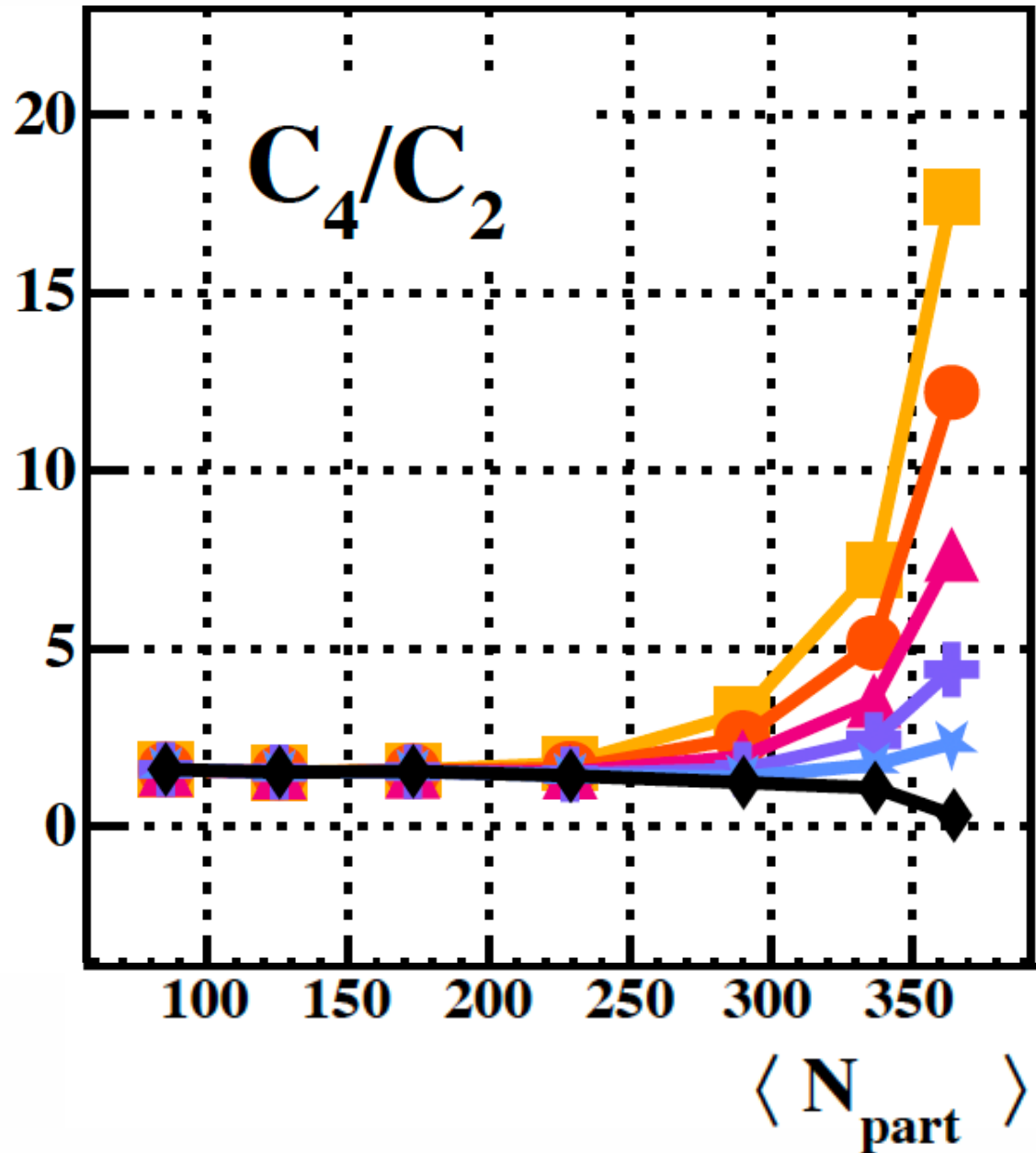


C_4/C_2



How Not to Measure a False Critical Point

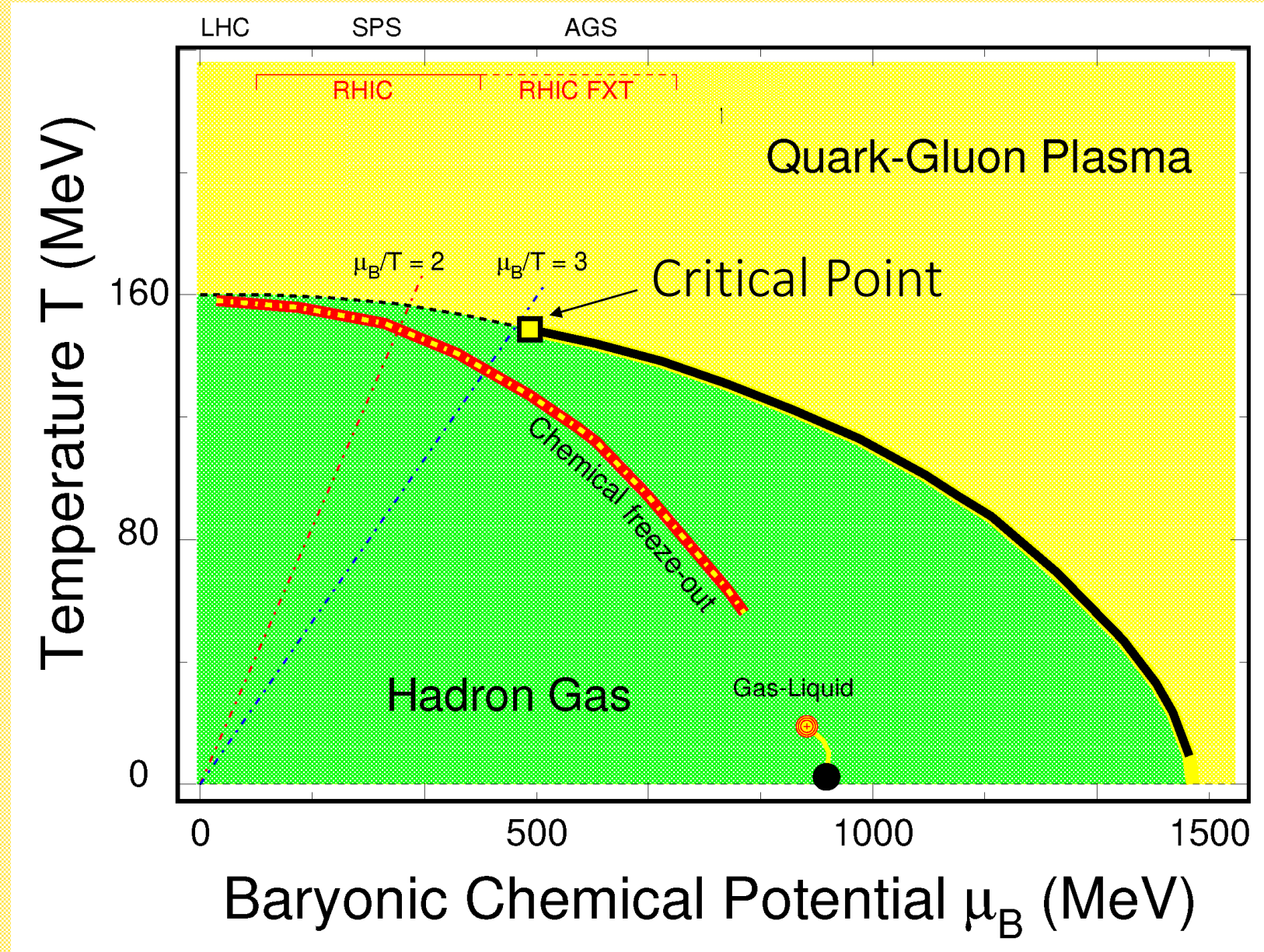
Zachary Sweger
University of California, Davis



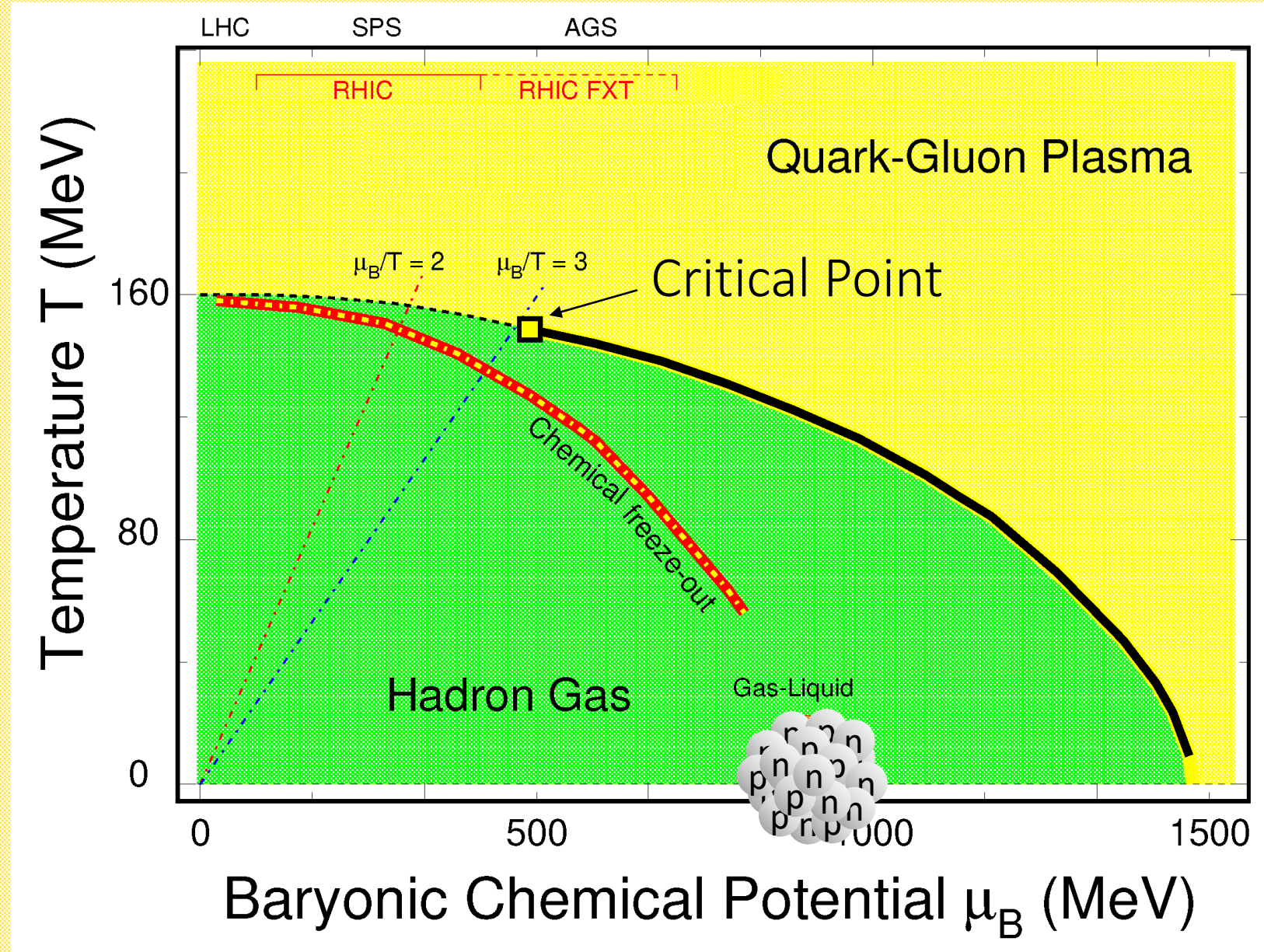
Overview

- QCD Critical Fluctuations
- Rare and spontaneous detector failure
- Toy model with UrQMD
- How not to measure a false signal

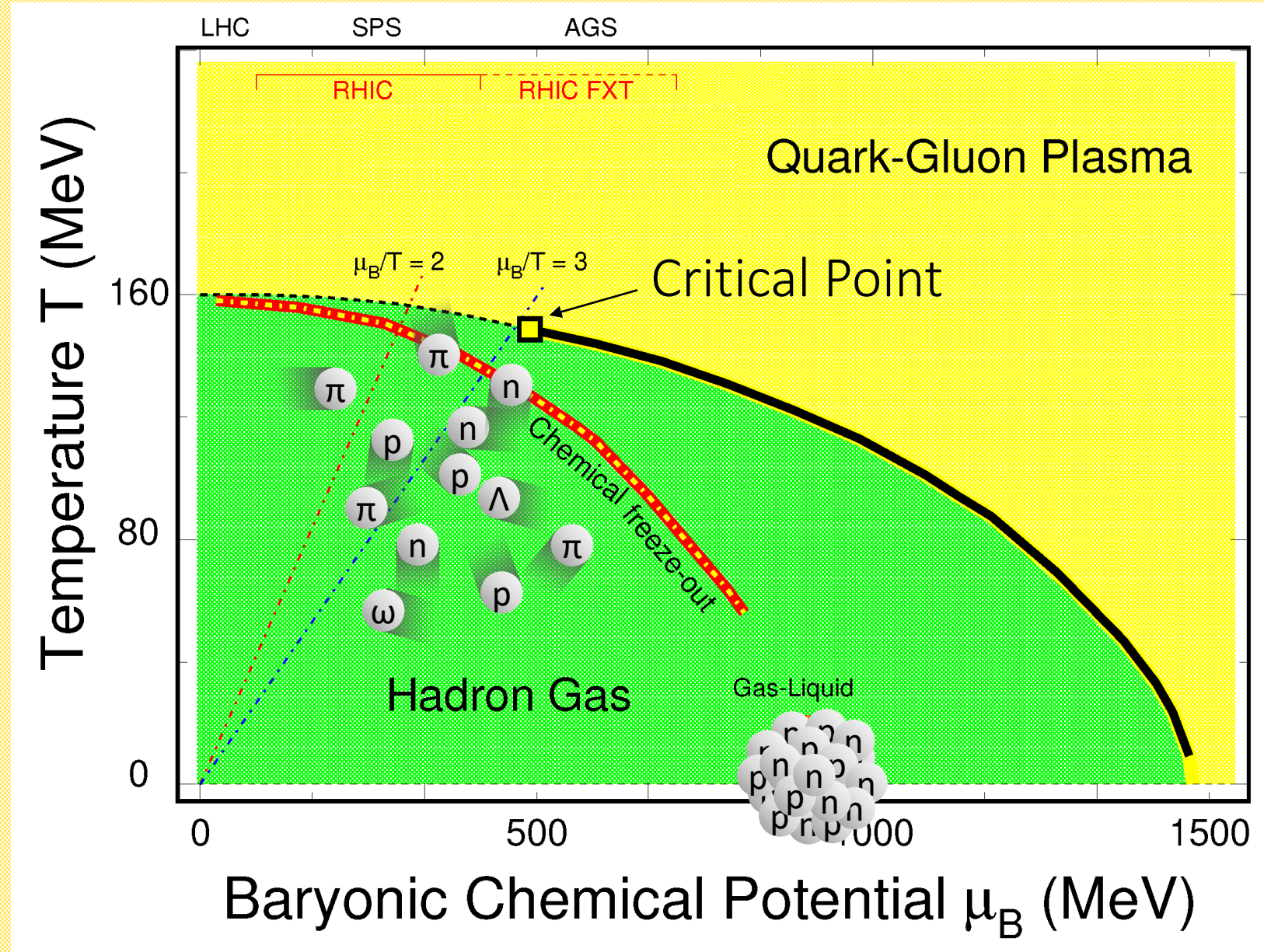
Phases of QCD Matter



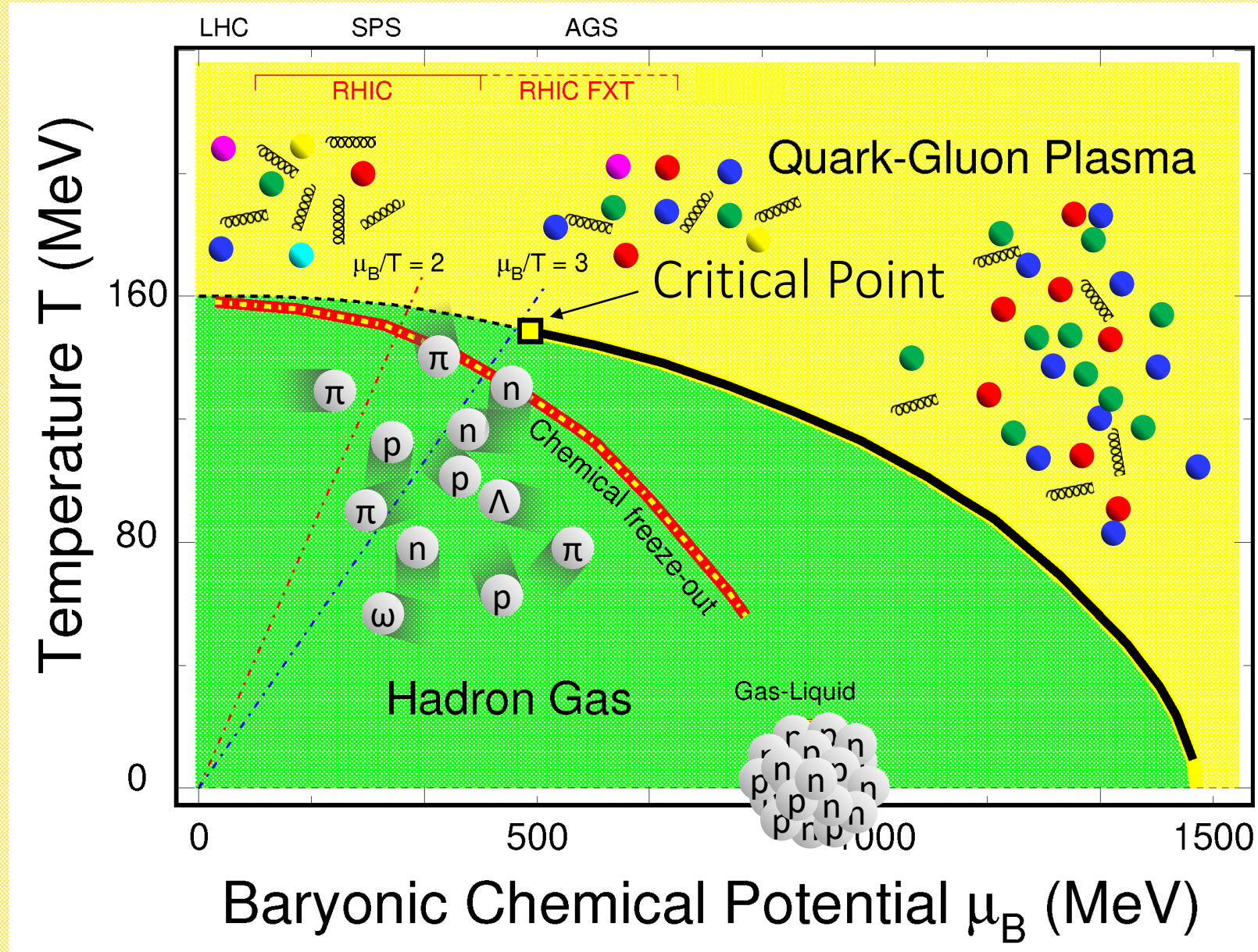
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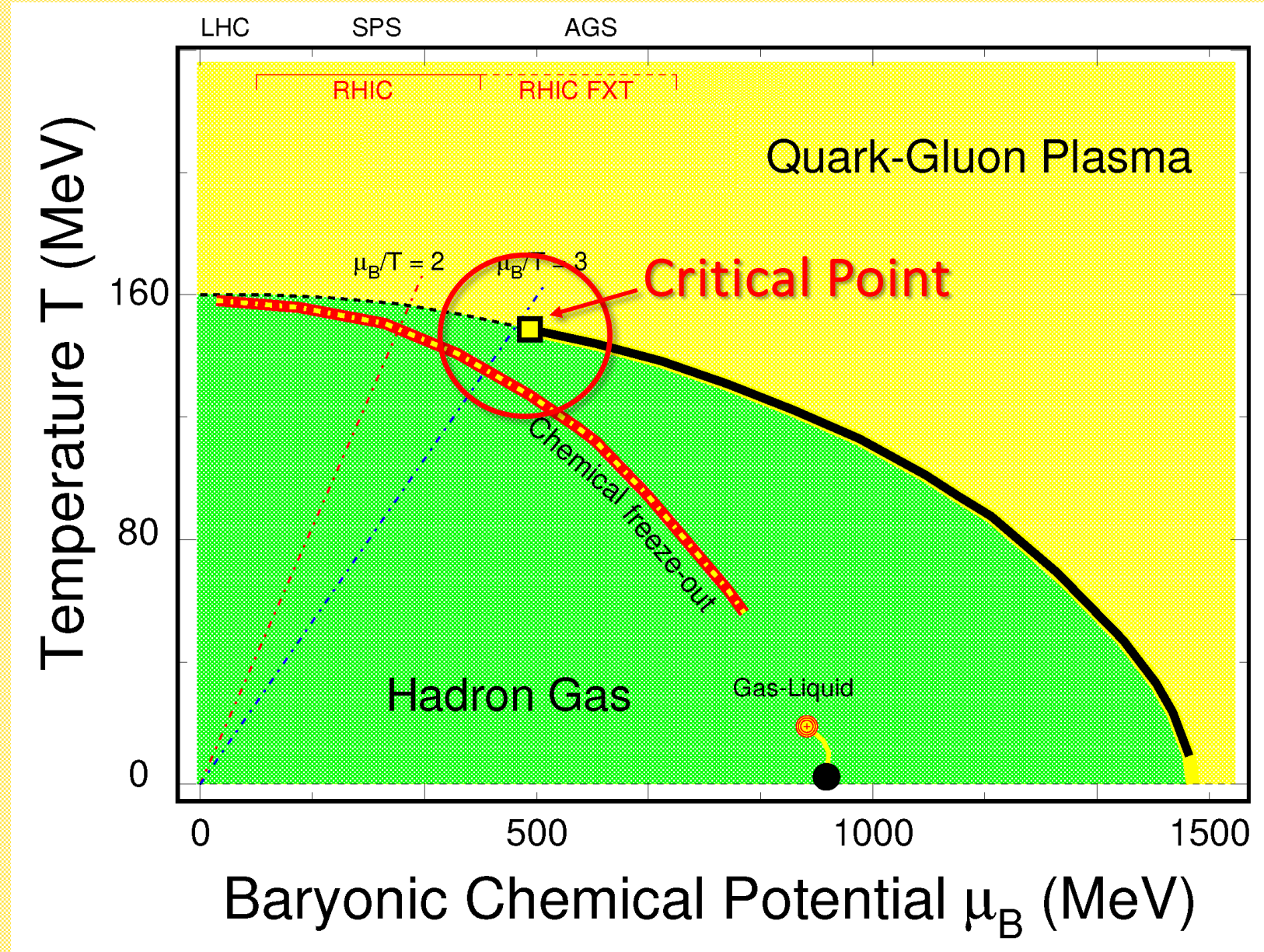
Phases of QCD Matter



Phases of QCD Matter

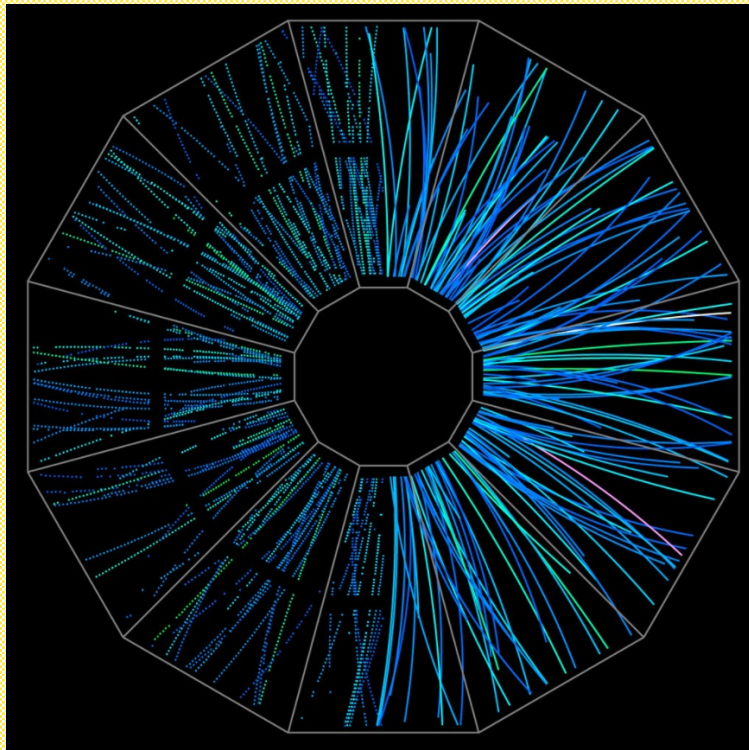


Phases of QCD Matter



Non-Gaussian fluctuations in baryon number

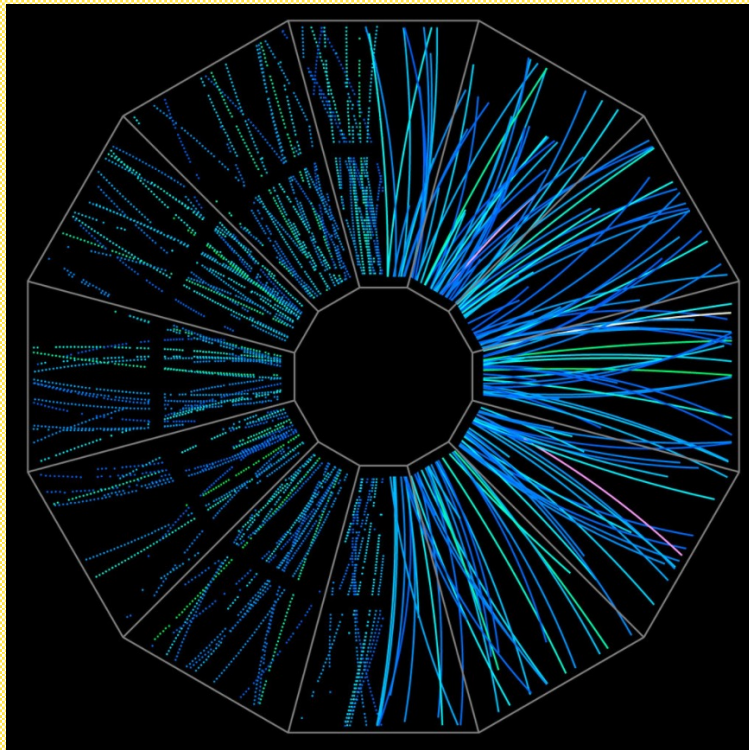
- Distribution of net-baryon number is expected to fluctuate near a critical point
- We measure events in which the nuclei collide head-on



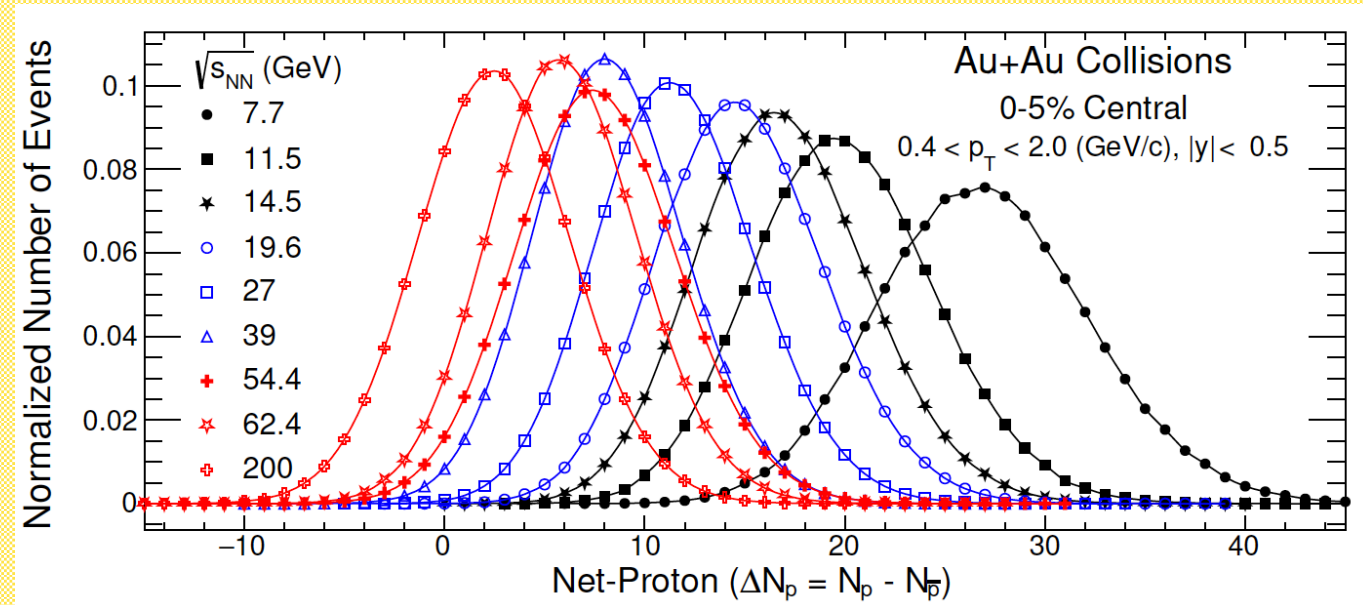
<https://www.bnl.gov/newsroom/news.php?a=214492>

Non-Gaussian fluctuations in baryon number

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Net-proton distributions at several energies as measured by STAR.
arXiv:2001.02852v2 [nucl-ex] 31 Jul 2020

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- We measure events in which the nuclei collide head-on
- Count the number of protons (N_p), antiprotons (N_{pbar}), net-protons ($N_p - N_{pbar}$)
- Measure the mean, variance, skewness, kurtosis...

cumulants

$$C_1 = \langle N \rangle \equiv \mu \text{ [mean]}$$

$$C_2 = \langle (N - \mu)^2 \rangle \equiv \sigma^2 \text{ [variance]}$$

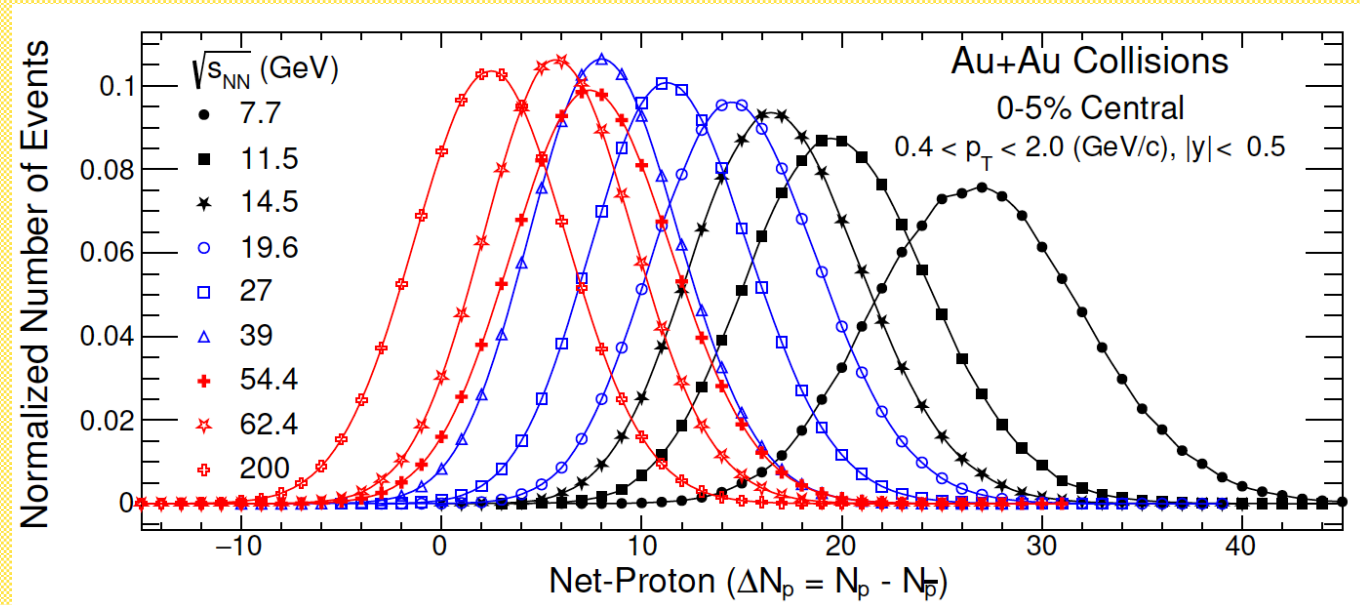
$$C_3 = \langle (N - \mu)^3 \rangle$$

$$C_4 = \langle (N - \mu)^4 \rangle - 3\langle (N - \mu)^2 \rangle^2$$

standardized moments

$$S\sigma = C_3/C_2 \text{ [skewness]}$$

$$\kappa\sigma^2 = C_4/C_2 \text{ [excess kurtosis]}$$

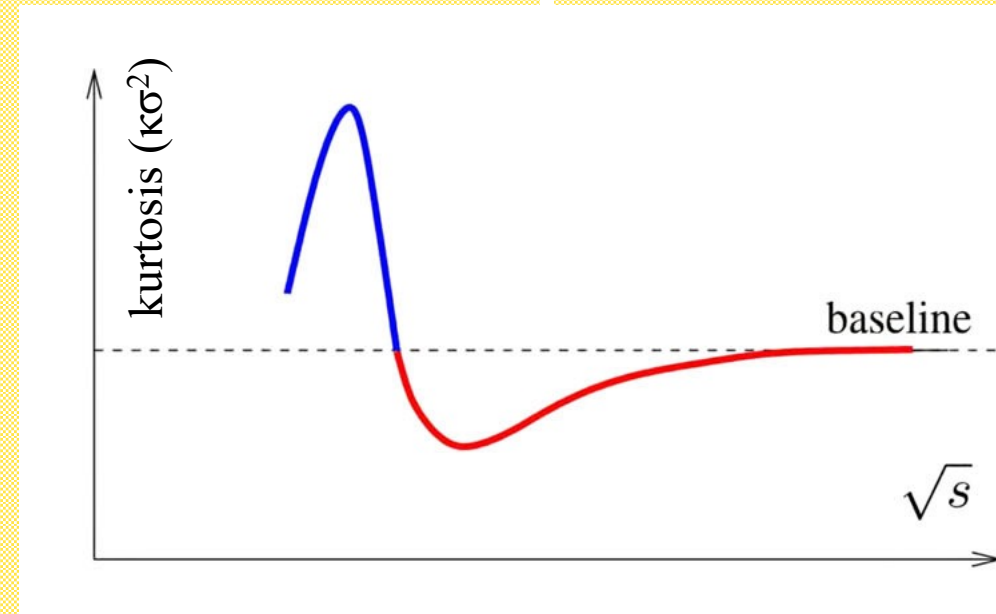


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Proximity to Critical Point \rightarrow Alternately Enhanced and Suppressed Kurtosis

Predicted fluctuation in kurtosis near critical point

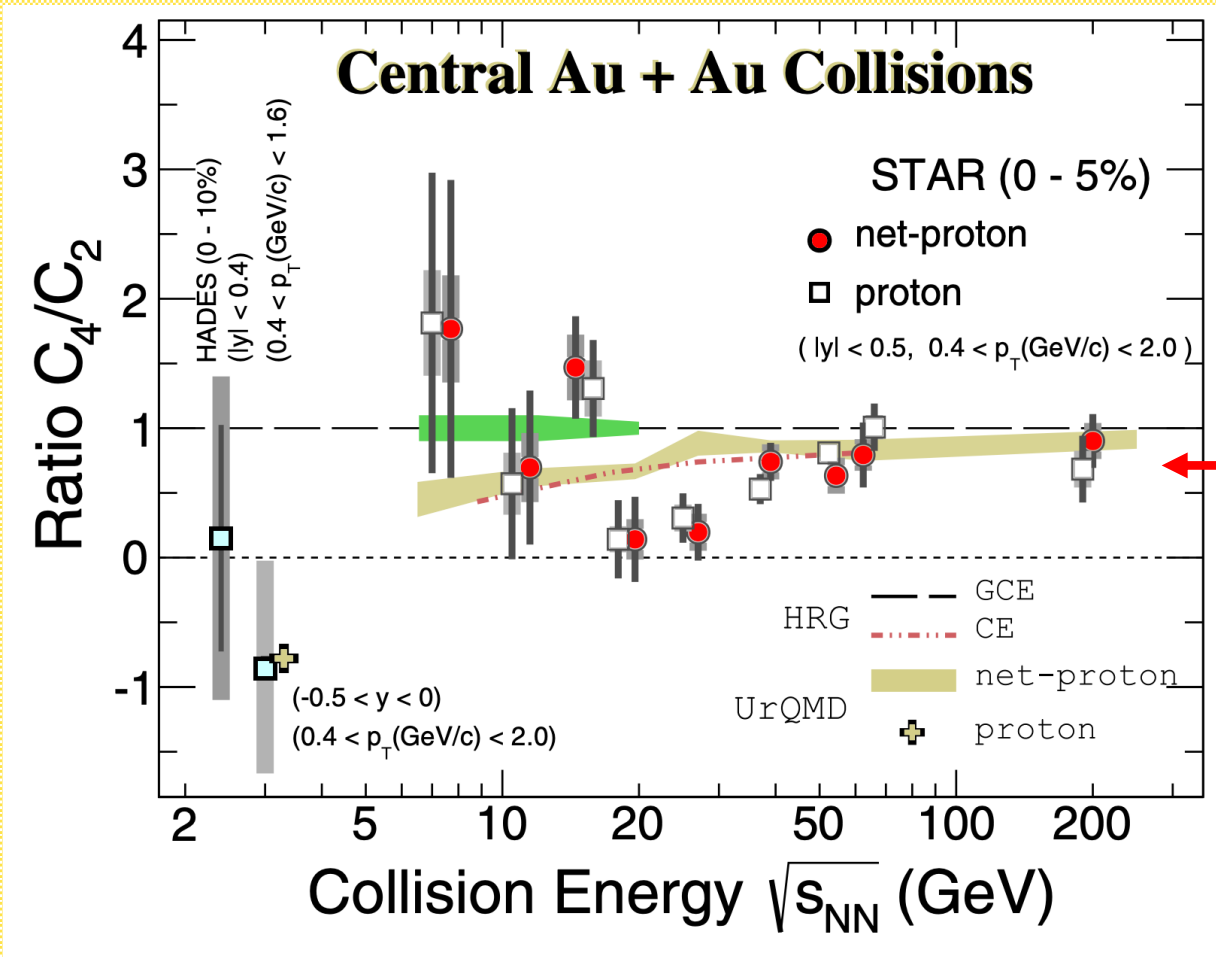
Fluctuation in kurtosis near critical point



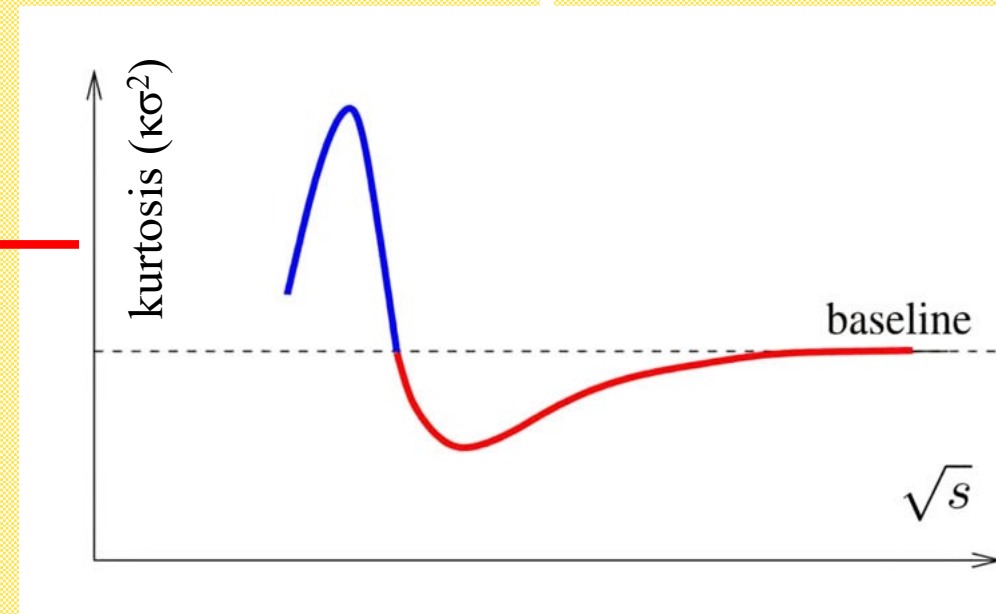
M. Stephanov. J. Physics G.: Nucl. Part. Phys. **38** (2011) 124147

Comparison to Published Beam Energy Scan Results

Published Results



Fluctuation in kurtosis near critical point



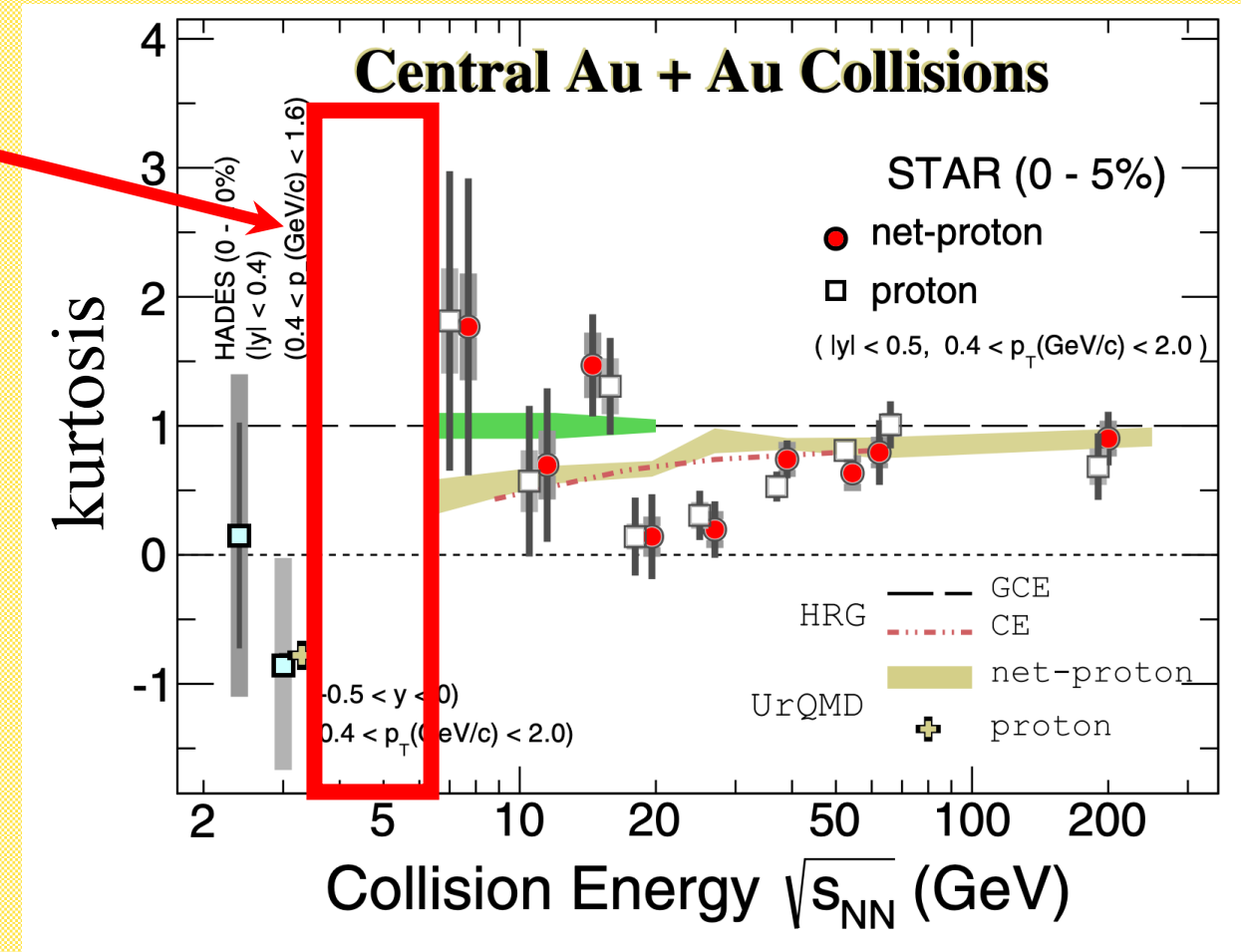
STAR, *Phys. Rev. Lett.* **128**, 202303 (2022); *Phys.Rev.C* **107**.024908 (2023).
Phys. Rev. Lett. **126**, 092301 (2021); *Phys. Rev. C* **104**, 024902 (2021)

M. Stephanov, *J. Physics G.: Nucl. Part. Phys.* **38** (2011) 124147

Remaining Results

- The remaining fixed-target datasets will cover much of the gap

Published Results

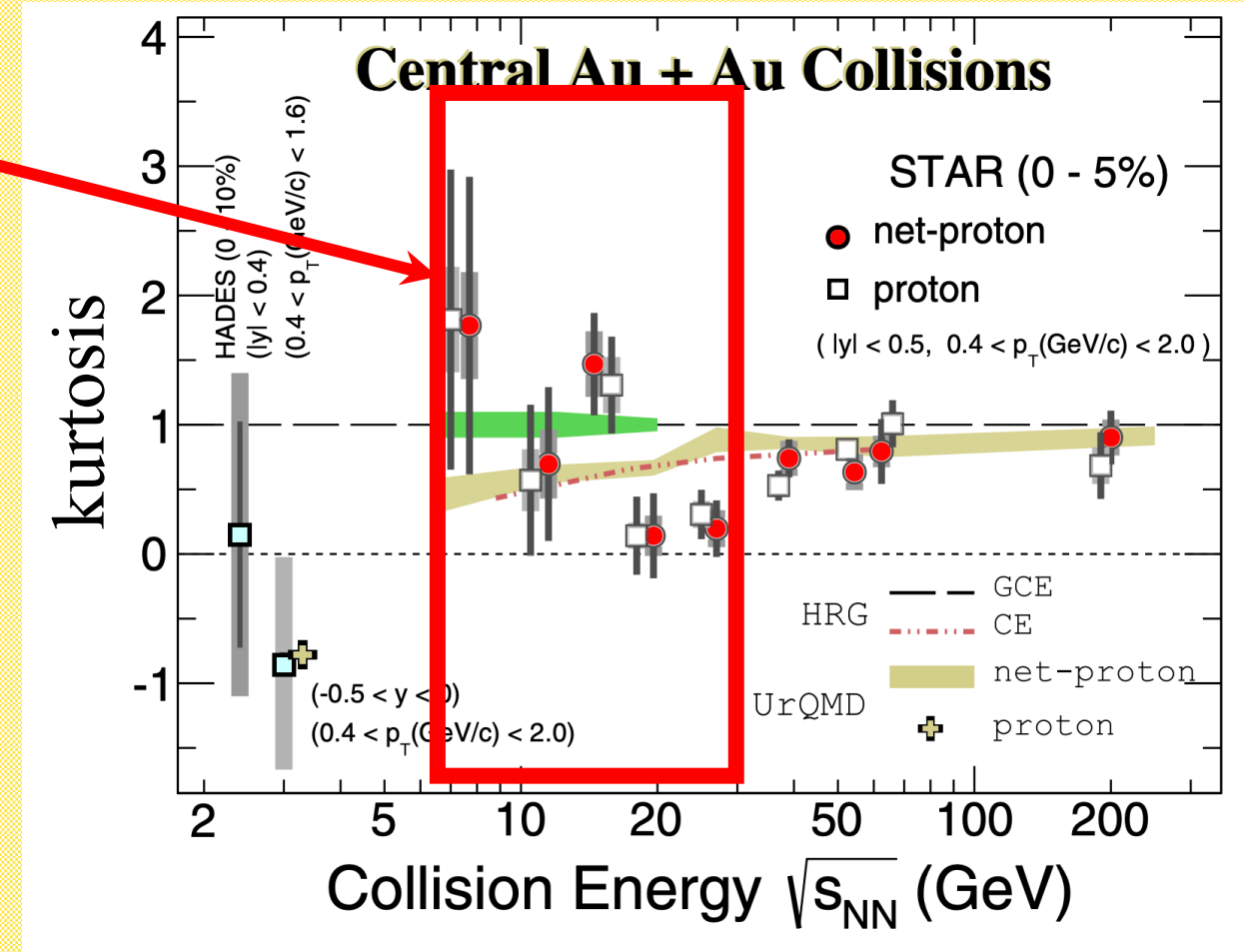


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Phys. Rev. Lett. 126, 092301 (2021); *Phys. Rev. C* 104, 024902 (2021)

Remaining Results

- The remaining fixed-target datasets will cover much of the gap
- High-statistics data re-collected below 27 GeV (BES-II)
 - High precision new results shown in Bappaditya Mondal's poster!
 - Waiting for publication

Published Results



STAR, *Phys. Rev. Lett.* 128, 202303 (2022); *Phys.Rev.C* 107.024908 (2023).
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Proton Number and Multiplicity Correlations

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Proton Number and Multiplicity Correlations

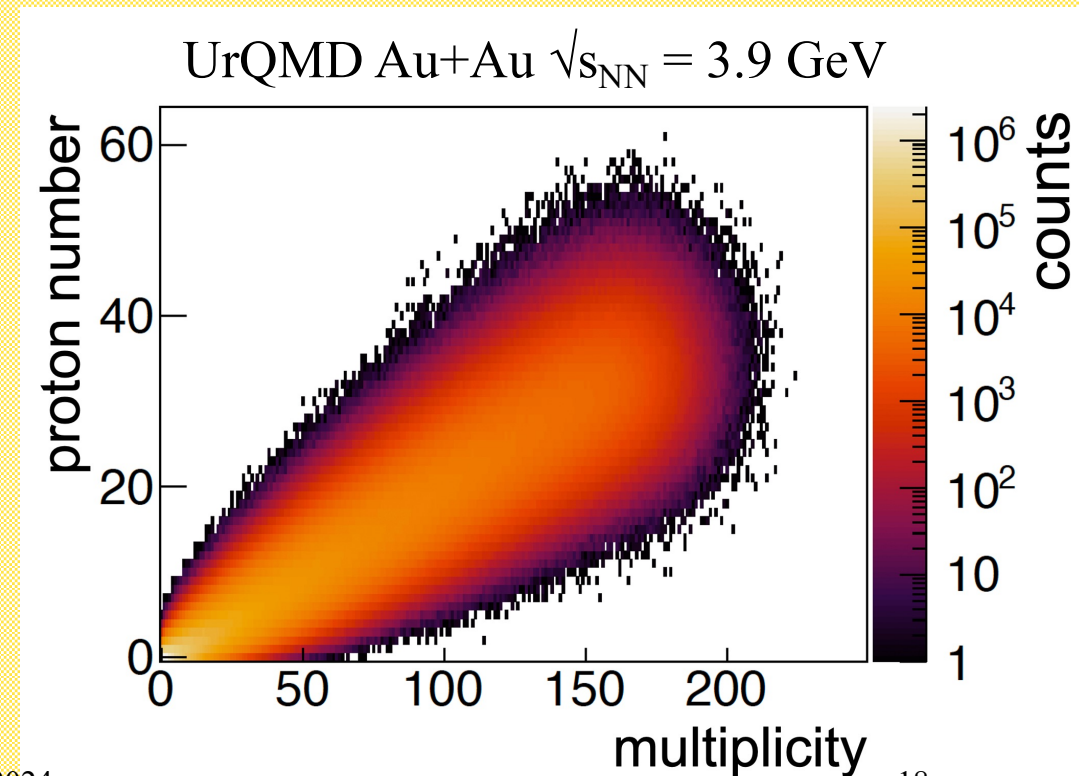
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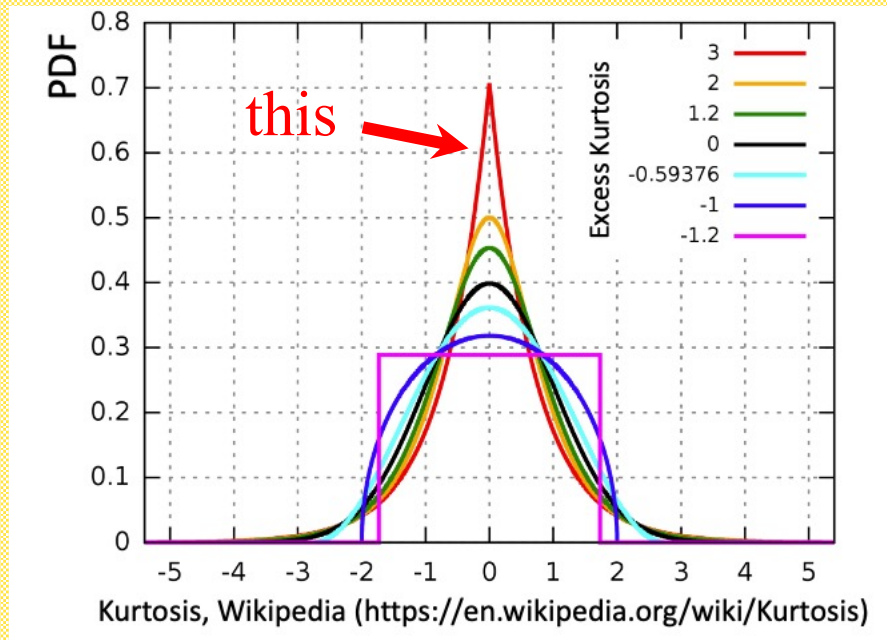
The takeaway: the correlation between proton number and multiplicity is our *entire* measurement



What does large kurtosis mean?

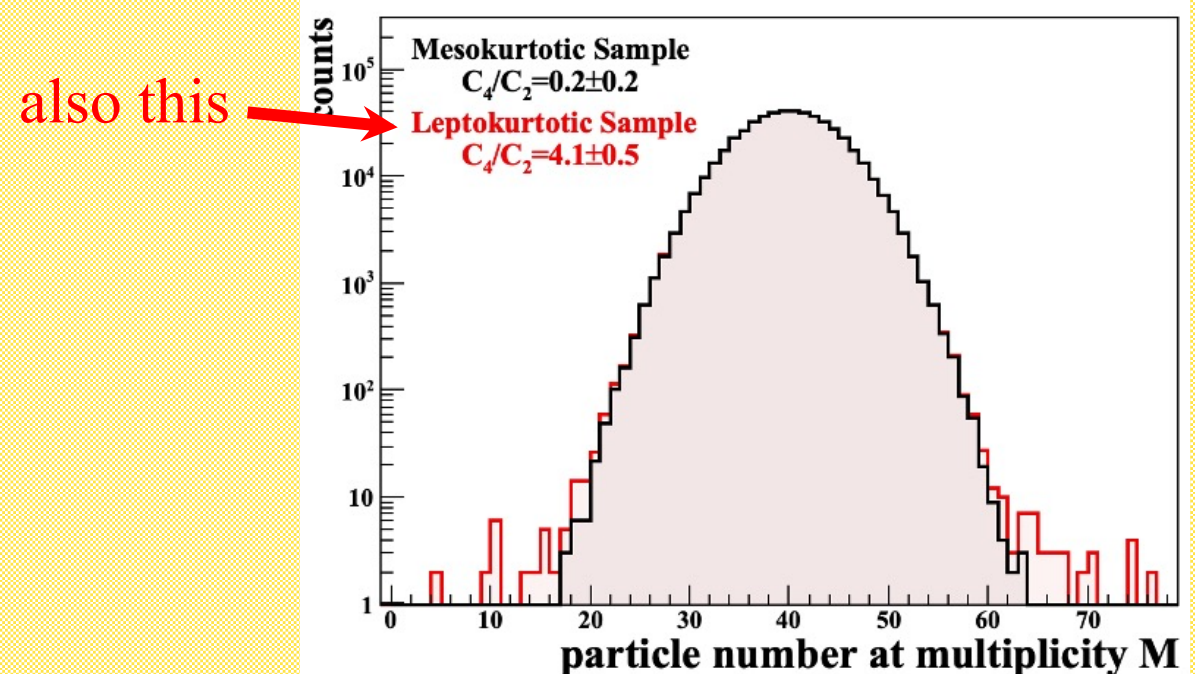
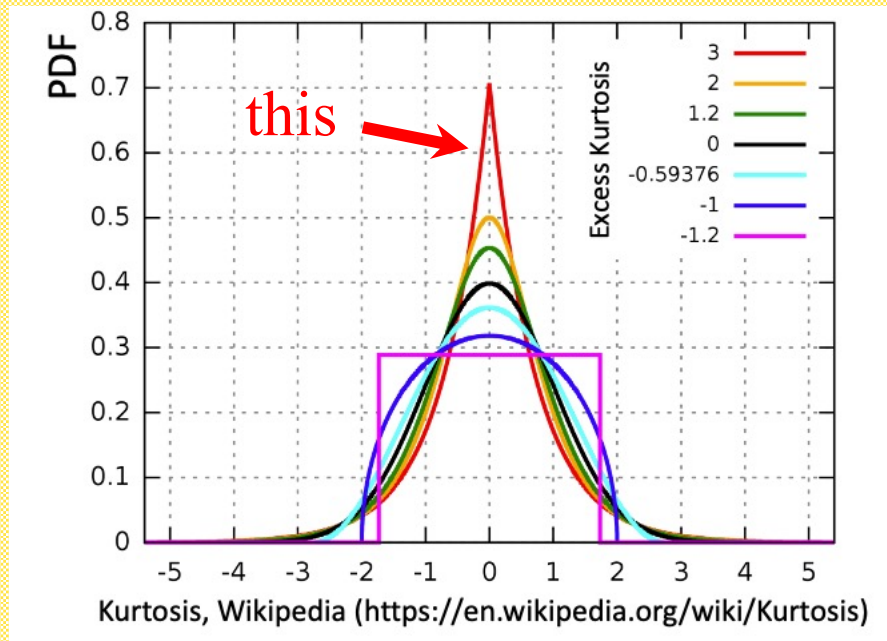
What does large kurtosis mean?

- Large kurtosis means large tails
- We use this figure which shows a highly-tailed distribution with a large kurtosis



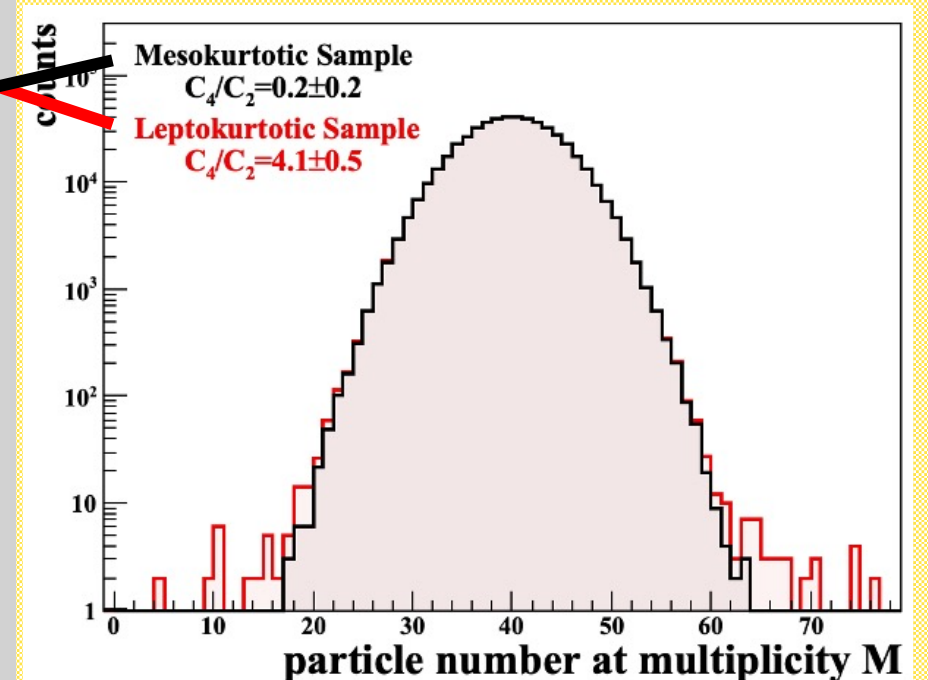
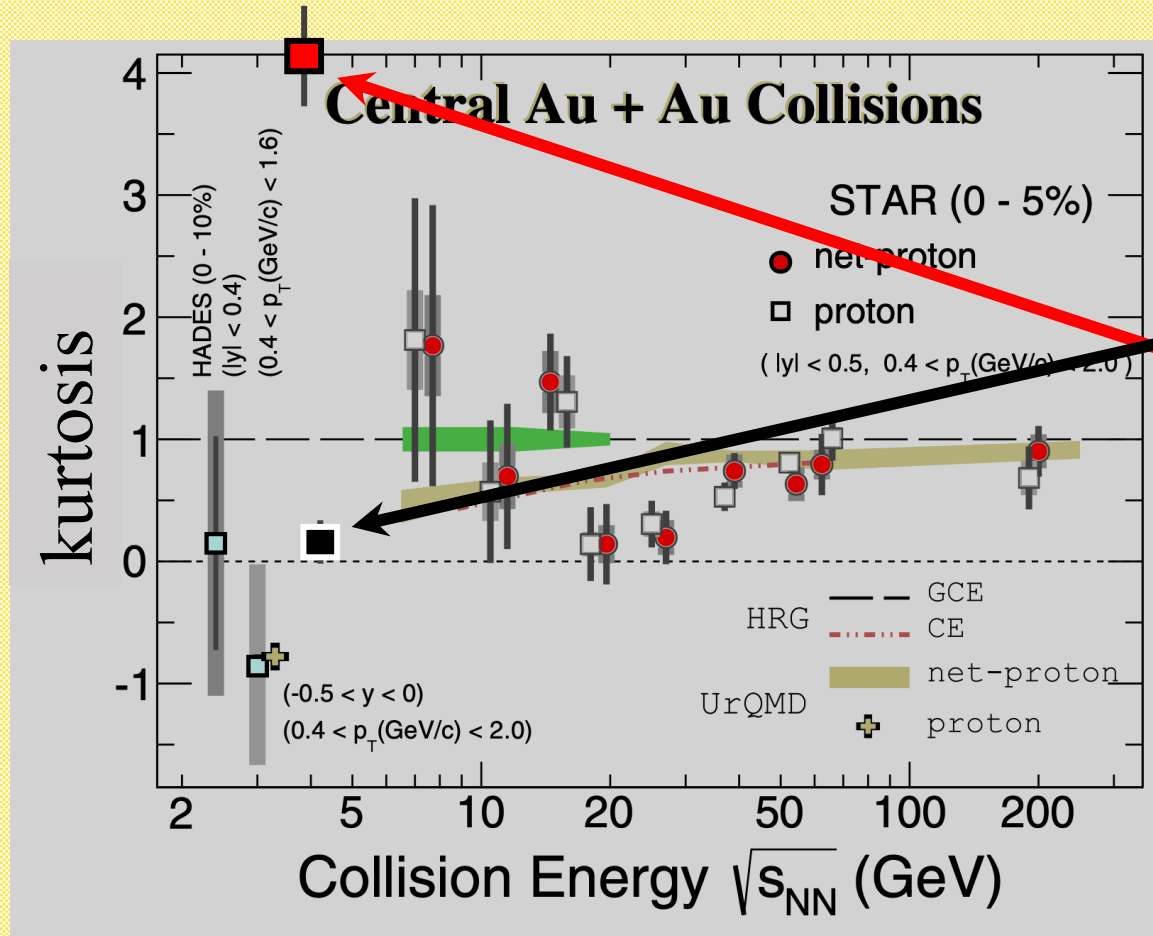
What does large kurtosis mean?

- Large kurtosis means large tails
- We use this figure which shows a highly-tailed distribution with a large kurtosis
- Long tails *many* decades down cause large kurtosis too!
- The black is a Gaussian. The red is the same Gaussian, but one in 10000 samples is from a Gaussian with twice the width



What does large kurtosis mean?

- This gives an idea of the scale of fluctuations

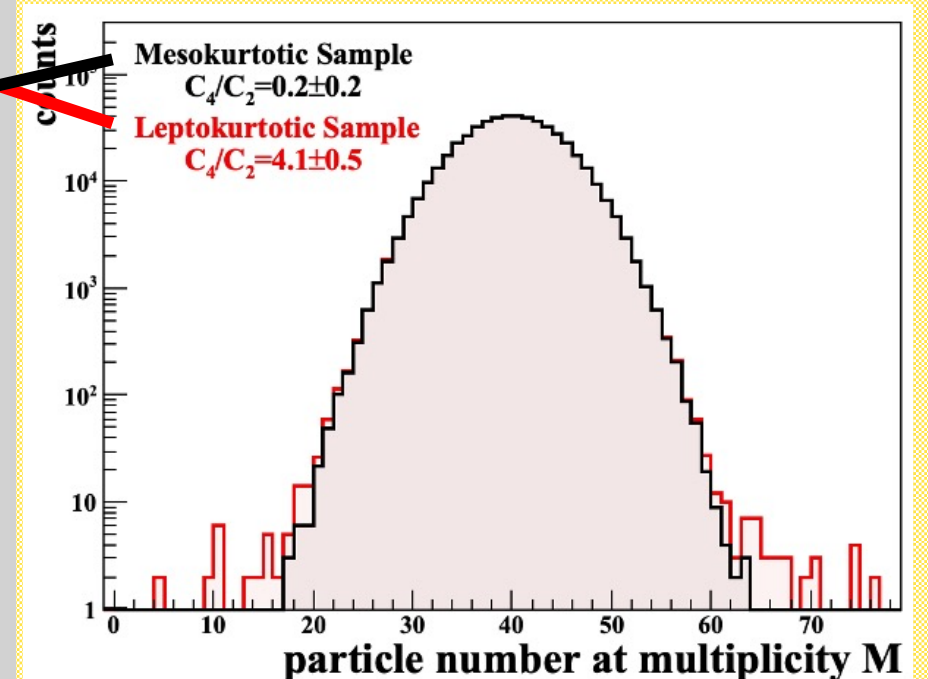
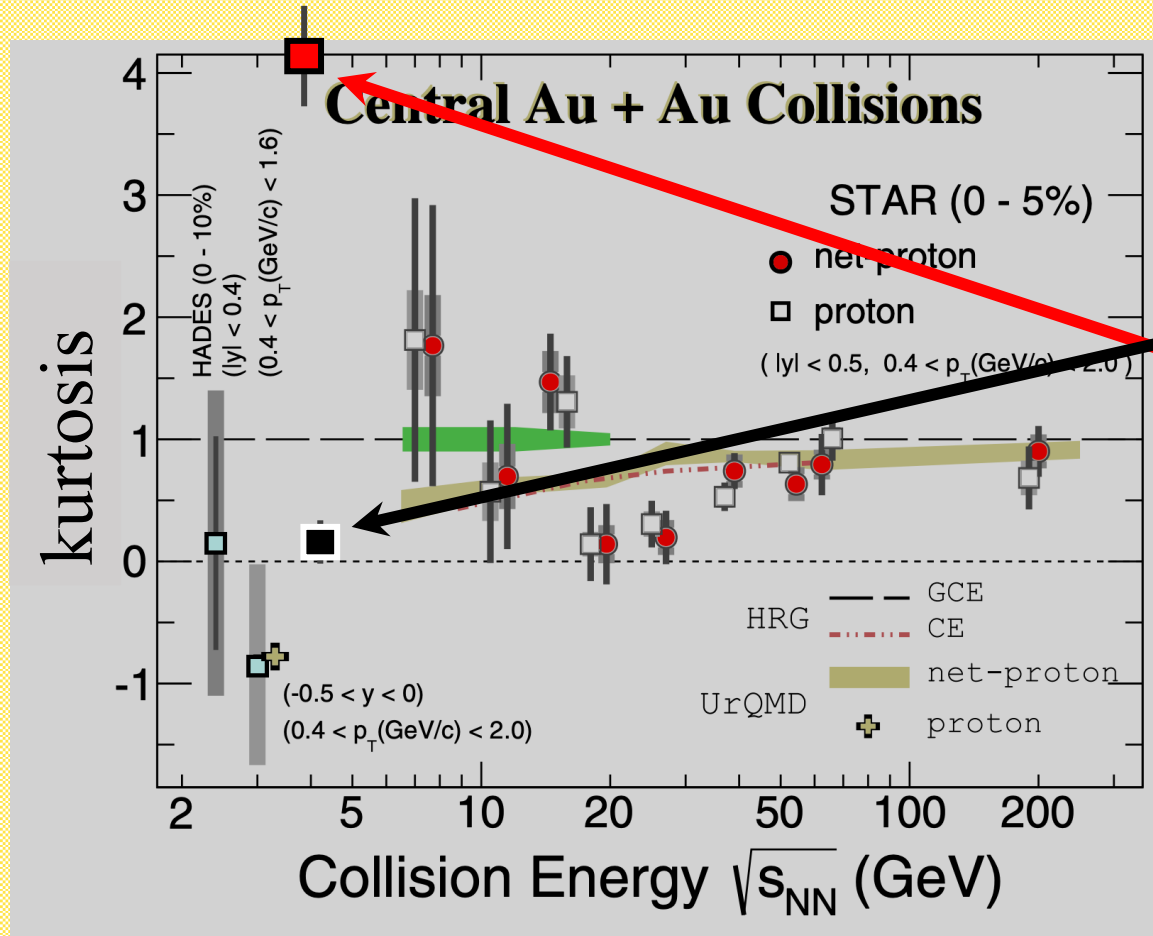


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What does large kurtosis mean?

- This gives an idea of the scale of fluctuations
- We need to understand all low-statistics outliers!

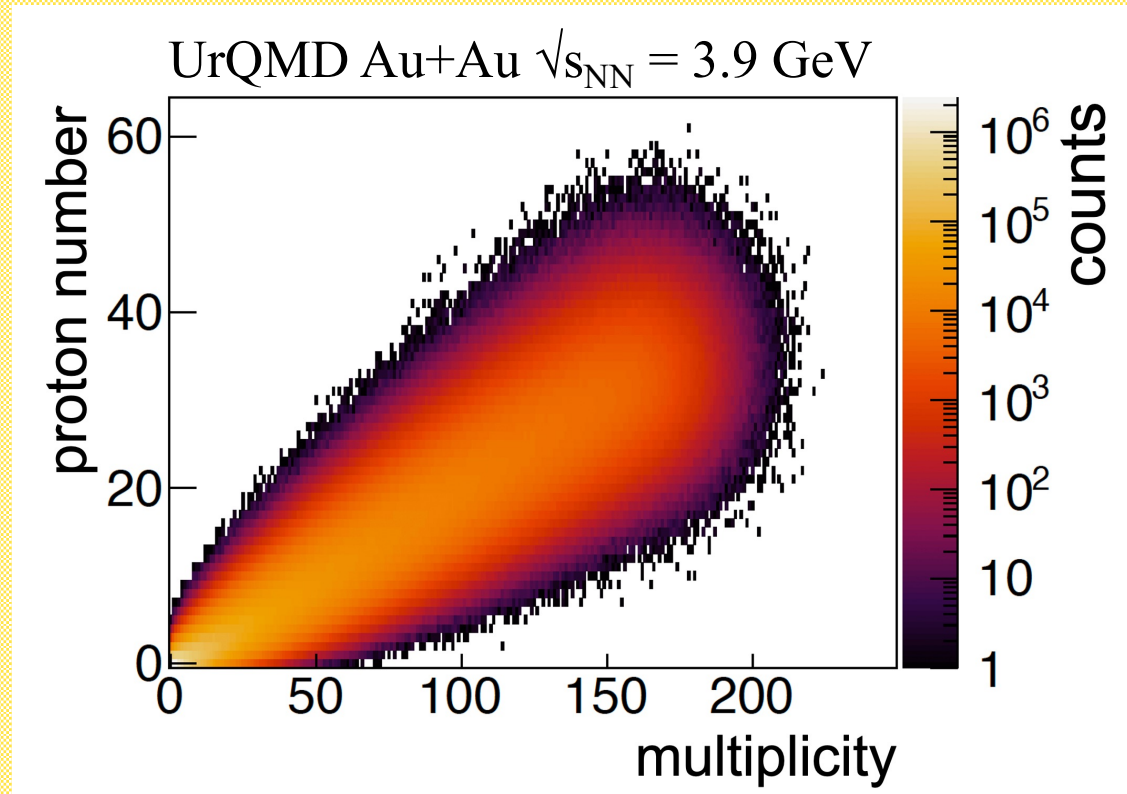


STAR, *Phys. Rev. Lett.* 128, 202303 (2022); *Phys. Rev. C* 107, 024908 (2023).

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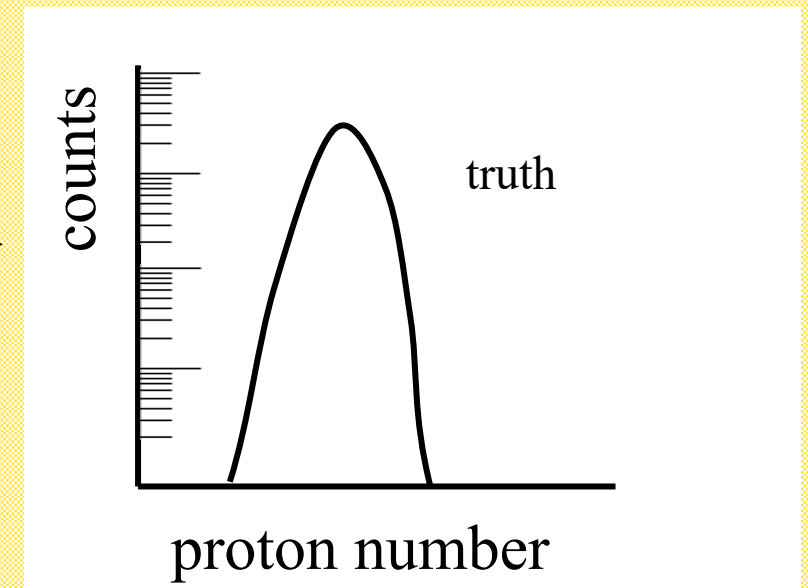
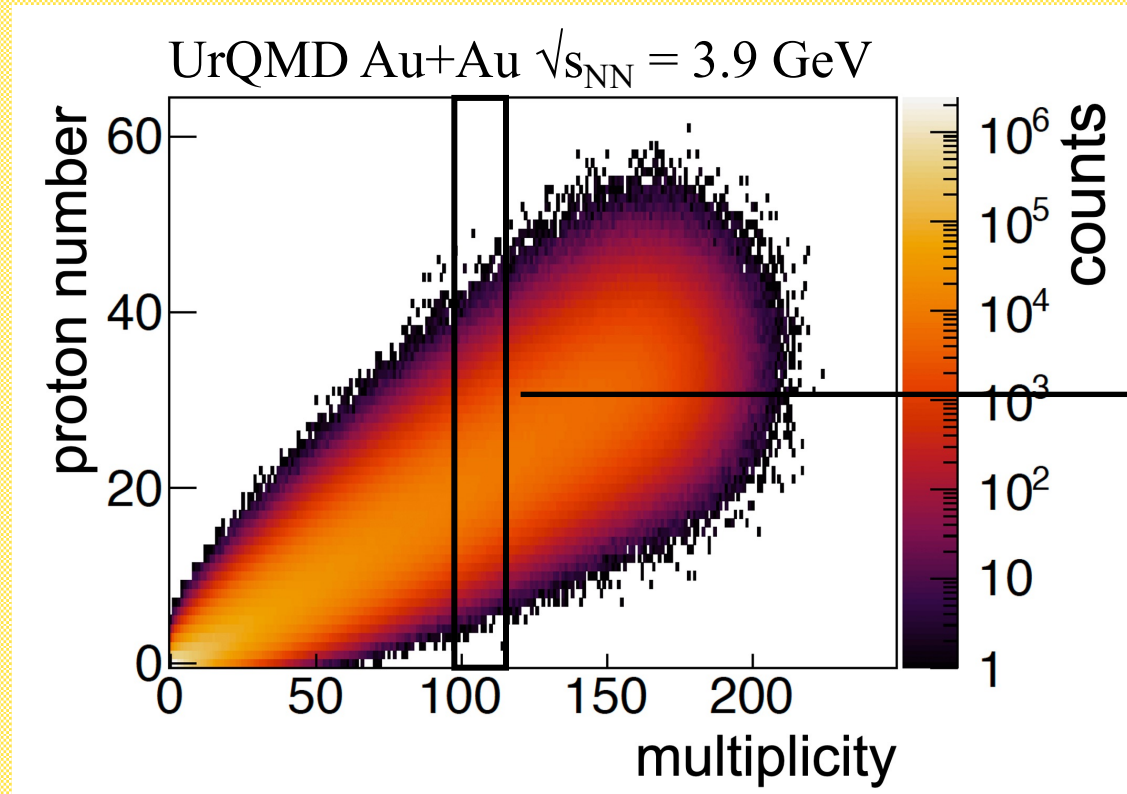
Rare and Spontaneous Detector Failure

- We often use different detectors to measure proton number and multiplicity
- This makes us vulnerable to the rare and spontaneous failure of either detector



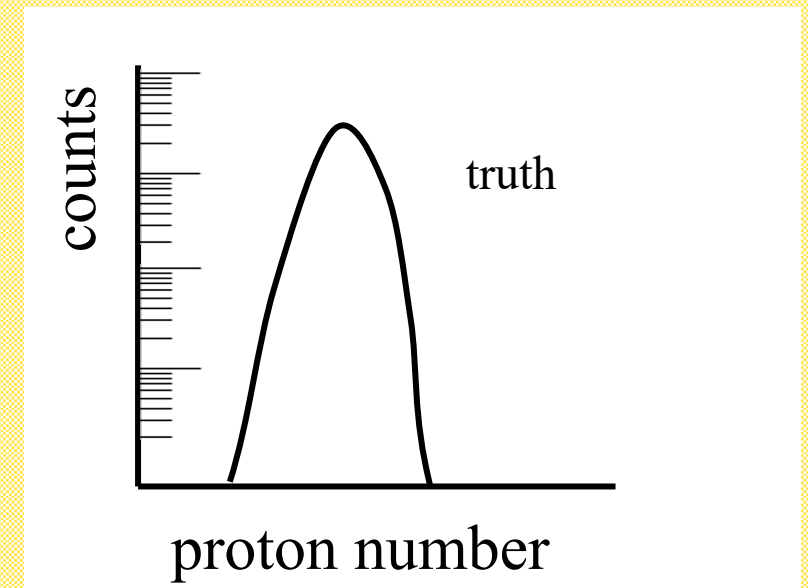
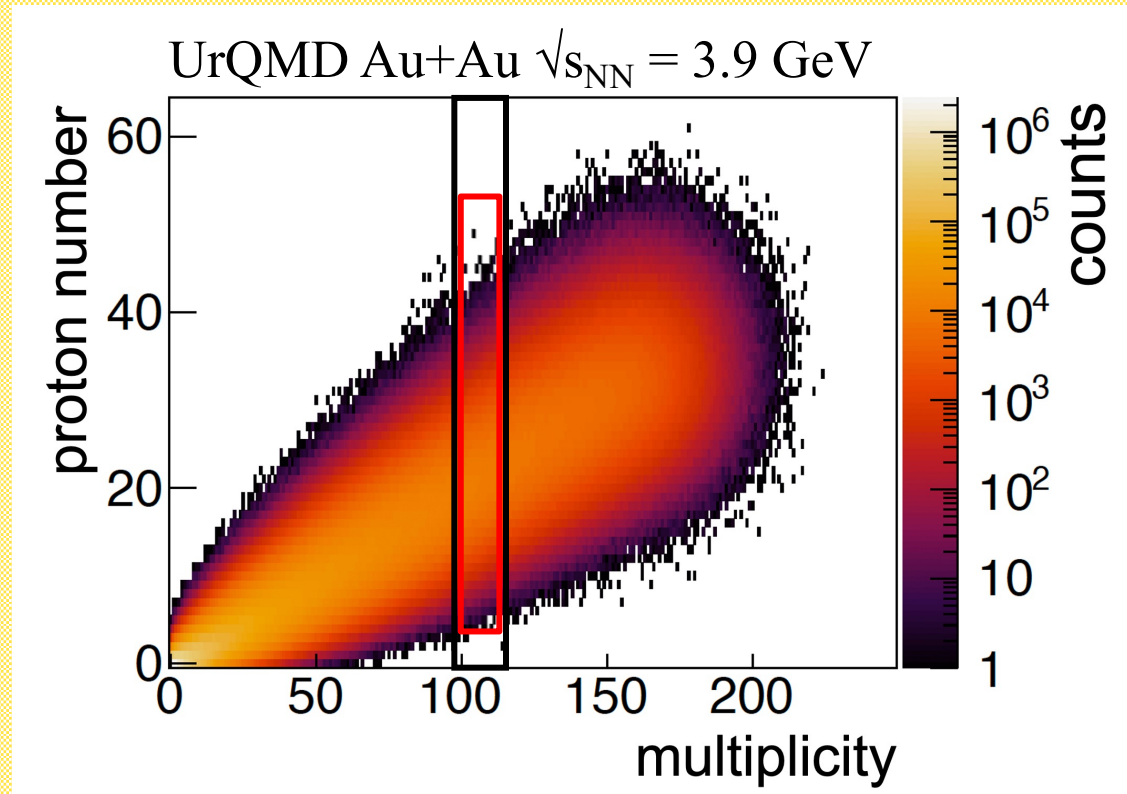
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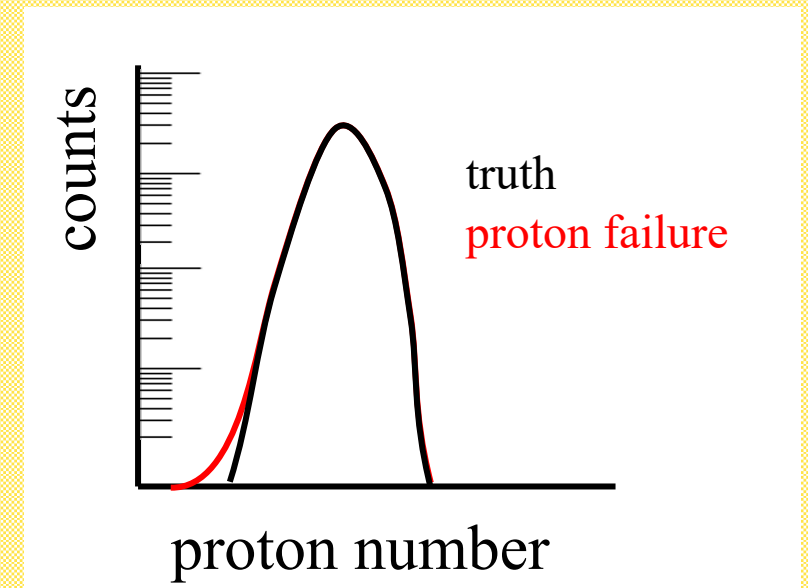
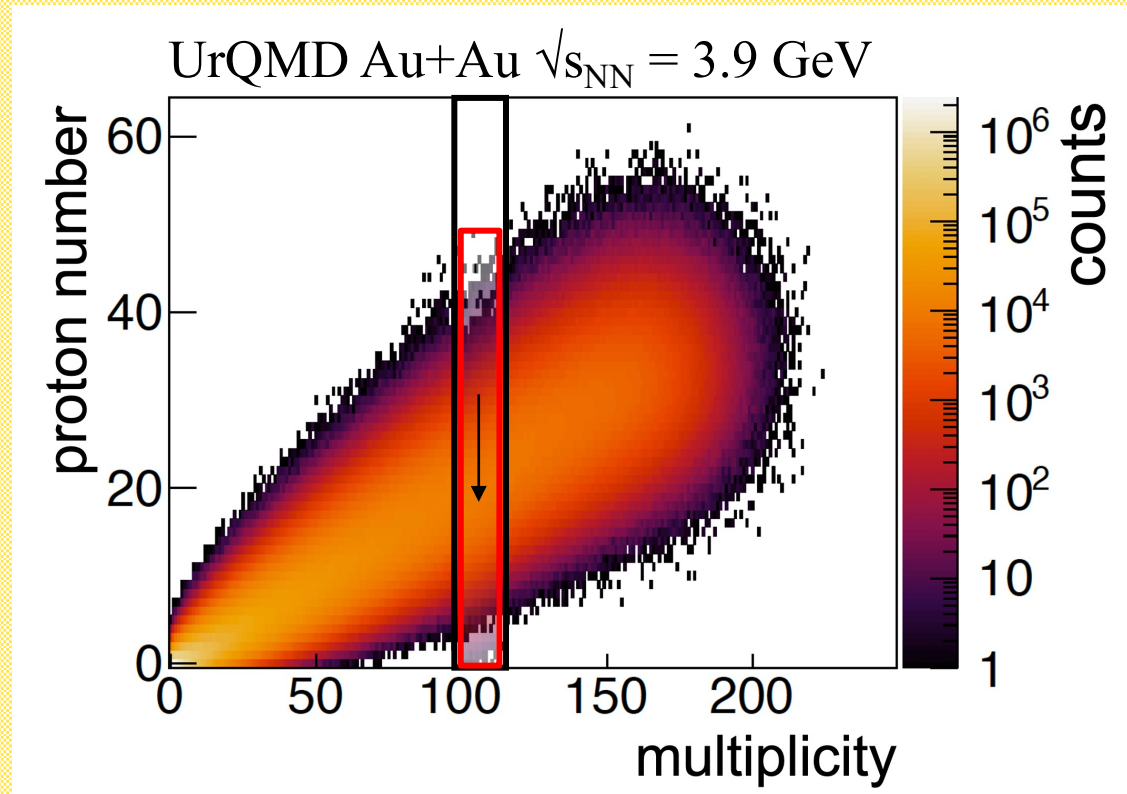
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- Consider a vertical slice in our correlation plot
 - If the proton detector ever fails



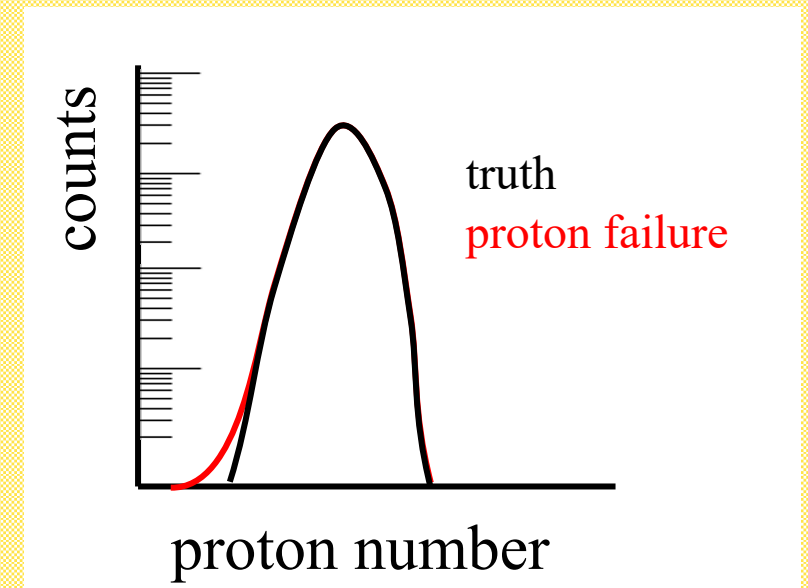
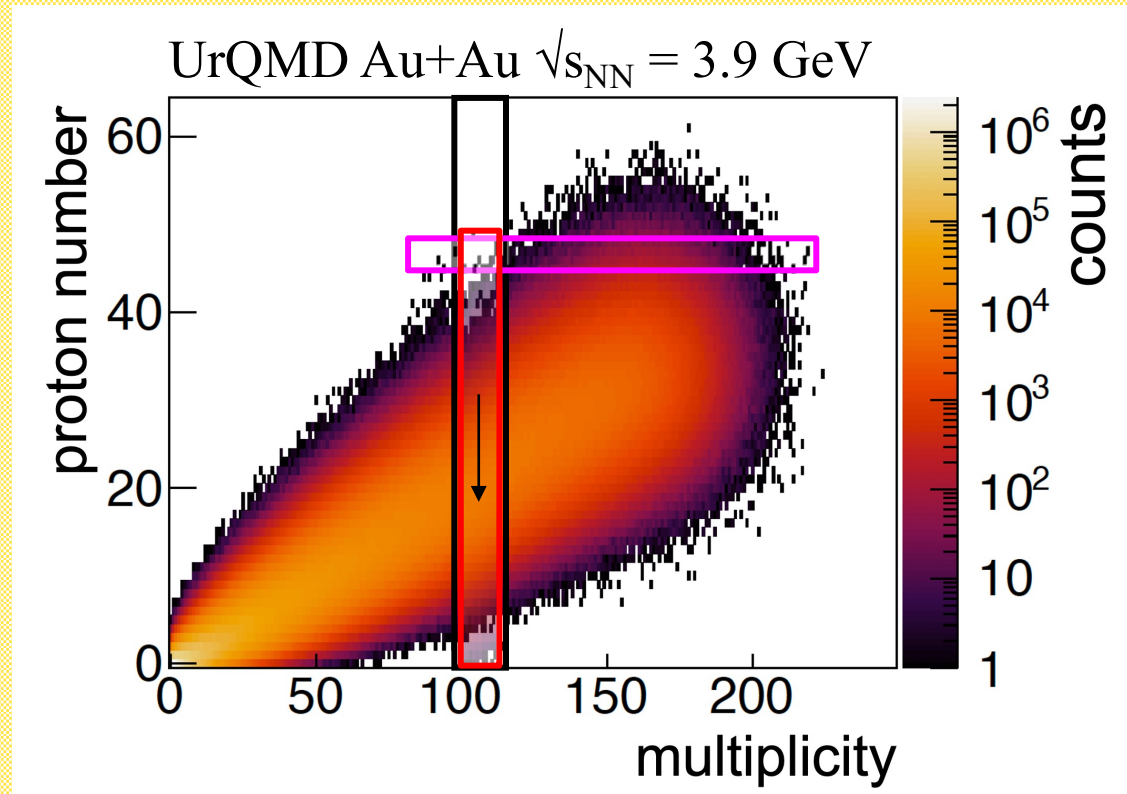
Rare and Spontaneous Detector Failure

- We often use different detectors to measure proton number and multiplicity
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- Consider a vertical slice in our correlation plot
 - If the proton detector ever fails, we get a sudden vertical shift adding to low tails



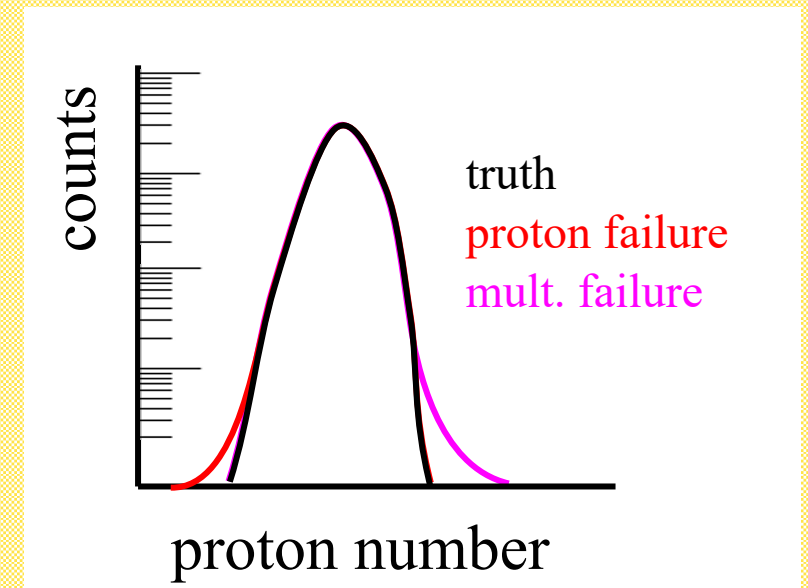
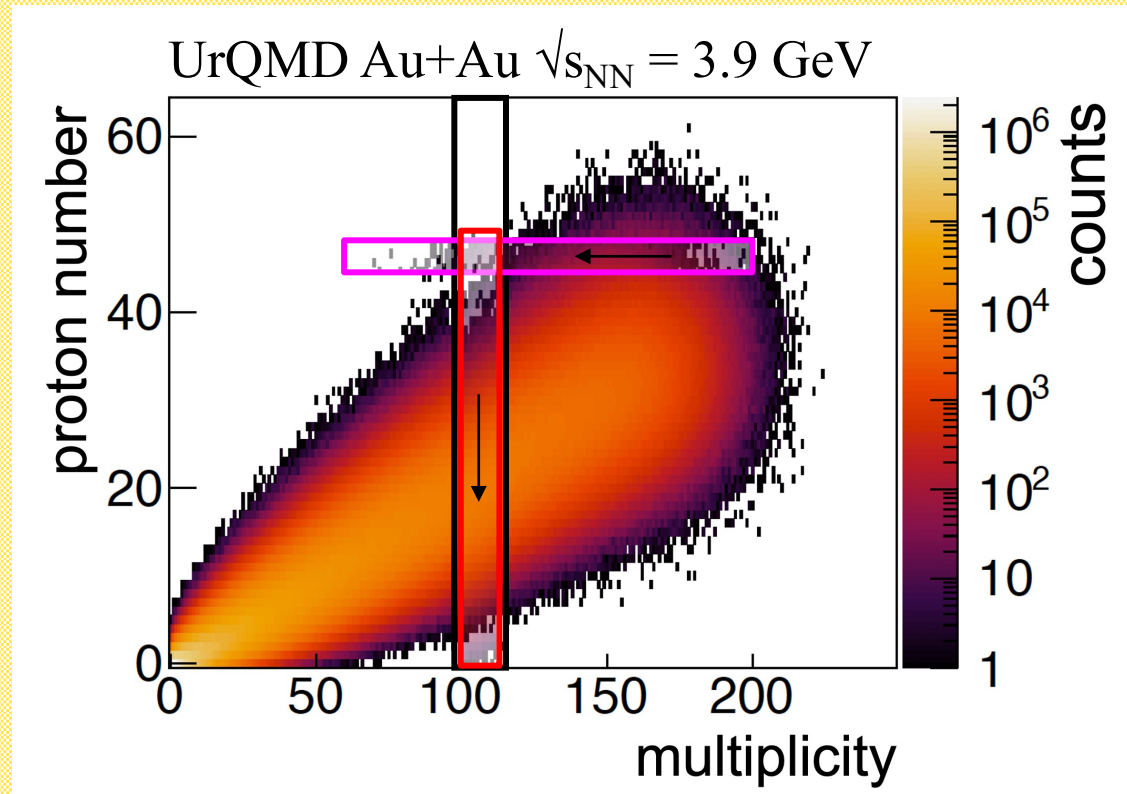
Rare and Spontaneous Detector Failure

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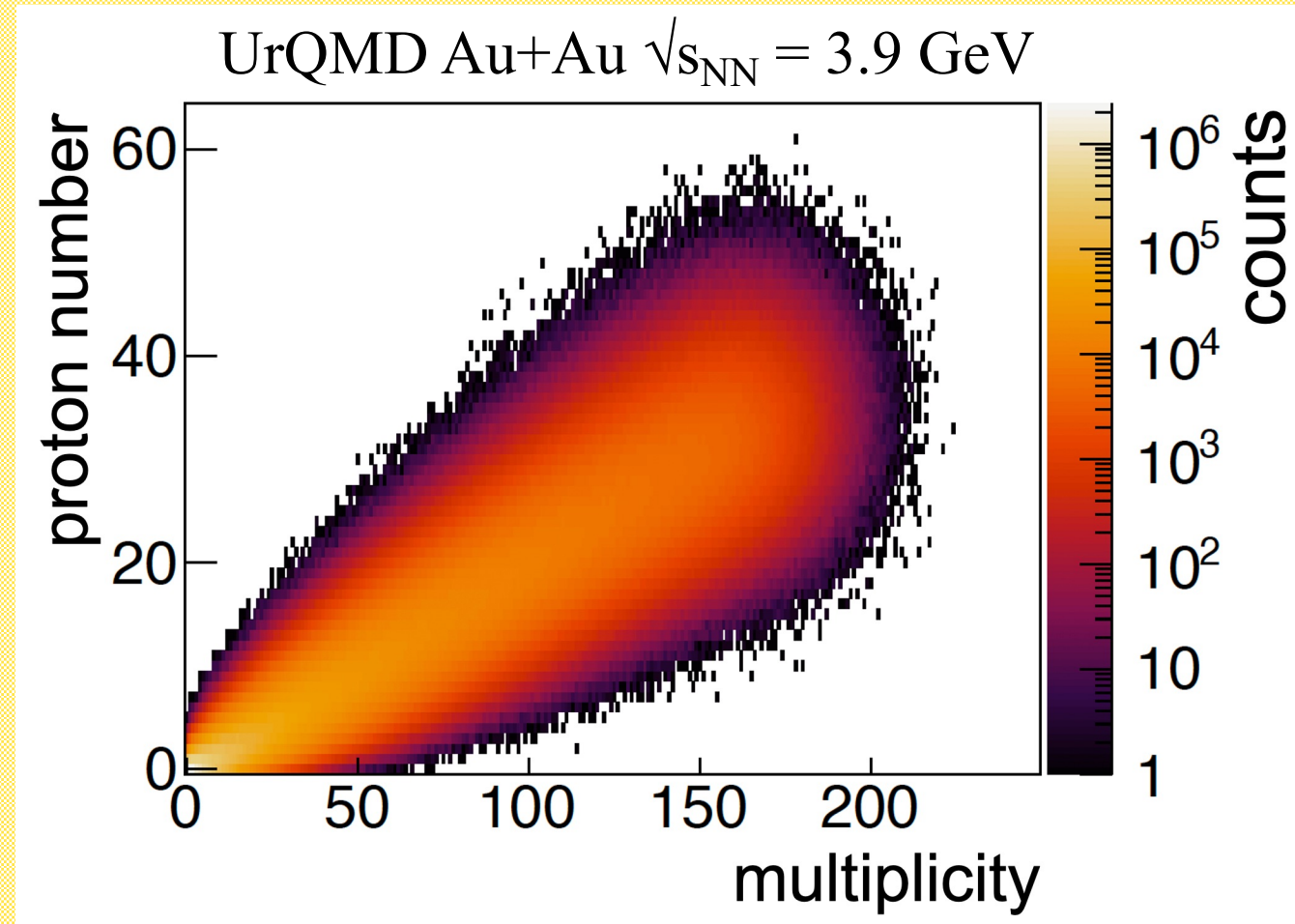
Rare and Spontaneous Detector Failure

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 - If the proton detector ever fails, we get a sudden vertical shift adding to low tails
 - If the multiplicity detection ever fails, sudden horizontal shift adds to high tails



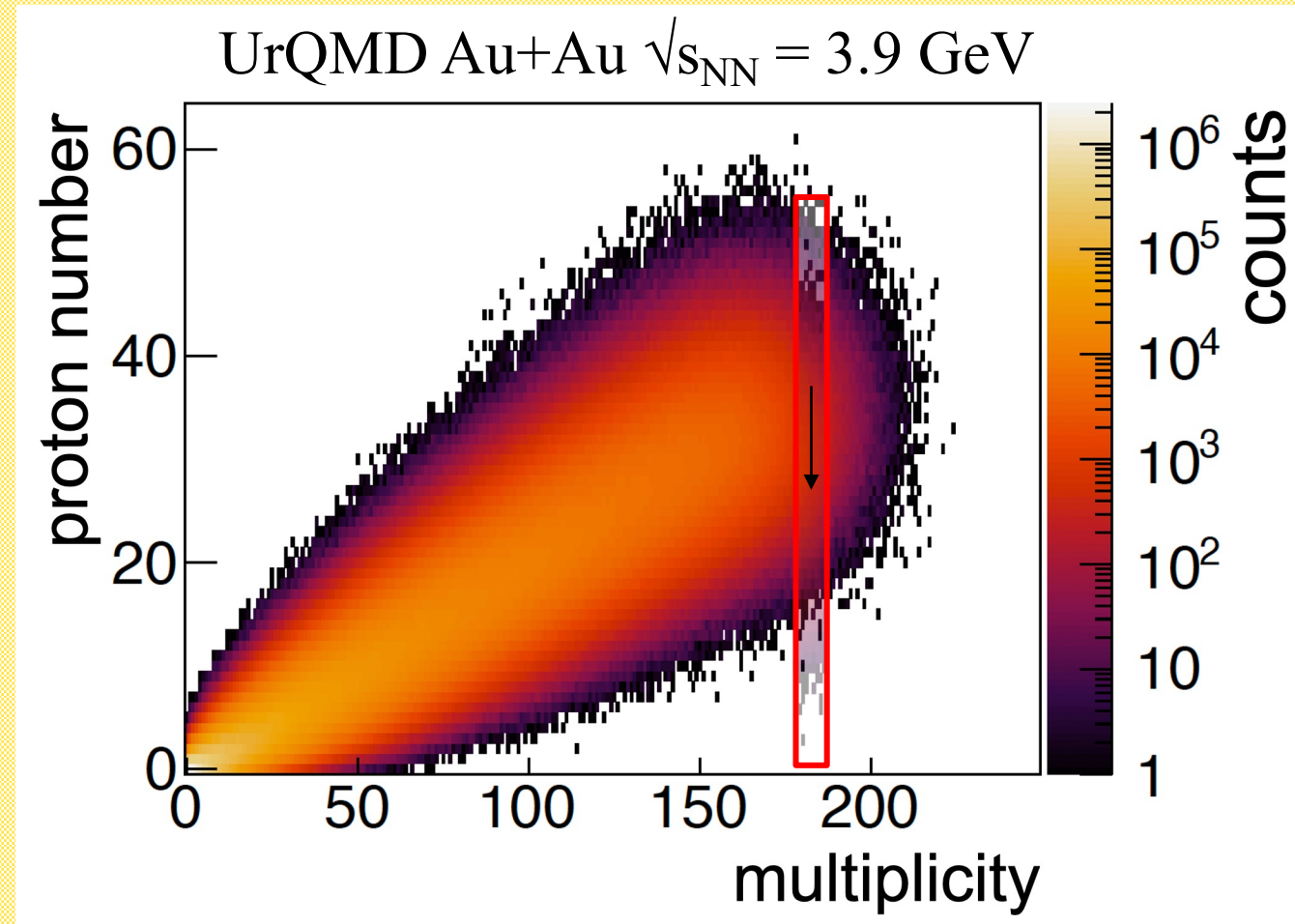
Rare and Spontaneous Detector Failure

- There are many potential sources of detector failure
- Proton detector:



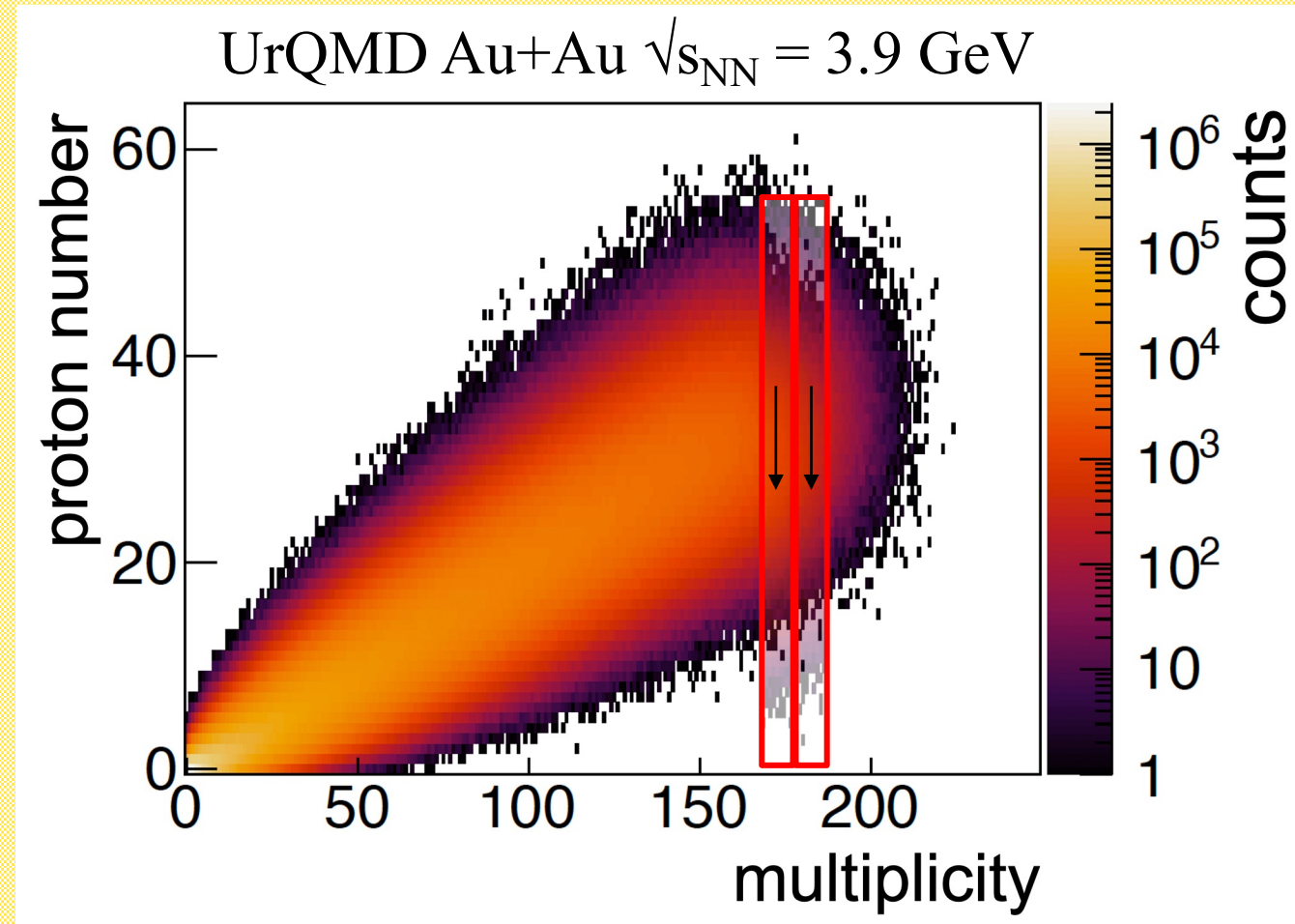
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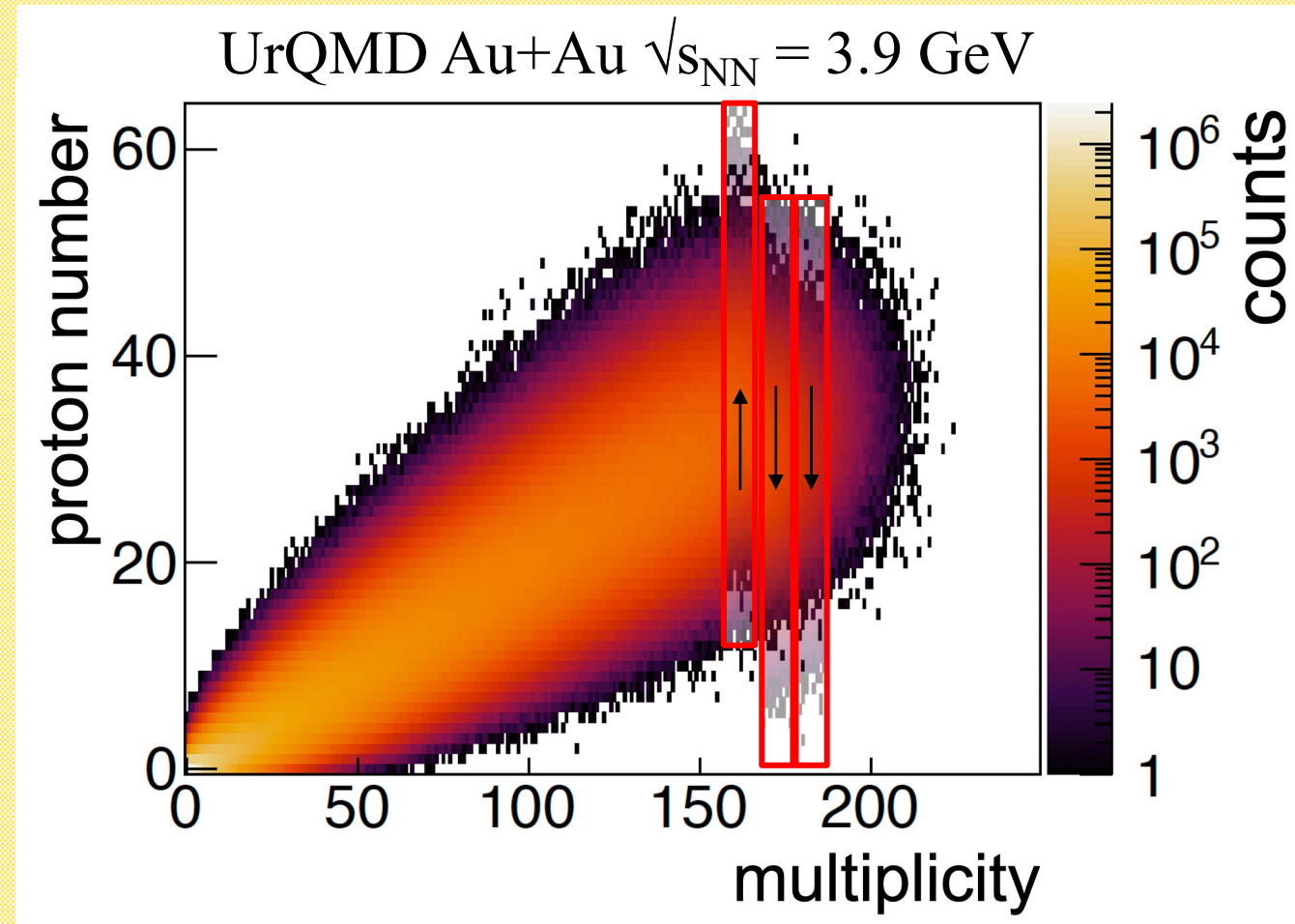
Rare and Spontaneous Detector Failure

- There are many potential sources of detector failure
- Proton detector:
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 - ❑ Algorithmic failure



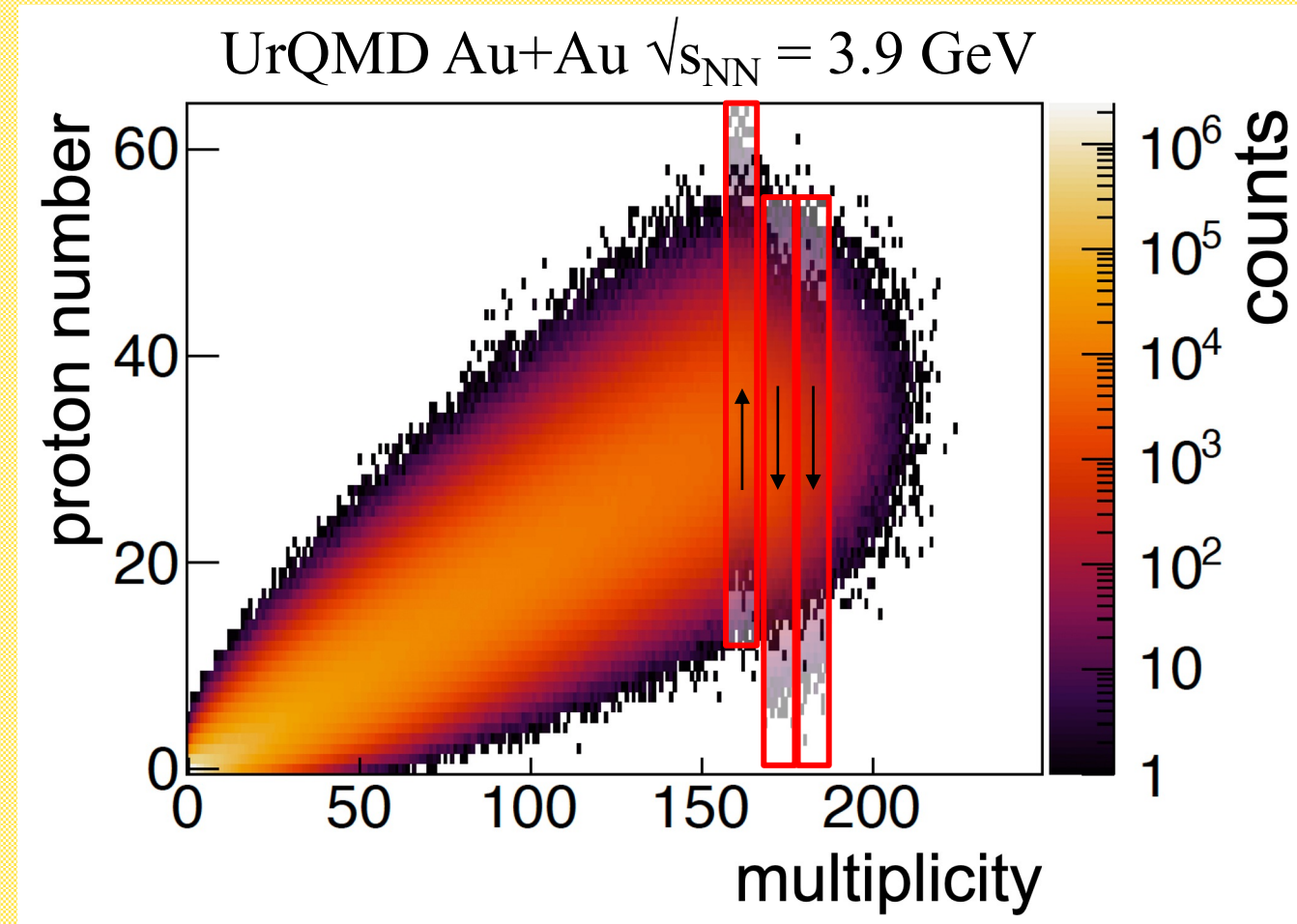
Rare and Spontaneous Detector Failure

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 - Sensitivity to pileup



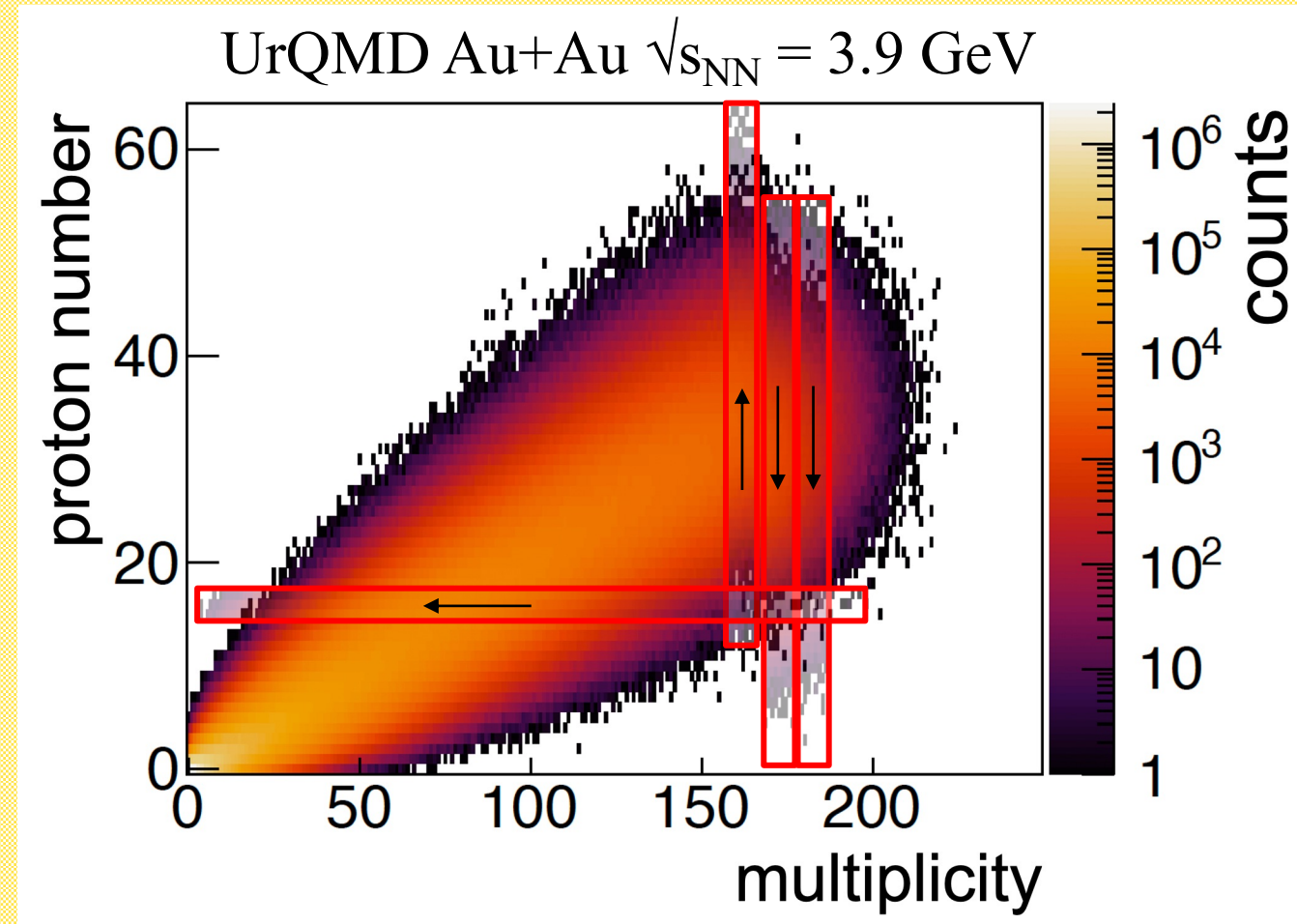
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- Multiplicity detector:



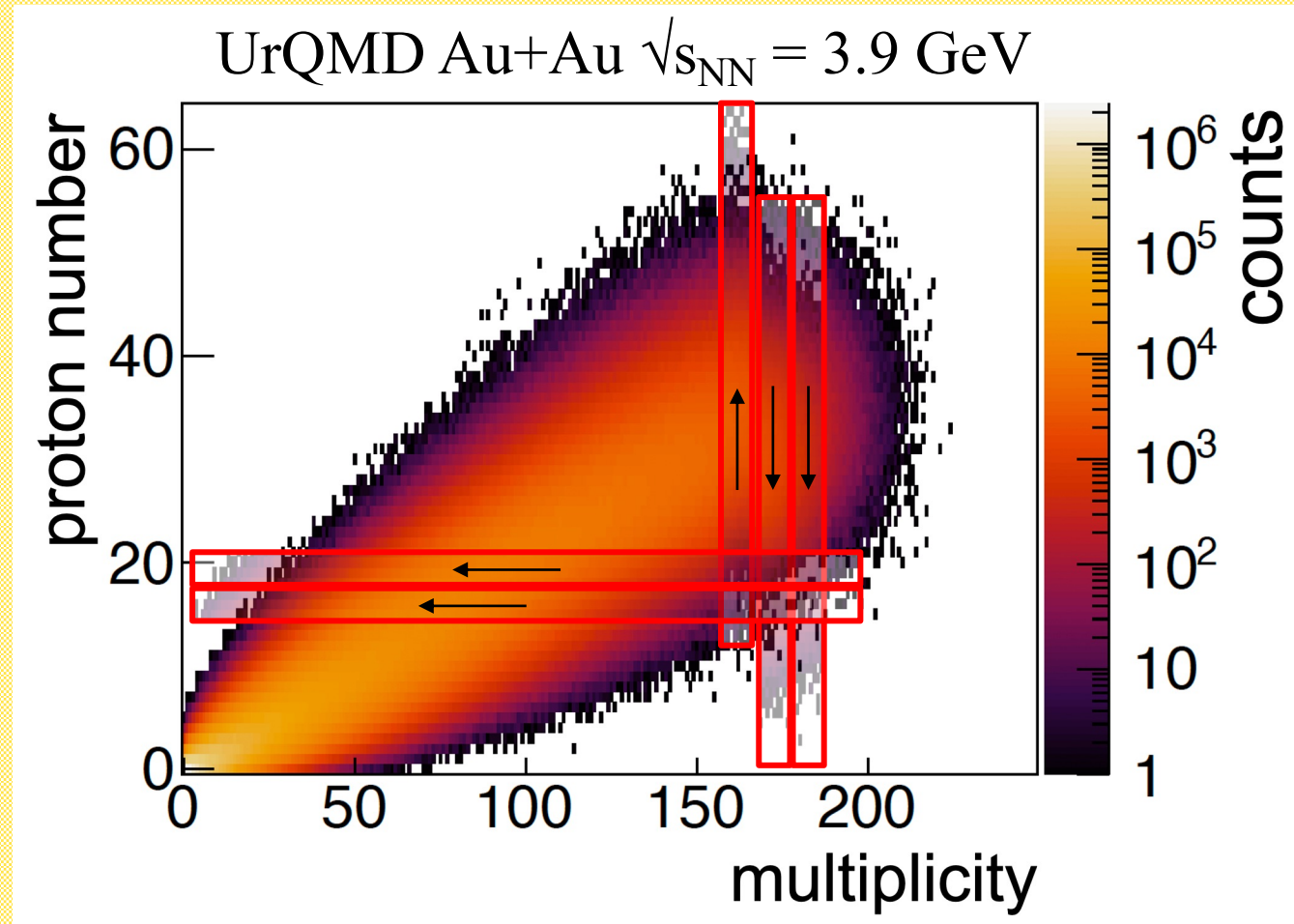
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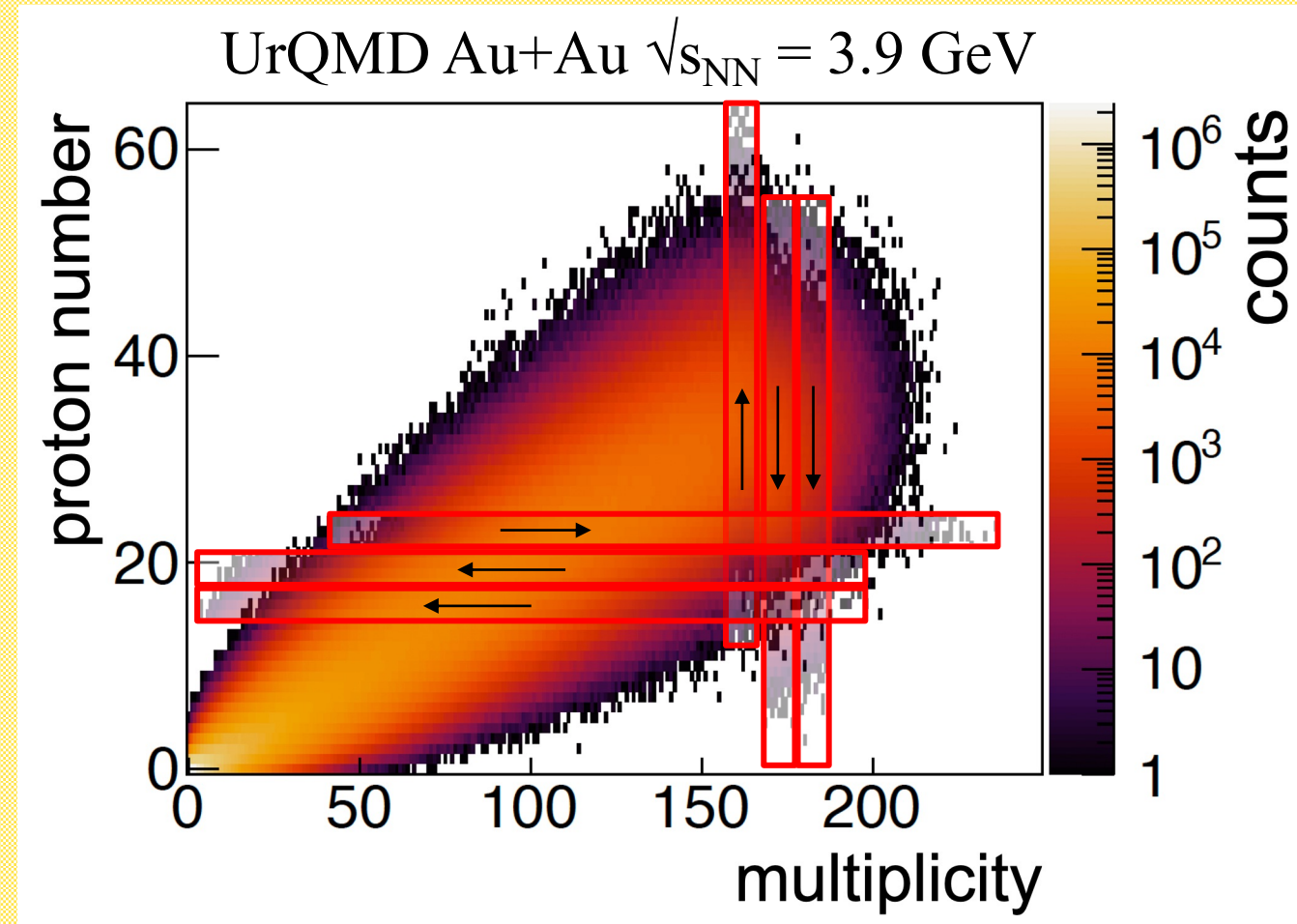
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Toy Model at $\sqrt{s_{\text{NN}}} = 3.9 \text{ GeV}$

The Toy Model

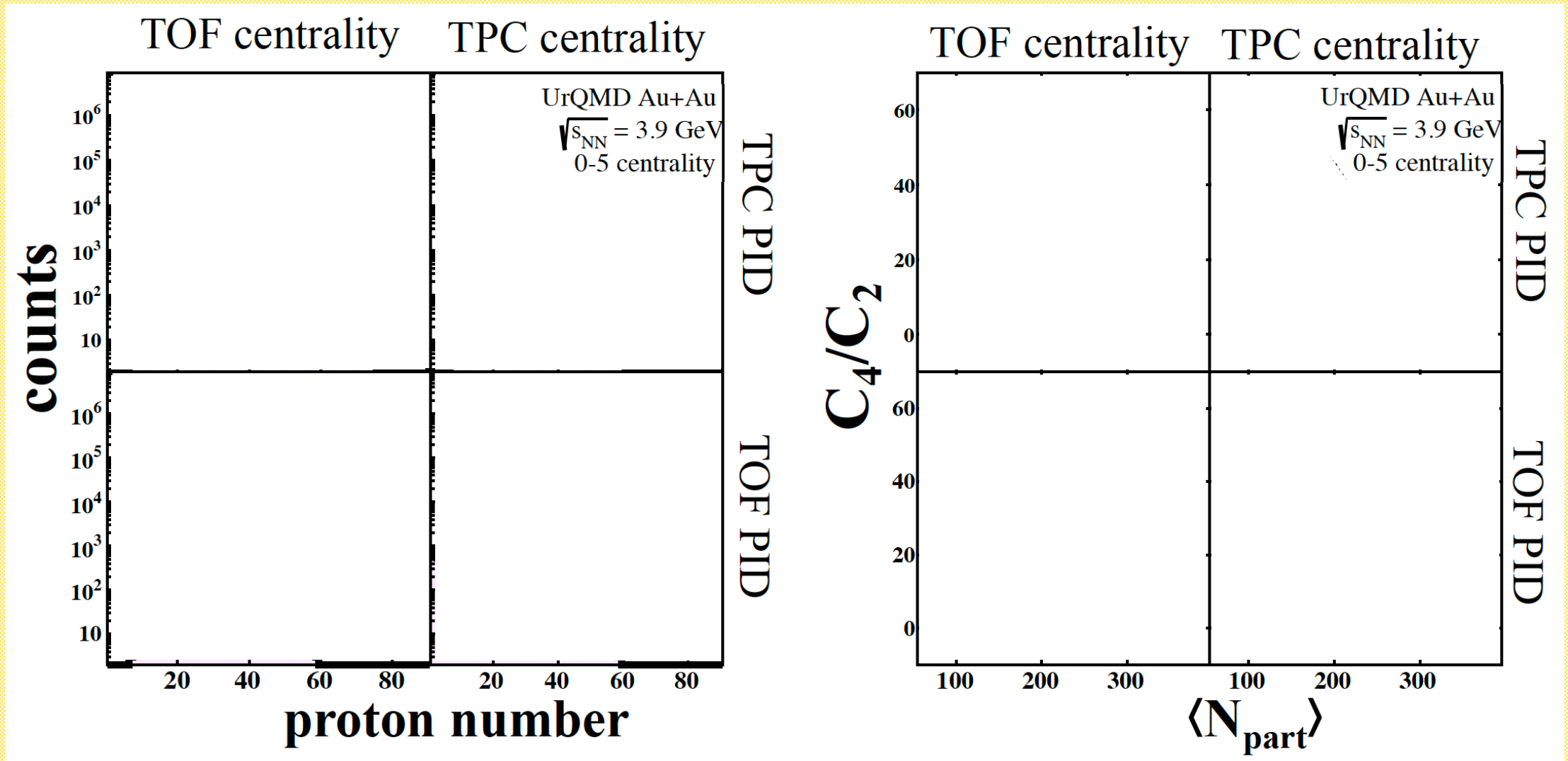
- UrQMD in cascade mode
- $\sqrt{s_{\text{NN}}} = 3.9 \text{ GeV}$
- Analysis window: $0.4 < p_{\text{T}} < 2 \text{ GeV}/c$, $-0.5 < y - y_{\text{CM}} < 0$
- Out-of-time pileup: double collisions at 0.01% rate

Toy Model at $\sqrt{s_{\text{NN}}} = 3.9 \text{ GeV}$

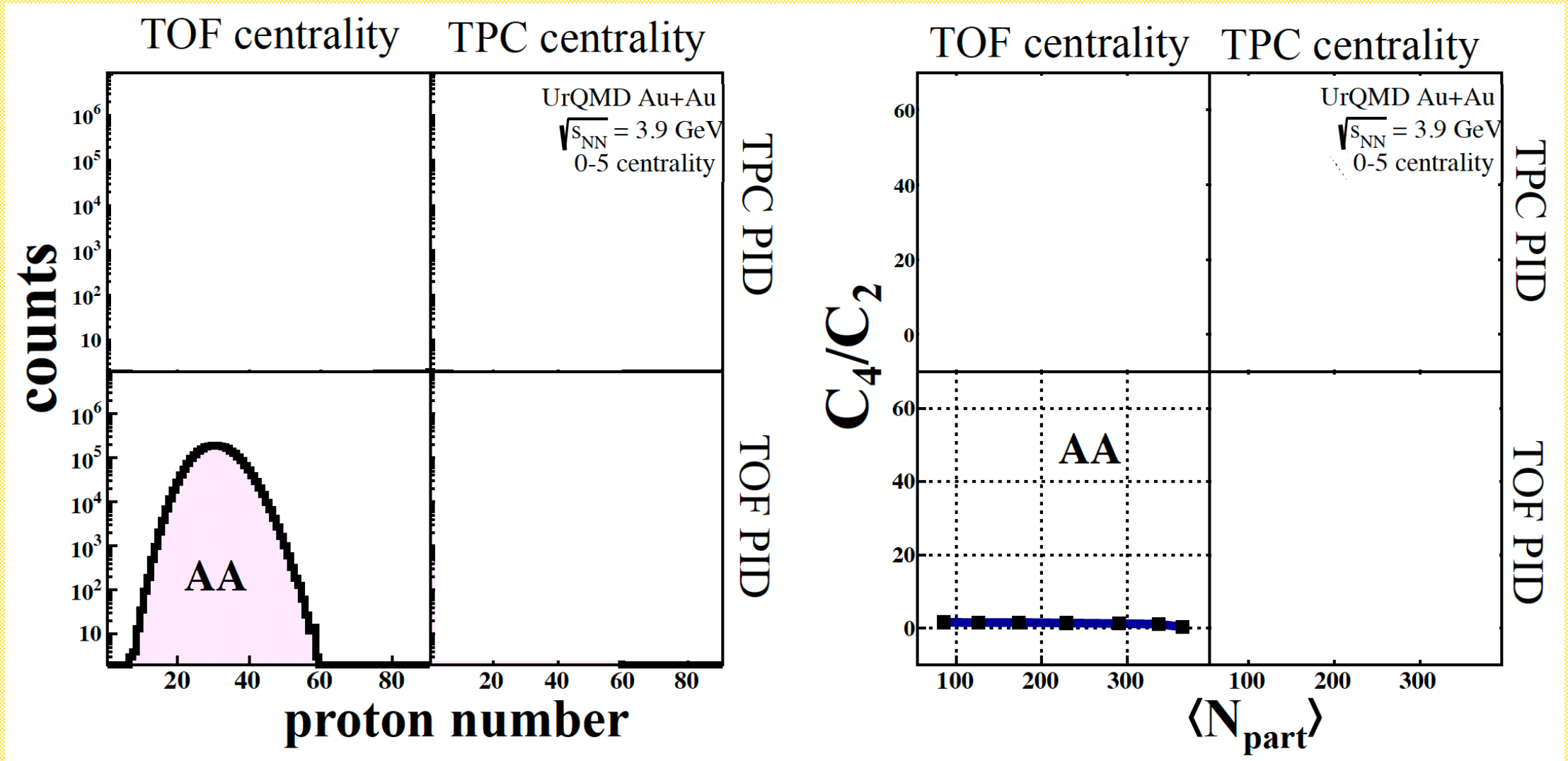
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- Out-of-time pileup: double collisions at 0.01% rate
- TPC toy model:
 - 100% efficiency for single collisions
 - 100% pileup efficiency (slow detector)
- TOF toy model:
 - 100% efficiency for single collisions
 - 0% pileup efficiency (fast detector)

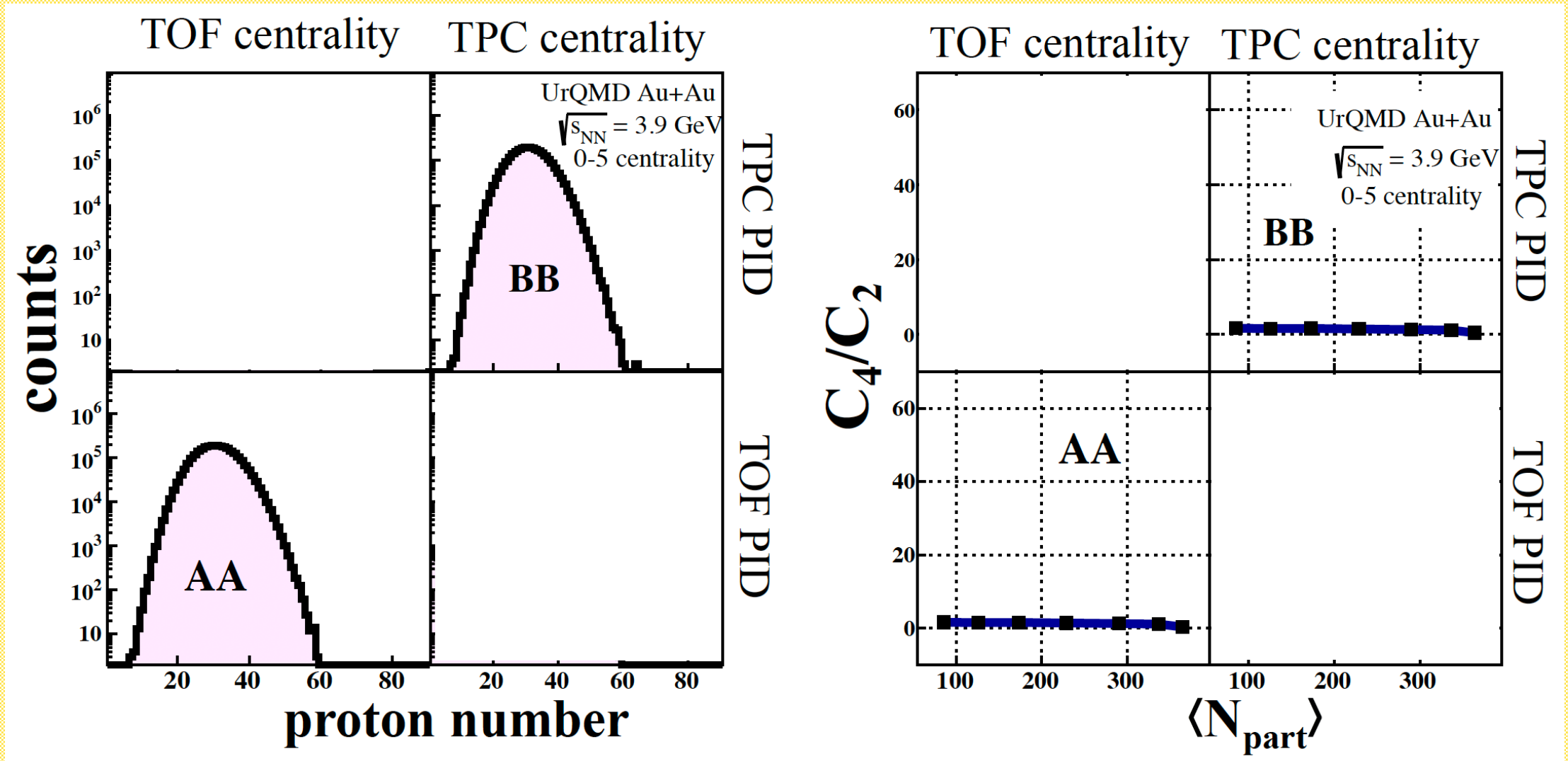
Pileup in the Toy Model



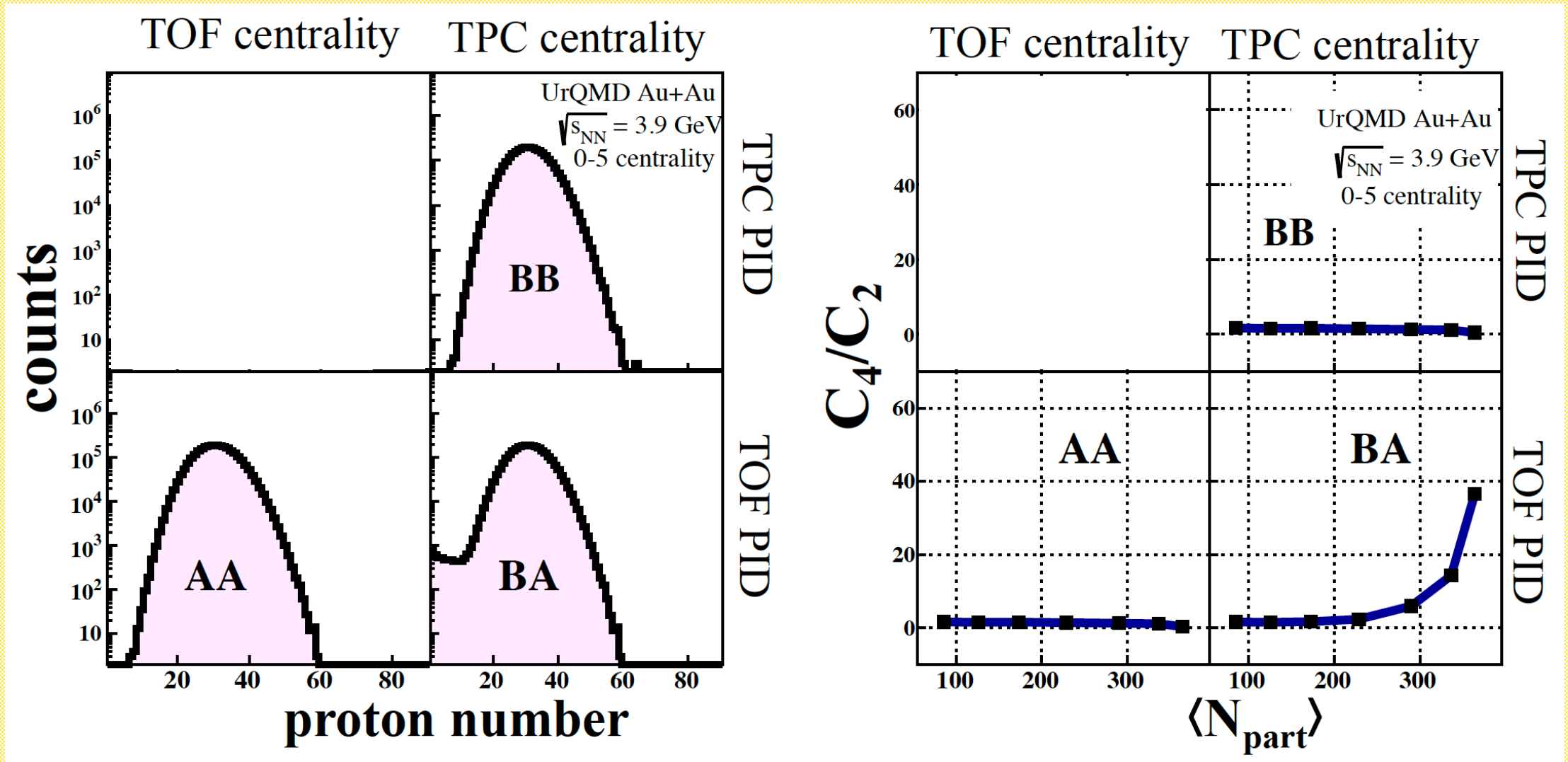
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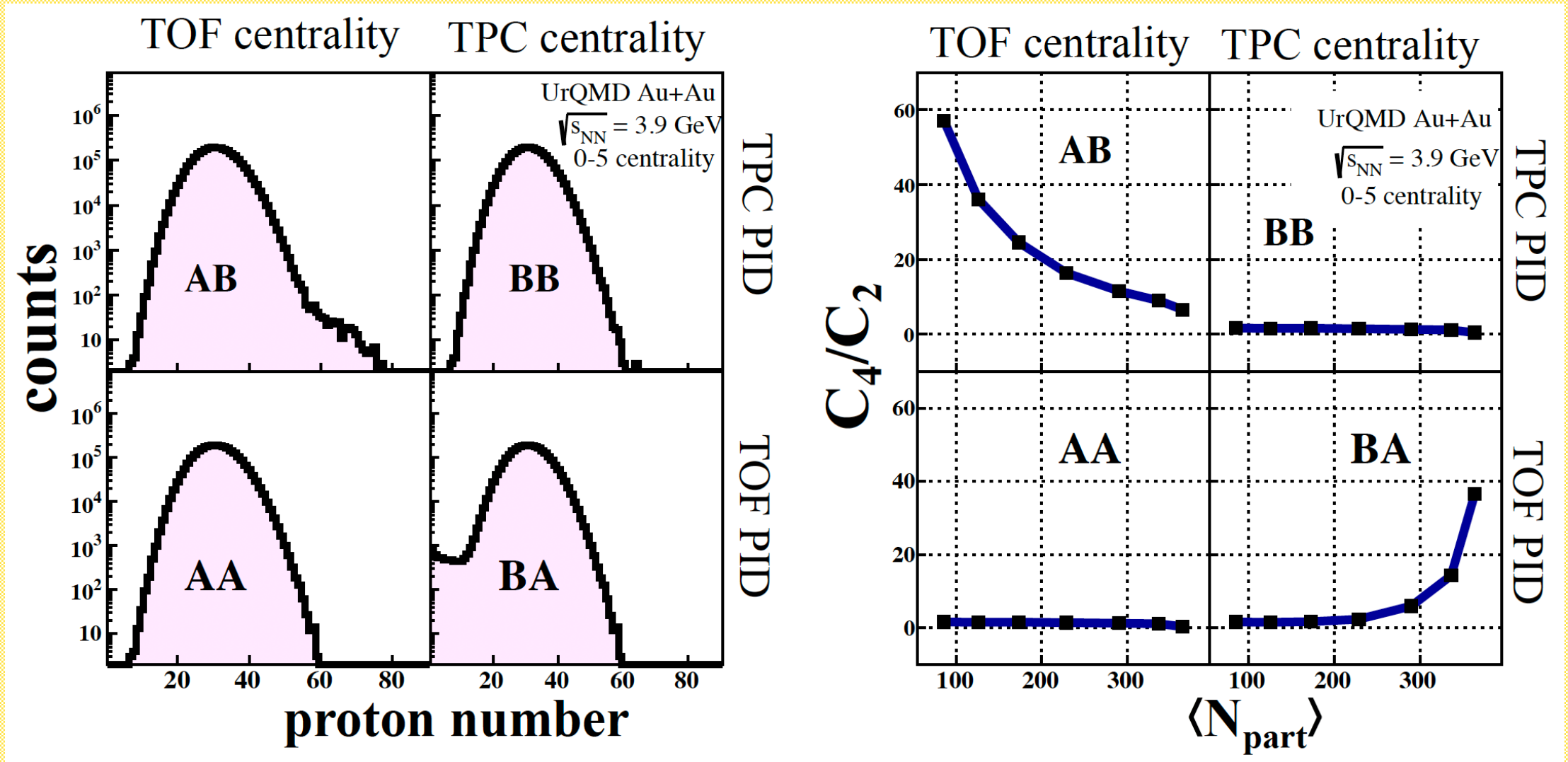
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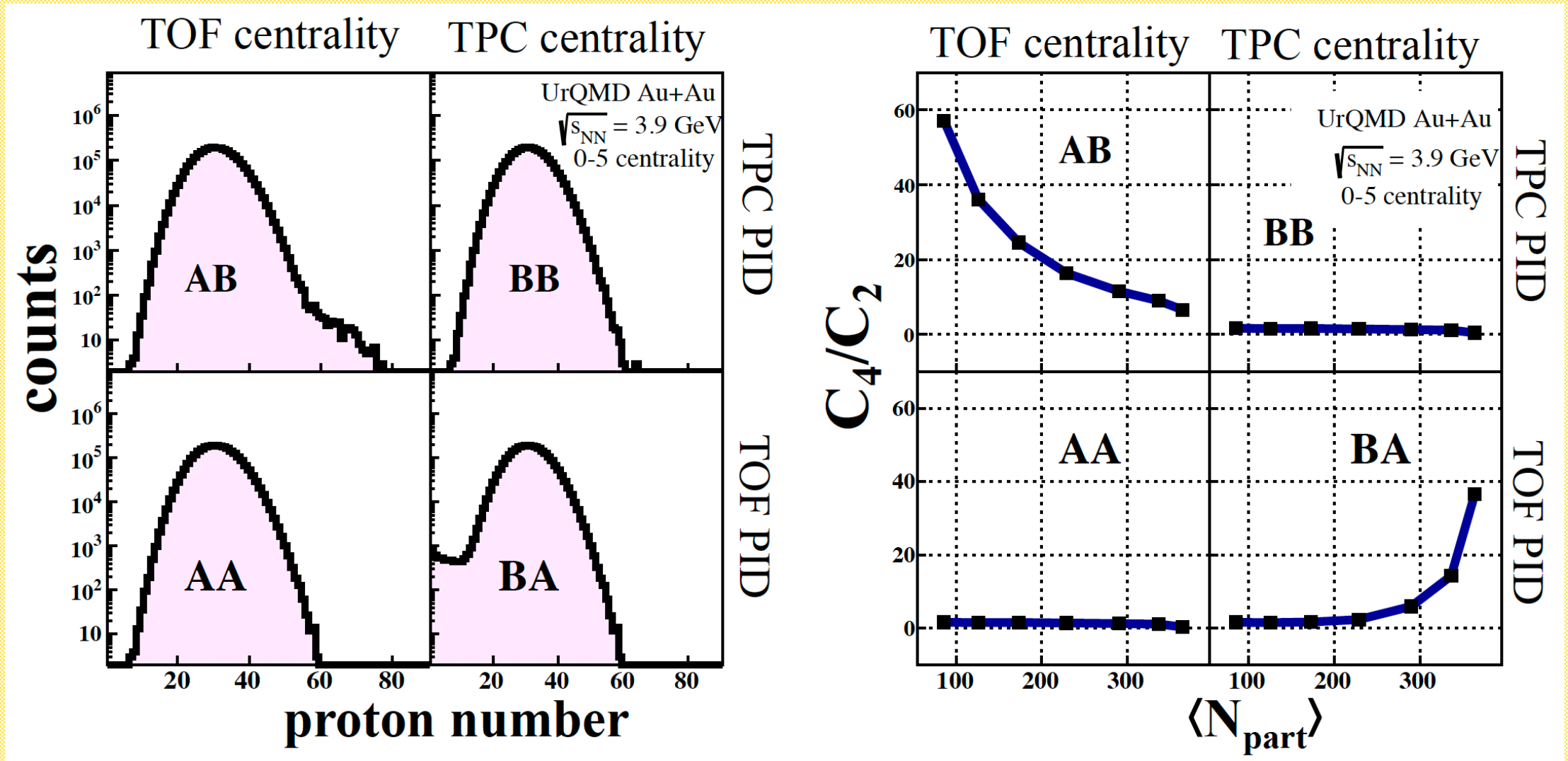


Pileup in the Toy Model



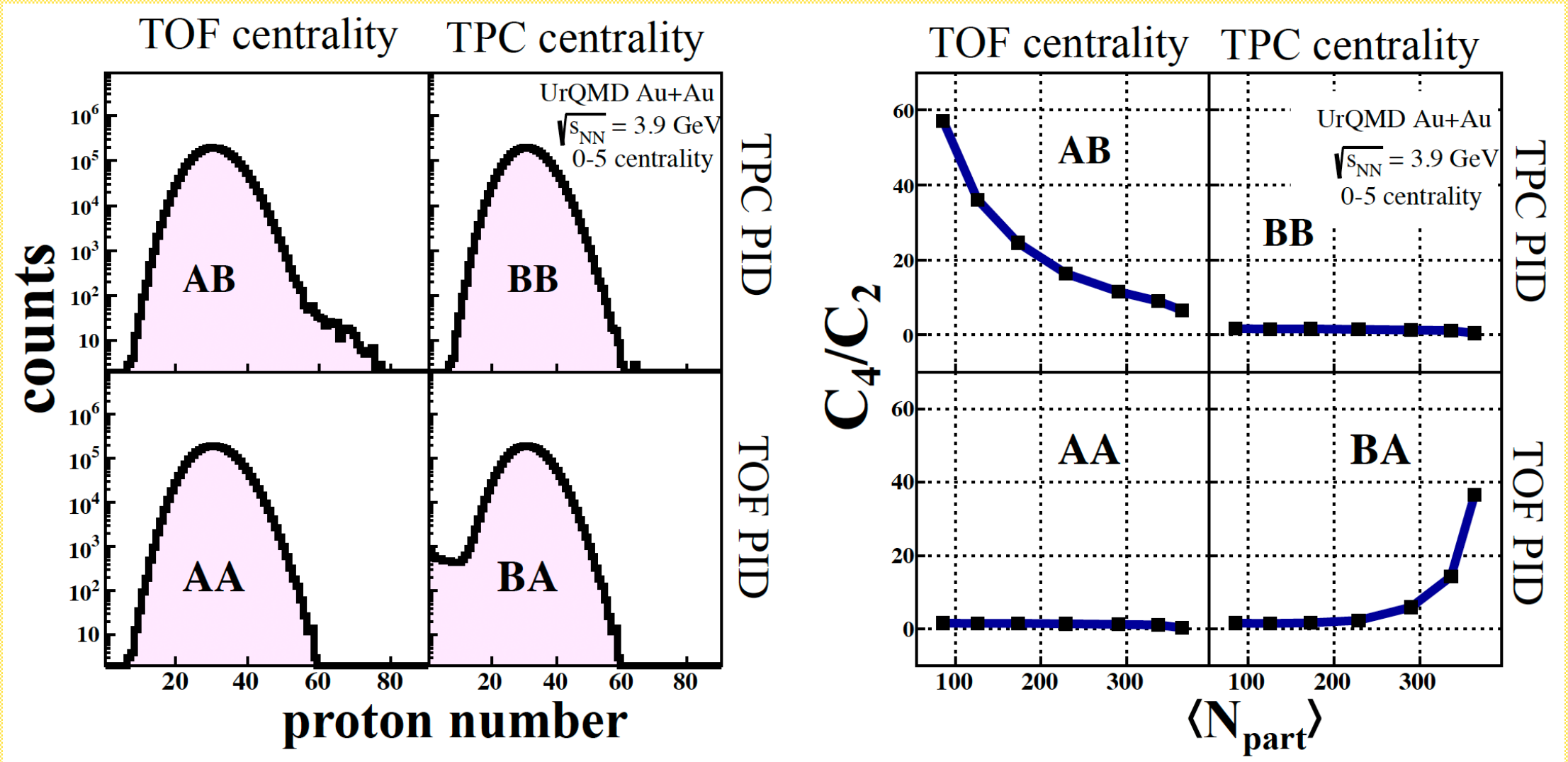
Pileup in the Toy Model

- When identical detectors are used tails are suppressed



Pileup in the Toy Model

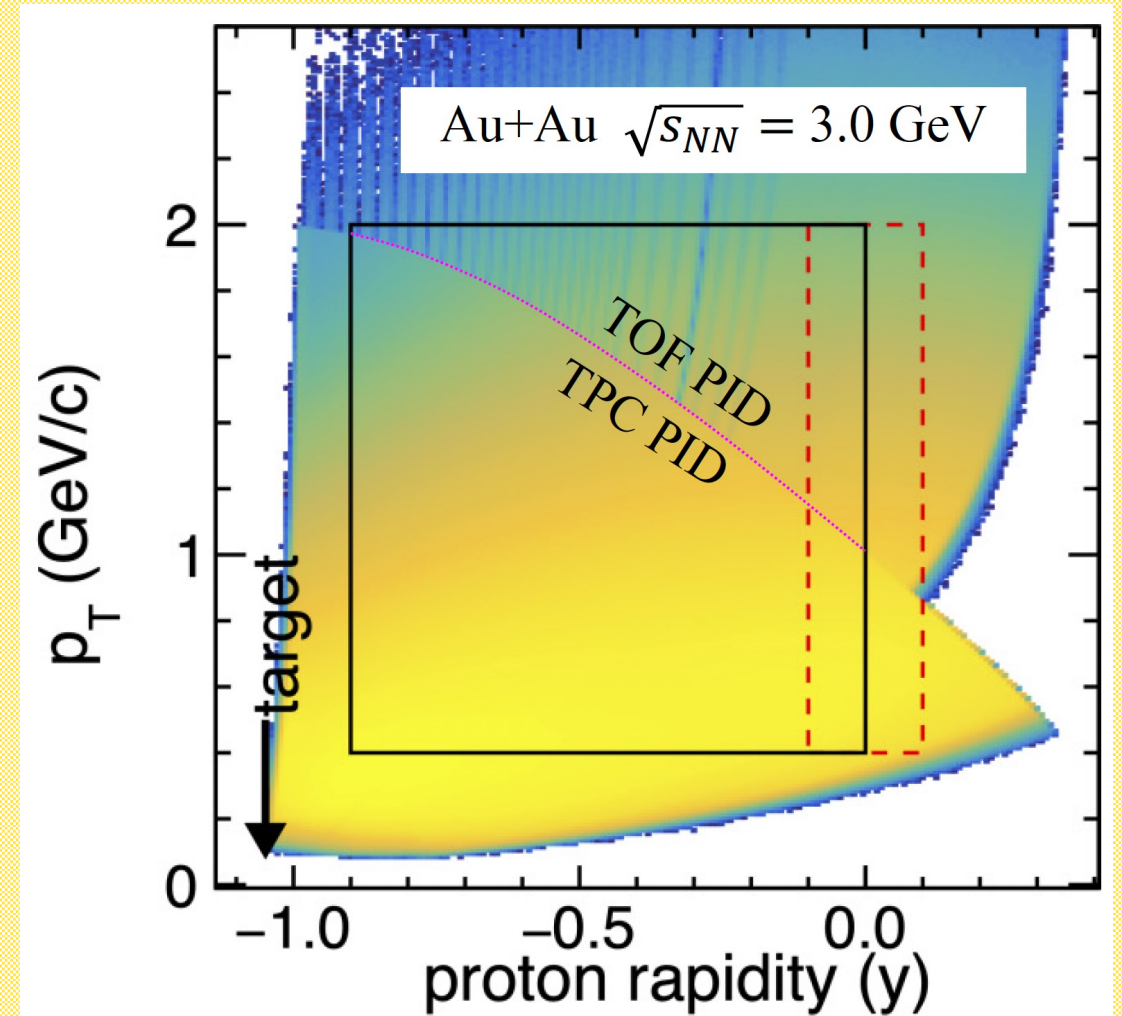
- When identical detectors are used tails are suppressed
- Pileup only causes large signal when we mix detectors



A More Realistic Toy Model

A More Realistic Toy Model

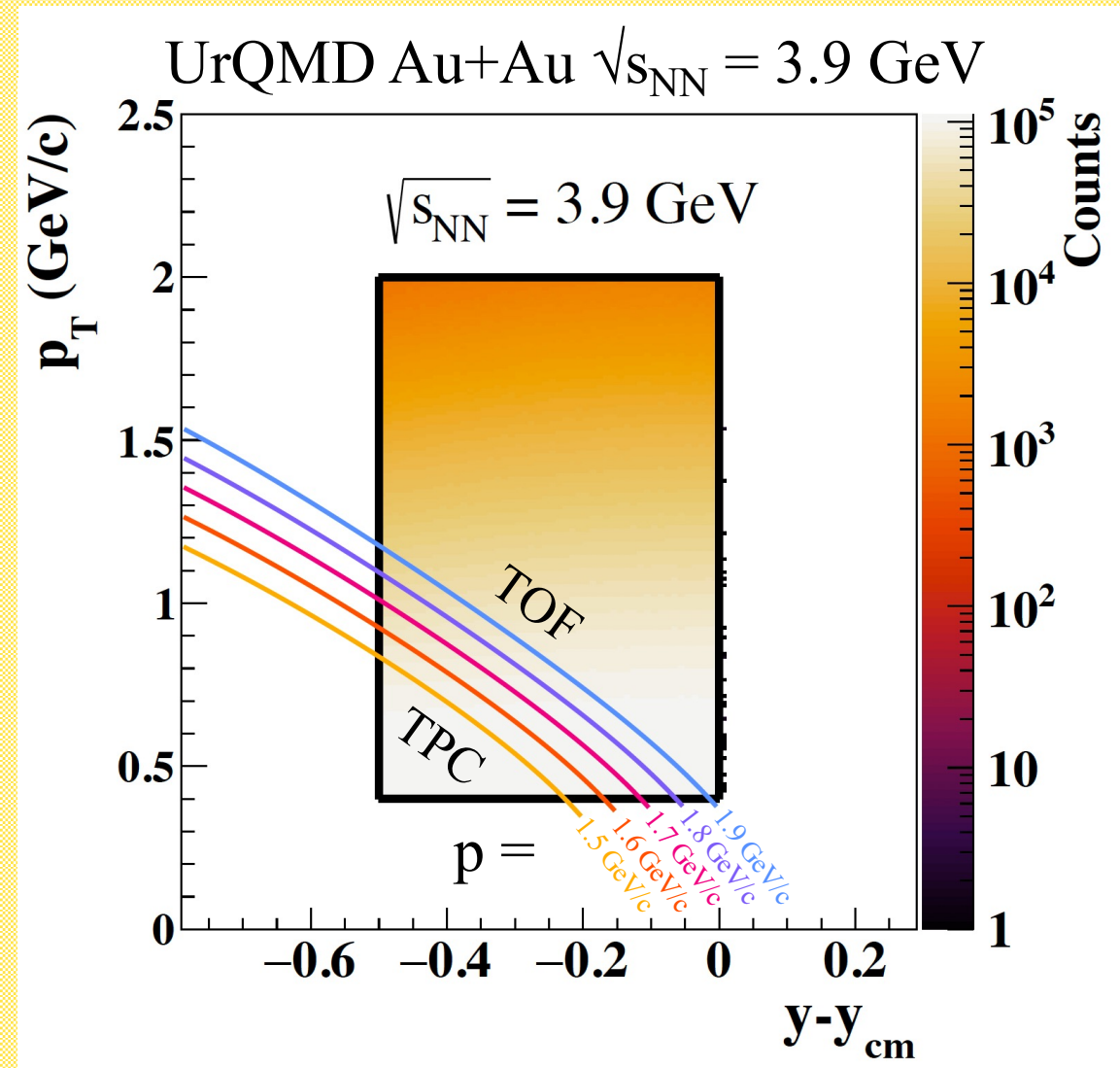
- Like at 3 GeV, we often don't use one detector to identify protons
- At some momentum, we switch from using the TPC to additionally requiring TOF



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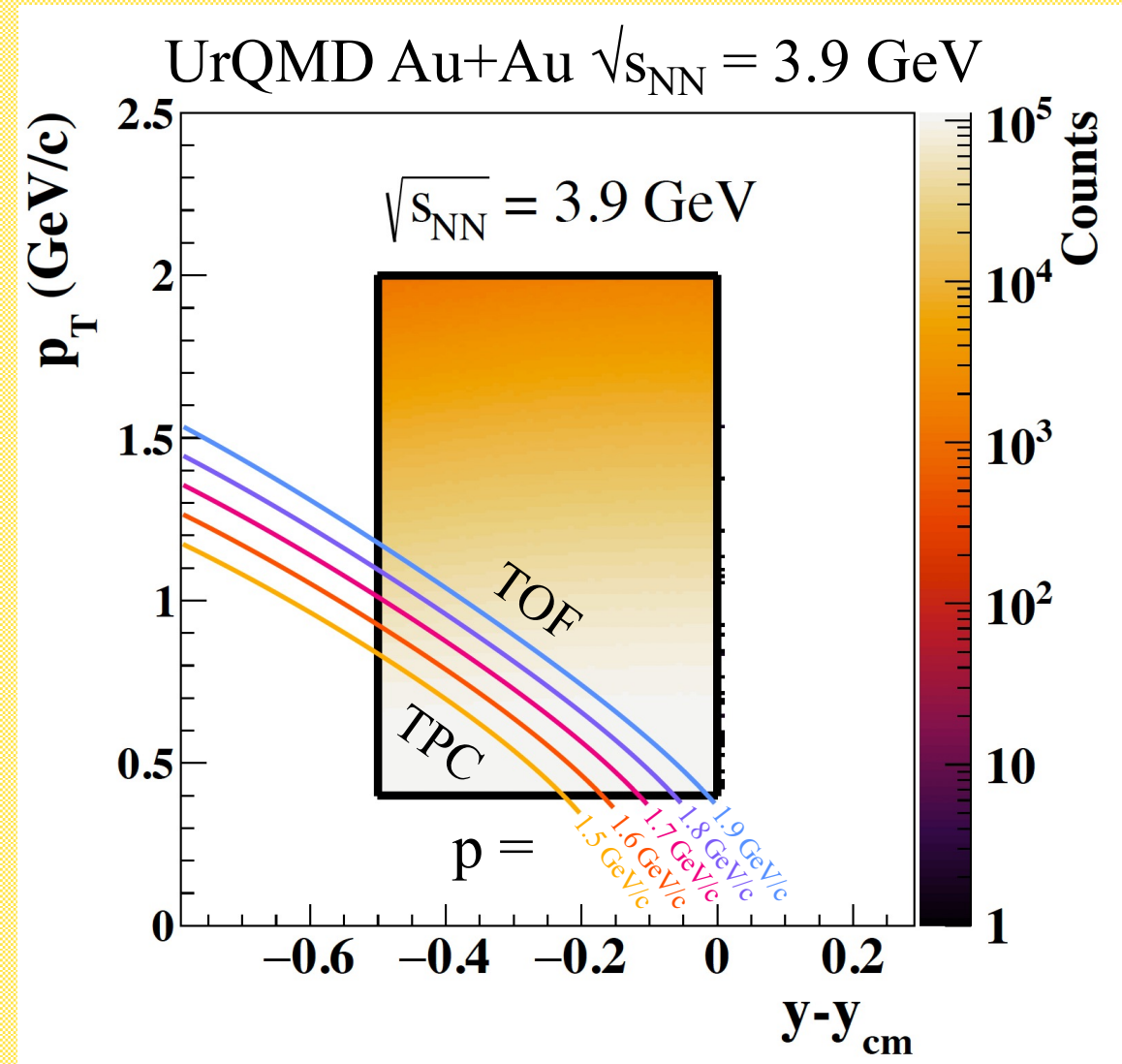
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- In toy model, we can similarly set a momentum above which we require TOF
- We can test effect of various momentum cutoffs

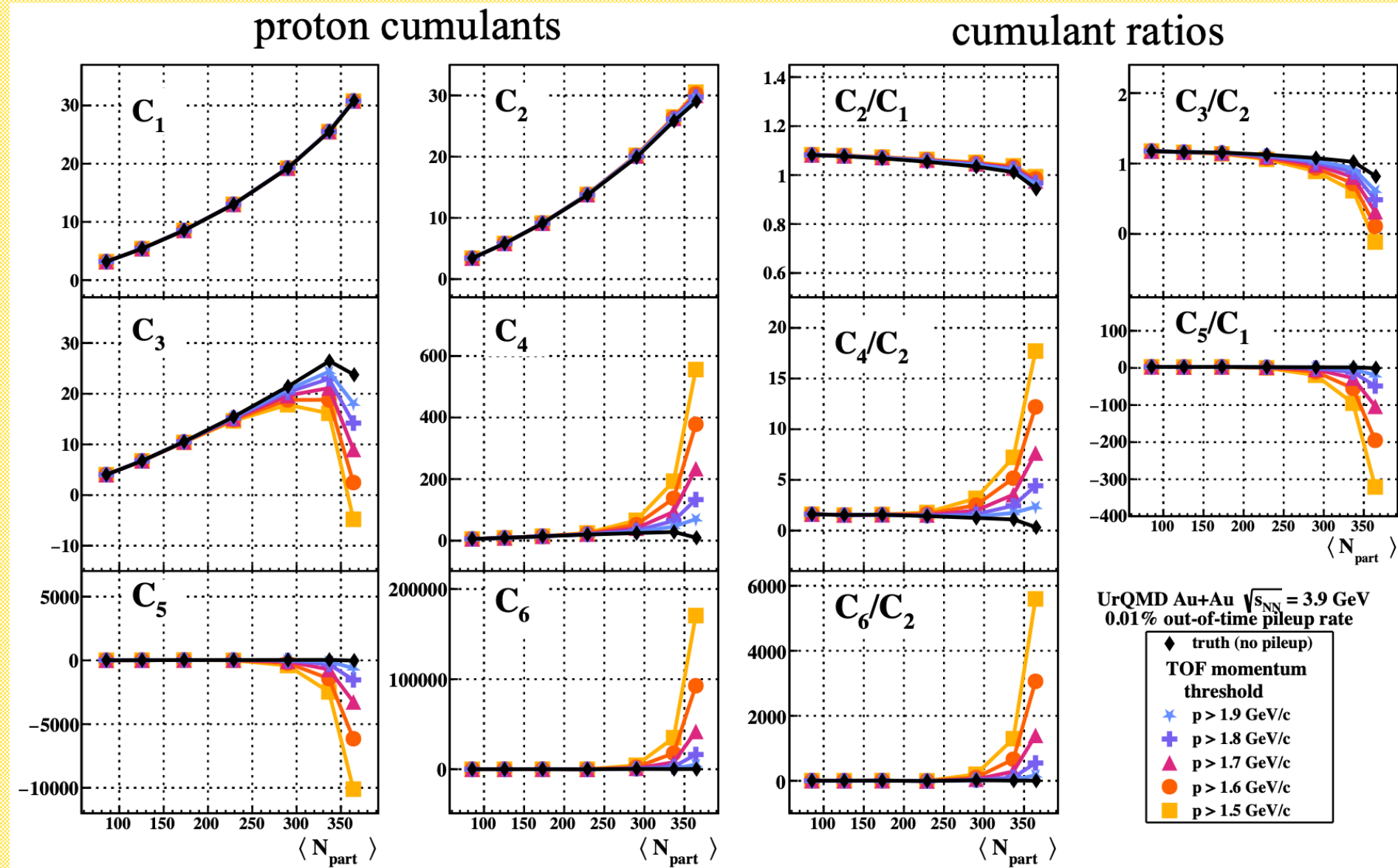


A More Realistic Toy Model

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- At some momentum, we switch from using the TPC to additionally requiring TOF
- In toy model, we can similarly set a momentum above which we require TOF
- We can test effect of various momentum cutoffs
- Strategy:
 - toy TPC measures multiplicity
 - toy TPC measures protons beneath momentum cutoff. Above cutoff, use toy TOF (no pileup)



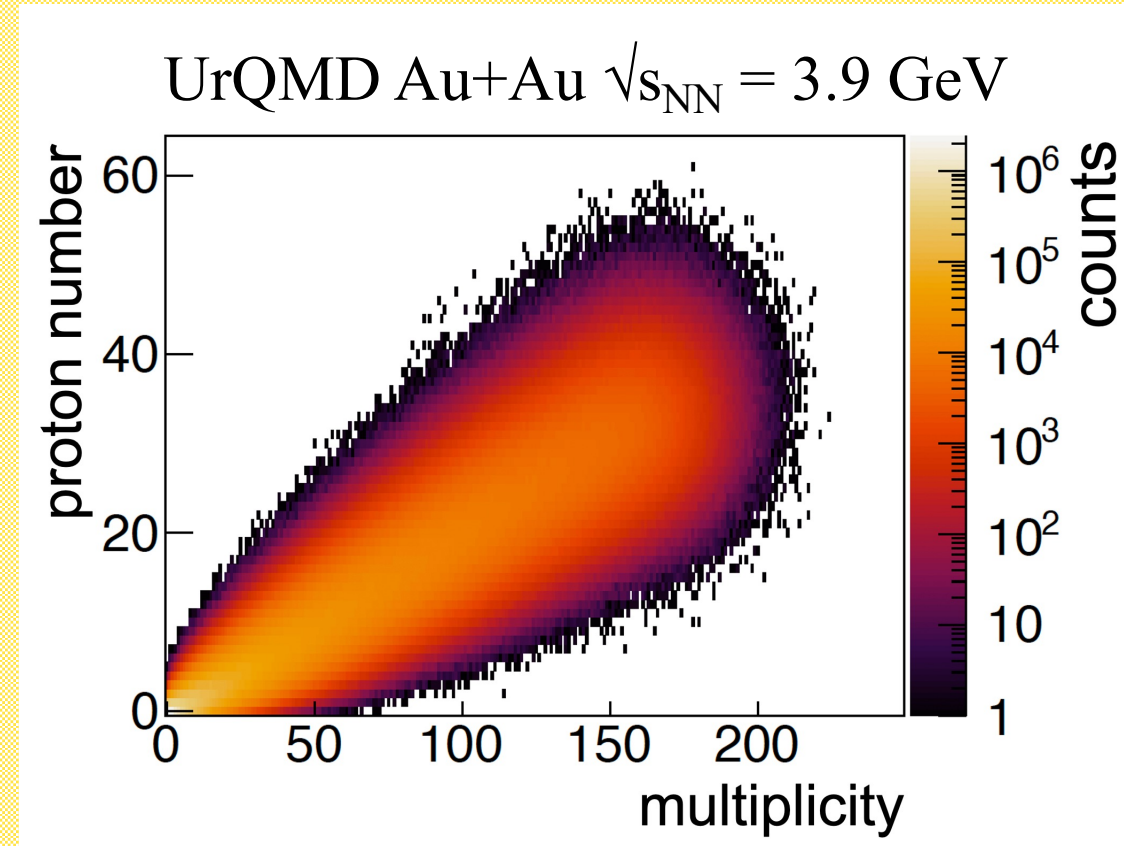
Cumulant Instability in the Toy Model



How to Measure a False Critical Point

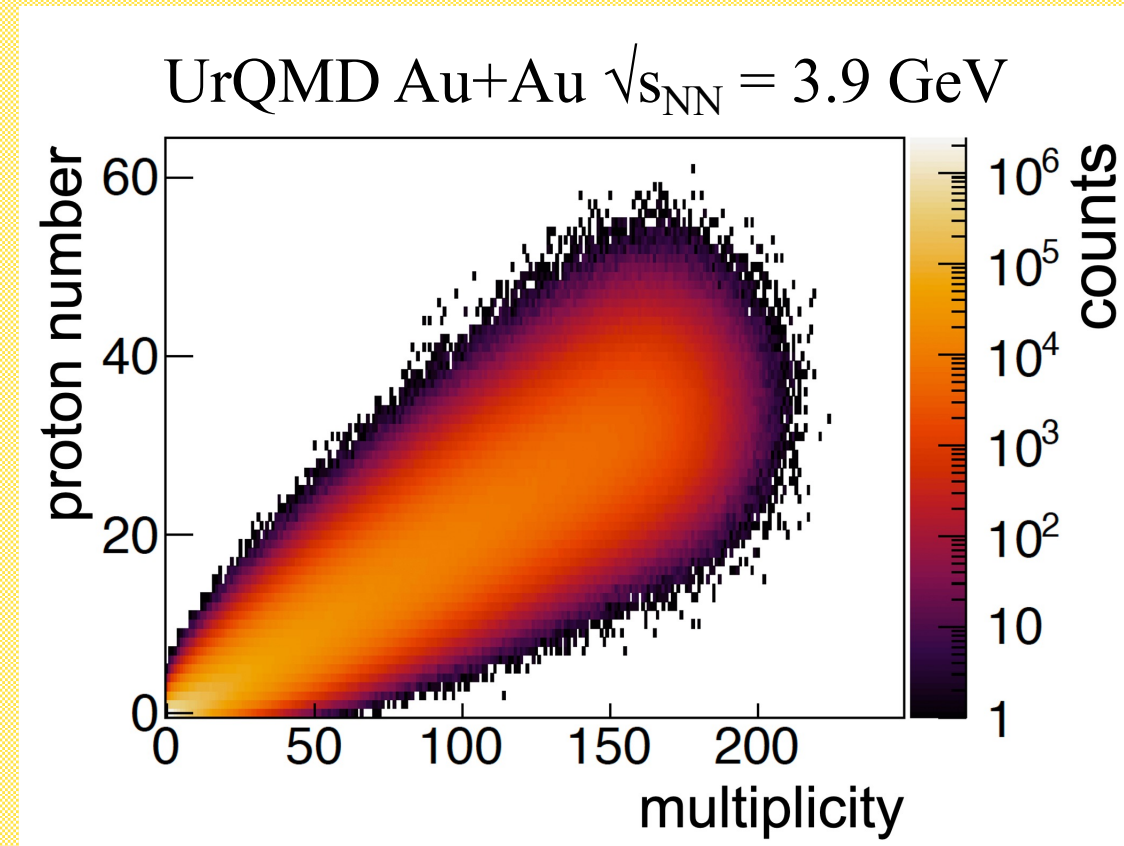
How to Measure a False Critical Point

- Again, the correlation between the multiplicity and proton number is the entire measurement



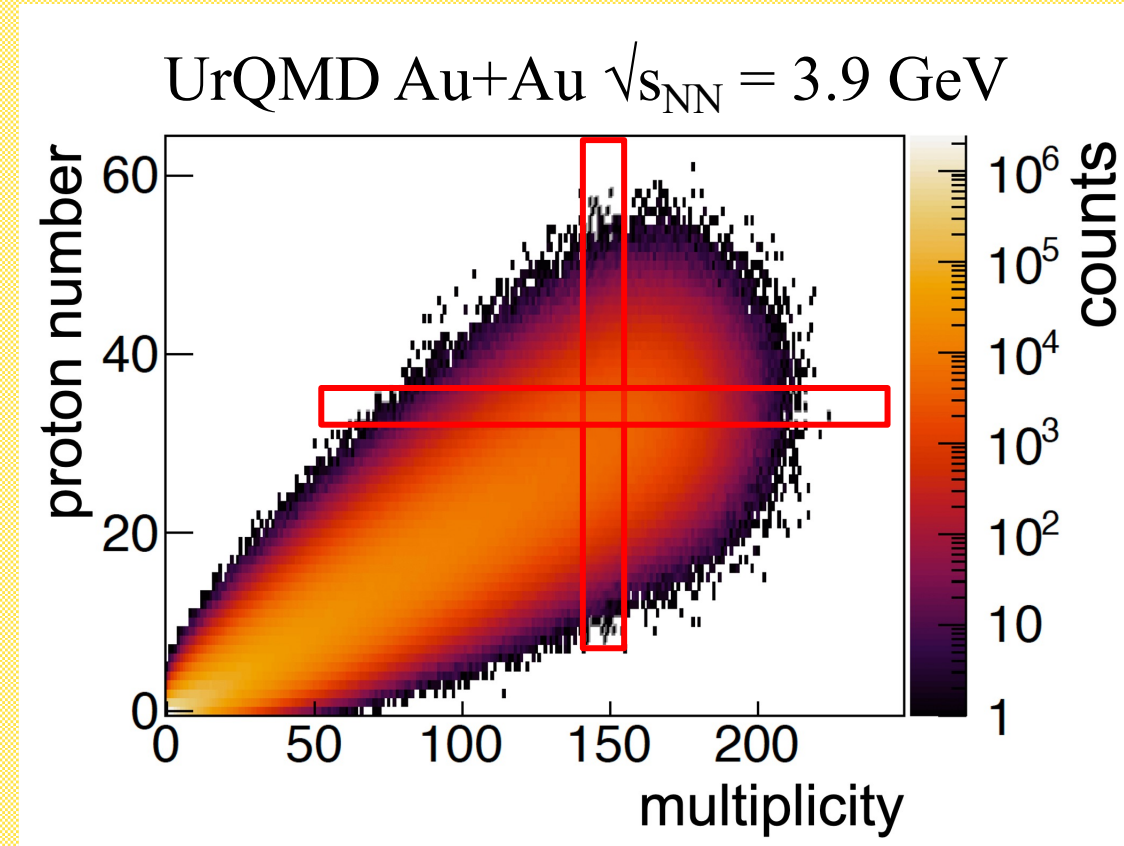
How to Measure a False Critical Point

- Again, the correlation between the multiplicity and proton number is the entire measurement
- To make a false signal



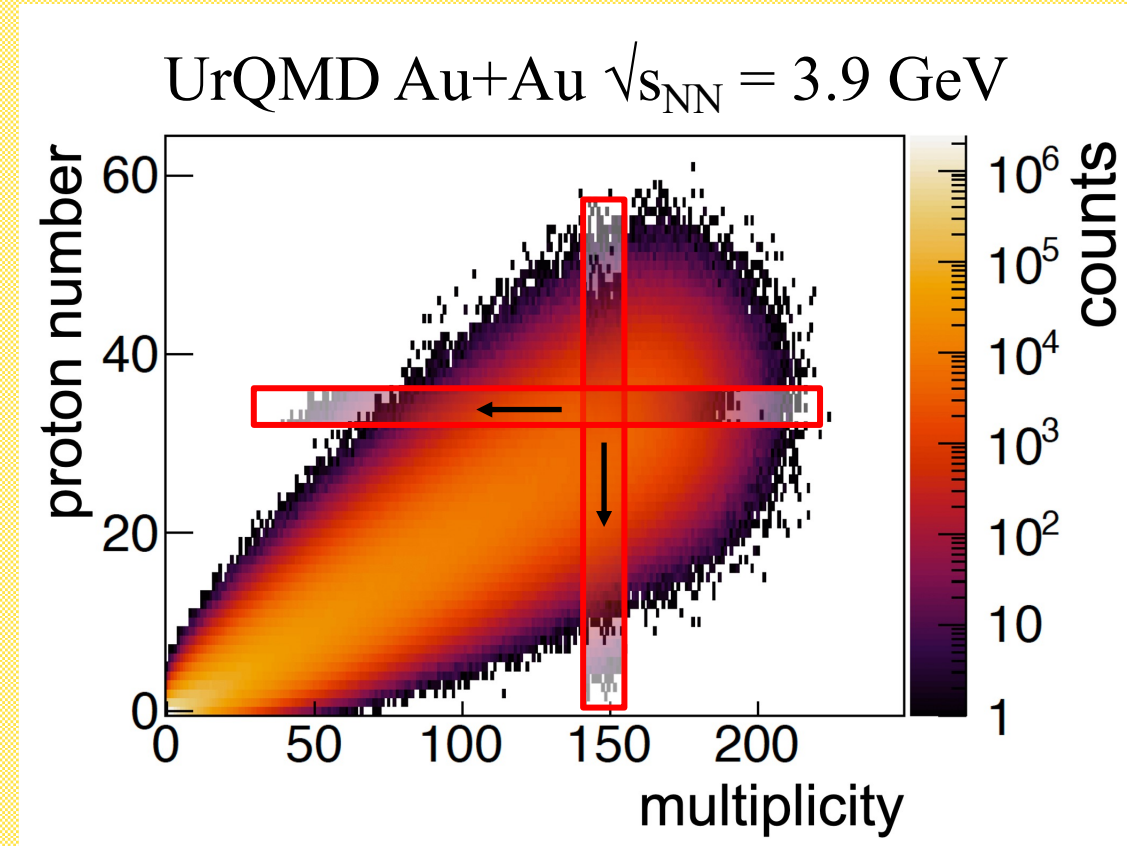
How to Measure a False Critical Point

- Again, the correlation between the multiplicity and proton number is the entire measurement
- To make a false signal
 1. Measure multiplicity and proton number with two different detectors/methods

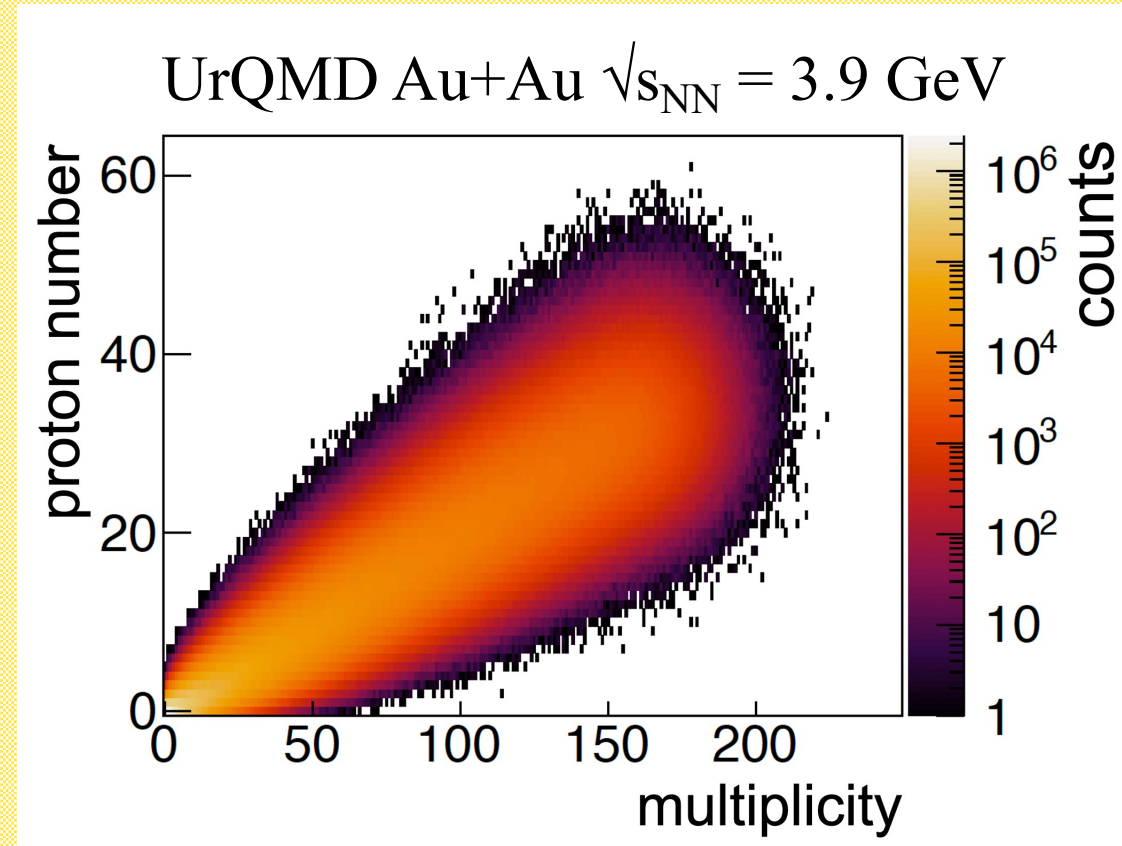


How to Measure a False Critical Point

- Again, the correlation between the multiplicity and proton number is the entire measurement
- To make a false signal
 1. Measure multiplicity and proton number with two different detectors/methods
 2. Ensure that a small fraction of the time, the methods become uncorrelated

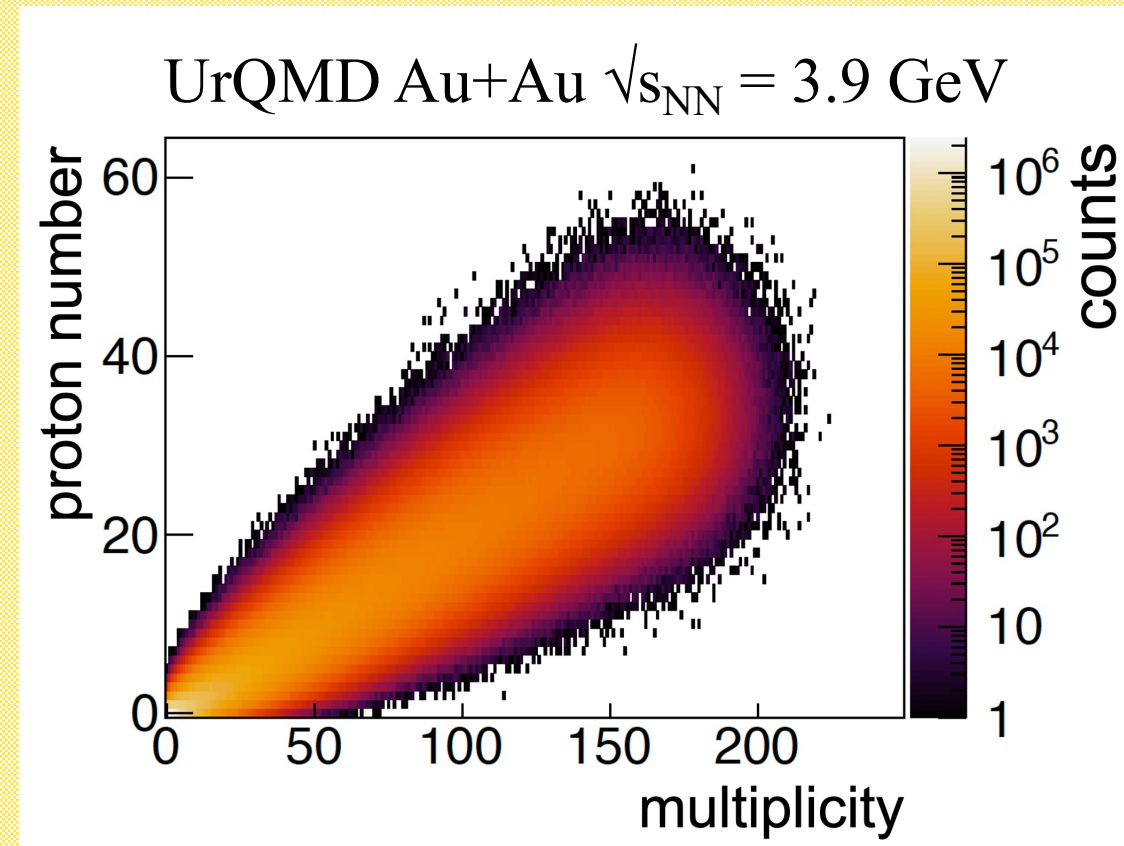


How *NOT* to Measure a False Critical Point



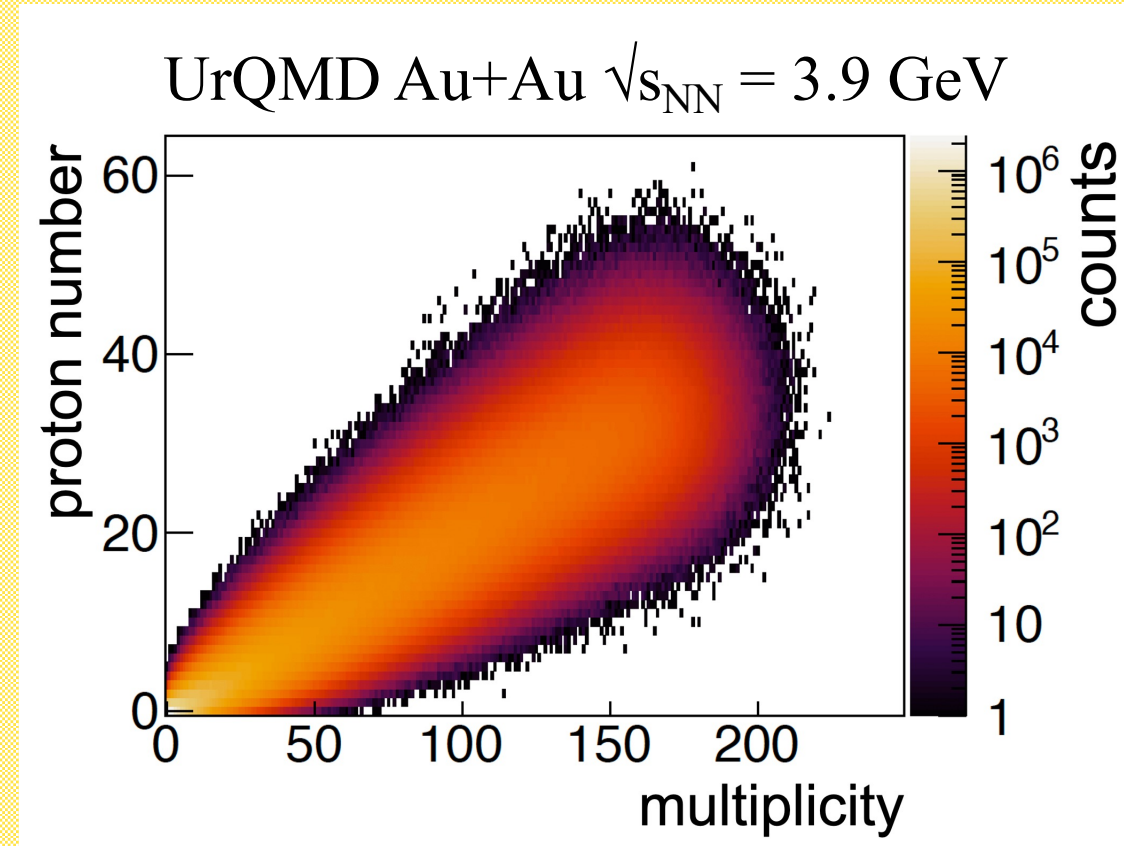
How *NOT* to Measure a False Critical Point

- To make a robust signal



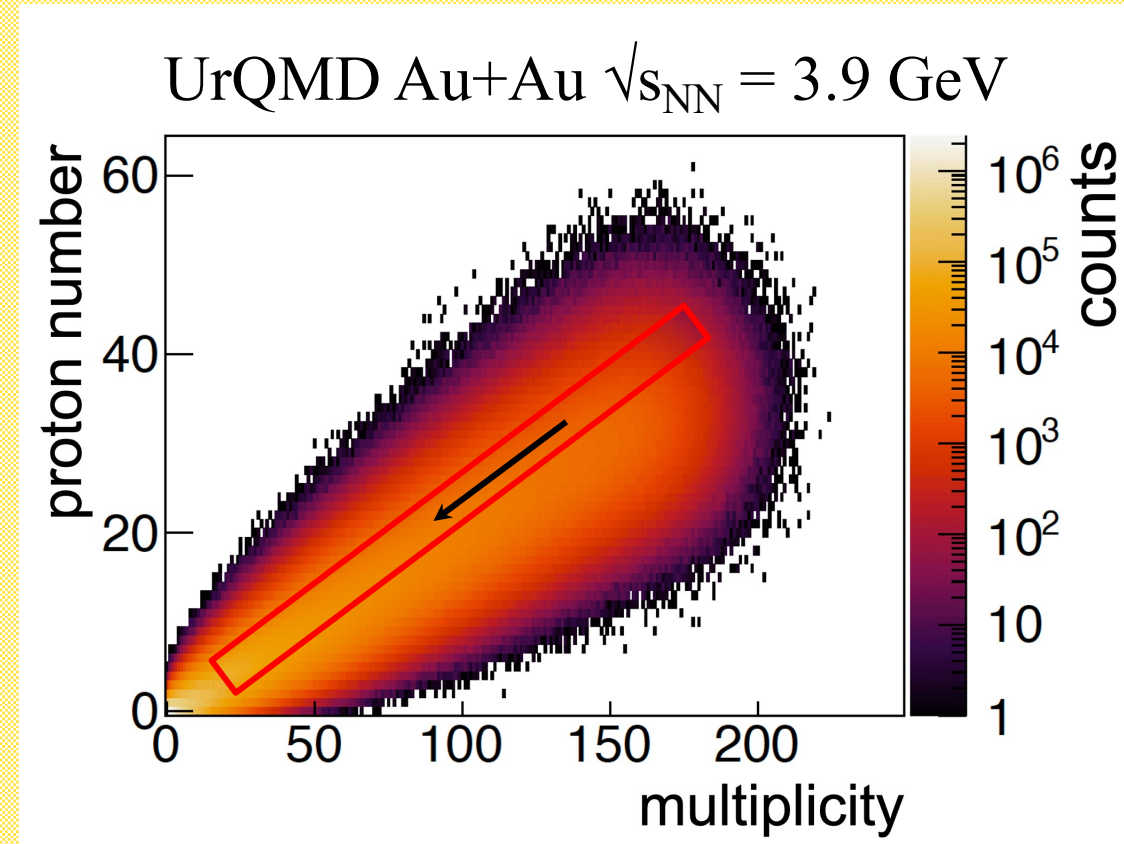
How *NOT* to Measure a False Critical Point

- To make a robust signal
 1. Maximize the similarities between multiplicity and proton-number measurements



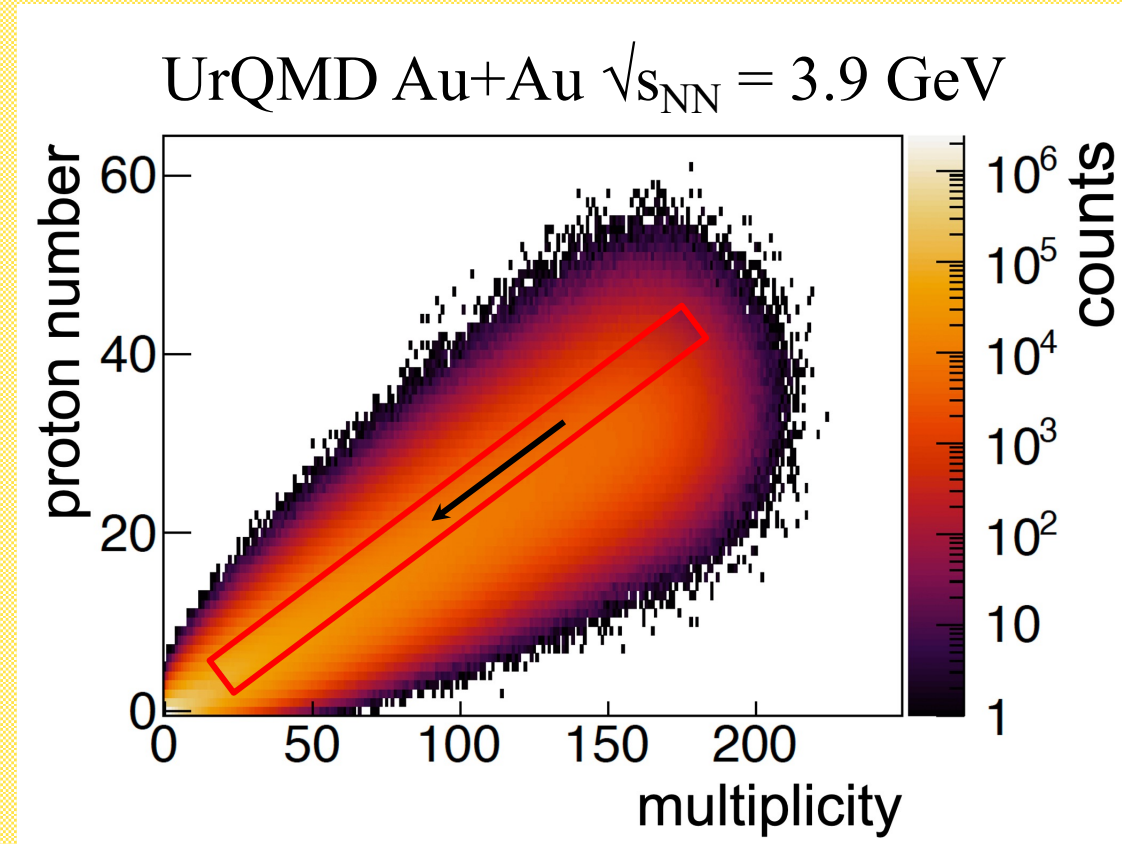
How *NOT* to Measure a False Critical Point

- To make a robust signal
 1. Maximize the similarities between multiplicity and proton-number measurements
- If both measurements fail, they fail in the same way

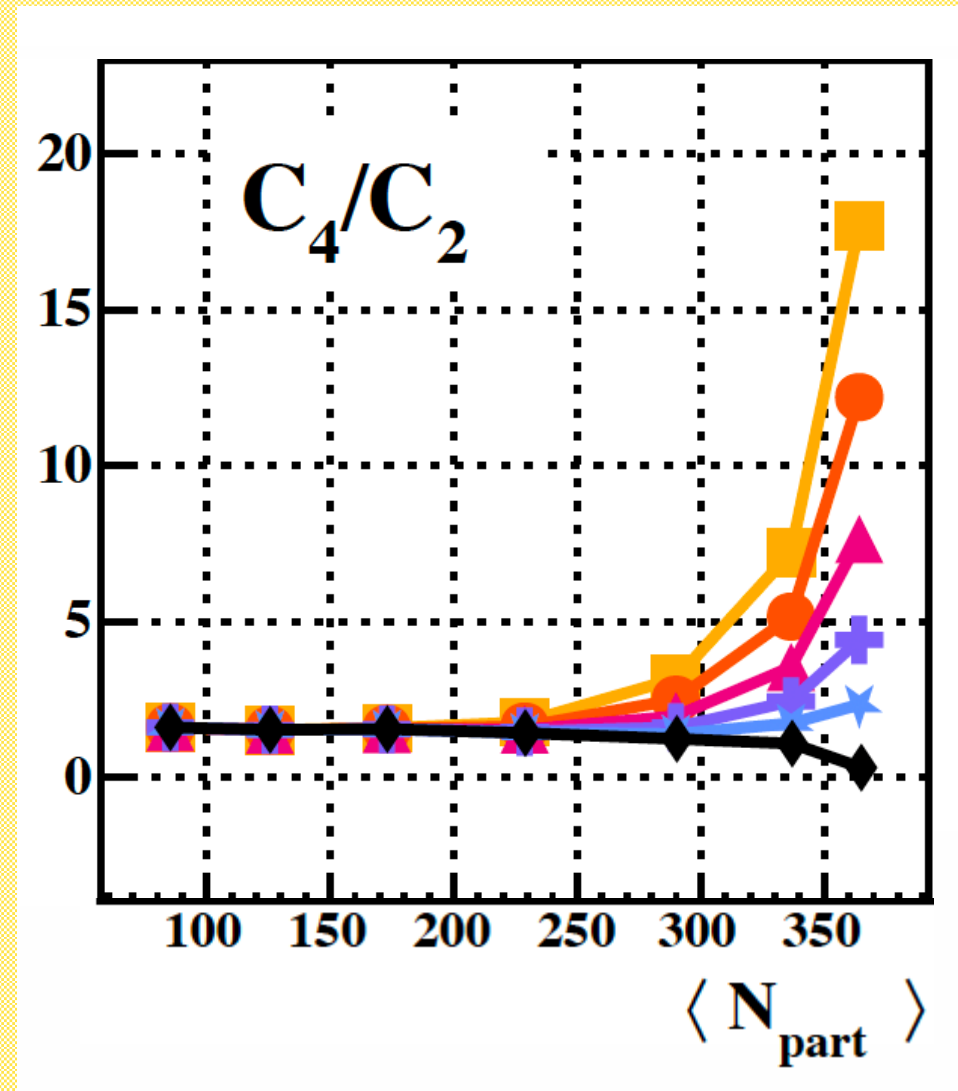


How *NOT* to Measure a False Critical Point

- To make a robust signal
 1. Maximize the similarities between multiplicity and proton-number measurements
- If both measurements fail, they fail in the same way
- Nearly all non-physics fluctuations are suppressed!

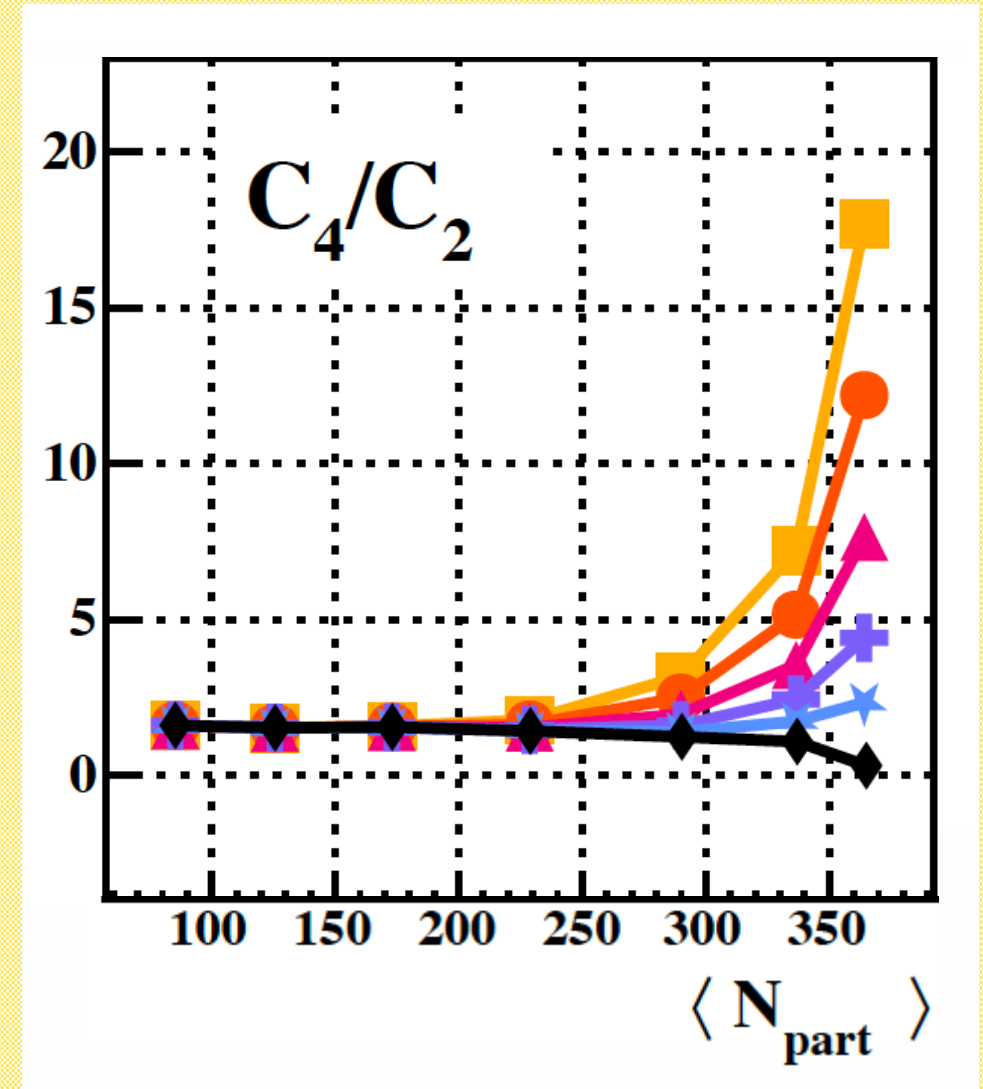


Conclusions



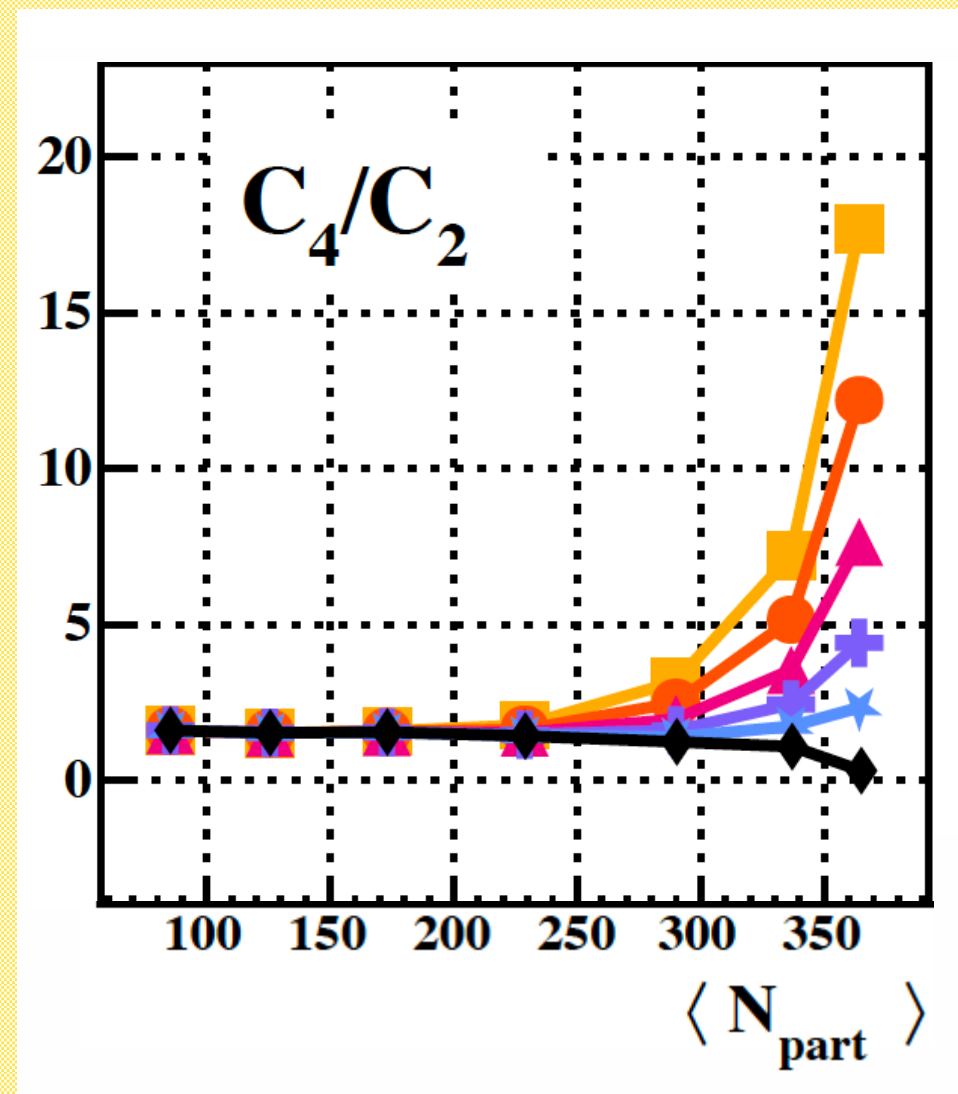
Conclusions

- The largest background of high-order cumulants is rare and spontaneous decorrelations between measurements of multiplicity and proton number



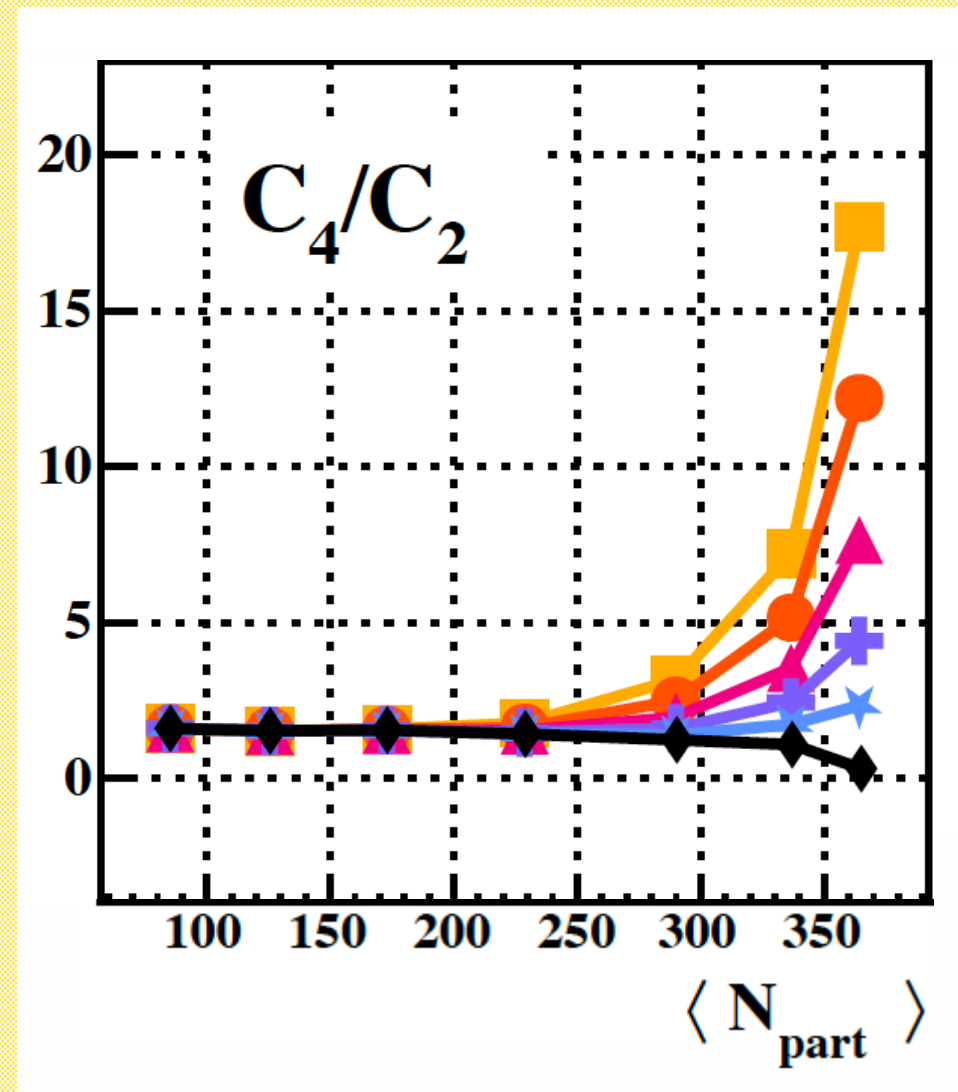
Conclusions

- The largest background of high-order cumulants is rare and spontaneous decorrelations between measurements of multiplicity and proton number
- These rare and spontaneous decorrelations can be caused by



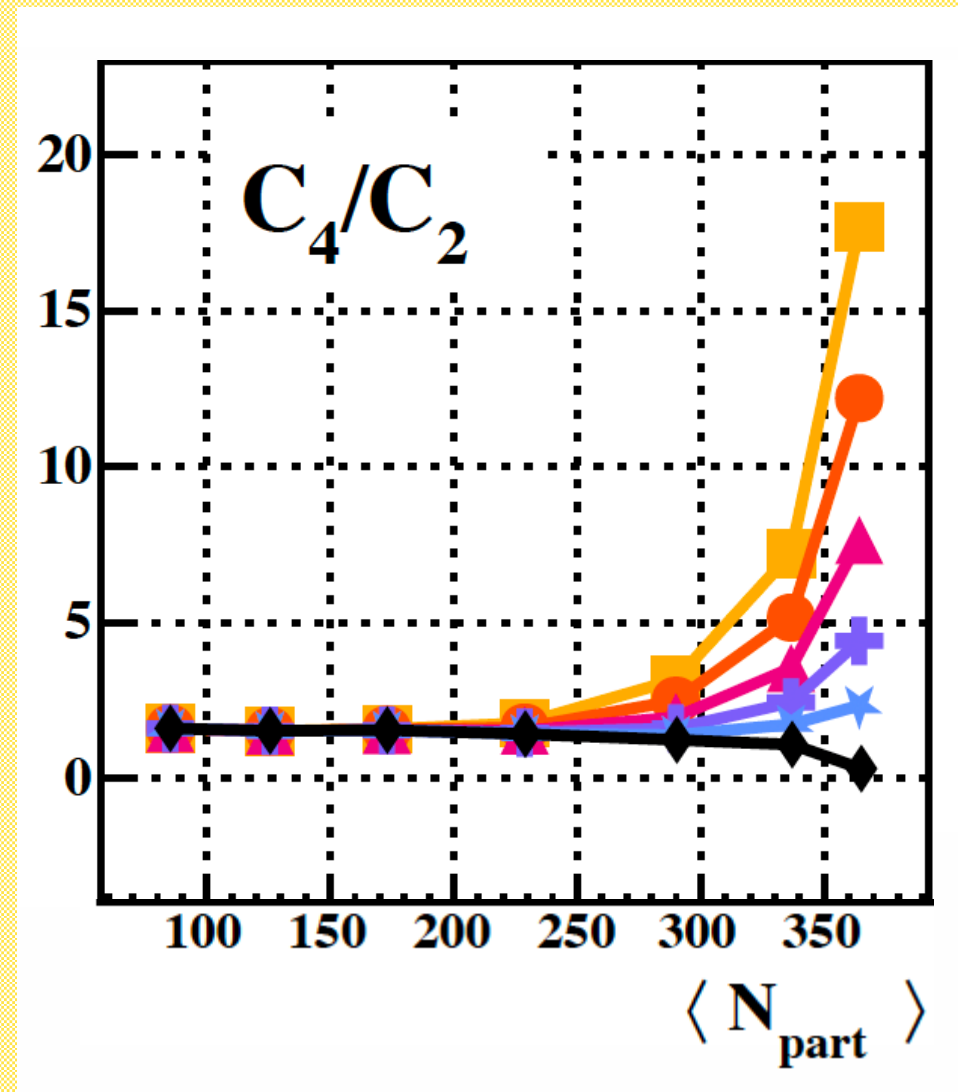
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 - ✓ algorithmic failure



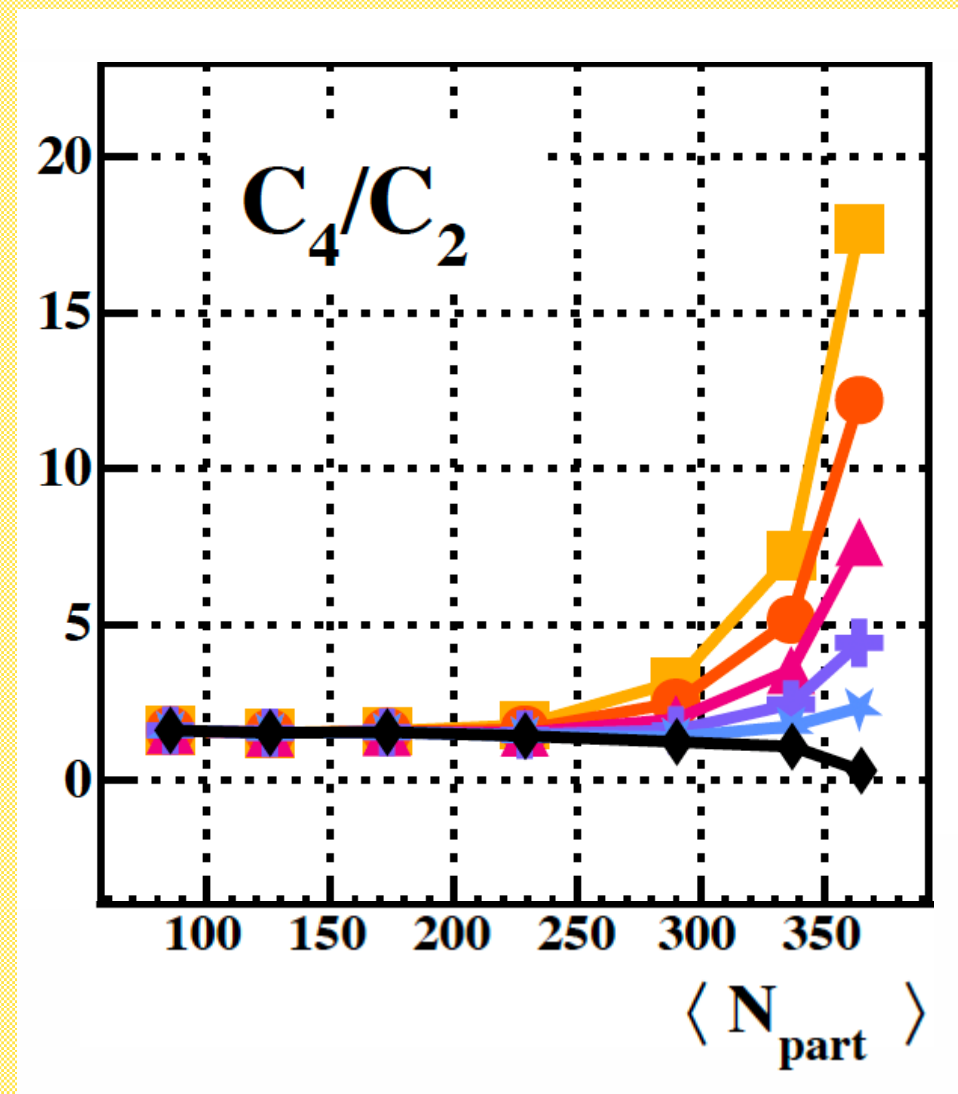
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 - ✓ algorithmic failure
 - ✓ fluctuations in acceptance



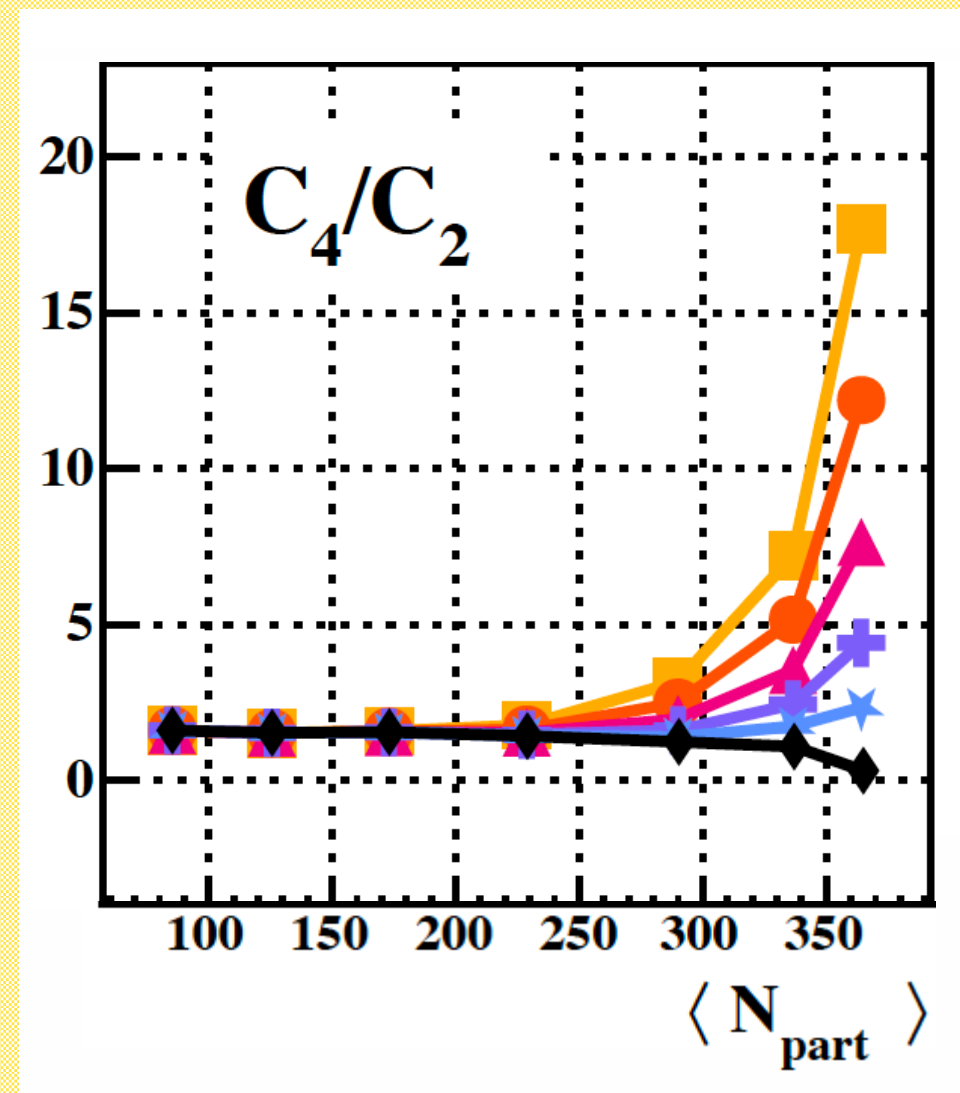
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 - ✓ pileup



Conclusions

- The largest background of high-order cumulants is rare and spontaneous decorrelations between measurements of multiplicity and proton number
- These rare and spontaneous decorrelations can be caused by
 - ✓ algorithmic failure
 - ✓ fluctuations in acceptance
 - ✓ pileup
- These can be suppressed by maximizing similarities between proton number and multiplicity measurements



Thank you!