CniPol Meeting
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The model utilized to determine the target polarization at HJET

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Outlook

- **The method used to determine the jet target polarization is reviewed.**
- **The main question behind the consideration:**

  is the polarization value $\boldsymbol{P_{\mathrm{jet}} = 0.958 \pm 0.001}$, evaluated in 2004 is still valid?

The Square Root Formula

If unpolarized proton beam is scattered off a vertically polarized target and recoil protons are detected in left/right symmetric detectors, the spin asymmetry can be calculated as

$$\boldsymbol{a} = \frac{N_R^+ - N_L^+}{N_R^+ + N_L^+} = A_\mathrm{N} P$$

The spin flip, allows us to eliminate the systematic errors due to possible acceptance asymmetry ϵ.

$$\begin{aligned} N_R^+ &= N_0(1+a)(1+\lambda)(1+\epsilon) \\ N_R^- &= N_0(1-a)(1-\lambda)(1+\epsilon) \\ N_L^+ &= N_0(1-a)(1+\lambda)(1-\epsilon) \\ N_L^- &= N_0(1+a)(1-\lambda)(1-\epsilon) \end{aligned}$$

The systematic error free solution if
$P_+ = P_-$, $A_\mathrm{N}^{(R)} = A_N^{(L)}$, **and** $\epsilon_+ = \epsilon_-$

$$\boldsymbol{a} = \frac{\sqrt{N_R^+ N_L^-} - \sqrt{N_L^+ N_R^-}}{\sqrt{N_R^+ N_L^-} + \sqrt{N_L^+ N_R^-}}$$

and similar for $\boldsymbol{\lambda}$ and $\boldsymbol{\epsilon}$

If $P_\pm = P \pm \delta P$ $\quad (a_\pm = a \pm \delta a)$:

$$\begin{aligned} N_R^+ &= N_0(1+a+\delta a)(1+\lambda)(1+\epsilon) \\ N_R^- &= N_0(1-a+\delta a)(1-\lambda)(1+\epsilon) \\ N_L^+ &= N_0(1-a-\delta a)(1+\lambda)(1-\epsilon) \\ N_L^- &= N_0(1+a-\delta a)(1-\lambda)(1-\epsilon) \end{aligned}$$

$$\begin{aligned} a_\mathrm{calc} &\approx a &&= A_\mathrm{N} P \\ \epsilon_\mathrm{calc} &\approx \epsilon + \delta a &&= \epsilon + A_\mathrm{N} \delta P \\ \lambda_\mathrm{calc} &\approx \lambda \end{aligned}$$

For the recoil spectrometer, only average value of the jet target spin up and down polarizations, $P = (|P_+| + |P_-|)/2$, is essential.

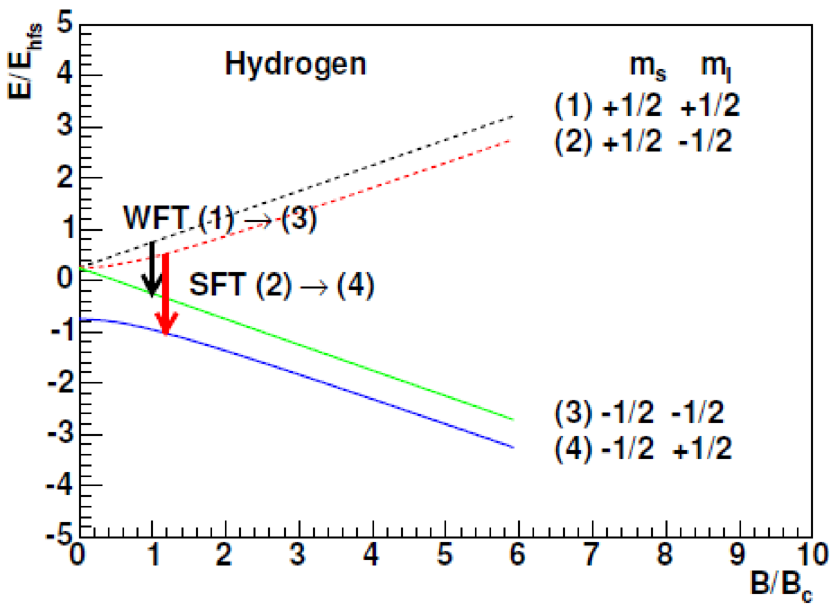
Hyperfine states of the atomic hydrogen

The hyperfine (discriminating by electron ↑↓ and proton ± spins) states of the S-wave ground states of hydrogen atom:

$$\psi_1 = |\uparrow +\rangle$$
$$\psi_2 = \cos\theta\ |\uparrow -\rangle + \sin\theta\ |\downarrow +\rangle$$
$$\psi_3 = |\downarrow -\rangle$$
$$\psi_4 = \cos\theta\ |\downarrow +\rangle - \sin\theta\ |\uparrow -\rangle$$

$\tan 2\theta = B_c/B$
$B_c = 50.7\ \mathrm{mT}$
$B = 120\ \mathrm{mT}$ **(the holding field)**

$$\cos 2\theta = \frac{B_{hold}}{\sqrt{B_{hold}^2 + B_c^2}} = 0.921$$



The hyperfine states flow

Dissociator	$n_1\psi_1 + n_2\psi_2 + n_3\psi_3 + n_4\psi_4$
Separating magnet	$n_1\psi_1 + n_2\psi_2$

RF transitions:

SFT=ON WFT=OFF	$n_1\epsilon_{1\to3}\psi_1 + n_2\psi_2 + n_1(1-\epsilon_{1\to3})\psi_3$	$\Rightarrow P_-$
SFT=OFF WFT=ON	$n_1\psi_1 + n_2\epsilon_{2\to4}\psi_2 + n_2(1-\epsilon_{2\to4})\psi_4$	$\Rightarrow P_+$
SFT=ON WFT=ON	$n_1\epsilon_{1\to3}\psi_1 + n_2\epsilon_{2\to4}\psi_2 + n_1(1-\epsilon_{1\to3})\psi_3 + n_2(1-\epsilon_{2\to4})\psi_4$	$\Rightarrow P_0$
SFT=OFF WFT=OFF	$n_1\psi_1 + n_2\psi_2$	

Separating Magnet (BRP)

SFT=ON WFT=OFF	$n_1\epsilon_{1\to3}\psi_1 + n_2\psi_2$	$\Rightarrow m_-$
SFT=OFF WFT=ON	$n_1\psi_1 + n_2\epsilon_{2\to4}\psi_2$	$\Rightarrow m_+$
SFT=ON WFT=ON	$n_1\epsilon_{1\to3}\psi_1 + n_2\epsilon_{2\to4}\psi_2$	$\Rightarrow m_0$
SFT=OFF WFT=OFF	$n_1\psi_1 + n_2\psi_2$	

$\epsilon_{1\to3}$ and $\epsilon_{2\to4}$ are inefficiencies of the RF transitions
$P_{-,+,0}$ are jet polarizations in the scattering chamber
$m_{-,+,0}$ are counts in the Breit-Rabi polarimeter

The Jet Polarization

$$-P_- = \frac{n_1 + n_2\cos 2\theta - 2n_1\epsilon_{1\to3}}{n_1+n_2} \approx 96\%$$

$$P_+ = \frac{n_1 + n_2\cos 2\theta - 2n_2\cos 2\theta\,\epsilon_{2\to4}}{n_1+n_2} \approx 96\%$$

$$-P_0 = \frac{n_1 - n_2\cos 2\theta - 2n_1\epsilon_{1\to3} + 2n_2\cos 2\theta\,\epsilon_{2\to4}}{n_1+n_2} \approx 4\%$$

The (fixed) typos in Hiromi's thesis are highlighted.

The BRP counts

$$m_- = n_1\epsilon_{1\to3} + n_2$$

$$m_+ = n_1 + n_2\epsilon_{2\to4}$$

$$m_0 = n_1\epsilon_{1\to3} + n_2\epsilon_{2\to4}$$

$$\epsilon_{1\to3} = \frac{m_-/(m_+ - m_0) - n_2/n_1}{1 + m_-/(m_+ - m_0)}$$

$$\epsilon_{2\to4} = \frac{m_+/(m_- - m_0) - n_1/n_2}{1 + m_+/(m_- - m_0)}$$

It is hardcoded in **HjetPolManager**:
$\cos 2\theta = 0.92$, $n_2/n_1 = 1.00239$

Simplified expression for the jet target polarization

$$\boldsymbol{P}_{\mathbf{jet}} = \frac{|P_+| + |P_-|}{2} = 1 - \frac{n_2}{n_1 + n_2} \times [1 - \cos 2\theta + \epsilon_{1\to3} + \epsilon_{2\to4} \cos 2\theta]$$

$$= \frac{\mathbf{1 + \cos 2\theta}}{\mathbf{2}} - \frac{\boldsymbol{m_0}}{\boldsymbol{m_+ + m_-}} + \mathcal{O}(\mathbf{0.01\%})$$

$\mathbf{96.1\%}$
(defined by the holding field magnet)

$\sim\mathbf{0.3\%}$
(measured by the Breit-Rabi polarimeter)

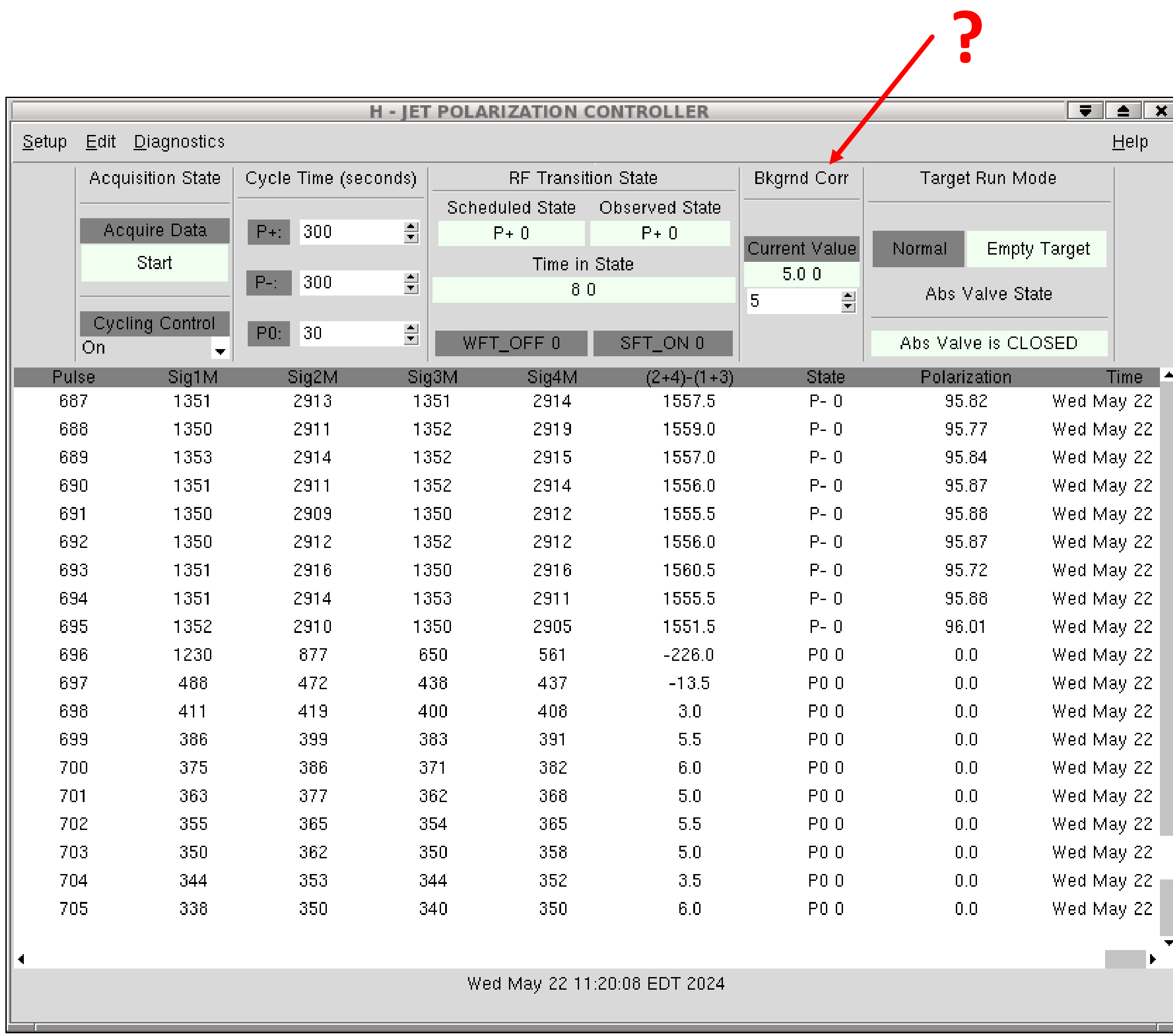
- **The HJET Breit-Rabi polarimeter does not measure the jet target polarization.**
- **It only monitors average RF transition inefficiency (which is small)**
- **Evaluation of $\boldsymbol{m_0 = m_0^{\mathrm{ON}} - m_0^{\mathrm{OFF}} \approx 305 - 300}$ is the dominant (the only) source of the systematic uncertainty in value of the inefficiency.**

jetPolDisplay

$\epsilon = 5.3/(1555 \times 2)$
$= 0.17\%$,

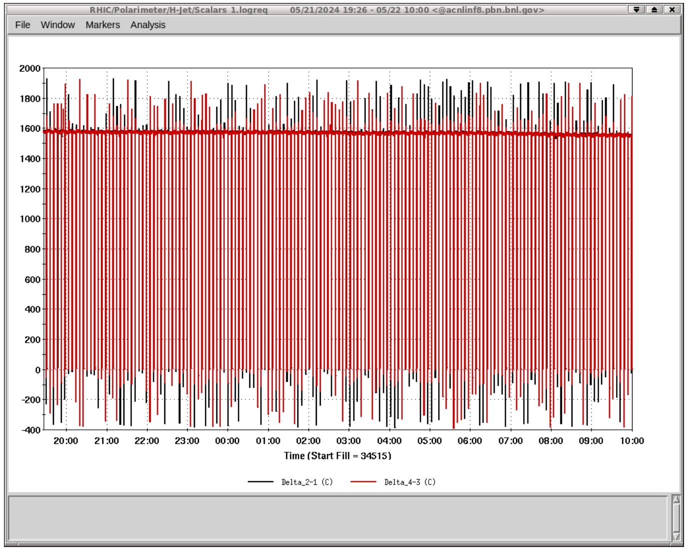
but what actually is Bkrnd. Corr (5.0), which have reduced ϵ by $\epsilon \rightarrow \epsilon - 0.15\%$?

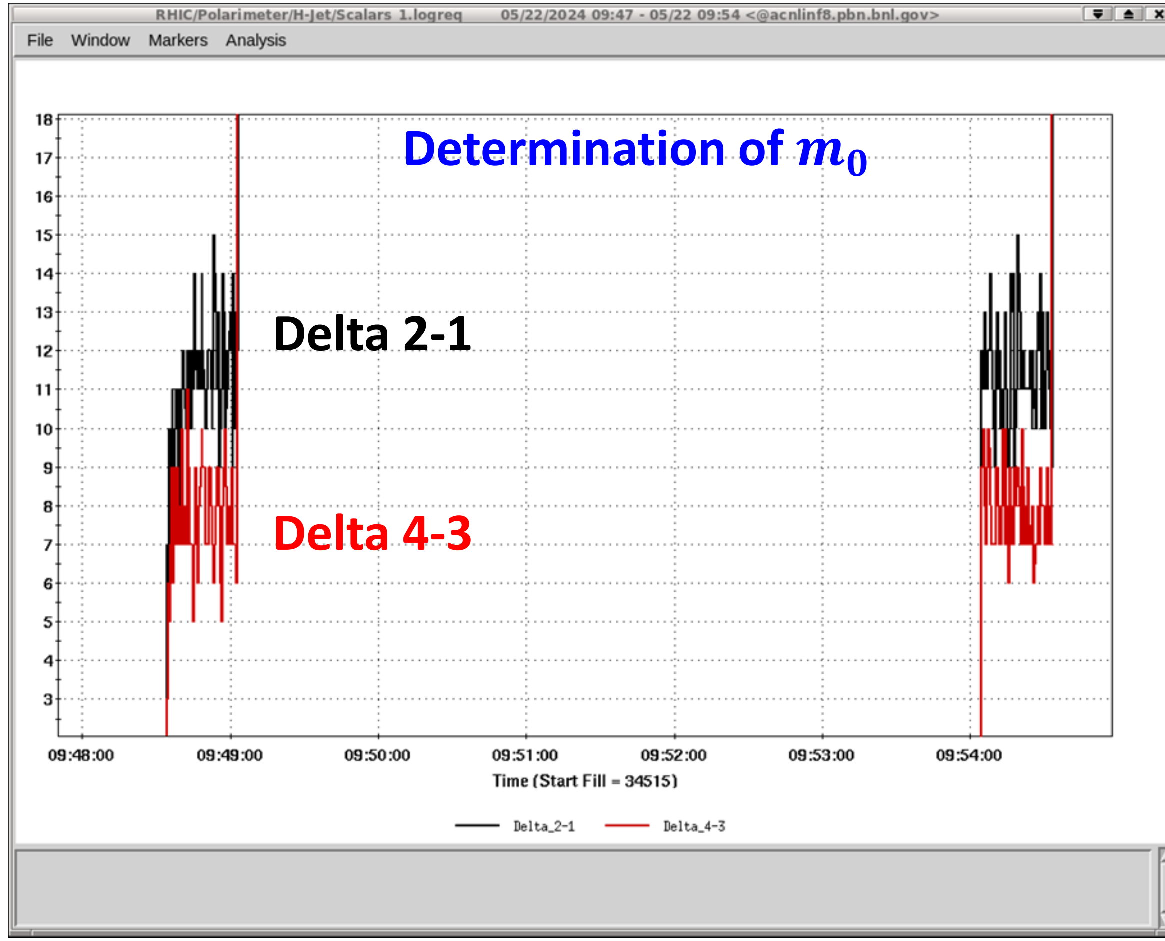
Value of ϵ, with or without Bkrnd. Corr., is in good agreement with that determined 20 years ago.



Pulse	Sig1M	Sig2M	Sig3M	Sig4M	(2+4)-(1+3)	State	Polarization	Time
687	1351	2913	1351	2914	1557.5	P- 0	95.82	Wed May 22
688	1350	2911	1352	2919	1559.0	P- 0	95.77	Wed May 22
689	1353	2914	1352	2915	1557.0	P- 0	95.84	Wed May 22
690	1351	2911	1352	2914	1556.0	P- 0	95.87	Wed May 22
691	1350	2909	1350	2912	1555.5	P- 0	95.88	Wed May 22
692	1350	2912	1352	2912	1556.0	P- 0	95.87	Wed May 22
693	1351	2916	1350	2916	1560.5	P- 0	95.72	Wed May 22
694	1351	2914	1353	2911	1555.5	P- 0	95.88	Wed May 22
695	1352	2910	1350	2905	1551.5	P- 0	96.01	Wed May 22
696	1230	877	650	561	-226.0	P0 0	0.0	Wed May 22
697	488	472	438	437	-13.5	P0 0	0.0	Wed May 22
698	411	419	400	408	3.0	P0 0	0.0	Wed May 22
699	386	399	383	391	5.5	P0 0	0.0	Wed May 22
700	375	386	371	382	6.0	P0 0	0.0	Wed May 22
701	363	377	362	368	5.0	P0 0	0.0	Wed May 22
702	355	365	354	365	5.5	P0 0	0.0	Wed May 22
703	350	362	350	358	5.0	P0 0	0.0	Wed May 22
704	344	353	344	352	3.5	P0 0	0.0	Wed May 22
705	338	350	340	350	6.0	P0 0	0.0	Wed May 22

LogView





- **Brnd. Corr.** are not applied in LogView
- **Is the discrepancy between Delta_2-1 and Delta_4-3 due to a drawback in the chopper design?**
- **If so, the systematic error (Bgrnd.Corr.) of about 5 is not unreasonable.**

Could systematic errors in value of m_0 be improved?

Is second RF available?

$$m_0 = n_1\epsilon_{1\to3} + n_2\epsilon_{2\to4}$$

$$m_0' = n_1\epsilon_{1\to3}\epsilon_{1\to3}^{(2)} + n_2\epsilon_{2\to4}\epsilon_{2\to4}^{(2)} \approx 0$$

Polar.	RF transitions		Second RF		BRP
	SFT	WFT	SFT	WFT	
P_+	OFF	ON	OFF	OFF	m_+
P_0	ON	ON	OFF	OFF	m_0
P_-	ON	OFF	OFF	OFF	m_+
P_0	ON	ON	ON	ON	$m_0' = 0$

$$P_{\text{jet}} = 0.961 - \frac{m_0 - m_0'}{m_+ + m_-}$$

To verify the method, we should compare
$$(m_0 - m_0')_{2-1} = (m_0 - m_0')_{4-3}$$

Magnetic Field

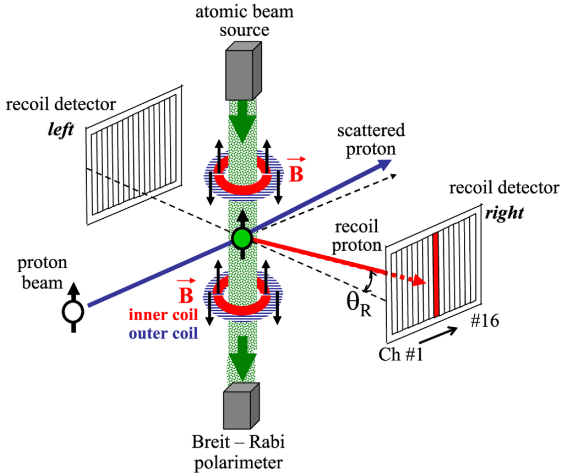


FIG. 1 (color online). Layout of the pp elastic scattering setup with an example of parallel proton spins, $p^{\uparrow}p^{\uparrow} \rightarrow pp$. The target protons cross the RHIC beam from above.

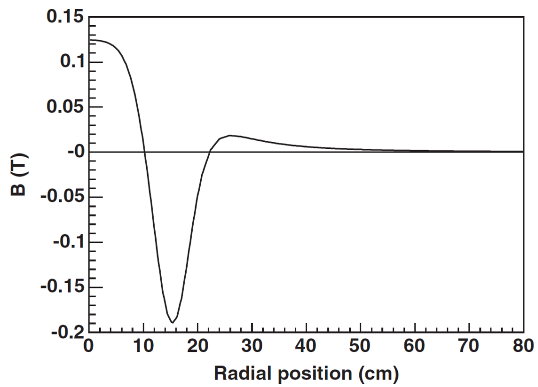
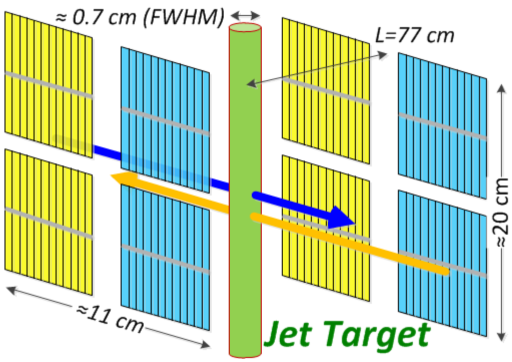


FIG. 3. The H-jet target holding magnetic field calculated by the OPERA program with the experimental setting: inner coil 349 A (N = 56); outer coil 275 A (N = 40). The recoil proton detectors sit at ~78 cm from the H-jet target center.

$$P_{\text{jet}}^{\text{MF}} = \frac{1}{2}\left(1 + \frac{B}{\sqrt{B^2 + B_c^2}}\right) \quad \Longrightarrow \quad \Delta B = 20\ \text{G} \quad \Longrightarrow \quad \Delta P/P = 0.1\%$$

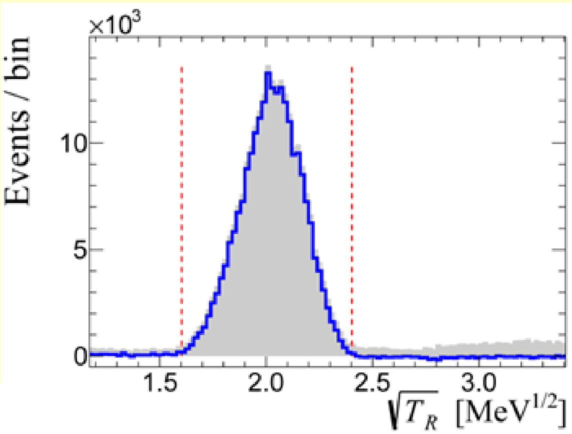
To keep the jet polarization within $\pm 0.1\%$,
(long term) stability of the magnetic field should be $\pm 2\%$

Indirect control for the magnetic field stability

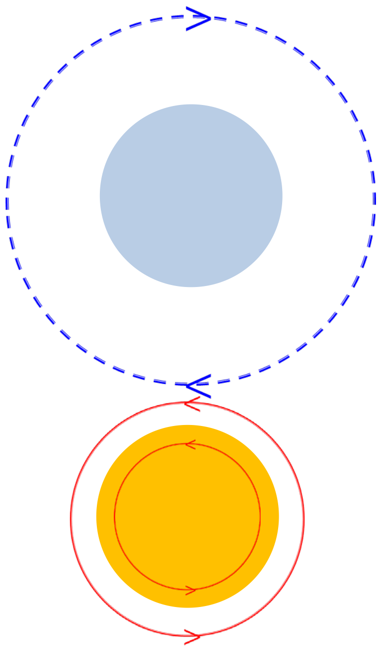


$$z_R = z_{\text{jet}} + L\sqrt{\frac{T_R}{2m_p}\frac{E_{\text{beam}}+m_p}{E_{\text{beam}}-m_p+T_R}} \pm \frac{\boldsymbol{b}_{\mathbf{MF}}}{\sqrt{T_R}}$$

- The magnetic field tracking correction factor $b_{\text{MF}} = \frac{qL}{c} \times \int_0^L \left(1-\frac{r}{L}\right)\frac{B(r)dr}{\sqrt{2m_p}}$ has different signs for left and right detectors
- The currents in the Helmholtz coils were adjusted to eliminate b_{MF}
- Nevertheless, $b_{\text{MF}} \approx 0.7\ \text{mm MeV}^{1/2}$,
- b_{MF} effectively alter $\langle T_R \rangle$ in a Si strip (depending o the strip location)
- No variations of b_{MF} has been detected. However, for numerical estimates more detailed analysis should be done.



The beam magnetic field



Since $\langle B_y \rangle = 0$, this magnetic field should not affect the P_{jet}.

Possible visible effect of the beam magnetic field

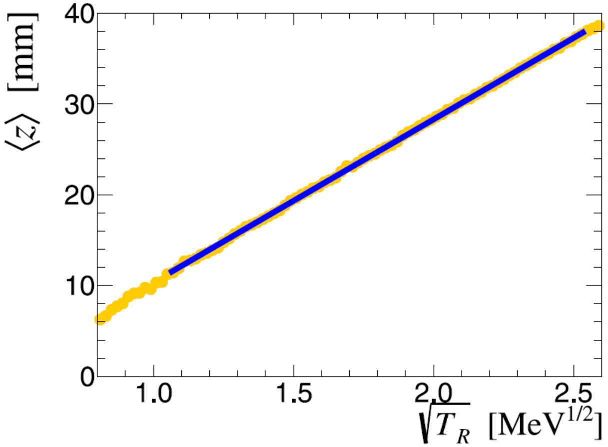


Fig. 12. The average (all HJET detectors) jet center coordinate $\langle z_0 \rangle$ dependence on the recoil proton energy T_R. The solid line indicates the result of linear fit in the 1.1 –6.5 MeV energy range.

- The b_{MF} corrections should cancel due to left/right average.
- Extrapolating to $\sqrt{T_R} \to 0$, one can find the gap between blue and yellow detectors.
- The unexplained result was 17 mm instead of 18 mm measured by a ruler.
- Since $B_y(-x) = -B_y(x)$, the beam magnetic field can **potentially** explain the puzzle.

Summary

- I did not find sufficient reasons to concern about the value of the jet target polarization, ~~$P_{\text{jet}} = 0.958 \pm 0.001$~~ $P_{\text{jet}} = 0.957 \pm 0.001$ and its stability.
- Nevertheless,
  - Better understanding of the systematics in determination of m_0 is needed. In particular, the value of Bgrnd. Corr. should be understood.
  - New measurements of the holding magnet field are needed.
  - What is the efficiency of the beam separation?
  - Effect of the bunch frequency was not considered.