U.S. DEPARTMENT OF ENERGY | Office of Science

ILLINOIS

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BEAM ENERGY SCAN THEORY OVERVIEW

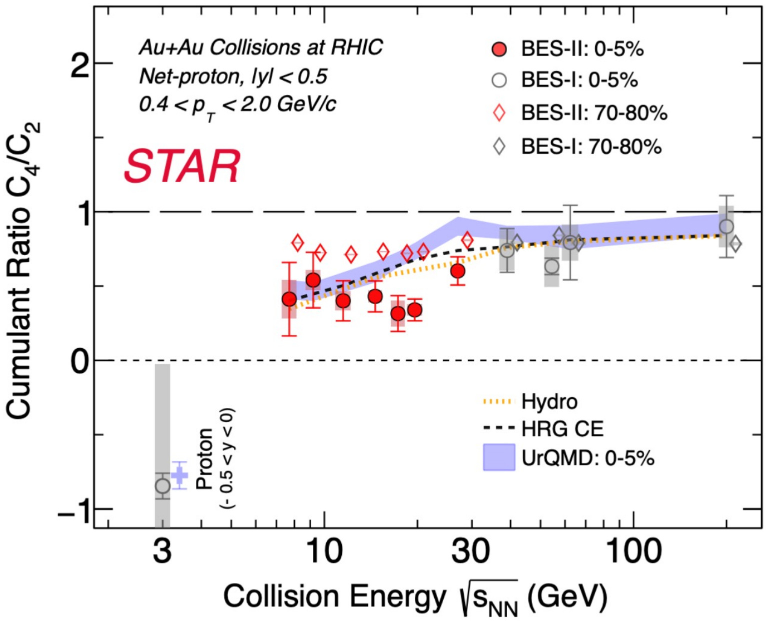
2024 RHIC/AGS ANNUAL USERS' MEETING

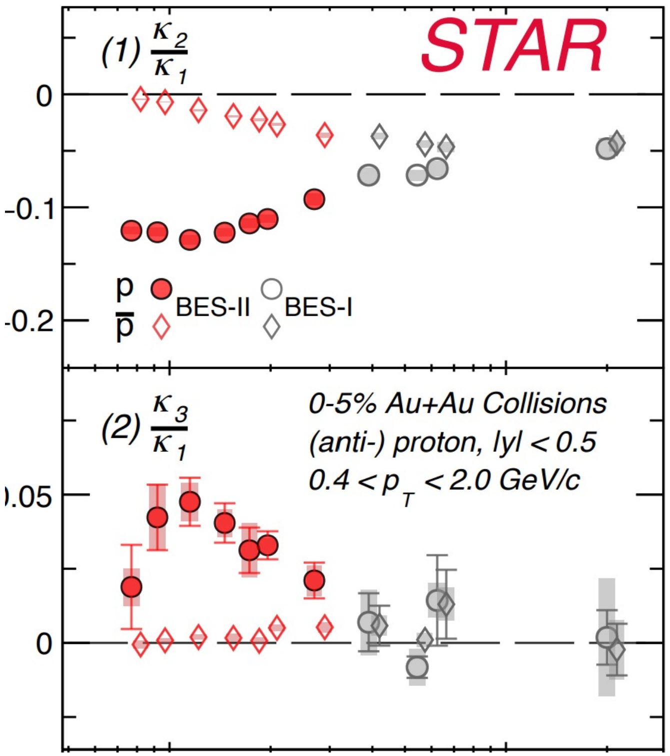
Jun 11-14, 2024

Illinois Center for Advanced Studies of the Universe

The task: integrated framework tools to connect experimental data to theory measurements.

- New experimental results from BES-II at STAR on the signatures of the critical point in Au+Au (7.7 – 27 GeV) collisions.
- $C4/C2$ shows minimum around ~ 20 GeV comparing to models without CP.
- Factorial cumulants show promising behaviour in agreement with signature.

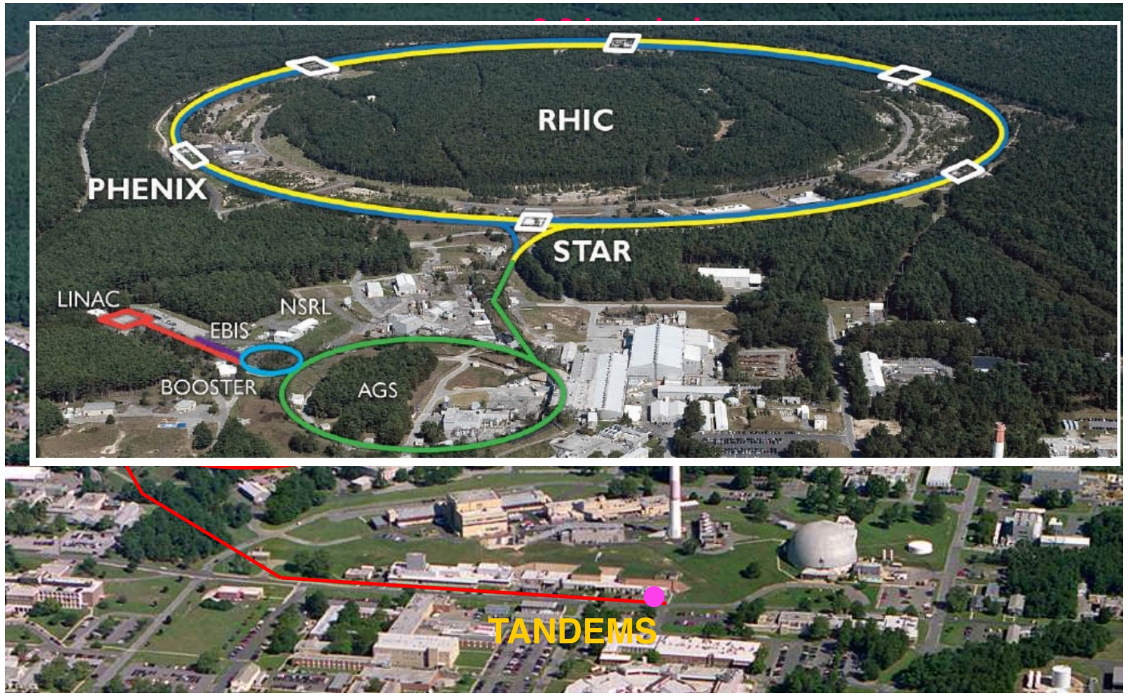




Can theory meet experiment at high $25 < \mu_q < 750\,\mathrm{MeV}$ region?.

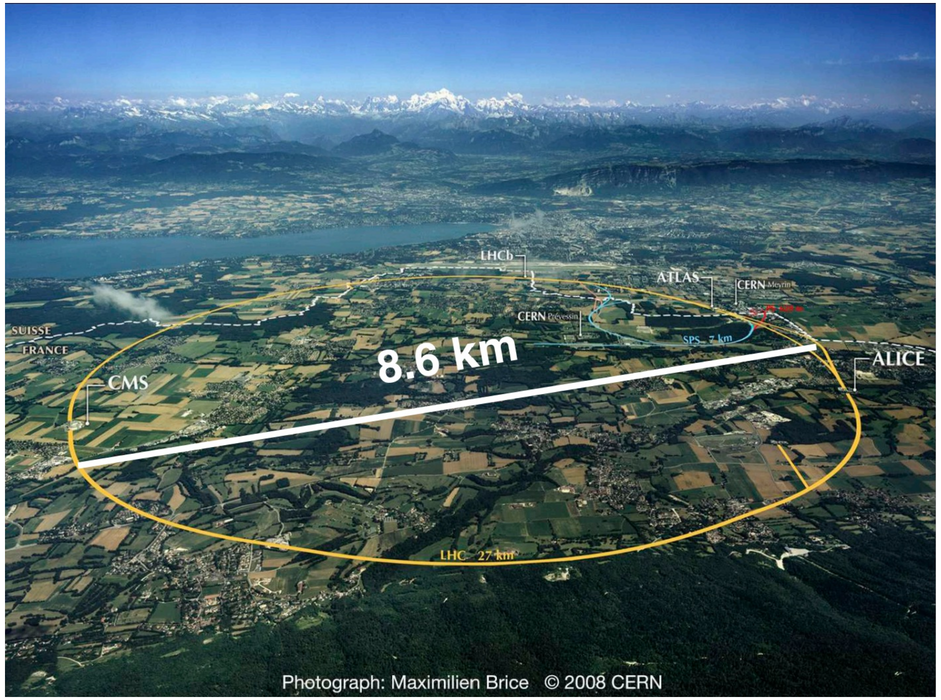
EXPLORING THE SMALLEST BITS OF MATTER AT EXTREME CONDITIONS

Creating the deconfined matter of quark gluon plasma in the lab



Relativistic Heavy Ion Collider (RHIC)

[Brookhaven National Lab BNL, Long Island, NY]

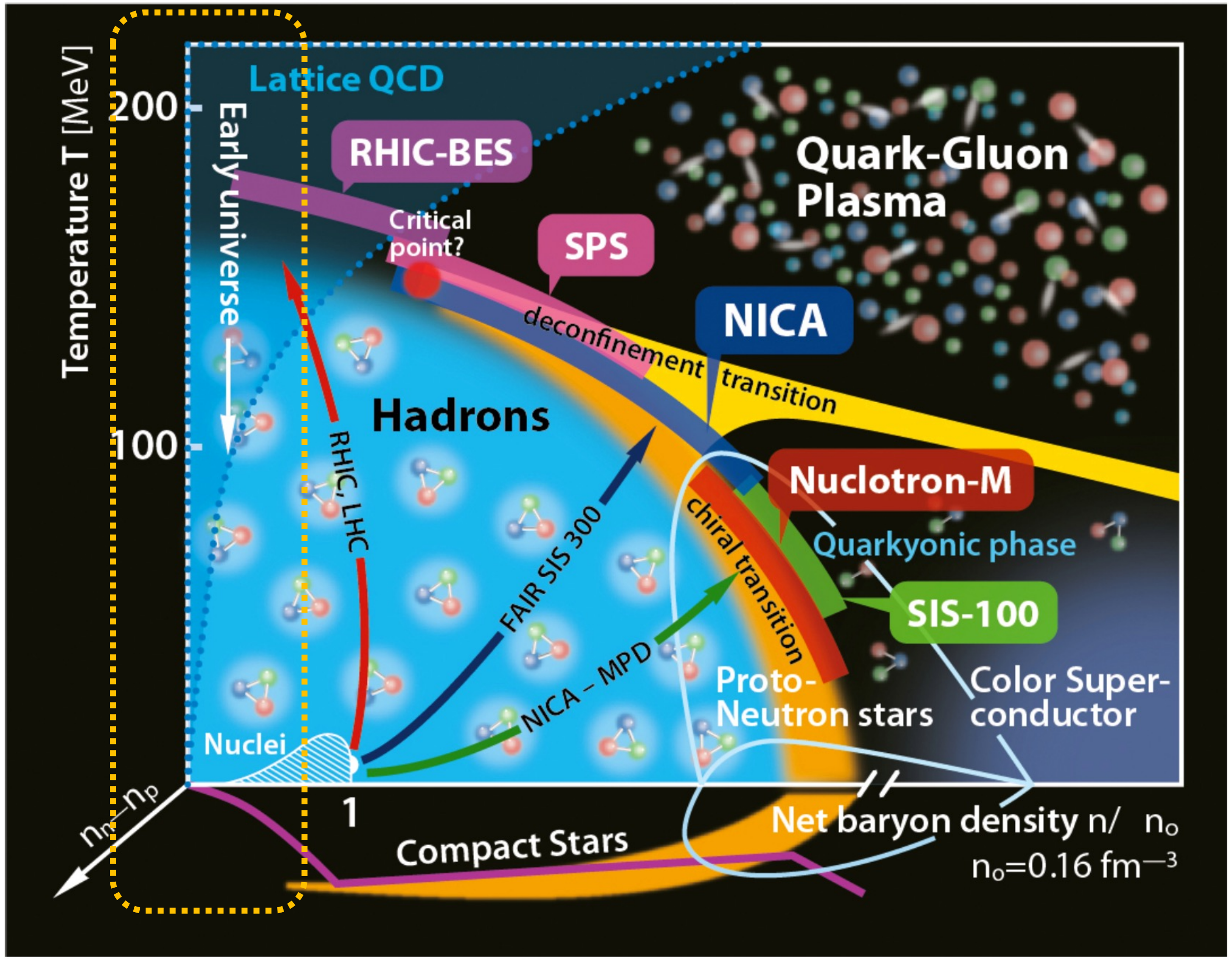


Large Hadron Collider (LHC) at CERN

[CERN, Geneva, Switzerland/France]

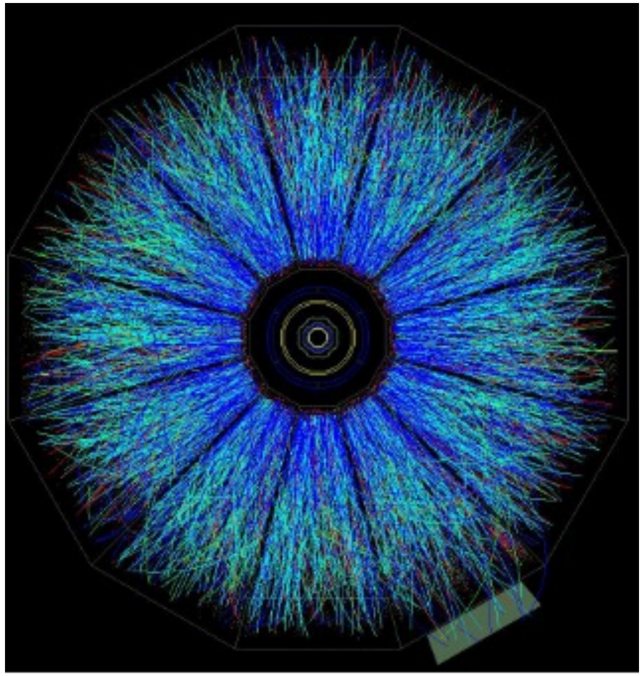
BIG QUESTIONS | THE PHASE DIAGRAM OF STRONGLY COUPLED MATTER | | TOWARDS HIGH DENSITY

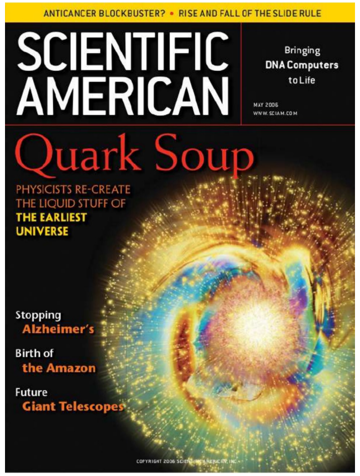
- What are the relevant degrees of freedom in different regions of the phase diagram?
- What is the nature of the transition between these phases?. Is it a cross over or critical point?
- Understanding the **transport properties** of the strongly interacting matter of QCD
- A lot of the Physics in the phase diagram currently are **theoretical predictions**
- Great progress made possible through the **interplay** between theory and experiment!



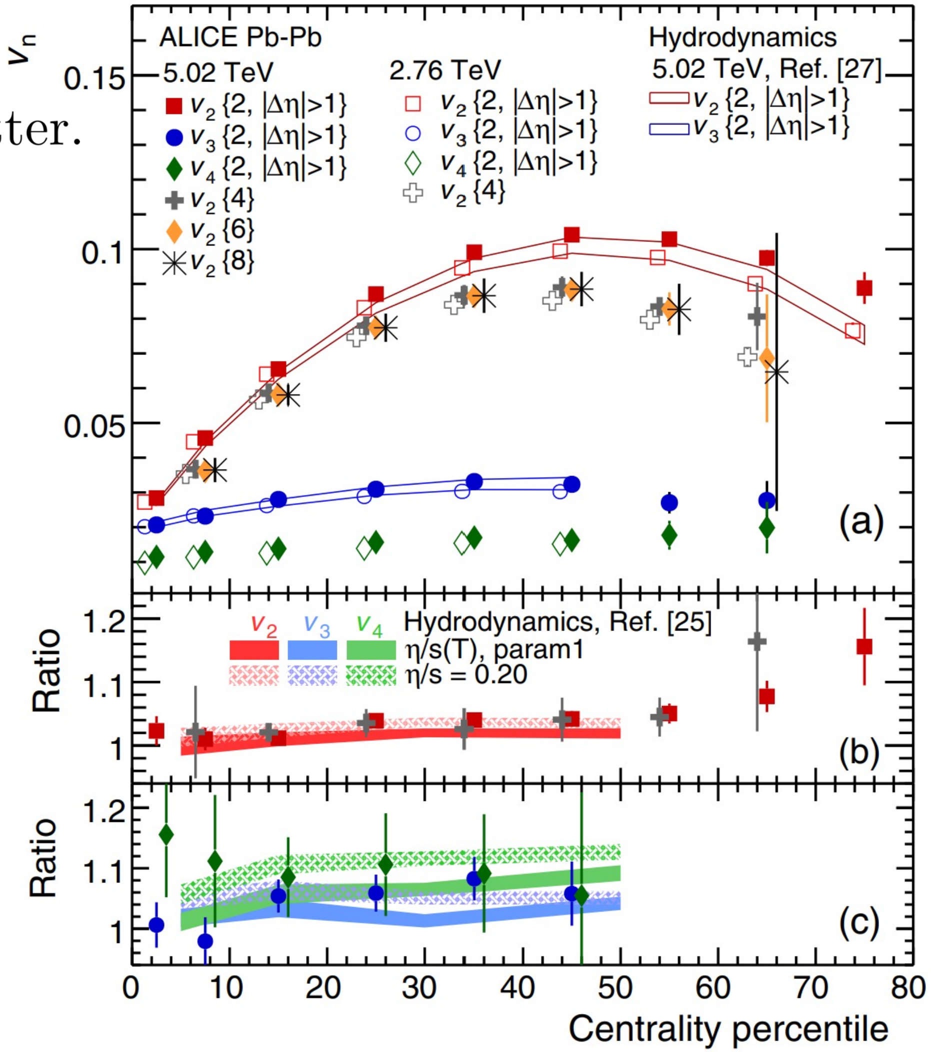
WHY DOES IT WORK? | | PHENOMENOLOGICAL SUCCESS OF HYDRODYNAMICS | | MAJOR DISCOVERIES

- Precise predictive power of observables.
- Extraction of the detailed properties of the QGP state of matter.





$$\eta/s \approx 1/4\pi$$



Huovinen, Kolb, Heinz, Ruuskanen ,Voloshin, Physics Letters B 99,(2001)

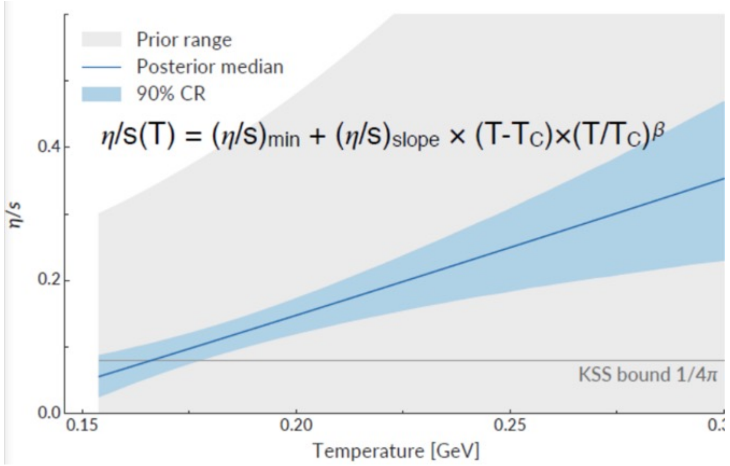
Noronha-Hostler, Luzum, and Ollitrault PRC 93 (2016)

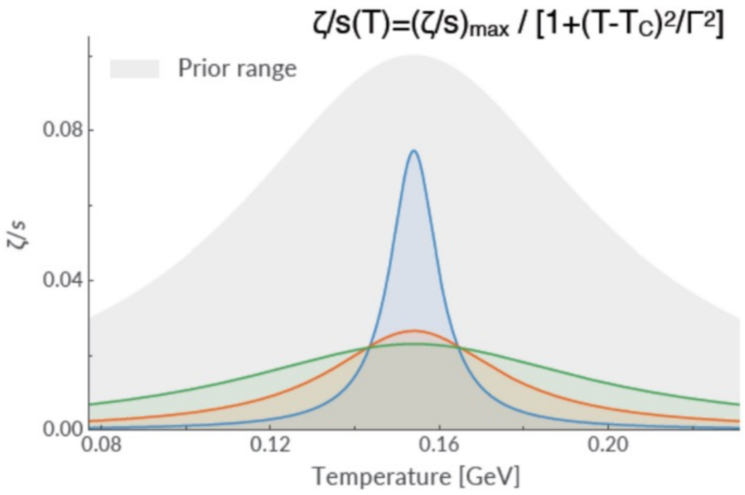
Niemi, Eskola, Paatelainen, and K. Tuominen PRC 93, 2016

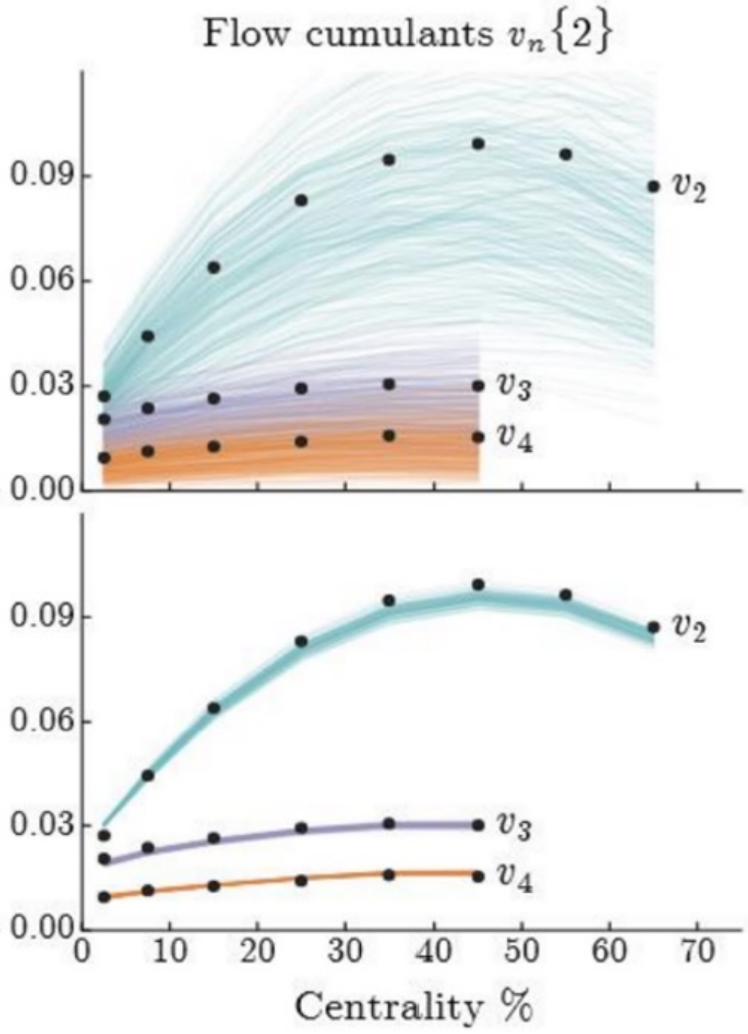
The ALICE Collaboration PRL 116, (2016)

EXTRACTING THE QGP TRANSPORT PROPERTIES

- Bayesian global fitting to extract the QGP shear and bulk viscosity.
- Good constraining power on η/T and ζ/T in PbPb at the LHC.
- The extracted η/s is close to the KSS bound of $1/4\pi$.



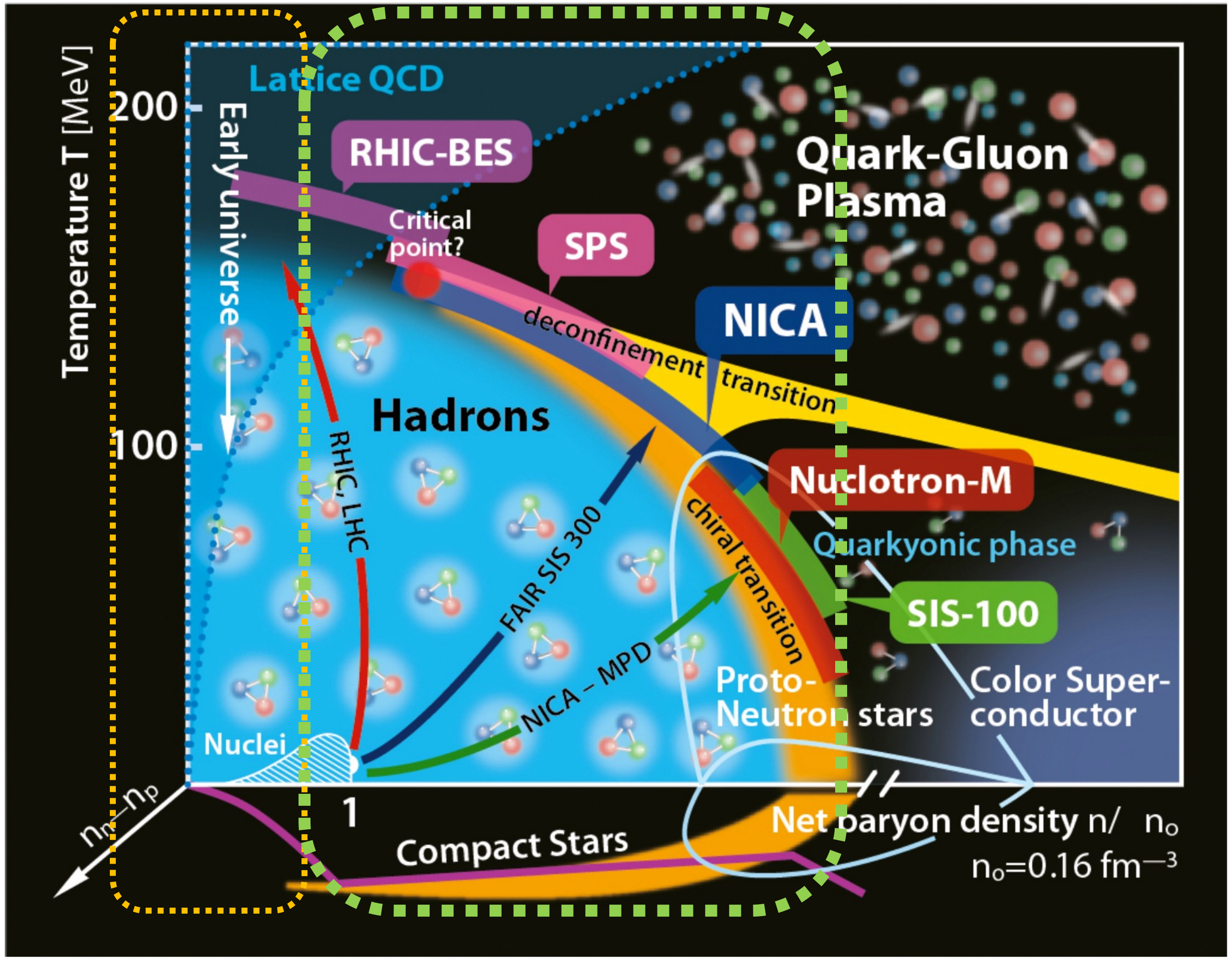




Bernhard, Moreland, Bass, Nature Physics 15, 1113 (2019)

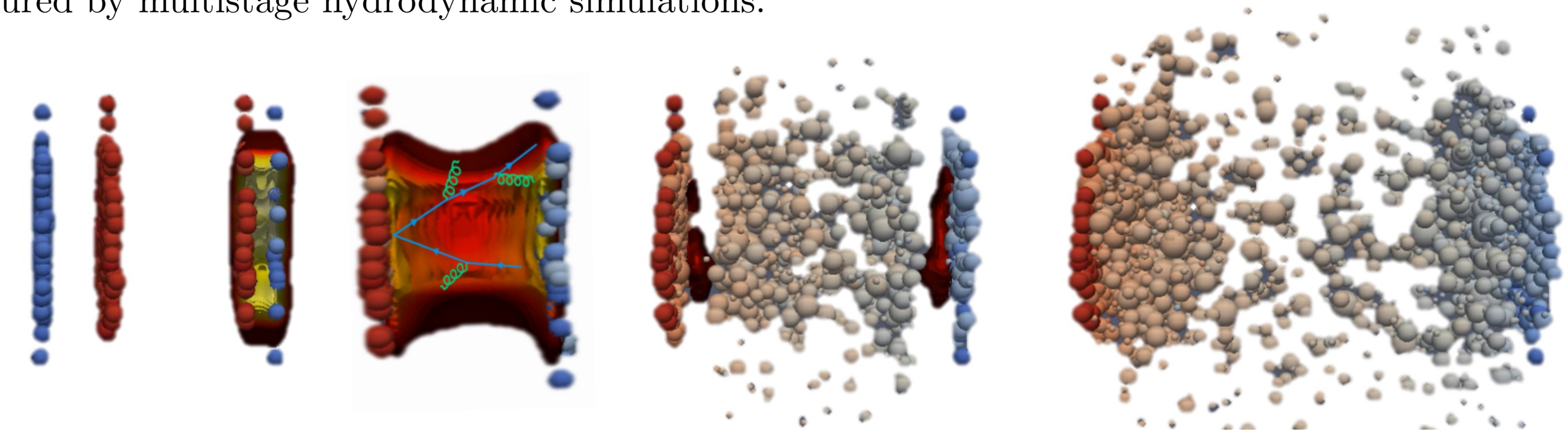
	$\hat{q}$	η/s	T_{init}	T_c
RHIC 200 GeV	1.2 ± 0.3	0.12	> 300 MeV	~160 MeV
LHC 2.67 TeV	1.9 ± 0.7	0.2	> 400 MeV	
	GeV²/fm	Ratio = 1.6	From Photons T = 220 MeV & >300 MeV	

- Heavy-ion collisions have the potential to answer the question of the existence of the QCD critical point and to determine its location?
- By scanning the QCD phase diagram via varying the crucial experimental control parameter: the beam energy, or the center-of-mass collision energy per nucleon pair $\sqrt{s}_{NN}$.
- This is the major motivation behind the Beam Energy Scan (BES) program at the Relativistic Heavy Ion Collider (RHIC).



Standard Model of Heavy-Ion Collisions|| Multistage description of bulk dynamics

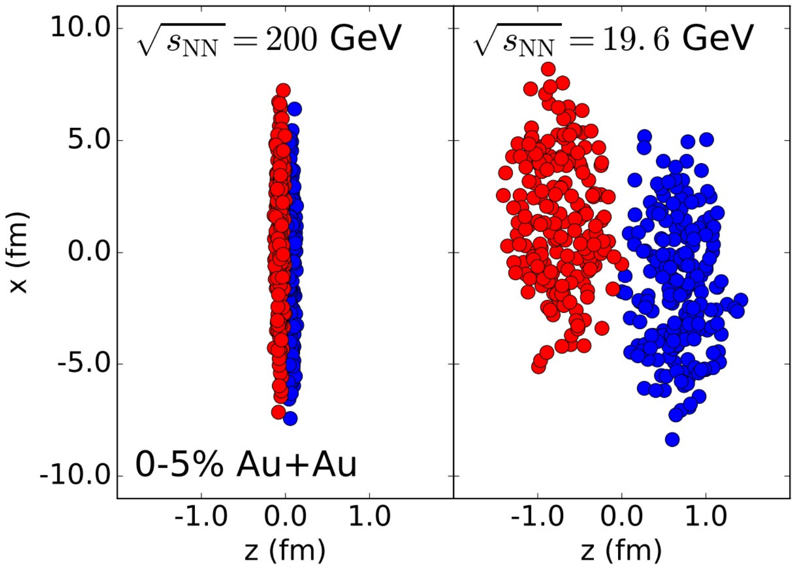
- At intermediate and high energies ($\sqrt{s}_{NN} \geq 7GeV$), the bulk dynamics captured by multistage hydrodynamic simulations.

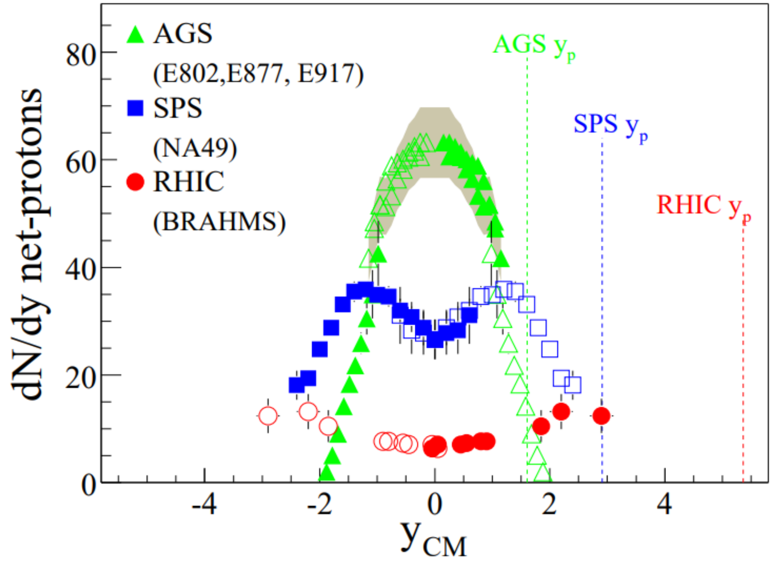


initial state + pre-equilibrium $\tau = 0.1\,\mathrm{fm/c}$ | Hydrodynamics $\tau = 1\,\mathrm{fm/c}$ | Freezeout/ Particalization $\tau = 10\,\mathrm{fm/c}$ | Hadronic afterburner $\tau = 20\,\mathrm{fm/c}$

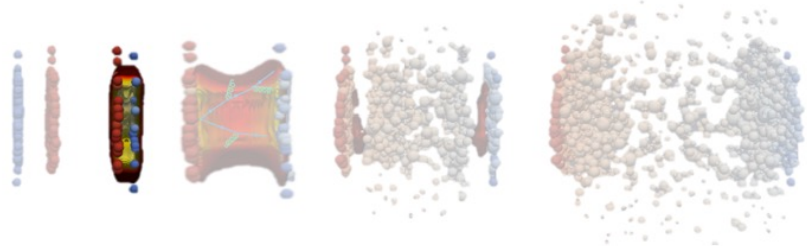
- Understanding the bulk dynamics forms the foundation to study:
  - **Critical phenomena.**
  - **EoS constraints.**
  - **Transport properties.**
  - **Hard probes.**
  - **Electromagnetic probes**

THEORETICAL MODELING || INITIAL STATE, BARYON STOPPING, AND BARYON JUNCTION





$$t_{\text{overlap}} = \frac{2R}{\gamma v_z} = \frac{2R}{\sinh(y_b)}\,.$$



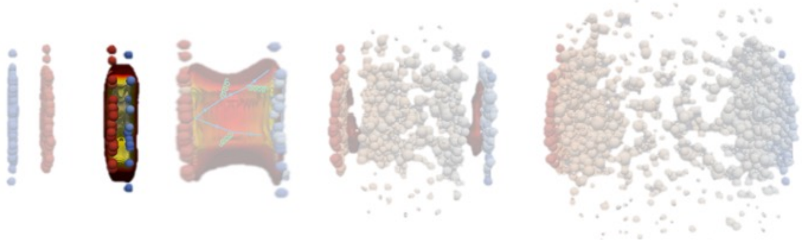
- High energy: instantenous collisions allow for space dependent deposition of enrgy or entropy in the initial state with $\epsilon(x, y)$ or $s(x, y)$.
- low/intermediate energy: Construction of **dynamic** space-time $(3+1)$-dimensional Initial State.
- Net baryon $\neq 0$ and highly non-uniform.
- **Primary challenge** is understanding the mechanisms governing baryon stopping and energy deposition during the pre-hydrodynamic phase.

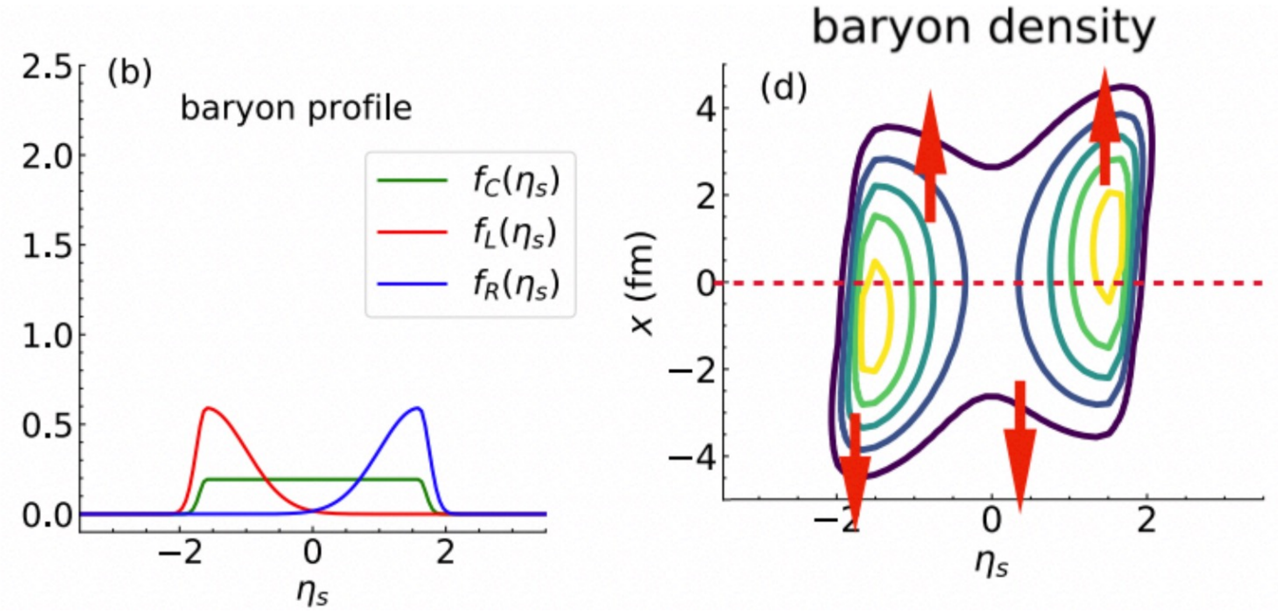
Bearden et al., PRL 93 (2004)

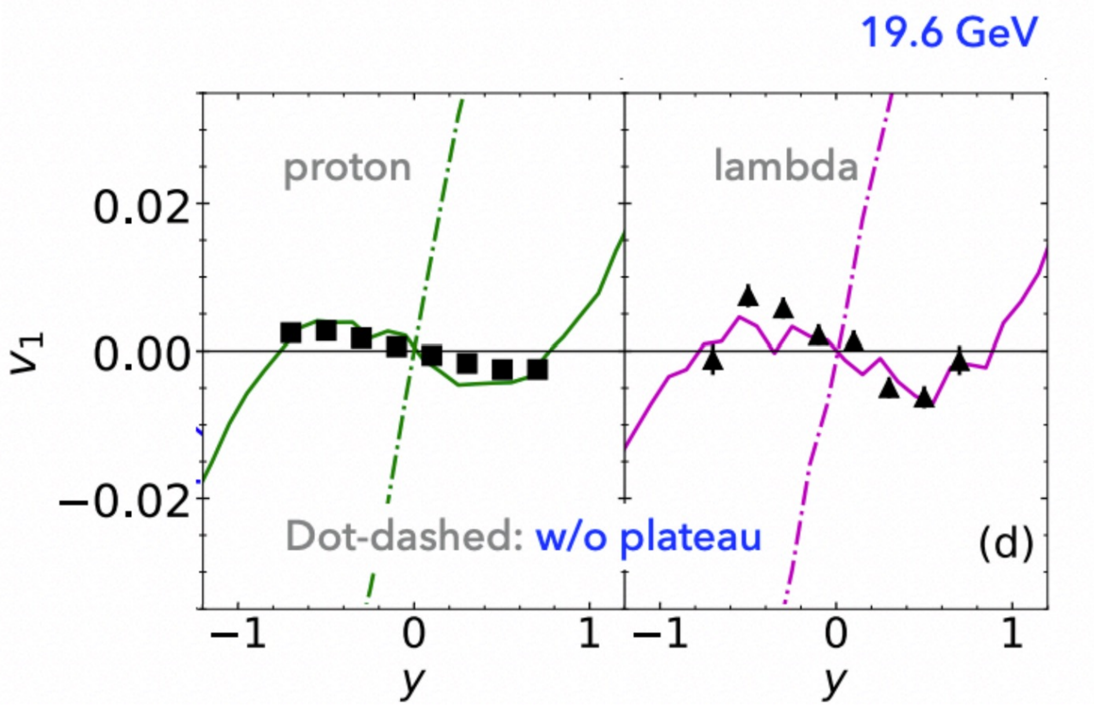
Miller et al, Ann. Rev. Nucl. Part. Sci. 57, (2007)

Paramteric initial conditions

- Phenomenologically, longitudinal profiles are parameterized to account for specific observables sensitive to longitudinal bulk dynamics, e.g. longitudinal decorrelation and rapidity-dependent identified particle yields.





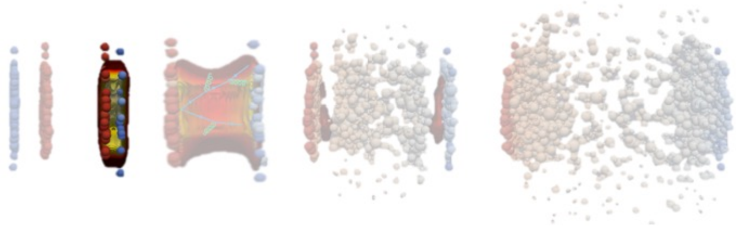


Du et al, PRC 108 (2023)

Ryu, Jupic, and Shen PRC 104

Dr Xiang-Yu Wu Wed @3:45pm

Dynamical initialisation

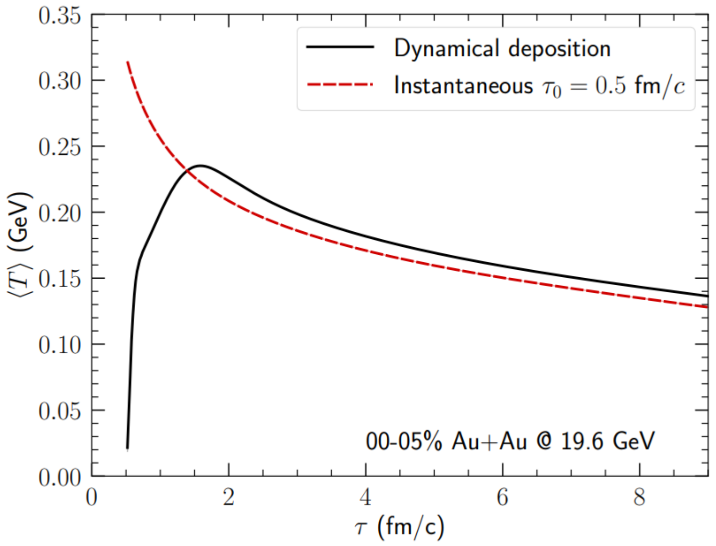


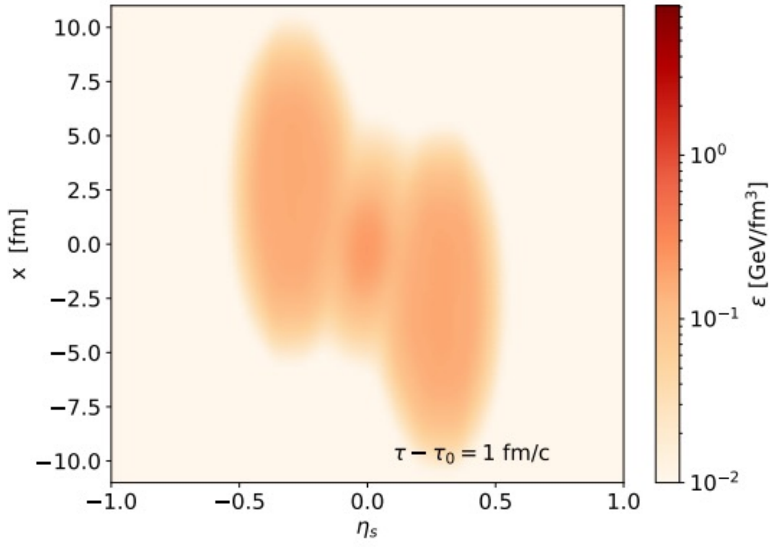
- A spacetime-dependent transition from a microscopic partonic description to macroscopic hydrodynamic.

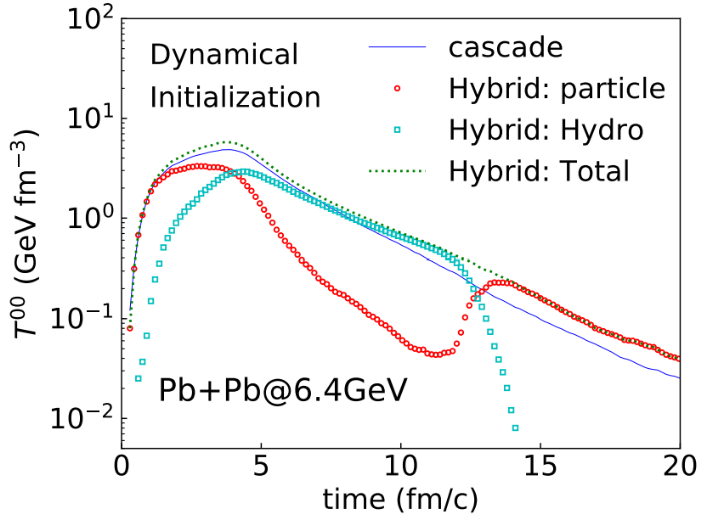
$$\partial_\mu T^{\mu\nu}_{\rm fluid}(x) = J^\nu_{\rm source}(x) \equiv -\partial_\mu T^{\mu\nu}_{\rm part}(x) \ ,$$
$$\partial_\mu N^{\mu}_{\rm fluid}(x) = \rho_{\rm source}(x) \equiv -\partial_\mu N^{\mu}_{\rm part}(x) \ ,$$

Chen and Schenke, PRC97(2018).

Akamatsu et al. ,PRC. 98 (2018)







Shen et al., Hard Probes 2023.

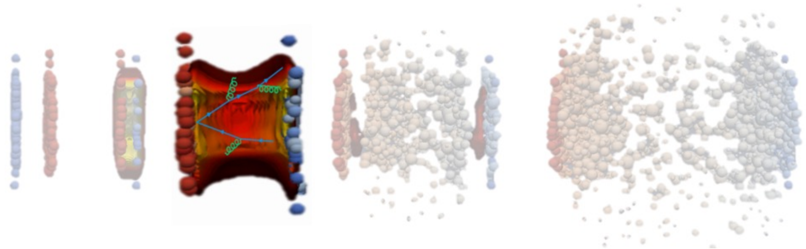
Cimerman et al., PRC 107 (2023)

- These approaches are unable to elucidate the mechanisms of thermalization or even hydrodynamization.
- The criteria for the time-dependent transition between initial conditions and hydrodynamics are somewhat ad hoc

DISSIPATIVE HYDRODYNAMICS: AN EFFECTIVE FIELD THEORY

The state of the system is given by the fields

$$\varphi_i = \{u^\mu(\mathbf{x}), \varepsilon(\mathbf{x}), \rho_q(\mathbf{x}), \Pi(\mathbf{x}), \pi^{\mu\nu}(\mathbf{x}), n_q^\mu(\mathbf{x})\}$$



Conservation laws

$$\boxed{\begin{aligned} D_\mu T^{\mu\nu} &= 0\,, \\ D_\mu N_q^\mu &= 0 \end{aligned}}$$

(Energy-momentum conservation)

(Charge conservation)

Constitutive relations

$$\boxed{\begin{aligned} T^{\mu\nu} &= \varepsilon u^\mu u^\nu - p\Delta^{\mu\nu} + \Pi^{\mu\nu} \\ N_q^\mu &= \rho_q u^\mu + n_q^\mu \end{aligned}}$$

$$(u^\mu u_\mu = -1\,,\ \Delta_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu\,)$$

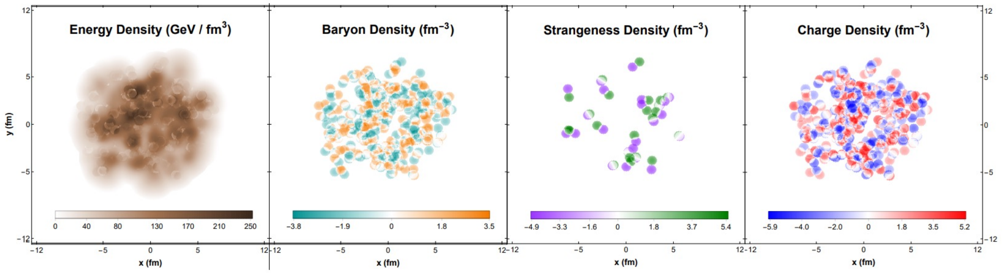
Hydrodynamics frame

$$u_\mu T^{\mu\nu} = \varepsilon u_\nu\,,$$

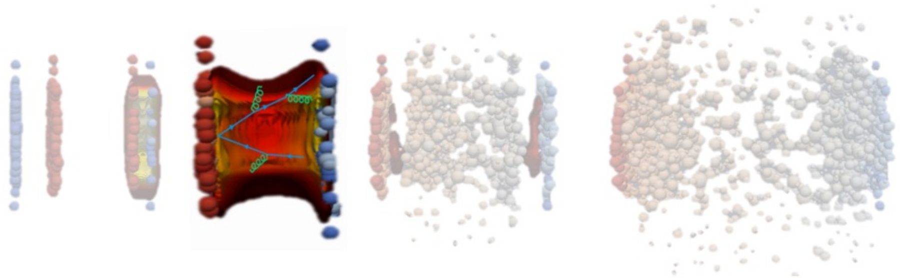
(Landau Frame)

$$T^{\mu\nu} = \begin{array}{c} \begin{array}{cc} \text{energy density} & \text{energy flux} \end{array} \\ \begin{pmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{pmatrix} \\ \begin{array}{ccc} \text{momentum density} & \text{momentum flux} & \text{isotropic pressure} \end{array} \end{array}$$

Initial fields



How does one evolve the dissipative fields $(\Pi, \pi^{\mu\nu}, n^\mu)$?



Maximum entropy principle: 2nd order

Israel, Stewart, Ann. Phys. 118 (1979)

DA, Dore, Noronha-Hostler arxiv.2209-11210

$$S^\mu = su^\mu - \sum_q^{B,S,Q} \alpha_q n_q^\mu - \frac{1}{2}u^\mu\left(\beta_\Pi \Pi^2 + \beta_\pi \pi^{\mu\nu}\pi_{\mu\nu} + \sum_q^{B,S,Q} \beta_n^{qq'} n_q^\mu n_\mu^{q'}\right) - \sum_q^{B,S,Q}\left(\gamma_{n\Pi}^q n_q^\mu \Pi + \gamma_{n\pi}^q n_q^\nu \pi_\nu^\mu\right) - \frac{1}{2}\left(u^\nu \beta_{\Pi\pi}\Pi\pi_{\mu\nu}\right)$$

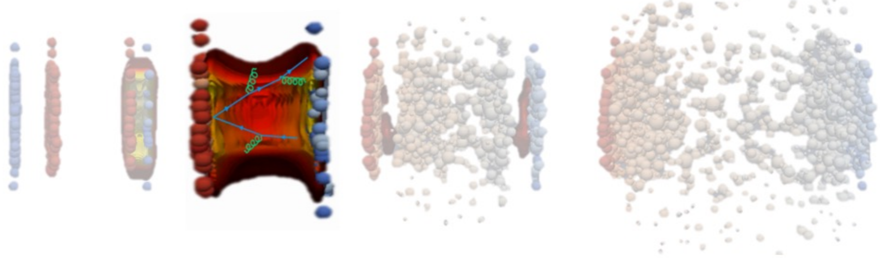
$$\tau_\Pi \dot{\Pi} + \Pi = -(\zeta + \frac{\tau_\Pi}{2}\Pi)\theta - \underbrace{\frac{\tau_\Pi}{2\beta_\Pi}\dot{\beta}_\Pi \Pi}_{\dot{\beta}} + \underbrace{\frac{\zeta\lambda_\Pi^q}{\beta} n_q^\mu \nabla_\mu \delta_{n\Pi}^q - \frac{\zeta\delta_{n\Pi}^q}{\beta}\partial_\mu n_q^\mu + \frac{\zeta\delta_{\Pi\pi}}{2\beta}\pi^{\mu\nu}\sigma_{\mu\nu}}_{\text{dissipative couplings}},$$

$$\tau_\pi \dot{\pi}^{\mu\nu} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \frac{\tau_\pi}{2}\pi^{\mu\nu}\theta + \frac{\tau_\pi \dot{\beta}_\pi}{2\beta_\pi}\pi^{\mu\nu} - \underbrace{\frac{2\eta\delta_{n\pi}^q}{\beta}\nabla^{\langle\mu} n_q^{\nu\rangle}}_{\dot{\beta}} + \underbrace{\frac{2\eta\lambda_\pi^q}{\beta} n_q^{\langle\mu}\nabla^{\nu\rangle}\delta_{n\pi}^q - \frac{\eta\delta_{\Pi\pi}}{\beta}\Pi\sigma_{\mu\nu}}_{\text{dissipative couplings}},$$

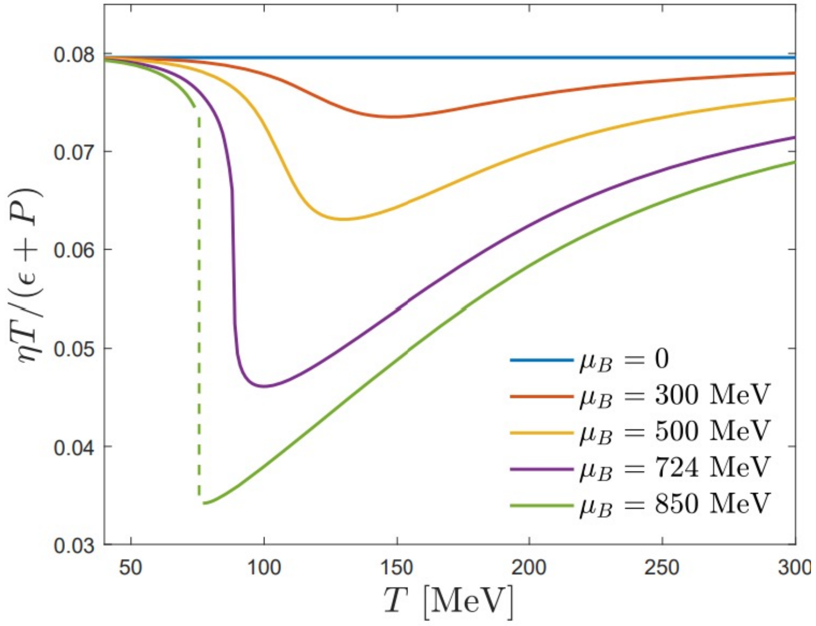
$$\tau_{qq'}\dot{n}_{q'}^\mu + n_q^\mu = -\kappa_{qq'}\nabla^\mu \alpha_{q'} + \frac{\tau_{qq'} n_{q'}^\mu}{2\beta}\theta - \underbrace{\frac{\tau_{qq'}}{2\beta}\dot{\beta}_{q'l} n_l^\mu}_{\dot{\beta}} - \underbrace{\left[\frac{\kappa_{qq'}\delta_{n\Pi}^{q'}}{\beta}\nabla^\mu \Pi + \frac{\kappa_{qq'}\delta_{n\pi}^{q'}}{\beta}\nabla_\nu \pi^{\mu\nu}\frac{\kappa_{qq'}\tilde{\lambda}_\Pi^{q'}}{\beta}\Pi\nabla^\mu \delta_{n\Pi}^{q'} + \frac{\kappa_{qq'}\tilde{\lambda}_\pi^{q'}}{\beta}\pi^{\mu\nu}\nabla_\nu \delta_{n\pi}^{q'}\right]}_{\text{dissipative couplings}}.$$

TRANSPORT|| HYDRODYNAMICS WITH MULTIPLE CONSERVED CHARGES

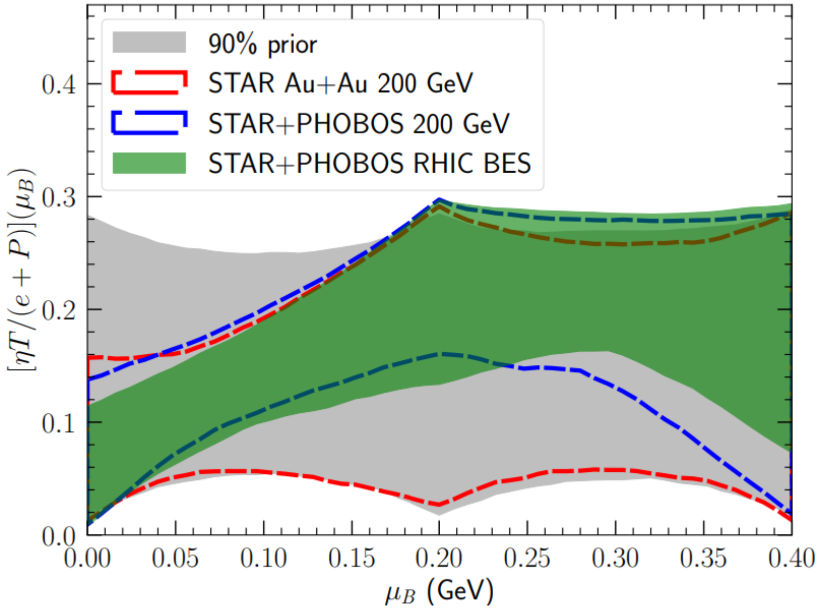
- Obtaining transport coefficients poses a great challenge and often demands sophisticated calculations and models.
- kinetic transport, holography, and PQCD provided some limits on η, ζ, κ.
- Bayesian analysis at finite μ_q constrained transport.



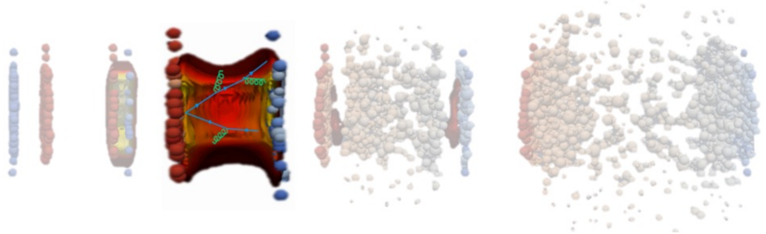
Grefa et al, PRD 106 (2022)



Shen, Schenke, and Zhao PRL132 (2024)



BULK DYNAMICS || HYDRODYNAMICS WITH MULTIPLE CONSERVED CHARGES

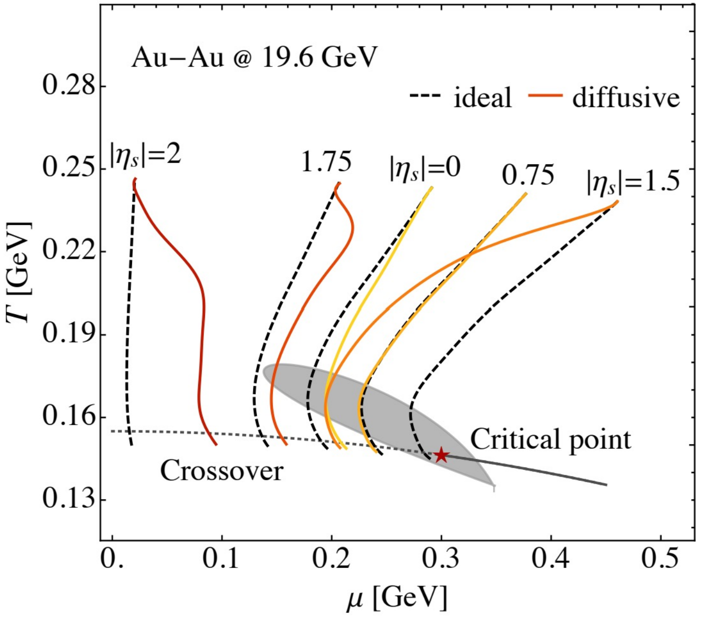


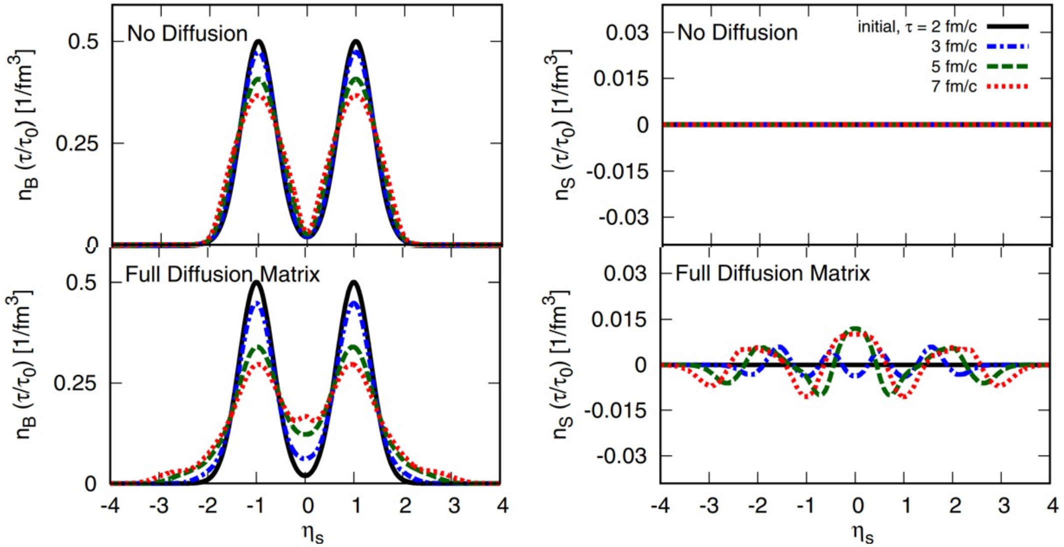
Charge Diffusion

$$\tau_q \dot{n}_q^{\langle\mu\rangle} + n_q^\mu = \kappa_q \nabla^\mu \left(\frac{\mu_q}{T}\right) + (\text{higher-order terms}),$$

$$\sum_{q'} \tau_{qq'} \dot{n}_{q'}^{\langle\mu\rangle} + n_q^\mu = \sum_{q'} \kappa_{qq'} \nabla^\mu \left(\frac{\mu_{q'}}{T}\right) + (\text{higher-order terms}),$$

- Due to the presence of off-diagonal elements in the diffusion matrix $\kappa_{qq'}$, the diffusion current of a particular charge can receive contributions from gradients of the chemical potentials of all charge types.





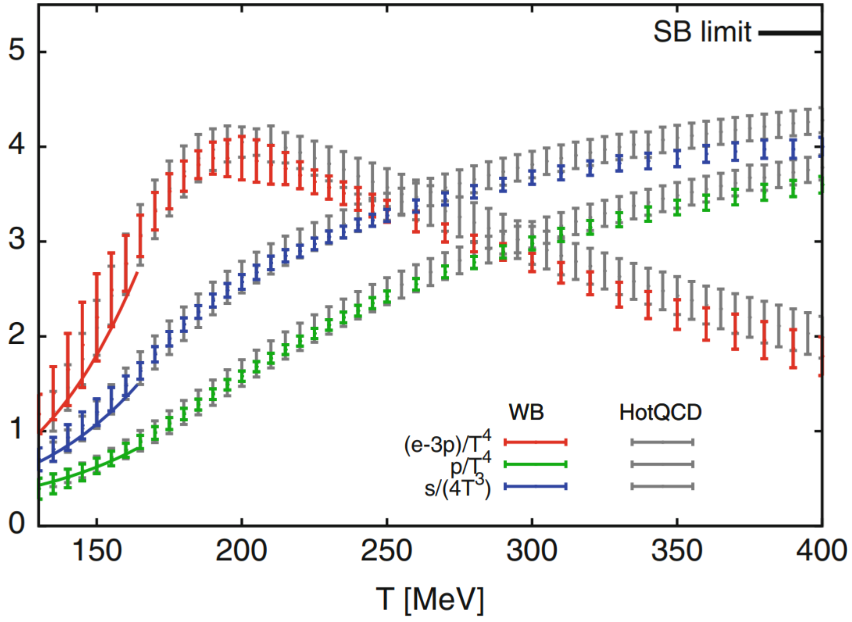
- Dynamics of the baryon current plays a pivotal role in determining the evolution of baryon rich QCD matter within this diagram and in the search for the CP signatures.
- Its significance lies in determining the trajectory of systems across the QCD phase diagram and is particularly relevant for interpreting potential signatures of the QCD critical point, especially concerning proton cumulant measurements.

Greif, Fotakis et al., PRL 120, (2018) Fotakis, Greif et al., PRD 101, (2020) Denicol et al PRC 98 (2018)

THERMODYNAMICS || LATTICE EOS AT ZERO AND FINITE CHARGE DENSITY

Exact EoS from lattice QCD at $\mu_B = 0$

Quantum Chromo Dynamics QCD predicts a crossover phase transition from hadrons to a system of quark-gluon plasma around $T \sim 155\,\mathrm{MeV}$ or ($T \sim 10^{12}$ Kelvin)



What is the nature of the phase transition at $\mu_B \neq 0$

- LQCD disfavors the existence of the CP at $\mu/T \leq 2$

Bazavov et al. PRD95 (2017)

Taylor expansion around $\mu_B = 0$

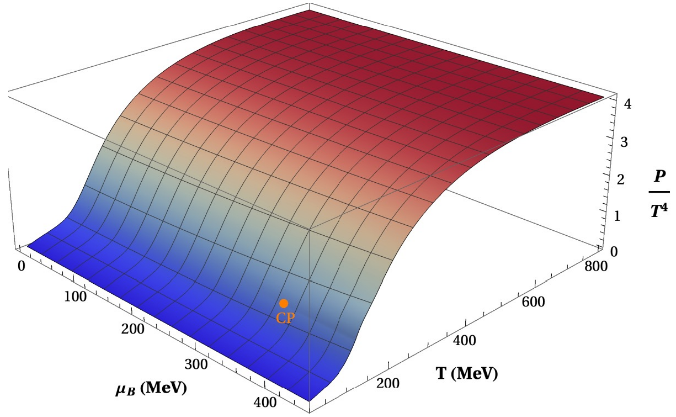
Noronha-Hostler et al PRC 100 (2019), P. Alba et al., PRD 96(2017)

Monnai, Schenke, Shen, PRC 100 (2019)

$$\frac{P_0(T,\mu_B,\mu_Q,\mu_S)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!}\chi^{BQS}_{i,j,k}\left(\frac{\mu_B}{T}\right)^i\left(\frac{\mu_Q}{T}\right)^j\left(\frac{\mu_S}{T}\right)^k$$

$$\chi^{BQS}_{ijk} = \frac{\partial^{i+j+k}(p/T^4)}{\partial(\frac{\mu_B}{T})^i\partial(\frac{\mu_Q}{T})^j\partial(\frac{\mu_S}{T})^k}\bigg|_{\mu_B,\mu_Q,\mu_S=0}$$

(valid only until about ($\mu_B/T \leq 2$))



Karthein, Mroczek et al Eur.Phys.J.Plus 136 (2021)

TESTING LATTICE EOS AT FINITE DENSITY

- **Tylor expansion of the QCD pressure** $\frac{P}{T^4} = \frac{1}{VT^3}\, ln\mathcal{Z}(T,\mu_q)$

$$\frac{P_0(T,\mu_B,\mu_Q,\mu_S)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!}\chi^{BQS}_{i,j,k}\left(\frac{\mu_B}{T}\right)^i\left(\frac{\mu_Q}{T}\right)^j\left(\frac{\mu_S}{T}\right)^k$$

$$\chi^{BQS}_{ijk} = \left.\frac{\partial^{i+j+k}(p/T^4)}{\partial(\frac{\mu_B}{T})^i\partial(\frac{\mu_Q}{T})^j\partial(\frac{\mu_S}{T})^k}\right|_{\mu_B,\mu_Q,\mu_S=0}$$

- **At low T, the χ^{BQS}_{ijk} are mathched to HRG**

$$P(T,\mu_B,\mu_S,\mu_Q) = \sum_i T\frac{(-1)^{B_i+1}g_i}{2\pi^2}\int_0^\infty dp\, p^2 \ln\left[1+(-1)^{B_i+1}e^{-\frac{\varepsilon_i-\mu_i}{T}}\right]$$
$$\simeq \sum_i T\frac{g_i}{2\pi^2}\int_0^\infty dp\, p^2 e^{-\frac{\varepsilon_i-\mu_i}{T}} \simeq \sum_i Te^{\mu_i/T}\frac{g_i}{2\pi^2}\int_0^\infty dp\, p^2 e^{-\varepsilon_i/T}$$

- **At high T, reproduce the PQCD calculations**

$$P^{SB} = \frac{\pi^2}{45}T^4\left(N_c^2-1\right) + \sum_f \frac{N_c}{3\pi^2}\left[\frac{7\pi^4T^4}{60} + \frac{\mu_f\pi^2T^2}{2} + \frac{\mu_f^4}{4}\right],$$

Noronha-Hostler et al PRC 100 (2019), P. Alba et al., PRD 96(2017)

Monnai, Schenke, Shen PRC 100(2019),

Monnai, Schenke, Shen, Mod.Phys.A 36 (2021)

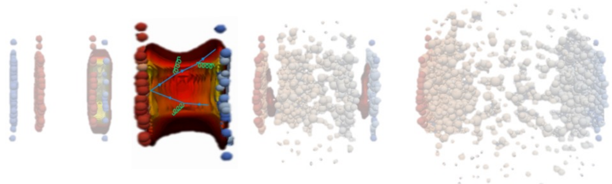
(valid only until about ($\mu_B/T \leq 2$))

- Sources of uncertainties associated with the EOS:

1. Uncertainties in the lattice QCD results.
2. Different methods of interpolating the EOS between lattice QCD and the HRG.

- These uncertainties can propagate into the results of model calculations.

TESTING LATTICE EOS AT FINITE DENSITY



- **4D interpolation and root-finding**:

  Equation of State ⇒ Hydrodynamics fields
  $P_0(\varepsilon, \rho_B, \rho_S, \rho_q) \rightarrow P_0(T, \mu_B, \mu_S, \mu_Q)$

- **Update to the EOS code**

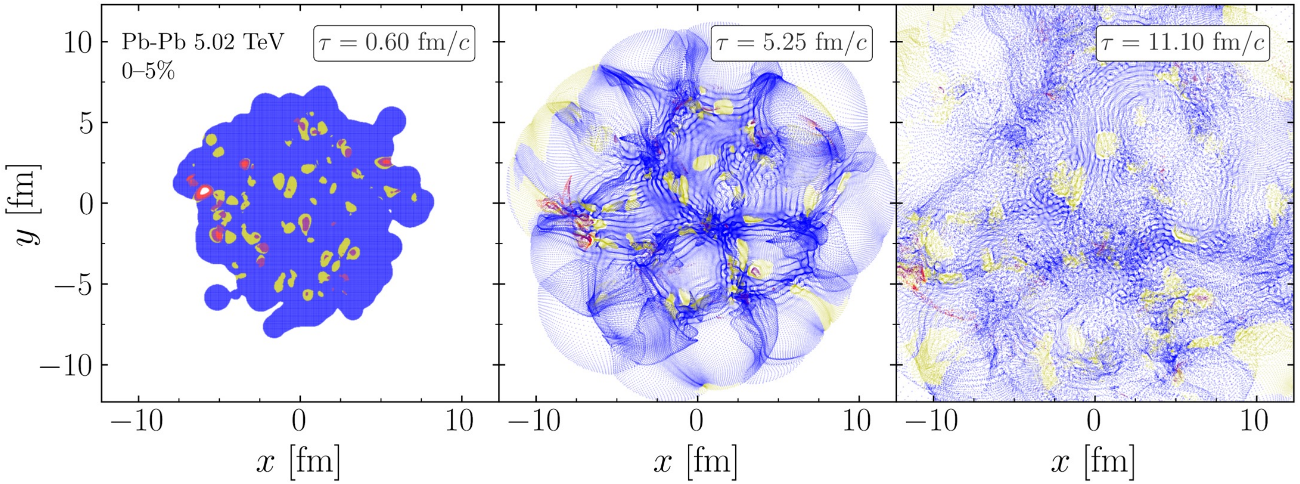
  Incorporate thermodynamics derivatives required by the hydro simulation:

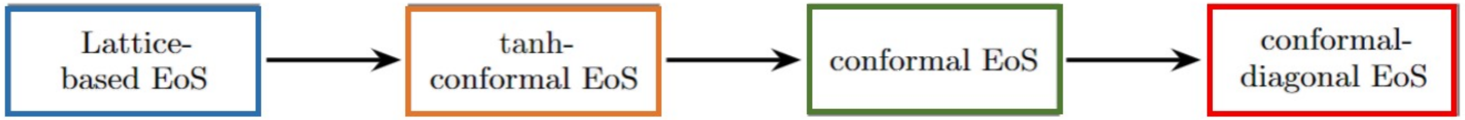
$$\frac{dw}{d\tau} = \sum_{\varphi \in \Phi} \frac{\partial w}{\partial \varphi} \frac{d\varphi}{d\tau}$$

- **Fallbacks EoSs**

  Taylor expansion around $\mu_B = 0$
  Noronha-Hostler et al PRC 100 (2019),
  P. Alba et al., PRD 96(2017)

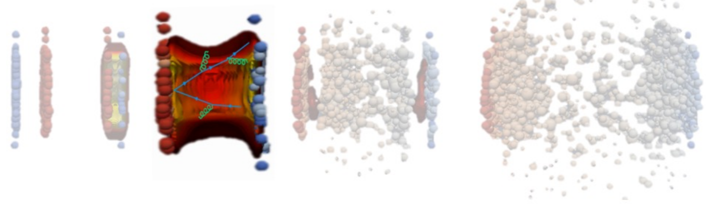
Plumberg, **DA** , Dore , Mroczek , Carzon , Salina San Martin, Spychalla , Sievert, and Noronha-Hostler, arXiv:2405.09648 (2024





OPEN SOURCE/ easily adapted to new EoS options

CONSTRAINTS ON THE EOS

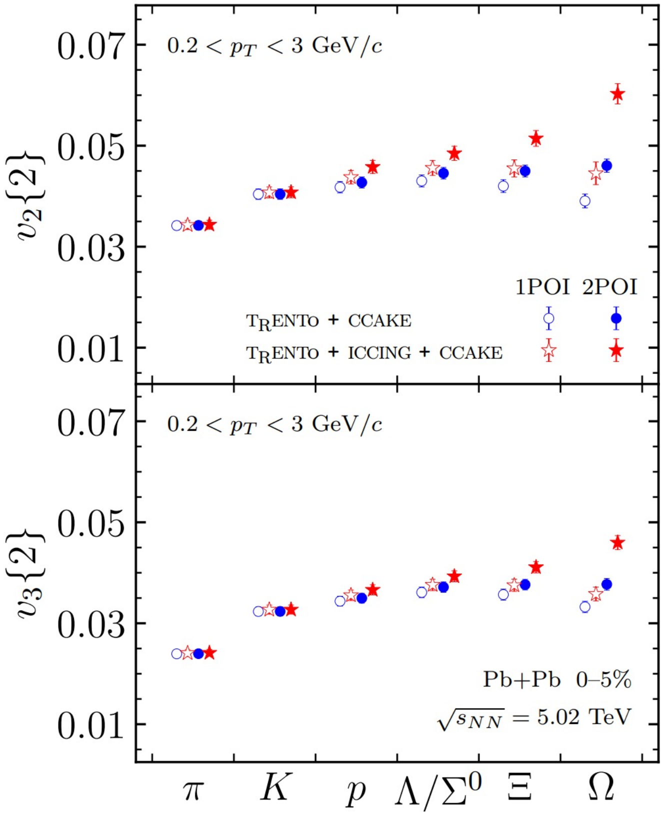


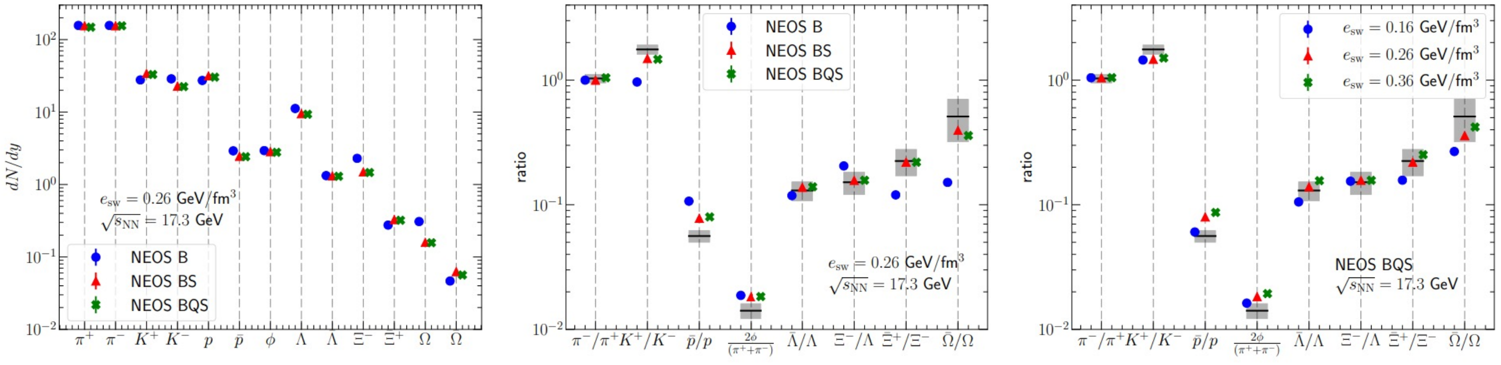
- Distinct differences in identified particle yields around midrapidity obtained from incorporating different charge sectors.

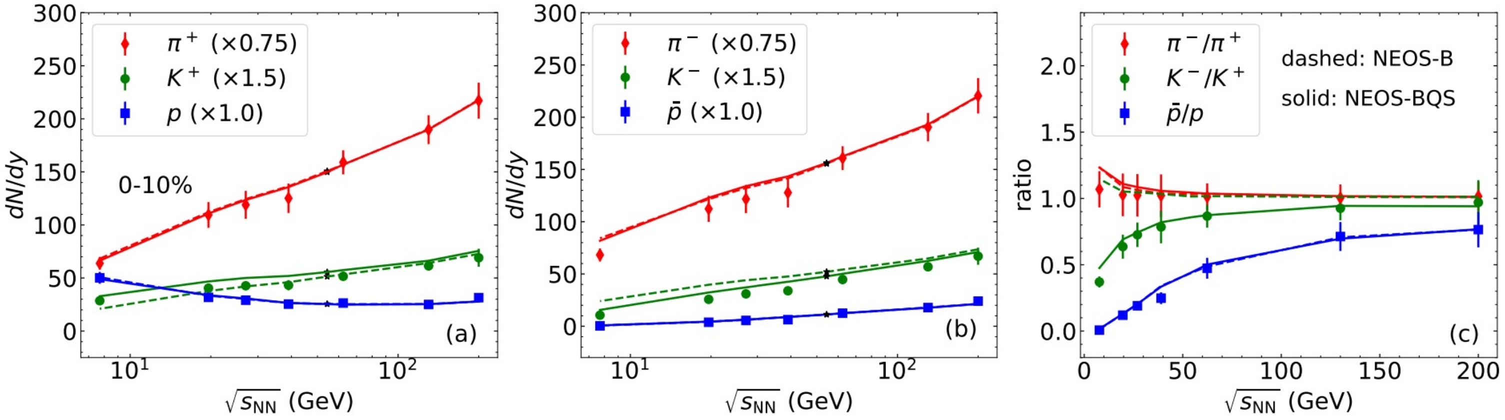
Monnai, Schenke, and Shen PRC 100 (2019)

Du arXiv:2401.00596 (2024)

Plumberg, DA, et al arXiv:2405.09648 (2024)





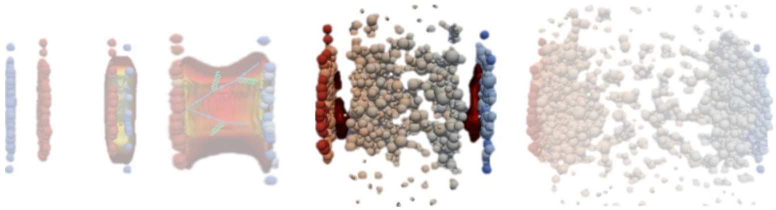


FREEZEOUT AND HADRONIZATION

- A freeze-out hypersurface denoted as $\Sigma_\mu(x)$, which is defined based on various criteria.

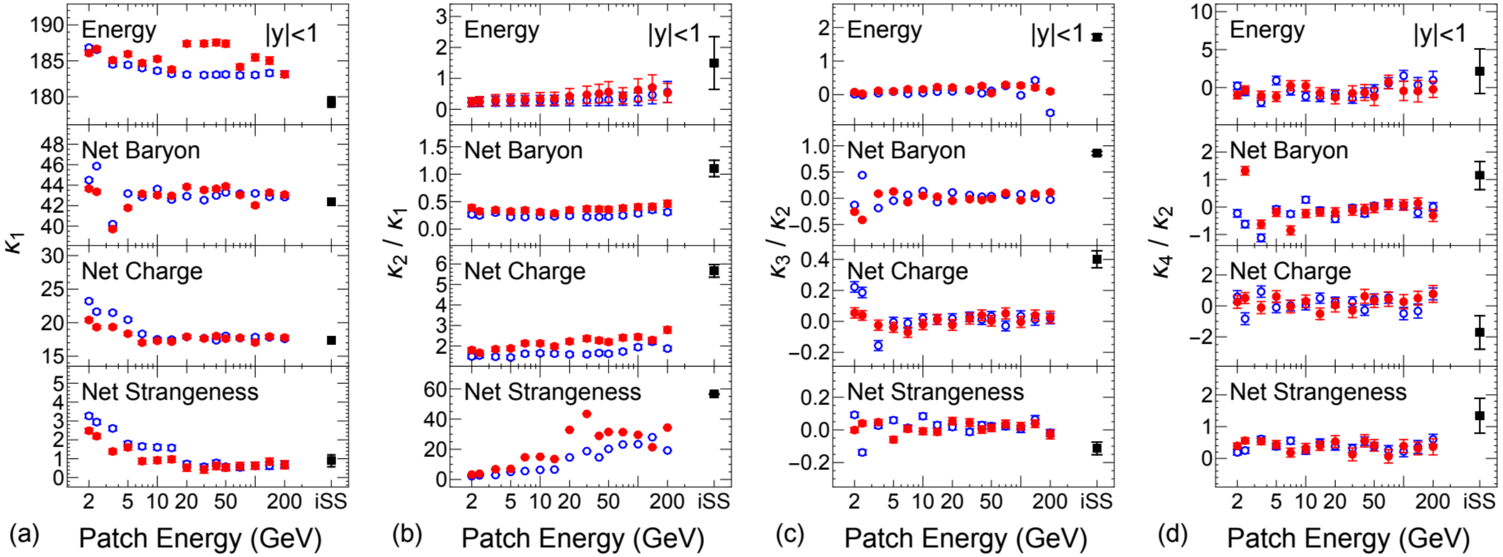
$$p^0 \frac{d^3 N_i}{d^3 p} = \frac{1}{(2\pi)^3} \int_\Sigma d^3\sigma_\mu(x) p^\mu f_i(x,p)\,.$$

$$f_i(x,p) \equiv f_{\mathrm{eq},i} + \delta f_i = f_{\mathrm{eq},i} + \delta f_{\pi,i} + \delta f_{\Pi,i} + \delta f_{n_q,i}\,.$$

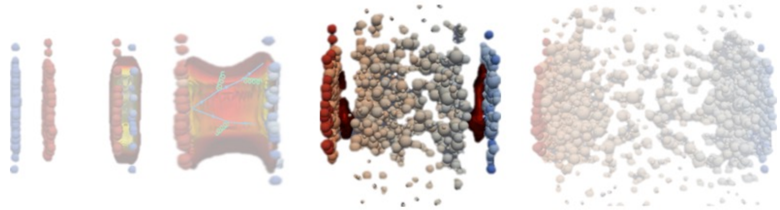


Oliinychenko, Shi, and Koch PRC 102 (2020),

- Energy-momentum and charge conservation imposed at freezeout
- Crucial to maintain the critical correlations in coordinate space, which are propagated during the hydrodynamic evolution, and translate them into correlations in momentum space among the particles.
- Translation of hydrodynamic fluctuations into fluctuations of observables has been developed.



FREEZEOUT AND HADRONIZATION



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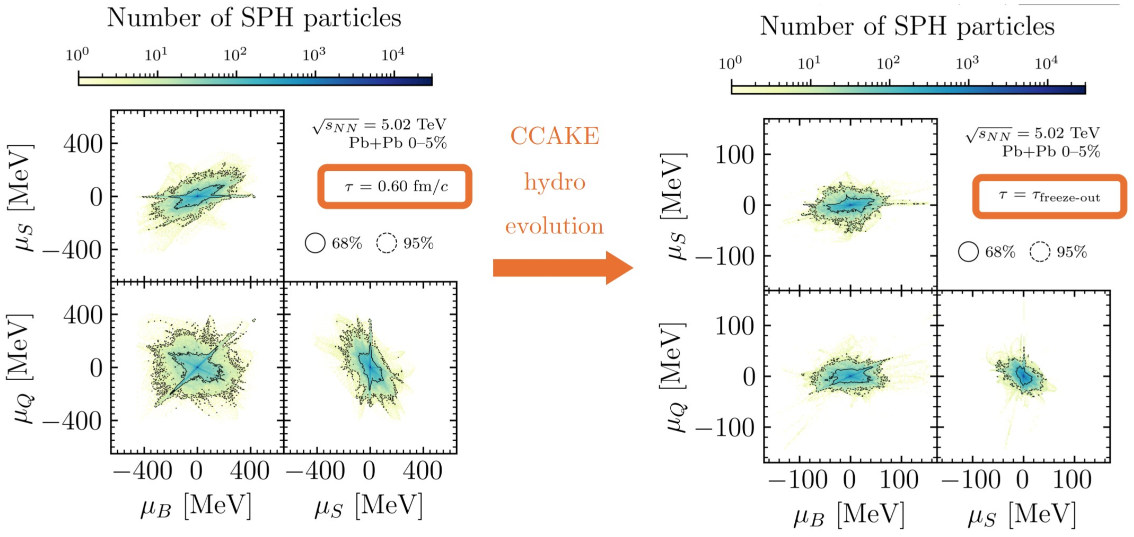
Oliinychenko, Shi, and Koch PRC 102 (2020),

Can one explore the systemiatics at LHC?

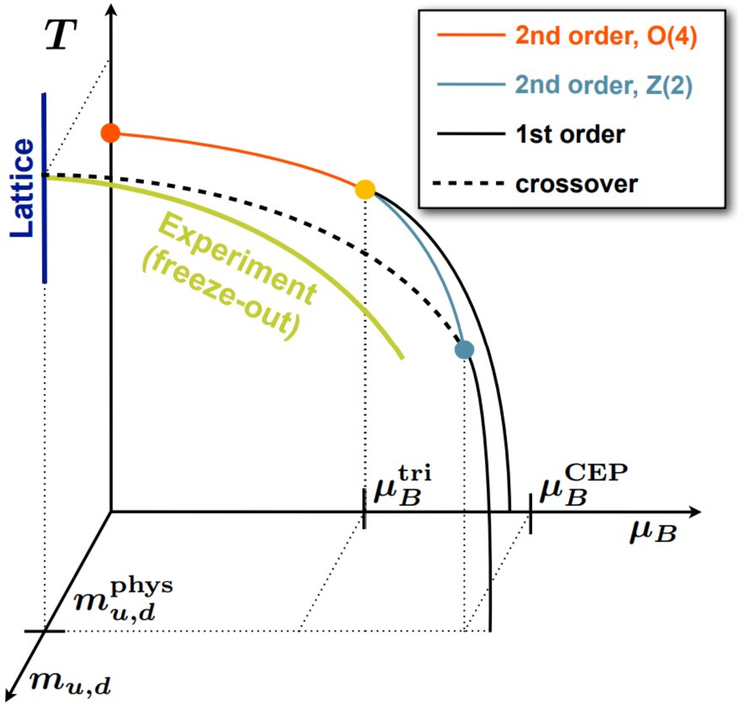


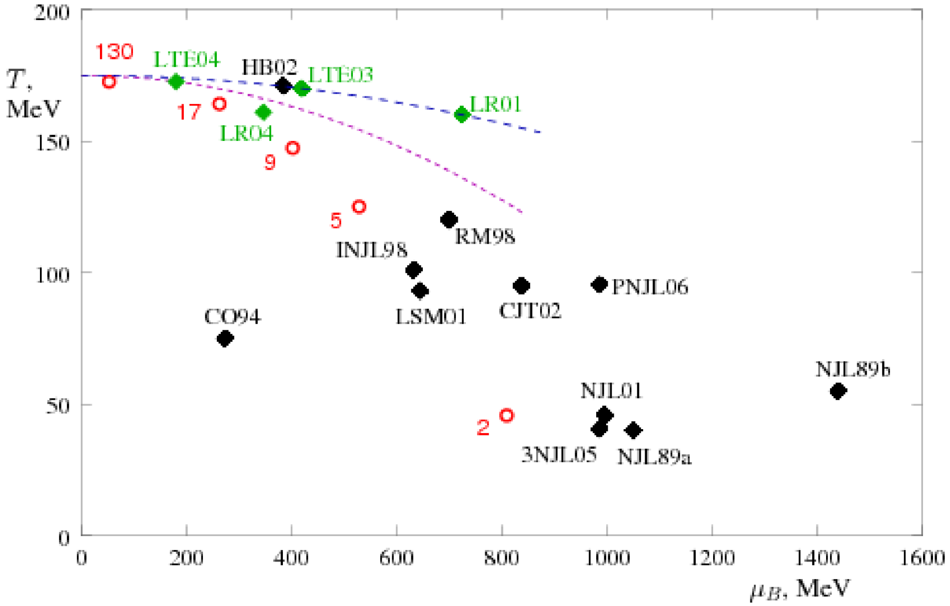
Chemical potentials at initial time

Chemical potentials freeze-out

Jordi Salinas Tue @4:10pm

- theoretical approaches — from RMM to FRG to AdS/CFT — consistently indicate that the transition becomes discontinuous above a certain critical baryon chemical potential, where, the QCD critical point is located.

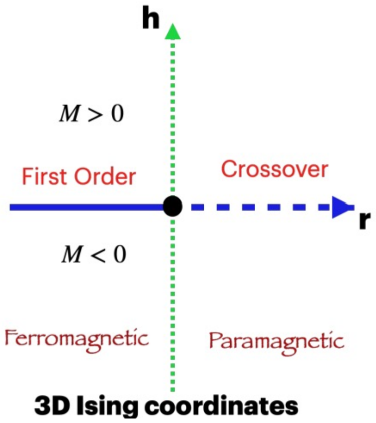




CRITICALITY FROM LATTICE QCD

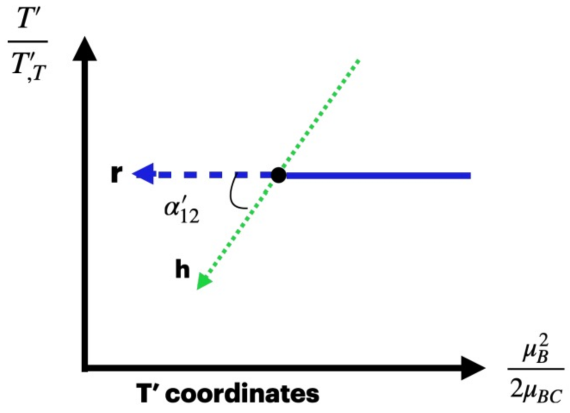
LQCD+ Ising model

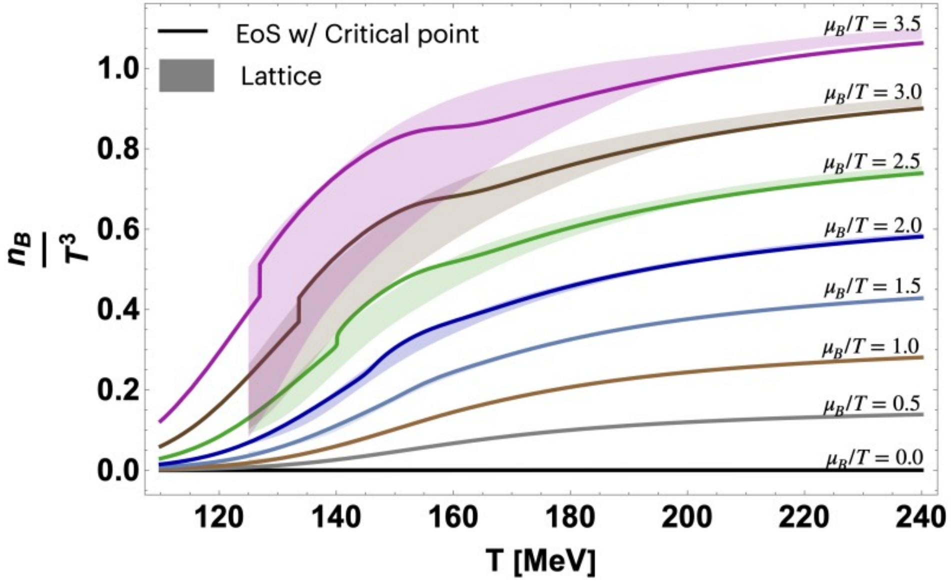
kahangirwe et al, arXiv:2402.08636 (2024)



$$\frac{T' - T_0}{T_C T'_{,T}} = - w' h \sin \alpha'_{12}$$

$$\frac{\mu_B^2 - \mu_{BC}^2}{2\mu_{BC} T_C} = w' (-r\,\rho' - h \cos \alpha'_{12})$$





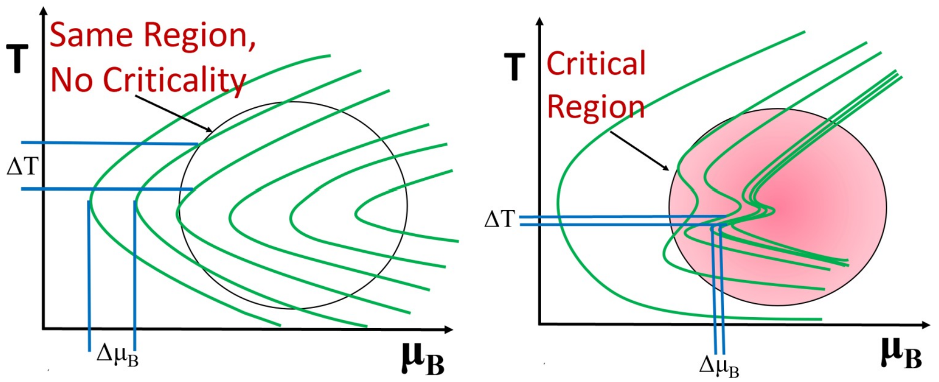
- **Criticality and critical point**:
  - Implementation in multiple charge sectors. Little has been explored for multi-component hydrodynamic evolution in the presence of critical region, and it is demanding to keep up with the BES new results on cumulants ..etc
  - Control over the transition regimes for different EoS when matched. This is useful for Bayesian analysis tools.

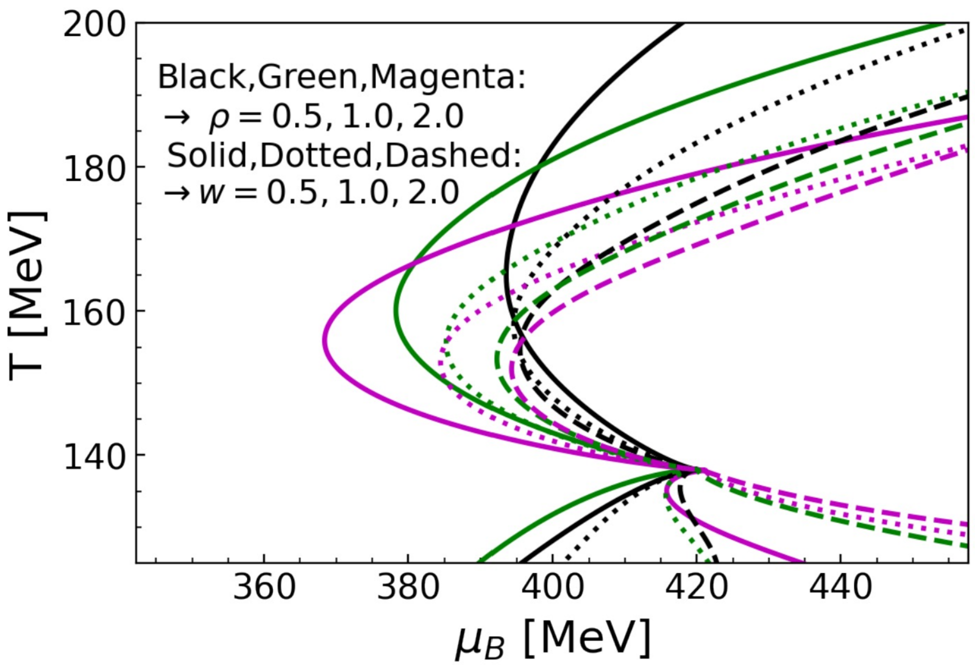
- Adding the critical point in an EOS deforms hydrodynamics trajectories, leading them to converge toward the critical point, a phenomenon known as the **critical lensing**
- How do the size and shape of the critical region affects the isentropic trajectories?.
- Whether or not the dissipatice corrections present at FO manifest themselves?

Dore, Noronha-Hostler, and McLaughlin, PRD. 102(2020).

Dore et al. PRD. 106(2022)

Chattopadhyay et al. ,PRC. 107 (2023).

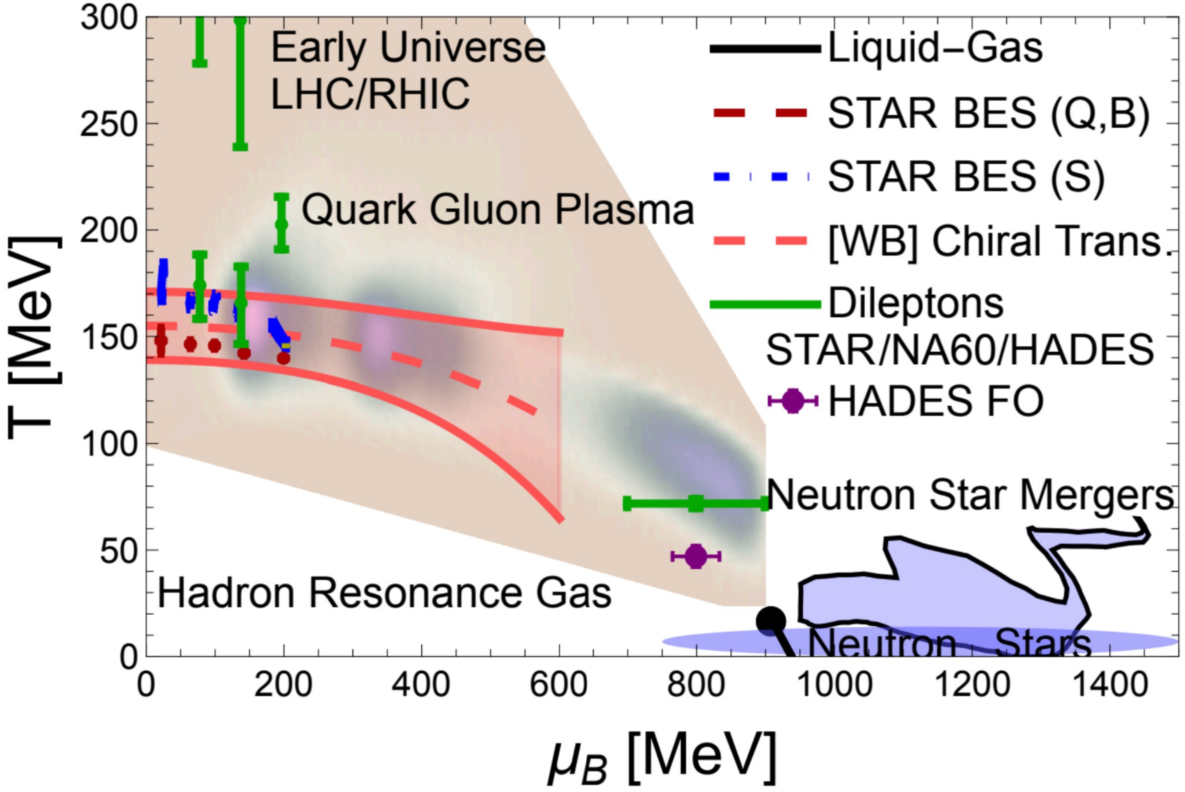




NEUTRON STARS

- Gravitational wave observations of neutron star mergers is a potential lab for exploration of the QCD phase diagram.
- NS and NS mergers is a regime complementary to that explored by heavy-ion collisions, that is at high baryon densities and low temperatures as well as at substantial isospin fractions.

Lovato et al, 2211.02224 [nucl-th]



Conclusions and Remarks

- Direct comparison with experiment is fundamental to make progress on theoretical predictions for the BES.
- Understanding the mechanisms for baryon charge deposition and equilibration is essential for initial state generators.
- The particular choice of dissipative dynamics and transport directly impact the extraction of observables by affecting the dynamical trajectories near the CP.
- How far are we from hydrodynamics simulations with the critical point and the correct dynamics?
- What is the key factor missing from our current simulations?
  Stochastic and thermal fluctuations?

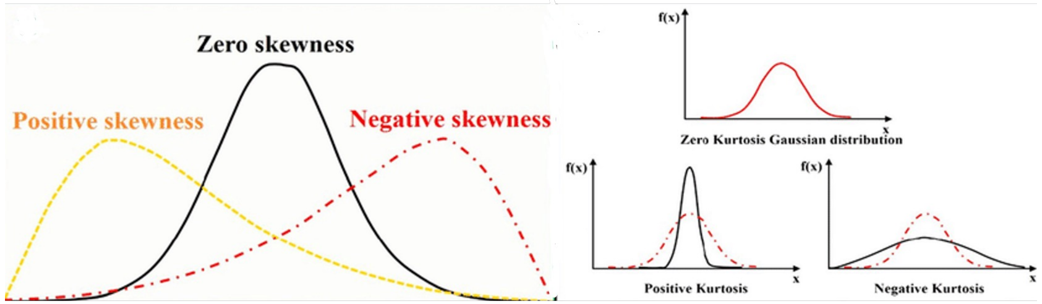
THANK YOU FOR YOUR ATTENTION !!

CRITICALITY

- Cumulant of a conserved quantity

Cumulants	**Factorial Cumulant**
$C_1 =< n >$	$\kappa_1 = C_1$
$C_2 =< \delta n^2 >$	$\kappa_2 = -C_1 + C_2$
$C_3 =< \delta n^3 >$	$\kappa_3 = 2C_1 - 3C_2 + C_3$
$C_4 =< \delta n^4 > -3 < \delta n^2 >$	$\kappa_4 = -6C_1 + 11C_2 - 6C_3 + C_4$

where $\delta n = n - < n >$, **n** is the **Net proton multiplicity** in an event

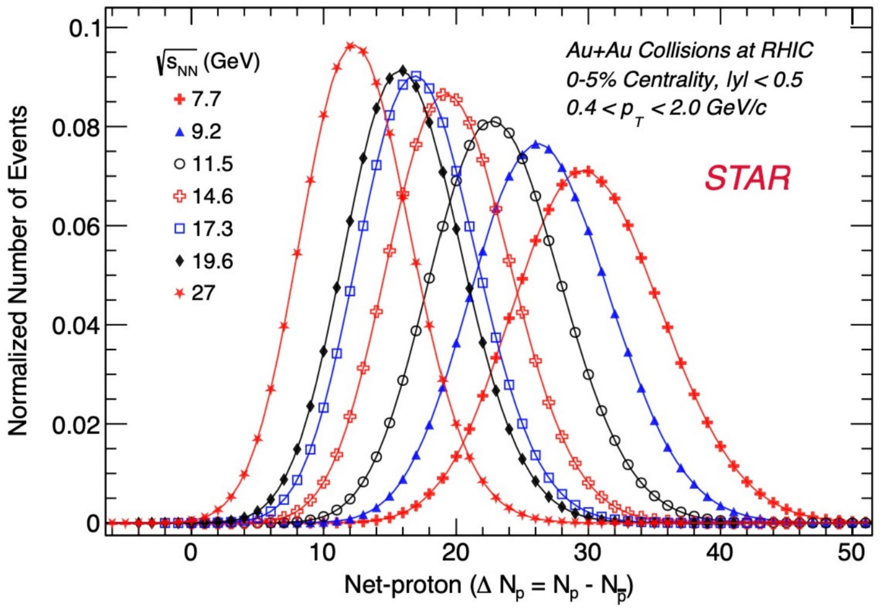


- Connection to correlation length

$$C_2 \propto \xi^2; \qquad ; C_3 \propto \xi^{9/2}; \qquad C_4 \propto \xi^7$$

- Connection to susceptibilities

$$C_{4q}/C_{2q} \propto \chi_4/\chi_2; \qquad C_{6q}/C_{2q} \propto \chi_6/\chi_2$$

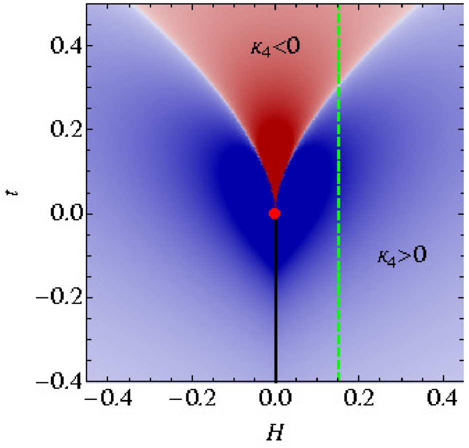


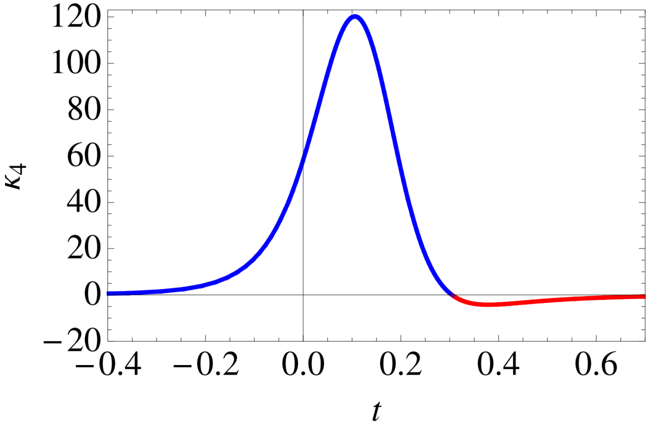
Gupta, PoS CPOD2009:025,2009

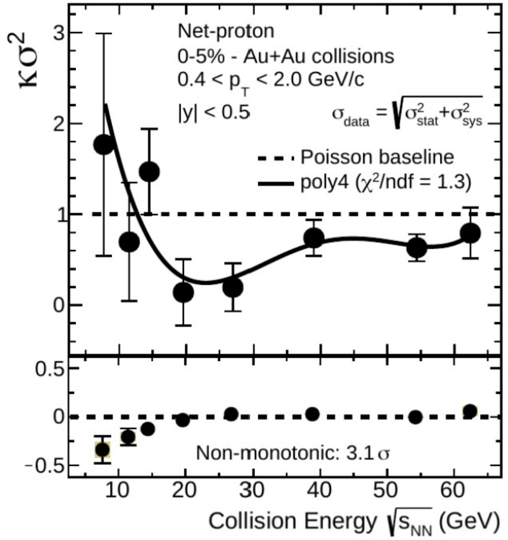
CRITICALITY

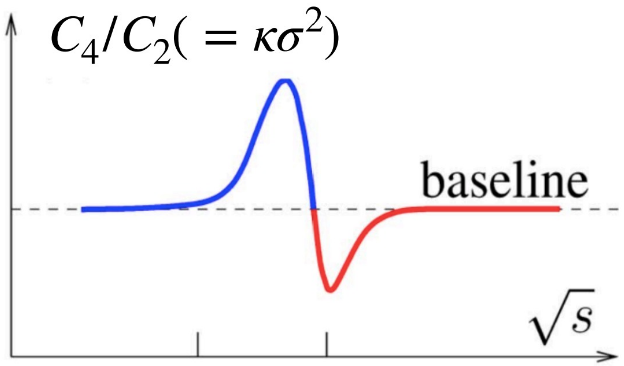
M. A. Stephanov
Department of Physics, University of Illinois, Chicago, Illinois 60607, USA

We point out that the quartic cumulant (and kurtosis) of the order parameter fluctuations is universally *negative* when the critical point is approached on the crossover side of the phase separation line. As a consequence, the kurtosis of a fluctuating observable, such as, e.g., proton multiplicity, may become smaller than the value given by independent Poisson statistics. We discuss implications for the Beam Energy Scan program at RHIC.







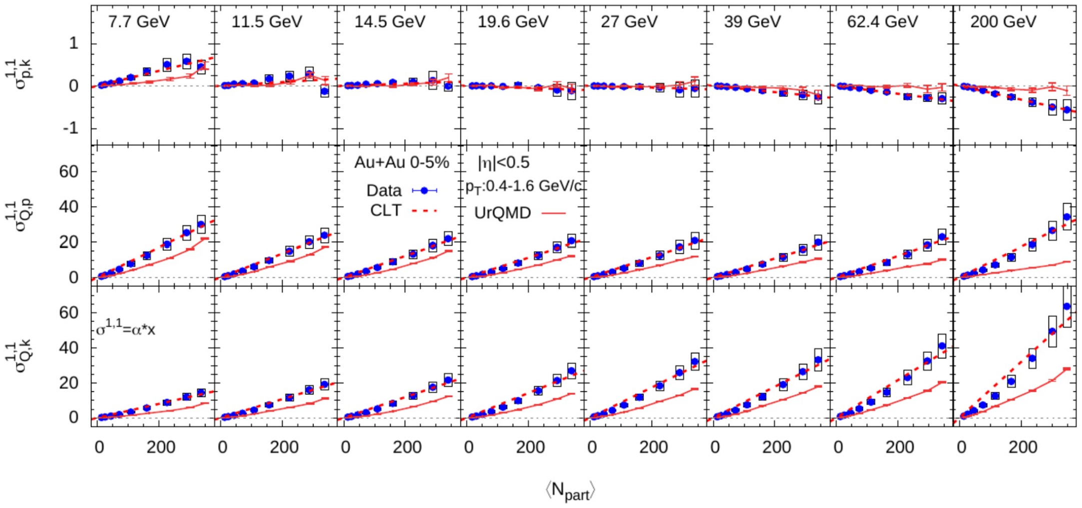


- The amplitude of the critical fluctuations which would be seen if the system created in a HIC reached the CEP has to be estimated properly, taking into account critical slowing-down and finite-size effects of the system.
- The correlation length ξ (and thus the related cumulants) are expected to diverge to infinity at the vicinity of the CEP, when performing lQCD simulations of an infinite size system during an infinite time.
- However, in a HIC, the system has a finite size, what will limit the growth of ξ as it can logically not be larger than the system itself [225].
- This finite size effect is even more strengthen by the fact that, as the system is inhomogeneous, only a part of the whole system would approach the CEP.
- In addition, the system would reach the conditions of criticality only for a nite time as it cools down and cannot thus stay in equilibrium in this region, what would slow down the growth of ξ.
- Taking into account those two eects, the amplitude of ξ would then be limited to the order of $2 \sim 3$ fm [226].

Path to observation of the CP…. if it exists || Susceptibilities

In a grand-canonical ensemble, to what a heavy-ion collision can be compared to, they are defined as derivatives of the partition function Z

$$\chi_{i,j}^{X,Y} = \frac{1}{VT^3} \left. \frac{\partial^{i+j} Z(T,V,\mu)}{(\partial\hat{\mu}_X)^i(\partial\hat{\mu}_Q)^j} \right|_{\mu_{X,Y}=0} \qquad (\hat{\mu} = \frac{\mu}{T})$$



- As we are searching for radical changes in the state of nuclear matter, i.e. phase transition, these derivatives of Z should reveal them.

- Susceptibilities can be written as a function of the net-charge cumulants ($N_X = n_X - n_X$).

$$\chi_{11}^{XY} = \frac{1}{VT^3}\sigma_{XY}^{11} = \frac{< N_X N_Y > - < N_X >< N_Y >}{VT^3}$$

$$\chi_2^X = \frac{1}{VT^3}\sigma_X^2 = \frac{< N_X^2 > - < N_X >^2}{VT^3}$$

- Also, in order to have observables independent from volume or temperature, which cannot be measured directly in experiments, ratios are often used.

$$C_{BS} = \frac{\sigma_{BS}^{11}}{\sigma_S^2}; \qquad C_{QS} = \frac{\sigma_{QS}^{11}}{\sigma_S^2}; \qquad C_{QB} = \frac{\sigma_{QB}^{11}}{\sigma_B^2}.$$

TESTING LATTICE EOS AT FINITE DENSITY

- **Tylor expansion of the QCD pressure** $\frac{P}{T^4} = \frac{1}{VT^3}\, ln\mathcal{Z}(T, \mu_q)$

$$\frac{P_0(T,\mu_B,\mu_Q,\mu_S)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi^{BQS}_{i,j,k} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

$$\chi^{BQS}_{ijk} = \left. \frac{\partial^{i+j+k}(p/T^4)}{\partial(\frac{\mu_B}{T})^i \partial(\frac{\mu_Q}{T})^j \partial(\frac{\mu_S}{T})^k} \right|_{\mu_B,\mu_Q,\mu_S=0}$$

- **At low T, the χ^{BQS}_{ijk} are mathched to HRG**

$$P(T,\mu_B,\mu_S,\mu_Q) = \sum_i T \frac{(-1)^{B_i+1} g_i}{2\pi^2} \int_0^\infty dp\, p^2 \ln\left[1 + (-1)^{B_i+1} e^{-\frac{\varepsilon_i - \mu_i}{T}}\right]$$
$$\simeq \sum_i T \frac{g_i}{2\pi^2} \int_0^\infty dp\, p^2 e^{-\frac{\varepsilon_i - \mu_i}{T}} \simeq \sum_i T e^{\mu_i/T} \frac{g_i}{2\pi^2} \int_0^\infty dp\, p^2 e^{-\varepsilon_i/T}$$

- **At high T, reproduce the PQCD calculations**

$$P^{SB} = \frac{\pi^2}{45} T^4 \left(N_c^2 - 1\right) + \sum_f \frac{N_c}{3\pi^2} \left[\frac{7\pi^4 T^4}{60} + \frac{\mu_f \pi^2 T^2}{2} + \frac{\mu_f^4}{4}\right],$$

Noronha-Hostler et al PRC 100 (2019), P. Alba et al., PRD 96(2017)

Monnai, Schenke, Shen PRC 100(2019),

Monnai, Schenke, Shen, Mod.Phys.A 36 (2021)

(valid only until about ($\mu_B/T \leq 2$))

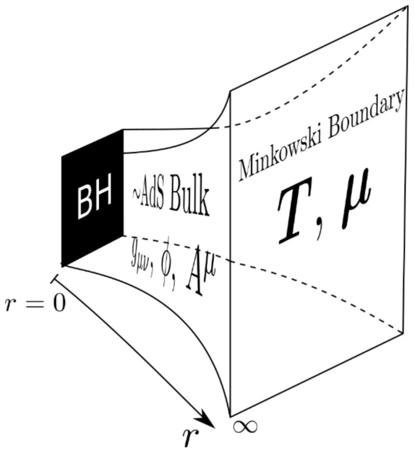
- Sources of uncertainties associated with the EOS:

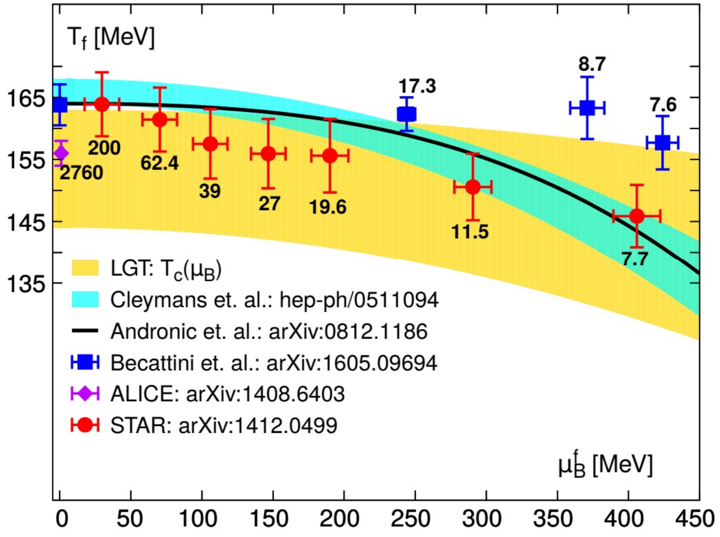
1. Uncertainties in the lattice QCD results.
2. Different methods of interpolating the EOS between lattice QCD and the HRG.

- These uncertainties can propagate into the results of model calculations.

CRITICALITY

- observables
- Are these the only, all observables?. Are they correct?
- Costructing proper observables vs measureing them vs theoretically correctly put the models together
- Any ideas for short cuts or simple models for what it should look like if we do have a CP?





h1: O. DeWolfe et al., PRD (2011)
h2: J. Knaute et al., PLB (2018)
h3: R. Critelli et al., PRD (2017)
h4: R.-G. Cai, PRD (2022)
h5: M. Hippert, C. R. et al, arXiv:2309.00579
h6: X. Chen, M. Huang, PRD (2024)
h7: Q. Fu et al., arXiv:2404.12109
h8: N. Jokela et al., arXiv: 2405.02394
h9: N. Jokela et al., arXiv: 2405.02394