

Proton cumulants and EoS theory overview

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2024 RHIC/AGS Annual Users' Meeting

June 12, 2024

2024 RHIC/AGS ANNUAL USERS' MEETING

A New Era of Discovery
Guided by the New Long Range Plan
for Nuclear Science

June 11–14, 2024



QCD under extreme conditions

What we know

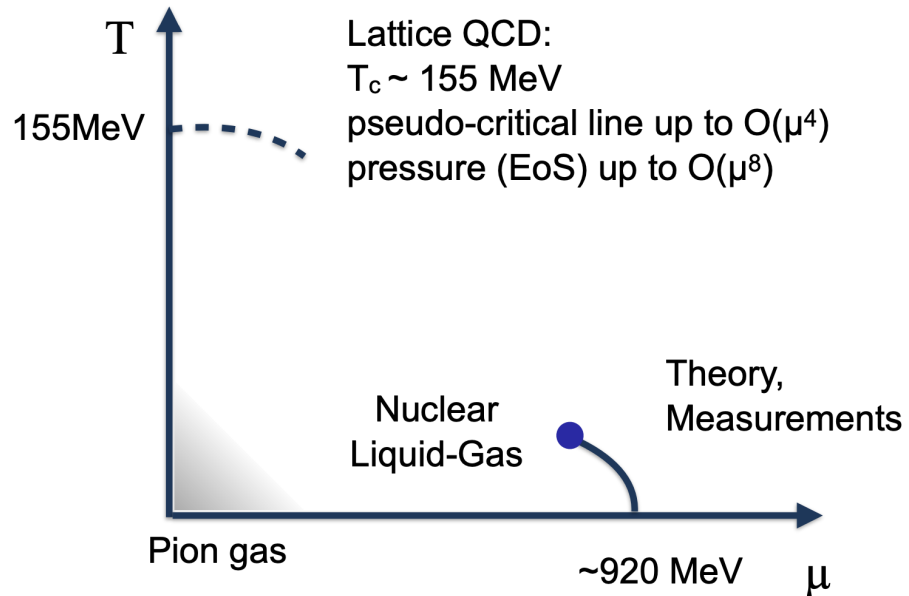


Figure courtesy of V. Koch

- Dilute hadron gas at low T & μ_B due to confinement, quark-gluon plasma high T & μ_B
- Nuclear liquid-gas transition in cold and dense matter, lots of other phases conjectured
- Chiral crossover at $\mu_B = 0$

What we know

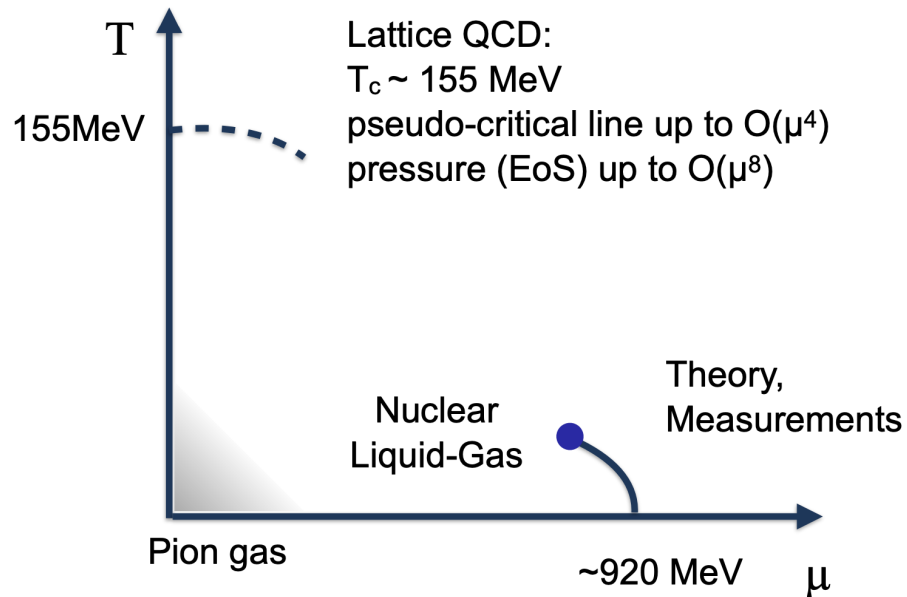


Figure courtesy of V. Koch

What we hope to know

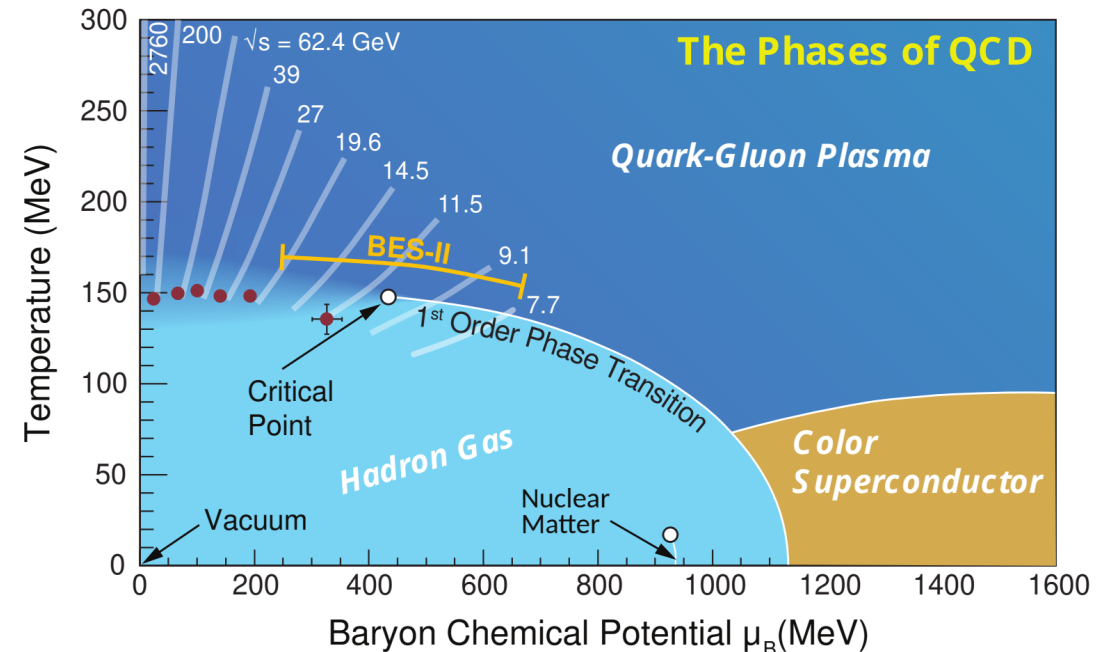


Figure from Bzdak et al., Phys. Rept. '20 & 2015 Long Range Plan

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QCD under extreme conditions

What we know

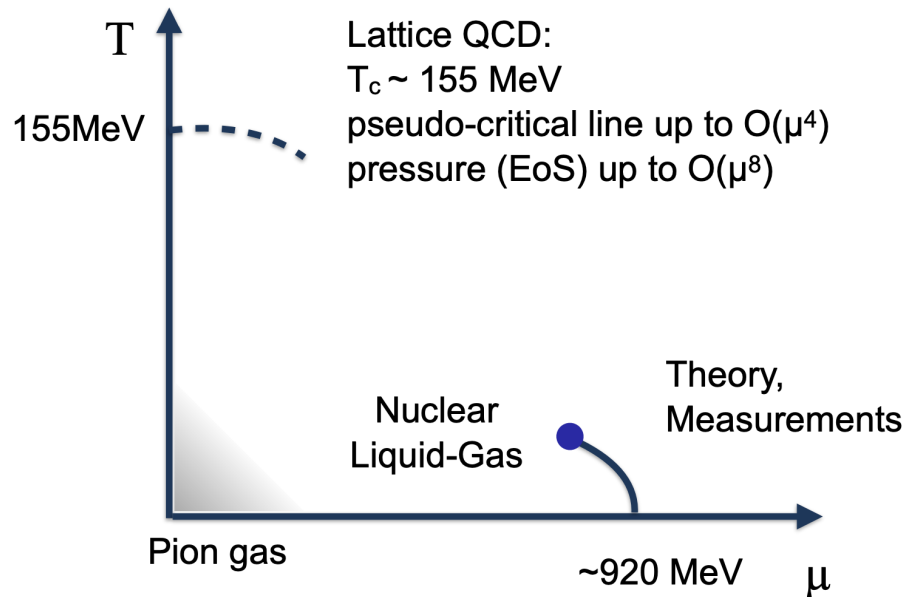


Figure courtesy of V. Koch

What we hope to know

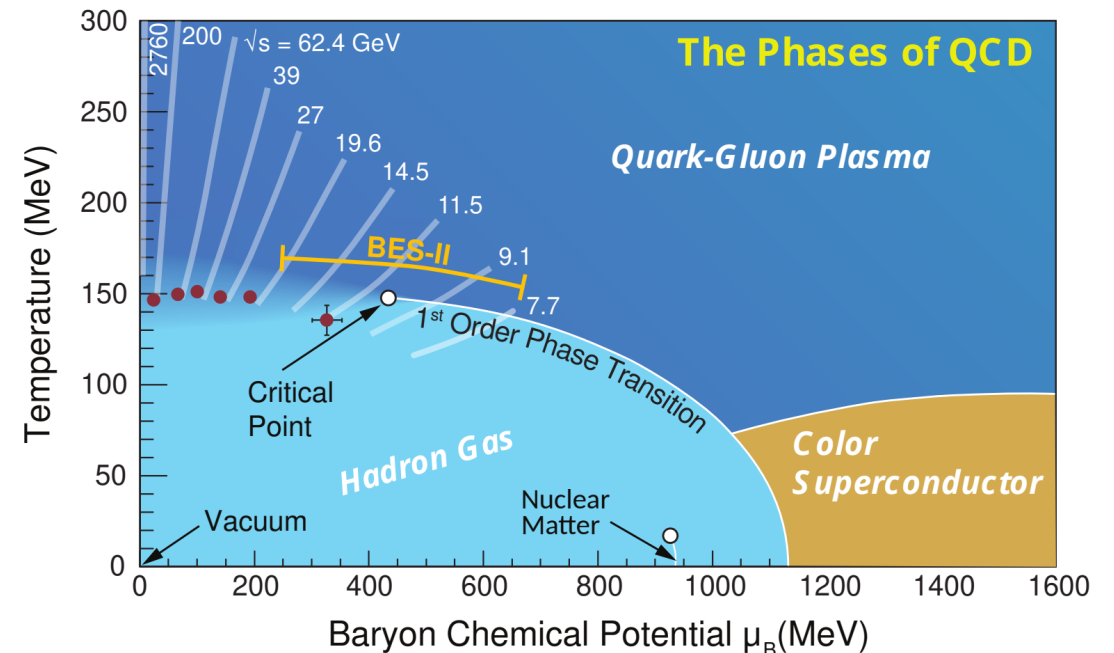


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Key question: *Is there a QCD critical point and how to find it?*

Critical point predictions as of a few years ago

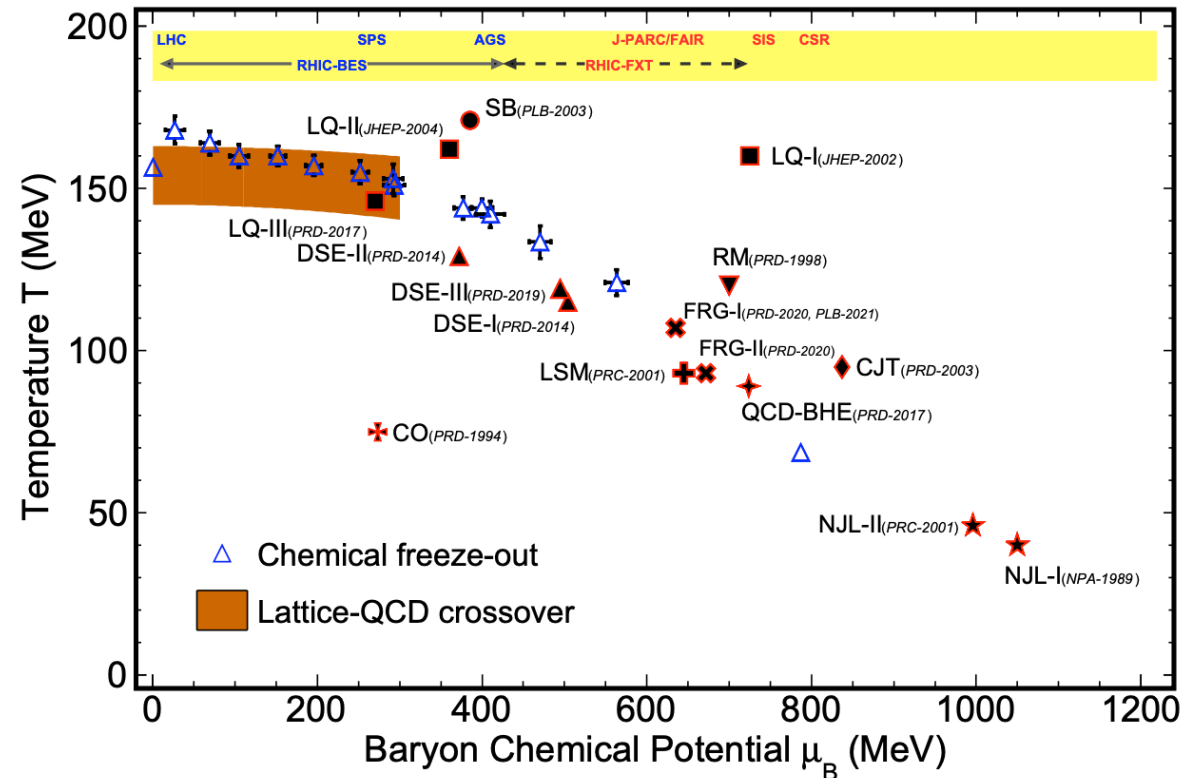


Figure adapted from A. Pandav, D. Mallick, B. Mohanty, Prog. Part. Nucl. Phys. 125 (2022)

Including the possibility that the QCD critical point does not exist at all

de Forcrand, Philipsen, JHEP 01, 077 (2007); VV, Steinheimer, Philipsen, Stoecker, PRD 97, 114030 (2018)

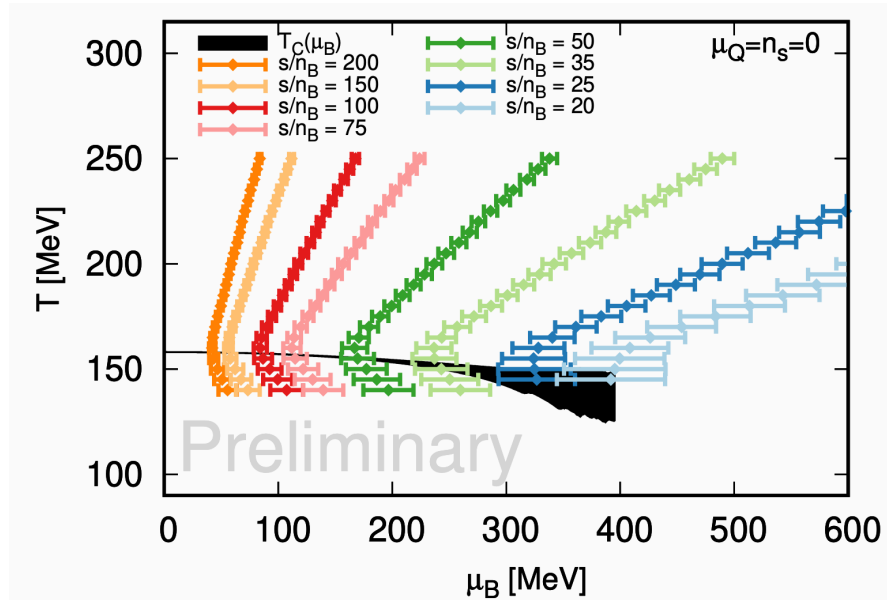
Extrapolations from lattice QCD at $\mu_B = 0$

Ideally, find the critical point through first-principle **lattice QCD** simulations at finite μ_B

- Challenging (sign problem), but perhaps not impossible? [Borsanyi et al., Phys. Rev. D 107, 091503L (2023)]

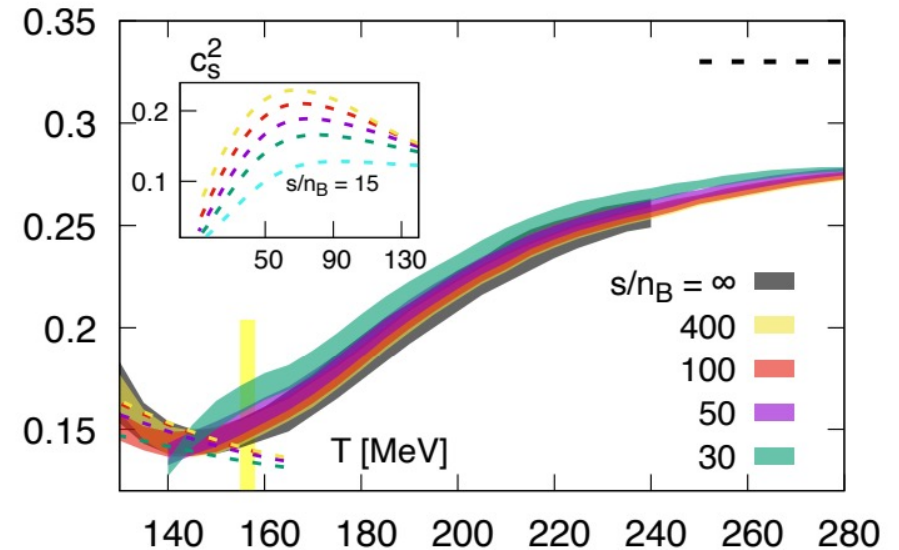
Taylor expansion + various resummations and extrapolation schemes from $\mu_B = 0$

alternative expansion scheme



[Borsanyi et al. (WB), Phys. Rev. D 105, 114504 (2022)]

Padé approximants



[Bollweg et al. (HotQCD), Phys. Rev. D 108, 014510 (2023)]

No indications for the strengthening of the chiral crossover or critical point signals

Disfavors QCD critical point at $\frac{\mu_B}{T} < 3$

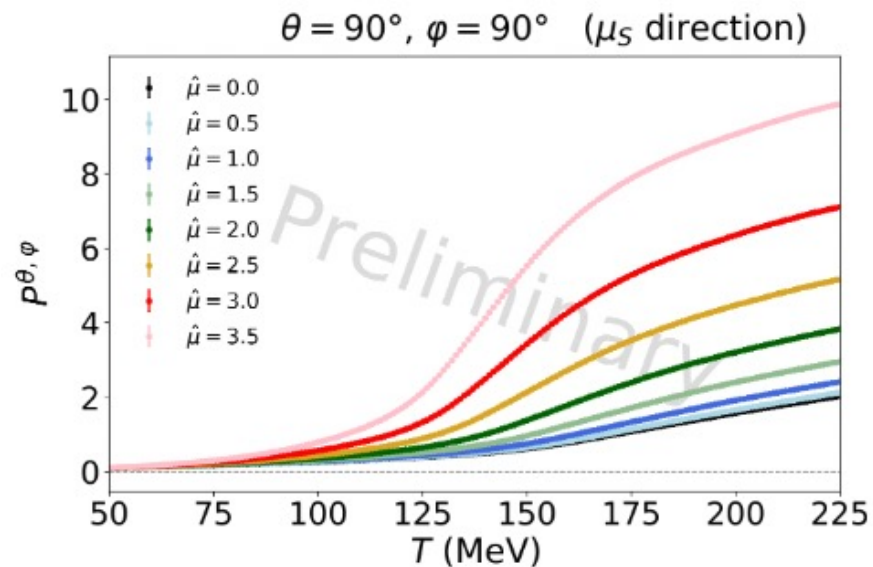
Extrapolations from $\mu_B = 0$: 4D-TEoS EoS

4D-TEoS EoS: alternative expansion scheme in three chemical potentials [J. Jahan, talk at SQM2024]

- Maps densities at finite μ 's to susceptibilities at $\mu = 0$
- Extended density coverage (whole RHIC-BES)
- Assumes no CP

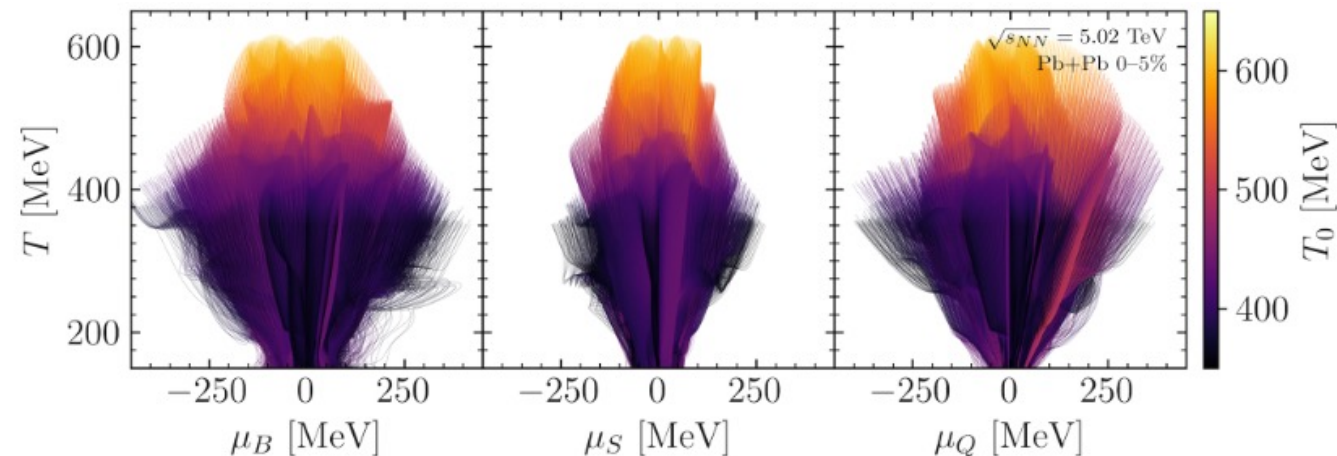
$$X_1^{\theta,\varphi}(T, \hat{\mu}) = \frac{\bar{X}_1^{\theta,\varphi}(\hat{\mu})}{\bar{X}_2^{\theta,\varphi}(0)} \times X_2^{\theta,\varphi}(T^{\theta,\varphi}(T, \hat{\mu}), 0)$$

$$\begin{aligned} \hat{\mu}_B &= \hat{\mu} \cdot \cos(\theta) & \hat{\mu} &= \sqrt{\hat{\mu}_B^2 + \hat{\mu}_Q^2 + \hat{\mu}_S^2} \\ \hat{\mu}_Q &= \hat{\mu} \cdot \sin(\theta) \cos(\varphi) & \iff & \varphi = \arccos\left(\frac{\hat{\mu}_Q}{\sqrt{\hat{\mu}_Q^2 + \hat{\mu}_S^2}}\right) \\ \hat{\mu}_S &= \hat{\mu} \cdot \sin(\theta) \sin(\varphi) & \theta &= \arccos\left(\frac{\hat{\mu}_B}{\hat{\mu}}\right) \end{aligned}$$



Required for BQS hydro simulations

[Plumberg, Almaalol et al., arXiv:2405.09648]

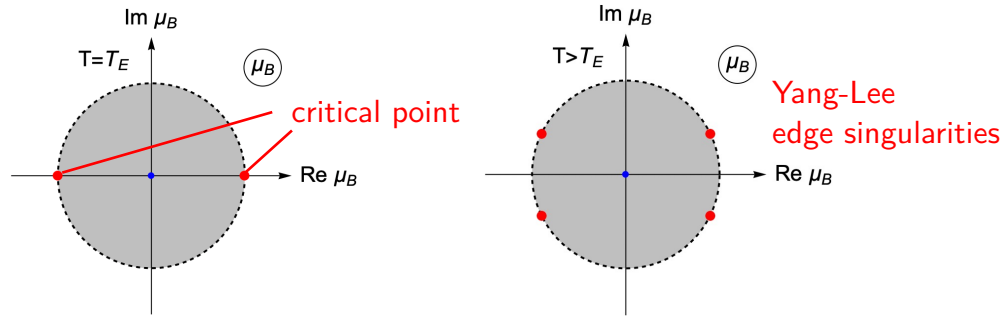


Searching for singularities in the complex plane

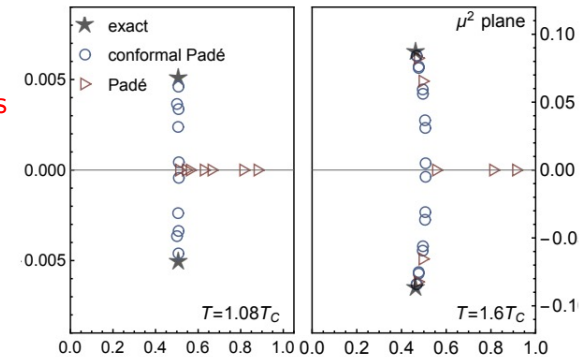
Critical point:

- singularity in the partition function
- $T=T_E$: real μ_B axis
- $T>T_E$: Yang-Lee edge singularities in the complex μ_B plane

[M. Stephanov, Phys. Rev. D 73, 094508 (2006)]

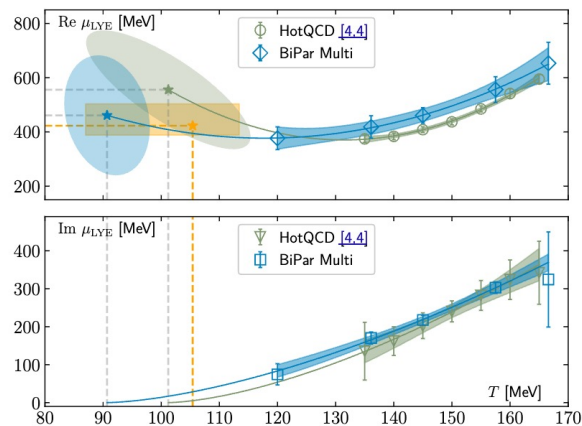


Lattice QCD is at $T > T_E$:



[G. Basar, arXiv:2312.06952]

- Extract YL edge singularity through (multi-point*)/(conformal**) Padé fits
- See if it approaches the real axis as temperatures decreases



Critical Point: 3D-Ising scaling inspired fit:

$$\text{Im } \mu_{LY} = c(T - T_{CEP})^\Delta$$

$$\text{Re } \mu_{LY} = \mu_{CEP} + a(T - T_{CEP}) + b(T - T_{CEP})^2$$

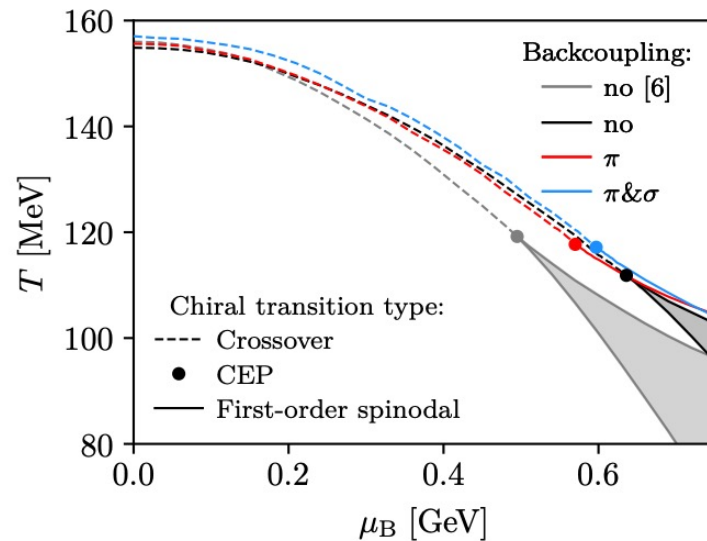


Extrapolated CP estimate:
 $T \sim 90-110 \text{ MeV}$, $\mu_B \sim 400-600 \text{ MeV}$

NB: many things have to go right, systematic error still very large (up to 100%), no continuum limit (likely large cut-off effects)

Dyson-Schwinger equations

truncation errors

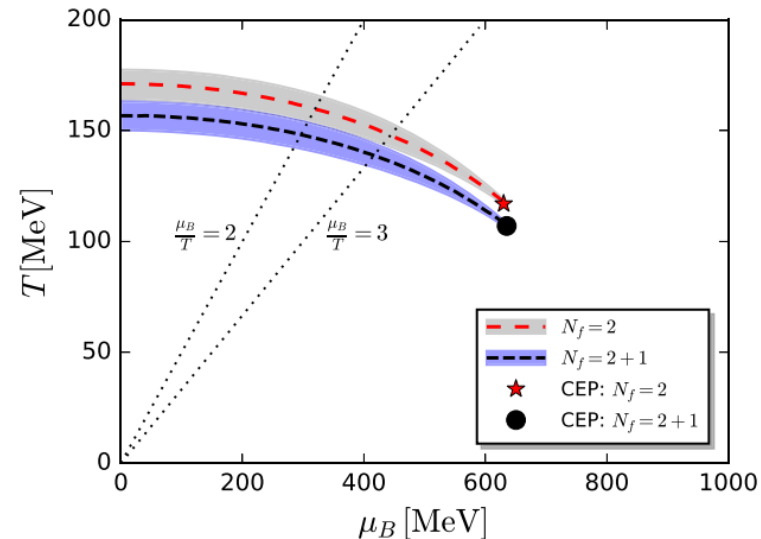


Gunkel, Fischer, PRD 104, 054202 (2021)

$T \sim 120 \text{ MeV}$ $\mu_B \sim 600 \text{ MeV}$

Functional renormalization group

truncation errors

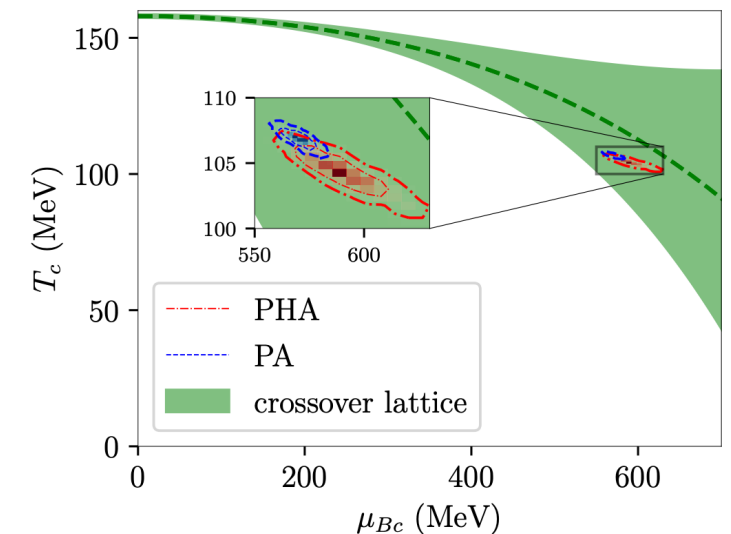


Fu, Pawlowski, Rennecke, PRD 101, 053032 (2020)

$T \sim 100 \text{ MeV}$ $\mu_B \sim 600 - 650 \text{ MeV}$

Black-hole engineering

strongly-coupled only ($\eta/s=1/4\pi$)



Hippert et al., arXiv:2309.00579

$T \sim 105 \text{ MeV}$ $\mu_B \sim 580 \text{ MeV}$

All in excellent agreement with lattice QCD at $\mu_B = 0$
and predict QCD critical point in a similar ballpark of $\mu_B/T \sim 5-6$

If true, reachable in heavy-ion collisions at $\sqrt{s_{NN}} \sim 3 - 5 \text{ GeV}$

Search for critical point with heavy-ion collisions

Control parameters

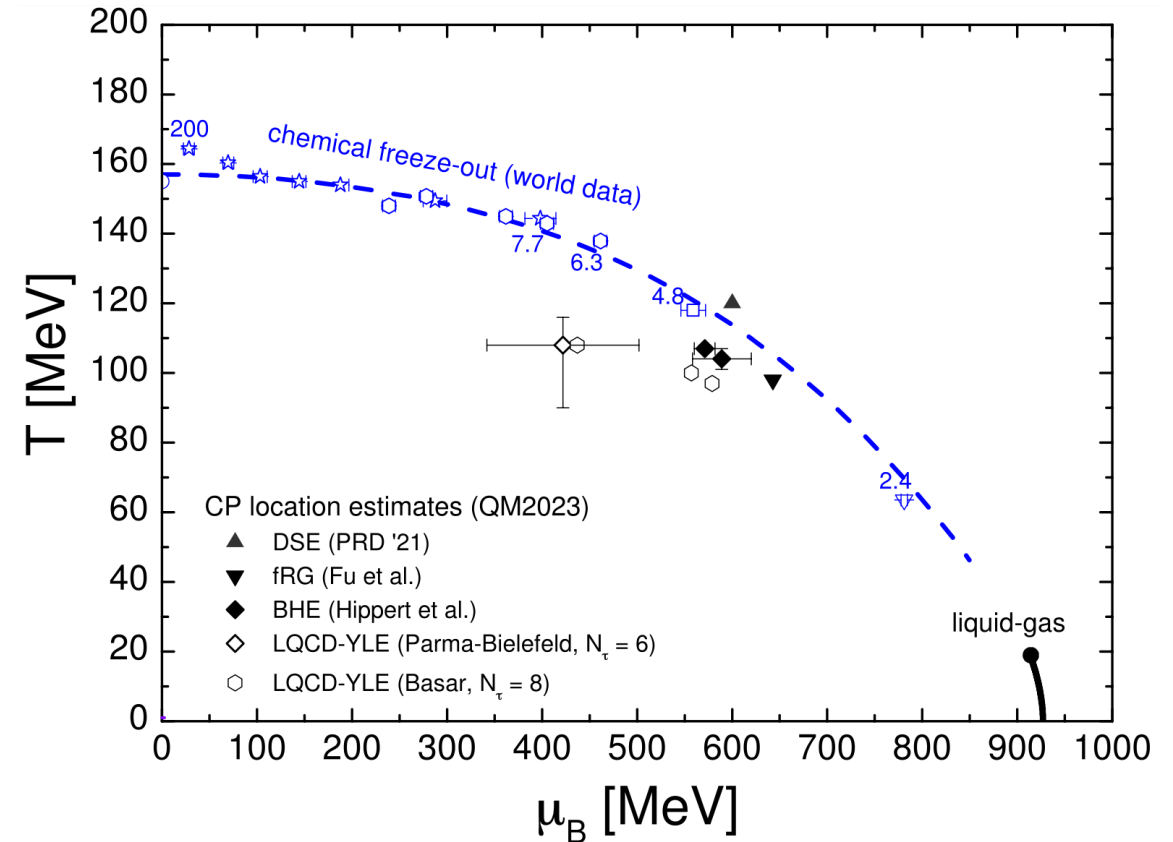
- Collision energy $\sqrt{s_{NN}} = 2.4 - 5020$ GeV
 - Scan the QCD phase diagram
- Size of the collision region
 - Expect stronger signal in larger systems

Measurements

- Final hadron abundances and momentum distributions **event-by-event**

Chemical freeze-out curve and CP

- Sets lower bound on the temperature of the CP
- **Caveats:** strangeness neutrality ($\mu_S \neq 0$), uncertainty in the freeze-out curve



A. Lysenko, Poberezhnyuk, Gorenstein, VV, in preparation

Event-by-event fluctuations and statistical mechanics

Cumulant generating function

$$K_N(t) = \ln \langle e^{tN} \rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

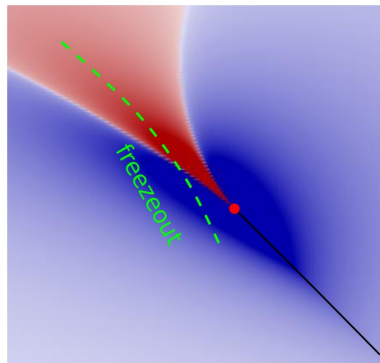
$$\kappa_n \propto \frac{\partial^n (\ln Z^{\text{gce}})}{\partial \mu^n}$$

Grand partition function

$$\ln Z^{\text{gce}}(T, V, \mu) = \ln \left[\sum_N e^{\mu N/T} Z^{\text{ce}}(T, V, N) \right]$$

Cumulants measure chemical potential derivatives of the (QCD) equation of state

- **(QCD) critical point:** large correlation length and fluctuations



M. Stephanov, PRL '09, '11
Energy scans at RHIC (STAR)
and CERN-SPS (NA61/SHINE)

$$\kappa_2 \sim \xi^2, \quad \kappa_3 \sim \xi^{4.5}, \quad \kappa_4 \sim \xi^7$$

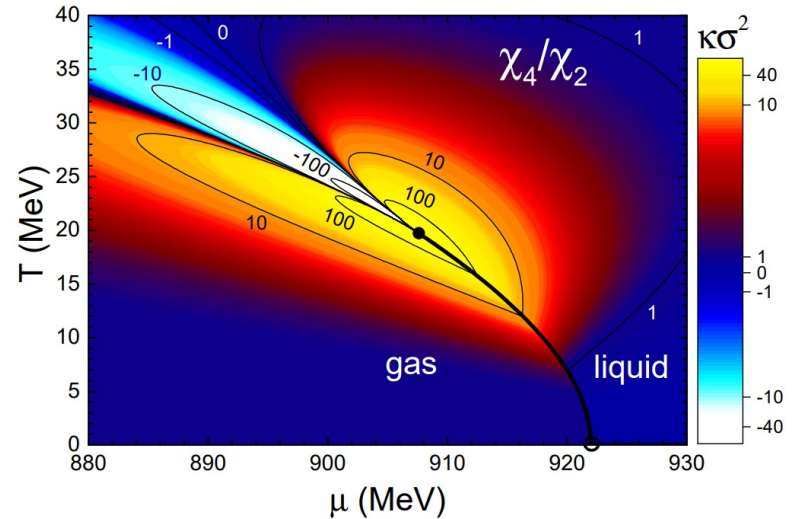
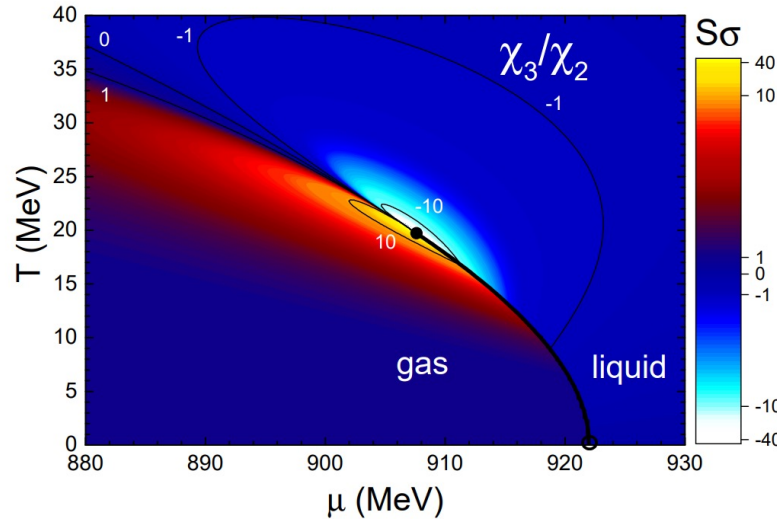
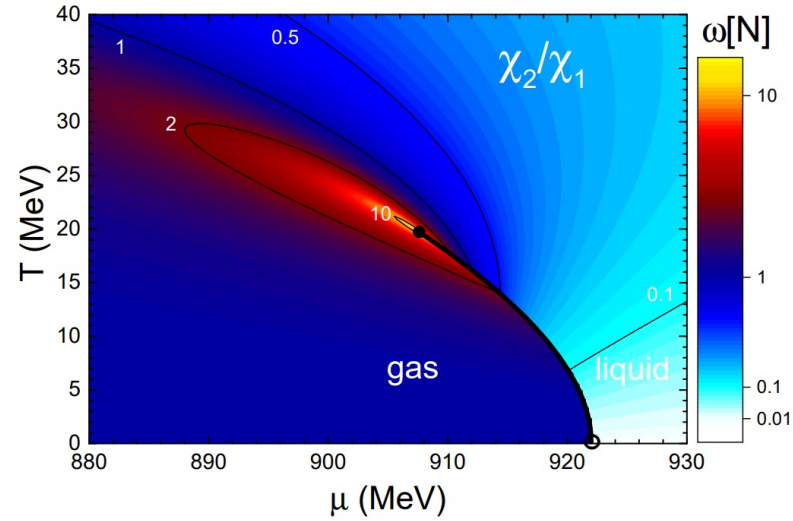
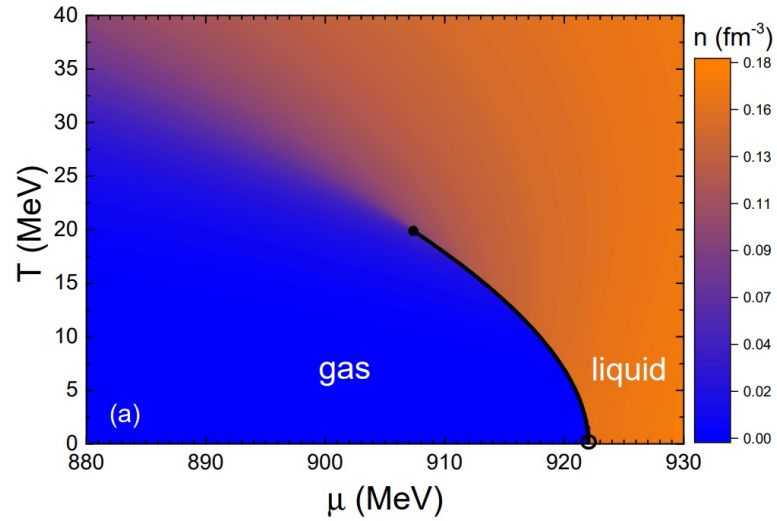
$$\xi \rightarrow \infty$$

Looking for enhanced fluctuations
and non-monotonocities

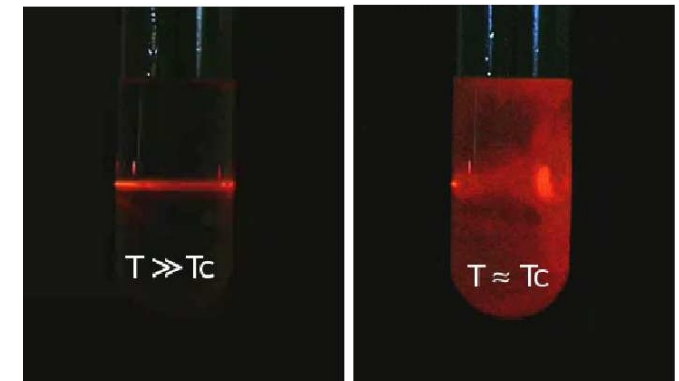
Other uses of cumulants:

- QCD degrees of freedom
[Jeon, Koch, PRL 85, 2076 \(2000\)](#)
[Asakawa, Heinz, Muller, PRL 85, 2072 \(2000\)](#)
- Extracting the speed of sound
[A. Sorensen et al., PRL 127, 042303 \(2021\)](#)
- Conservation volume
[VV, Donigus, Stoecker, PRC 100, 054906 \(2019\)](#)

Example: (Nuclear) Liquid-gas transition



Critical opalescence



$$\langle N^2 \rangle - \langle N \rangle^2 \sim \langle N \rangle \sim 10^{23}$$

in equilibrium

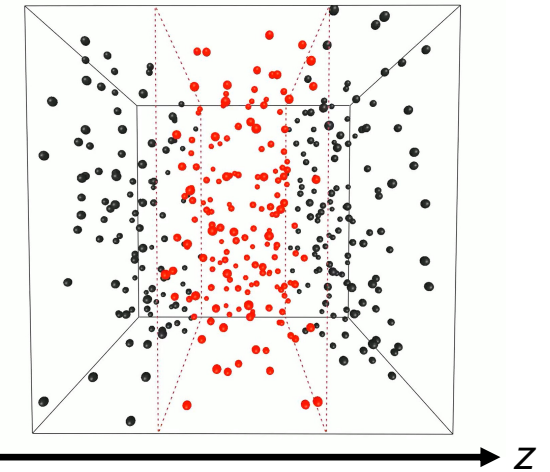
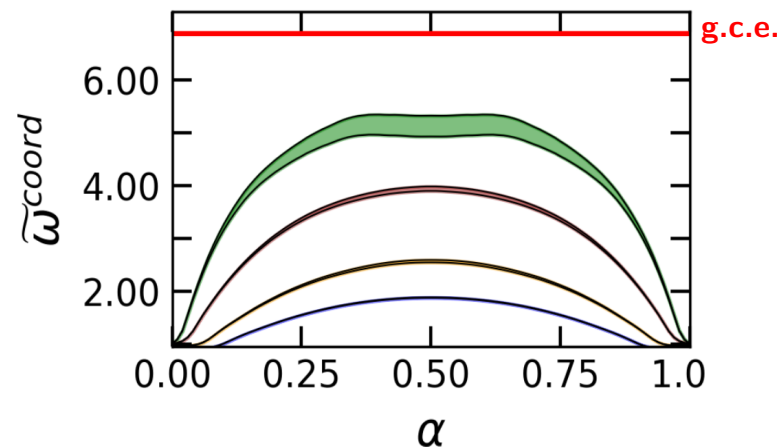
Example: Critical fluctuations in a microscopic simulation

V. Kuznetsov et al., Phys. Rev. C 105, 044903 (2022)

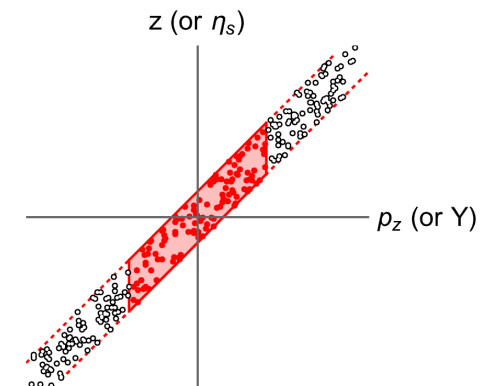
Classical molecular dynamics simulations of the **Lennard-Jones fluid** near Z(2) critical point ($T \approx 1.06T_c$, $n \approx n_c$) of the liquid-gas transition

Scaled variance in coordinate space acceptance $|z| < z^{max}$

$$\tilde{\omega}^{coord} = \frac{1}{1 - \alpha} \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle}$$



Heavy-ion collisions:
flow correlates p_z and z cuts

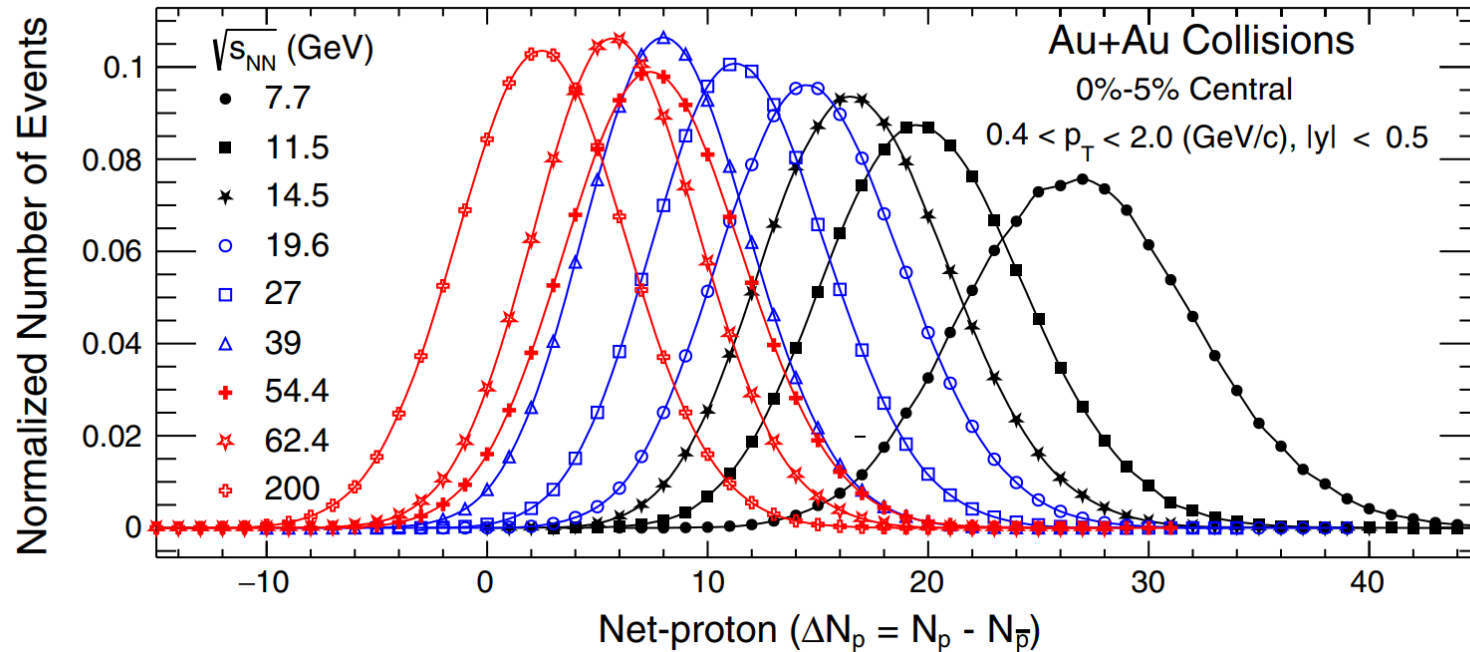


- Large fluctuations survive despite strong finite-size effects
- Need coordinate space cuts (collective flow helps)
- Here no finite-time effects

Measuring cumulants in heavy-ion collisions

Count the number of events with given number of e.g. (net) protons $P(\Delta N_p) \sim \frac{N_{\text{events}}(\Delta N_p)}{N_{\text{events}}^{\text{total}}}$

STAR Collaboration, Phys. Rev. Lett. 126, 092301 (2021)



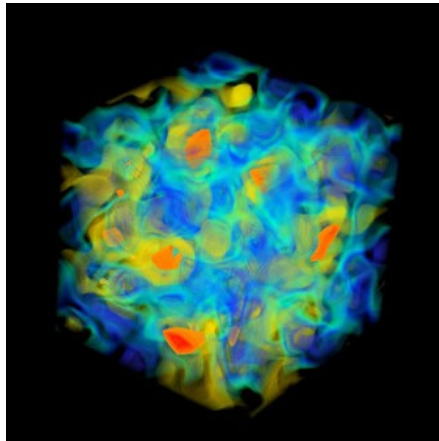
Cumulants are extensive, $\kappa_n \sim V$, use ratios to cancel out the volume

$$\frac{\kappa_2}{\langle N \rangle}, \quad \frac{\kappa_3}{\kappa_2}, \quad \frac{\kappa_4}{\kappa_2}$$

Look for subtle critical point signals (tails of the distribution)

Theory vs experiment: Challenges for fluctuations

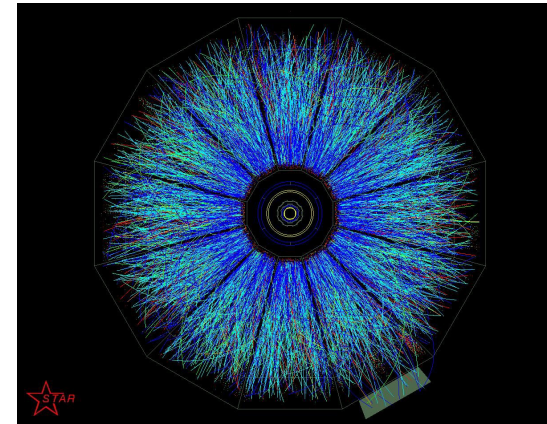
Theory



© Lattice QCD@BNL

- Coordinate space
- In contact with the heat bath
- Conserved charges
- Uniform
- Fixed volume

Experiment



STAR event display

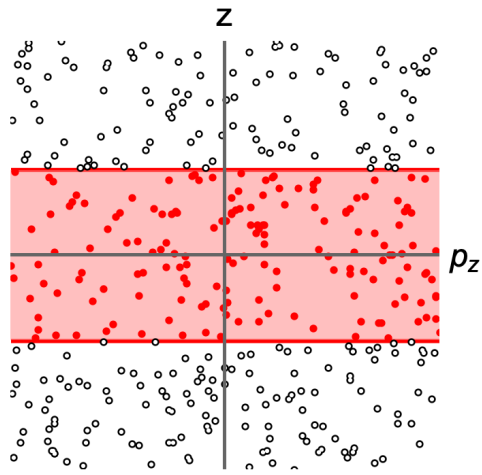
- Momentum space
- Expanding in vacuum
- Non-conserved particle numbers
- Inhomogenous
- Fluctuating volume

Need dynamical description

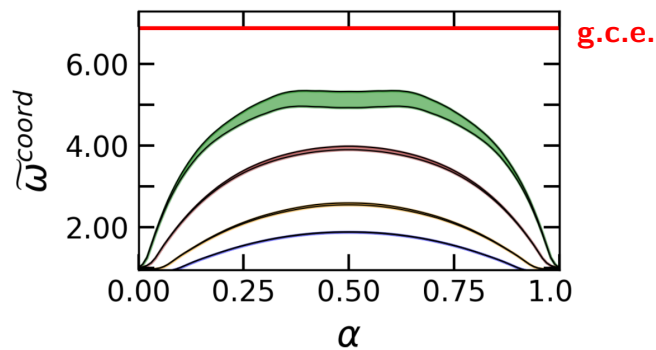
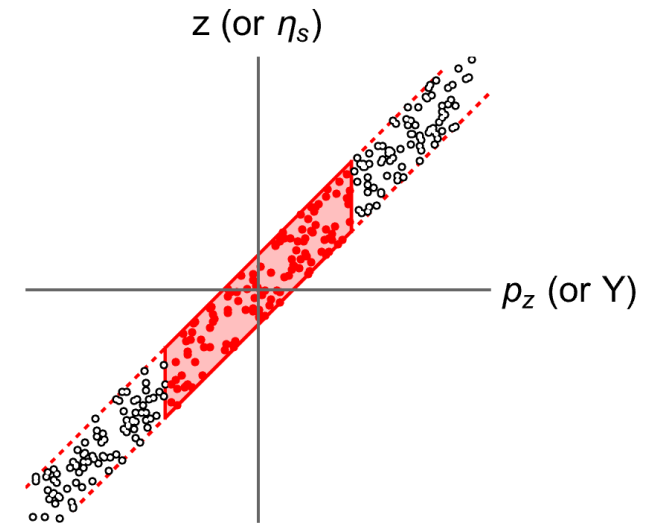
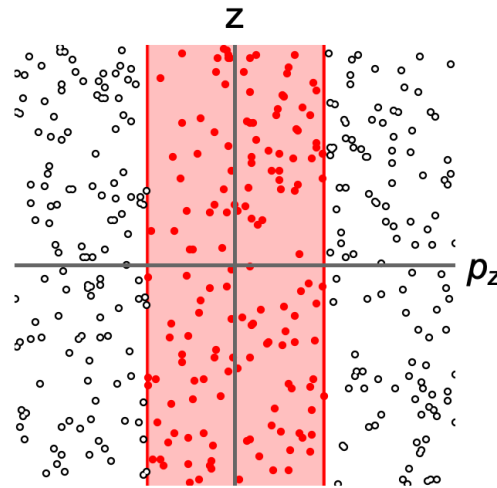
Box setup: Coordinates and momenta are uncorrelated

HICs: Flow (e.g. Bjorken)

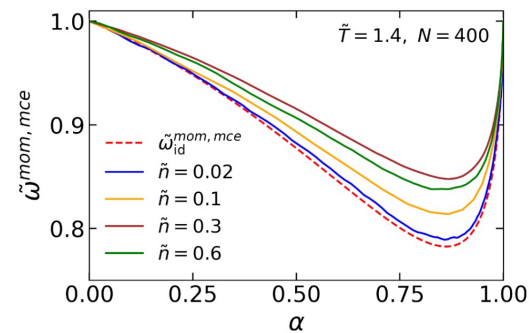
Coordinate space cut



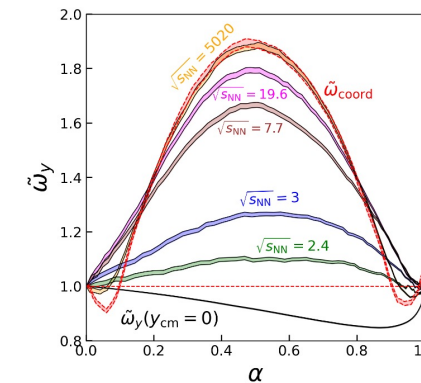
Momentum space cut



Large correlations



Nothing left



momentum cut ~ coordinate cut + smearing

Dynamical approaches to the QCD critical point search

1. Dynamical model calculations of critical fluctuations



[X. An et al., Nucl. Phys. A 1017, 122343 (2022)]

- Fluctuating hydrodynamics (hydro+) and (non-equilibrium) evolution of fluctuations
- Equation of state with a tunable critical point [P. Parotto et al, PRC 101, 034901 (2020); J. Karthein et al., EPJ Plus 136, 621 (2021)]
- Generalized Cooper-Frye particlization [M. Pradeep, et al., PRD 106, 036017 (2022); PRL 130, 162301 (2023)]

Alternatives at high μ_B : hadronic transport/molecular dynamics with a critical point

[A. Sorensen, V. Koch, PRC 104, 034904 (2021); V. Kuznetsov et al., PRC 105, 044903 (2022)]

2. Deviations from precision calculations of non-critical fluctuations

- Non-critical baseline is not flat [Braun-Munzinger, Rustamov, Stachel, NPA 1008, 122141 (2021)]
- Include essential non-critical contributions to (net-)proton number cumulants
- Exact **baryon conservation** + **hadronic interactions** (hard core repulsion)
- Based on realistic hydrodynamic simulations tuned to bulk data

[VV, C. Shen, V. Koch, Phys. Rev. C 105, 014904 (2022)]

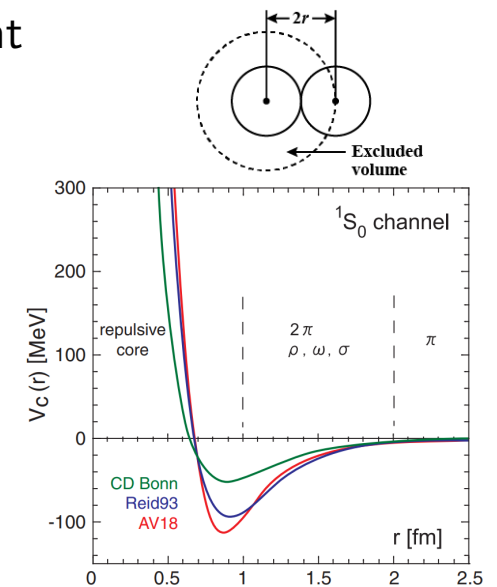
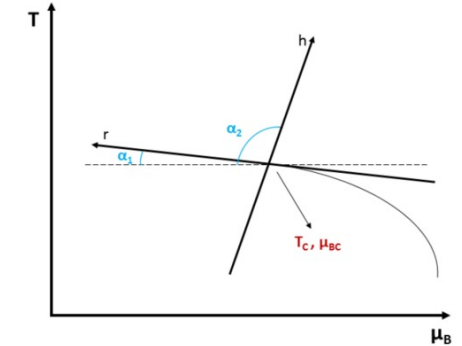
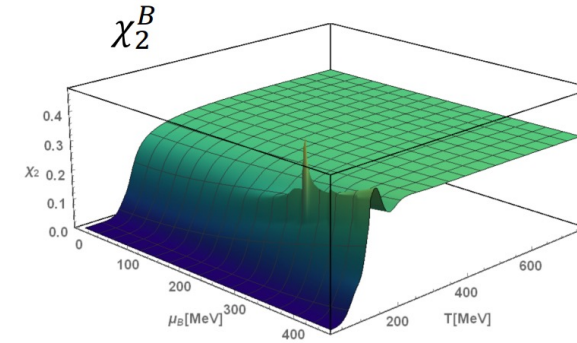


Figure from Ishii et al., PRL '07

Equation of state with a tunable critical point

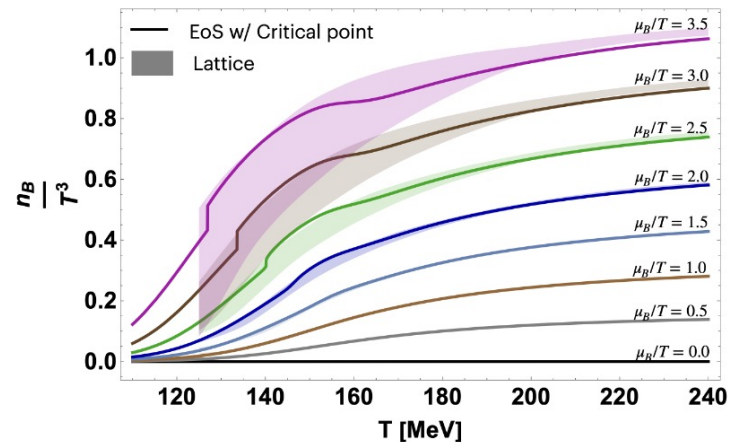
BEST equation of state: [P. Parotto et al, PRC 101, 034901 \(2020\)](#)

- 3D-Ising CP mapped onto the QCD
- Tunable CP location along the pseudocritical line
- Matched to lattice data at $\mu_B = 0$



New development: [M. Kahangirwe et al, PRD 109, 094046 \(2024\)](#)

Match to **alternative expansion scheme** from lattice QCD instead of Taylor expansion, extending the range to whole BES range



$$p(T, \mu_B) = \underbrace{p^{\text{non-Ising}}(T, \mu_B)}_{\text{regular}} + \underbrace{p^{\text{Ising}}(T, \mu_B)}_{\text{critical}}$$

Alternative ways to embed the critical point:

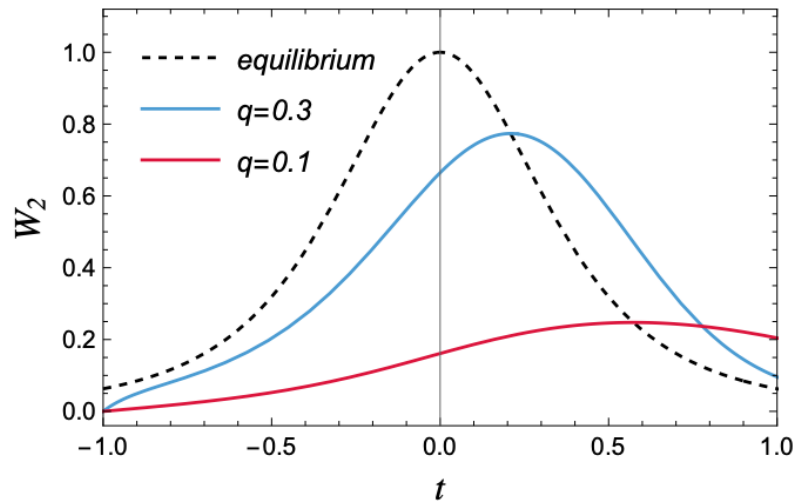
[\[J. Kapusta, T. Welle, C. Plumberg, PRC 106, 014909 \(2022\); PRC 106, 044901 \(2022\)\]](#)

Equilibrium expectations for fluctuations:

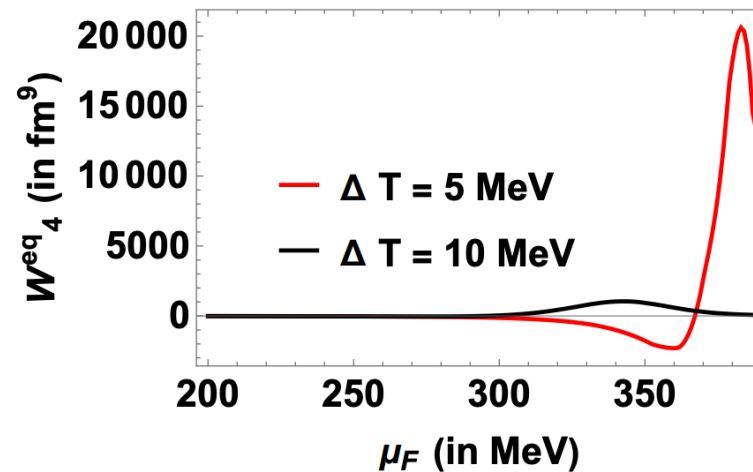
[\[J.M. Karthein et al., 2402.18738; SQM2024\]](#)

Non-equilibrium evolution and critical slowing down

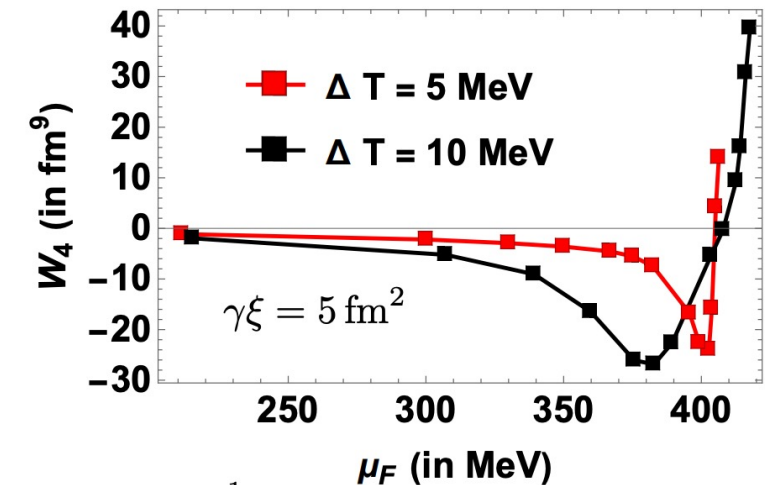
- Non-equilibrium evolution of (non-)Gaussian fluctuations
 - Strong suppression of critical point signals due to critical slowing down and (local) conservation



[X. An et al., PRL 127, 072301 (2021)]



[M. Pradeep et al., Phys. Rev. D 106 (2022) 036017; QM2024]



- Generalized Cooper-Frye particlization: maximum entropy freeze-out of fluctuations

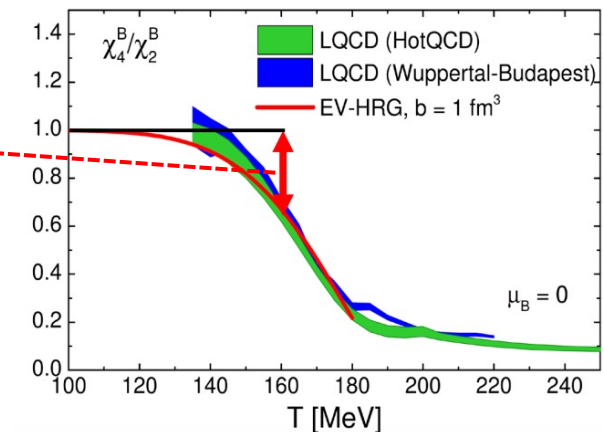
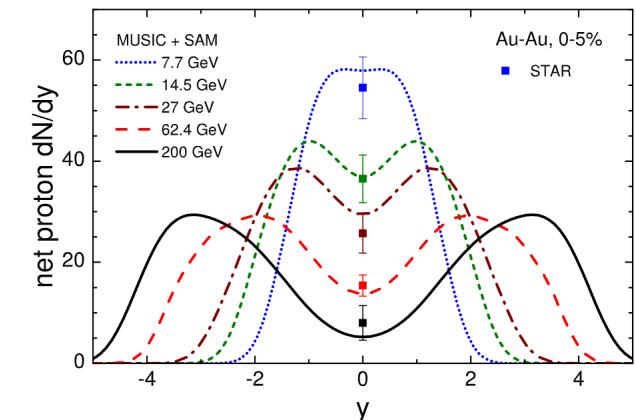
[M. Pradeep, M. Stephanov, PRL 130, 162301 (2023)]
- Diffusion and cross-correlations of multiple conserved charges and energy-momentum, balancing conservation laws

[O. Savchuk, S. Pratt, PRC 109, 024910 (2024)]

Calculation of non-critical contributions

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

- (3+1)-D viscous hydrodynamics evolution (MUSIC-3.0)
 - Collision geometry-based 3D initial state [Shen, Alzhrani, PRC 102, 014909 (2020)]
 - Crossover equation of state based on lattice QCD [Monnai, Schenke, Shen, Phys. Rev. C 100, 024907 (2019)]
 - Cooper-Frye particlization at $\epsilon_{sw} = 0.26 \text{ GeV}/\text{fm}^3$
- Non-critical contributions are computed at particlization
 - QCD-like baryon number distribution via excluded volume $b = 1 \text{ fm}^3$ [VV, V. Koch, Phys. Rev. C 103, 044903 (2021)]
 - Exact global baryon conservation* (and other charges)
 - Subensemble acceptance method 2.0 (analytic) [VV, Phys. Rev. C 105, 014903 (2022)]
 - or FIST sampler (Monte Carlo) [VV, Phys. Rev. C 106, 064906 (2022)]
<https://github.com/vlvovch/fist-sampler>
- **Absent:** critical point, local conservation, initial-state/volume fluctuations

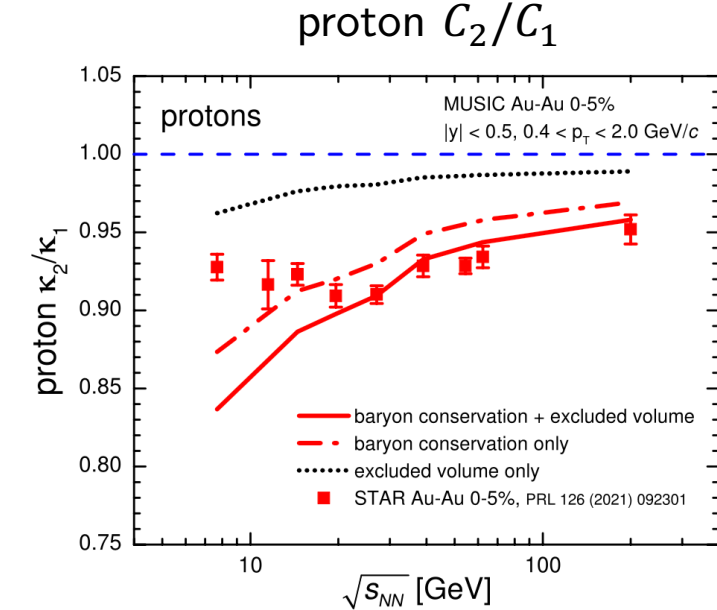
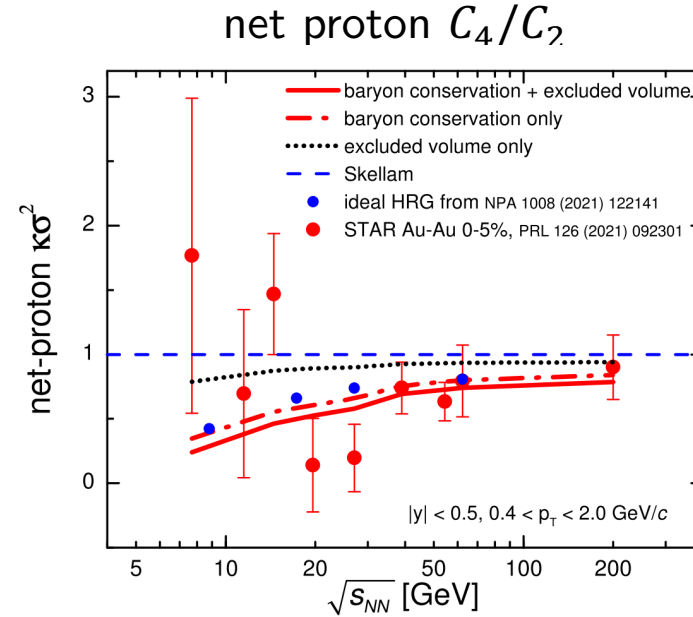
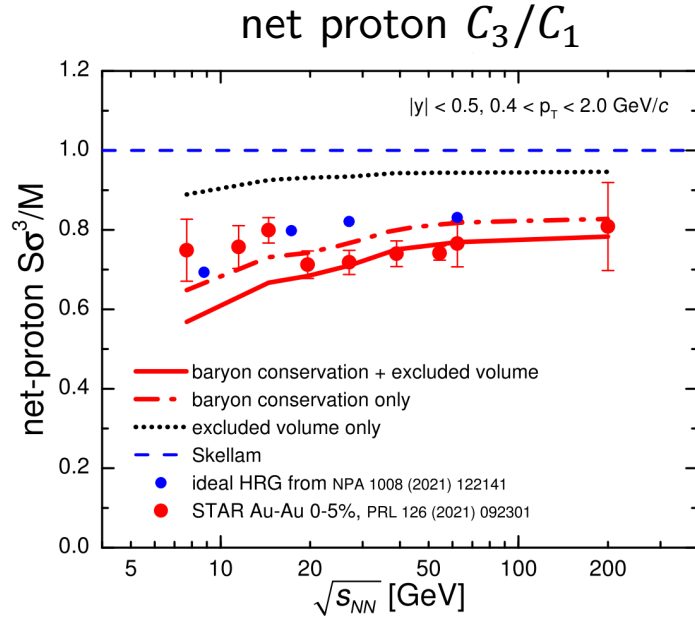


*If baryon conservation is the only effect (no other correlations), non-critical baseline can be computed without hydro

Braun-Munzinger, Friman, Redlich, Rustamov, Stachel, NPA 1008, 122141 (2021)

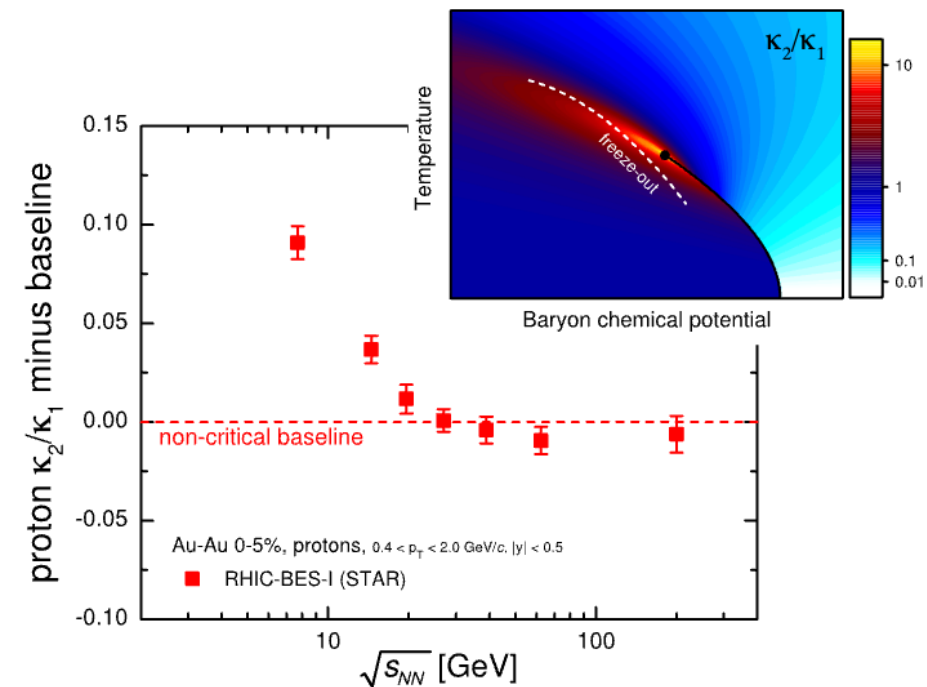
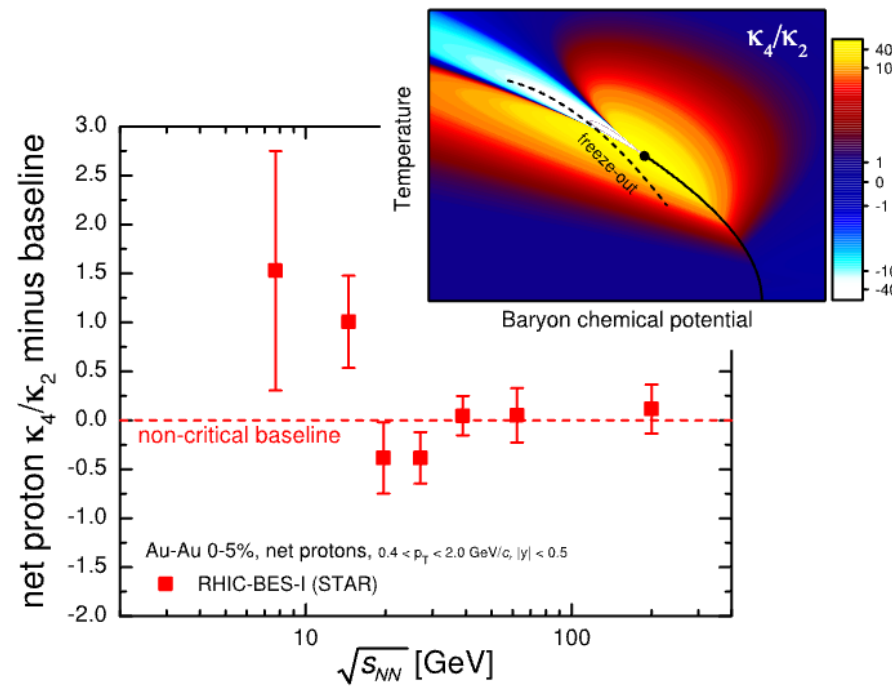
RHIC-BES-I: Net proton cumulant ratios (MUSIC)

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

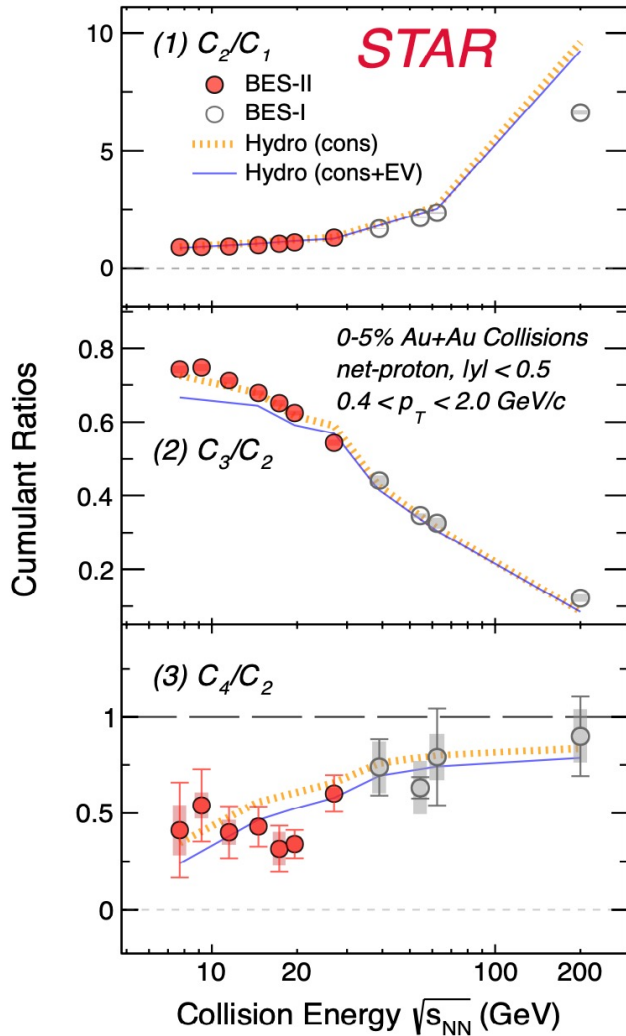


- Data at $\sqrt{s_{NN}} \geq 20 \text{ GeV}$ consistent with non-critical physics (BQS conservation and repulsion)
- Effect from baryon conservation is stronger than repulsion but both are required at $\sqrt{s_{NN}} \geq 20 \text{ GeV}$
- Deviations from baseline at lower energies?

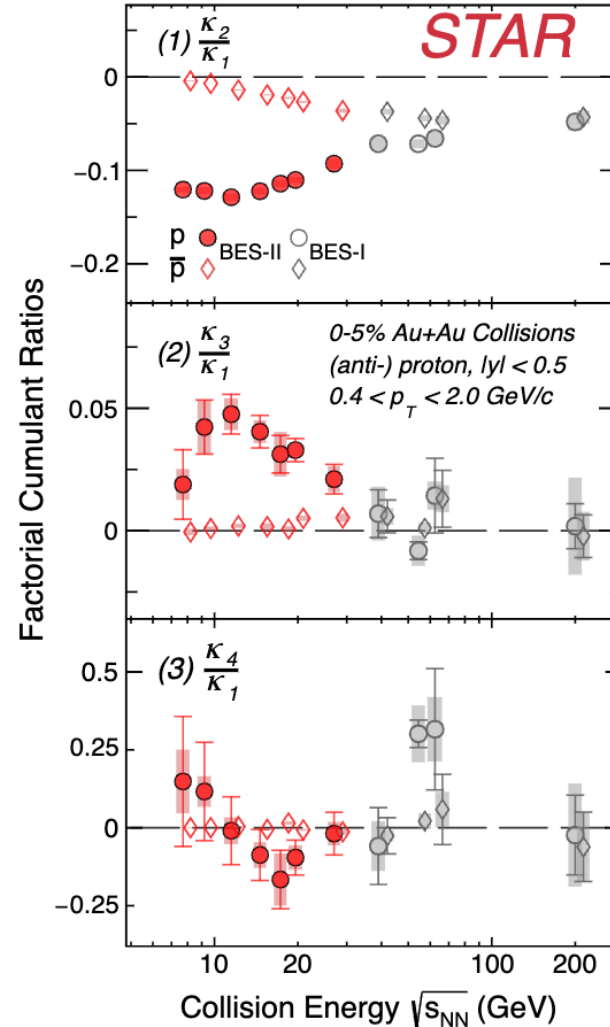
Subtracting the hydro baseline



Net-proton cumulant ratios



Proton/antiproton factorial cumulant ratios



- No smoking gun signature for CP
- More structure seen in factorial cumulants
 - What are they?

Factorial cumulants \hat{C}_n vs ordinary cumulants C_n

Factorial cumulants: ~irreducible n-particle corr.

$$\hat{C}_n \sim \langle N(N-1)(N-2)\dots \rangle_c$$

$$C_1 = \hat{C}_1$$

$$C_2 = \hat{C}_2 + \hat{C}_1$$

$$C_3 = \hat{C}_3 + 3\hat{C}_2 + \hat{C}_1$$

$$C_4 = \hat{C}_4 + 6\hat{C}_3 + 7\hat{C}_2 + \hat{C}_1$$

Ordinary cumulants: mix corrs. of different orders

$$C_n \sim \langle \delta N^n \rangle_c$$

$$\hat{C}_1 = C_1$$

$$\hat{C}_2 = C_2 - C_1$$

$$\hat{C}_3 = C_3 - 3C_2 + 2C_1$$

$$\hat{C}_4 = C_4 - 6C_3 + 11C_2 - 6C_1$$

[Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017)]

Factorial cumulants and different physics mechanisms

- Baryon conservation
[Bzdak, Koch, Skokov, EPJC '17]

$$\hat{C}_n^{\text{cons}} \propto a^n \quad \textit{small}$$

- Excluded volume
[VV et al, PLB '17]

$$\hat{C}_n^{\text{EV}} \propto b^n \quad \textit{small}$$

- Volume fluctuations
[Holzman et al., arXiv:2403.03598]

$$\hat{C}_n^{\text{CF}} \sim (\hat{C}_1)^n \kappa_n[V] \quad \textit{depends on Vfluc}$$

- Critical point
[Ling, Stephanov, PRC '16]

$$\hat{C}_2^{\text{CP}} \sim \xi^2, \quad \hat{C}_3^{\text{CP}} \sim \xi^{4.5}, \quad \hat{C}_4^{\text{CP}} \sim \xi^7 \quad \textit{large}$$

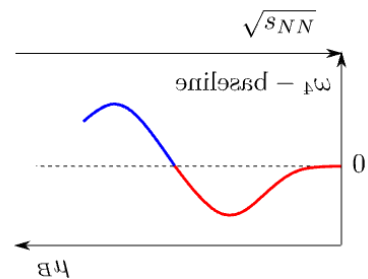
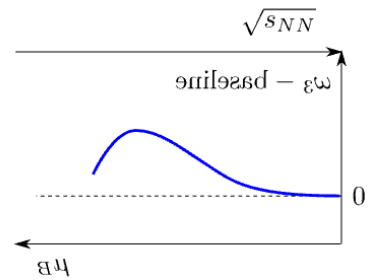
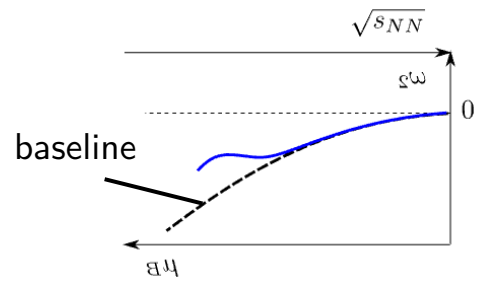
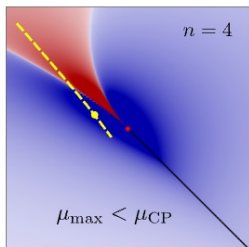
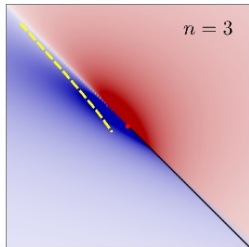
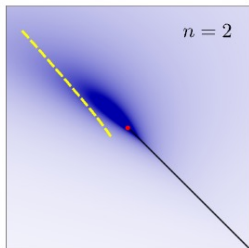
- proton vs baryon $\hat{C}_n^B \sim 2^n \times \hat{C}_n^p$ **same sign!**
[Kitazawa, Asakawa, PRC '12]

Factorial cumulants from RHIC-BES-II

From M. Stephanov (SQM2024):

$$\omega_n = \hat{C}_n / \hat{C}_1$$

(universal EOS) critical χ_n :



Bzdak et al review 1906.00936

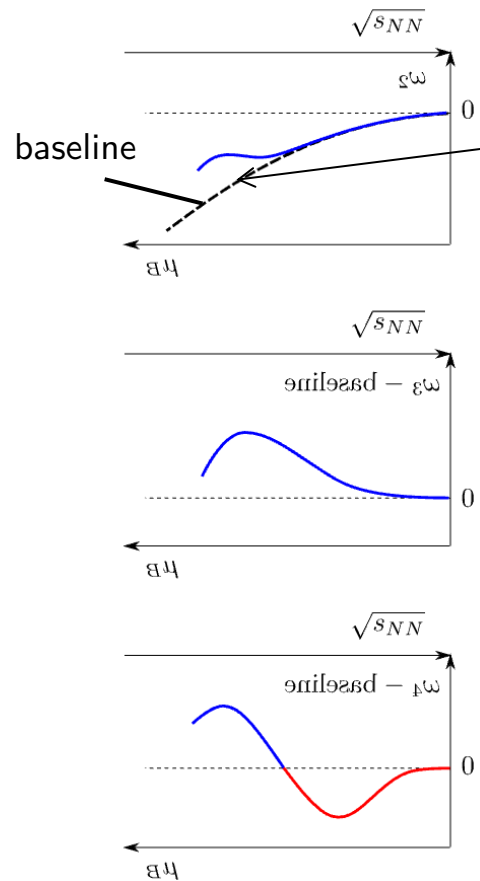
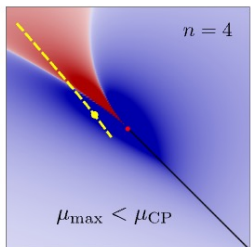
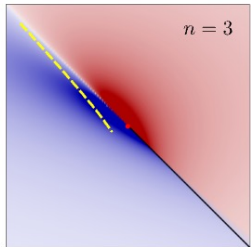
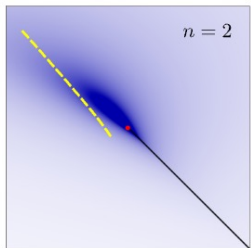
Expected signatures: **bump** in ω_2 and ω_3 , **dip** then **bump** in ω_4 for CP at $\mu_B > 420$ MeV

Factorial cumulants from RHIC-BES-II

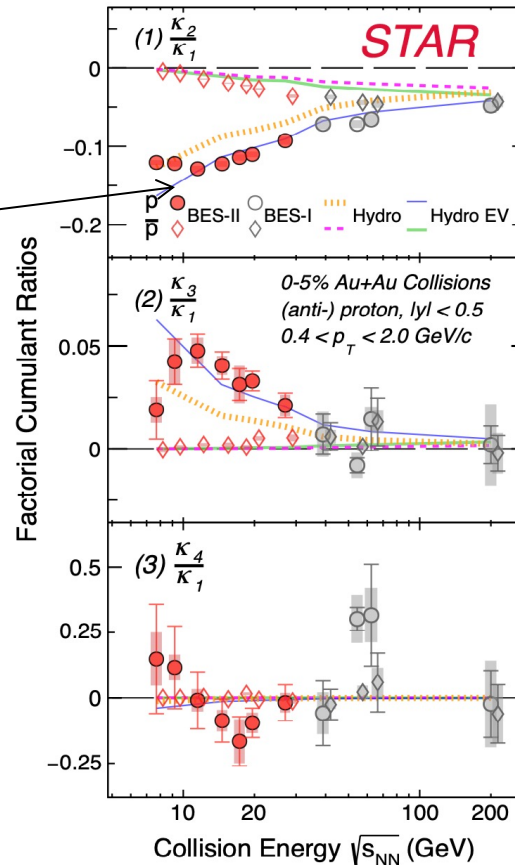
From M. Stephanov (SQM2024):

$$\omega_n = \hat{C}_n / \hat{C}_1^n$$

(universal EOS) critical χ_n :



STAR data:



A. Pandav, CPOD2024

baseline (hydro):

VV, V. Koch, C. Shen, PRC 105, 014904 (2022)

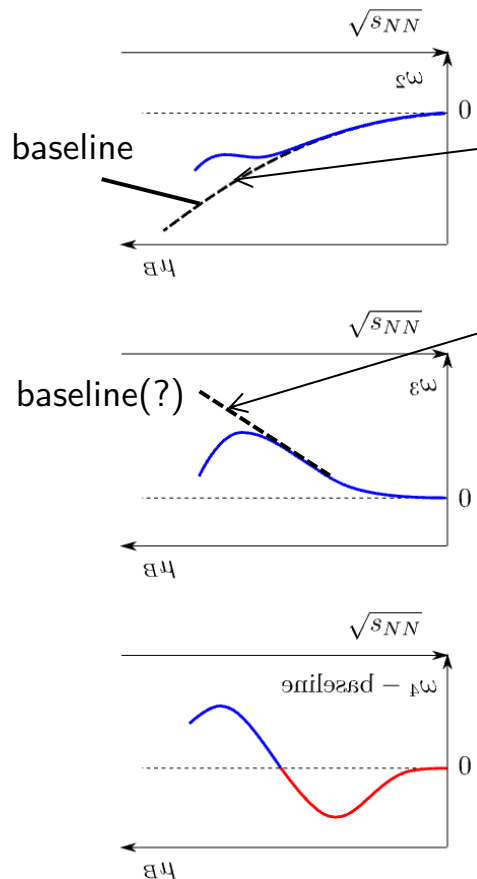
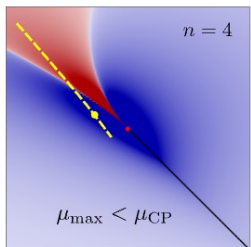
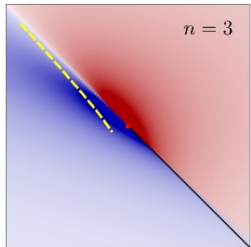
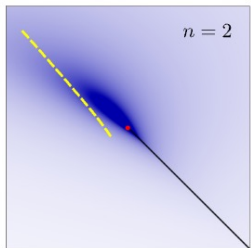
Expected signatures: **bump** in ω_2 and ω_3 , **dip** then **bump** in ω_4 for CP at $\mu_B > 420$ MeV

Factorial cumulants from RHIC-BES-II

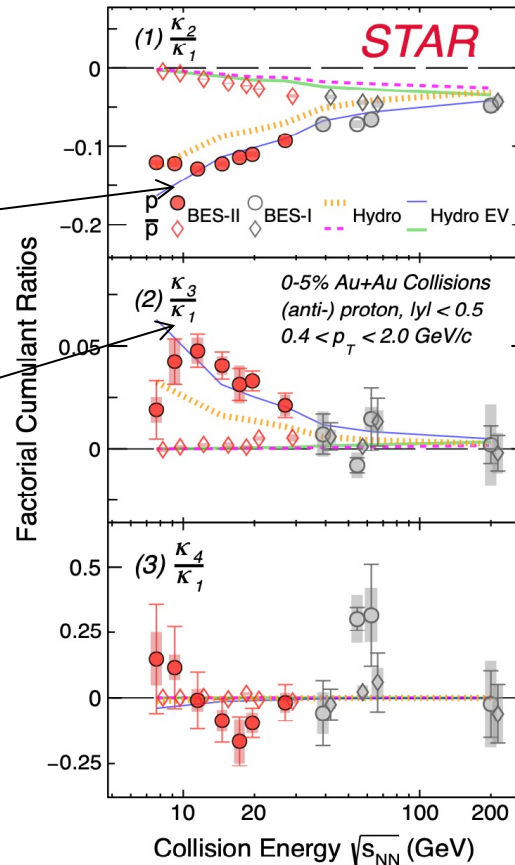
From M. Stephanov (SQM2024):

$$\omega_n = \hat{C}_n / \hat{C}_1^n$$

(universal EOS) critical χ_n :



STAR data:



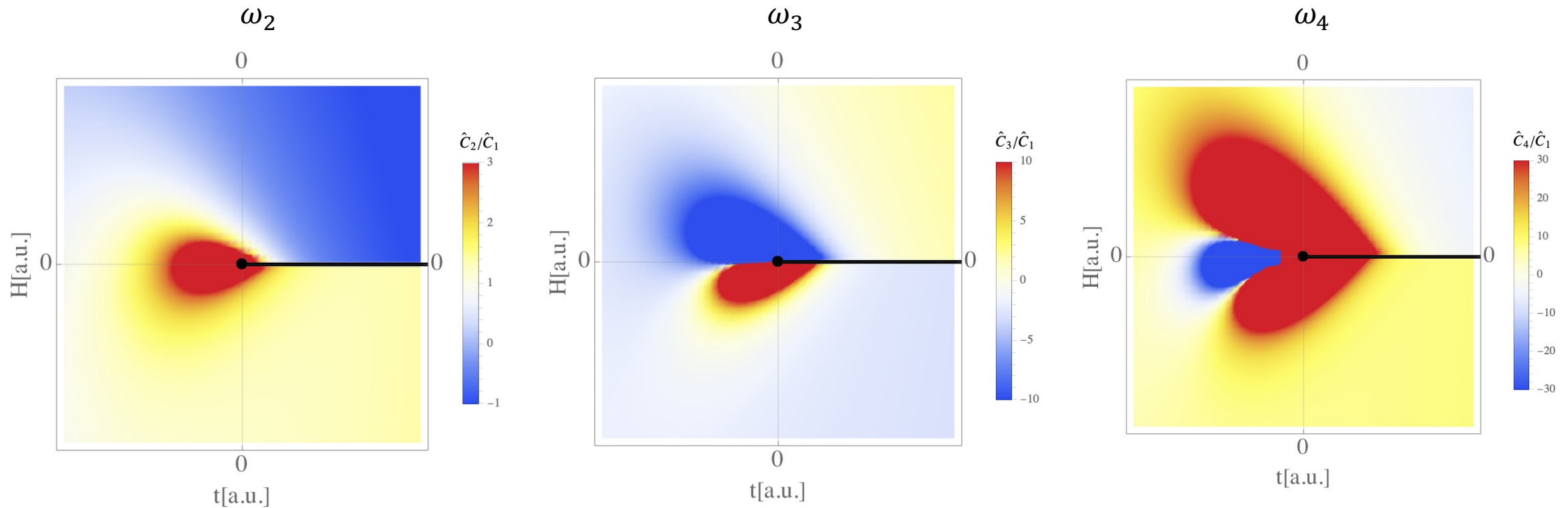
A. Pandav, CPOD2024

baseline (hydro):

VV, V. Koch, C. Shen, PRC 105, 014904 (2022)

- describes right side of the peak in \hat{C}_3
- implies
 - *positive* \hat{C}_2 – baseline > 0
 - *negative* \hat{C}_3 – baseline < 0

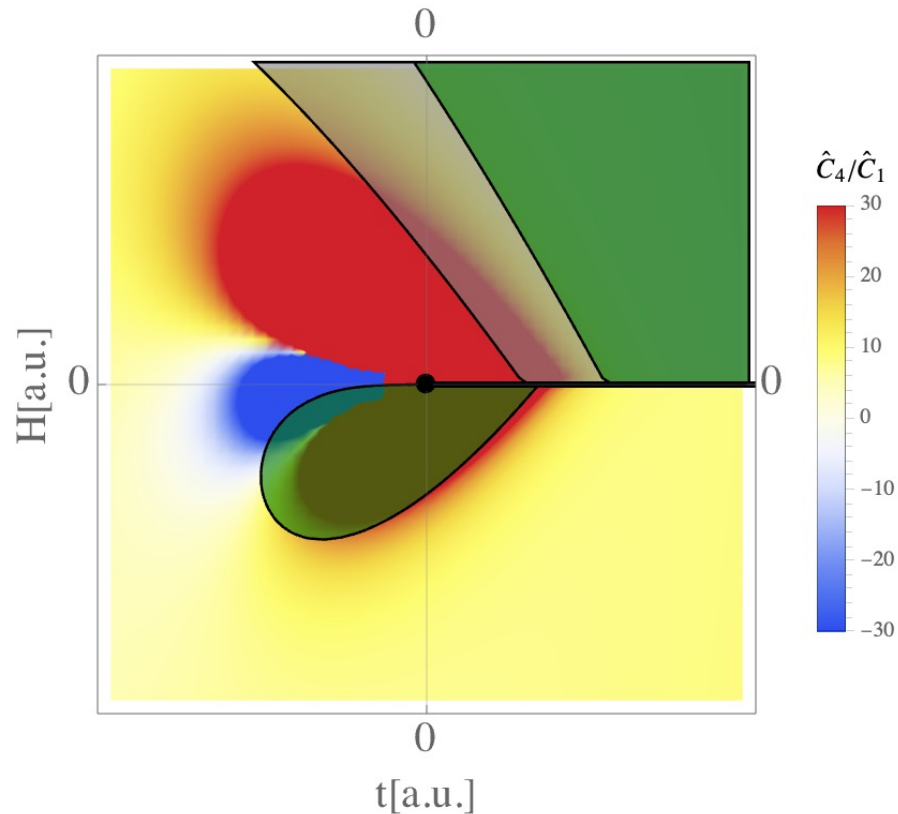
Factorial cumulants in Ising model



Adapted from Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017)

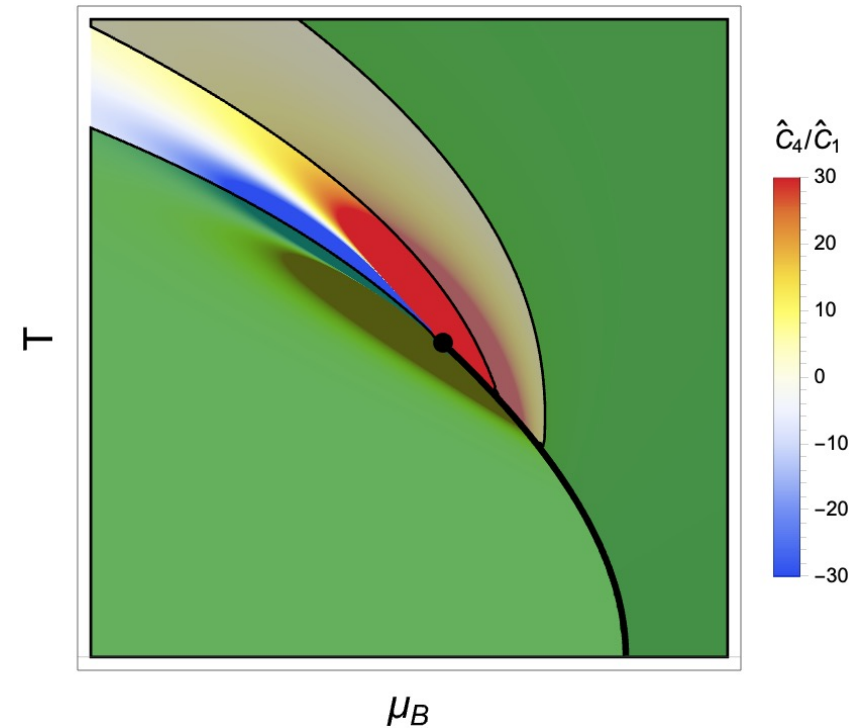
Factorial cumulants from RHIC-BES-II and CP

Exclusion plots



Shaded regions exclude $\hat{C}_2 < 0$ & $\hat{C}_3 > 0$

How it may look like in $T - \mu_B$ plane



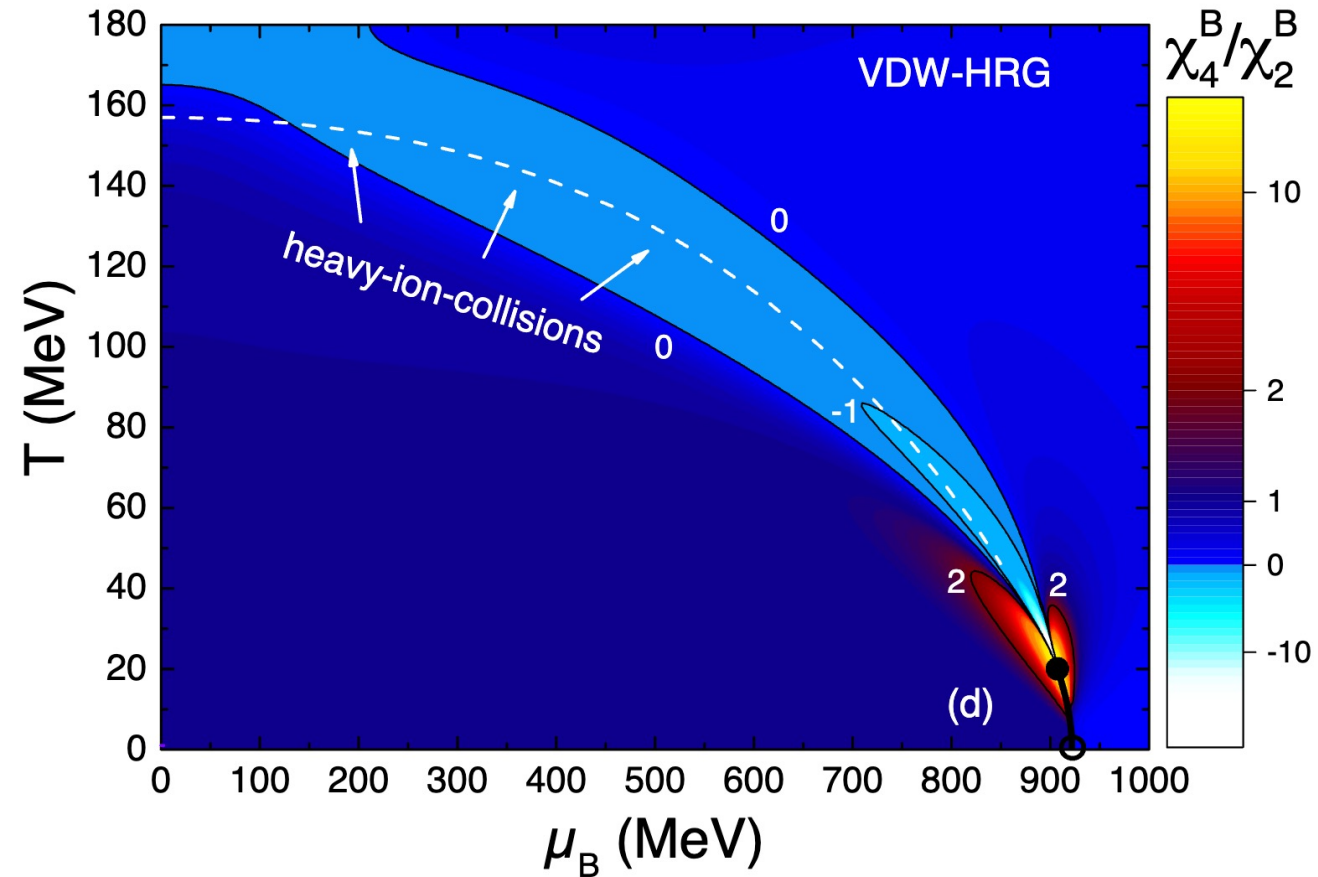
Based on QvdW model of nuclear matter

VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)

Freeze-out of fluctuations of the QGP side of the crossover?

Nuclear liquid-gas transition

HRG with attractive and repulsive interactions among baryons

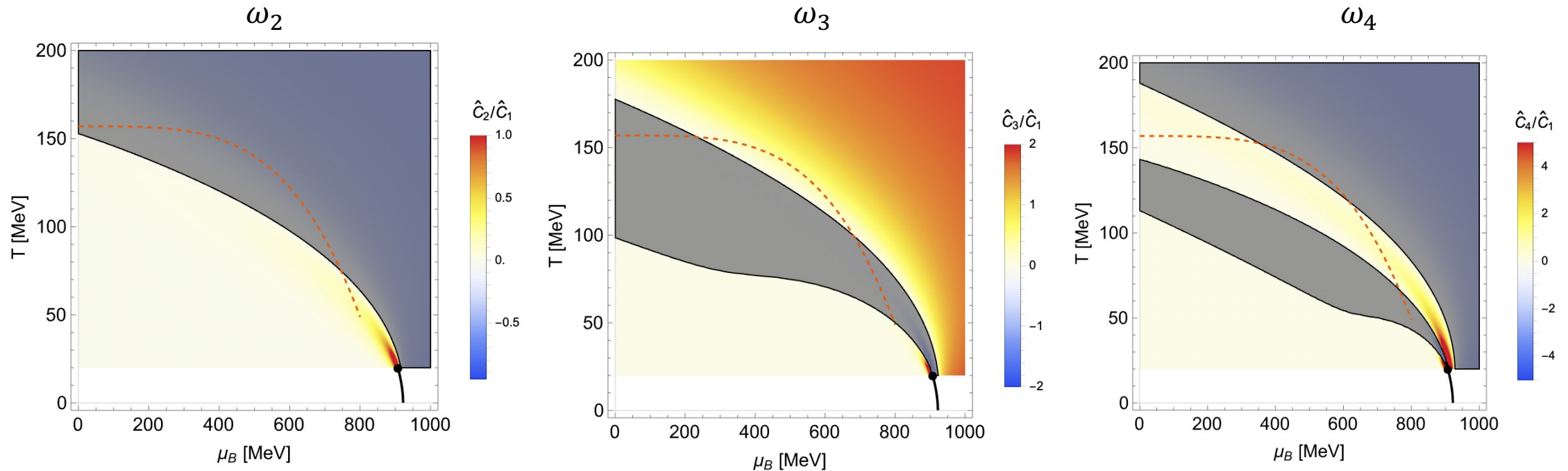


VV, Gorenstein, Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

Factorial cumulants and nuclear liquid-gas transition

Calculation in a van der Waals-like HRG model

VV, Gorenstein, Stoecker, EPJA 54, 16 (2018)

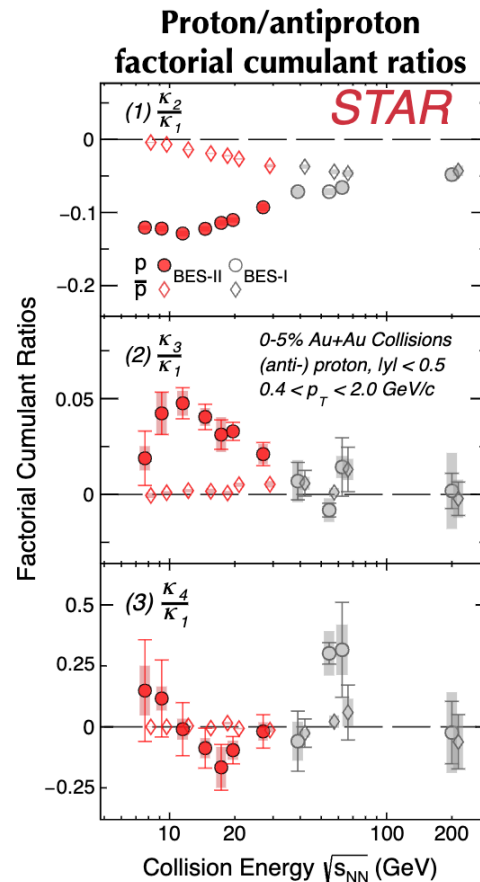
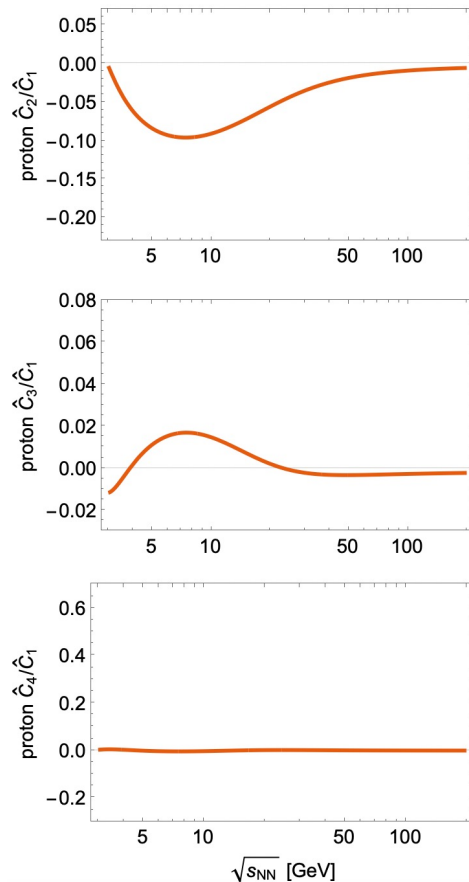


Shaded regions: negative values

Factorial cumulants and nuclear liquid-gas transition

Calculation in a van der Waals-like HRG model along the freeze-out curve*

VV, Gorenstein, Stoecker, EPJA 54, 16 (2018)



NB: The calculation is grand-canonical

*Poberezhnyuk et al., PRC 100, 054904 (2019)

- QCD equation of state
 - Well controlled at small baryon densities with lattice QCD where the transition is a chiral crossover
 - New extrapolation schemes extend the coverage to whole BES range assuming there is no CP
 - New developments point to the possible CP location at $T \sim 90\text{-}120$ MeV and $\mu_B \sim 500 - 650$ MeV
- Proton cumulants are uniquely sensitive to the CP but challenging to model dynamically
 - factorial cumulants are especially advantageous
- BES-II data
 - Consistent with non-critical physics at $\sqrt{s_{NN}} \geq 20$ GeV (as was BES-I data)
 - Shows (non-monotonic) structure in factorial cumulants
 - Positive \hat{C}_2 and negative \hat{C}_3 after subtracting non-critical baseline at $\sqrt{s_{NN}} < 10$ GeV
 - Improved understanding of non-critical effects, volume fluctuations, and nuclear interactions is crucial

Thanks for your attention!

Backup slides

Lower energies $\sqrt{s_{NN}} \leq 7.7$ GeV

- Intriguing hints from HADES@2.4 GeV and STAR-FXT@3GeV: huge excess of two-proton correlations!

[HADES Collaboration, Phys. Rev. C 102, 024914 (2020)]

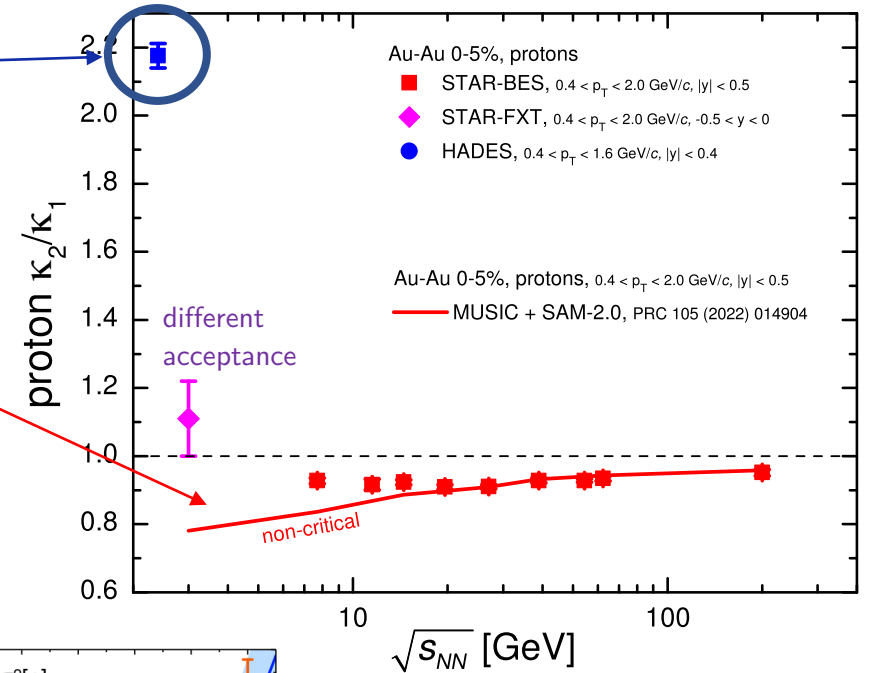
[STAR Collaboration, Phys. Rev. Lett. 128, 202303 (2022)]

- No change of trend in the non-critical reference
- Additional mechanisms:
 - Nuclear liquid-gas transition (the other QCD critical point)
 - Light nuclei formation/fragmentation
 - Stronger initial state, volume, and baryon stopping fluctuations

Talk by A. Bzdak, Wed 14:20; Poster by A. Rustamov

- Difference in acceptance ($-0.5 < y < 0$ vs $|y| < 0.5$)

- Improved modeling of lower energies required



VV, Phys. Rev. C 106, 064906 (2022)

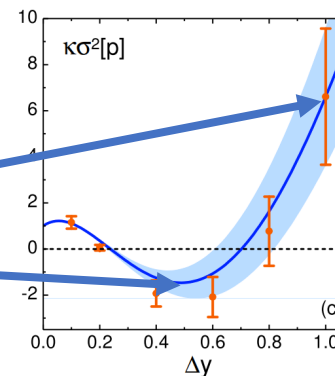
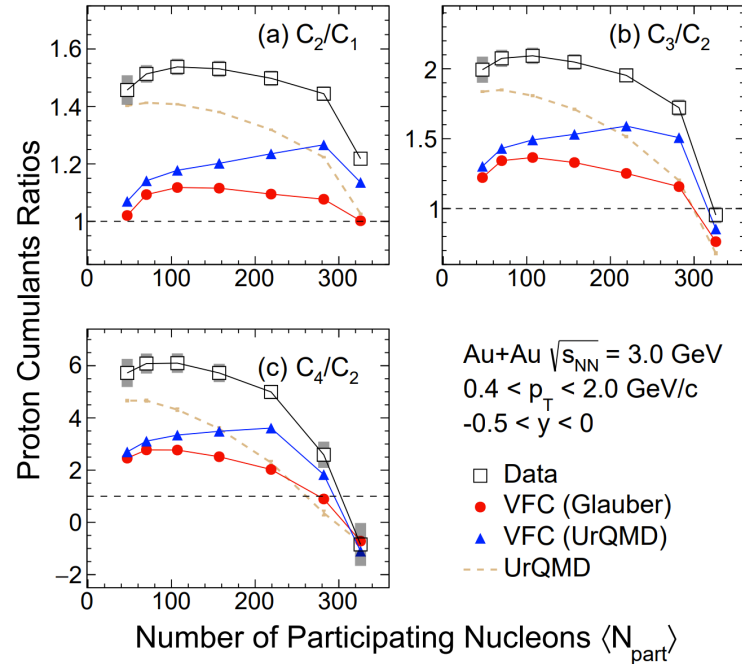


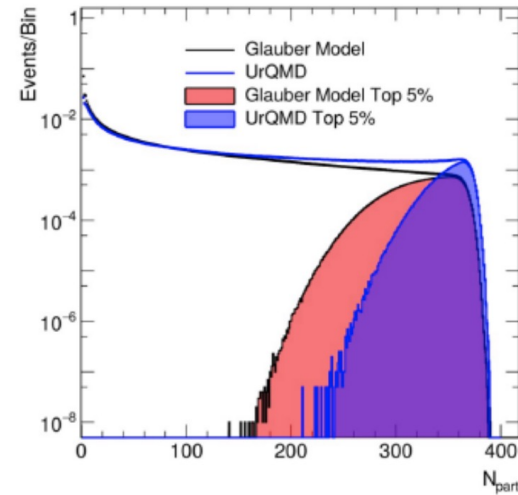
Figure from O. Savchuk et al., PLB 835, 137540 (2022)

We may want to understand κ_2 first

Lower energies $\sqrt{s_{NN}} \leq 7.7$ GeV



STAR-FXT



HADES

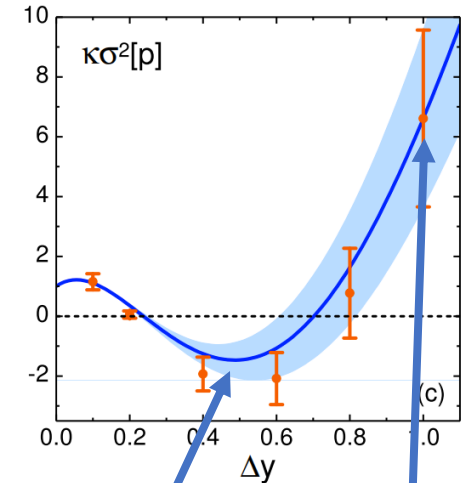


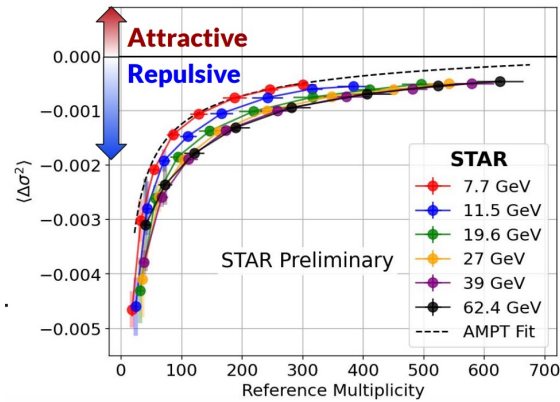
Figure from O. Savchuk et al., PLB 835, 137540 (2022)

- STAR Collaboration, Phys. Rev. Lett. 128 (2022) 202303
- Volume fluctuations/centrality selection appear to play an important role
 - UrQMD is useful for understanding basic systematics associated with it
- Indications for enhanced scaled variance, $\kappa_2/\kappa_1 > 1$
- κ_4/κ_2 negative and described by UrQMD (purely hadronic?), note $-0.5 < y < 0$ instead of $|y| < 0.5$

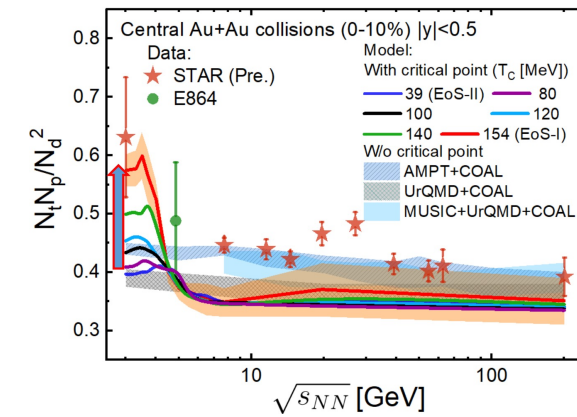
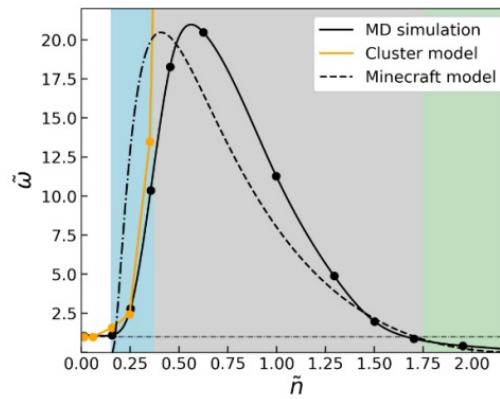
Proper understanding of $\kappa_2/\kappa_1 > 1$ in both HADES and STAR-FXT is missing

Other observables

- Azimuthal correlations of protons
 - points to repulsion at RHIC-BES



- Light nuclei
 - Spinodal/critical point enhancement of density fluctuations and light nuclei production

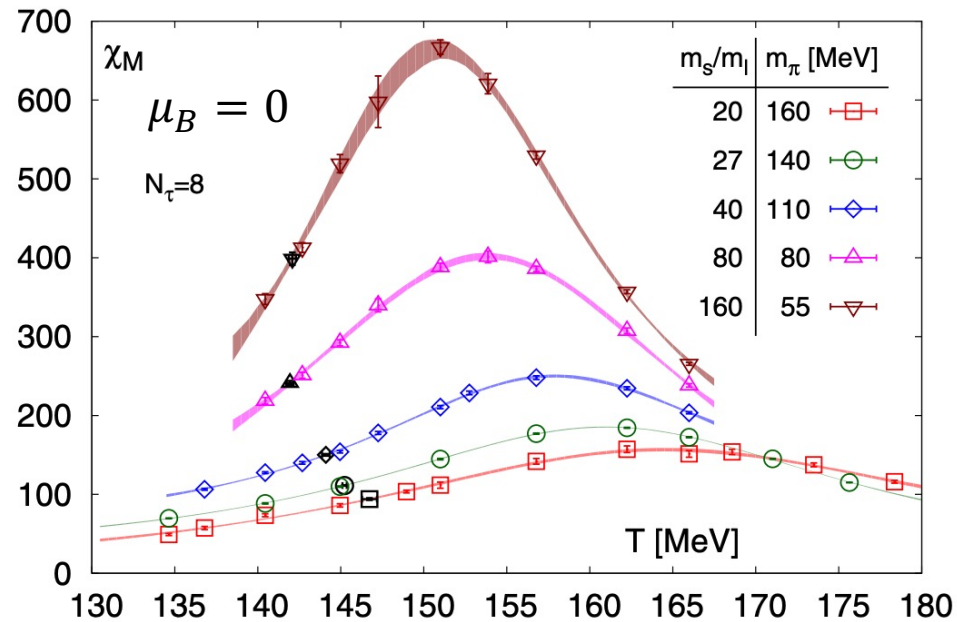


- Proton intermittency
 - No structure indicating power-law seen by NA61/SHINE
- Directed flow, speed of sound

Consistency in understanding all the observables is required

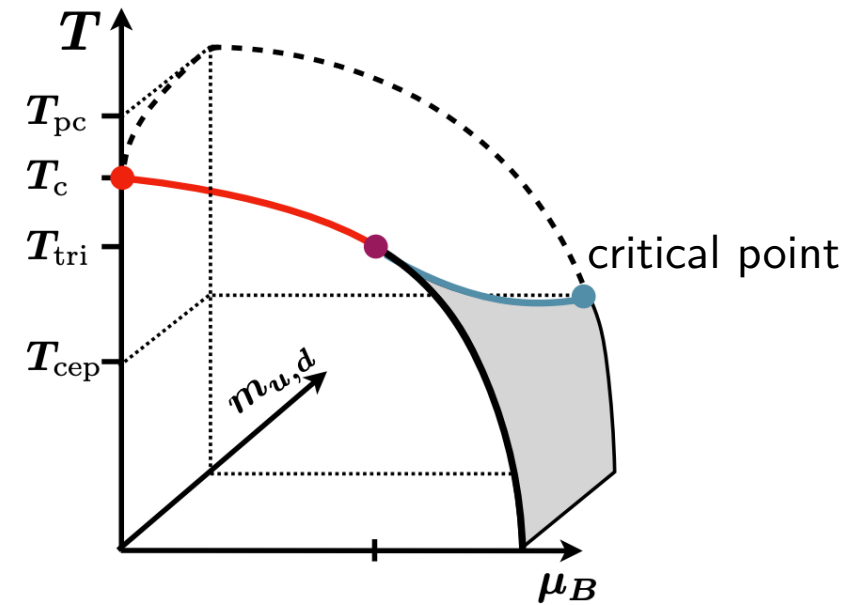
Hunting for the QCD critical point with lattice QCD

Remnants of $O(4)$ chiral criticality at $\mu_B = 0$ quite well established with lattice QCD



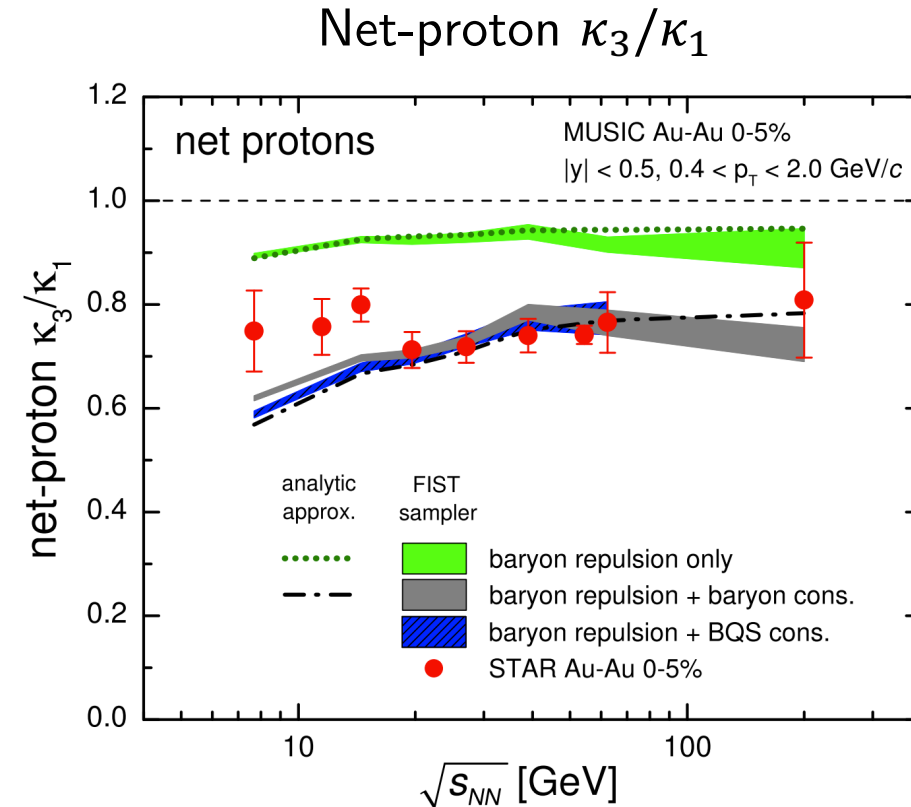
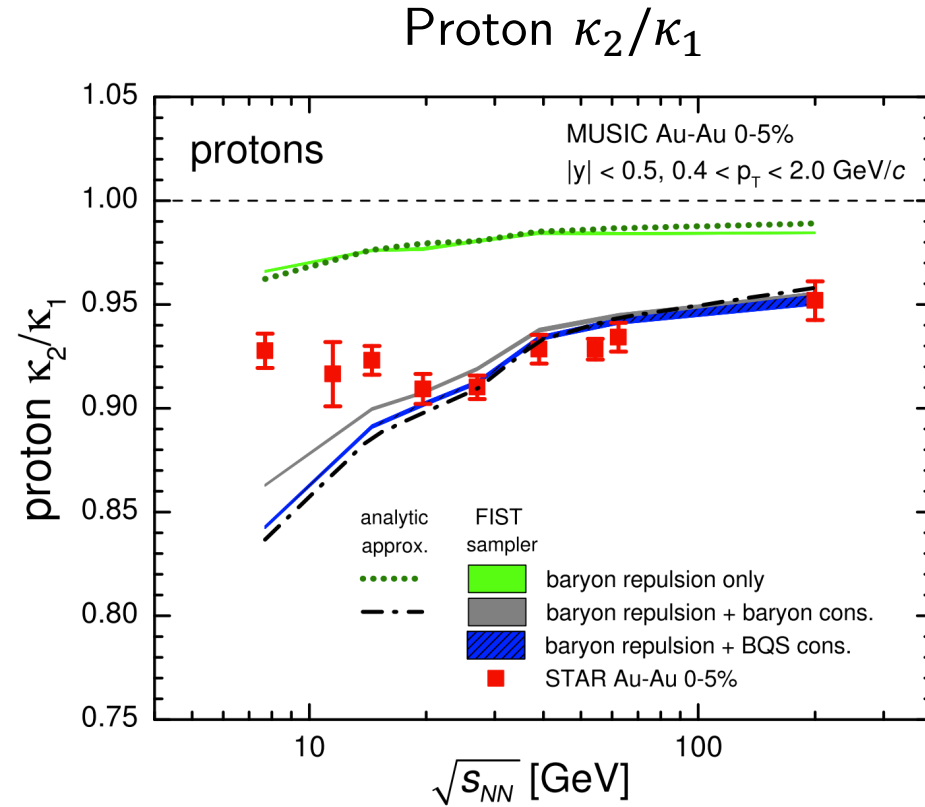
HotQCD Collaboration, PRL 123, 062002 (2019)

Physical quark masses away the chiral limit: Expect a $Z(2)$ critical point at finite μ_B

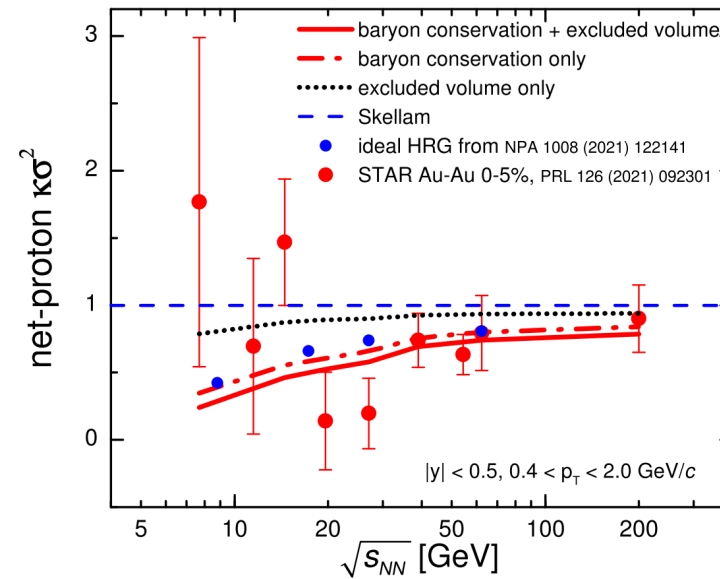
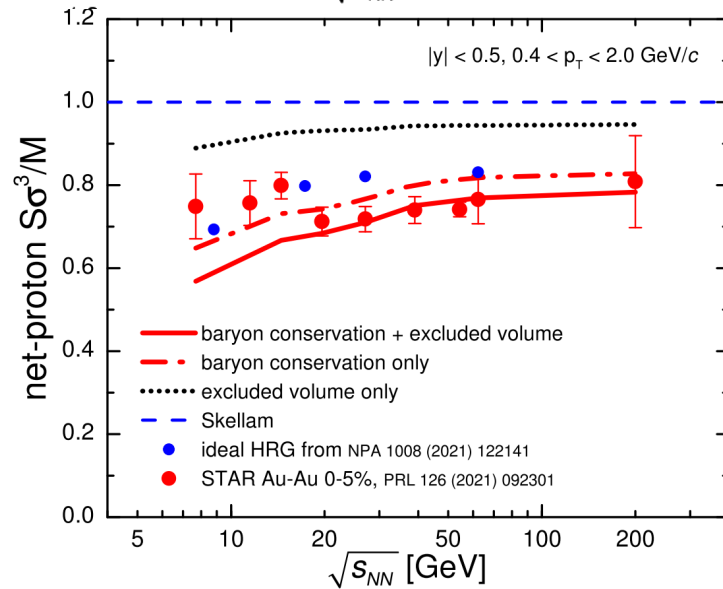
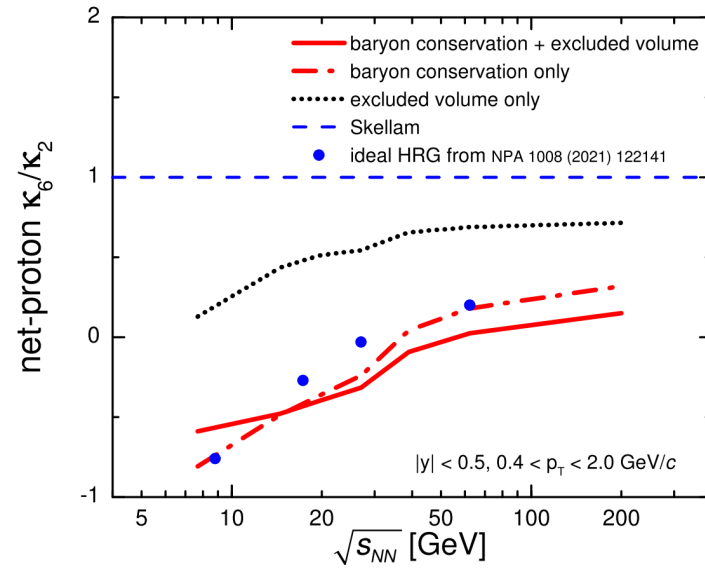
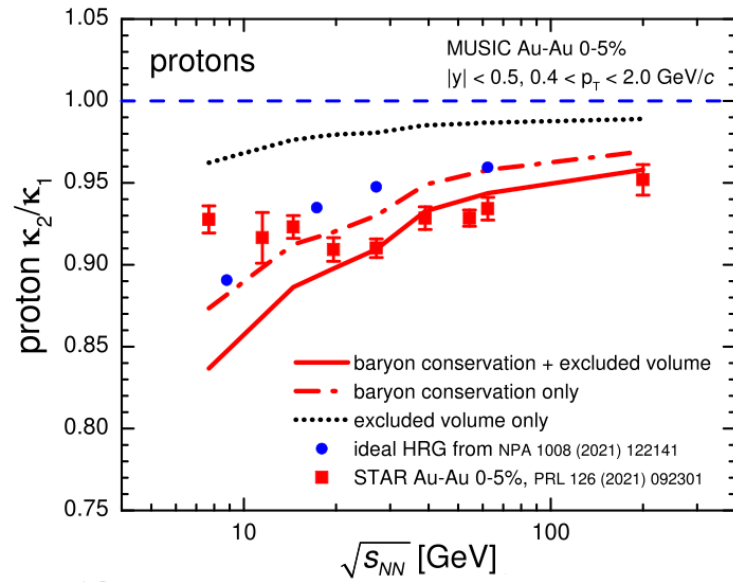


C. Schmidt

Non-critical cumulants: Analytic vs Monte Carlo

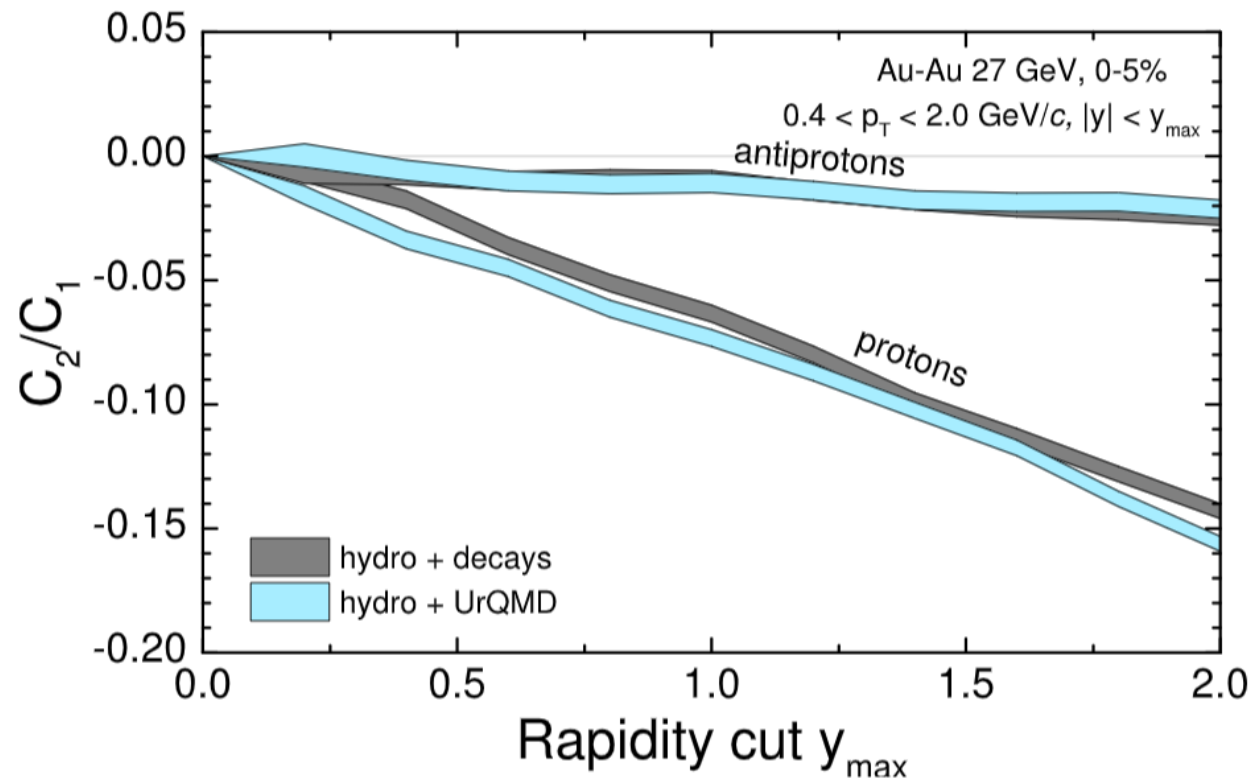


Non-critical cumulants

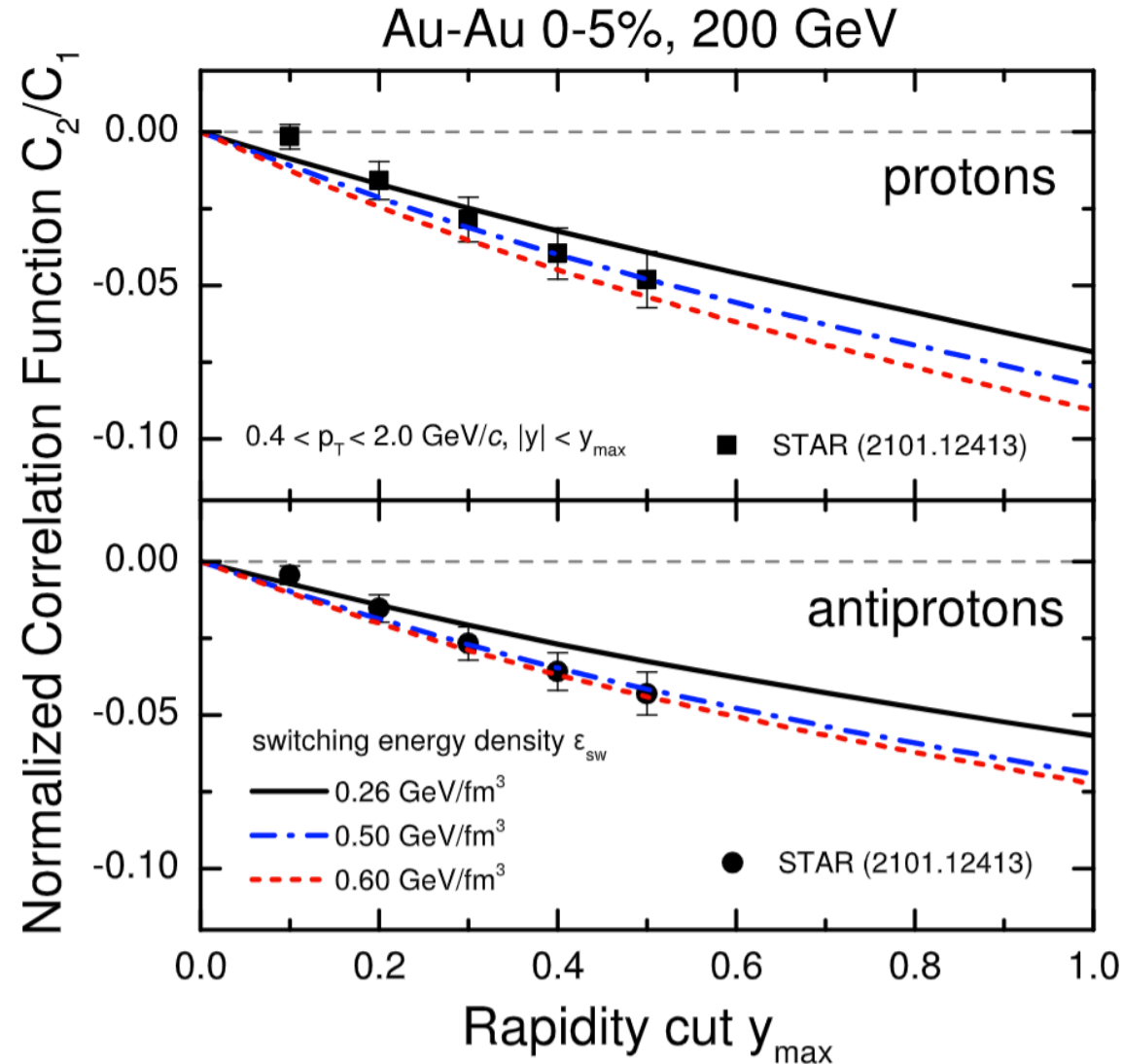


Effect of the hadronic phase

Sample ideal HRG model at particlization with exact conservation of baryon number using Thermal-FIST and run through hadronic afterburner UrQMD

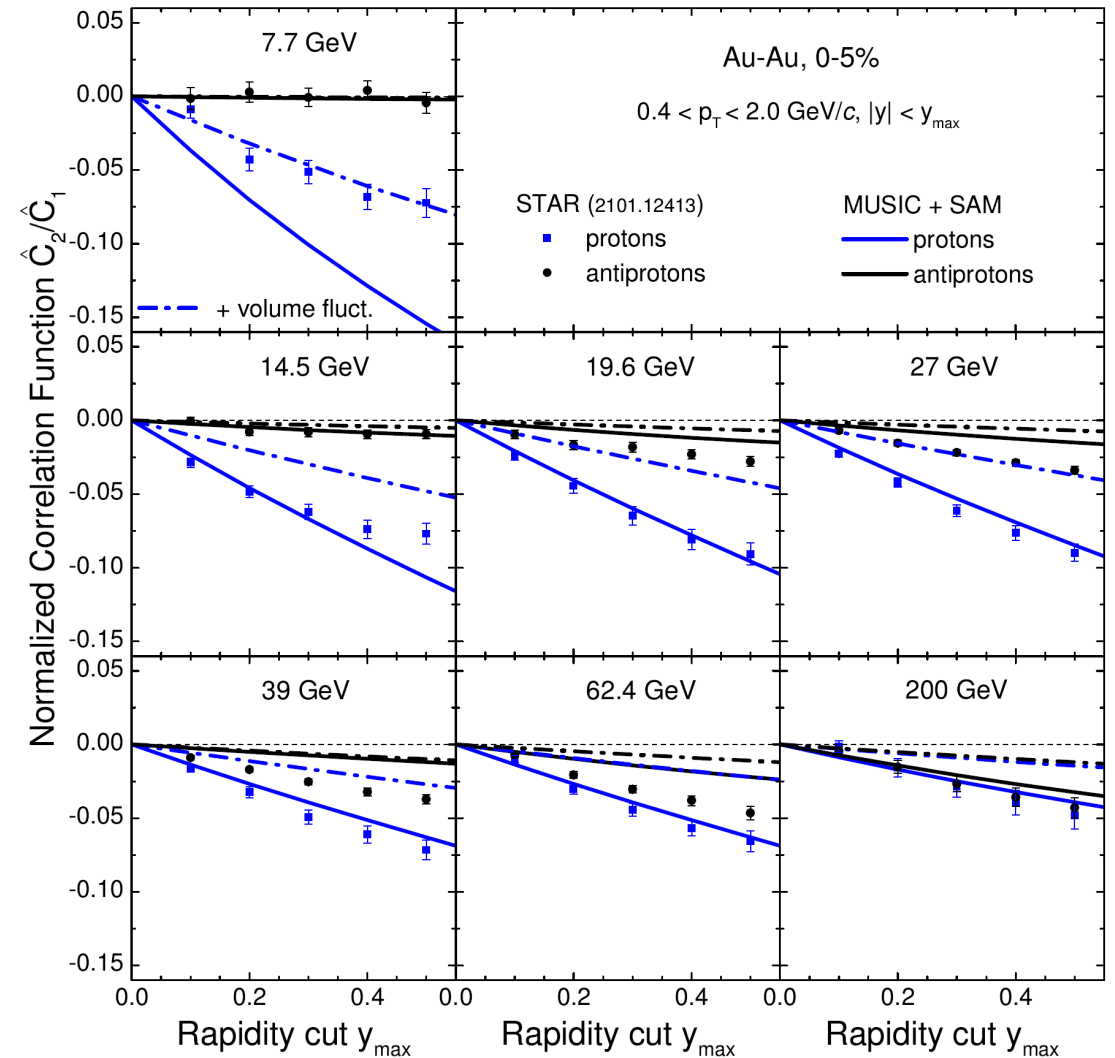


Dependence on the switching energy density



Acceptance dependence of two-particle correlations

- Changing y_{max} slope at $\sqrt{s_{NN}} \leq 14.5$ GeV?
- Volume fluctuations? [Skokov, Friman, Redlich, PRC '13]
 - $C_2/C_1 \neq C_1 * \Delta v^2$
 - Can improve low energies but spoil high energies?
- **Attractive interactions?**
 - Could work if baryon repulsion turns into attraction in the high- μ_B regime
 - **Critical point?**



Net baryon fluctuations at LHC

- Global baryon conservation distorts the cumulant ratios already for one unit of rapidity acceptance

e.g. $\frac{\chi_4^B}{\chi_2^B} \Big|_{T=160\text{MeV}}^{\text{GCE}} \stackrel{\text{"lattice QCD"}}{\simeq 0.67} \neq \frac{\chi_4^B}{\chi_2^B} \Big|_{\Delta Y_{\text{acc}}=1}^{\text{HIC}} \stackrel{\text{experiment}}{\simeq 0.56}$

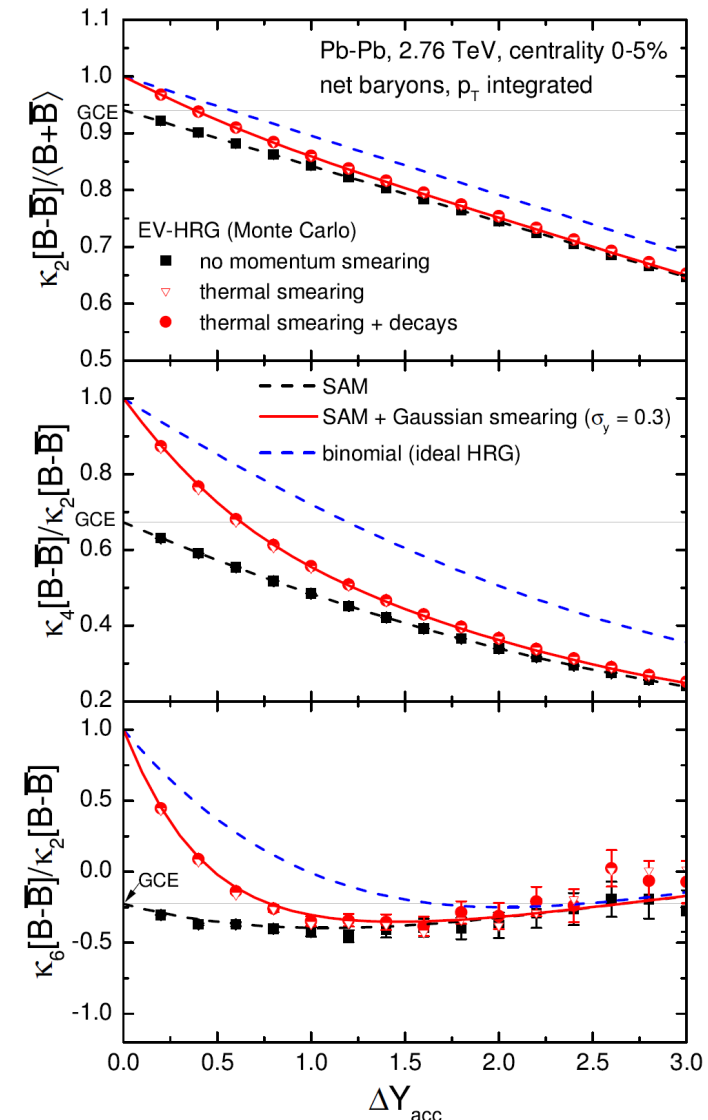
- Neglecting thermal smearing, effects of global conservation can be described analytically via SAM

$$\frac{\kappa_2}{\langle B + \bar{B} \rangle} = (1 - \alpha) \frac{\kappa_2^{\text{gce}}}{\langle B + \bar{B} \rangle}, \quad \alpha = \frac{\Delta Y_{\text{acc}}}{9.6}, \quad \beta \equiv 1 - \alpha$$

$$\frac{\kappa_4}{\kappa_2} = (1 - 3\alpha\beta) \frac{\chi_4^B}{\chi_2^B},$$

$$\frac{\kappa_6}{\kappa_2} = [1 - 5\alpha\beta(1 - \alpha\beta)] \frac{\chi_6^B}{\chi_2^B} - 10\alpha(1 - 2\alpha)^2\beta \left(\frac{\chi_4^B}{\chi_2^B} \right)^2$$

- Effect of resonance decays is negligible



Net baryon vs net proton



- Thermal smearing distorts the signal at $\Delta Y_{accept} \leq 1$. Net baryons converge to model-independent SAM result at larger ΔY_{accept}
- net baryon \neq net proton, e.g.

$$\left. \frac{\chi_4^B}{\chi_2^B} \right|_{\Delta Y_{acc}=1}^{\text{HIC}} \simeq 0.56 \neq \left. \frac{\chi_4^p}{\chi_2^p} \right|_{\Delta Y_{acc}=1}^{\text{HIC}} \simeq 0.83$$

- Baryon cumulants can be reconstructed from proton cumulants via binomial (un)folding based on isospin randomization [Kitazawa, Asakawa, Phys. Rev. C 85 (2012) 021901]
 - Requires the use of joint factorial moments, only experiment can do it model-independently



unfolding \longrightarrow

