High Energy Cooling R&D

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Response to recommendation

R13 (Ring cooler feasibility study for EIC):

The Dept. and the study group are encouraged to assess and provide the effort needed to determine the required design details in line with the EIC project needs.

We have continued efforts to address most challenging questions regarding Ring-based electron cooler design. Progress with design will be discussed in this presentation. Also, we have started to evaluate ERL-based High-Energy Cooler (HEC) which will be described in this presentation.

As part of HEC R&D, we plan to develop most promising scheme, evaluate risks and costs and choose most reliable and cost-effective approach for the EIC.

We estimate that about 2-3 Accelerator Physicist FTE efforts over several years will be needed to develop robust HEC system for the EIC.

Note: The EIC Change Control Process is presently underway to include Low-Energy Cooler (LEC) in project scope and remove high-energy CeC-based cooler from the scope. Continuing HEC accelerator physics and accelerator design R&D is needed to develop a feasible and efficient scheme for the HEC (not necessarily CeC-based).

Introduction

Cooling requirements for Electron Ion Collider (EIC)

Low-Energy Cooling (LEC):

Cooling of protons and ions at injection energy of protons (24 GeV):

The goal of cooling at proton injection energy is to obtain initial proton parameters by cooling the vertical emittance from ~2 um to 0.3-0.5 um (rms normalized). This requires a 13 MeV electron beam.

The Change Control Process is presently underway to include LEC in the EIC project baseline.

High-Energy Cooling (HEC) of protons:

At EIC proton top collision energies, cooling should counteract the longitudinal and transverse emittance growth and maintain close to initial beam emittances.

Robust HEC system capable of fully counteracting emittance growth at collision energies would greatly improve luminosity in the EIC.

High-Energy Cooling for EIC

Previously, the HEC system for the EIC was based on a novel method of micro-bunched Coherent Electron Cooling (CeC). While recent years R&D studies have made significant progress, there are crucial unresolved issues related with extremely tight tolerances on timing electron and hadron beams in the cooler and cooling diagnostic.

An alternative HEC systems based on well-established technique of Electron Cooling were considered in the past and some of the approaches are being explored in greater detail here at CAD.

Electron Cooling at high energies

$$\lambda \propto \frac{r_e^2 m_e c Z^2 \Lambda_c}{A_i m_p} \cdot \frac{1}{\gamma^2} \cdot N_e \cdot \frac{L_{CS}}{C_{ring}} \cdot \frac{1}{\left(\frac{\varepsilon_{ne}}{\beta_e} + \frac{\varepsilon_{ni}}{\beta_i}\right)(\varepsilon_{ne}\beta_e + \varepsilon_{ni}\beta_i)\sqrt{\sigma_{\delta e}^2 + \sigma_{\delta i}^2}\sqrt{\sigma_{ze}^2 + \sigma_{zi}^2}}$$

- Cooling rate drops quadratically with energy but grows linearly with number of electrons and length of the cooling section. Precooling helps (smaller ion emittances), so do small e-bunch emittances. Yet, we don't want to make e-emittances much smaller than p-emittances:
  - The gains in cooling rate become small when $\varepsilon_e \ll \varepsilon_i$
  - $\varepsilon_e \ll \varepsilon_i \quad \rightarrow$ core overcooling (bad for collider)

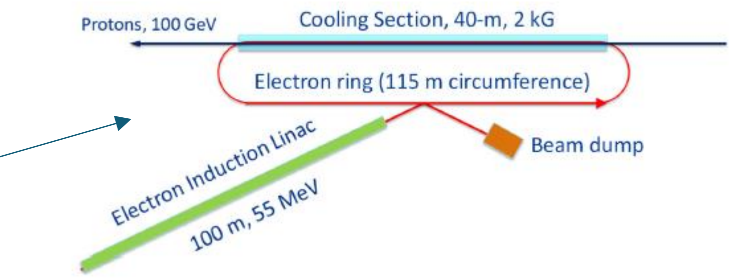
- With available $L_{CS} = 170$ m and $\varepsilon_{e(x,y,z)} \approx \varepsilon_{i(x,y,z)}$, the required cooling time (at $\gamma = 293$) of $\tau_{cool(x,y)} = 2,3$ h corresponds to average electron current in the cooling section of a few Amperes.
- **One needs to reutilize the same e-beam on several passes through the CS**

EC-based High-Energy Cooling

Several approaches based on conventional Electron Cooling were considered in the past:

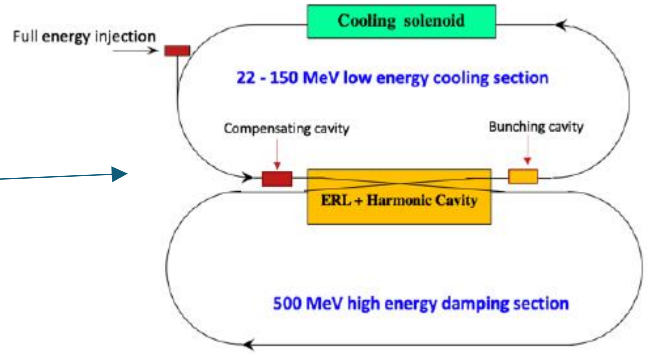
1. Induction linac based Ring cooler (FNAL):

V. Lebedev, S. Nagaitsev et al.,"CDR: A ring-based electron cooling for EIC", JINST 16 T01003 (2021).



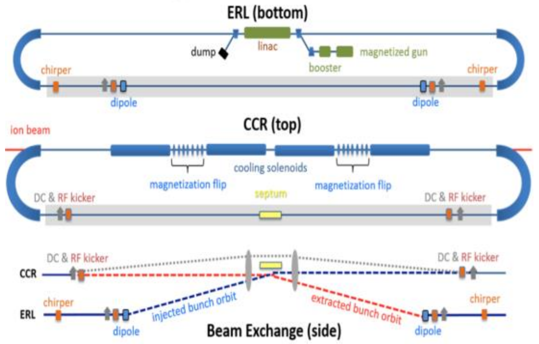
2. Dual-ring electron accelerator (JLAB):

B. Dhital et al., "Beam dynamics study in a dual energy storage ring for ion beam cooling", Proc. IPAC21, TUXA07 (2021).



3. ERL-based Circulator Ring (JLAB):

S. Benson et al., ERL19, LINAC20 presentations



4. Storage Ring electron cooler (BNL):

H. Zhao, J. Kewisch et al., "Ring-based electron cooler for high energy beam cooling", PRAB 24, 043501 (2021)

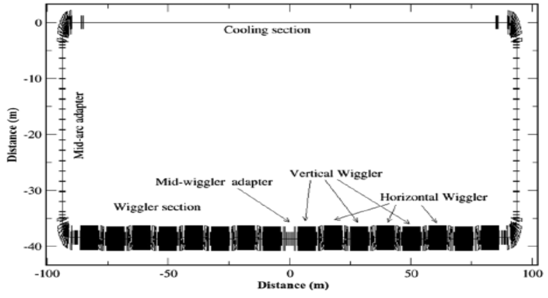


FIG. 2. Layout of the ring cooler.

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Recent HEC R&D

The focus of most recent R&D at CAD was on:

1. Design of storage Ring Electron Cooler (REC) where electron bunches which provide cooling of protons are being cooled themselves via synchrotron radiation.

Progress with REC design will be described in this presentation

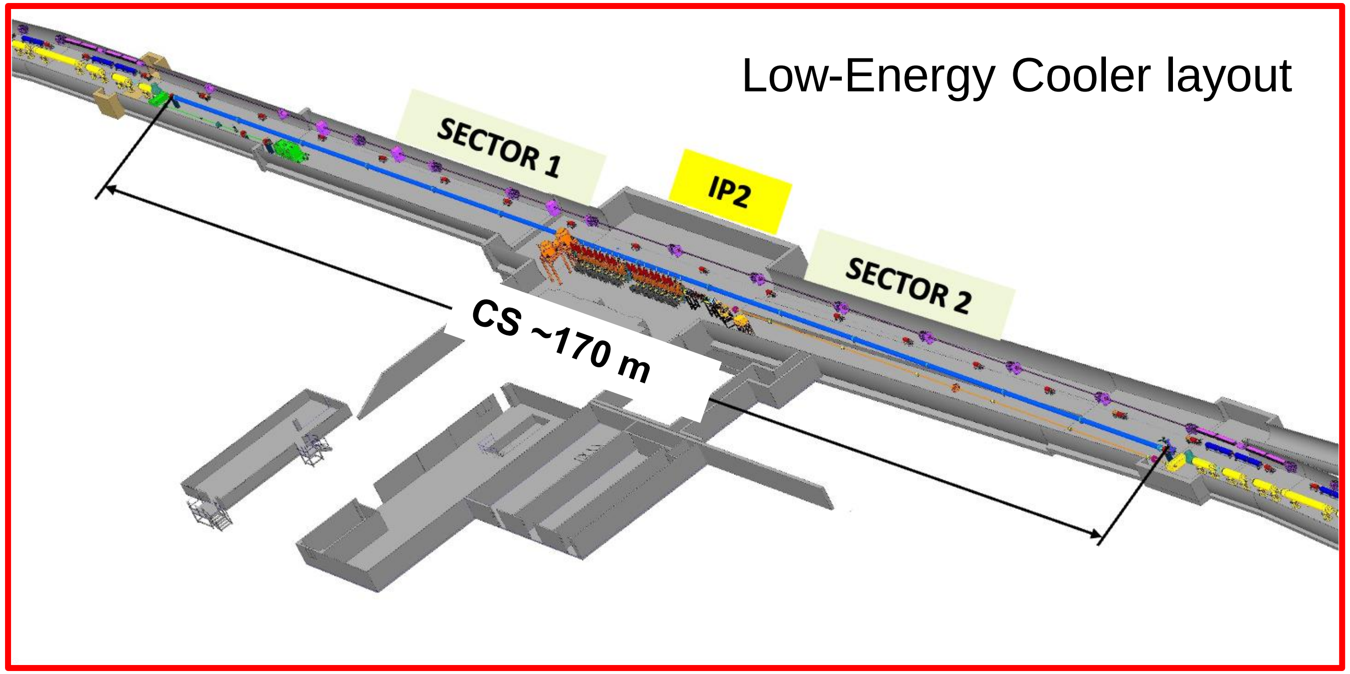
2. ERL-based Recirculator design where electron bunches are supplied by high-brightness electron source.

Initial concept of this approach and required parameters will be presented.

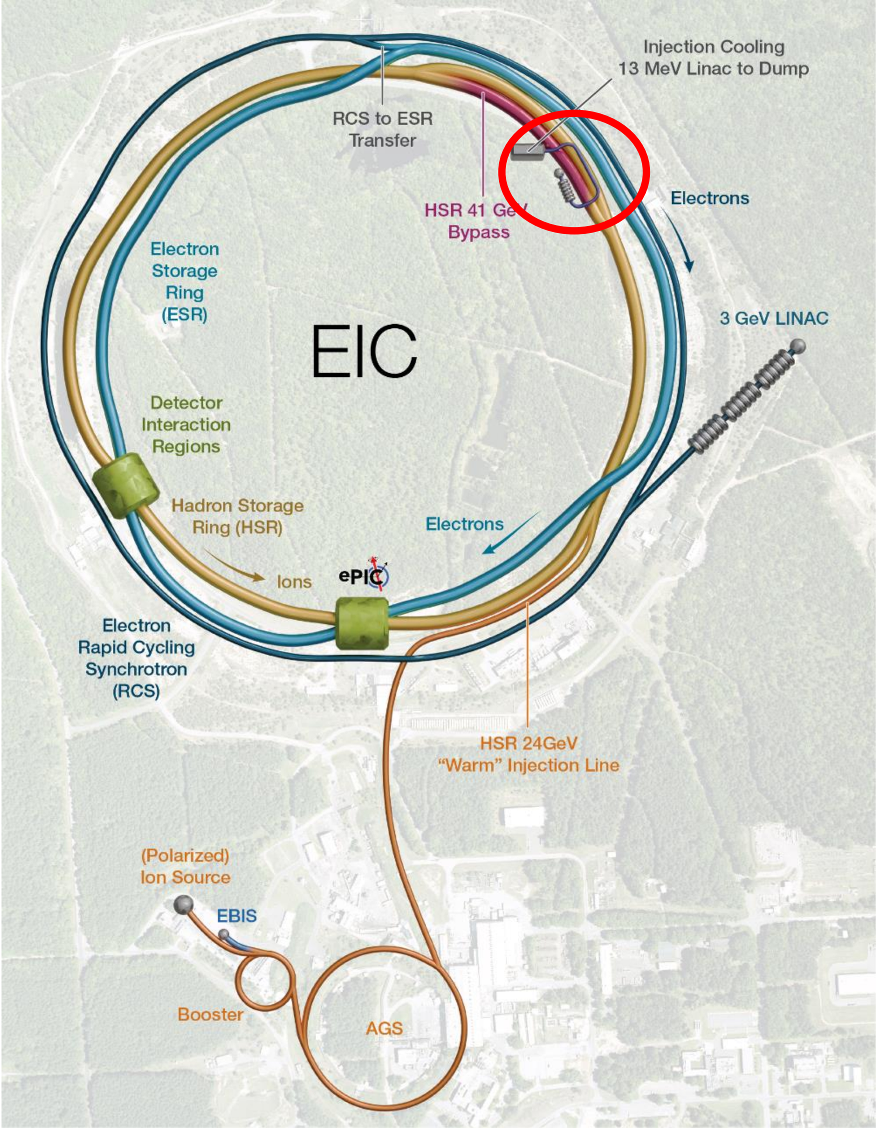
To simplify Technical Design of such electron accelerators, both approaches presently assume electron beam without any magnetization on the cathode and without need of continuous magnetic field in the cooling section.

Cooler’s location

- Both the Low-Energy Cooler and potential top energy cooler are located at a 2 o’clock hall.
- They must share the same section of the Hadron Storage Ring (HSR)
- About 170 m of the HSR are available for the cooling section



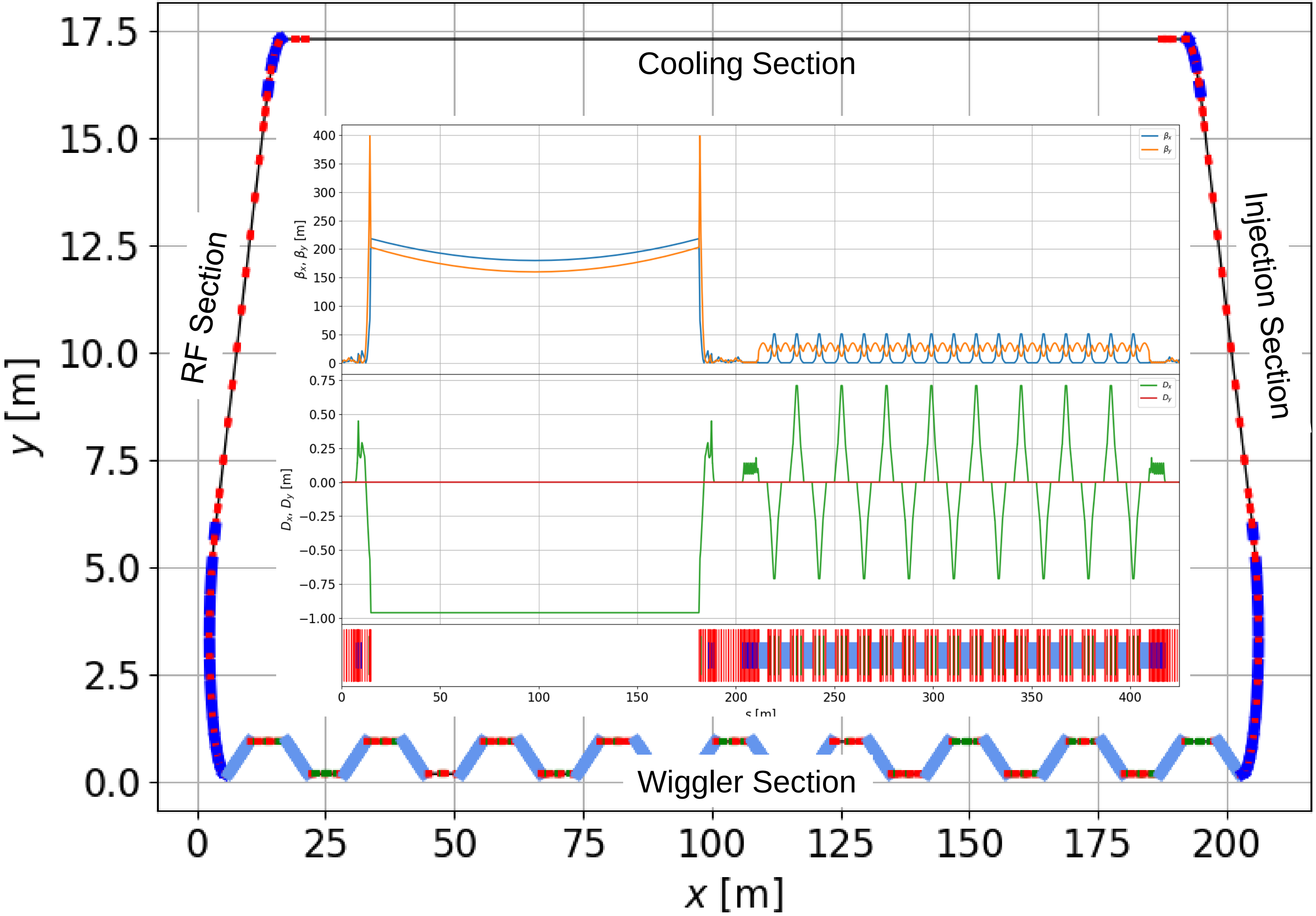
Low-Energy Cooler layout



Ring Electron Cooler

Conceptual design in progress

Basic idea and layout



- 170 m long cooling section (layout is compatible with EIC LEC)
- Ring circumference 426 m
- 140 e-bunches in the ring, each e-bunch makes 9 turns per 1 turn of hadrons in HSR
- Electrons are cooled by radiation damping in a wiggler section (18 damping wigglers, each wiggler is 4 m long)

What factors drive REC design parameters

- Consider cooling of 275 GeV protons with $\varepsilon_{px} = 11.3$ nm, $\varepsilon_{py} = 1$ nm, $\sigma_{p\delta} = 6.8 \cdot 10^{-4}$, $\sigma_{pz} = 6$ cm
- The required cooling times (balancing out the IBS-driven heating) are $\tau_{cool(x)} = 2$ h, $\tau_{cool(z)} = 3$ h
- **Our goal is to achieve $\boldsymbol{\tau_{cool(x)}} = 2$ h, $\boldsymbol{\tau_{cool(z)}} = 3$ h with as small e-bunch charge as possible**

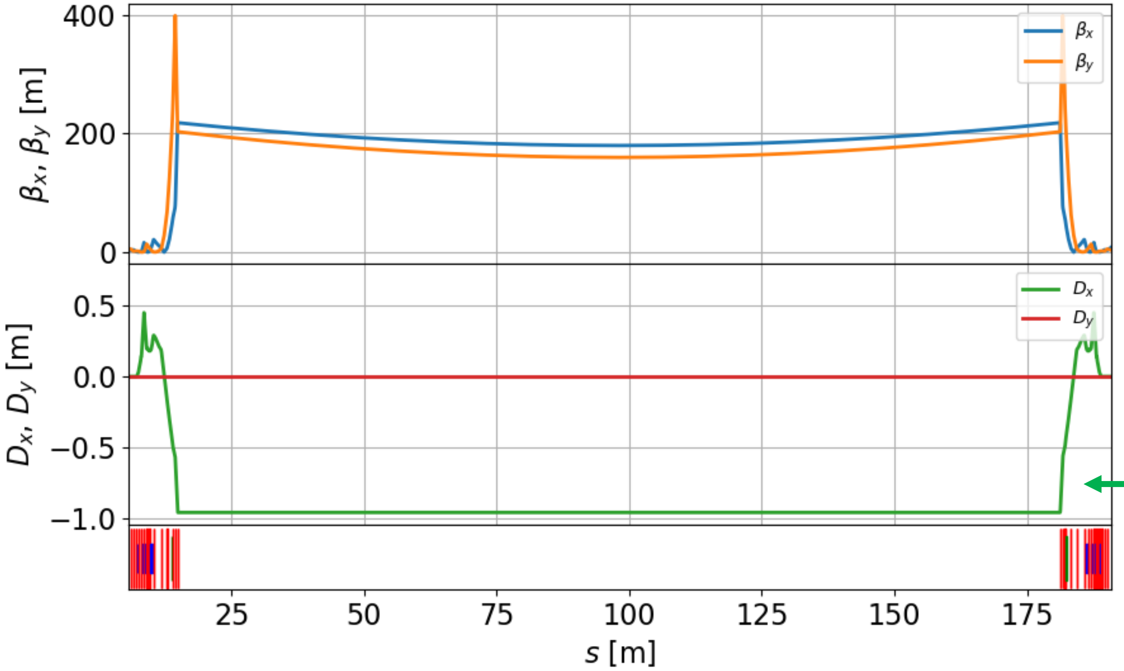
- Equilibrium e-bunch emittances are determined by a balance of the IBS rate, beam-beam scattering (BBS) rate, quantum excitations, and a rate of radiation damping (IBS and radiation damping are the main contributors):

$$\frac{d\varepsilon}{dt} = \left(-\lambda_{damp} + \lambda_{IBS} + \lambda_{BBS}\right)\varepsilon + C_q$$

$$C_q = \lambda_{damp}\, \varepsilon_{nat}$$

- Considerations of electron beam dynamics in the REC must include
  - optimization of momentum and dynamic apertures (strongly affected by the choice of a wiggler field profile)
  - proton-electron beam-beam,
  - self space charge,
  - optimization of electron and proton beams optical functions in the cooling section to both maximize the cooling and to minimize BBS rate and beam-beam parameter

Cooling section and cooling optimization (I)



- For any realistically achievable e-bunch parameters the longitudinal cooling is much faster than the transverse one.
- Therefore, **we utilize $z \rightarrow x$ redistribution by introducing electron and ion horizontal dispersions into the cooling section.**
- The combination of cooling and DA optimizations resulted in the lattice with $D_{ex} = 1$ m

- Larger N_e improves the cooling rate, but it also increases IBS, thus giving worse e-bunch emittances (for a given wiggler section), and larger emittances make cooling worse. We use a dedicated code (getrad), which allows us to find an optimal combination of bunch charge and 6-D phase space volume

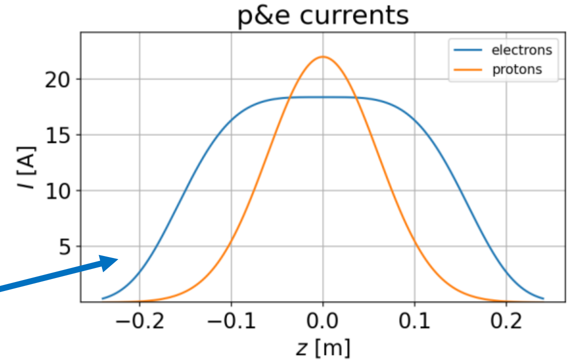
- **For optimized lattice we achieved:**

  $N_e = 1.3 \cdot 10^{11}$ ($Q_e = 21\ nC$)

  $\varepsilon_{xe} = \varepsilon_{ye} = 7.8\ nm$

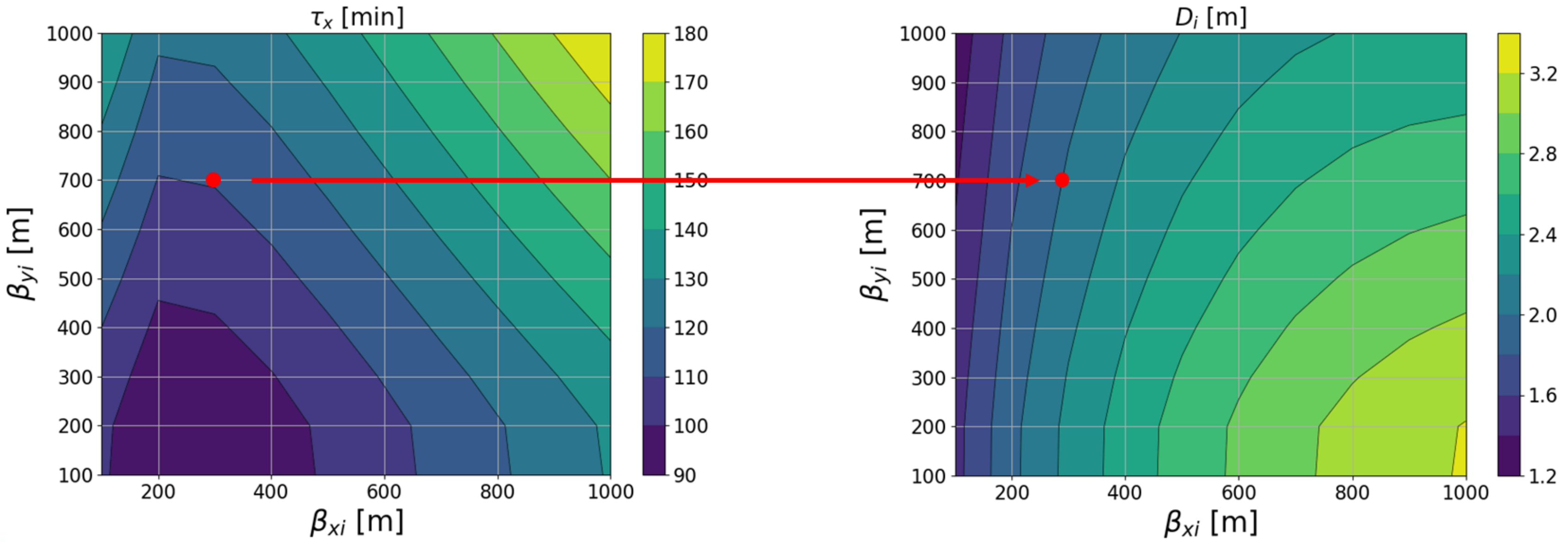
  $\sigma_{pe} = 9.7 \cdot 10^{-4}$

  $L_{FWHM} = 34\ cm$ (double-RF ⇔ flat-top e-bunch, which helps to limit the space charge)



Cooling section and cooling optimization (II)

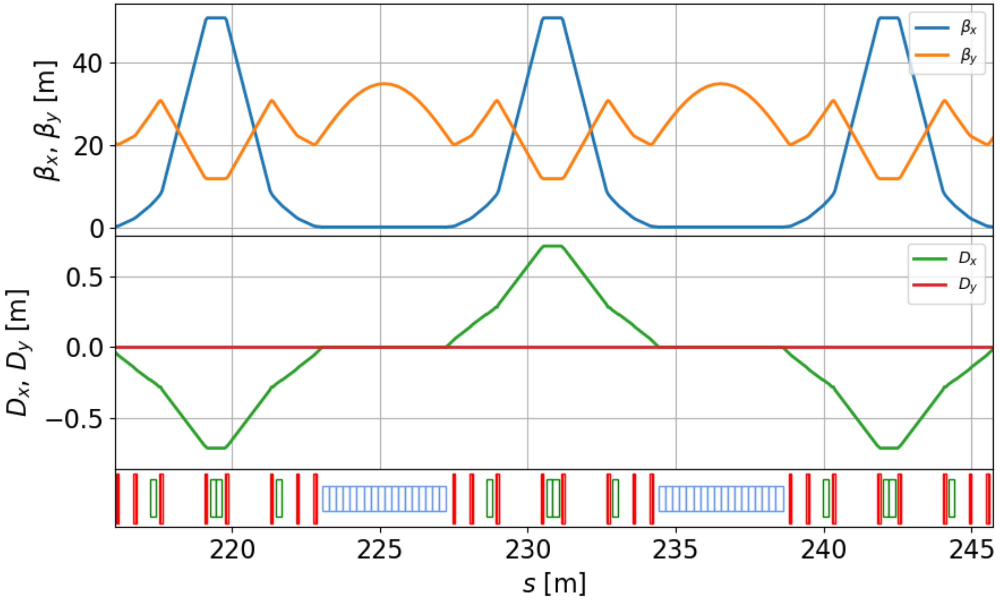
- Finalized choice of e- and p- dispersions and β-functions in the CS, as well as the fine-tuning of the e-bunch charge is driven by cooling optimization.
- We use another dedicated code to optimize electron and proton optical functions in the CS, alternating it with getrad code.

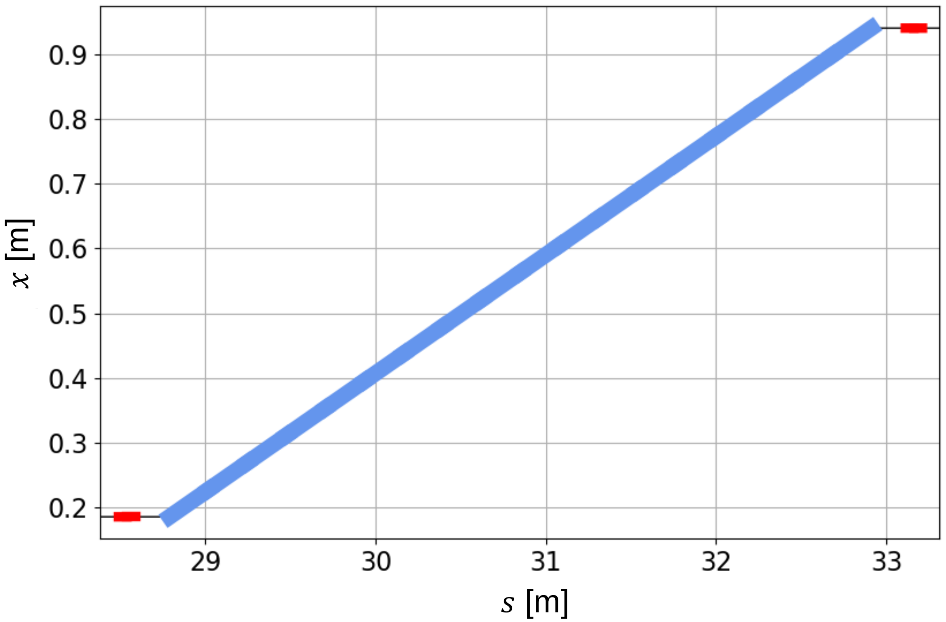


In this example the optimizer keeps $\tau_z = 3$ h and minimizes τ_x by varying D_i for each combination of (β_{xi}, β_{yi})

CS parameter	electrons	protons
β_x [m]	180	300
β_y [m]	160	700
D_x [m]	1	2.1

REC wigglers

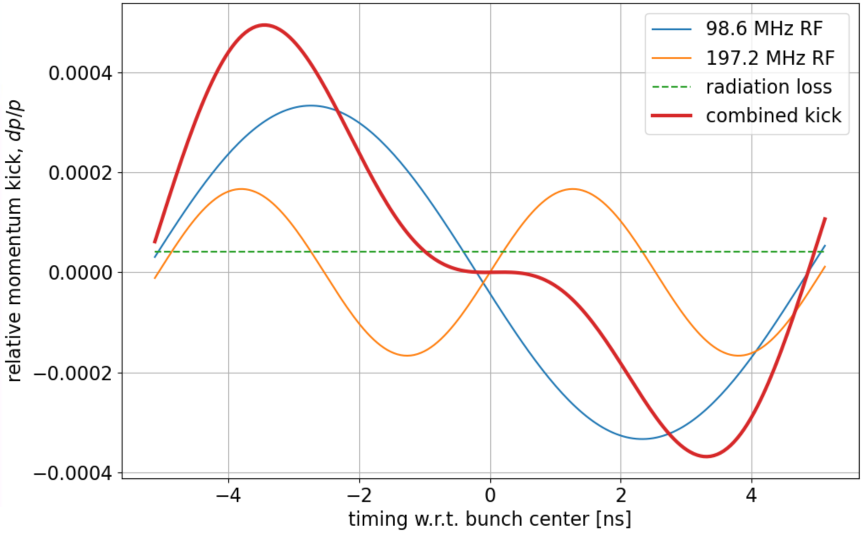


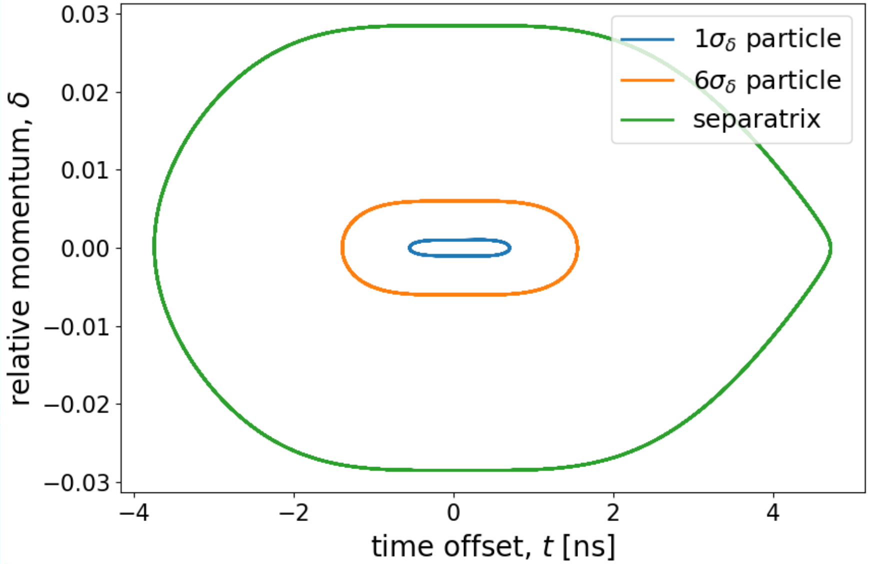


- We have 18 wigglers, 4-m long each, with peak field of 2.4 T.
- We enter and exit wigglers with non-zero angle. The regions with large dispersion between wigglers are used for chromaticity correction.
- To minimize the IBS-driven emittance growth in wigglers we need a tight focusing in horizontal direction (it minimizes H-function for small D_x, large D_x' case)
- Because we work with high field / low energy wigglers, a specific field profile is needed to minimize chromaticity
- **It was confirmed that required wigglers parameters are achievable, and a preliminary design was created.**

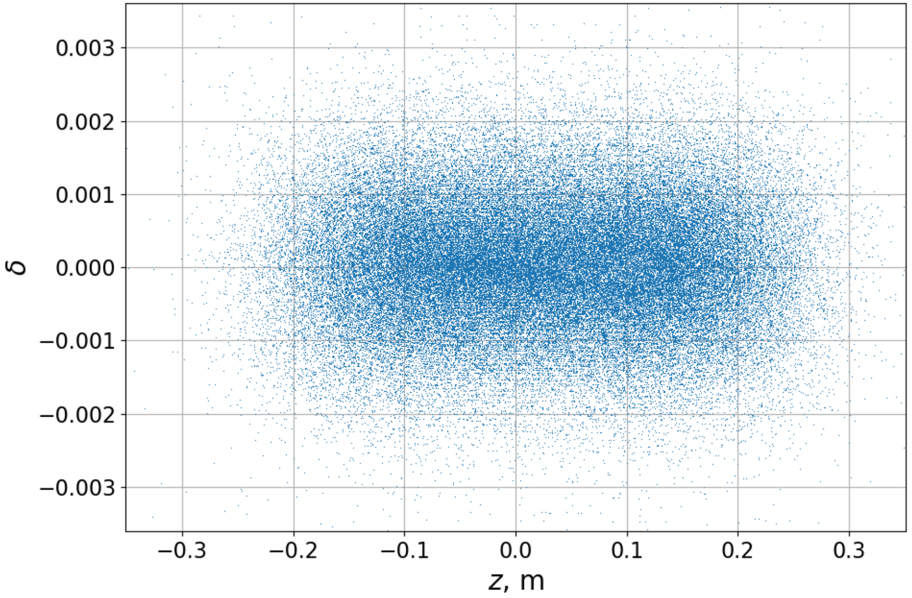
Wiggler parameter	value
Number of wigglers	18
Length [m]	4.2
Peak field [T]	2.38
Period [m]	0.2
Gap [cm]	2
Radiated power (per wiggler) [W]	674
Radiation angle [mrad]	3.4

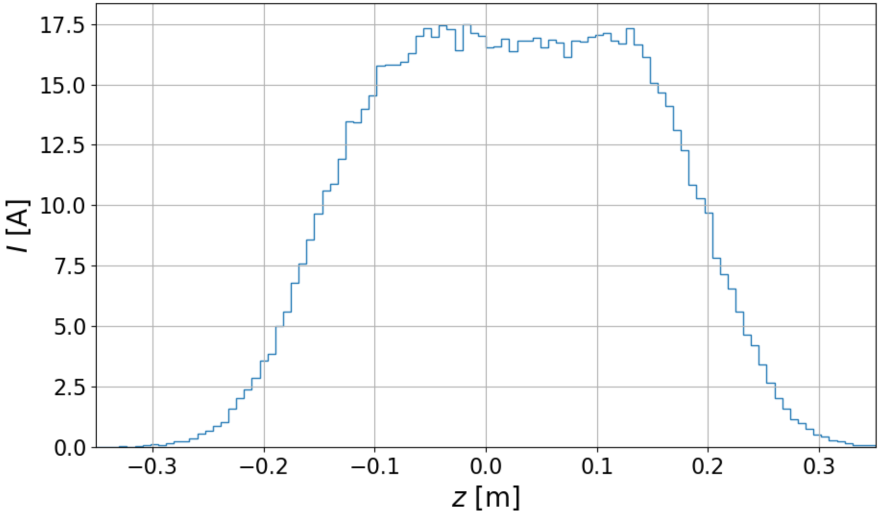
REC RF system



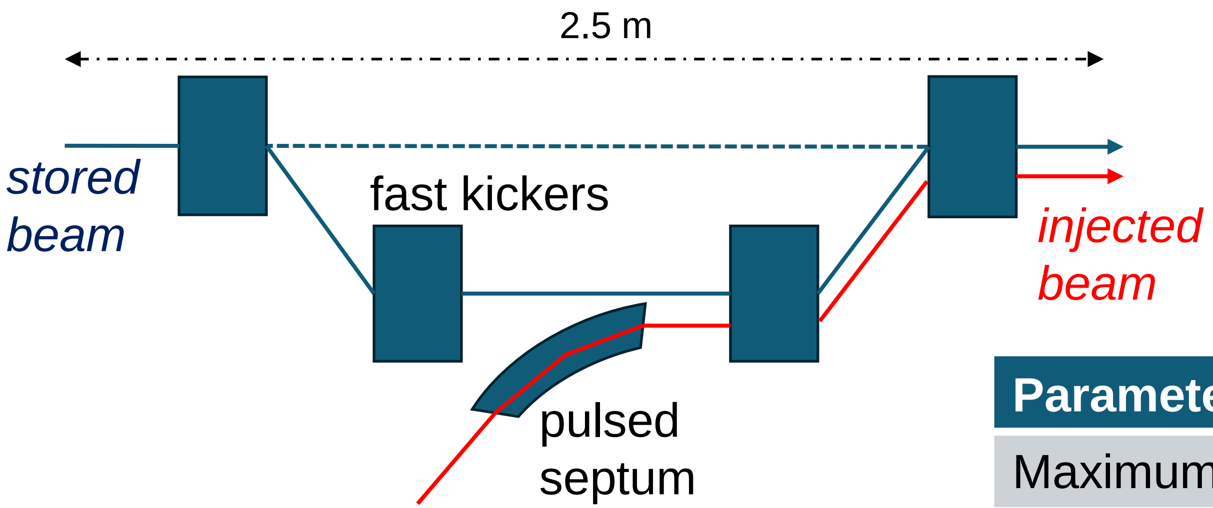


- The ring utilizes a dual RF system with fundamental frequency of 98.6 MHz and voltage of 50 kV and the 2nd harmonic (25 kV).
- To compensate for the radiation loss of 6 kV/turn, the fundamental phase is shifted by 7.24 degrees.
- The resulting RF bucket corresponds to the flat-top e-bunch with FWHM length L_{FWHM}=34 cm and $\sigma_\delta = 9.8 \cdot 10^{-4}$.
- For $1.3 \cdot 10^{11}$ electrons per bunch, the peak current is $I_p = 17.5\ A$





Injection scheme

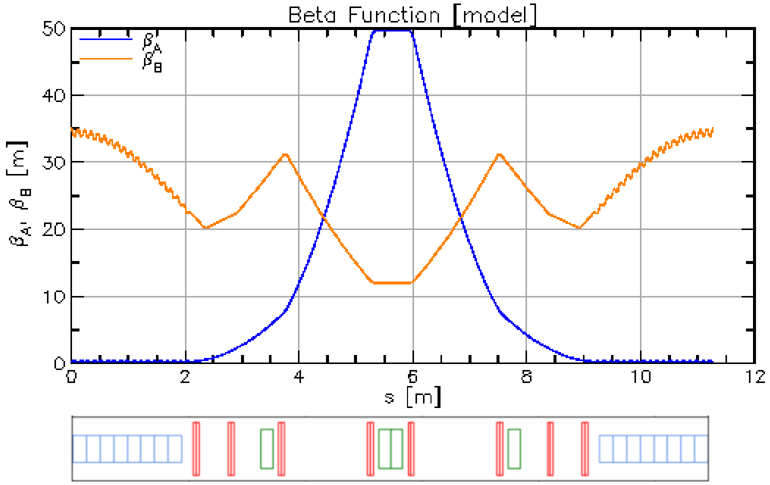


Beam parameter	Stored	Injected
β_x [m]	60	20
ε_x [nm]	8	5
ε_{nx} [μm] (out of the gun)		1.5
Q_b [nC]	21	1.75

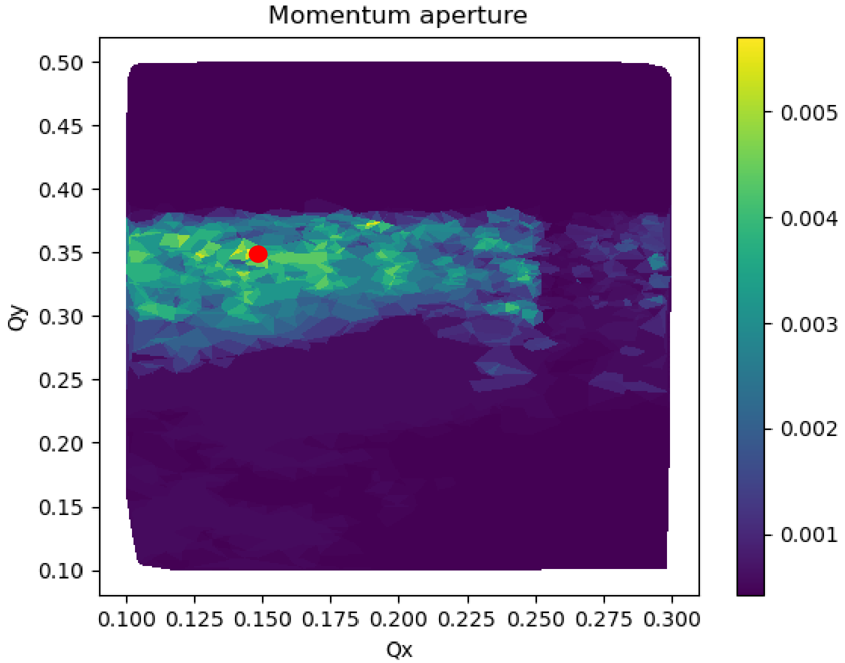
Parameter	Kicker	Septum
Maximum Field (B_k) [G]	760	7000
Magnetic Length (L_k) [m]	0.2	≥ 0.38
Pulse Shape	trapezoid	full $\sin$ - wave
Rise/Fall time [ns]	200	N/A
Flat-top duration [ns]	284	N/A
Wavelength (λ) [μs]	N/A	200
Repetition rate (f_k) [Hz]	3	3

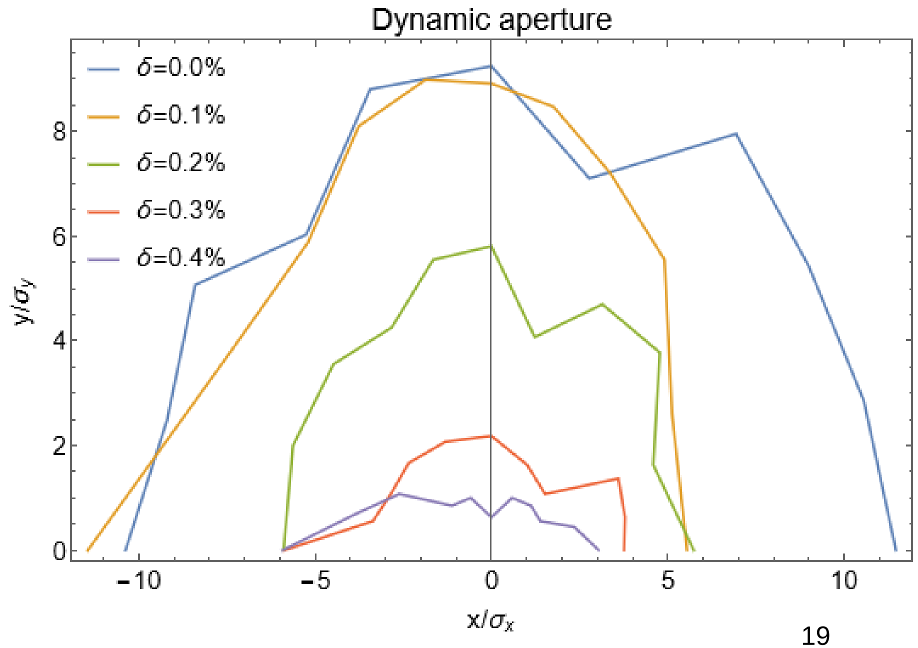
- We are planning to have a top-off injection replenishing 10% of each bunch every 1.6 s.
- Four kickers will create a closed bump bringing the stored beam closer to the injection septum. At the exit of the bump the injected beam will be displaced by 6 mm from the stored beam trajectory.
- The injector will be running with $f_{inj} = 3$ Hz injecting into 1/5 of the ring (28 bunches) each time.
- Initial injection can be performed with $f_0 = 5$ Hz frequency, filling up the ring in 20 s.

Dynamic and momentum aperture optimization



- Optimization of wigglers’ field profile allowed to substantially reduce chromaticity and utilize weaker sextupoles
- Phase advance over wiggler and sextupole blocks is an important parameter for dynamic aperture
- Two families of octupoles were optimized to reduce non-linear motion
- Tune was adjusted to maximize MA
- **Resulting DA/MA are:**
  - $A_x = 10\sigma_x$
  - $A_y = 9\sigma_y$
  - $A_\delta = 0.5\ \%$





Parameters

Table 1: The REC parameters (electron storage ring)

relativistic γ	293
ring circumference [m]	426
cooling section length [m]	170
horizontal dispersion in the CS [m]	1
number of damping wigglers	18
damping wiggler length [m]	4.2
damping wiggler field [T]	2.4
wiggler gap [cm]	2
wiggler period [cm]	20
momentum compaction	$-1.5 \cdot 10^{-3}$
main RF frequency [MHz]	98.6
main RF voltage [kV]	50
2nd harmonic RF voltage [kV]	25
number of bunches	140
number of particles per bunch	$1.3 \cdot 10^{11}$
charge per bunch [nC]	21
peak current [A] (flat top e-bunch)	17.5
average current [A]	2
geometric emittance (x, y) [nm]	7.8, 7.8
CS β-function (x, y) [m]	180, 160
rms relative momentum spread	$9.8 \cdot 10^{-4}$
FWHM bunch length (flat top e-bunch) [cm]	34
space charge tune shift (x,y)	0.14, 0.14
p-e focusing tune shift (x,y)	0.04, 0.09
radiation damping rate (x,y,z) [s^{-1}]	31, 31, 62
BBS rate (x,y,z) [s^{-1}]	0.8, -0.3, 12
IBS rate (x,y,z) [s^{-1}]	31, 31, 48

Table 2: The REC parameters (protons)

relativistic γ	293
number of particles per bunch	$6.9 \cdot 10^{10}$
geometric emittance (x, y) [nm]	11.3, 1
CS β-function (x, y) [m]	300, 700
rms relative momentum spread	$6 \cdot 10^{-4}$
rms bunch length (Gaussian p-bunch) [cm]	6
horizontal dispersion in the CS [m]	2.1
e-p focusing tune shift (x,y)	$1 \cdot 10^{-4}$, $1.7 \cdot 10^{-4}$
cooling time (x,y,z) [hrs]	**2, 4, 3**

Challenges and ongoing work

- REC is a low-energy high-current electron storage ring
- Careful considerations of collective effects and instabilities are needed
- Due to careful lattice optimization the bunch charge required for cooling was significantly reduced. This helps with collective effects.
- Touschek lifetime for the full lattice and obtained momentum aperture is 47 s, while vacuum lifetime is 16 s (assuming 0.5 nTorr vacuum)
- Vacuum requirements are tight, and worse vacuum will require increasing injection frequency
- The ongoing studies include:
  - Beam-beam (p-e focusing) and self space charge effects
  - Magnets' setting errors and misalignments
  - Tolerances to errors in various subsystems (injection, RF, wigglers) based on cooling requirements

Work to be done

- Systematic studies of instabilities must be performed (coherent wiggler instability, transverse mode-coupling instability, transverse and longitudinal coupled-bunch instability, electron-ion instability etc.)
- Beam loading in the RF and its effects on longitudinal bunch distribution need to be studied.
- Feedbacks (both to keep e-beam quality in the cooling section and to counteract possible instabilities) have to be considered.
- Various engineering systems (diagnostic, vacuum, machine protection) must be devised
- Bringing REC design to lower energies

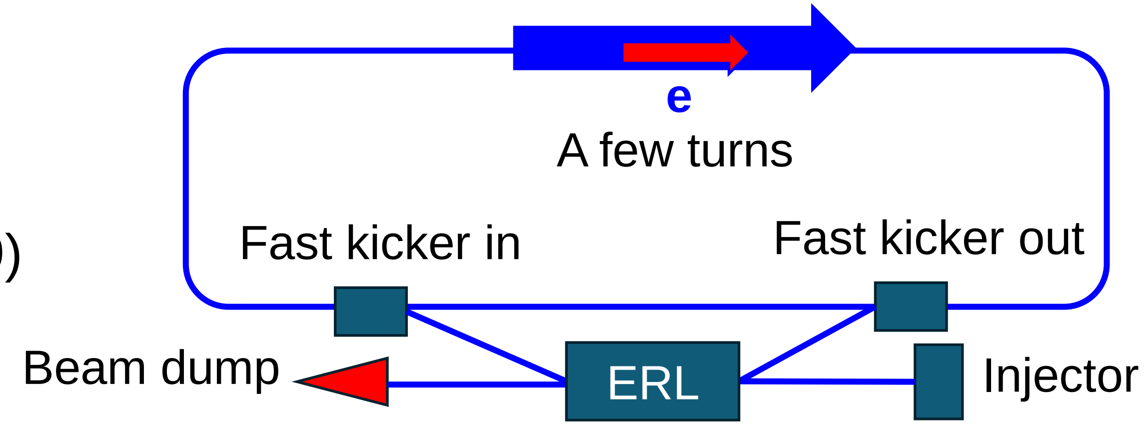
REC conceptual design status

- The realistic lattice for the Ring Electron Cooler was developed
- The lattice includes proper model of damping wigglers, dual RF system, and injection magnets.
- Dynamic and momentum aperture were optimized, 10σ horizontal, 9σ vertical and 0.5% longitudinal DA/MA were achieved.
- Careful optimization of the cooling section parameters allowed to achieve the required cooling times with substantially reduced electron bunch charge.

ERL based Recirculator

Feasibility studies

Recirculator concept



- Electron bunches are accelerated in the ERL
- Recirculated in the ring for just a few turns (1-9)
- Decelerated and sent to a beam dump
- Non-magnetized electron beam is used

- This scheme has some advantages:
  - IBS and BBS are not an issue
  - Self space charge in the ring must be considered, but it is a small effect
  - Proton-electron beam-beam is rather forgiving and allows to optimize β functions in the cooling section in a much wider range
  - There is no need for enhanced radiation damping, the bunch parameters are defined by dynamics in injector and ERL

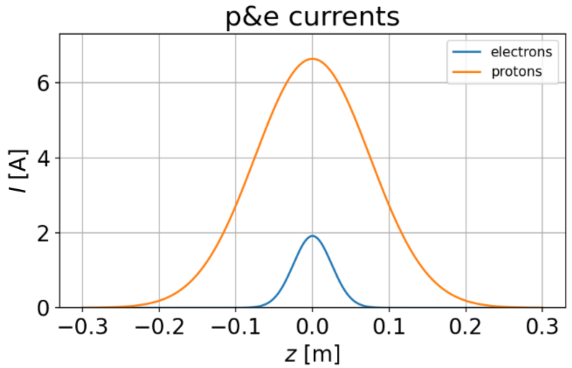
- There are some challenges associated with this approach:
  - High current Gun and high current ERL
  - Good quality bunches with high charge
  - Fast kickers

***Note**: Similar cooling scheme was suggested at JLAB for JLEIC (https://epaper.kek.jp/erl2019/talks/mocozbs01_talk.pdf).*
It utilized magnetized electron beam.

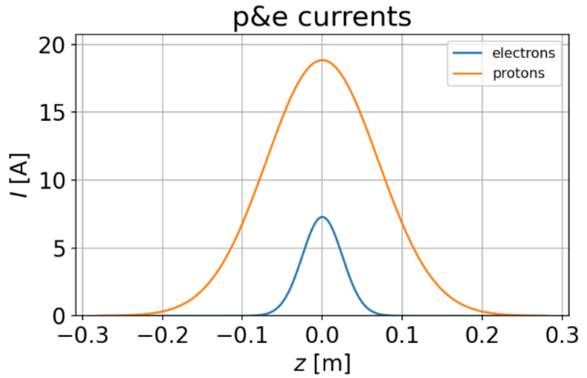
Recirculator parameters

Proton Energy , GeV	**275**	**100**	**41**
N_e	3.00E+10	1.25E+10	4.00E+09
Q_e, nC	5	2	0.64
Rms bunch length, cm	2.5	2.5	2.5
Peak Current, A	24	10	3
Repetition rate, MHz	98	98	98
$\langle I \rangle$ in cooling section, mA	490	196	63
Number of recirculations	9	4	1
$\langle I \rangle$ from gun, mA	54	49	63
Rms energy Spread in CS	3.00E-04	3.00E-04	3.00E-04
RMS Normilized Emittance, m	2.00E-06	1.50E-06	1.50E-06
Cooling Time (τ_x), hrs	1.8	1.9	2
Cooling Time (τ_y), hrs	3.6	3.9	1.8
Cooling Time (τ_z), hrs	2.9	1.6	1

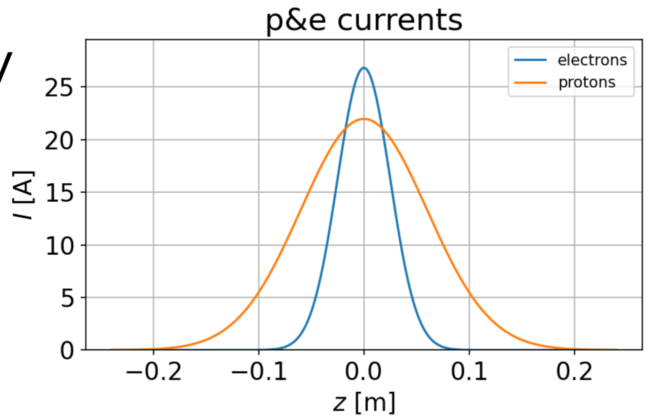
41 GeV protons



100 GeV protons



275 GeV protons

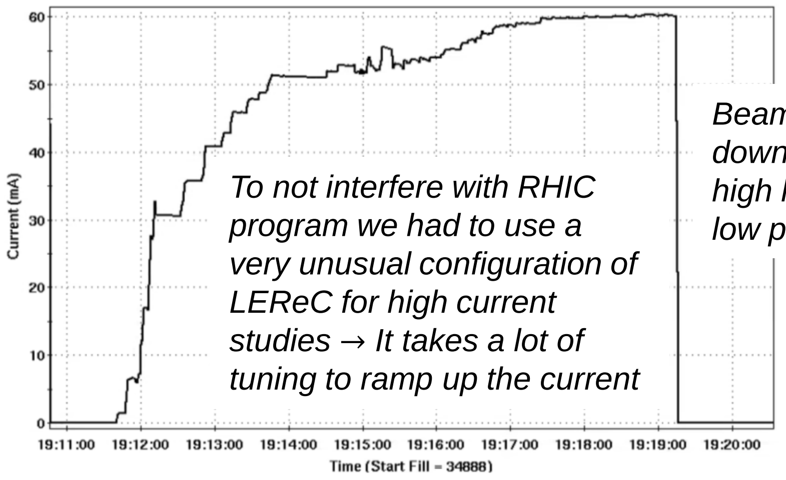


Challenges and mitigation

- **High current injector (up to 60 mA)**

LEReC: Stable 50 mA operation was achieved in 2022. In 2024, 60 mA operation was achieved. The duration of 60 mA test was limited by a specific machine setup, which we had to employ to avoid interference with RHIC program. High current studies with a more practical configuration are planned for 2025.

Cornell: 65 mA current was demonstrated



Beam was ramped down because of high losses in the low power dump

- **Obtaining low emittance and low energy spread bunches with high charge**

We can start with longer bunches and lower RF frequency (98 MHz for $Q_b = 5$ nC) in the injector, and then compress bunches and accelerate them with 591 MHz cavity.
Beam dynamics simulations for 2.5 nC bunches in Low Energy Cooler injector (13 MeV) using 197 MHz RF showed good results ($\varepsilon_n < 2\ \mu$m, $\sigma_\delta = 4.5 \cdot 10^{-4}$).

- **150 MeV Energy Recovery Linac operating at ~60 mA**

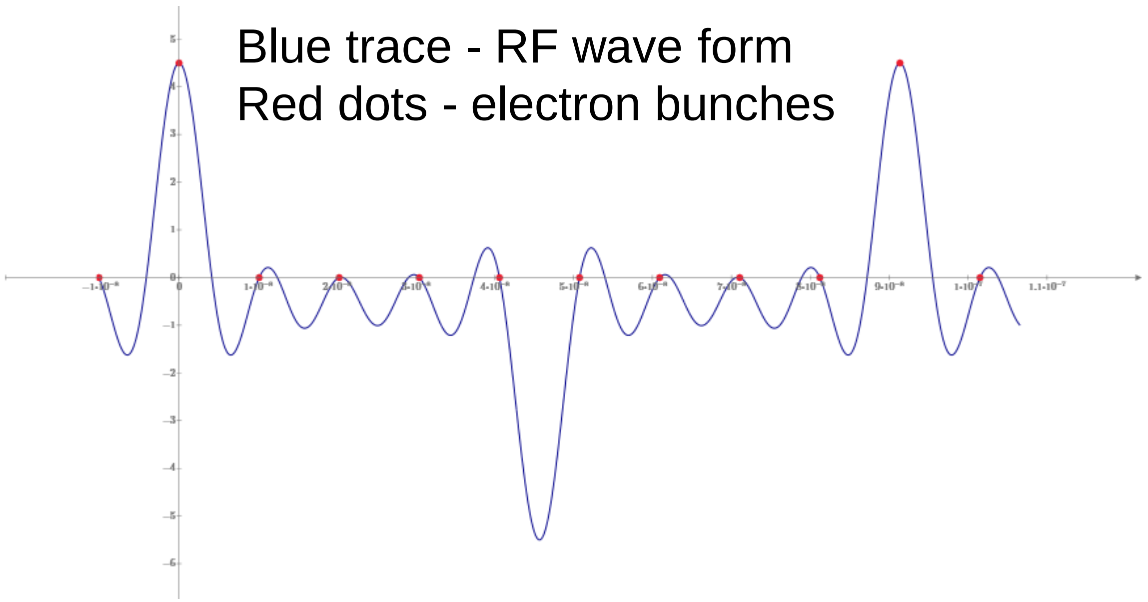
Both technical feasibility of such an ERL and relevant beam dynamics must be explored

Brookhaven National Laboratory

What about kickers?

For the highest energy we would need to kick every 9th bunch in and out

Blue trace - RF wave form
Red dots - electron bunches



Kicker fundamental 10.95 MHz (1/9th of the proton bunches rep. rate 98 MHz)

Operation of harmonic kicker with beam was recently demonstrated at Jlab

12th Int. Beam Instrum. Conf. IBIC2023, Saskatoon, Canada JACoW Publishing
ISBN: 978-3-95450-236-3 ISSN: 2673-5350 doi:10.18429/JACoW-IBIC2023-TUP031

BEAM TEST OF A HARMONIC KICKER CAVITY*

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Abstract

A harmonically resonant kicker cavity designed for beam exchange in a circulator cooler was built and successfully tested at the Upgraded Injector Test Facility (UITF) at Jefferson Lab. This type of cavity is being considered for the injection scheme of the Rapid Cycling Synchrotron at the Electron-Ion Collider, where the spacing of neighboring bunches demands very short kicks. Operating with five transversely deflecting modes simultaneously that resonate at 86.6 MHz and consecutive odd harmonics thereof, the prototype cavity selectively deflects 1 of 11 electron bunches while leaving the others unperturbed. An RF driver was developed to synthesize phase- and amplitude-controlled harmonic signals and combine them to drive the cavity while also separating the modes from a field-probe antenna for RF feedback and dynamic tuning. Beam deflection was measured by sweeping the cavity phase; the deflection waveform agrees with expectations, having sub-nanosecond rise and

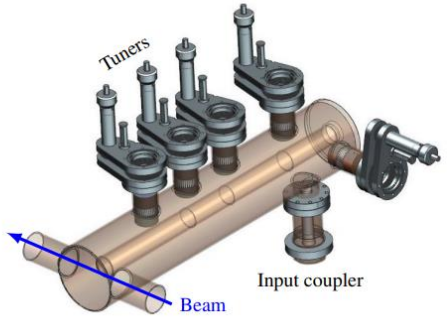

Figure 1: CAD model of a 5-mode harmonic kicker cavity. Five stub tuners are needed to tune all modes. The RF signal is coupled in through a single port; another port serves as the field probe (not shown here).

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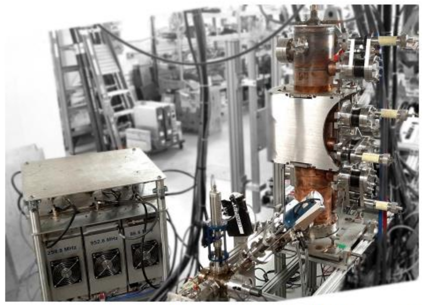


Figure 5: Harmonic kicker installed in the UITF beam line with the HAWG next to it.

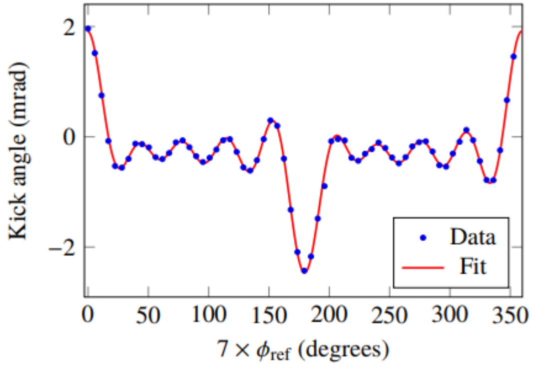


Figure 7: Deflection waveform with all five modes powered and optimized.

Feasibility of ERL based Recirculator

- The ERL based Recirculator with a few passes through the cooling section mitigates several beam physics problems associated with the electron storage ring
- We established requirements to electron bunch quality and outlined general Recirculator parameters from electron cooling considerations
- At a very first look we believe that with proper design of RF, longitudinal gymnastics, and transport system the required bunch quality can be achieved
- There are both engineering and physics questions that still must be answered before one can reach a conclusion on feasibility of this approach

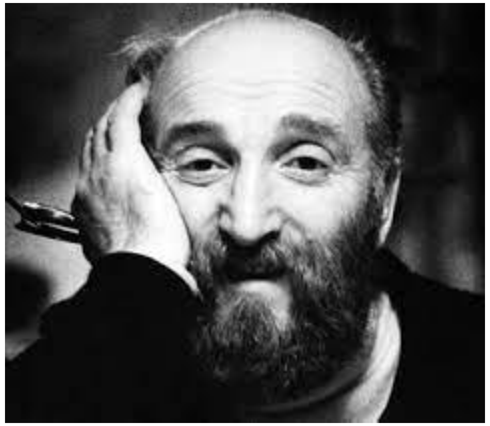
Summary

- Robust HEC system capable of fully counteracting emittance growth at collision energies would greatly improve luminosity in the EIC.
- Recently, significant progress was achieved with conceptual design of Ring-based electron cooler: The realistic lattice for the Ring Electron Cooler was developed including proper model of damping wigglers, dual RF system, and injection magnets. Dynamic and momentum aperture were optimized.
- Feasibility study of ERL-based Recirculator started.
- Proton beam cooling at high energy in the EIC is conceptually feasible. However, detailed comprehensive R&D is required to determine technical feasibility of various approaches.
- As part of HEC R&D, we plan to develop most promising scheme of HEC, evaluate risks and costs and choose most reliable and cost-effective approach for the EIC.
- Continuing accelerator design R&D for the HEC shall get us ready to add the HEC as an EIC upgrade.

Backup slides

Electron Cooling

- In electron cooling a bunch of hadrons co-travels with electrons (with matched γ) in a straight section of a hadron storage ring
- "Hot" hadron and "cold" electron gases exchange heat, which leads to reduction of the phase space volume occupied by hadrons
- Electron cooling was first demonstrated in 1974



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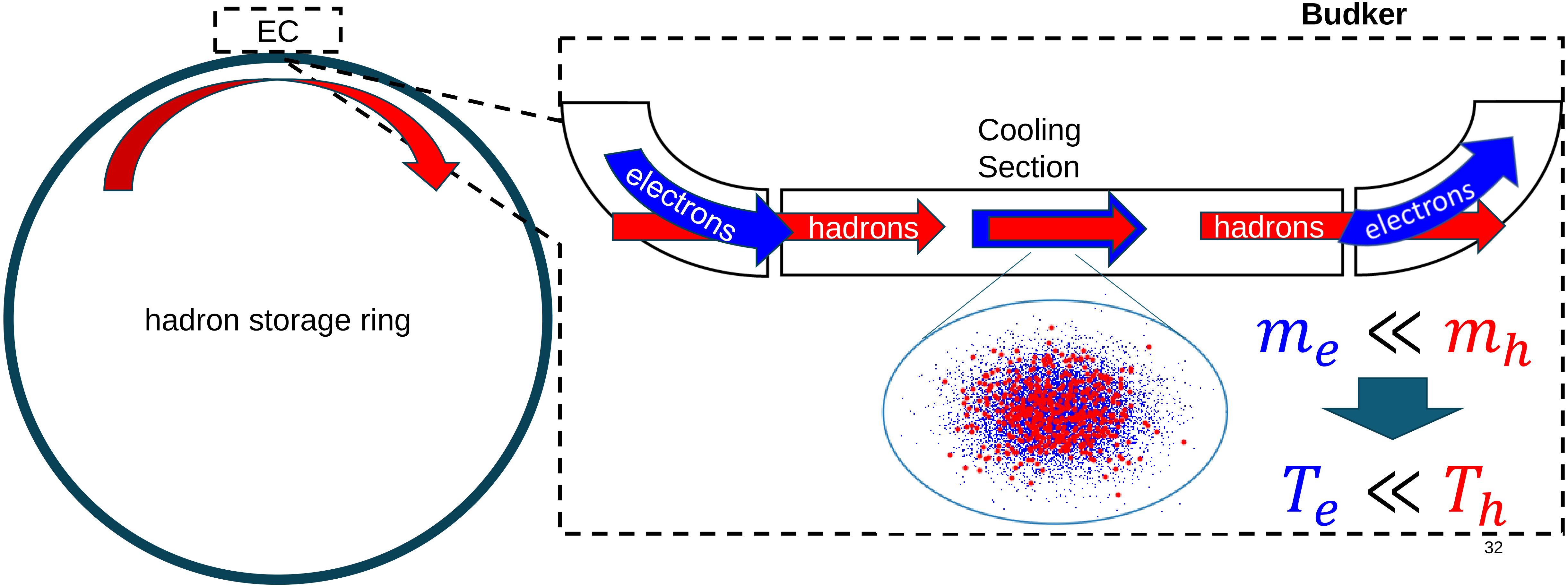


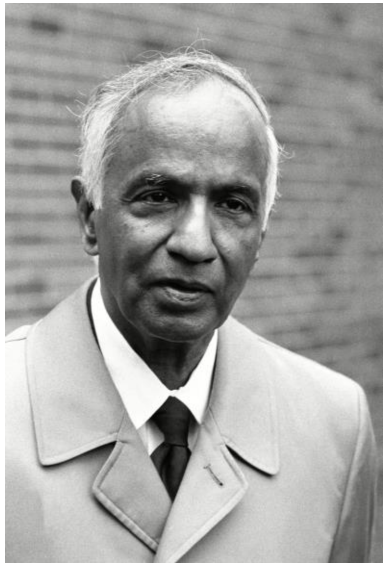
Dynamical friction

- A massive object (a star or an ion) moving through a cloud of lighter bodies (space dust or electrons) experiences Dynamical Friction - a pull from the cloud that slows down the object.

- If the force acting between bodies in the system follows the $F \propto 1/r^2$ law, then the dynamical friction force is given by:

$$\vec{F} = \frac{4\pi n_e e^4 Z^2}{m_e} \cdot \int \Lambda_c \frac{\overrightarrow{v_i} - \overrightarrow{v_e}}{|\overrightarrow{v_i} - \overrightarrow{v_e}|^3} f_{ve} d^3 v_e$$

- Each time an ion passes the cooling section (CS) it experiences a friction force which reduces its velocity deviation from an average bunch velocity



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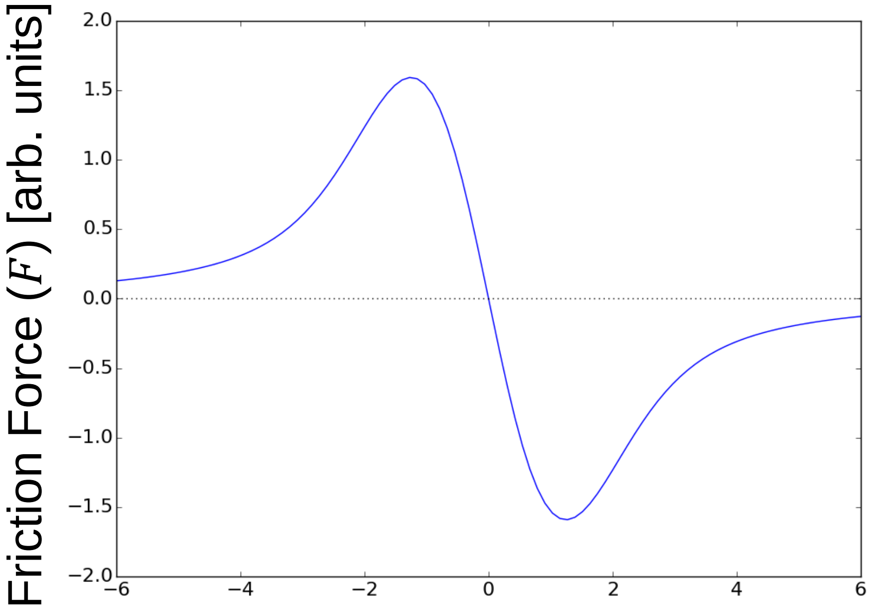
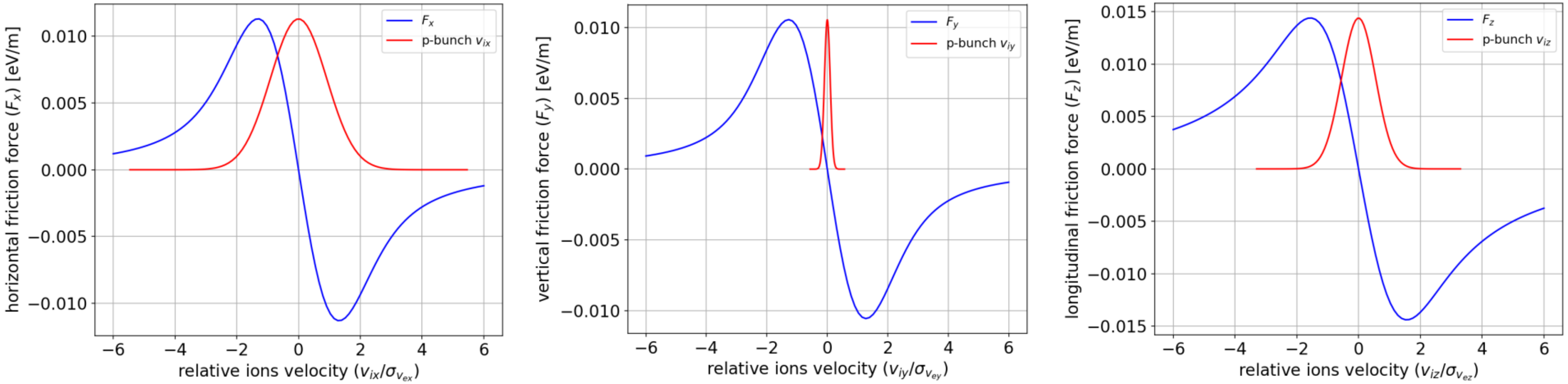


hadron velocity relative to electrons' velocity spread ($v_i/\Delta v_e$)

Fully integrated cooling rates



- While "small amplitude" formulas give good cooling times: $\tau_x = 50$ min and $\tau_z = 40$ min, the total cooling rate must be obtained by integrating the friction force over 6D distributions of both beams:

If both beams have Maxwell-Boltzmann velocity distributions, then integration over velocities can be reduced to 1D integrals

$$\lambda_{x,y,z} = -\frac{4\sqrt{2}}{\pi} C_1 \mathbb{S} \Psi_{x,y,z}$$

$$C_1 = \frac{N_e r_e^2 Z_i^2 m_e c^4 \Lambda_C \eta}{\gamma^2 A_i m_p}$$

$$\Psi_x = \int_0^\infty \frac{p^2 dp}{(1+2p^2 S_{\theta x}^2)^{3/2}(1+2p^2 S_{\theta y}^2)^{1/2}(1+2p^2 S_\delta^2)^{1/2}}$$

$$\Psi_y = \int_0^\infty \frac{p^2 dp}{(1+2p^2 S_{\theta y}^2)^{3/2}(1+2p^2 S_{\theta x}^2)^{1/2}(1+2p^2 S_\delta^2)^{1/2}}$$

$$\Psi_z = \int_0^\infty \frac{p^2 dp}{(1+2p^2 S_\delta^2)^{3/2}(1+2p^2 S_{\theta x}^2)^{1/2}(1+2p^2 S_{\theta y}^2)^{1/2}}$$

$$S_{\theta x,\theta y} = \gamma\beta c \sqrt{\sigma_{\theta xe,\theta ye}^2 + \sigma_{\theta xi,\theta yi}^2}$$

$$S_\delta = \beta c \sqrt{\sigma_{\delta e}^2 + \sigma_{\delta i}^2}$$

When both beams have 3D Gaussian density distribution:

$$\mathbb{S} = \frac{1}{S_x S_y S_z}$$

$$S_{x,y,z} = \sqrt{\sigma_{xe,ye,ze}^2 + \sigma_{xi,yi,zi}^2}$$

When e-bunch has a flat-top longitudinal distribution:

$$\mathbb{S} = \frac{1}{S_x S_y} \frac{\sqrt{2\pi}}{L_{ze}} \mathrm{erf}\left(\frac{L_{ze}}{2\sqrt{2}\sigma_{zi}}\right)$$

Redistribution of cooling rates

- To redistribute cooling between longitudinal and horizontal directions one needs:
  - Dependence of a longitudinal cooling force on a horizontal coordinate
  - Horizontal ion dispersion in the cooling section (dependence of ions' longitudinal velocity on their horizontal coordinate)
- The redistributed rates [2,3] can be calculated from:

$$\begin{aligned}
\lambda_{x1} &= \lambda_{x0} + k\lambda_{z0} \\
\lambda_{z1} &= \lambda_{z0} - k\lambda_{z0} \\
k &= \frac{D_i^2\sigma_{\delta i}^2 + D_i D_e \sigma_{\delta e}^2}{\sigma_{xi}^2 + \sigma_{xe}^2 + D_i^2\sigma_{\delta i}^2 + D_e^2\sigma_{\delta e}^2}
\end{aligned}$$

where λ_{x0} and λ_{z0} are given by formulas from the previous slide with the following substitutions:

$$\begin{aligned}
\sigma_{\delta e} &\rightarrow \sigma_{\delta e}\frac{\sigma_{xe}}{\sqrt{\sigma_{xe}^2 + D_e^2\sigma_{\delta e}^2}} \\
\sigma_{\delta i} &\rightarrow \sigma_{\delta i}\frac{\sigma_{xi}}{\sqrt{\sigma_{xi}^2 + D_i^2\sigma_{\delta i}^2}} \\
S_x &\rightarrow \sqrt{\sigma_{xi}^2 + \sigma_{xe}^2 + D_i^2\sigma_{\delta i}^2 + D_e^2\sigma_{\delta e}^2}
\end{aligned}$$

Space charge tune shift

- One can increase cooling by increasing e-bunch's charge while reducing its emittances:

$$\lambda \propto \frac{N_e}{\varepsilon_{x,y}^2 \varepsilon_z} \cdot \frac{L_{CS}}{C_{ring}}$$

- On the other hand, space charge tune shift can be estimated as:

$$\Delta\nu_{ex,ey} = \frac{I_e}{4\pi I_a \gamma^3} \int_0^{C_R} \frac{\beta(s)}{\sigma_e^2(s)} ds = \frac{I_e C_R}{4\pi I_a \gamma^3 \varepsilon_{x,y}}$$

- Depending on e-bunch longitudinal density distribution

Gaussian: $I_e = \frac{N_e e \beta c}{\sqrt{2\pi}\sigma_{ze}}$

$$\lambda_{x,y,z} = -\frac{4\sqrt{2}}{\pi} C_1 \mathbb{S} \Psi_{x,y,z}$$

Flat top: $I_e = \frac{N_e e \beta c}{L_{ze}}$

$$\mathbb{S} = \frac{1}{S_x S_y S_z}$$
$$S_{x,y,z} = \sqrt{\sigma_{xe,ye,ze}^2 + \sigma_{xi,yi,zi}^2}$$

$$\mathbb{S} = \frac{1}{S_x S_y} \frac{\sqrt{2\pi}}{L_{ze}} \mathrm{erf}\left(\frac{L_{ze}}{2\sqrt{2}\sigma_{zi}}\right)$$

For a fixed $\Delta\nu_e$, one gets a higher cooling rate for the same N_e with a flat top bunch if one makes $L_{ze} > \sqrt{2\pi}\sigma_{ze}$

Brookhaven National Laboratory

Optical features of REC wigglers (I)

- We are working in an unusual range of parameters – a large wiggler field $B_0 \approx 2.4\ [\mathrm{T}]$ and a relatively small beam energy $B\rho \approx 0.5\ [\mathrm{T \cdot m}]$
- Therefore, in our case $b \equiv \frac{B_0}{B\rho} \approx 4.8 \left[\frac{1}{\mathrm{m}}\right]$ is not small, unlike for the usual wigglers, where $b \ll 1$
- Let's consider an analytic representation of a wiggler field :

$$B_x = \frac{k_x}{k_y} B_0 \sinh(k_x x) \sinh(k_y y) \sin(kz)$$

$$B_y = B_0 \cosh(k_x x) \cosh(k_y y) \sin(kz)$$

$$B_z = \frac{k}{k_y} B_0 \cosh(k_x x) \sinh(k_y y) \cos(kz)$$

$$k_x^2 + k_y^2 = k^2; \quad k = \frac{2\pi}{\lambda} \approx 31.4 \left[\frac{1}{m}\right]$$

Optical features of REC wigglers (II)

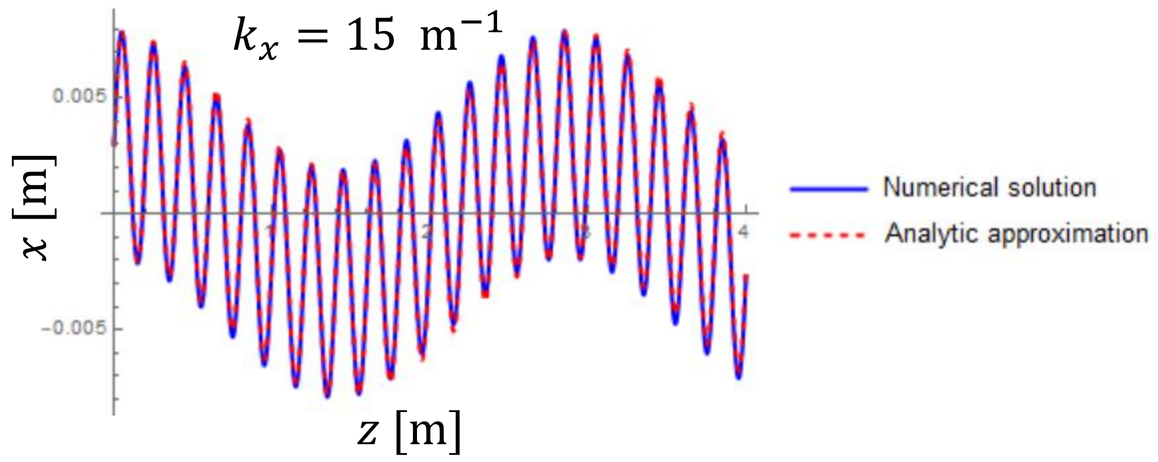
- Equations of motion:

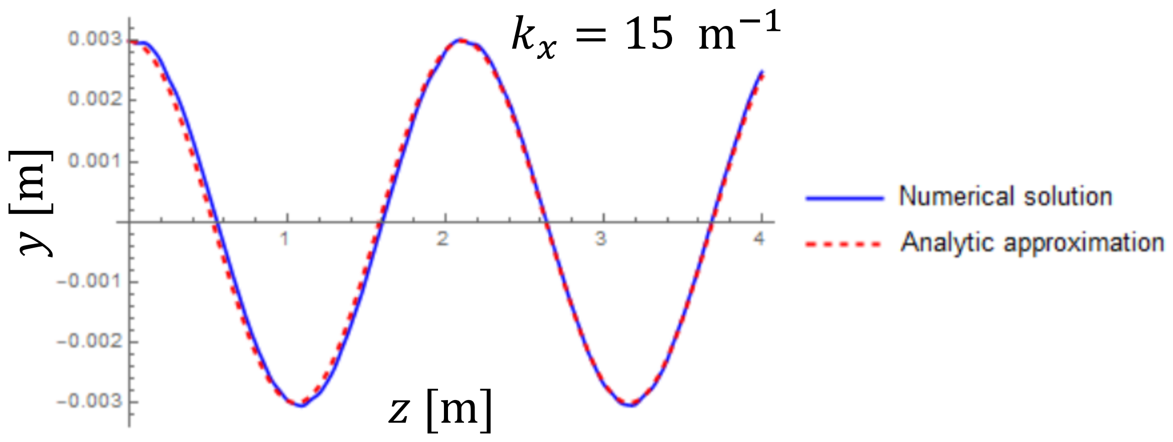
$$x'' = -\frac{B_y}{B\rho} + y'\frac{B_z}{B\rho}; \quad y'' = \frac{B_x}{B\rho} - x'\frac{B_z}{B\rho}$$

- Approximate analytic solution:

$$x = \frac{b}{k^2}\sin(kz) + x_0\cos\left(\frac{bk_x}{\sqrt{2}k}z\right) + \left(x_0' - \frac{b}{k}\right)\frac{\sqrt{2}k}{bk_x}\sin\left(\frac{bk_x}{\sqrt{2}k}z\right)$$

$$y = y_0\cos\left(\frac{bk_y}{\sqrt{2}k}z\right) + y_0'\frac{\sqrt{2}k}{bk_y}\sin\left(\frac{bk_y}{\sqrt{2}k}z\right)$$





- Our wigglers work as a thick lens in both $x\&y$ directions (there can be several full oscillations over the length of a wiggler)

Optical features of REC wigglers (III)

- Transport matrices:

$$M_x(z) = \begin{pmatrix} \cos\left(\frac{bk_x}{\sqrt{2}k}z\right) & \frac{\sqrt{2}k}{bk_x}\sin\left(\frac{bk_x}{\sqrt{2}k}z\right) \\ -\frac{bk_x}{\sqrt{2}k}\sin\left(\frac{bk_x}{\sqrt{2}k}z\right) & \cos\left(\frac{bk_x}{\sqrt{2}k}z\right) \end{pmatrix}; \quad M_y(z) = \begin{pmatrix} \cos\left(\frac{b\sqrt{k^2-k_x^2}}{\sqrt{2}k}z\right) & \frac{\sqrt{2}k}{b\sqrt{k^2-k_x^2}}\sin\left(\frac{b\sqrt{k^2-k_x^2}}{\sqrt{2}k}z\right) \\ -\frac{b\sqrt{k^2-k_x^2}}{\sqrt{2}k}\sin\left(\frac{b\sqrt{k^2-k_x^2}}{\sqrt{2}k}z\right) & \cos\left(\frac{b\sqrt{k^2-k_x^2}}{\sqrt{2}k}z\right) \end{pmatrix}$$

- This gives the expected focusing for small b: $\langle K_x \rangle = \frac{B_0^2}{2(B\rho)^2}\frac{k_x^2}{k^2}$, $\langle K_y \rangle = \frac{B_0^2}{2(B\rho)^2}\left(1-\frac{k_x^2}{k^2}\right)$

- Chromaticity is easy to calculate for the matched conditions $\left(\beta_x = \frac{\sqrt{2}k}{bk_x};\ \beta_y = \frac{\sqrt{2}k}{b\sqrt{k^2-k_x^2}}\right)$:

$$\frac{\partial Q_x}{\partial \delta} = -\frac{1}{2\pi}\frac{B_0 N_p \lambda}{\sqrt{2}B\rho}\frac{k_x}{k}; \quad \frac{\partial Q_y}{\partial \delta} = -\frac{1}{2\pi}\frac{B_0 N_p \lambda}{\sqrt{2}B\rho}\frac{\sqrt{k^2-k_x^2}}{k}$$

- Obtained analytic expressions were cross-checked by Bmad and GPT tracking
- For example, in the current lattice we want $\beta_x = 30\ \mathrm{cm}$. This sets $k_x = 30.7\ \mathrm{m}^{-1}$ and $\beta_y = 1.4\ \mathrm{m}$, $\frac{\partial Q_x}{\partial \delta} = -2.1$, $\frac{\partial Q_y}{\partial \delta} = -0.5$

Importance of proper wiggler focusing

- We started with a different wiggler field – a “s-bend wiggler”:

$$B_x = B_0 \cos(k_q x) \sinh(k_q y)$$
$$B_y = B_0 \cosh(ky) \sin(kz) + B_0 \sin(k_q x) \cosh(k_q y)$$
$$B_z = B_0 \sinh(ky) \cos(kz)$$

- For a thick lens case, such a wiggler produces a huge chromaticity in the direction orthogonal to the wiggling plane when $k_q \to \frac{B_0}{2B\rho}$:

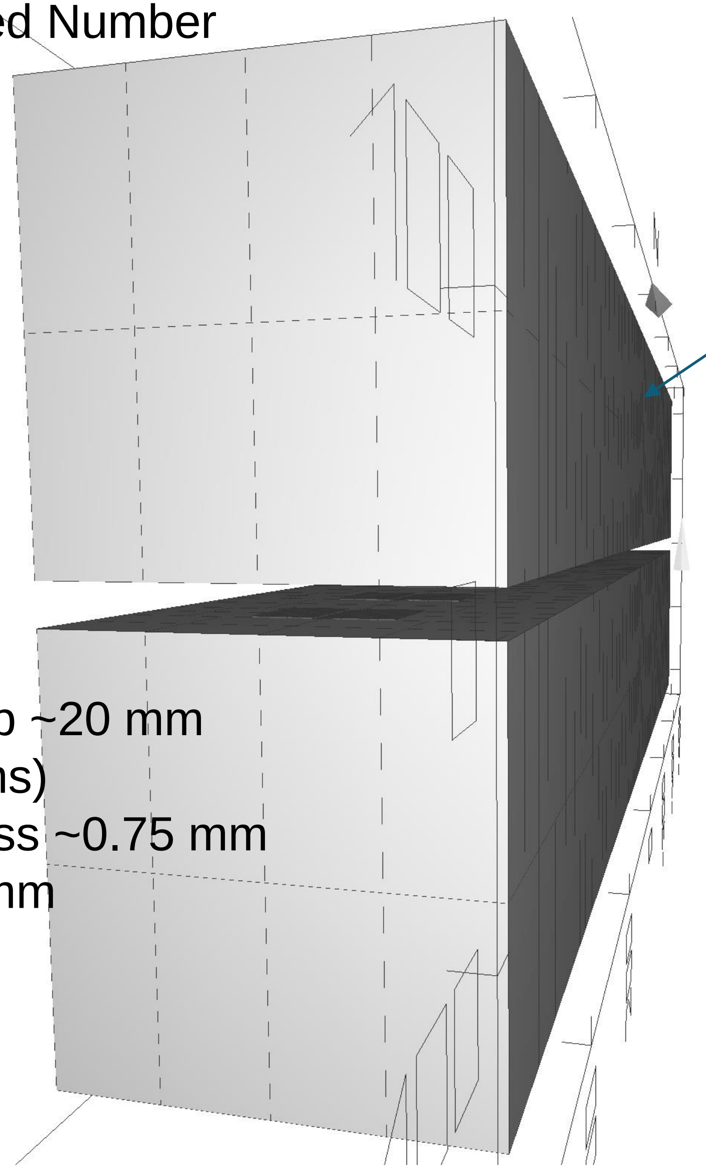
$$\frac{\partial Q_y}{\partial \delta} = -\frac{N\lambda B_0}{2\pi\sqrt{2}B\rho} \frac{1 - k_q B\rho/B_0}{\sqrt{1 - 2k_q B\rho/B_0}}$$

- Since the required focusing in the wiggling plane set our $k_q \approx 2.3\ \mathrm{m}^{-1}$, the chromaticity in a non-wiggling plane from a single wiggler became ~ -11, resulting in an extra chromaticity of ~ -200
- Switching to a wiggler with a “sextupole–like” field profile restored our momentum and dynamic aperture

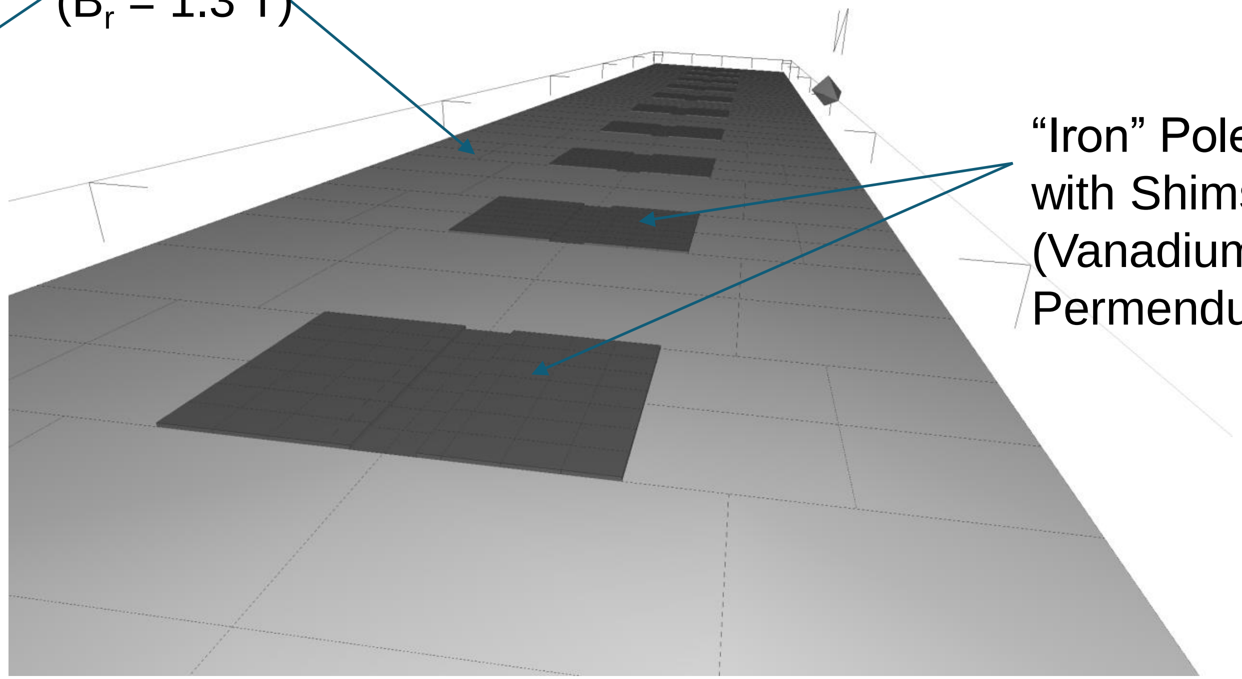
- Also, we found an element with a close to zero focusing and an arbitrary large chromaticity – this might be interesting

Preliminary (non-optimized) 3D Geometry of a Possible Damping Wiggler for EIC REC

Whole Geometry
(with reduced Number of Periods)



Lower Part



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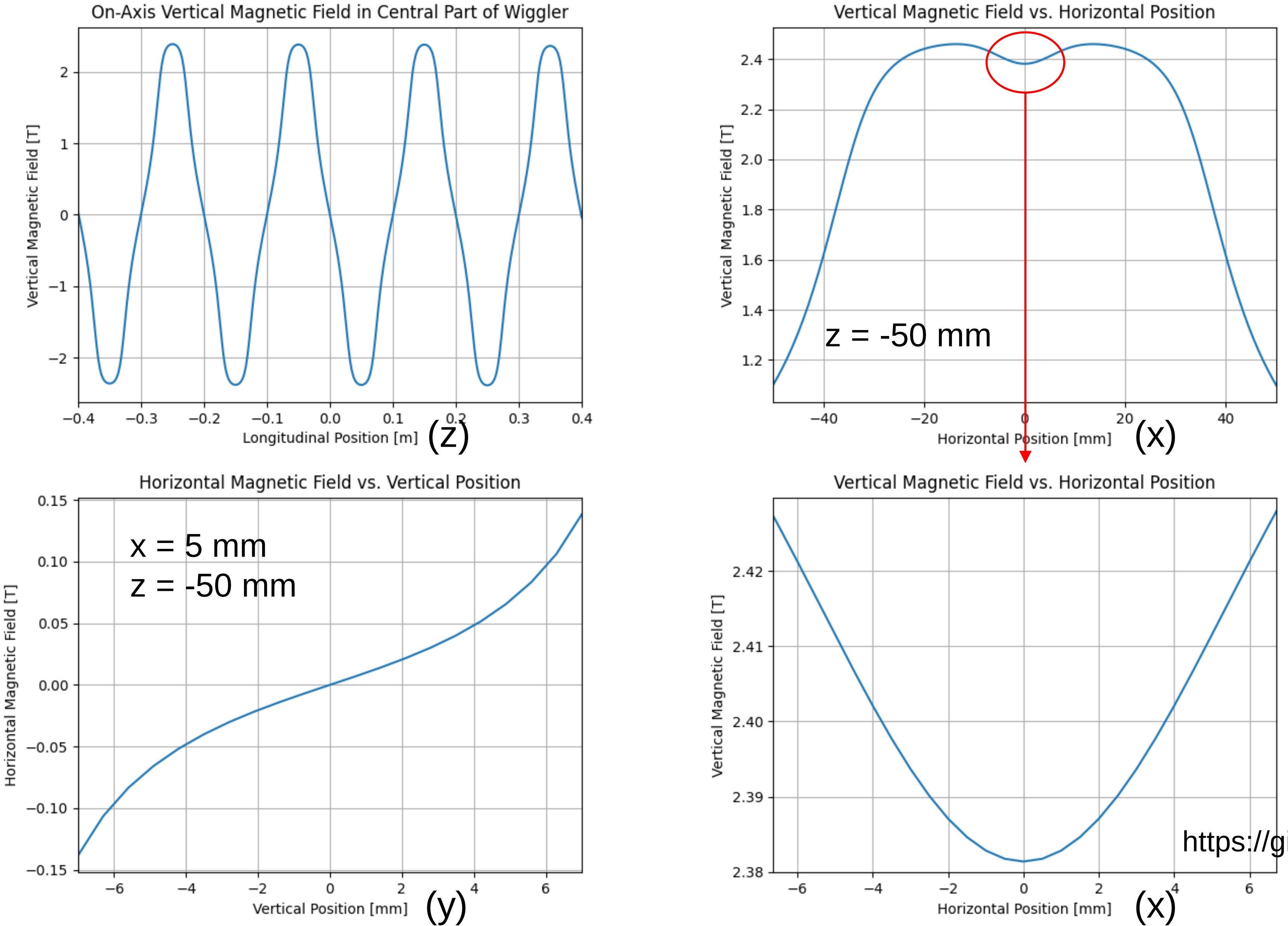
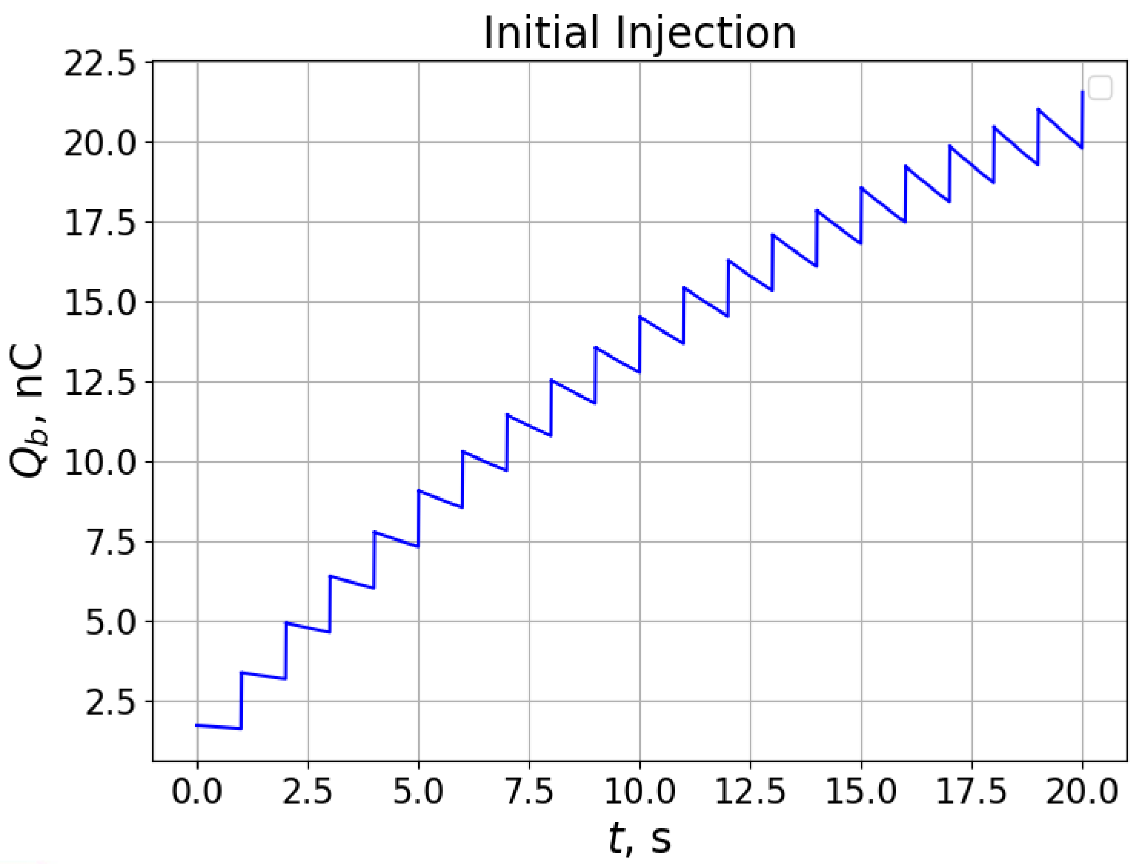
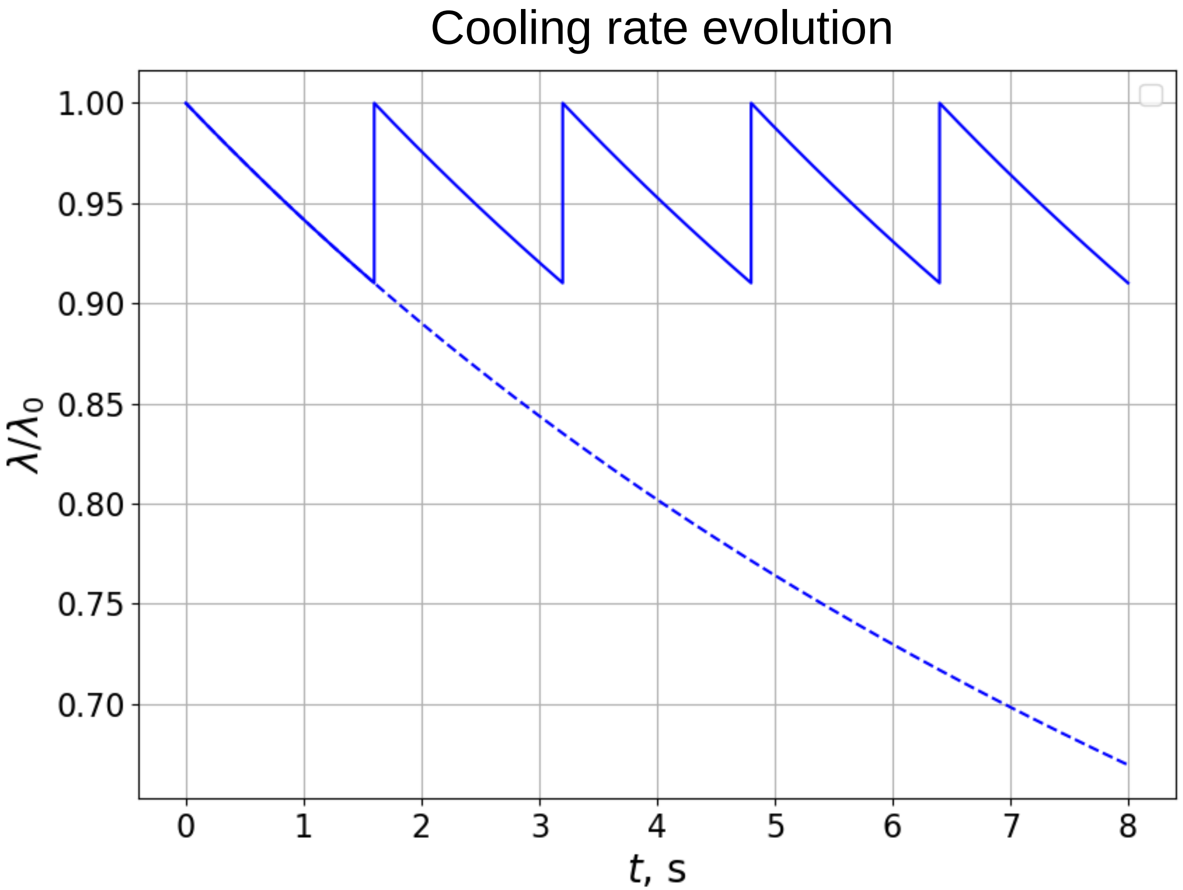
~Attainable Magnetic Field of a Possible Damping Wiggler for EIC REC (preliminary Radia calculations)



https://github.com/ochubar/Radia

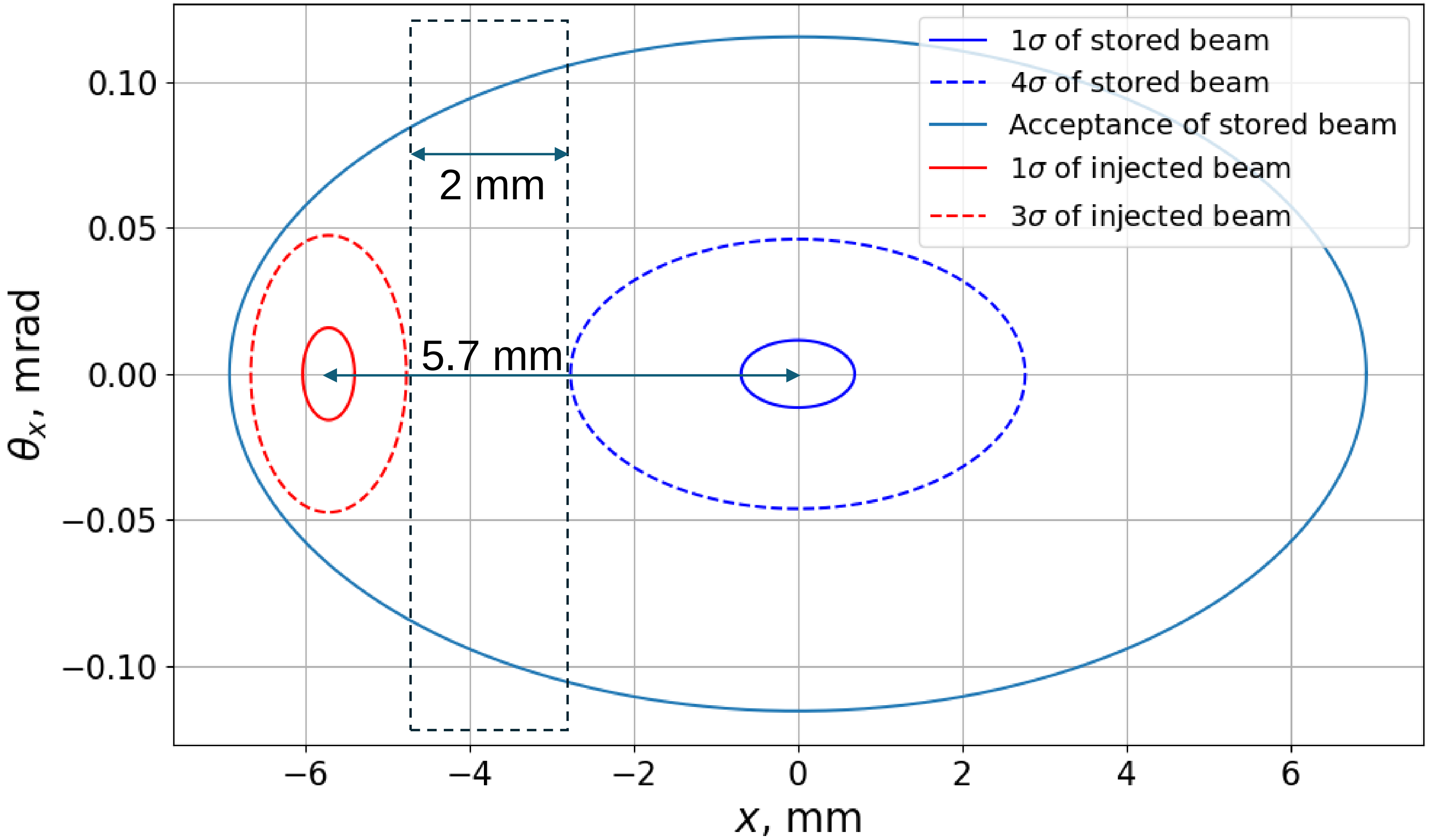
Injection (single bunch point of view)





Injection (transverse acceptance)

- We can safely assume the thickness of the septum to be 2 mm:



$\beta_{x(stored)} = 60$ m
$\beta_{x(injected)} = 20$ m

$\varepsilon_{stored} = 8$ nm
$\varepsilon_{injected} = 5$ nm
$\varepsilon_{n(injected)} = 1.5\ \mu$m

$4\sigma_{stored} = 2.8$ mm, therefore, if stored beam is 17 mm away from the inner edge of the septum knife, then the kicker bump must produce 14.2 mm displacement

Beam dynamics for 2.5 nC