Statistics, Data, and the Philosophy of Science

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1) Philosophy of Science: Questions for science. How to investigate?
2) Data: catchall for observations.
3) Statistics: Quantities to aid in interpreting data.
4) Hypothesis: Predicted behavior (of a statistic).

Example. Is my coin fair?

Philosophy of science: ? Question
Hypothesis: Do lots of coin flips, count heads, tails.
Expect: roughly equal Hs/Ts, if fair.

Data: 100 flips, get 41 heads, 59 tails.

Hypothesis: Each flip is a Bernoulli trials

$$X_i \sim \text{Bernoulli}\left(\frac{1}{2}\right) \qquad E(\bar{X}_i) = \frac{1}{2} \qquad \sigma(\bar{X}_i) = \frac{1}{2}$$

Central Limit Thm: $\boxed{\bar{X}} \sim \text{Normal}\left(E(\bar{X}_i), \frac{\sigma^2(\bar{X}_i)}{N}\right)$

$$= \text{Normal}\left(\frac{1}{2}, \frac{1}{400}\right)$$

Statistics: Heads = 1, Tails = 0

$\bar{x} = 0.41$

The p-value is a common metric to compare a statistic with a hypothesis. Specifically, the p-value is the probability that a more extreme data value is observed

$$p = 2\Phi\left[\frac{\bar{x}-\mu}{\sigma/\sqrt{N}}\right] = 2\Phi\left[\frac{0.41 - \frac{1}{2}}{1/20}\right] = 0.072$$

Interpretations:
- $p < 0.05$: statistical significance
  data is inconsistent with hypothesis
- $p > 0.05$: not statistical significance
  data is not inconsistent

Counterfactual: 2p Hs, 60 Ts. $p = 0.046$

p-hacking: "massaging" data to get $p < 0.05$

data dredging

Science estimates $P(H|D)$: probability that the hypothesis H is valid, given data D.

p-value: $P(D|H)$

If A, then B. Contrapositive: ~~If not B, then not A.~~

$P(H|D)$: If D, then H.

If not H_1, then not D;

$$P(H|D) = P(D^c|H^c)$$

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