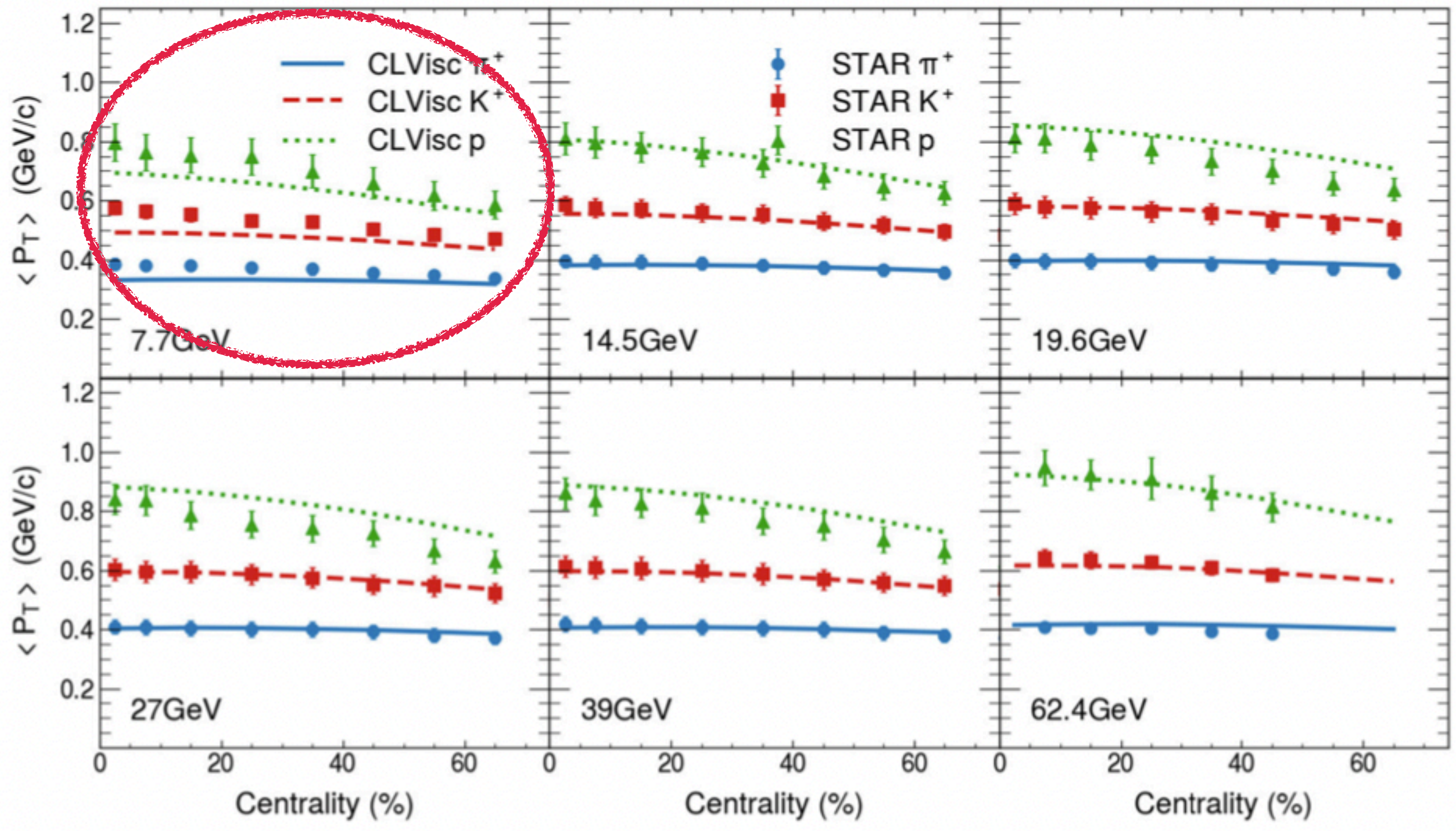
[Denicol, Gabriel S. et al. Phys.Rev. C98 (2018) no.3, 034916]

Mean transverse Momenta



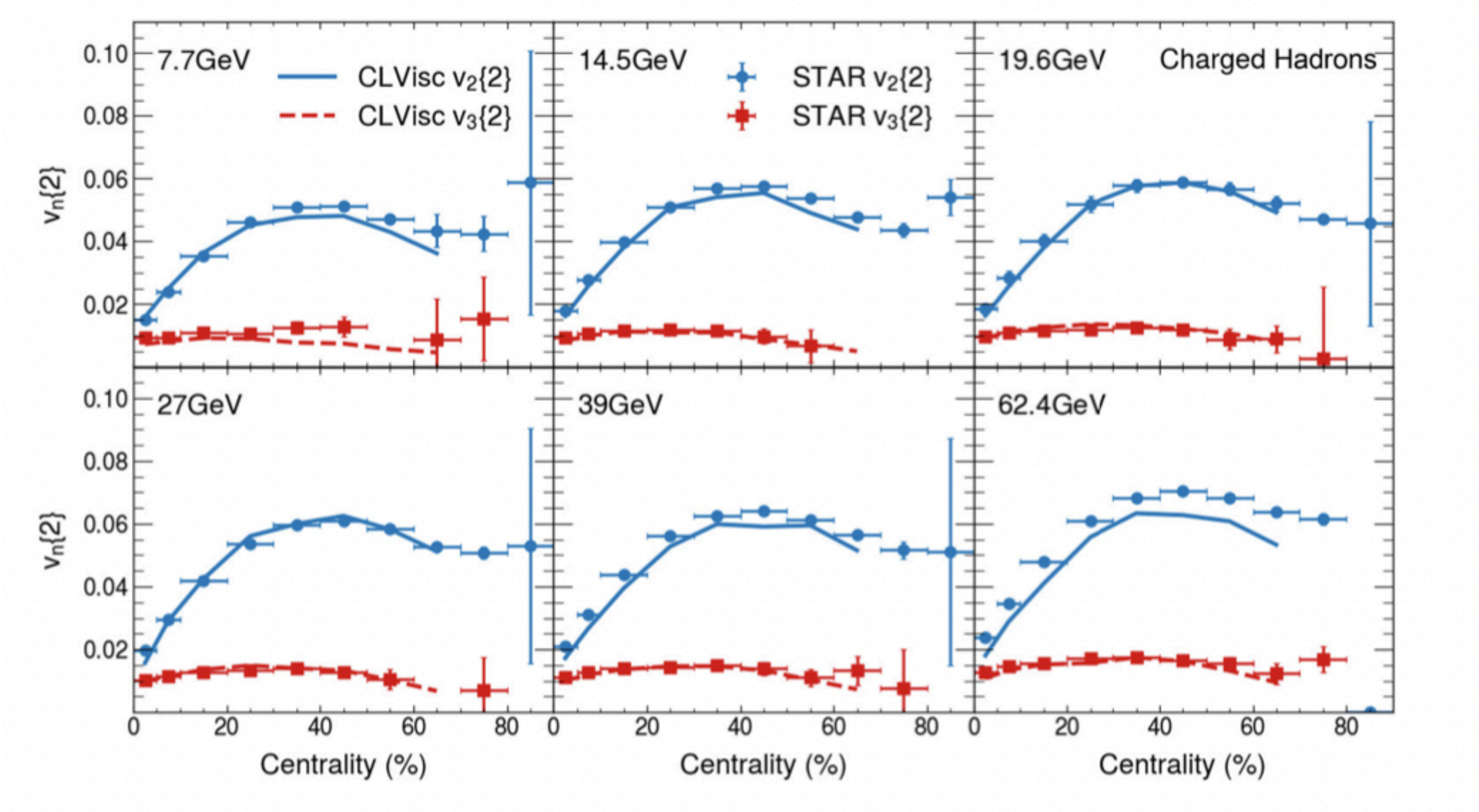
The CLVisc framework can describe the mean transverse momenta of identified particles from STAR.
One can clearly observe a larger blue shift effect for more massive particles.

Mean transverse Momenta



The CLVisc framework can describe the mean transverse momenta of identified particles from STAR.
One can clearly observe a larger blue shift effect for more massive particles.
The dynamical initial conditions and pre-equilibrium evolution should be considered

Anisotropic flows

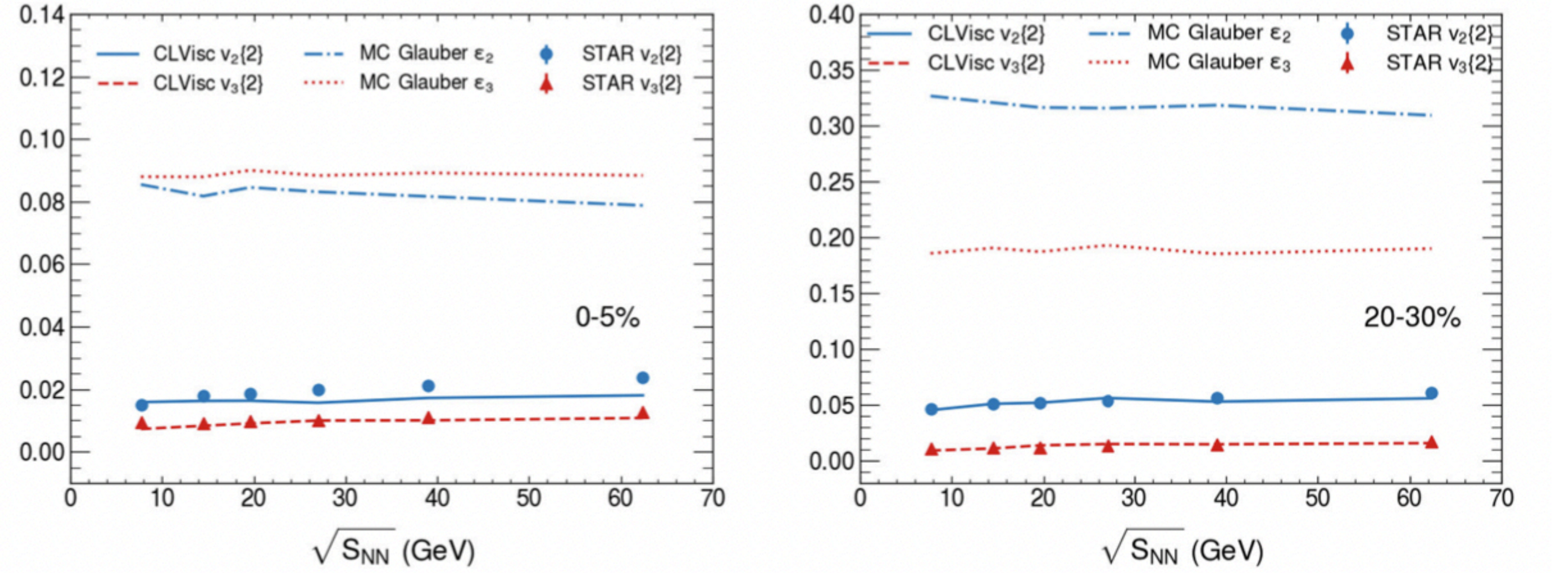


Our results are in good agreement with the experimental data from STAR:

$v_2\{2\}$: typical non-monotonic centrality dependences due to the combined effects of elliptic geometry, geometrical fluctuations, and system size.

$v_3\{2\}$: weak centrality dependence due to initial state geometrical fluctuations.

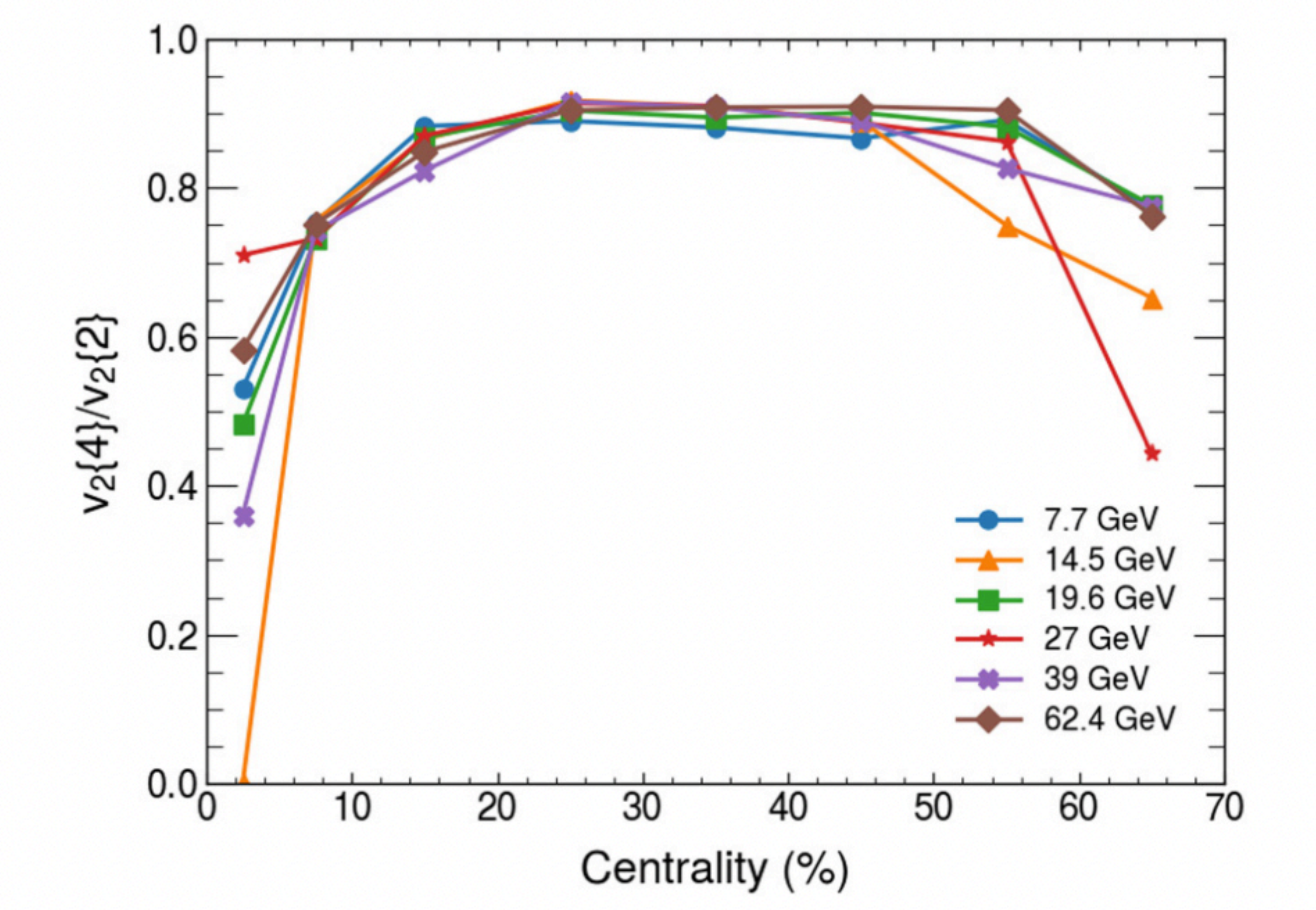
Anisotropic flows



We find that both elliptic and triangular flows increase slightly with the beam energy.

- the very weak collision energy dependence of eccentricities ε_2 and ε_3.
- the increase of radial flow due to the increase of initial energy density.

Flow fluctuations



Gaussian distribution assumption:

$$\frac{dP}{d^2\vec{v}_n} = \frac{1}{2\pi\sigma_n^2} e^{-\frac{(\vec{v}_n - \vec{\bar{v}}_n)^2}{2\sigma_n^2}}$$

$$v_n\{2\} \approx \langle v_n \rangle + \sigma_n^2 / \left(2\langle v_n \rangle\right),$$

$$v_n\{4\} \approx \langle v_n \rangle - \sigma_n^2 / \left(2\langle v_n \rangle\right),$$

$$v_n\{6\} \approx \langle v_n \rangle - \sigma_n^2 / \left(2\langle v_n \rangle\right),$$

The fluctuation of collective flow from multi-particle cumulant ratio

$\frac{v_n\{4\}}{v_n\{2\}} \ll 1$ Large fluctuation

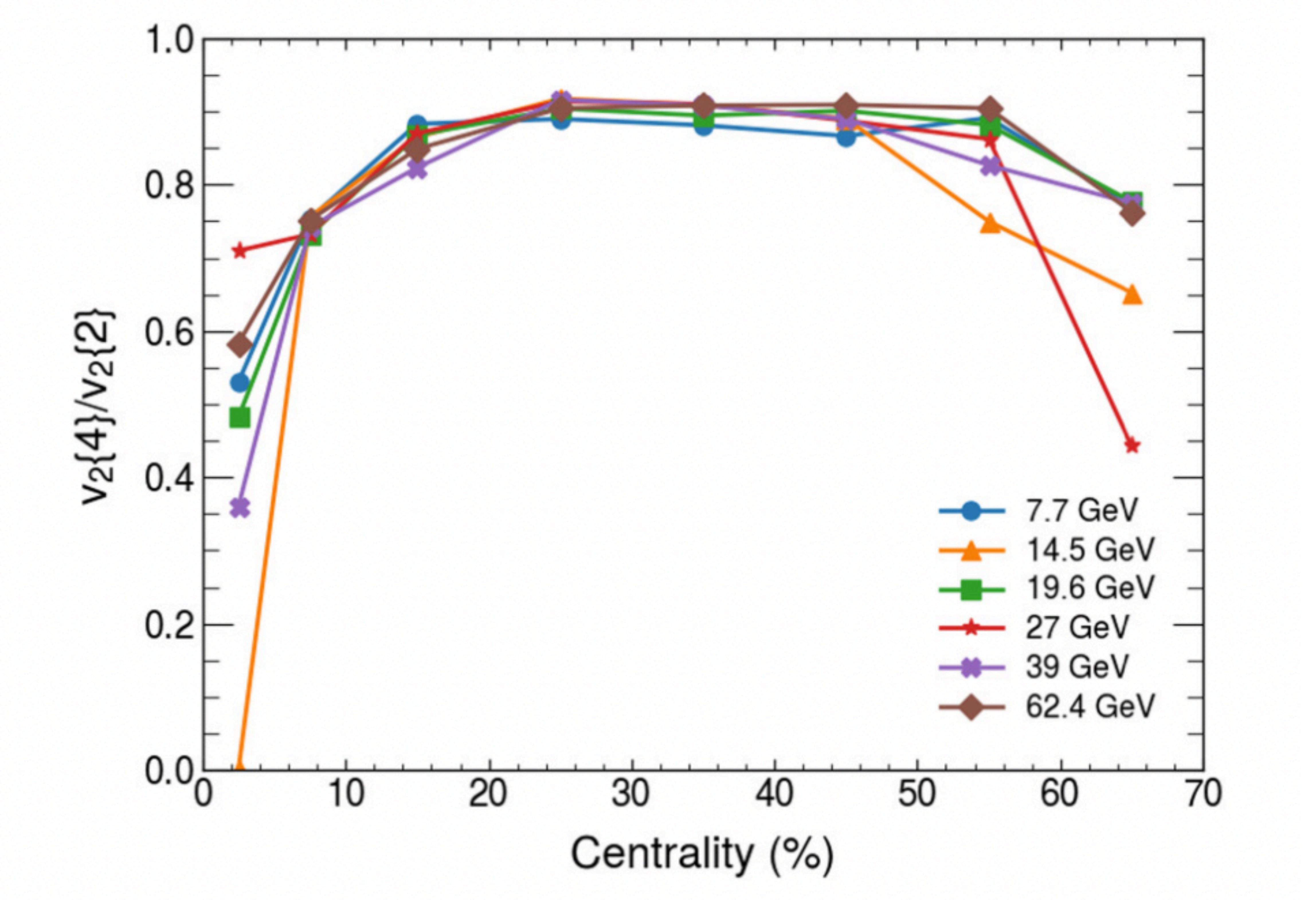
$\frac{v_n\{4\}}{v_n\{2\}} = 1$ No fluctuation

The multi-particle cumulant ratio $v_2\{4\}/v_2\{2\}$ first increases, and then decreases with centrality increased.

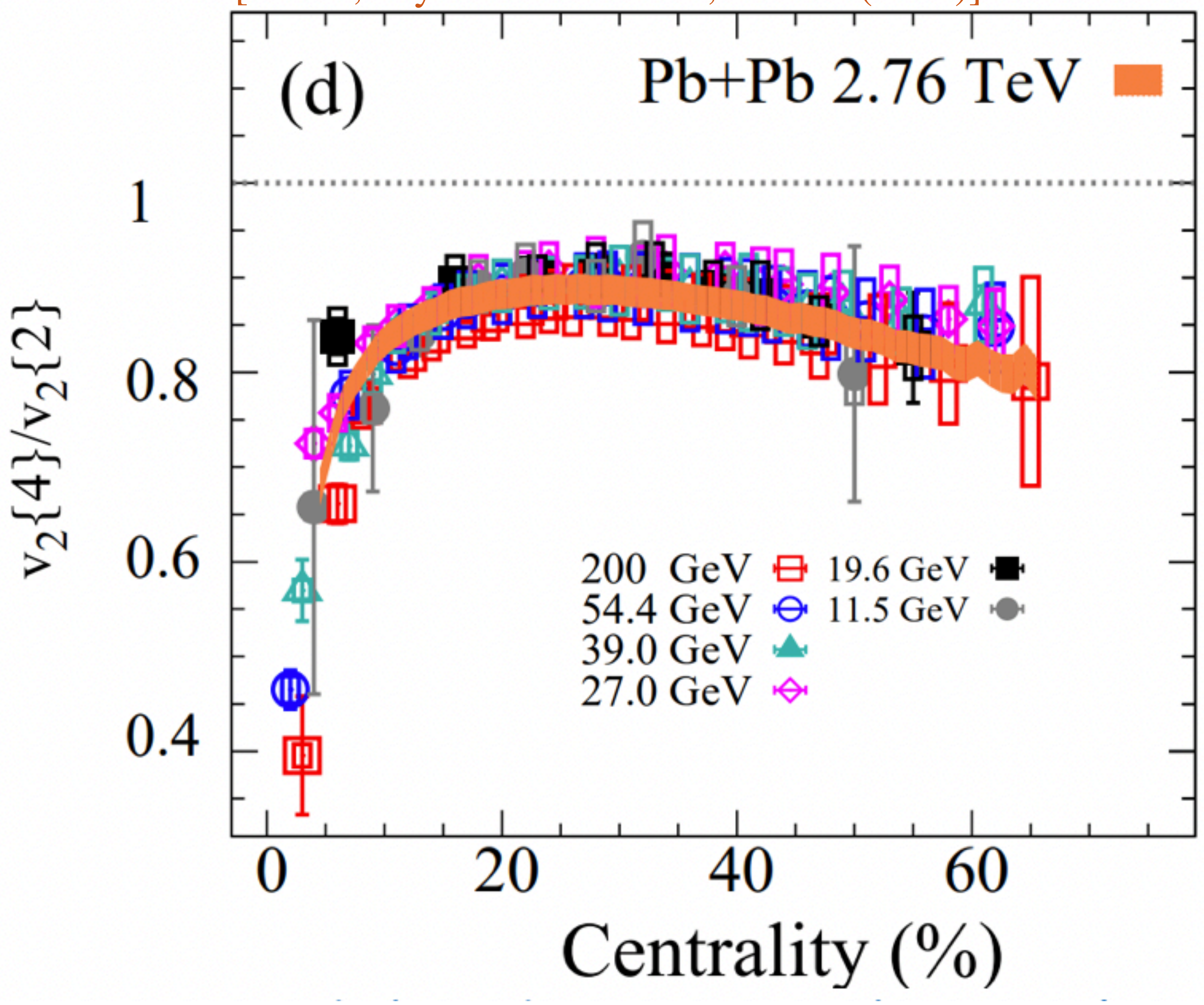
- initial collision geometry dominates in mid-central collisions.
- the fluctuations dominate in central and peripheral collisions.

The multi-particle cumulant ratio $v_2\{4\}/v_2\{2\}$ has weak collision energy dependence.

Flow fluctuations



[STAR, Phys. Rev. Lett. 129, 252301 (2022)]



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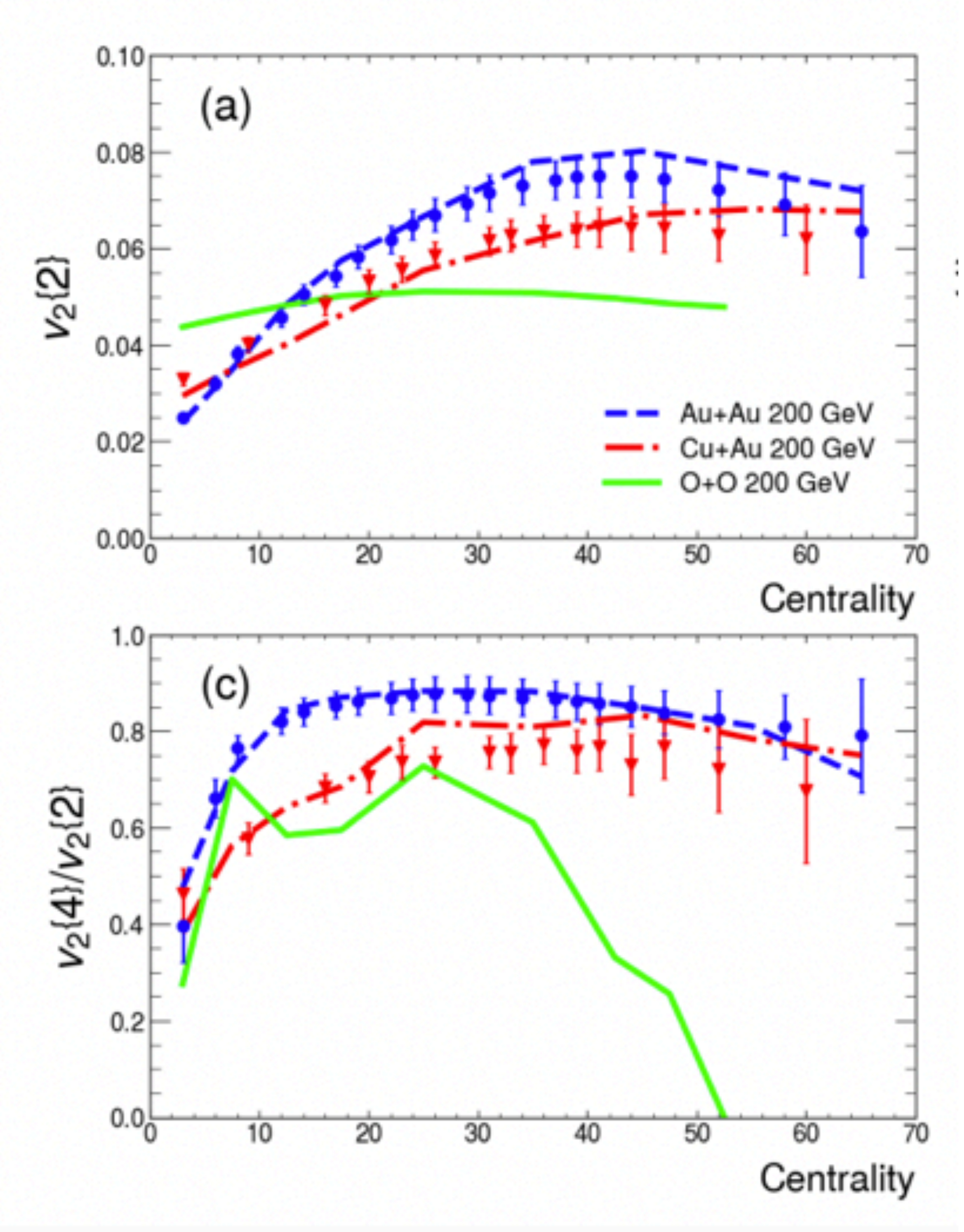
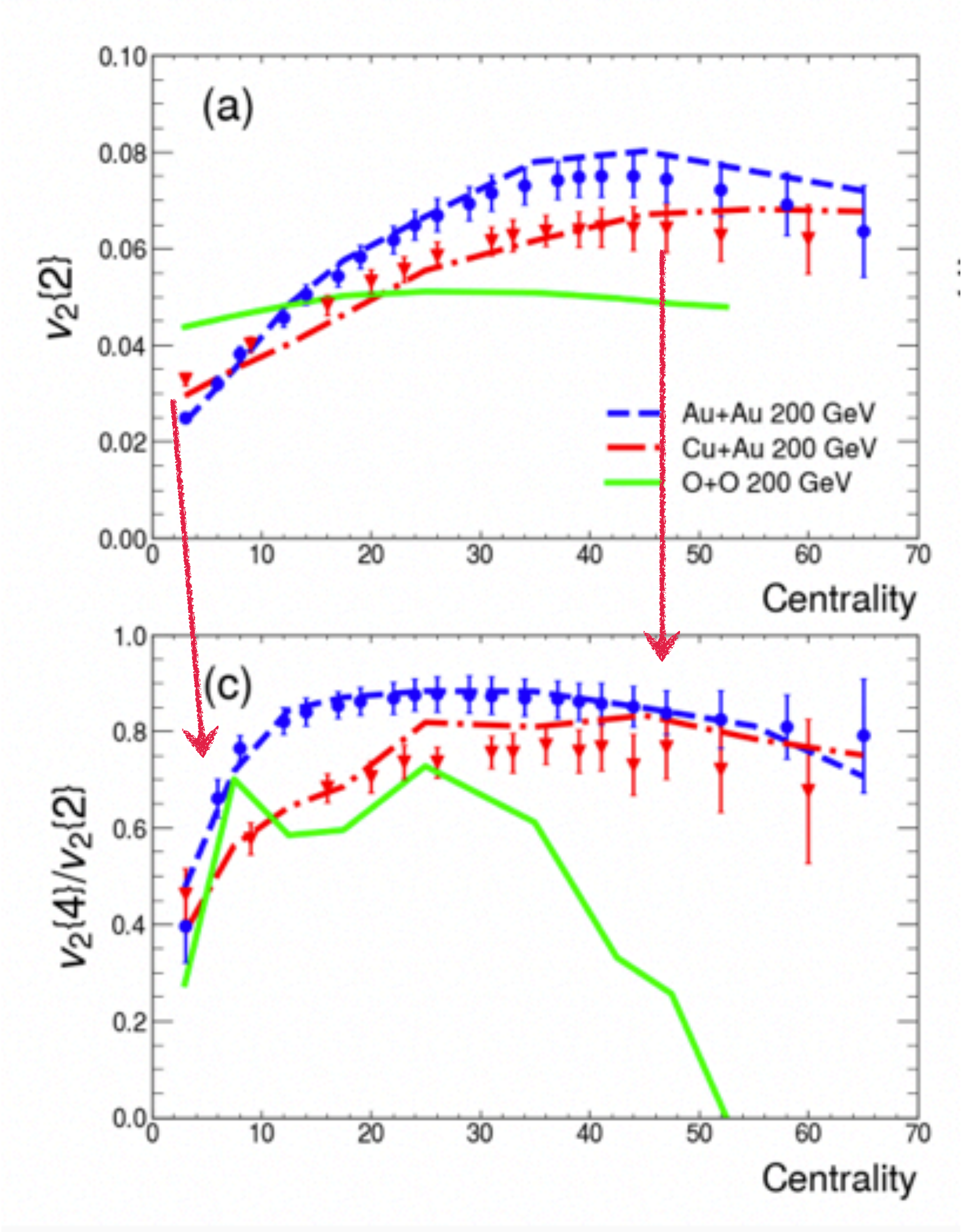
Anisotropic flow/Flow fluctuations in Au+Au, Cu+Au, O+O collision



Flow/flow fluctuation highly connects to collision systems.

Anisotropic flow/Flow fluctuations in Au+Au, Cu+Au, O+O collision



Flow/flow fluctuation highly connects to collision systems.

Cu+Au vs Au+Au:

- Larger $v_2\{2\}$ in the central centrality: fluctuations
- Smaller $v_2\{2\}$ in the larger centrality: smaller system size

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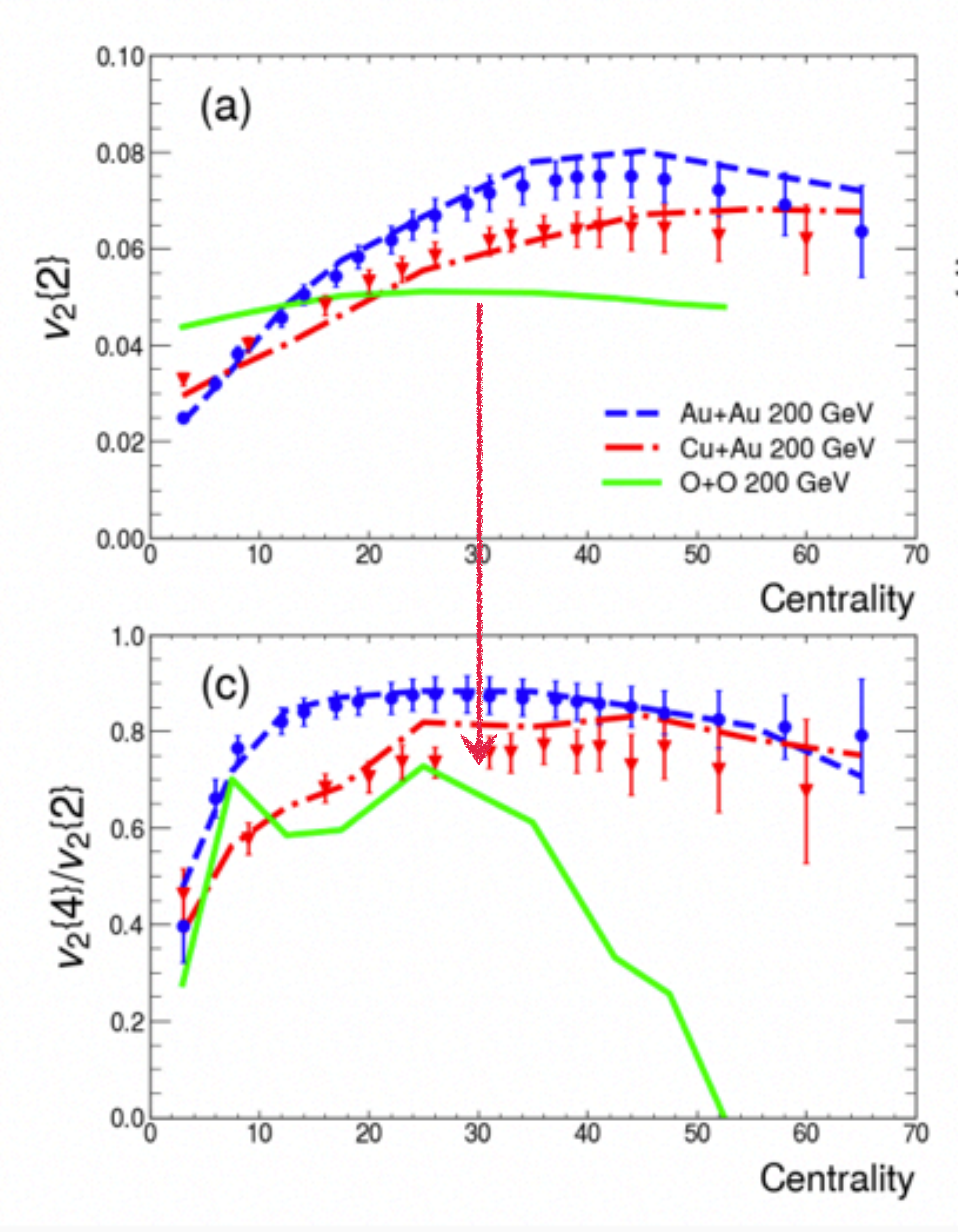

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O+O:

- $v_2\{2\}$: mainly driven by fluctuations
- Weak centrality dependence

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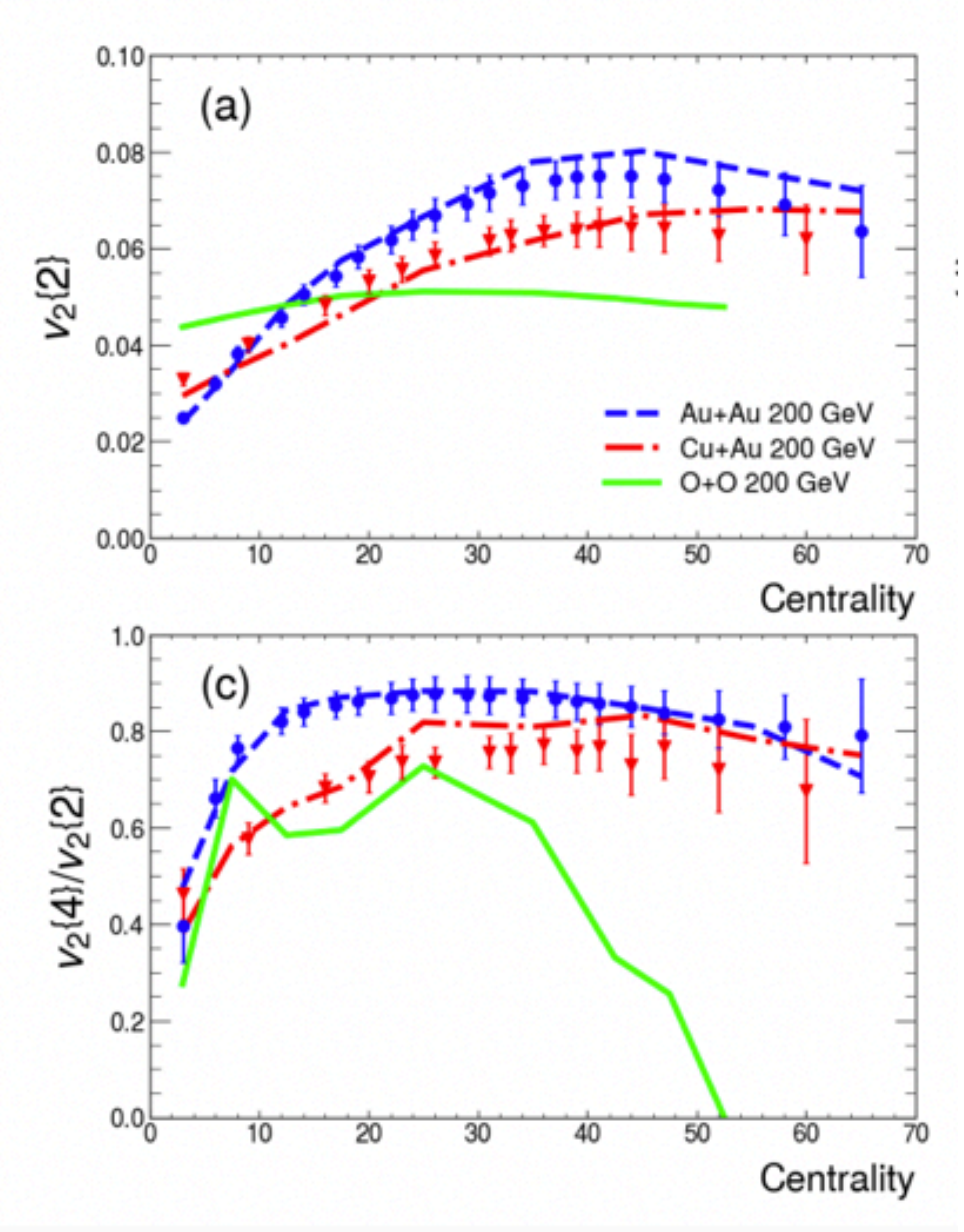

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O+O:

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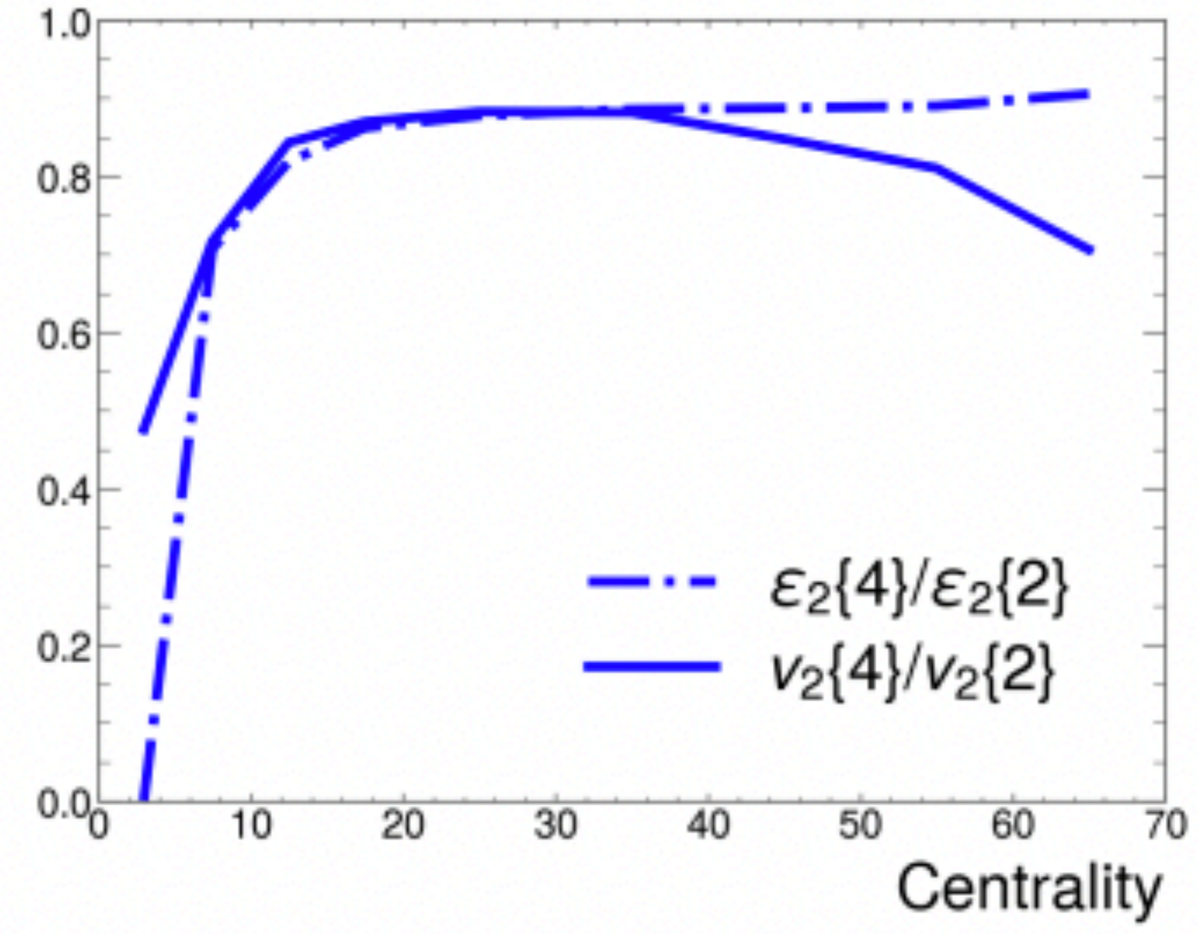
- Weak centrality dependence

Flow fluctuation:

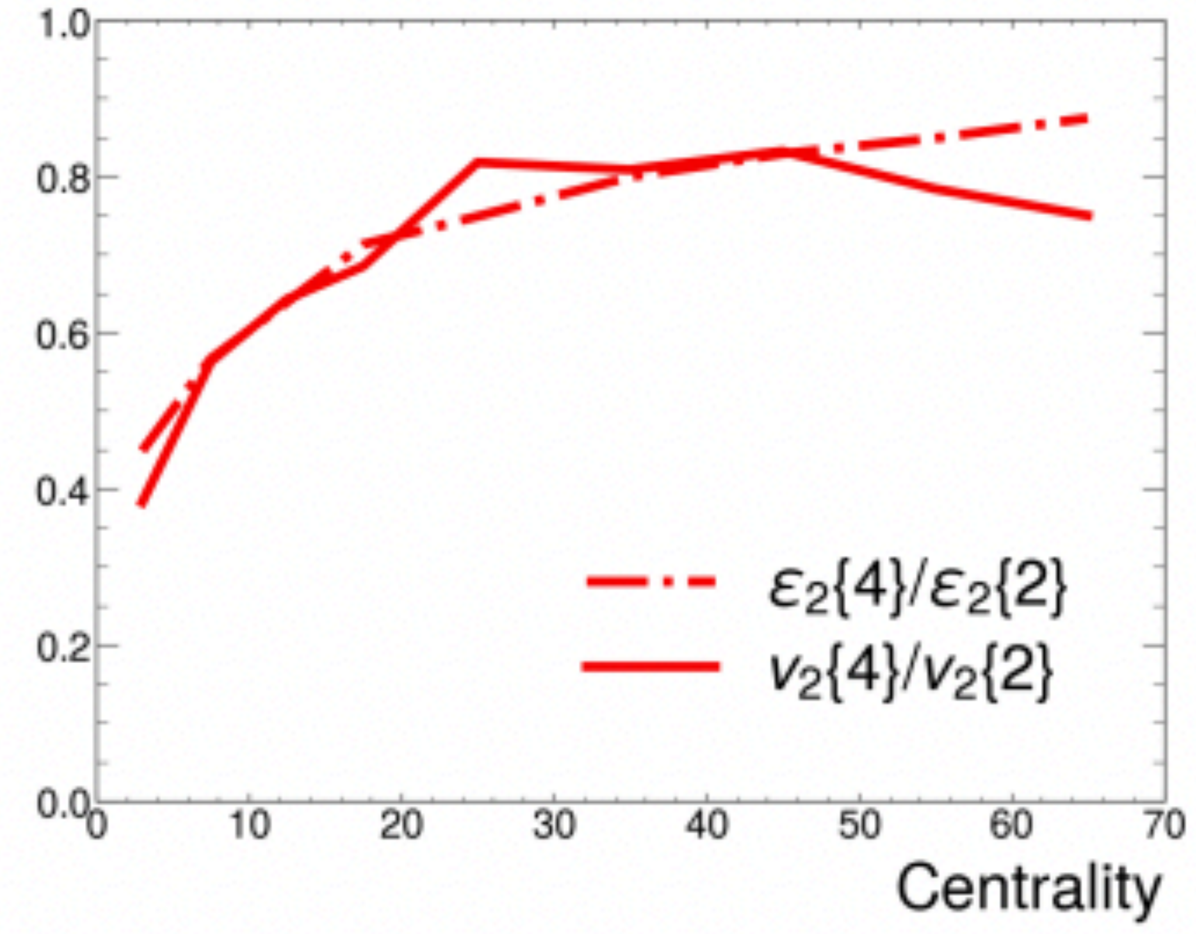
Au+Au< Cu+Au < O+O due to smaller system

Flow fluctuations on initial states

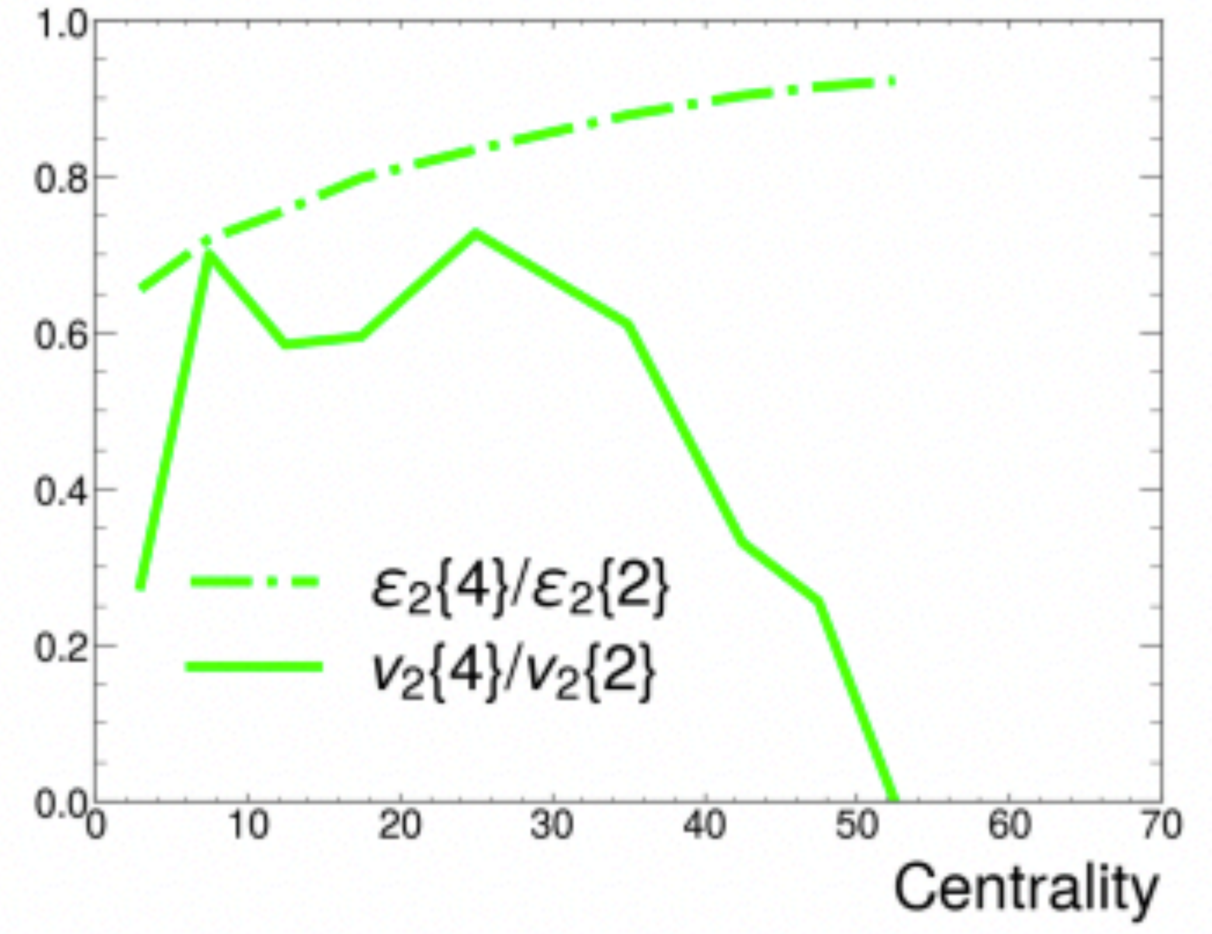
Au+Au@200 GeV



Cu+Au@200 GeV



O+O@200 GeV



The n-th order eccentricity: $\varepsilon_n = \frac{\sqrt{\langle r^n \cos(n\phi)\rangle^2 + \langle r^n \sin(n\phi)\rangle^2}}{\langle r^n \rangle}$

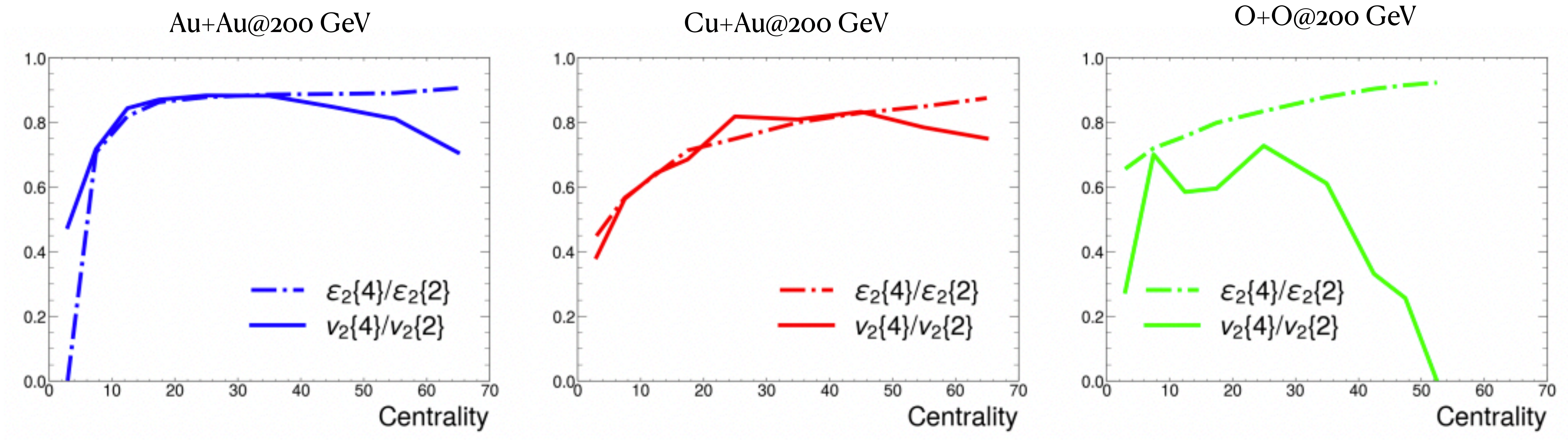
$$v_n \propto \varepsilon_n$$

The cumulants eccentricities

$$c_{\varepsilon_n}\{2\} = \left\langle \varepsilon_n^2 \right\rangle$$
$$c_{\varepsilon_n}\{4\} = \left\langle \varepsilon_n^4 \right\rangle - 2\left\langle \varepsilon_n^2 \right\rangle^2$$
$$c_{\varepsilon_n}\{6\} = \left\langle \varepsilon_n^6 \right\rangle - 9\left\langle \varepsilon_n^4 \right\rangle \left\langle \varepsilon_n^2 \right\rangle + 12\left\langle \varepsilon_n^2 \right\rangle^3$$

$$\varepsilon_n\{2\} = \sqrt{c_{\varepsilon_n}\{2\}}$$
$$\varepsilon_n\{4\} = \sqrt[4]{-c_{\varepsilon_n}\{4\}}$$
$$\varepsilon_n\{6\} = \sqrt[6]{c_{\varepsilon_n}\{6\}/4}$$

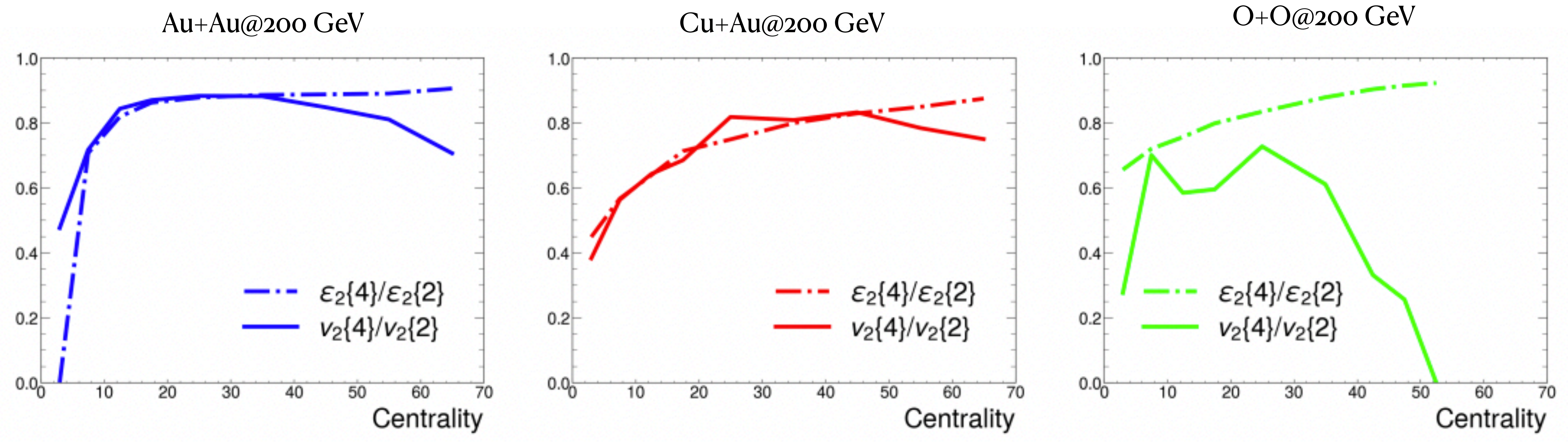
Flow fluctuations on initial states



A monotonic increase of $\varepsilon_2\{4\}/\varepsilon_2\{2\}$ for the three systems:

Central-> peripheral collision: initial fluctuation-> initial geometry

Flow fluctuations on initial states

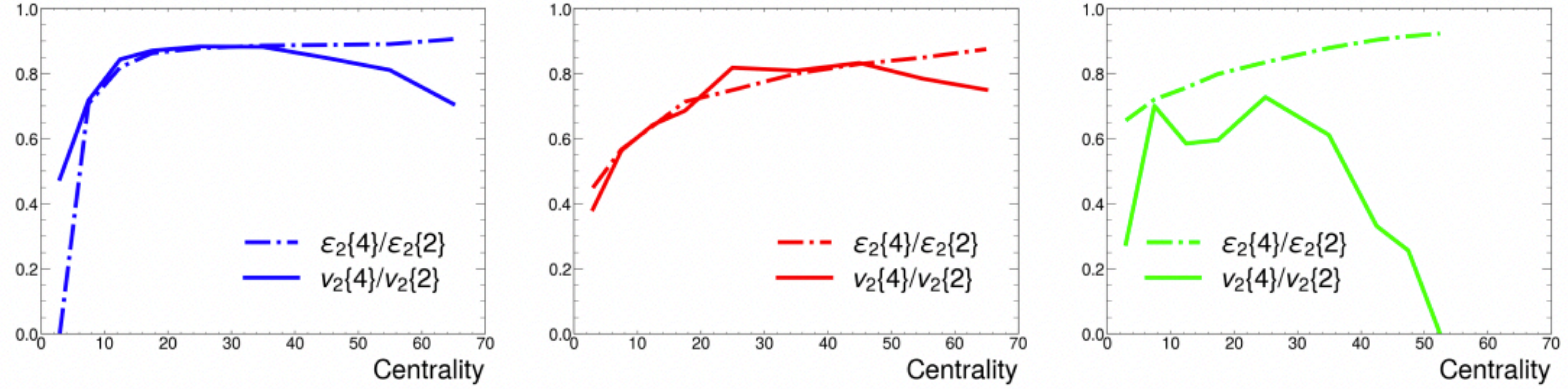


A monotonic increase of $\varepsilon_2\{4\}/\varepsilon_2\{2\}$ for the three systems:
Central-> peripheral collision: initial fluctuation-> initial geometry

The initial state fluctuations are the main source of the final state collective flow fluctuations in Au+Au, Cu+Au collision systems:
Semi-central: $\varepsilon_2\{4\}/\varepsilon_2\{2\} \approx v_2\{4\}/v_2\{2\}$
Peripheral: $\varepsilon_2\{4\}/\varepsilon_2\{2\} < v_2\{4\}/v_2\{2\}$ -> where is the fluctuations comes? hadronization and hadronic afterburner fluctuations.

Flow fluctuations on initial states

Au+Au@200 GeV Cu+Au@200 GeV O+O@200 GeV



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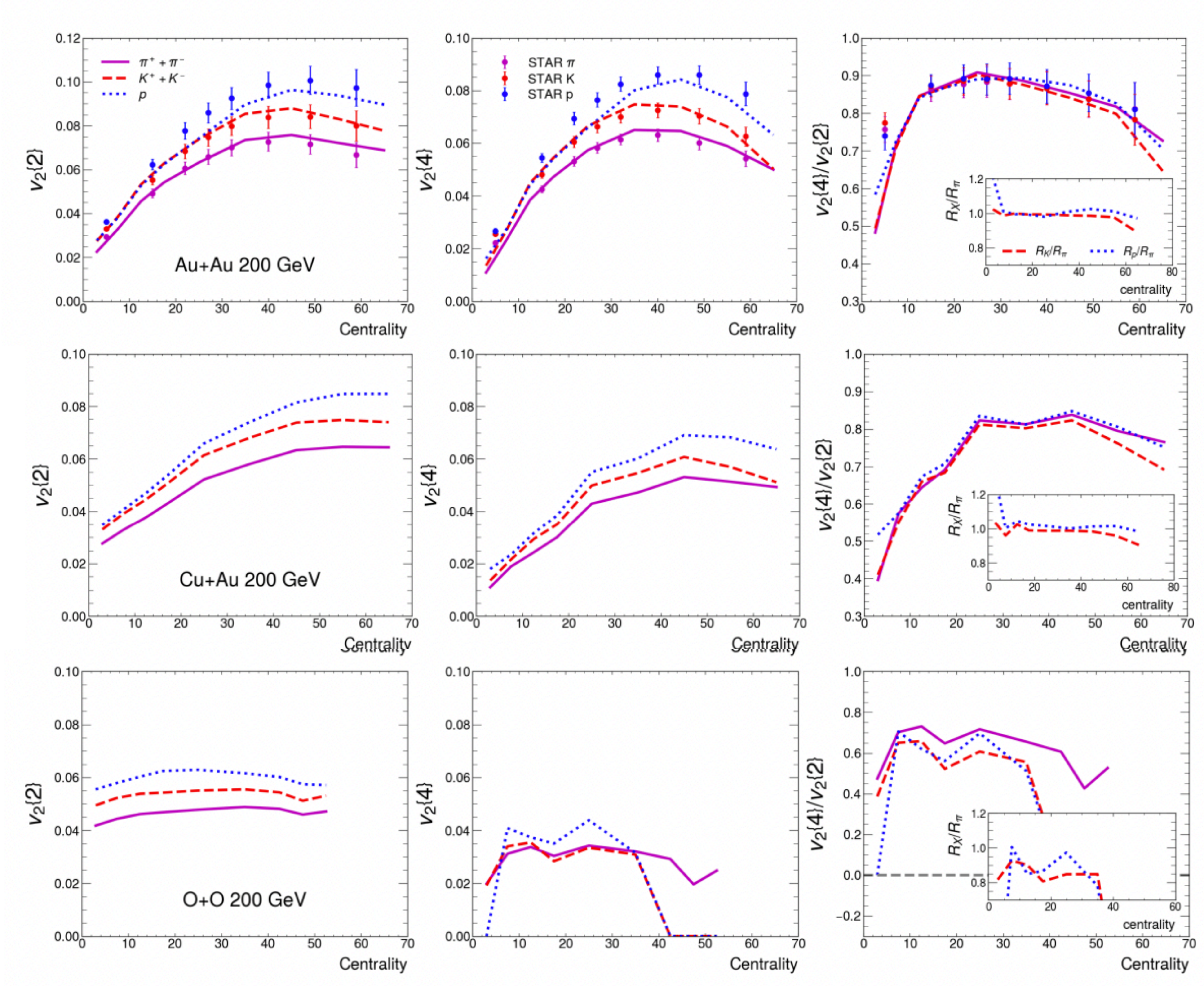
Peripheral: $\varepsilon_2\{4\}/\varepsilon_2\{2\} < v_2\{4\}/v_2\{2\}$ -> where is the fluctuations comes? hadronization and hadronic afterburner fluctuations.

Another sources instead of the initial state always have strong contributions to the flow fluctuations in O+O collision.

Flow fluctuations on identified particle

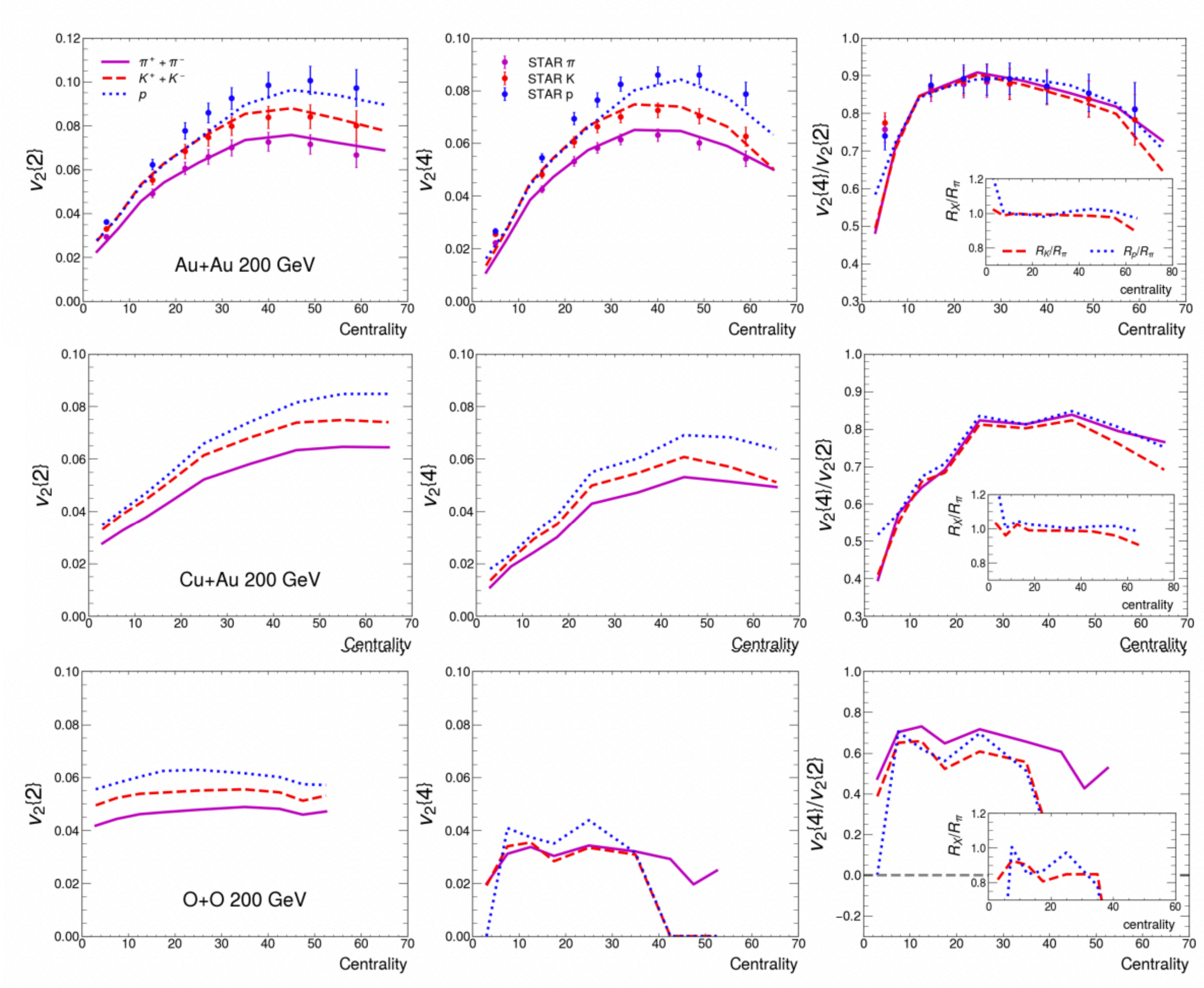


In Au+Au and Cu+Au collision systems:

different magnitudes of $v_2\{2\}$ and $v_2\{4\}$

similar $v_2\{4\}/v_2\{2\}$ for identified particle (π, k, p)

Flow fluctuations on identified particle

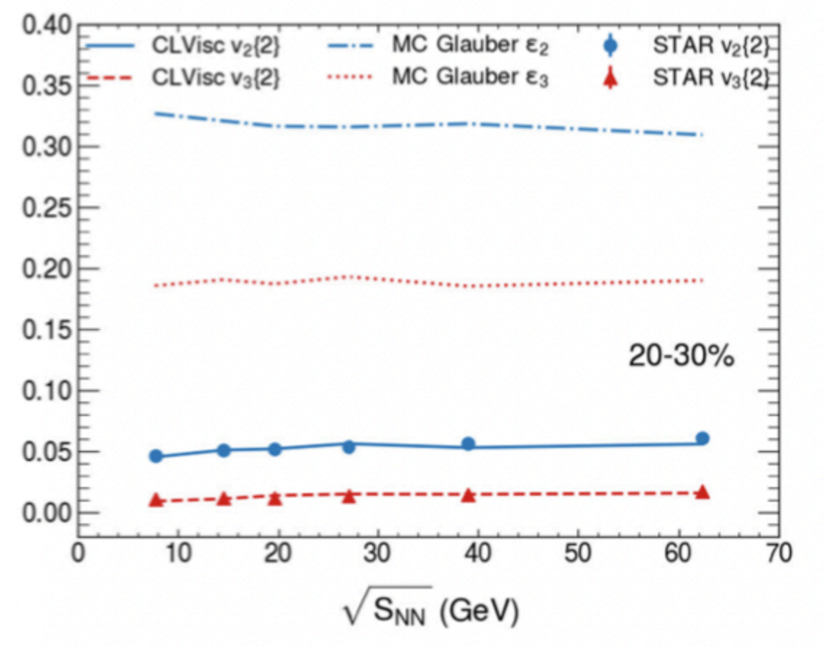


In Au+Au and Cu+Au collision systems:
different magnitudes of $v_2\{2\}$ and $v_2\{4\}$
similar $v_2\{4\}/v_2\{2\}$ for identified particle (π, k, p)

In O+O collision systems:
Fluctuation is no longer independent of particle species

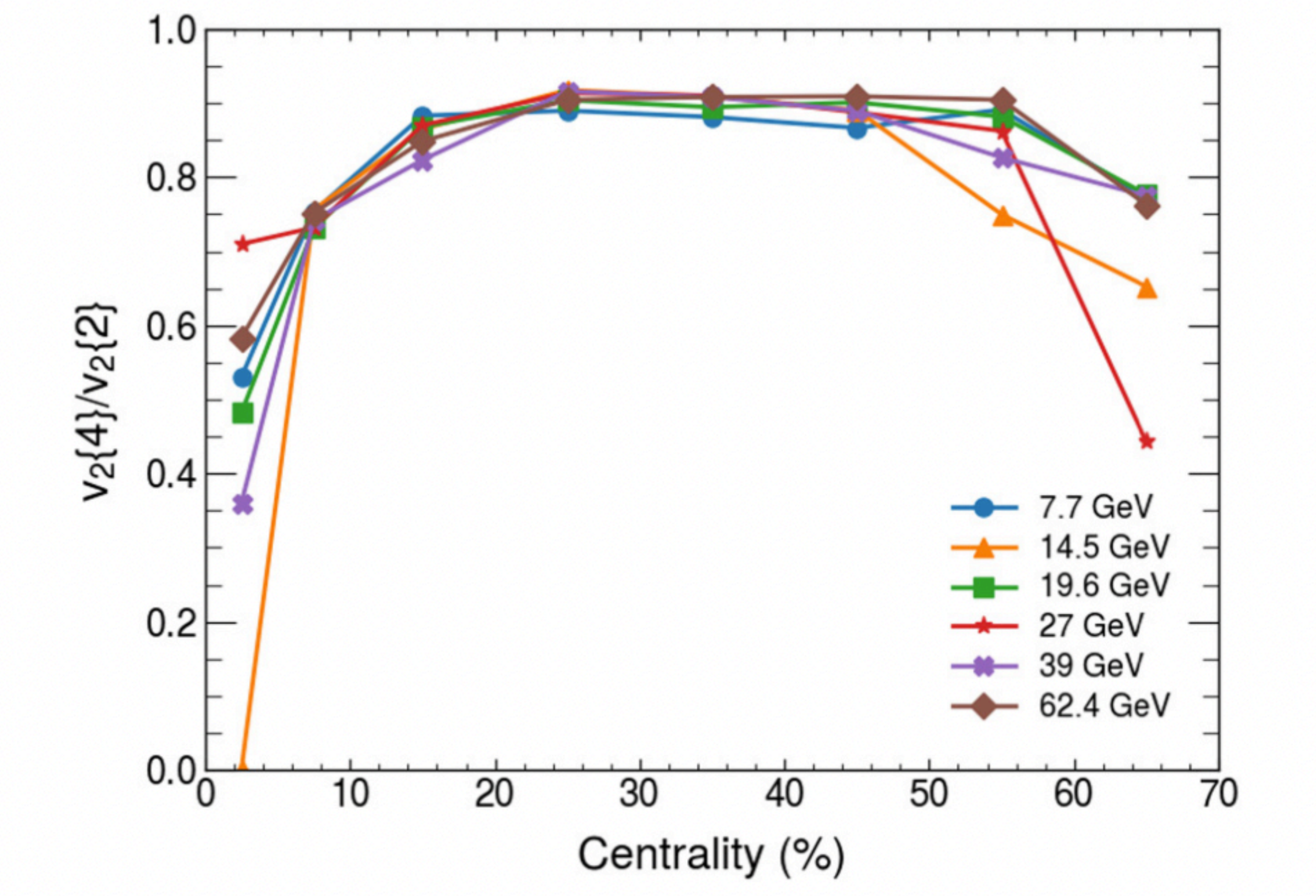
Summary

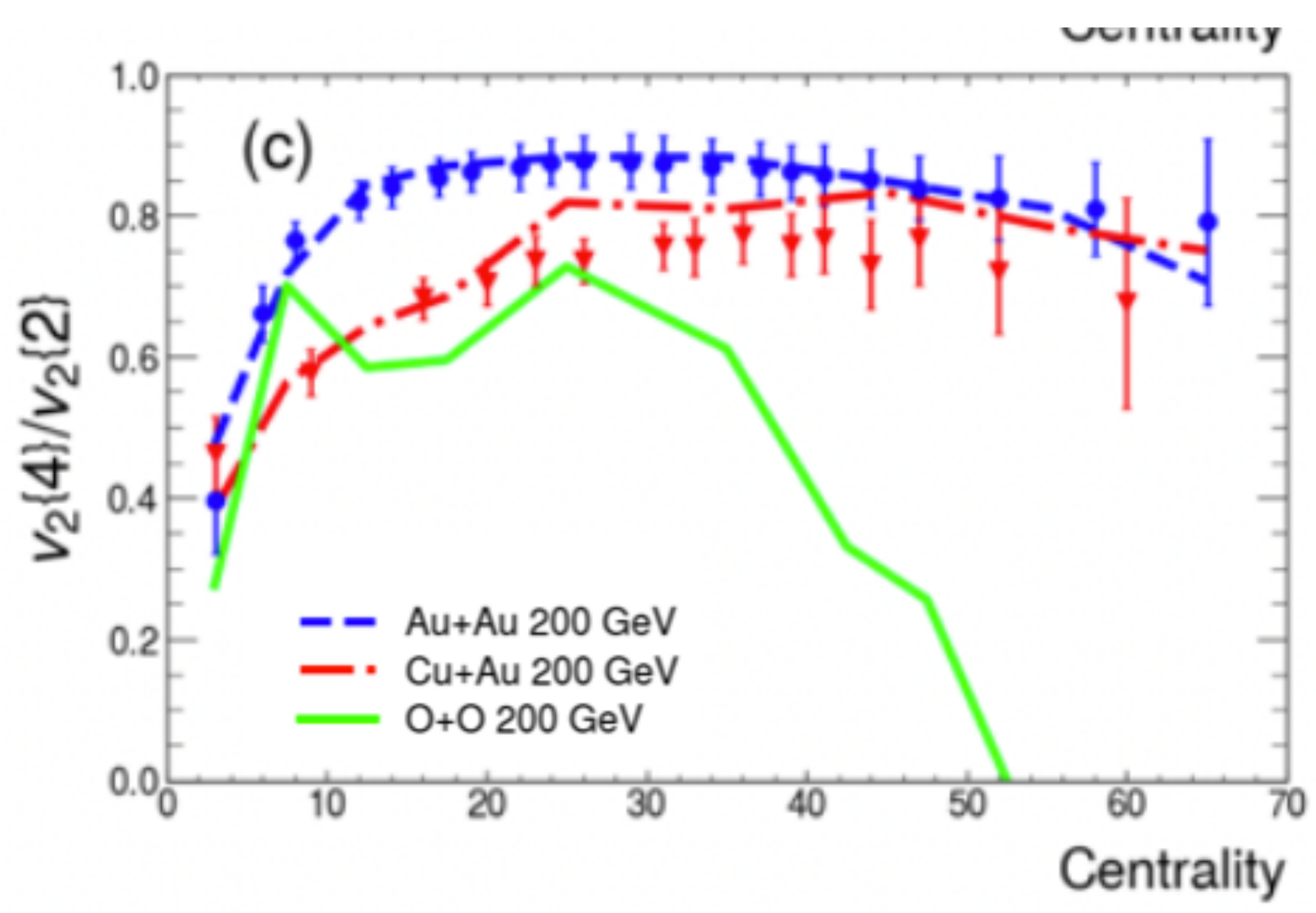
Anisotropic flow is a set of important observables used to constrain the initial conditions and transport properties of QGP.

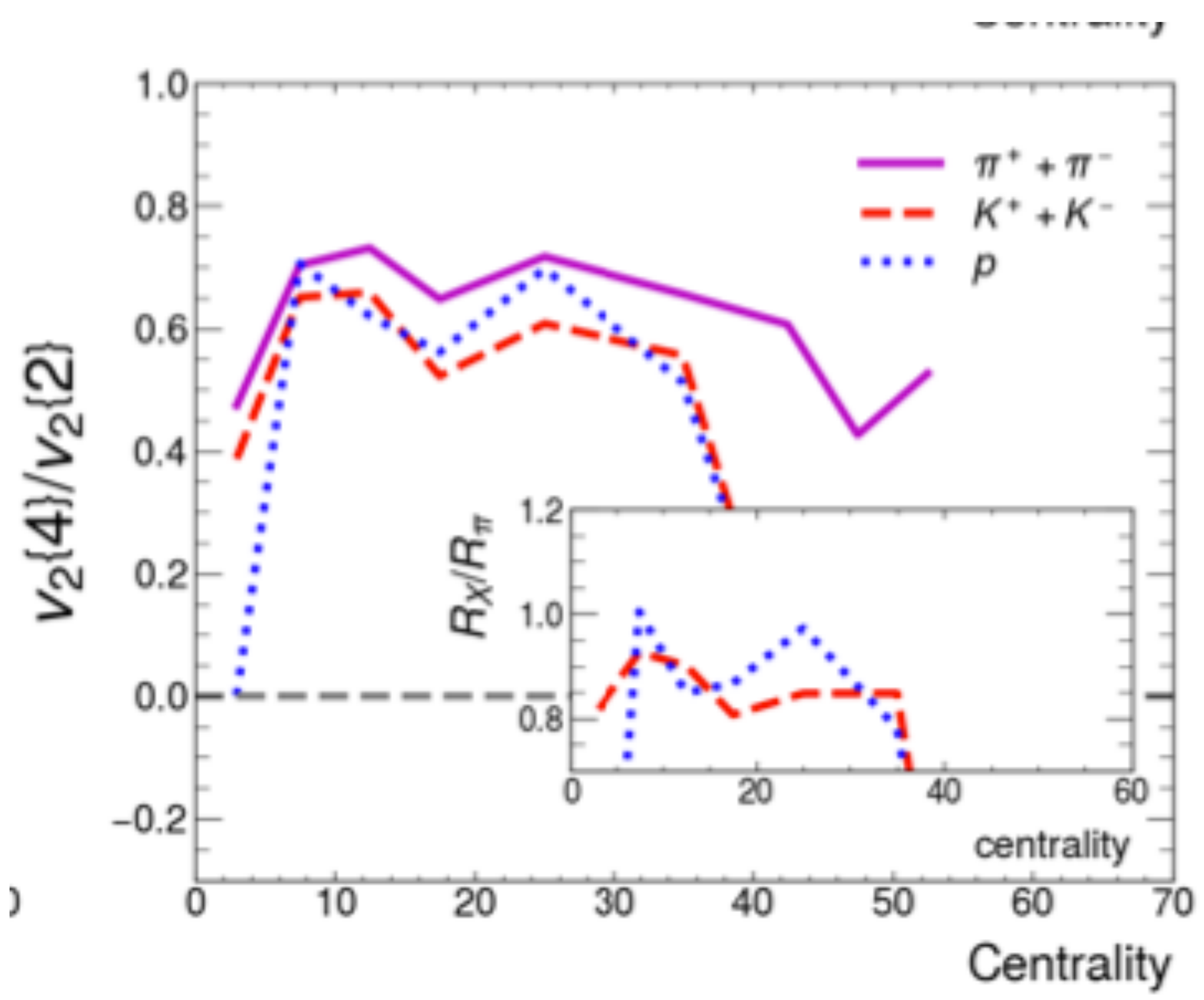


Our calculation provides a benchmark for understanding the RHIC-BES data.

- Elliptic and triangular flows increase slightly with the beam energy.
- The multi-particle cumulant ratio $v_2\{4\}/v_2\{2\}$ mainly origins from the fluctuations of initial conditions. It has weak collision energy dependence and strong collision system dependence.
- The ratio $v_2\{4\}/v_2\{2\}$ in small system (O+O) is no longer independent of particle species.



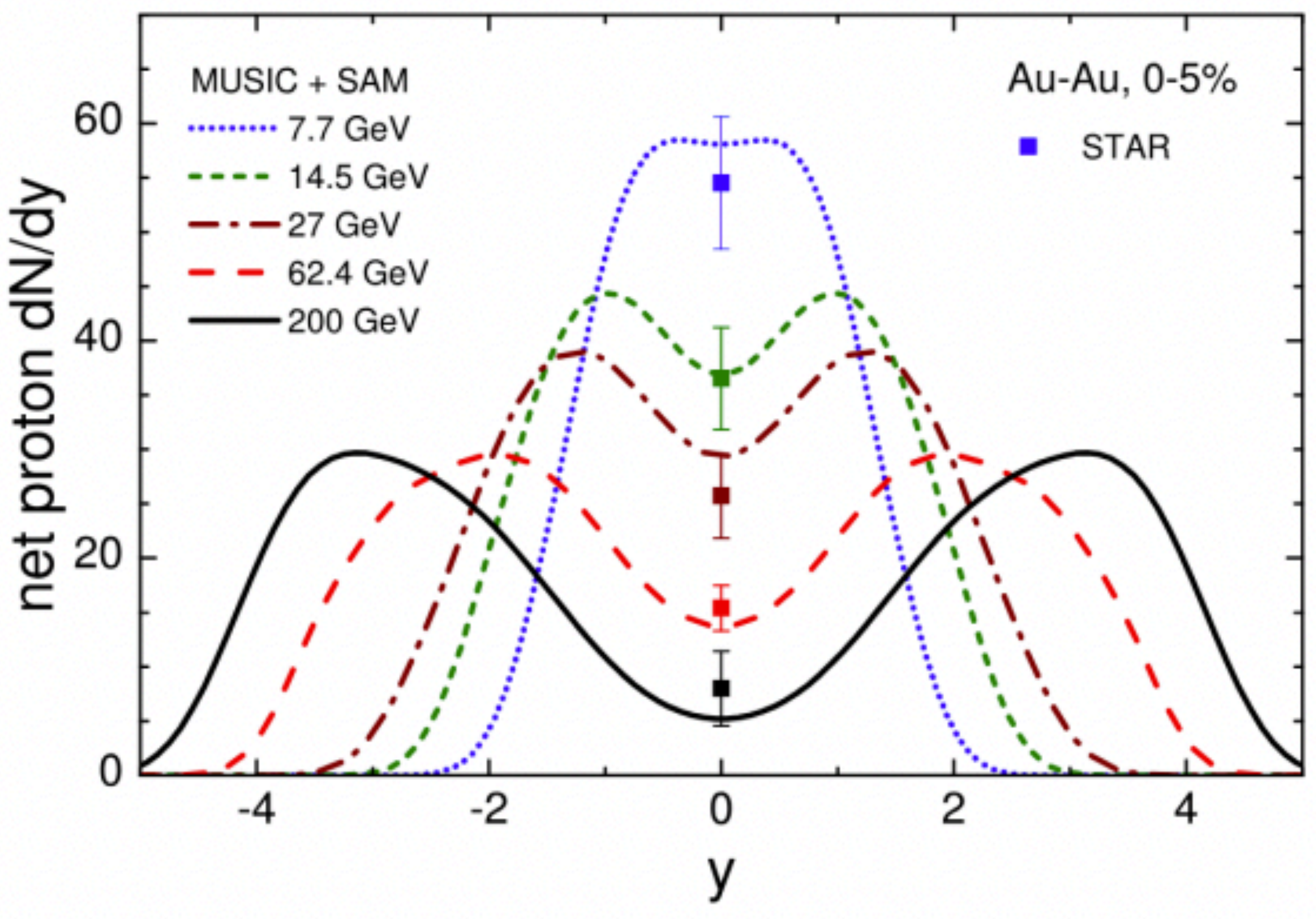




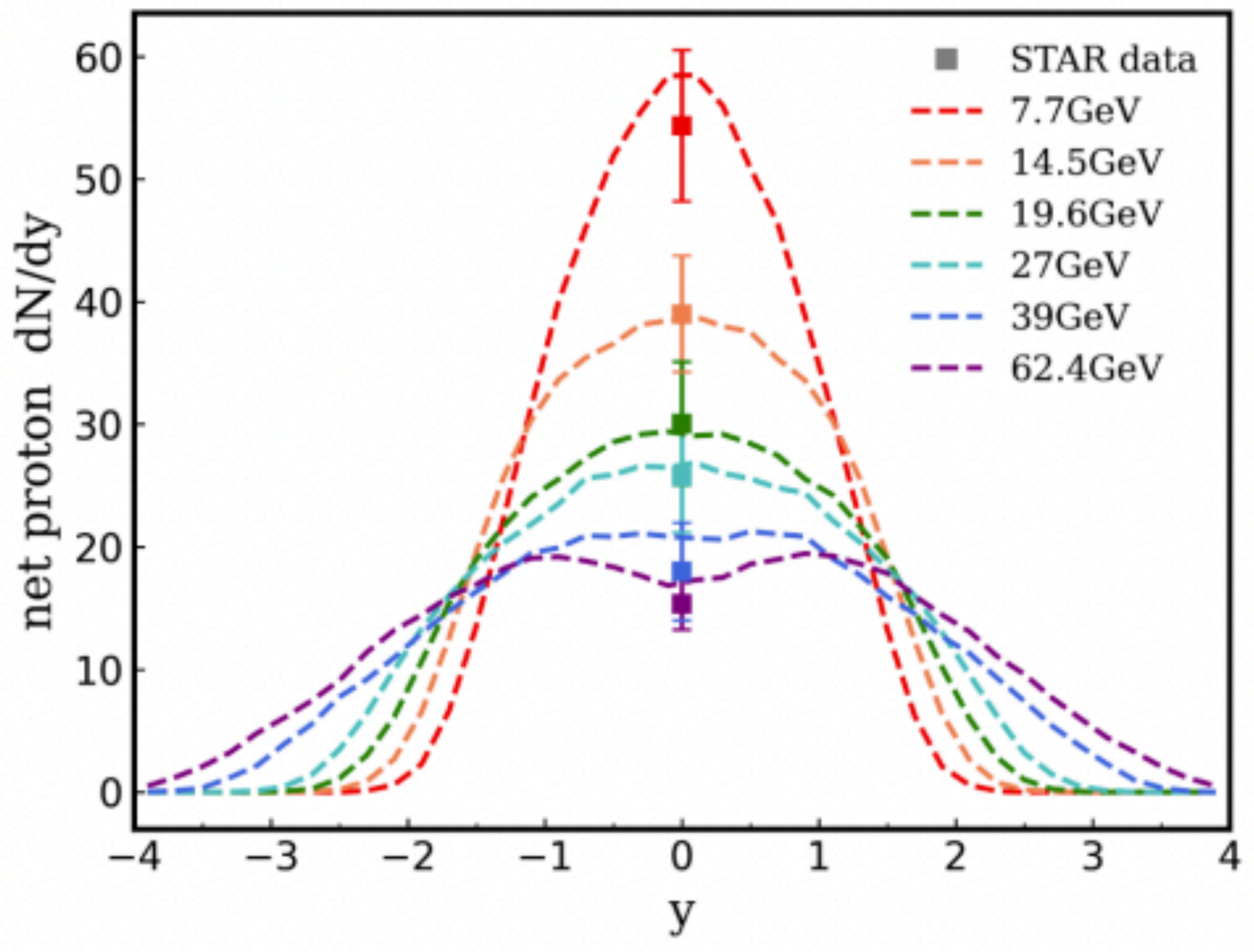
Outlook

There are still many things we do not know:

- The longitudinal dynamics.



Vovchenko, Koch, and C. Shen, PRC 105, 014904 (2022)



Y.-f. Shen, Chen, Wu, Xu, and Huang, 2404.02397

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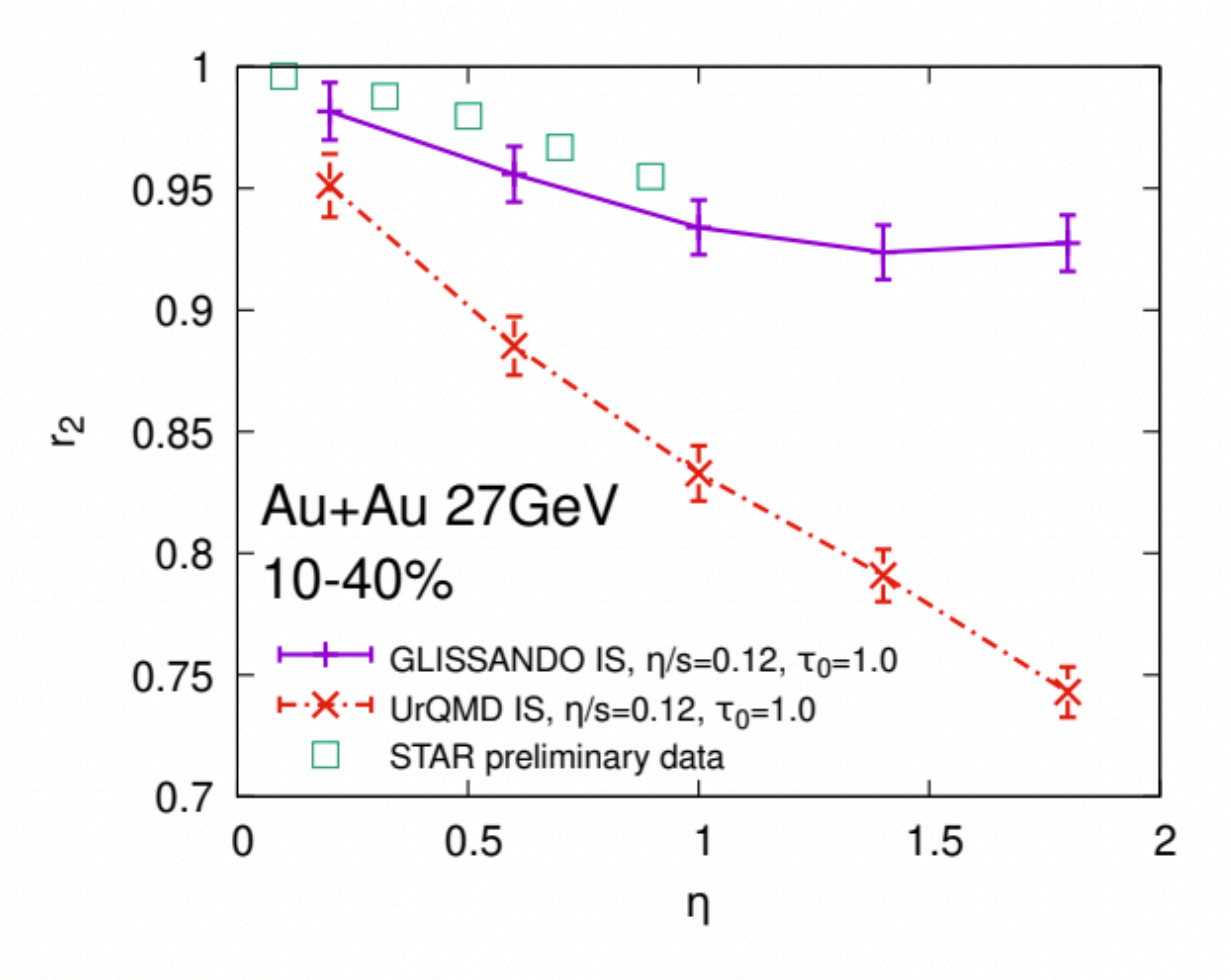
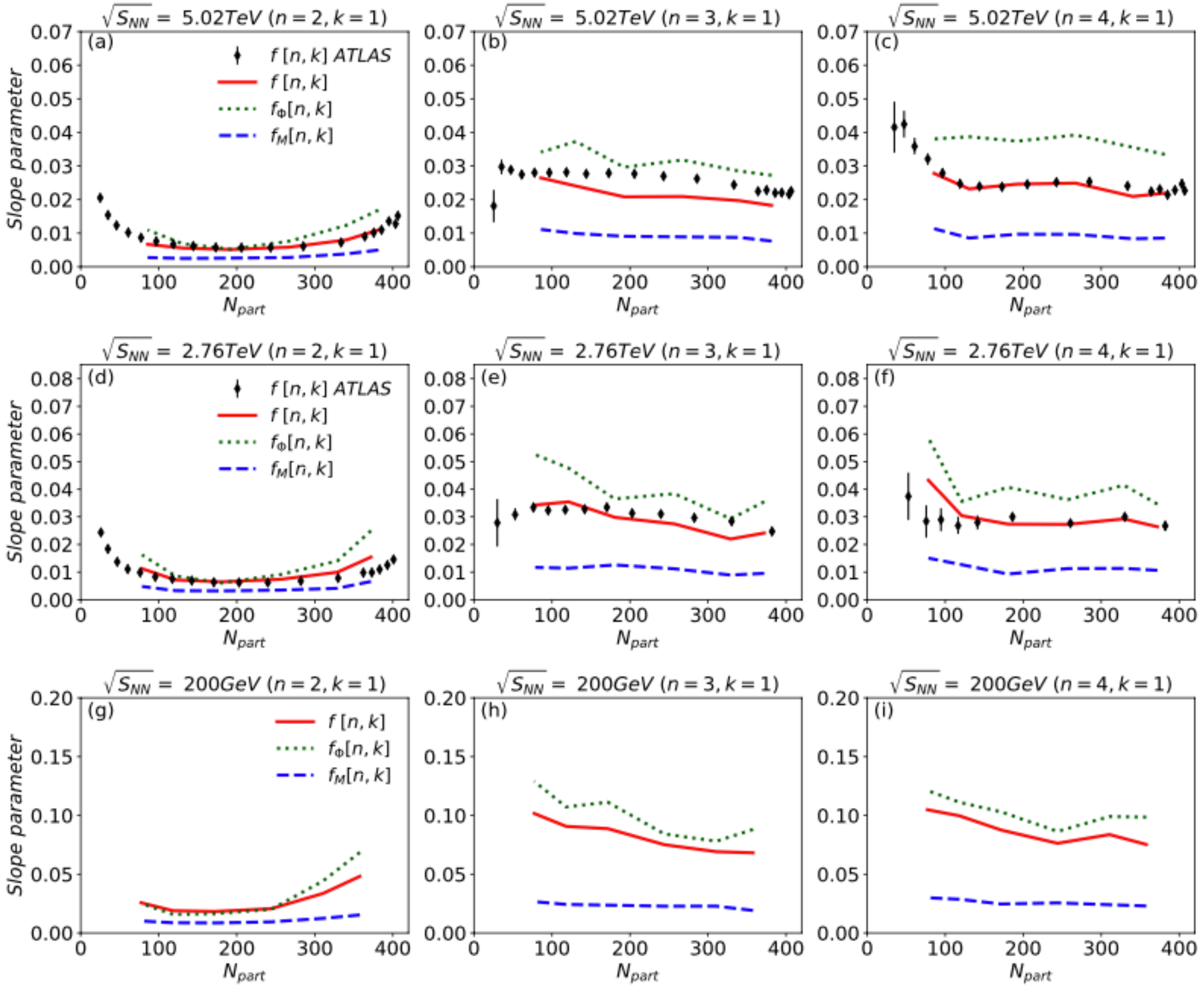


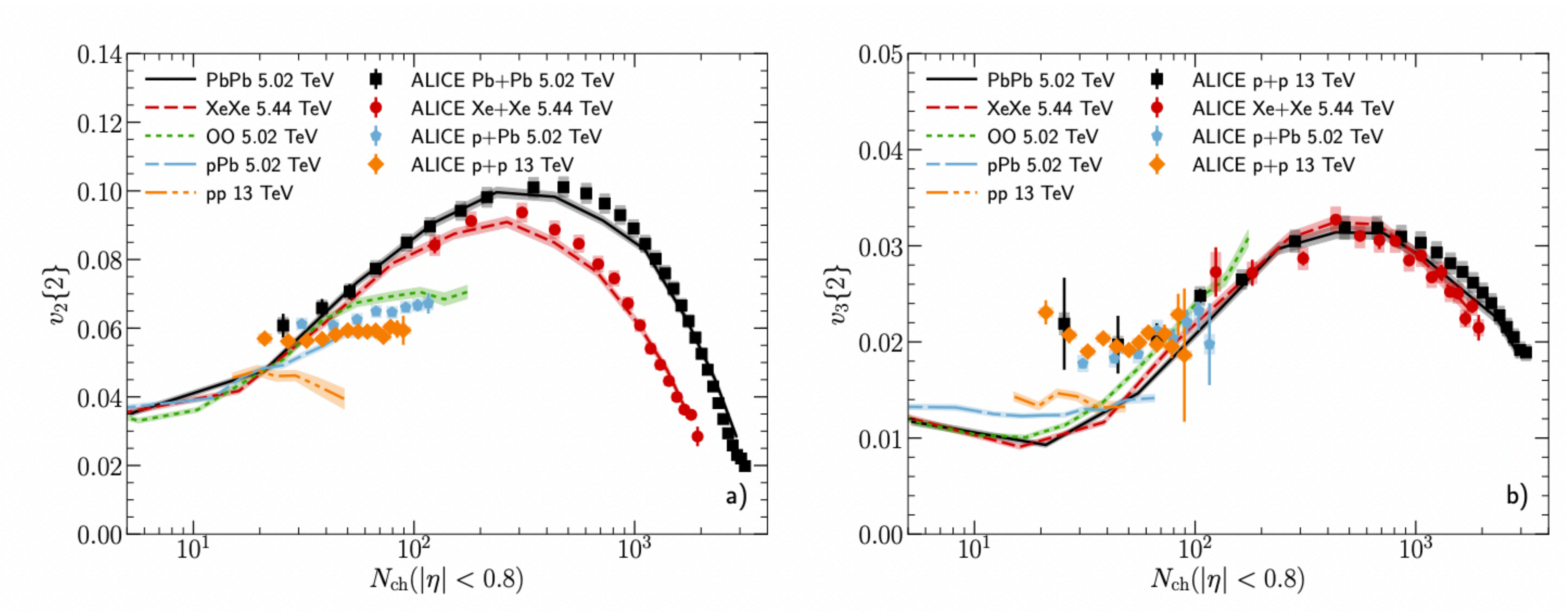


[Phys.Rev.C 104 (2021) 1, 014904]

Outlook

There are still many things we do not know:

- The longitudinal dynamics.
- The origins of anisotropic flow or flow fluctuations in small systems (p+p/p+A collisions).



[Phys.Rev.C 102 (2020) 4, 044905]

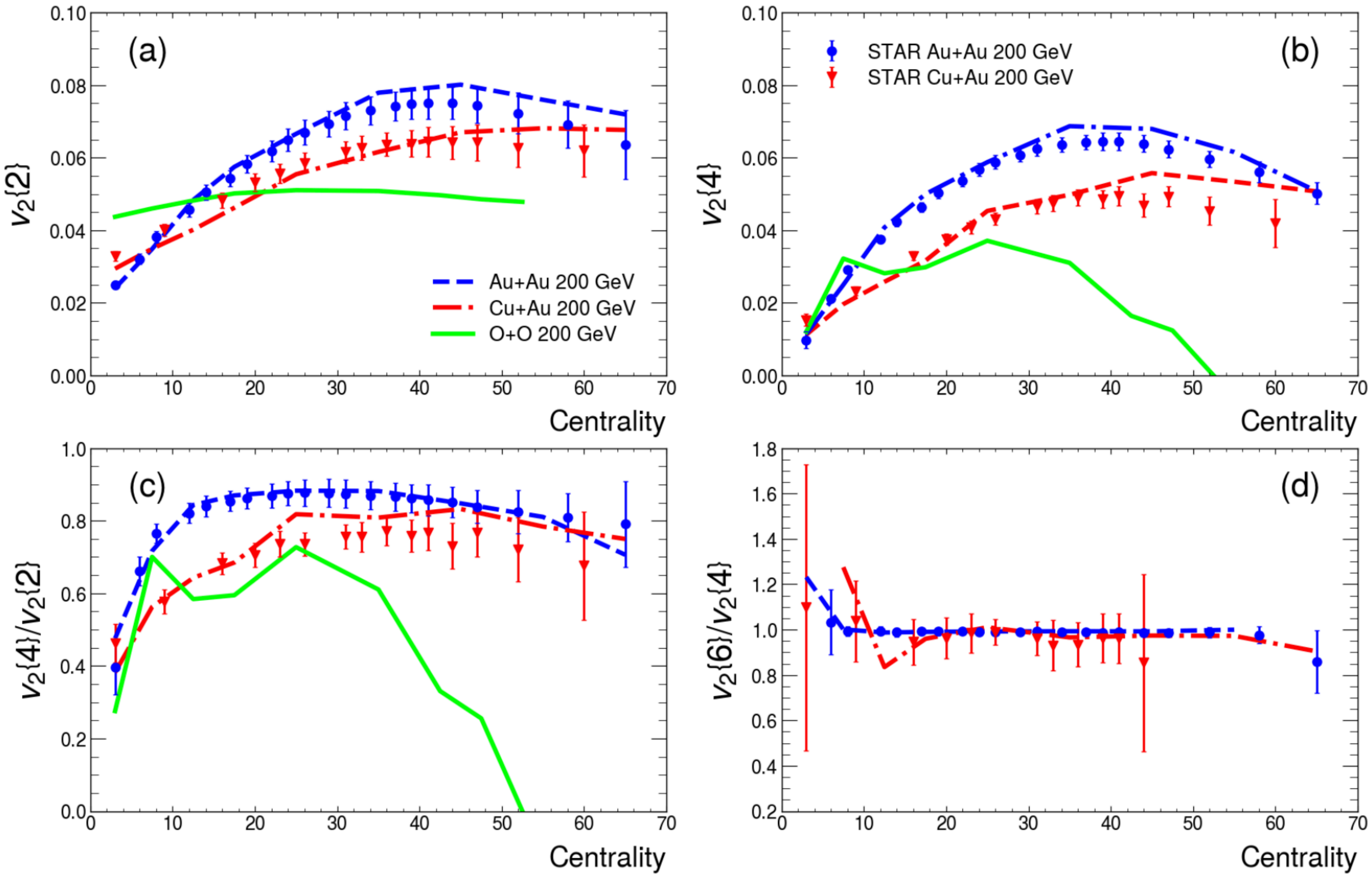
Outlook

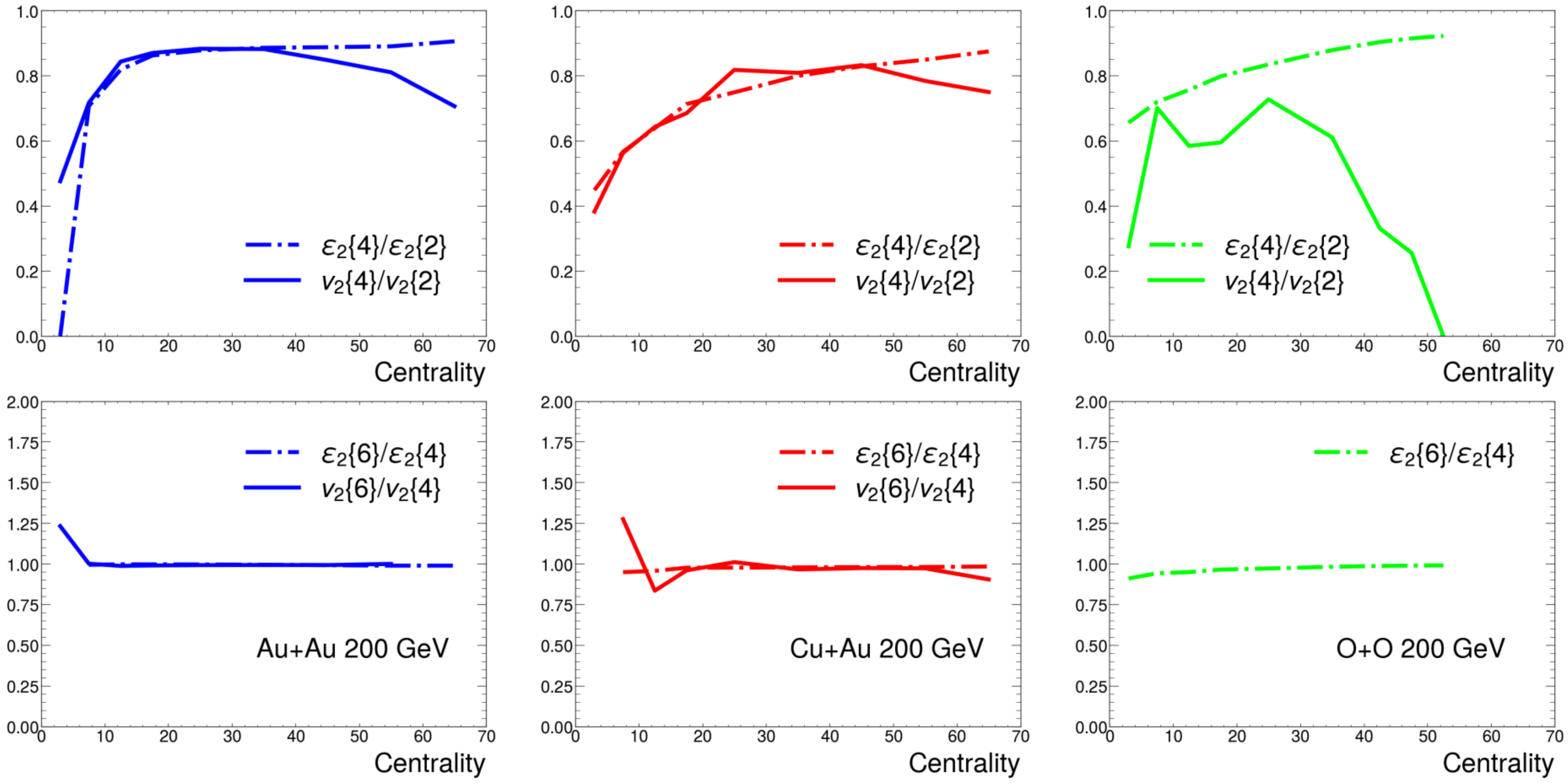
There are still many things we do not know:

- The longitudinal dynamics.
- The origins of anisotropic flow or flow fluctuations in small systems (p+A collisions).
- The evolution and diffusion of multiple conserved charges and the equation of state (EOS).

….

Thank you for your attention!


(a)
(b)
(c)
(d)
$v_2\{2\}$
$v_2\{4\}$
$v_2\{4\}/v_2\{2\}$
$v_2\{6\}/v_2\{4\}$
Centrality
Au+Au 200 GeV
Cu+Au 200 GeV
O+O 200 GeV
STAR Au+Au 200 GeV
STAR Cu+Au 200 GeV


$\varepsilon_2\{4\}/\varepsilon_2\{2\}$
$v_2\{4\}/v_2\{2\}$
Centrality
$\varepsilon_2\{6\}/\varepsilon_2\{4\}$
$v_2\{6\}/v_2\{4\}$
Centrality
Au+Au 200 GeV
Cu+Au 200 GeV
O+O 200 GeV

“Standard Model” of heavy ion collision