TALES FROM THE QUANTUM CRYPT

Exploring Quantum Many-Body Scars: Anomalies to Thermalization in Quantum Systems

Wildeboer et al., PRB 106, 205142 (2022)

… more has been done … … more to come …

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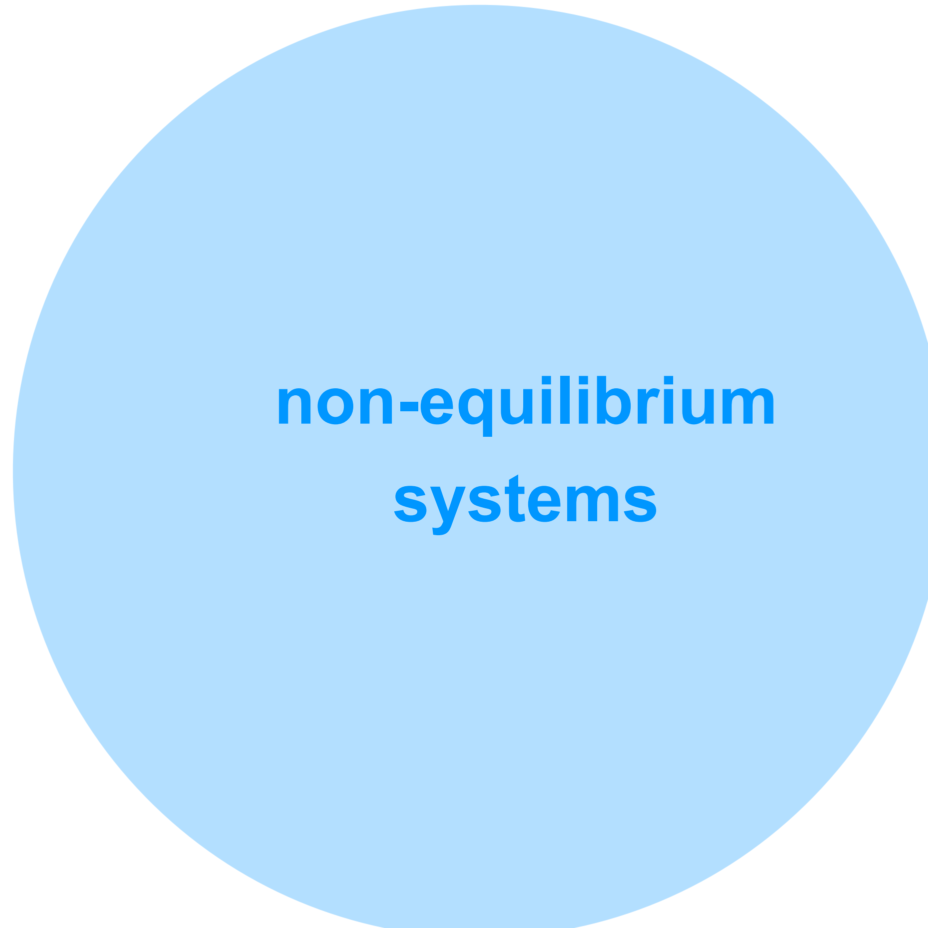
Physics Colloquium

September 17th 2024

Overarching Theme



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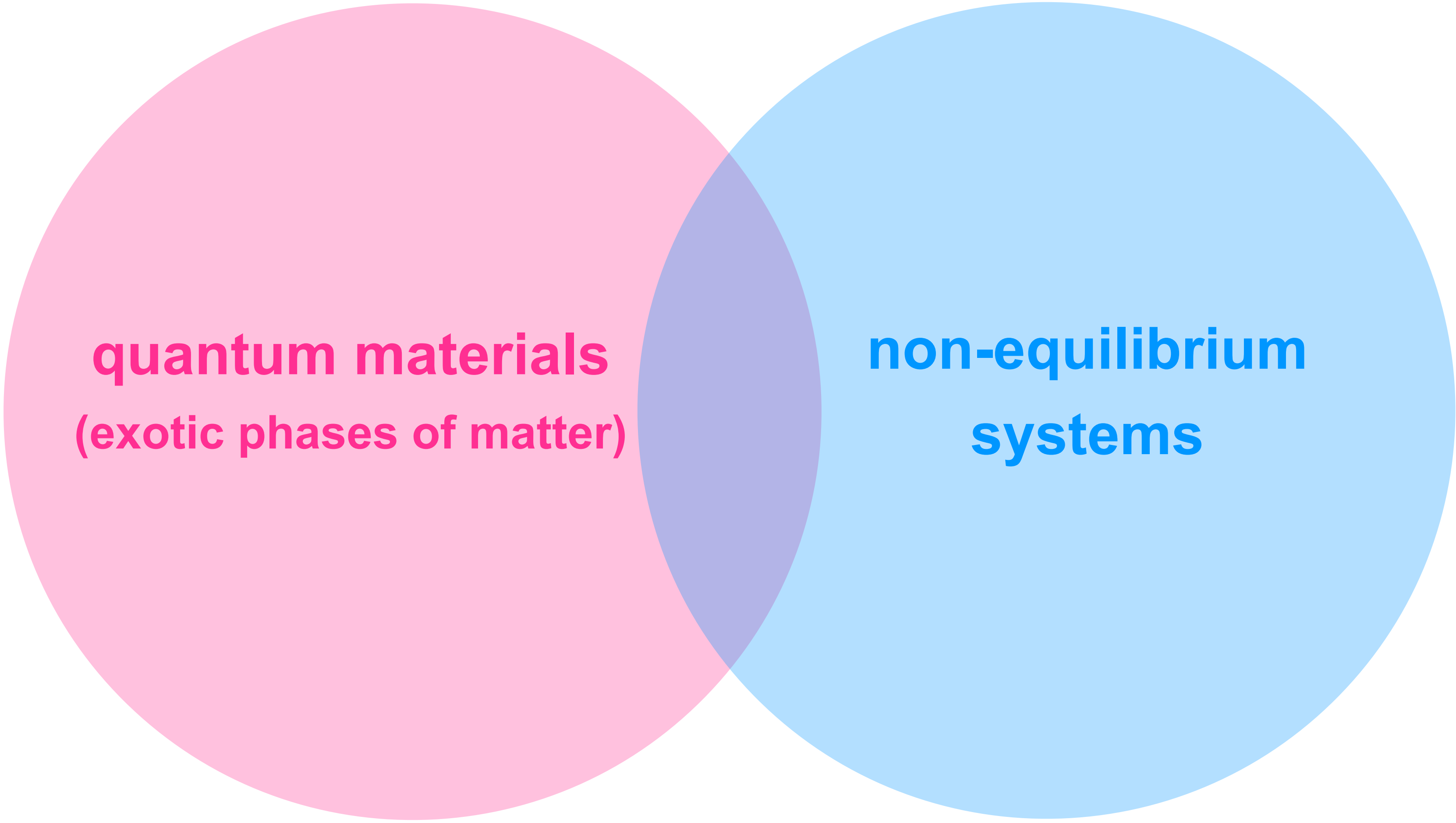

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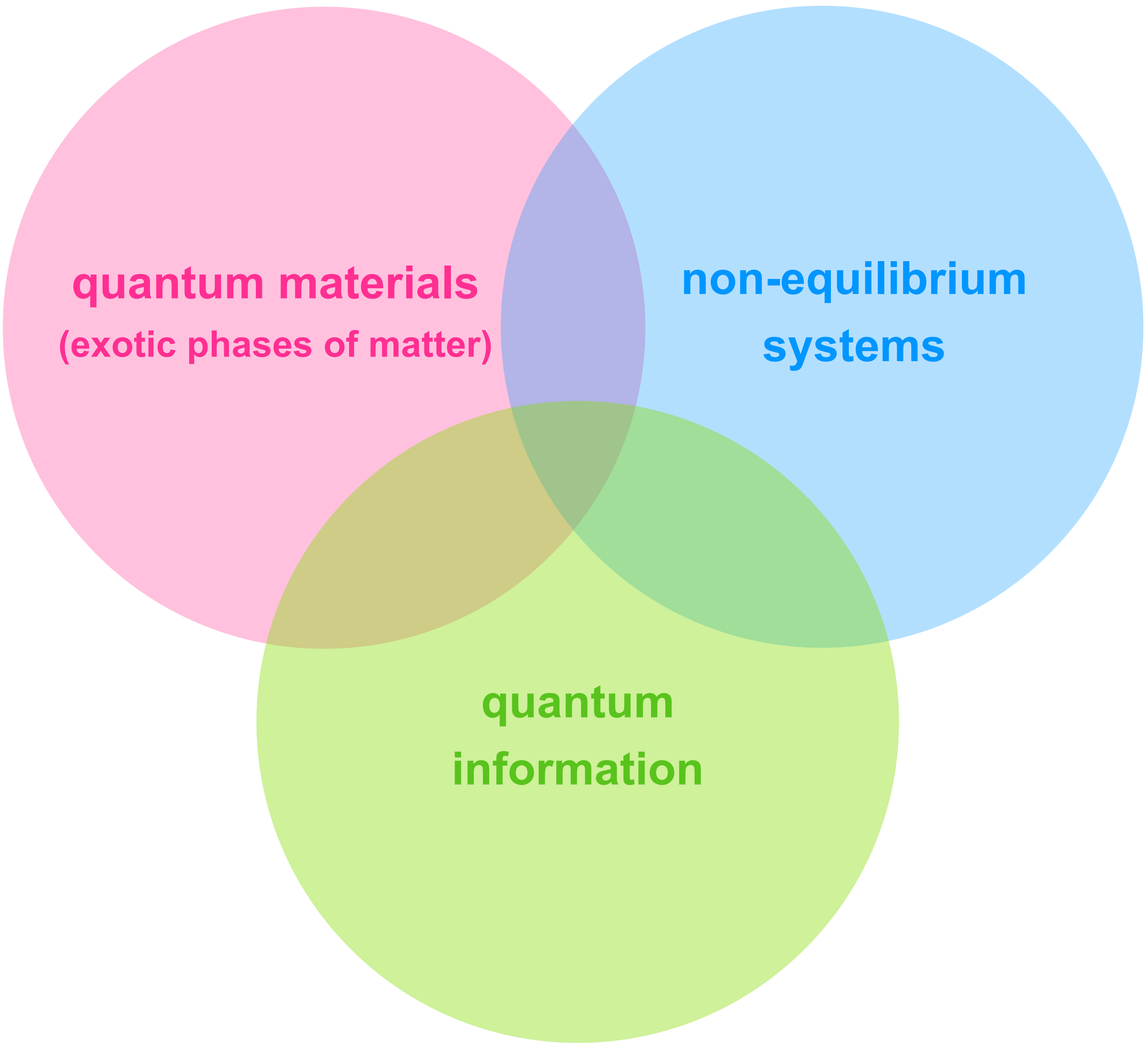

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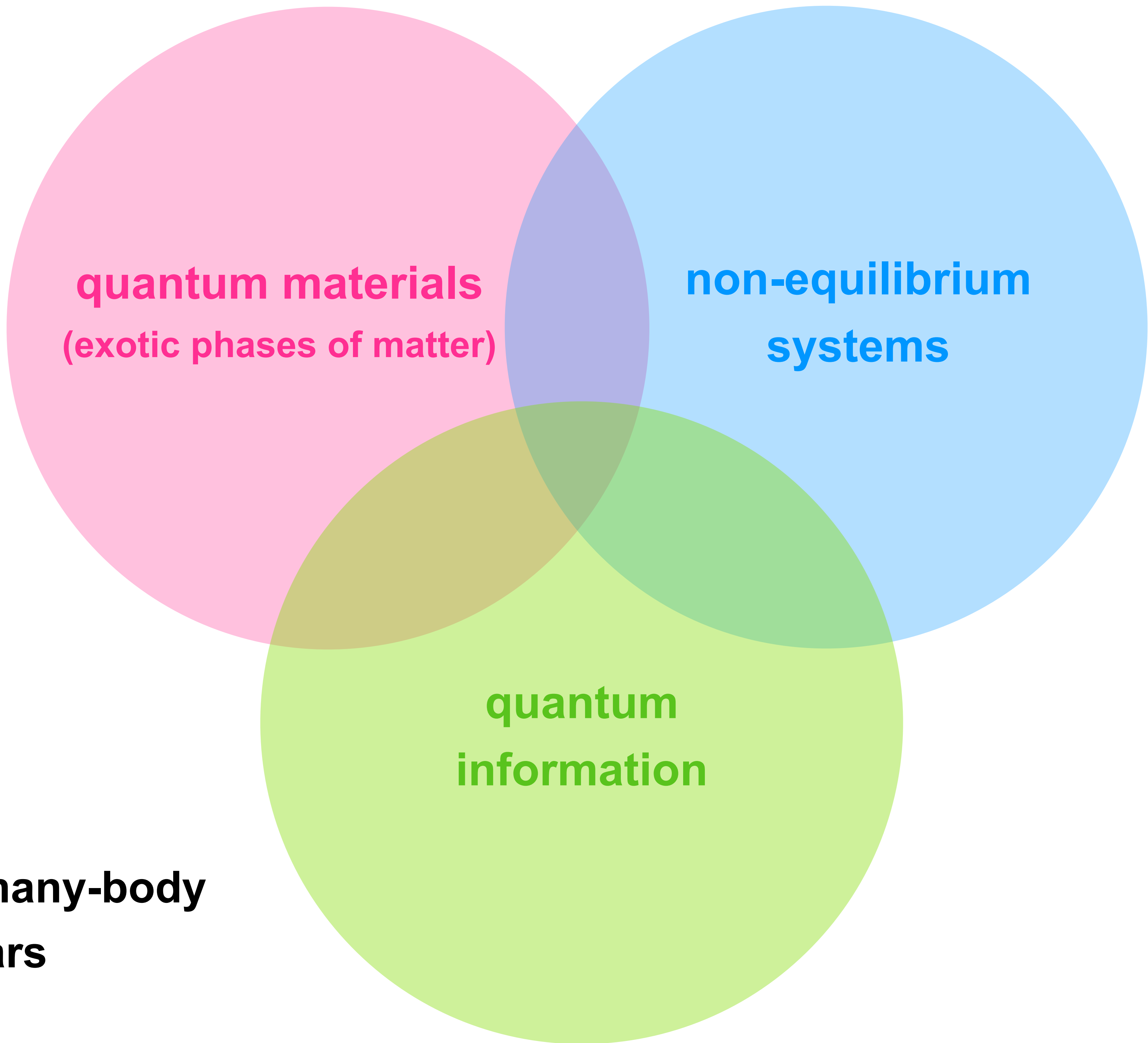
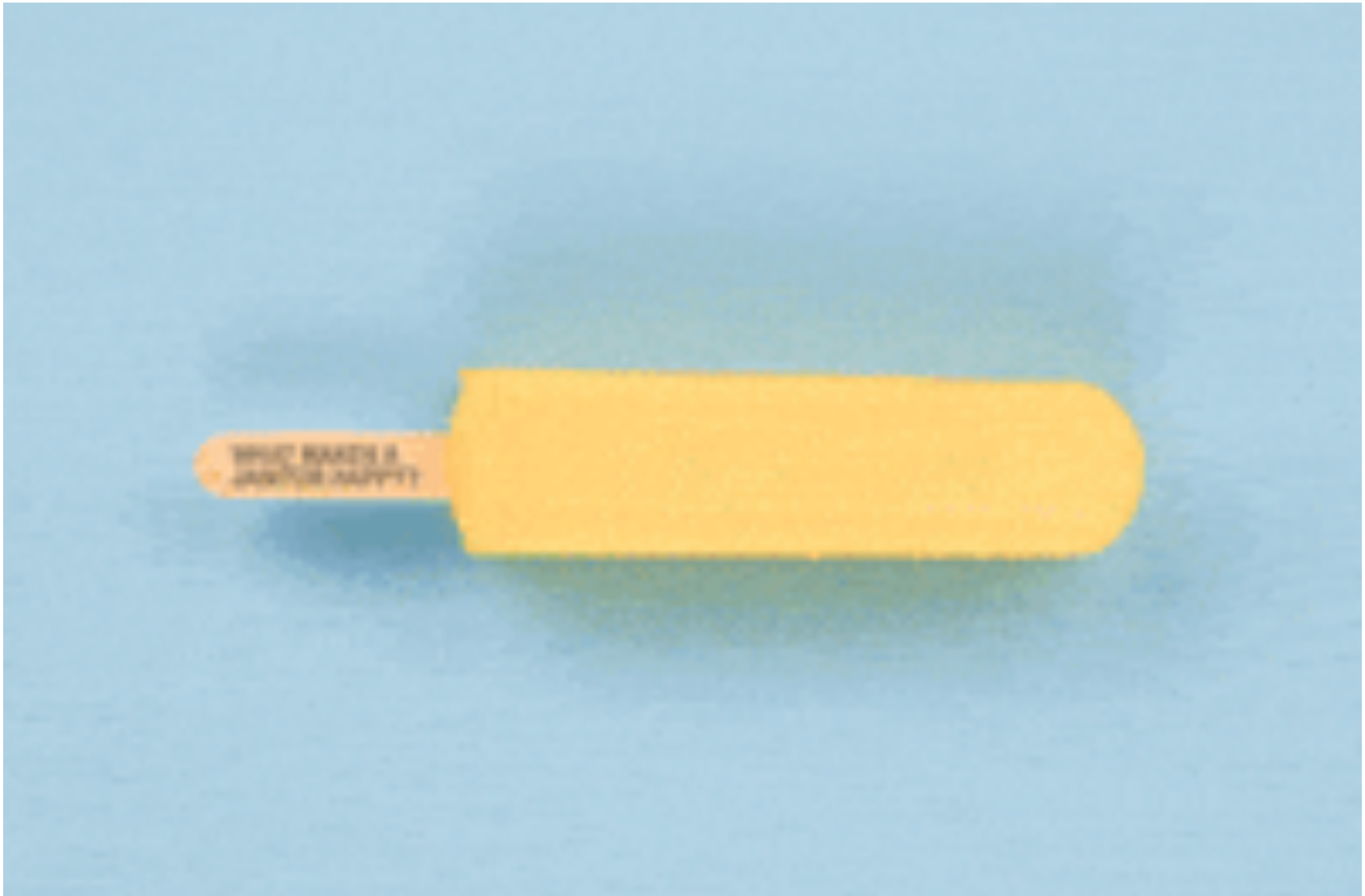


this talk:

quantum many-body scars

Enjoying the last days of summer …



What is a scar?

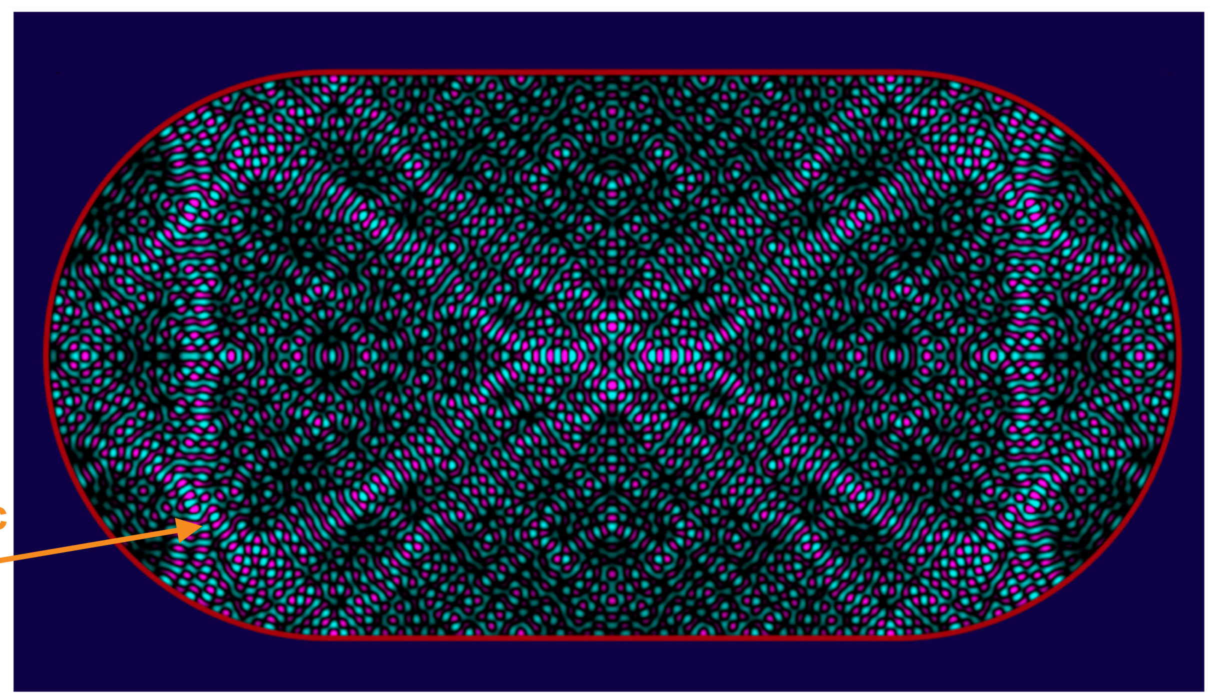
► **Quantum Chaos: Bunimovich Stadium**

- **chaotic system: ball will cover every possible trajectory inside the stadium**
- **if ball is started at a certain angle, it will instead retrace the same path forever**
- **same situation for if ball is replaced by quantum particle**

scarred wave function

$|\Psi|^2$ **Eric Heller 1980s**

unstable classical periodic orbit



billiard is quantum ergodic but not quantum unique ergodic (almost all eigenfunctions uniformly spread over the billiard

observable signatures in static/dynamic properties of the system

particle in a Bunimovich stadium can show scars along the trajectories where it is likely to be found

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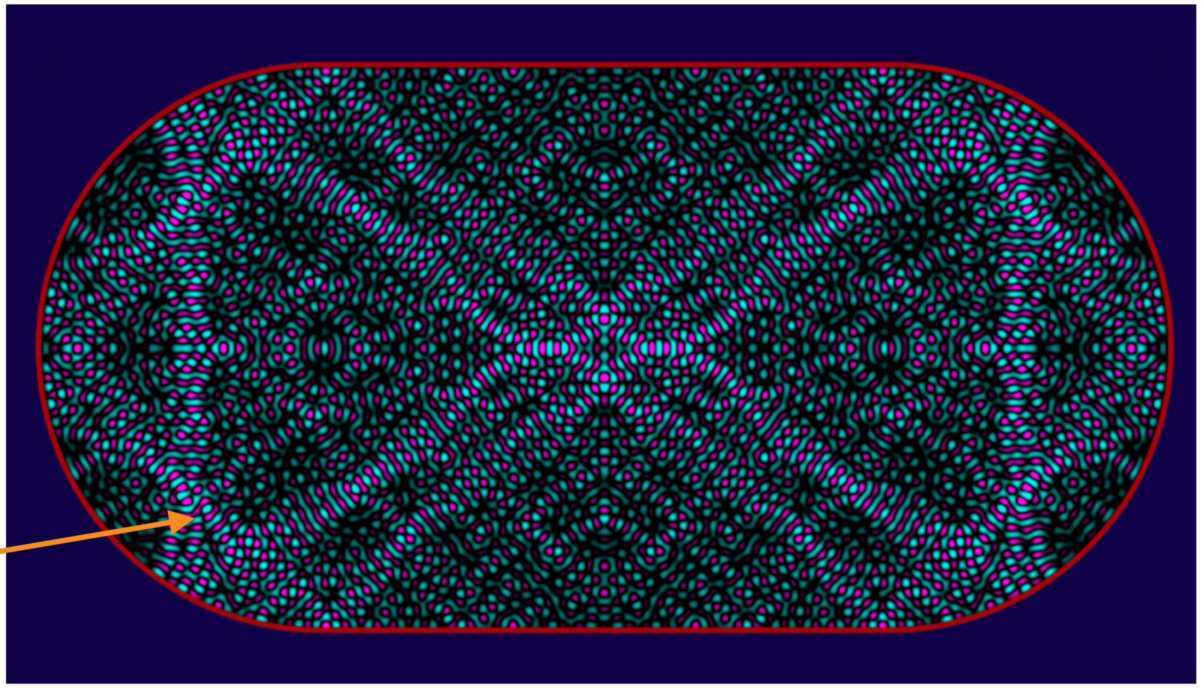
$\Longrightarrow$ Does this have an analogue in a quantum many-body system?

unstable classical periodic orbit



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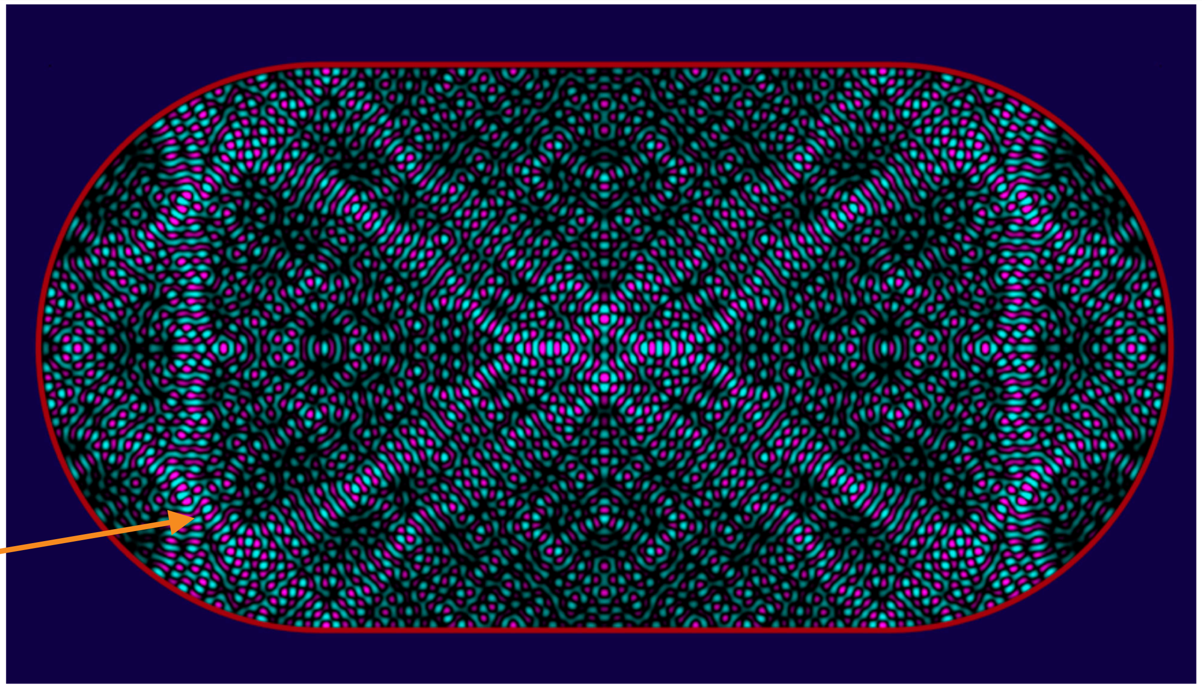
$\Longrightarrow$ **analogy with recurring alternating state of atoms: quantum-many body scars**



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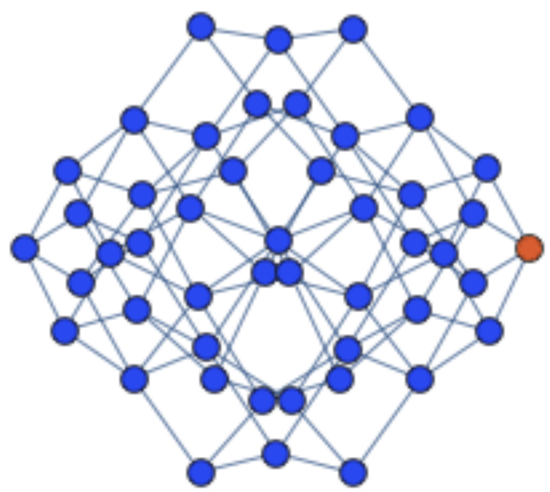
What is a quantum many-body scar?



- each configuration is a vertex
- vertices i and j are connected by $\langle \Psi_i | \mathcal{H} | \Psi_j \rangle$

► **10 atoms oscillating between ground state (black) and excited state (white). Atoms can be simultaneously in the superposition of all possible 47 configurations.**

► **Top plot shows different probabilities of individual configurations over time.**

Quantum Dynamics/Ergodicity

$\Longrightarrow$ **many-body physics beyond the ground states**

- **advances in experiments: isolated quantum systems**
- **fundamental questions: When and how is quantum information lost/retained?**

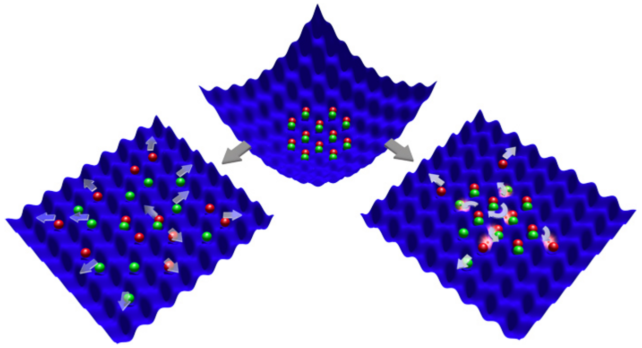
$\Longrightarrow$ **ergocity**

- **isolated system: quantum quench**

$$|\Psi\rangle \longrightarrow U(t) \longrightarrow |???\rangle$$

Motivation: Progress in Experiments

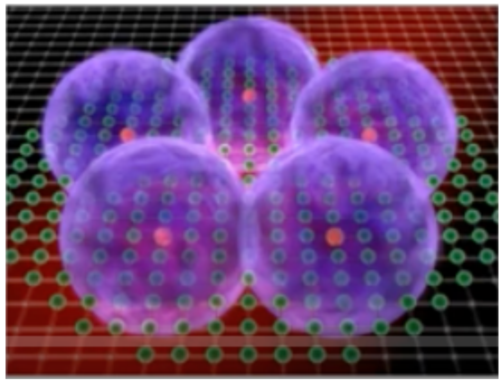
ultracold atomic systems



fermionic atoms in optical lattice: dynamics depend on (non)-interacting atoms

[Bloch group (MPQ, Munich)]

(ultracold) Rydberg atoms



alkali-metal atoms (lithium, sodium, potassium, rubidium, cesium, and francium)

[Harvard group]

- systems are isolated from environment
- non-equilibrium physics
- quantum dynamics: prepare precise initial states and observe the ensuing dynamics in real time

- experimental realization of topological matter

Ergodicity in Quantum Dynamics

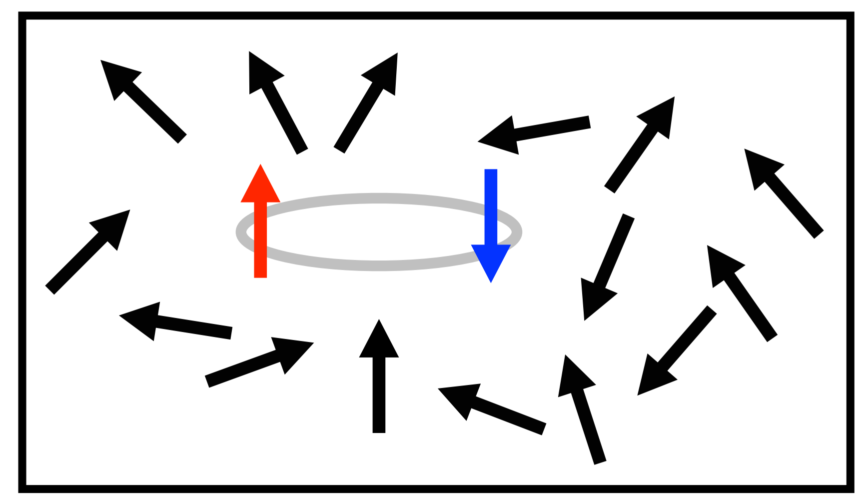
- **isolated system: quantum quench**

$$|\Psi\rangle \rightarrow U(t) \rightarrow |???\rangle$$

google

time

- ▶ **ergodic dynamics: system relaxes to locally thermal state regardless of initial condition**
- ▶ **mechanism: system acts as its own bath**
- ▶ **many-body time evolution washes away quantum correlations**
- ▶ **quantum information stored in local objects is rapidly lost as these get entangled with the rest of the systems.**
- ▶ **many-body system is essentially devoid of any remaining structure**



spin system: spins will get entangled with other spins as time progresses

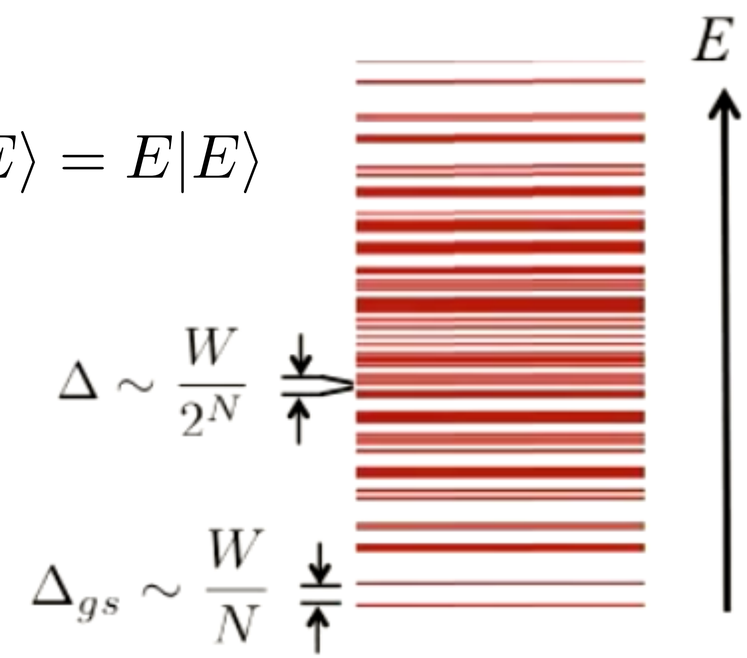
Ergodicity in Eigenstates

Eigenstate Thermalization Hypothesis (ETH)

Deutsch 1991, Srednicki 1994

- interested in generic high energy eigenstates $|E\rangle$ (finite energy density above ground state)

$$\mathcal{H}|E\rangle = E|E\rangle$$



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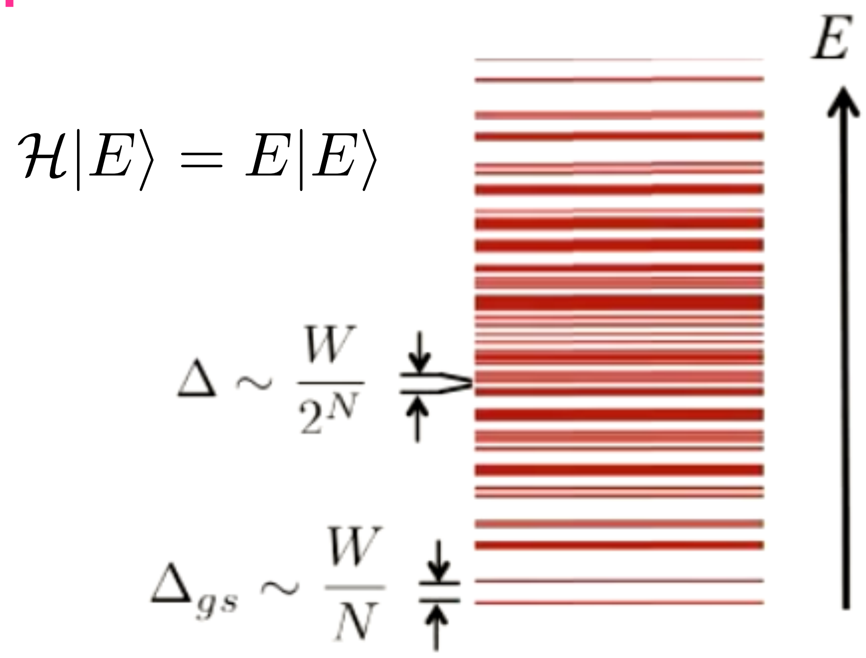
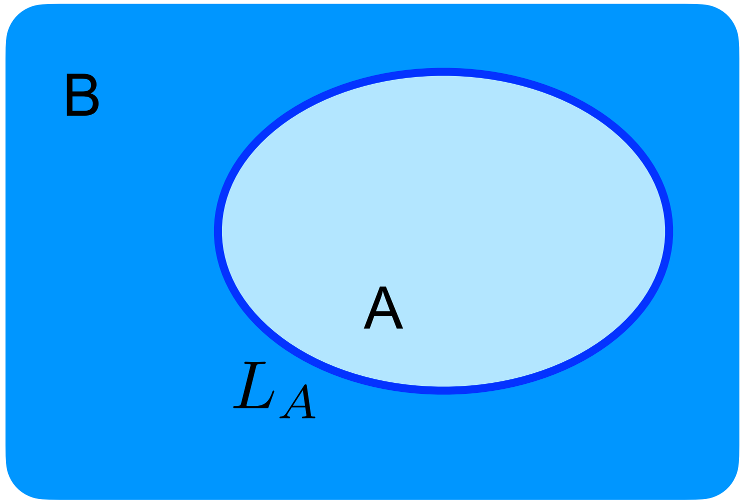
$$\rho_A = \mathrm{tr}_B |E\rangle\langle E| \longrightarrow \frac{1}{Z_A} e^{-\beta H_A}$$





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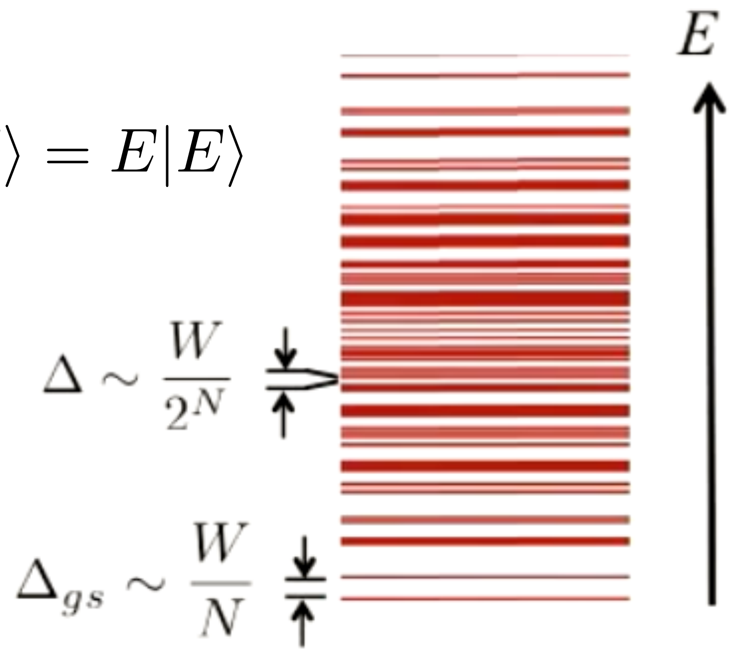

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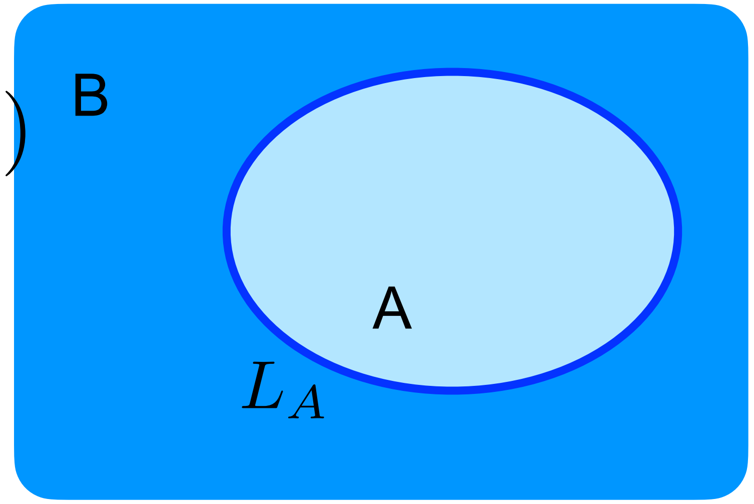
$$S_A = \mathrm{tr}[\rho_A \ln \rho_A] = s(E) L_A^d \propto S_{ther} \sim \mathrm{Vol}(A)$$

thermal entropy is extensive at finite temperature

$$\langle \mathcal{O}_A \rangle_E = Tr(\rho_A \mathcal{O}_A) \approx \langle \mathcal{O}_A \rangle_{E'}$$



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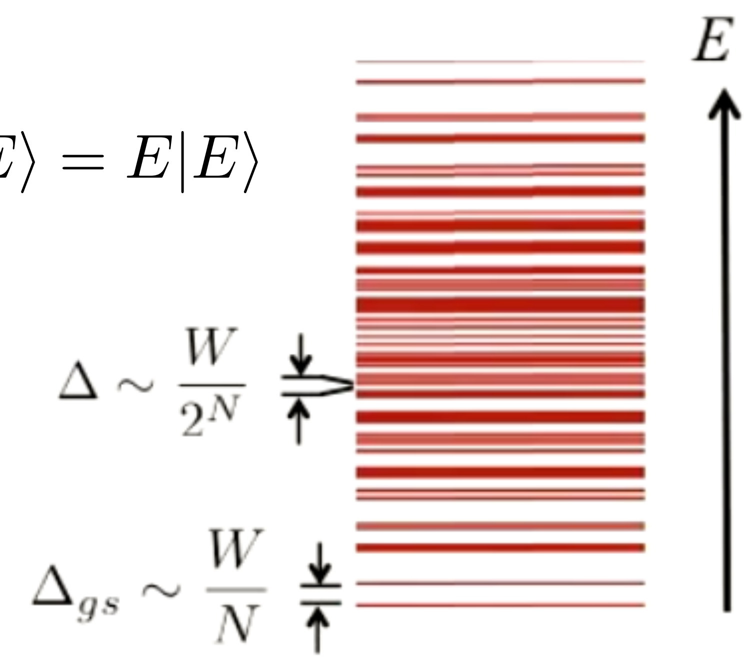

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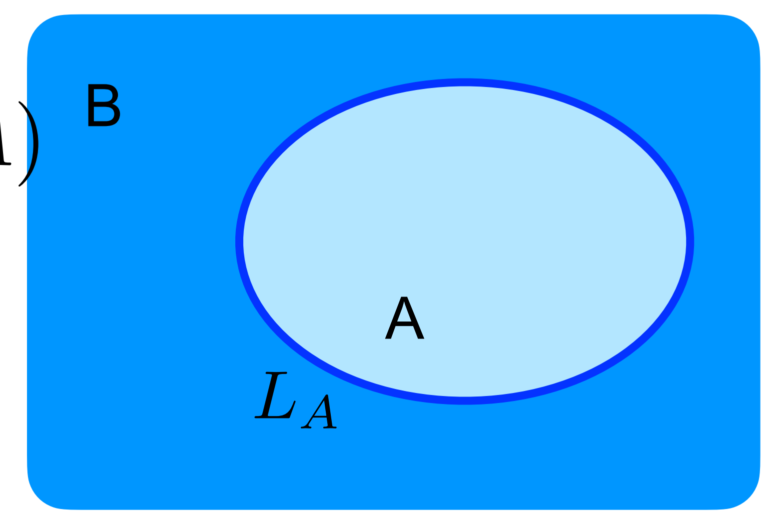

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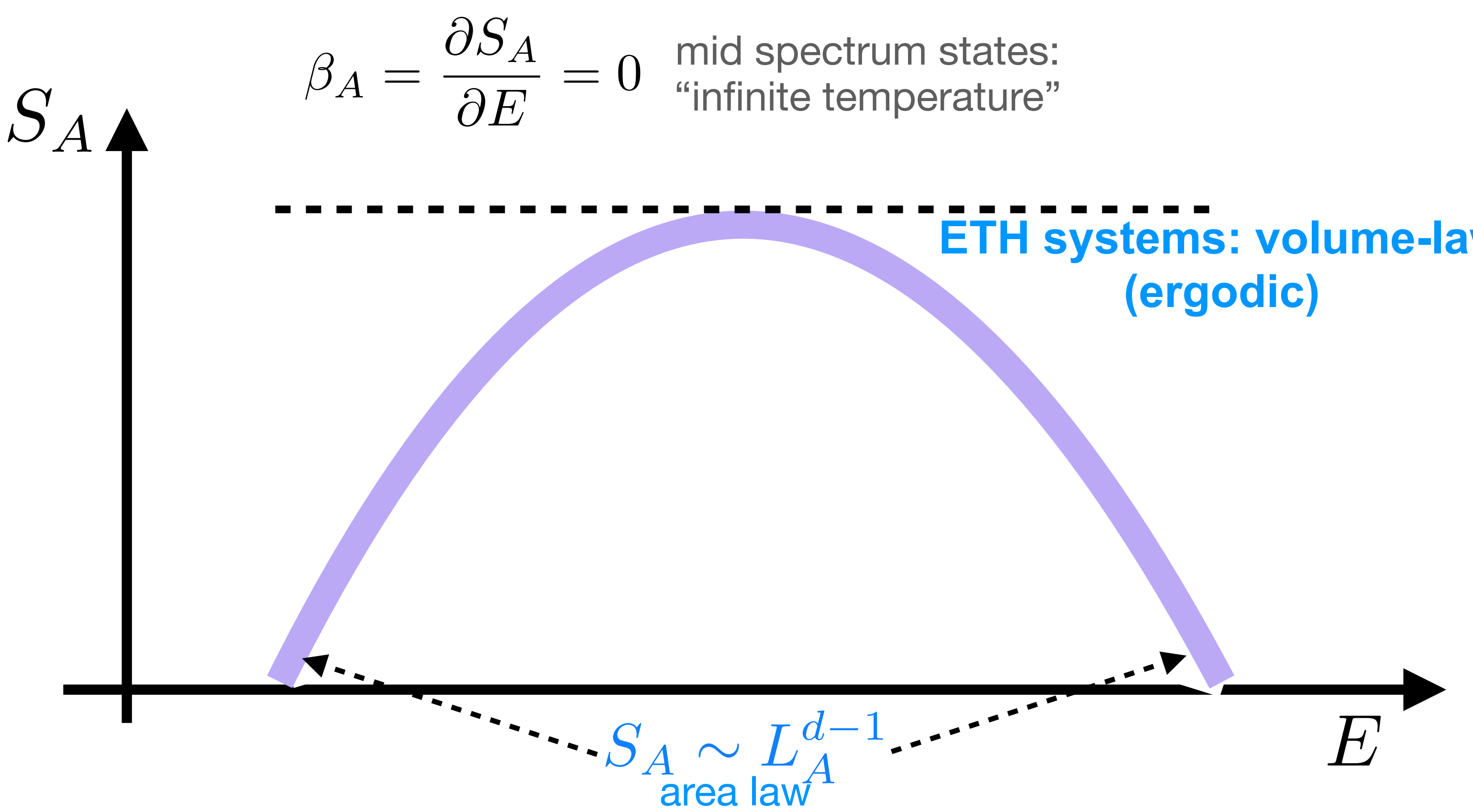
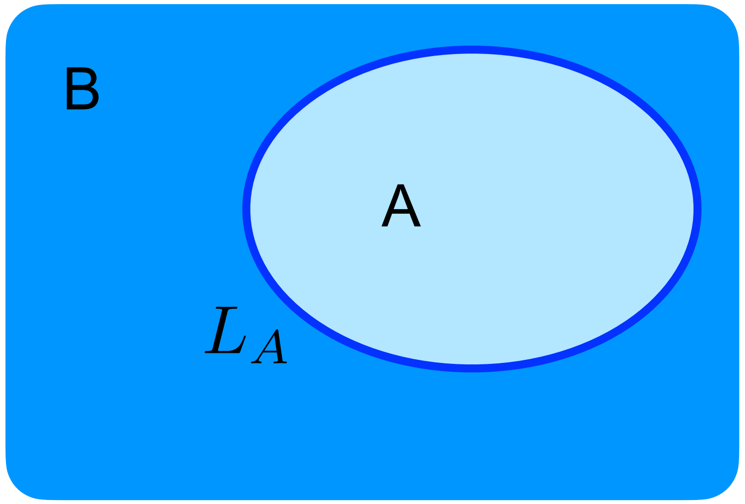
$\Longrightarrow$ **Are there non-ergotic quantum systems?**

Ergodicity in Eigenstates

- **Eigenstate Thermalization Hypothesis (ETH)** **Deutsch 1991, Srednicki 1994**
- **entanglement behavior of generic highly excited many-body states**





► **ETH systems: every eigenstate thermalizes, all finite energy density eigenstates exhibit volume-law**

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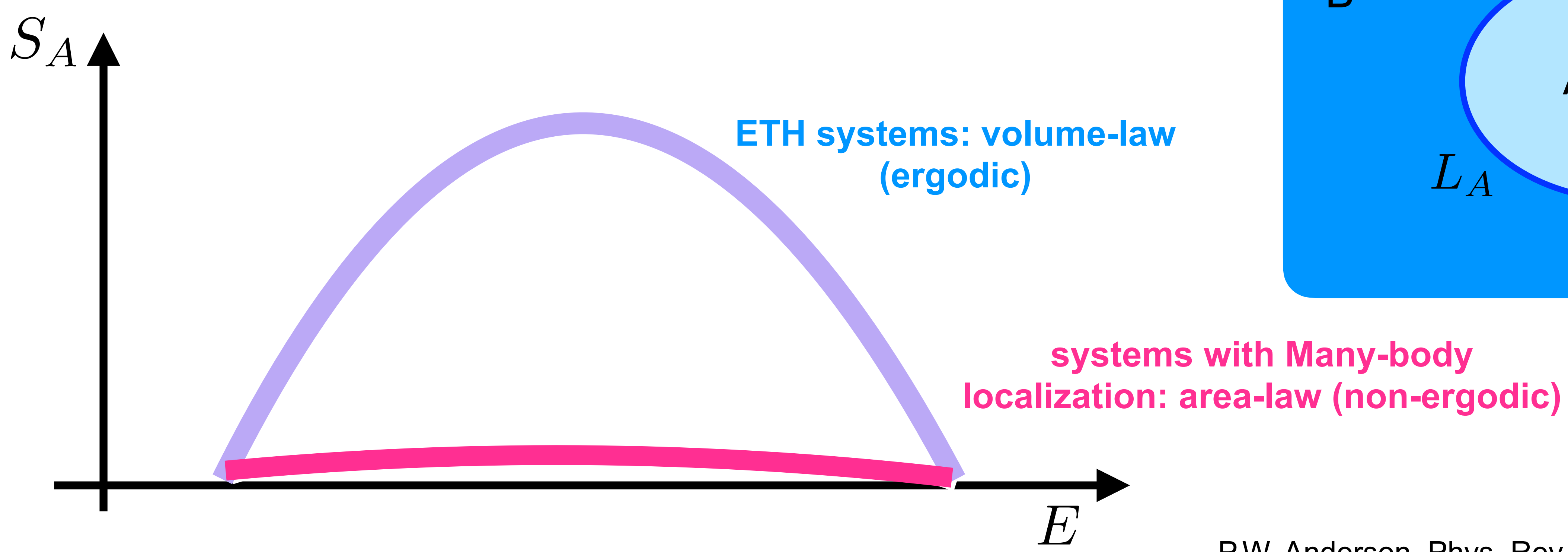

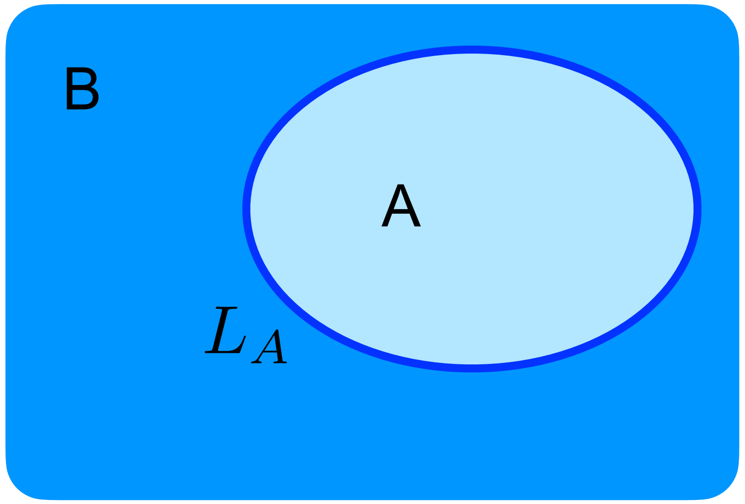

P.W. Anderson, Phys. Rev. 109, 1492 (1958)

Gornyi, Mirlin, Polyakov, PRL 95, 206603 (2005)

Pal and Huse, PRB 82, 174411 (2010)

Serbyn, Papic, Abanin, PRL 111, 12701 (2013)

Huse, Nandkishore, Oganesyan, PRB 90, 174202 (2014)

- **ETH systems: every eigenstate thermalizes, all finite energy density eigenstates exhibit volume-law**
- **Many-body localization (MBL) systems: all eigenstates have area-law entanglement**
- **Dynamics in MBL systems: all states retain memory of initial state (nonergodicity)**

Many-Body Localization phase:

strong ergodicity breaking

Ergodicity in Eigenstates

- **Eigenstate Thermalization Hypothesis (ETH)** **Deutsch 1991, Srednicki 1994**
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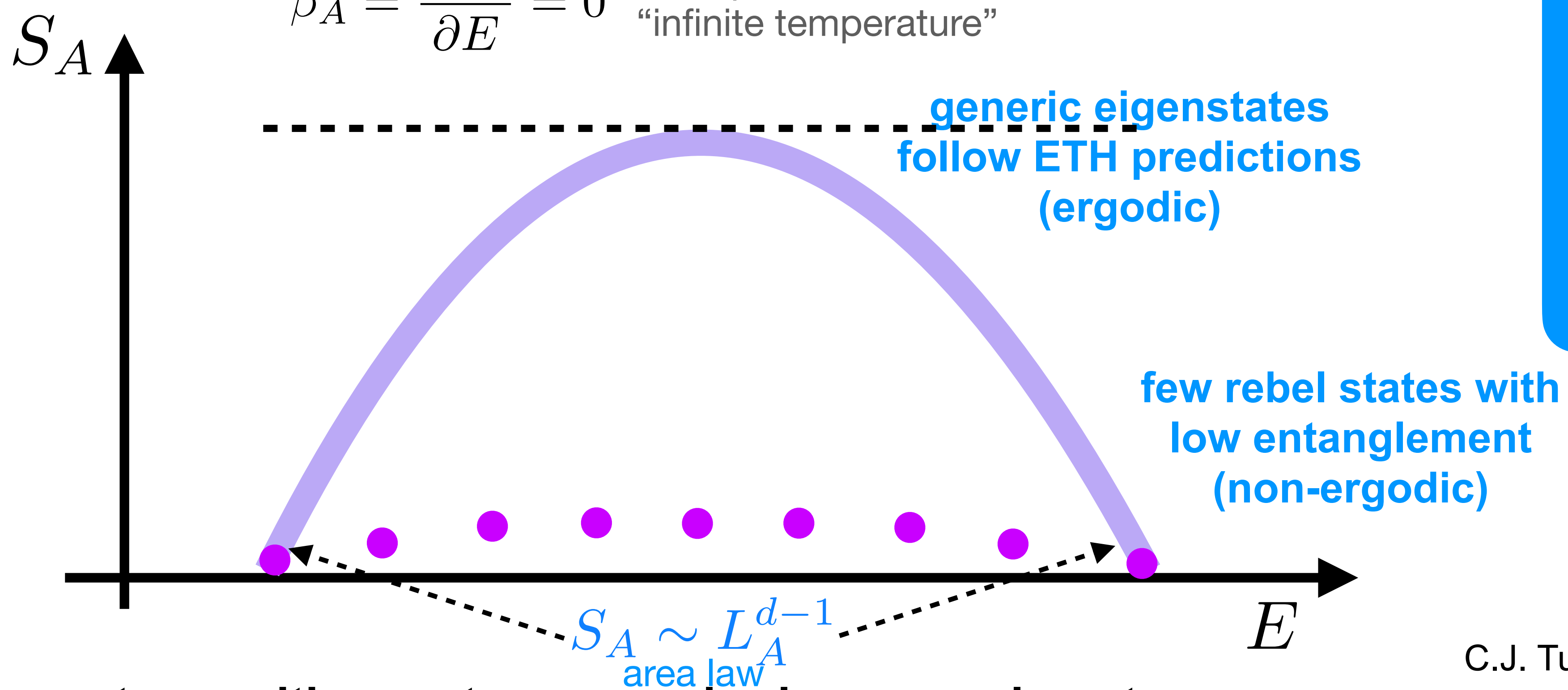
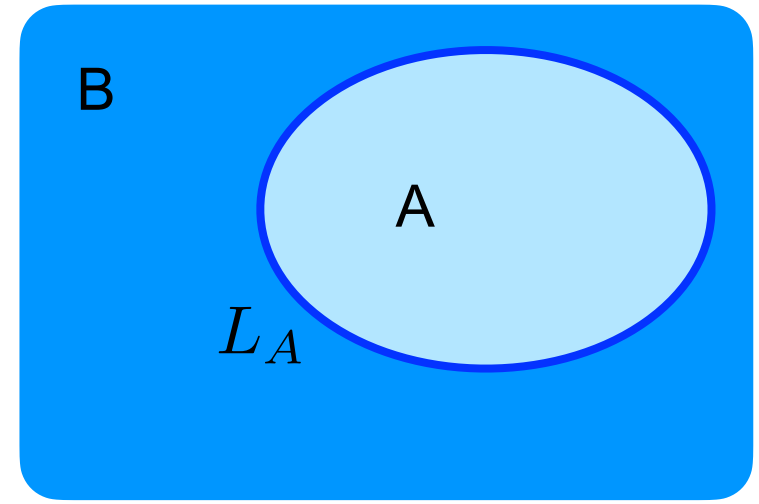
$$\beta_A = \frac{\partial S_A}{\partial E} = 0$$ mid spectrum states: "infinite temperature"





C.J. Turner et al., Nature 14, 745–749 (2018)

- ▸ **systems with quantum many-body scars: almost every eigenstate thermalizes**

$$\lim_{L\to\infty}\left(\frac{1}{\dim(\mathcal{H})}\times N_{\mathrm{non-thermal}}\right)=0$$

- ▸ **Dynamics: almost all states do NOT retain memory of initial state (ergodicity)**
- ▸ **systems with quantum many-body scars: a few eigenstates (scar states) exhibit sub-volume entanglement**
- ▸ **Dynamics: scar states DO retain memory of initial state (nonergodicity)**

quantum many-body scars:

weak ergodicity breaking

Quantum many-body Scars $\Longleftrightarrow$ Quantum Information

- **motivation in applications/advances in quantum information**

  - **Quantum Memory:** persistent long-lived oscillations relevant for quantum information storage, interest for applications in quantum memory and quantum error correction

  - **Entanglement Properties:** unique entanglement structures can be leveraged to study entanglement dynamics and correlations, critical for quantum information processing, understanding how entanglement evolves (entanglement dynamics) provides insights into non-equilibrium dynamics valuable for developing quantum algorithms/protocols

  - **Quantum Computing and Algorithms:** efficient state preparation in quantum computing, robustness against decoherence

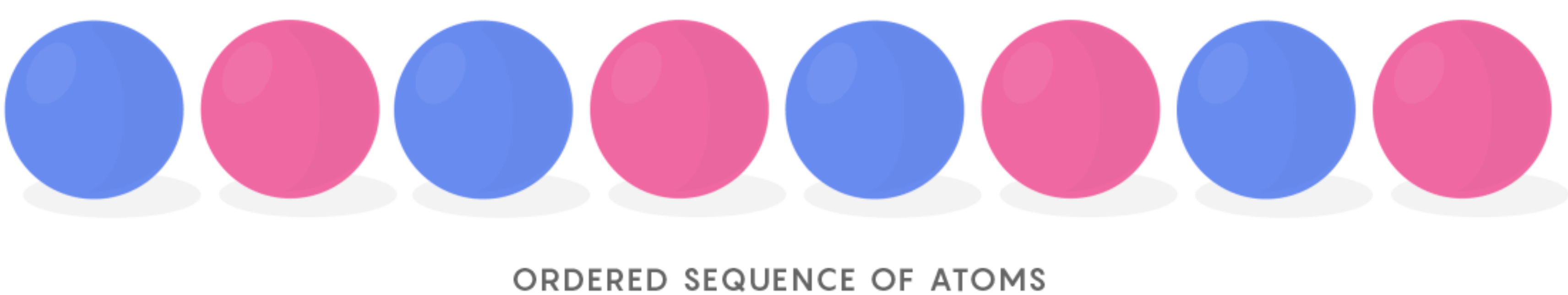
  - **Information Scrambling:** study of QMBS can contribute to understanding how information is scrambled in quantum systems, relevant for quantum communication and information security

  - **Exp. Realization in Cold Atoms:** providing platforms for exploring quantum information concepts in controlled environments, serve as testbeds for developing quantum information technologies

Experimental Realization

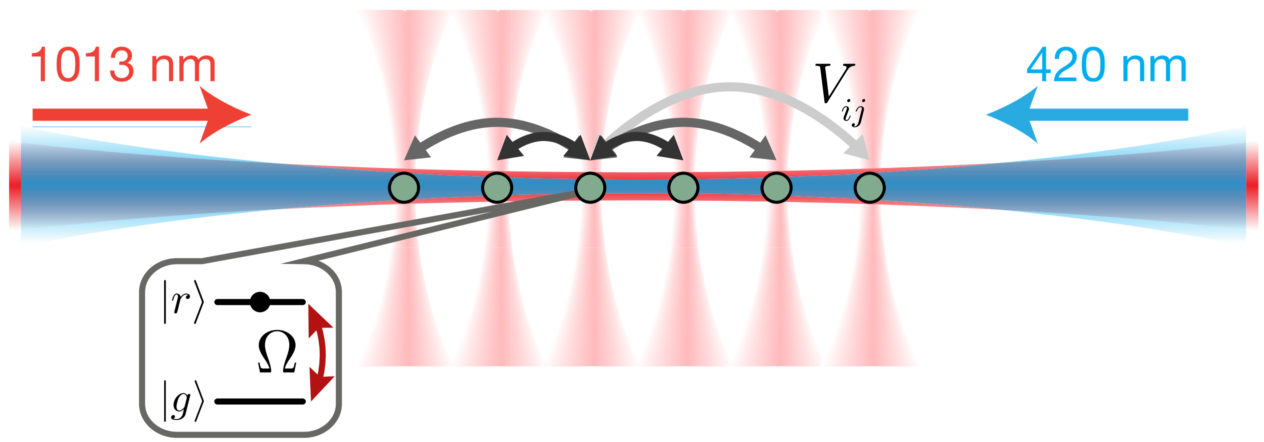
- **scars in a quantum generic system**
  - **51 (Rydberg) Rb atoms placed in a row: every other atom in either a high-energy excited state or a low-energy ground state**
  - **atoms reach equilibrium, then quickly revert to the original "antiferromagnet" state**

Rydberg experiment:



Experimental Realization

Many-body physics with Rydberg atoms



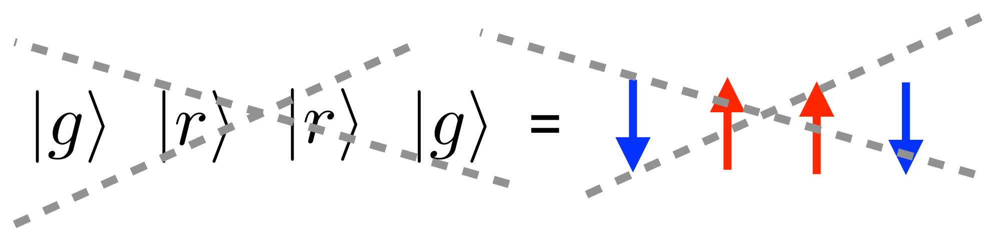
Bernien et al., Nature 551, 579 (2017)

- individual $^{87}\mathrm{Rb}$ atoms are trapped using optical tweezers (vertical red beams) and arranged into defect-free arrays

- coherent interactions V_{ij} between the atoms (arrows) are enabled by exciting them (horizontal blue and red beams) to a Rydberg state, with strength Ω

- strong van-der-Waals interaction between excited (spin-up) particles

- tune atomic spacing so that



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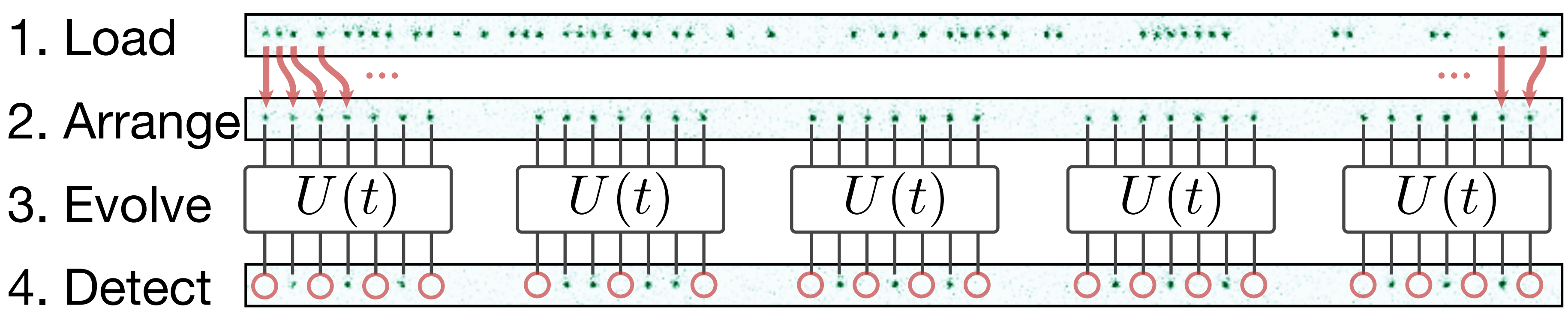

- **experimental protocol**

  1. atoms are loaded into a tweezer array

  2. atoms are re-arranged into a preprogrammed configuration

  3. the system evolves under U(t) with tunable parameters, this evolution can be implemented in parallel on several non-interacting sub-systems

  4. detect the final state using fluorescence imaging, atoms in state |g⟩ remain trapped, whereas atoms in state |r⟩ are ejected from the trap and detected as the absence of fluorescence (indicated with red circles)

Model for Rydberg Experiment

Many-body physics with Rydberg atoms

$\Longrightarrow$ **effective model for 1d chain of Rydberg atoms: spin-1/2 model**

$$\mathcal{H} = \sum_{i=1}^{L} P_i X_{i+1} P_{i+2}$$

strongly correlated paramagnet

$X_i,\ Y_i,\ Z_i$ are Pauli operators

local basis states at site i: $|\bullet\rangle = |\uparrow\rangle \quad |\circ\rangle = |\downarrow\rangle$

- $X_i = |\circ\rangle\langle\bullet| + |\bullet\rangle\langle\circ|$ creates or removes an excitation at site i
- $P_i = |\circ\rangle\langle\circ| = (1 - Z_i)/2$ projectors ensure that the nearby atoms are not simultaneously in the excited state

  $Z_i = |\bullet\rangle\langle\bullet| - |\circ\rangle\langle\circ|$

C.J. Turner et al., Nature 14, 745–749 (2018)

$$P_1 X_2 P_3 |\circ\circ\circ\rangle = |\circ\bullet\circ\rangle$$

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This model: C.J. Turner et al., Nature 14, 745–749 (2018)

$\Longrightarrow$ **able to describe unexpected revivals in certain states**
$|\mathbb{Z}_2\rangle = |\bullet\circ\bullet\circ\bullet\circ\ldots\rangle$

$\Longrightarrow$ **identifies special states responsible:**
quantum many-body scar states

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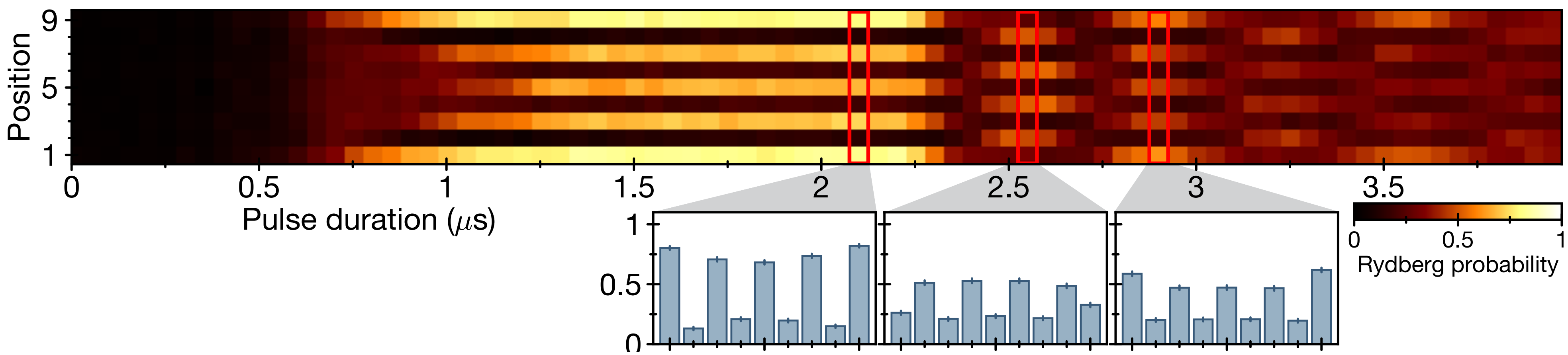


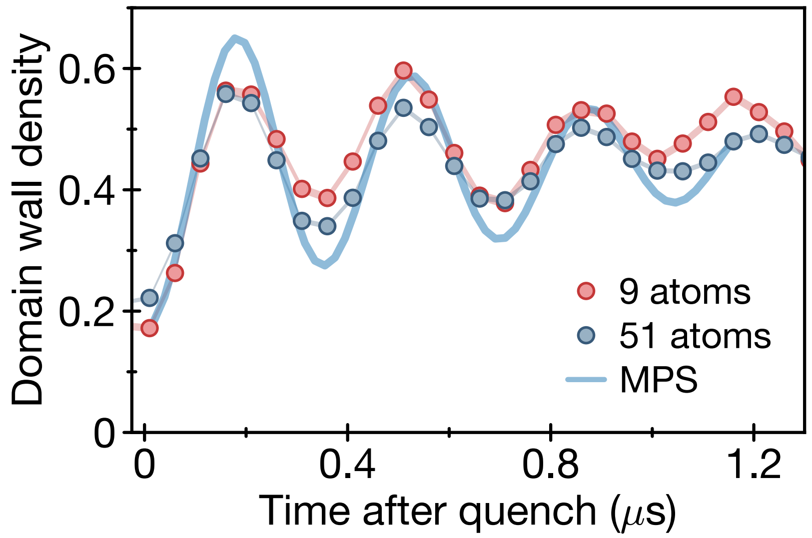
- strong coherent revivals after quench from Neel state $|\mathbb{Z}_2\rangle = |\bullet\circ\bullet\circ\bullet\circ\ldots\rangle$

Model for Rydberg Experiment

Many-body physics with Rydberg atoms

- start with antiferromagnetic initial state and evolve it for some time t: $|\mathbb{Z}_2\rangle = |\bullet \circ \bullet \circ \bullet \circ \ldots\rangle$

local basis states at site i: $|\bullet\rangle = |\uparrow\rangle$
$|\circ\rangle = |\downarrow\rangle$



$\Longrightarrow$ observe oscillations around a non-thermal value

C.J. Turner et al., Nature 14, 745–749 (2018)

- strong coherent revivals after quench from Neel state $|\mathbb{Z}_2\rangle = |\bullet \circ \bullet \circ \bullet \circ \ldots\rangle$

- no revivals for generic initial product states, they thermalize quickly
  example: $|\mathbb{Z}_0\rangle = |\circ \circ \circ \circ \circ \circ \ldots\rangle$

$\Longrightarrow$ **Highly unexpected! Model does not seem to satisfy ETH!!!**

$\Longrightarrow$ **strong dependence on initial state: ~~ETH or MBL~~**

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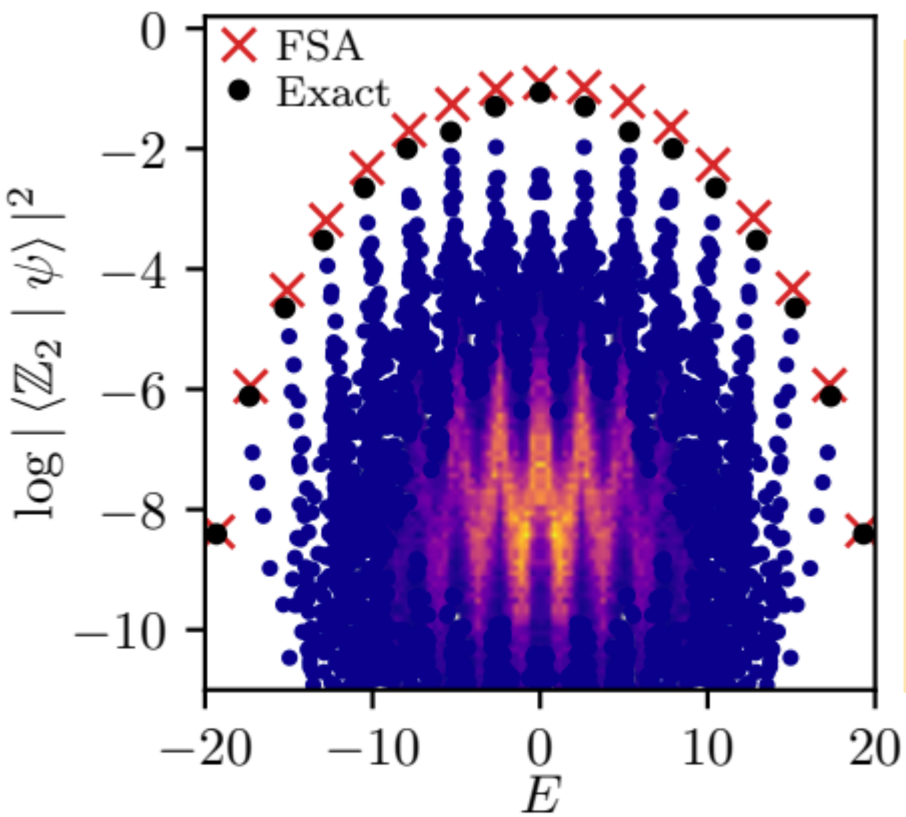
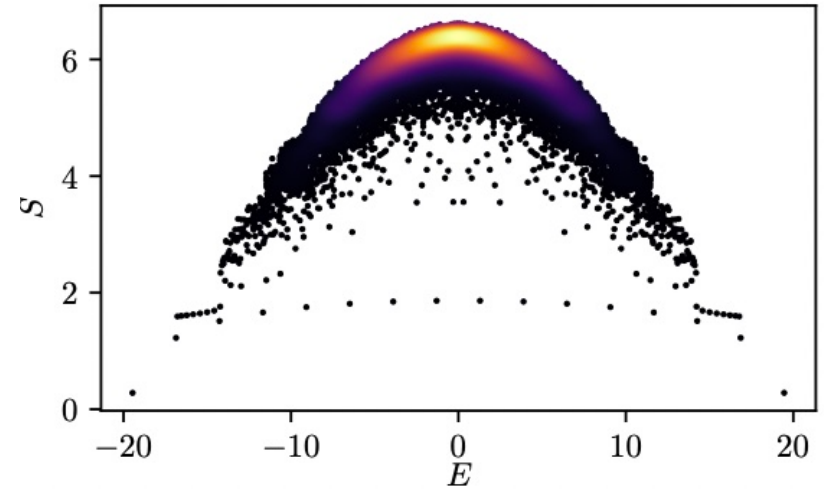


existence of (L+1)-states with atypically high overlap of $|\mathbb{Z}_2\rangle = |\bullet\circ\bullet\circ\bullet\circ\ldots\rangle$ state with each state of the spectrum of $\mathcal{H}$



subvolume law entanglement S for (L+1)-scars

Quantum Many-Body Scar States

Are there models (classes or families) that contain scar states that do not thermalize?

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Characteristics of systems with scars?

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General mechanism?

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General mechanism

1. Unconventional Symmetries or Conservation Laws:

- **Hidden Symmetries:** QMBS states often arise from hidden symmetries/conserved quantities not immediately apparent in the Hamiltonian. These symmetries can protect certain eigenstates from thermalizing and contribute to the scars' formation.
- **Strongly Broken Symmetries:** Some systems with QMBS have Hamiltonians that break certain symmetries, leading to a small subset of states that exhibit non-ergodic behavior.

2. Group-Theoretic Constructions:

- **Group-Theoretic Methods:** QMBS states can be constructed using group-theoretic methods, where special algebraic structures or symmetry groups lead to a discrete set of scar states. These constructions often reveal how such states can be embedded within the Hilbert space of a many-body system.

3. Entanglement Structure:

- **Special Entanglement Patterns:** QMBS states often exhibit unique entanglement properties, such as specific patterns of entanglement that prevent them from mixing with other states. These entanglement patterns can lead to slow dynamics and long-lived oscillations in observables.

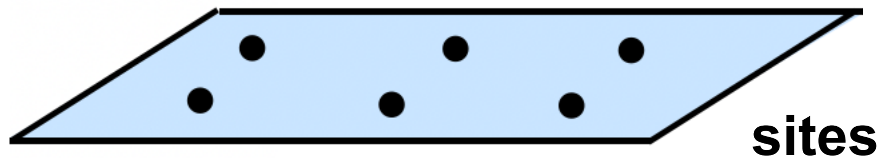
4. Perturbative Analysis

- **Perturbative Methods:** In some cases, QMBS can be understood as perturbations or excitations around exactly solvable points or models. These perturbative approaches help in identifying the conditions under which scar states persist.

5. Construction from Specific Models, Exact Solutions, Numerical and Experimental Observations

General Construction: Bilayer System

copy 1: $\mathcal{H}_1$



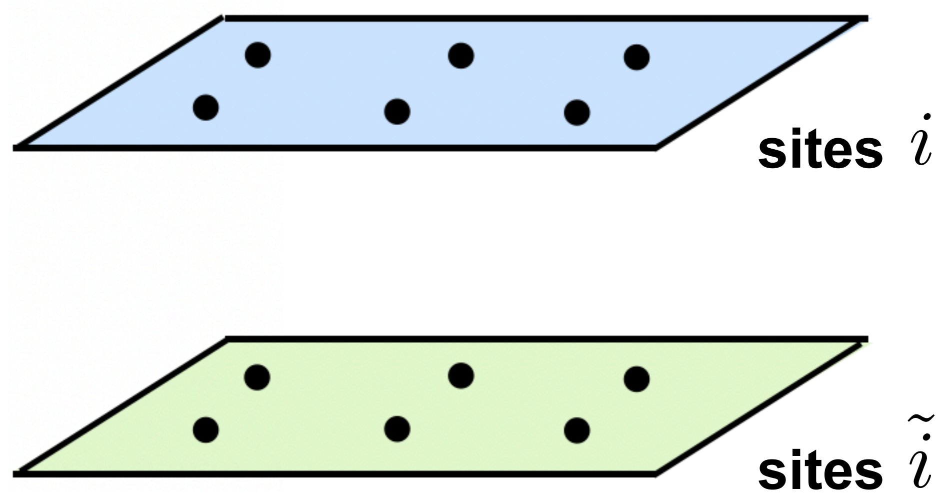
sites i

$\dim(\mathcal{H}_1) = d^N$

General Construction: Bilayer System

copy 1: $\mathcal{H}_1$

copy 2: $\mathcal{H}_2$

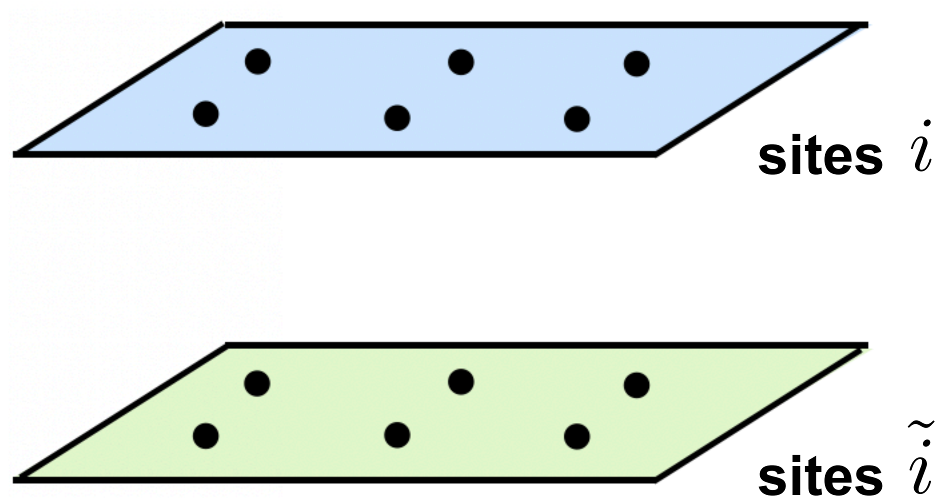


$\dim(\mathcal{H}_1) = d^N$

$\dim(\mathcal{H}_2) = d^N$

General Construction: Bilayer System

copy 1: $\mathcal{H}_1$



$\dim(\mathcal{H}_1) = d^N$

copy 2: $\mathcal{H}_2$

$\dim(\mathcal{H}_2) = d^N$

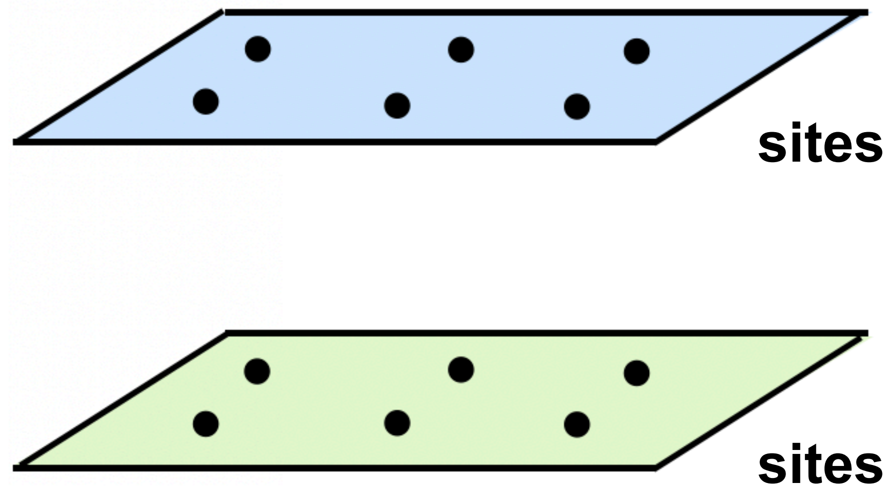
- $\mathcal{H} = \mathcal{H}_1 + \mathcal{H}_2$

$\mathcal{H}_2 = -\mathcal{H}_1$

mirror symmetry $\mathcal{M} : i \to \tilde{i}$

General Construction: Bilayer System

copy 1: $\mathcal{H}_1$



sites i

$\dim(\mathcal{H}_1) = d^N$

copy 2: $\mathcal{H}_2$

sites $\tilde{i}$

$\dim(\mathcal{H}_2) = d^N$

- $\mathcal{H} = \mathcal{H}_1 + \mathcal{H}_2$ $\quad\quad$ $\mathcal{H}_2 = -\mathcal{H}_1$

mirror symmetry $\mathcal{M} : i \to \tilde{i}$

$$\left.\begin{aligned} \mathcal{H}_1 &= \sum_{n=1}^{d^N} E_n |\Psi_n\rangle\langle\Psi_n| \\ \mathcal{H}_2 &= -\sum_{n=1}^{d^N} E_n |\Psi_n\rangle\langle\Psi_n| \end{aligned}\right\}$$

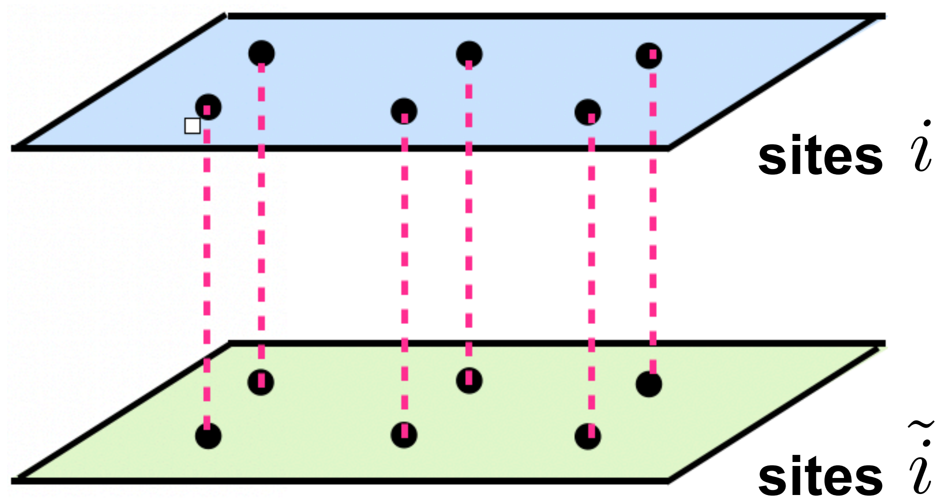
$$\mathcal{H}\,|\Psi_{nm}\rangle = (E_n - E_m)|\Psi_{nm}\rangle$$

$$\{|\Psi_{nm}\rangle = |\psi_n\rangle \otimes |\psi_m\rangle : \forall\ n, m = 1, \ldots d^N\}$$

with special case: $\mathcal{H}|\Psi_{nn}\rangle = 0$

$$\{|\Psi_{nn}\rangle = |\psi_n\rangle \otimes |\psi_n\rangle : \forall\ n = 1, \ldots d^N\}$$

General Construction: Bilayer System



$\dim(\mathcal{H}_1) = d^N$

$\dim(\mathcal{H}_2) = d^N$

- $\mathcal{H} = \mathcal{H}_1 + \mathcal{H}_2 + \mathcal{H}_{12}$ $\quad$ $\mathcal{H}_2 = -\mathcal{H}_1$

  mirror symmetry $\mathcal{M} : i \to \tilde{i}$

- **We demand in addition:** $\mathcal{H}_{12}|\Psi_{nn}\rangle = E_{12}|\Psi_{nn}\rangle$

Wildeboer et al., PRB 106, 205142 (2022)
Langlett et al., PRB 105, L060301 (2021)

$\{|\Psi_{nn}\rangle = |\psi_n\rangle \otimes |\psi_n\rangle : \forall\ n = 1, \ldots d^N\}$ **are quantum many-body scar states !**

$\Longrightarrow$ **Einstein-Podolsky-Rosen (EPR) scar states are born!**

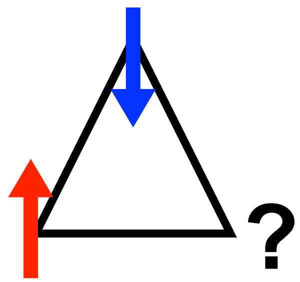
EPR Scar States

⟹ **examples:**

- **quantum dimer model as bilayer system on square lattice, …**
- **Bose-Hubbard model as bilayer system**
- **bilayer triangular lattice Heisenberg model with SU(2) symmetry**

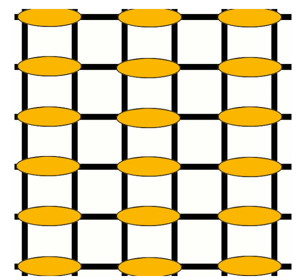
… and more see Wildeboer et al., PRB 106, 205142 (2022) and future work

Quantum dimer models

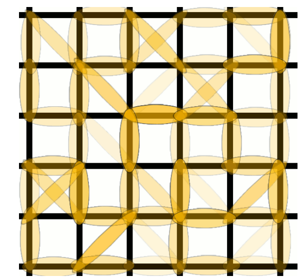


frustrated spin systems

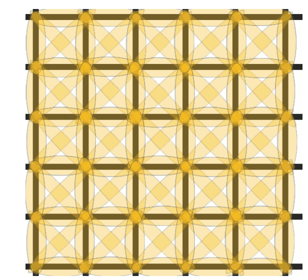
quantum fluctuations



valence bond solid order

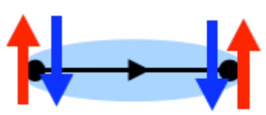


is melted by quantum fluctuations



giving rise to a quantum disordered state

dimers: **microscopic representations of spin singlets (valence bonds) assumed to be the building blocks of low energy subspace of an underlying quantum spin-1/2 problem with strong frustration**

$\Longrightarrow$  $= \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$ spin singlets = valence bonds

replaced by dimer (rod-like object)

Rokhsar, Kivelson, PRL 61, 2376 (1988)

Moessner, Sondhi, PRL 86, 1881 (2001)

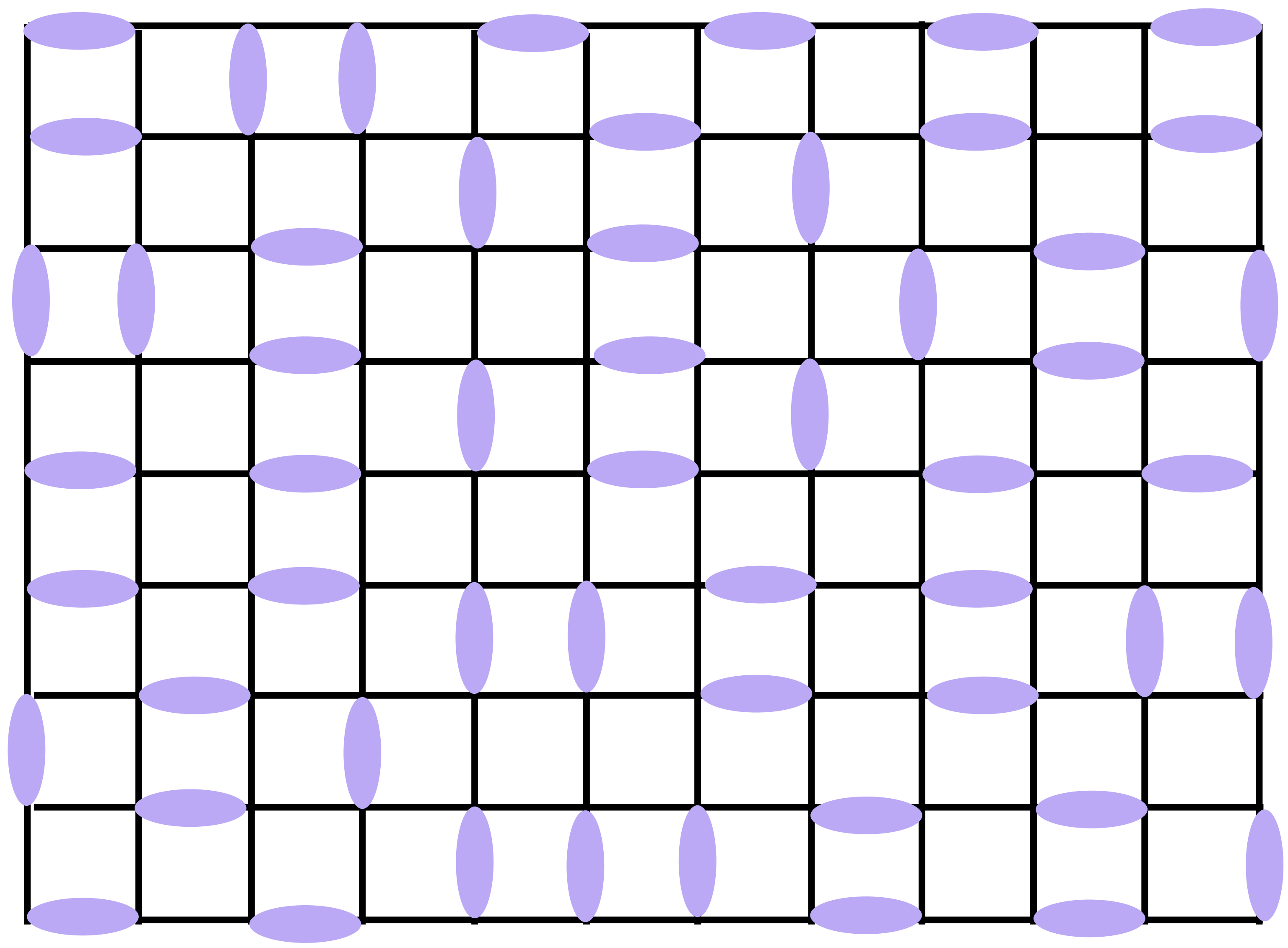
Misguich et al., PRL 89, 137202 (2002)

Wildeboer et. al, PRL 109, 147208 (2012)

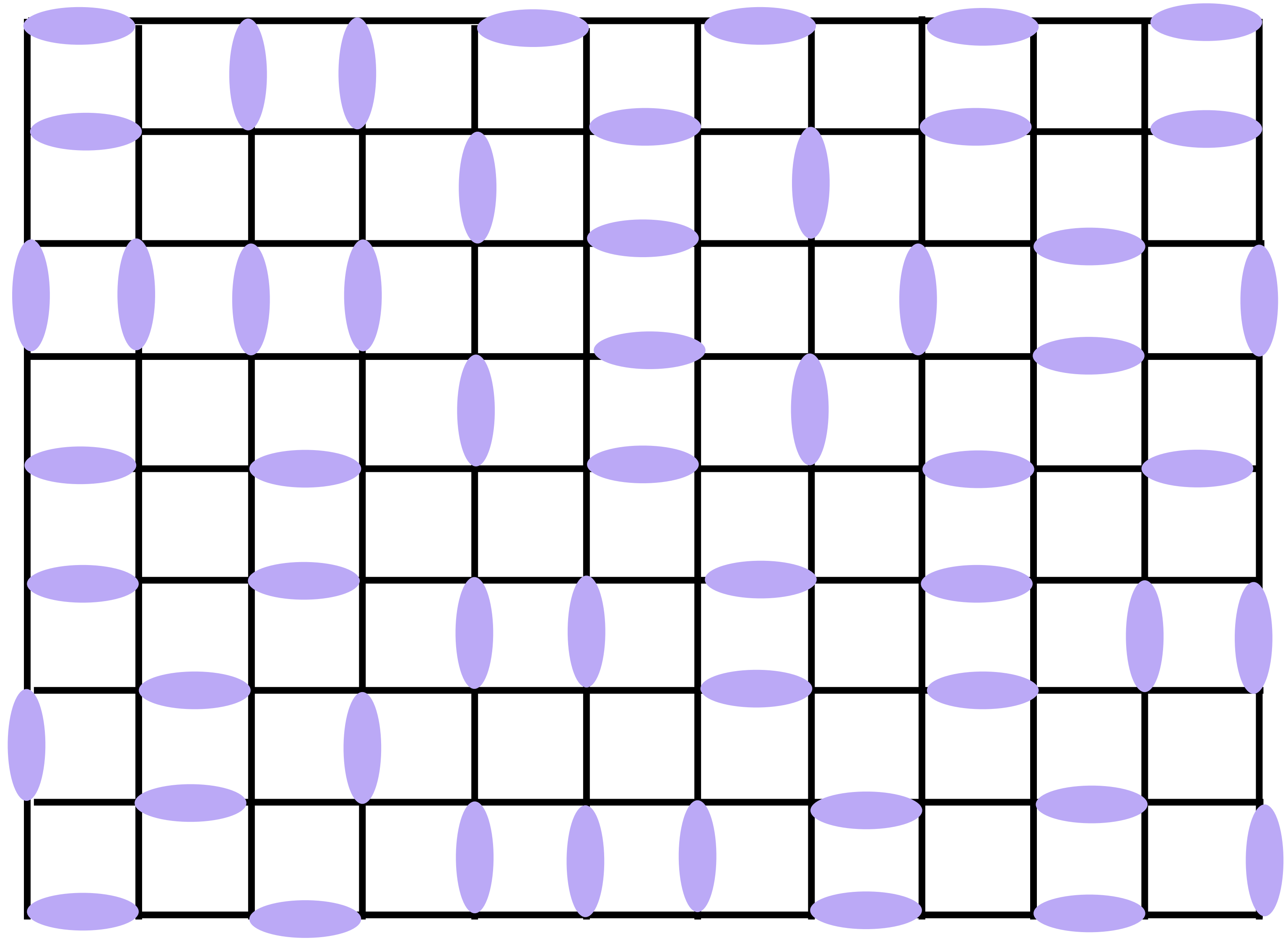
Wildeboer et. al, PRB 95, 100402 (2017)

Wildeboer et. al, PRB 102, 020401 (2020)

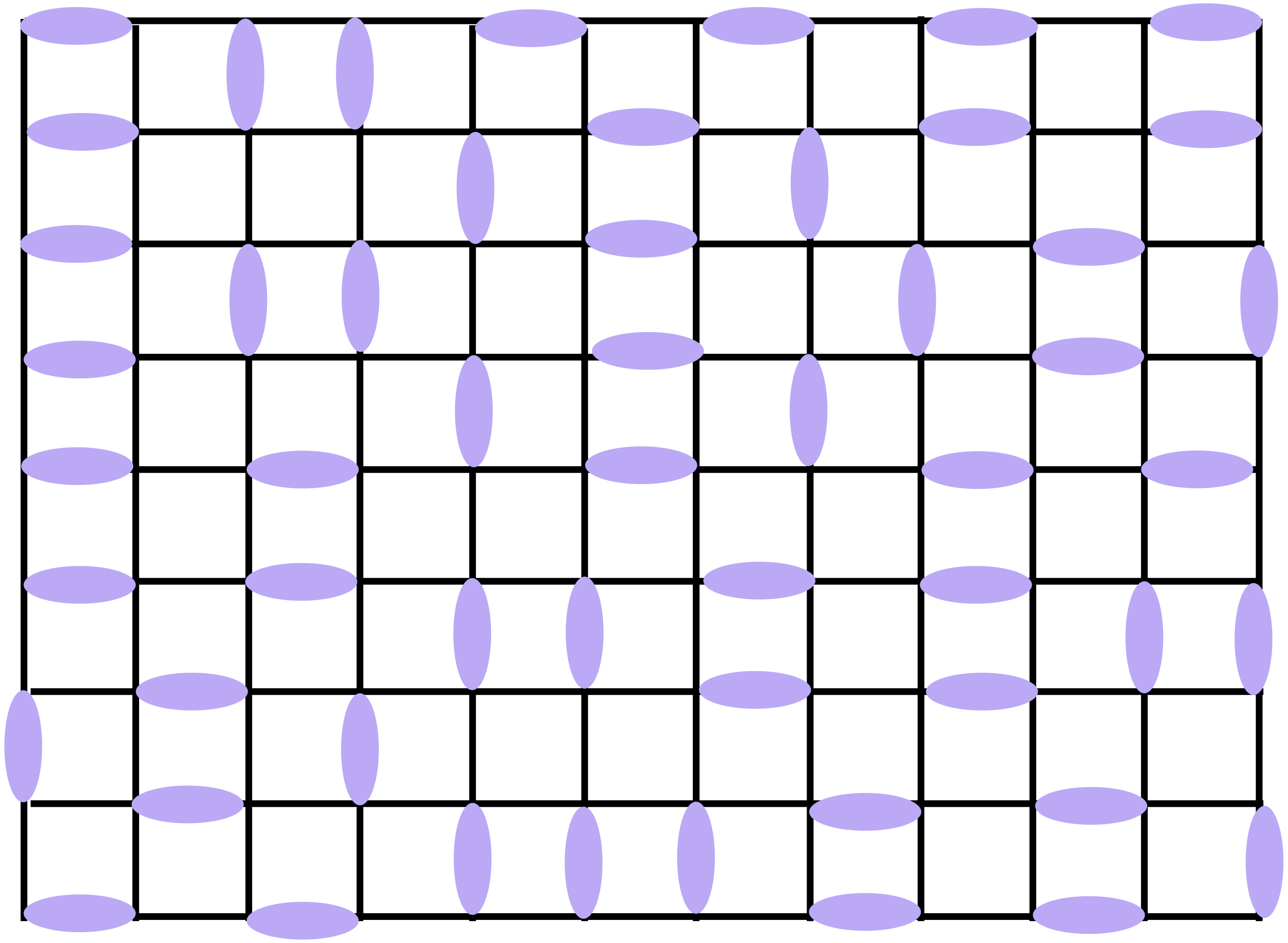
Dimers on the square lattice



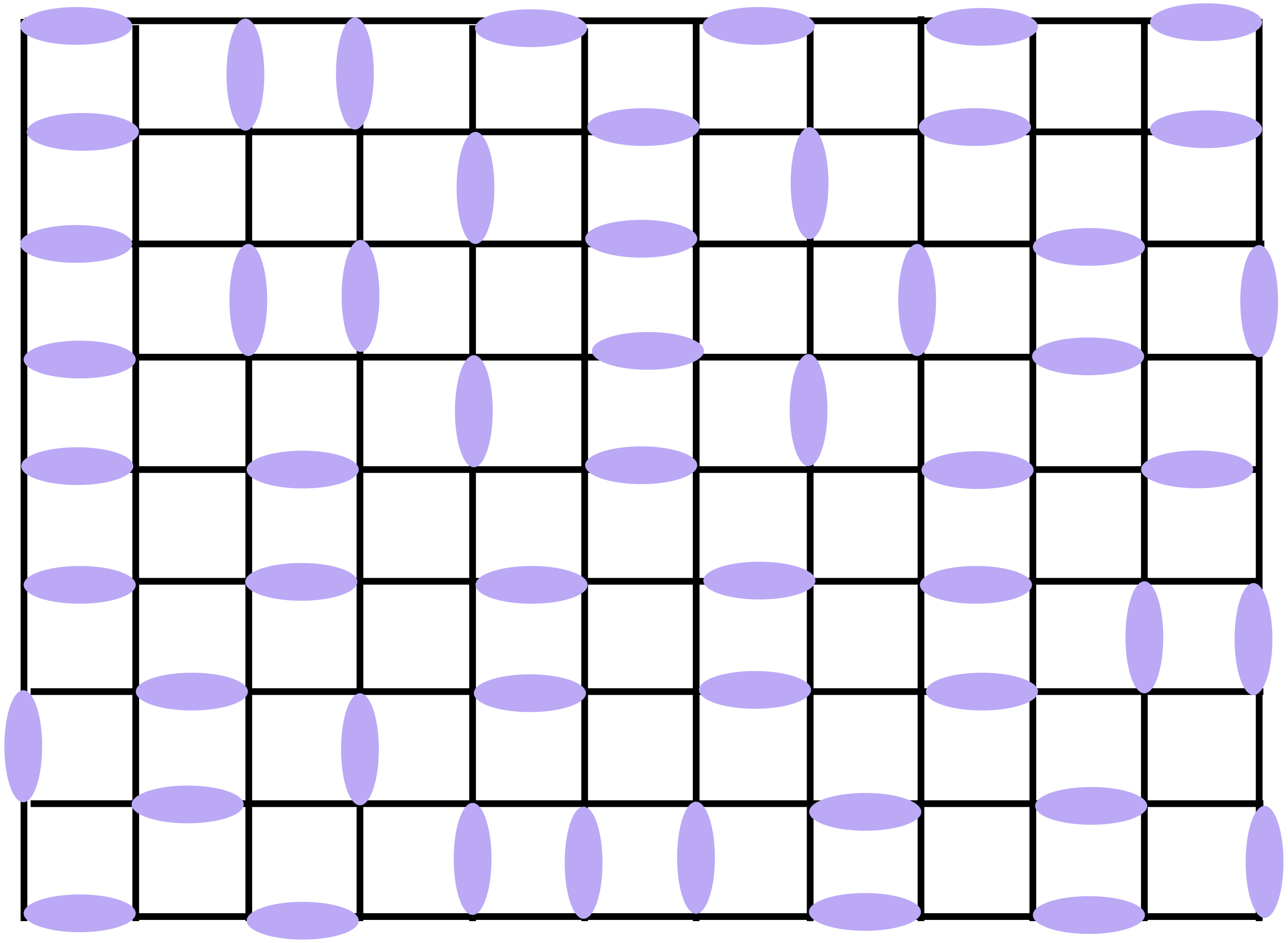
Dimers on the square lattice



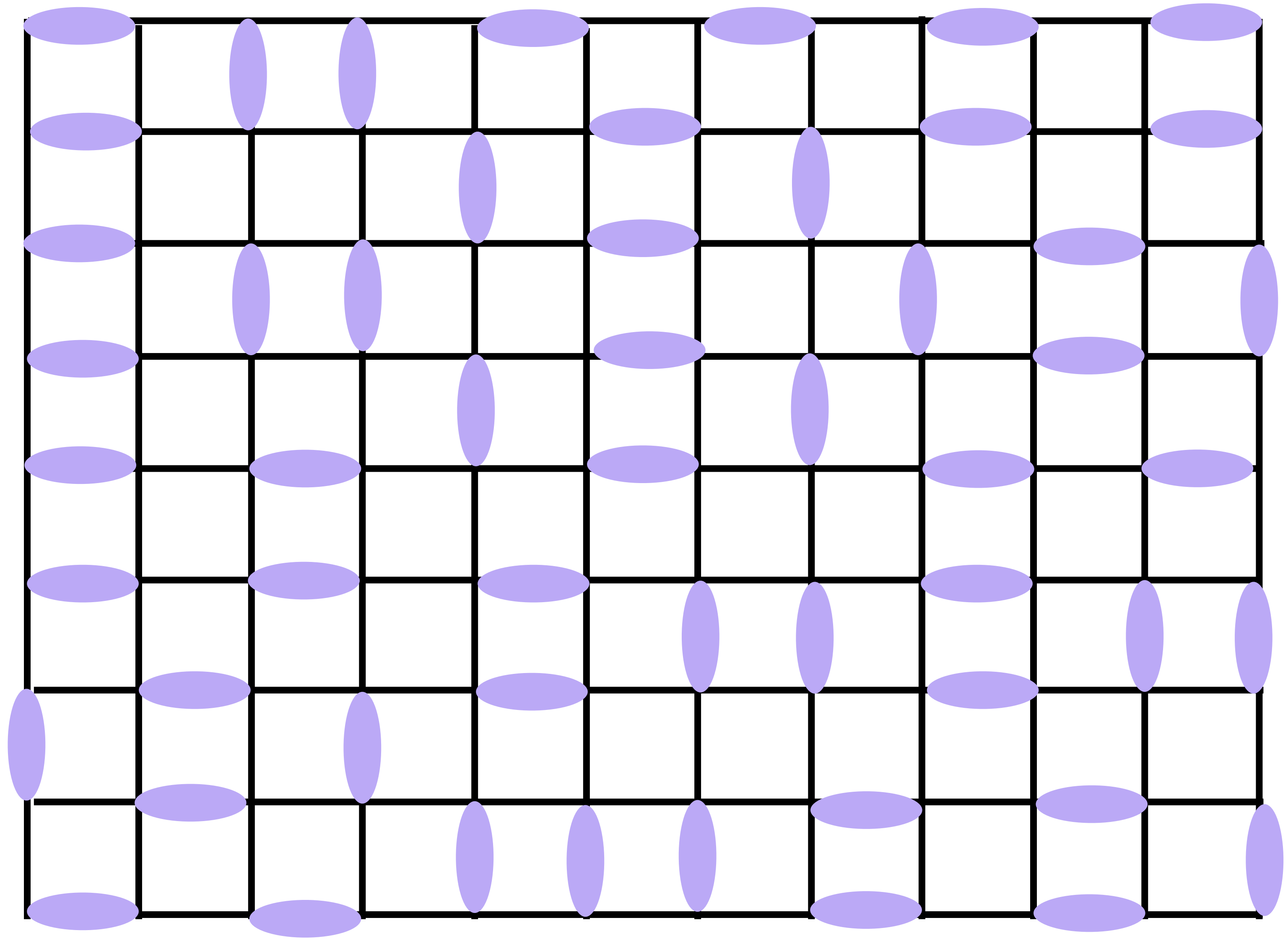
Dimers on the square lattice



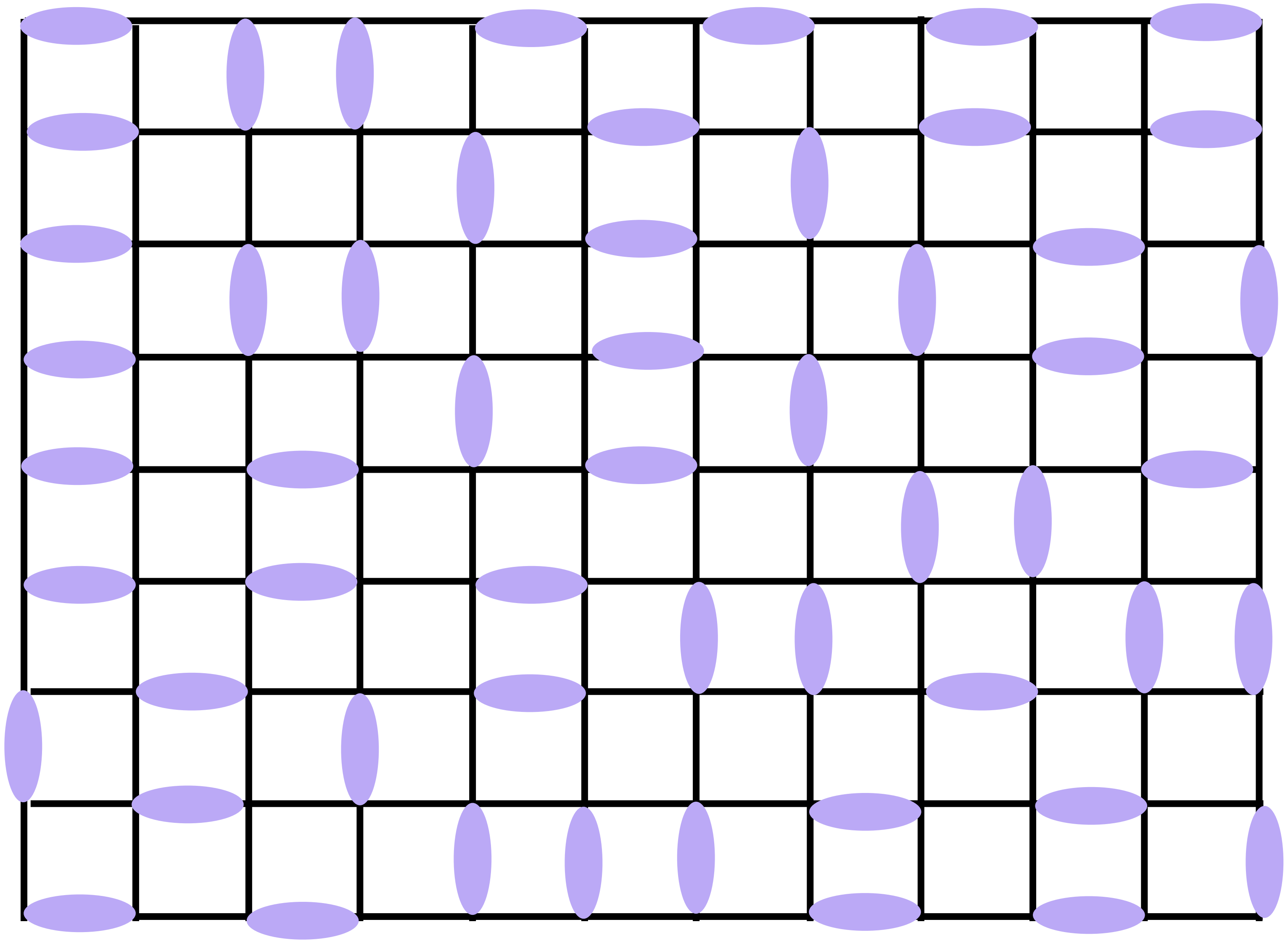
Dimers on the square lattice



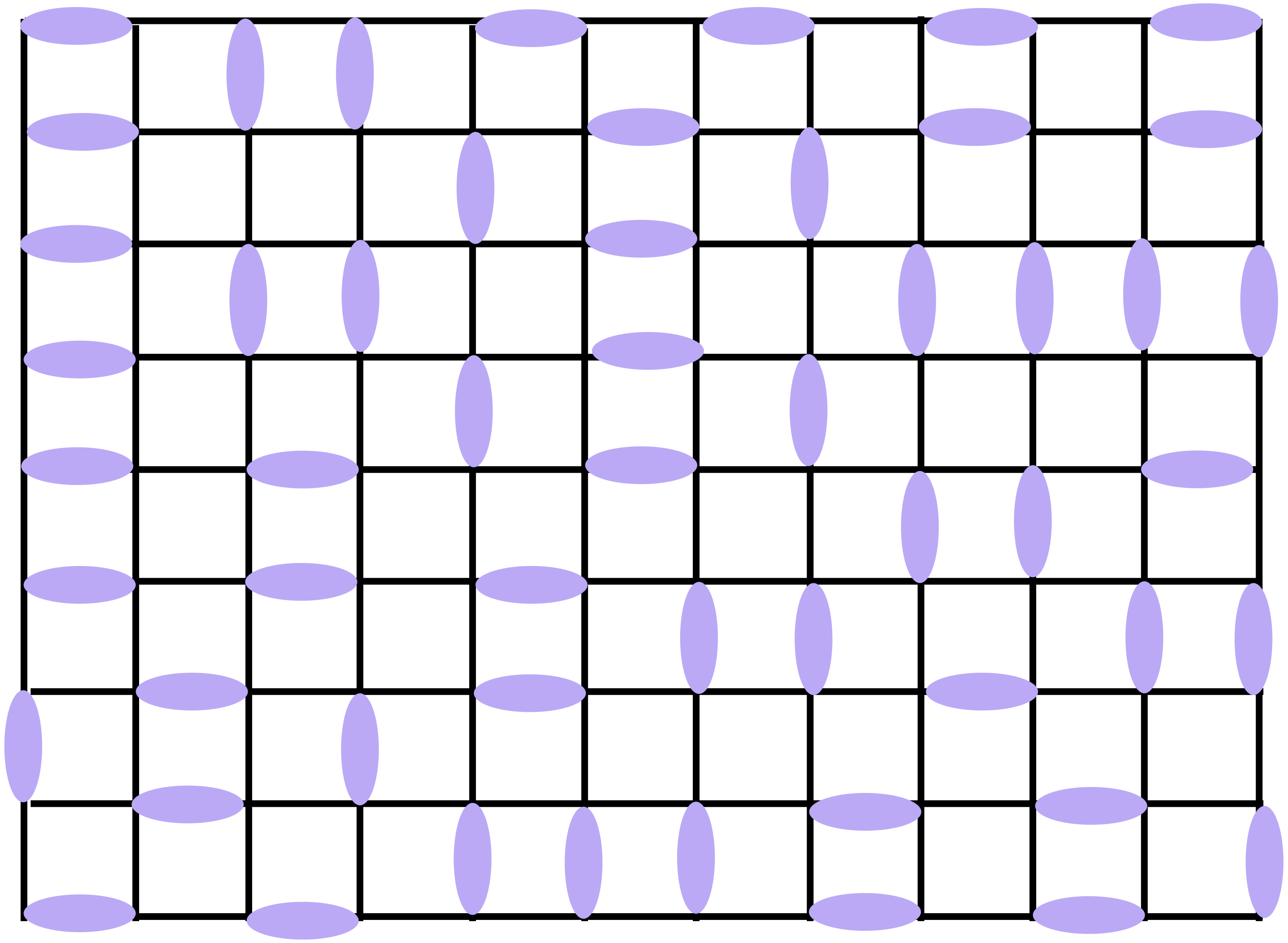
Dimers on the square lattice



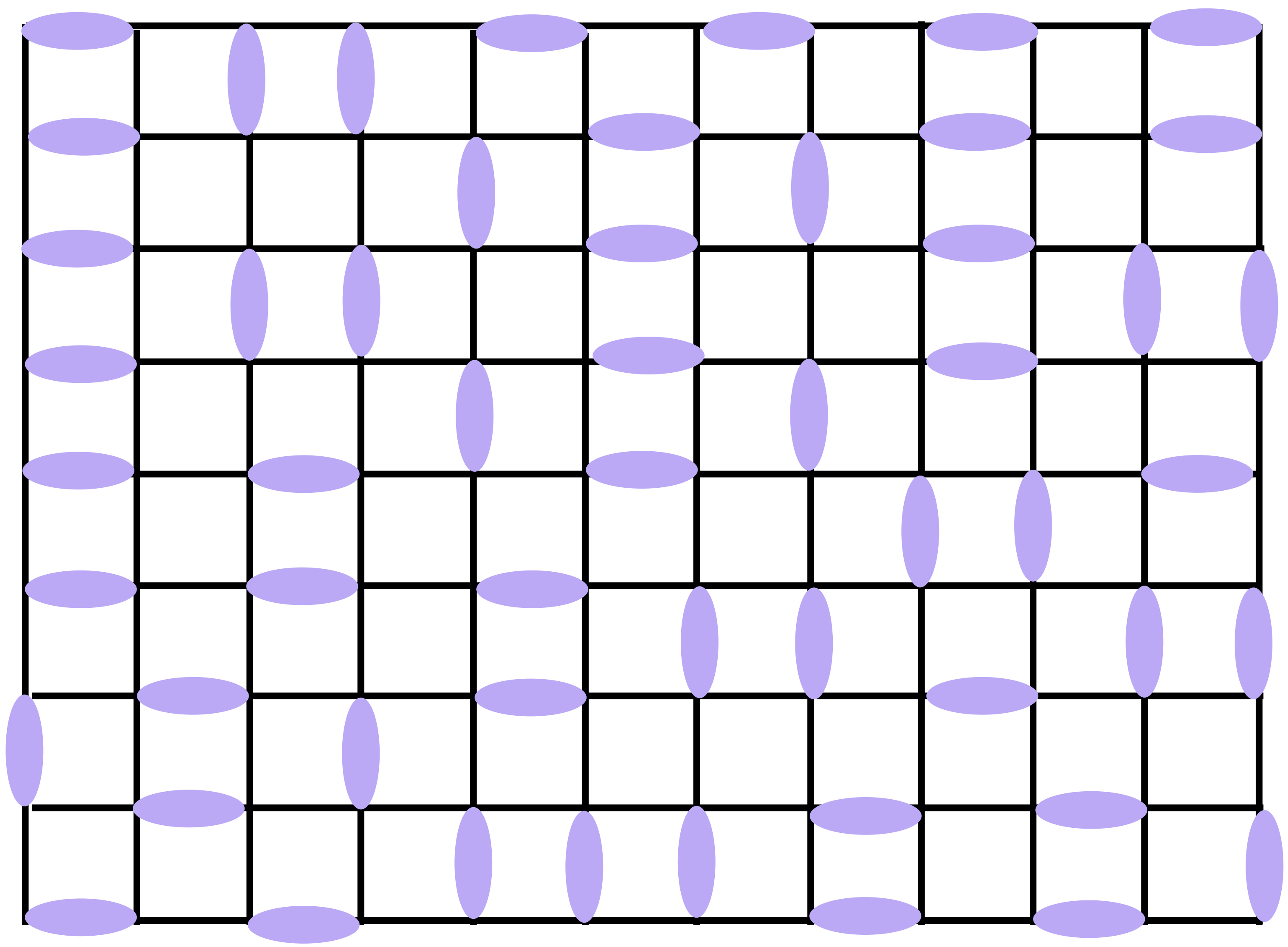
Dimers on the square lattice



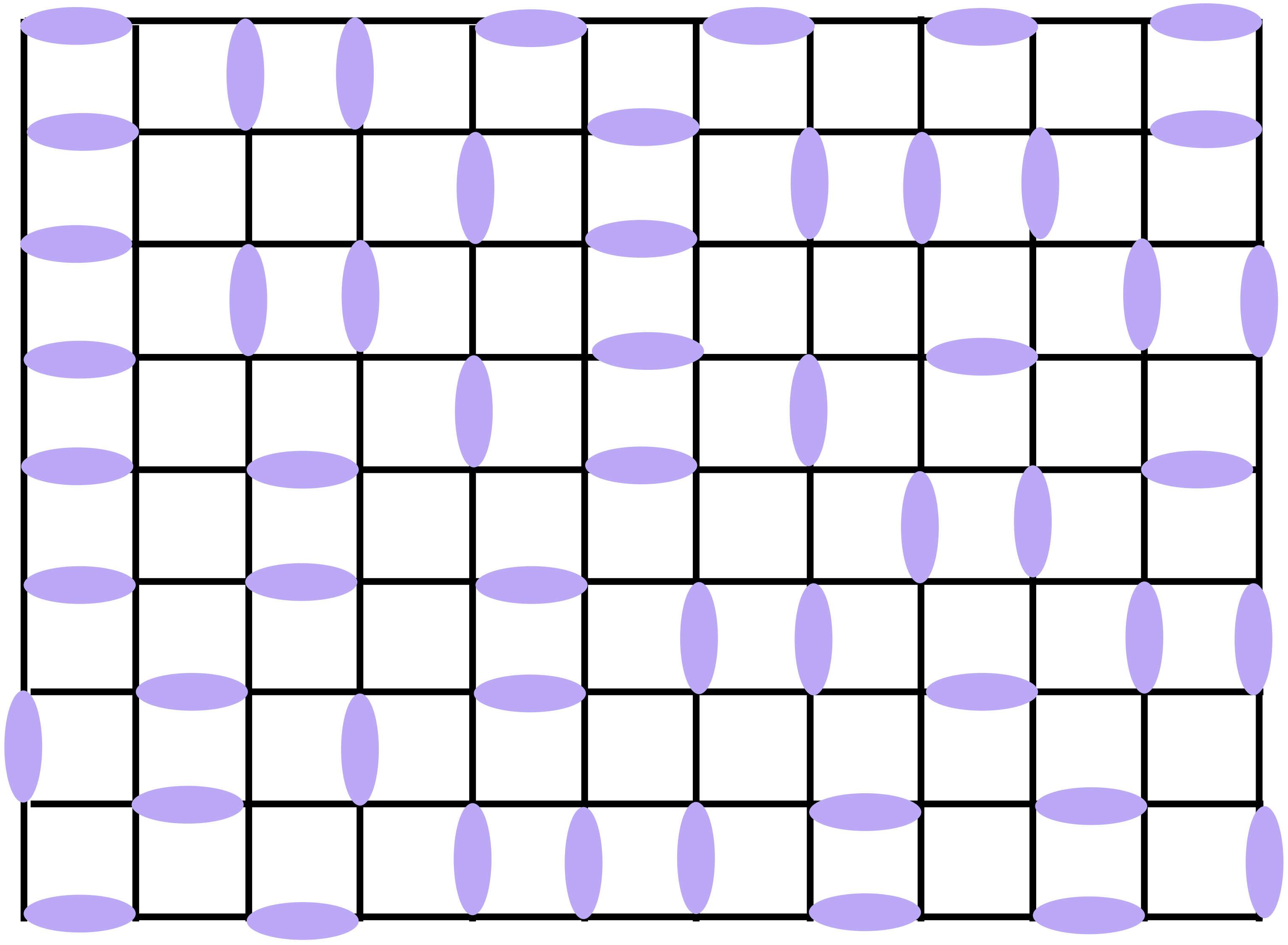
Dimers on the square lattice



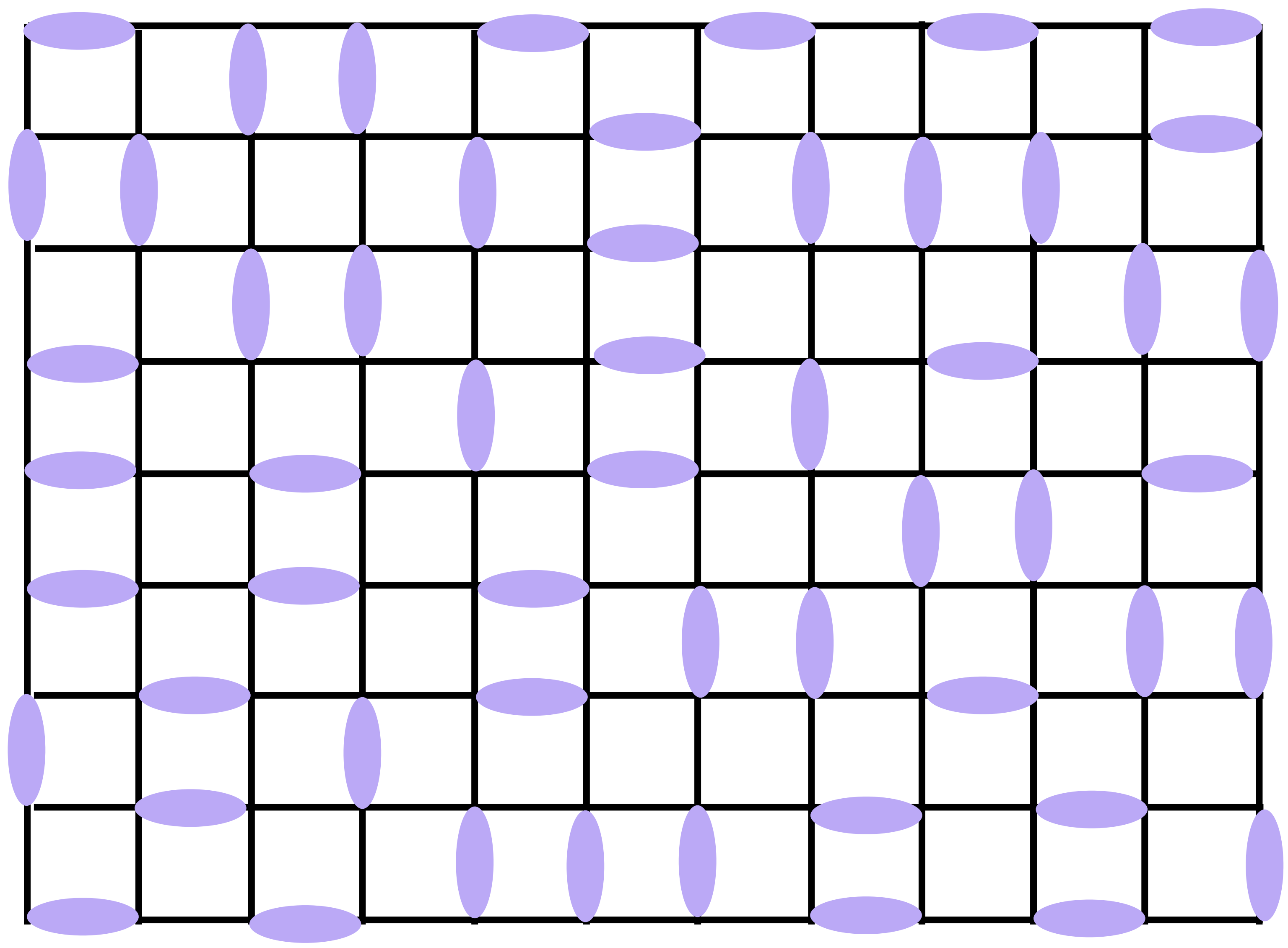
Dimers on the square lattice



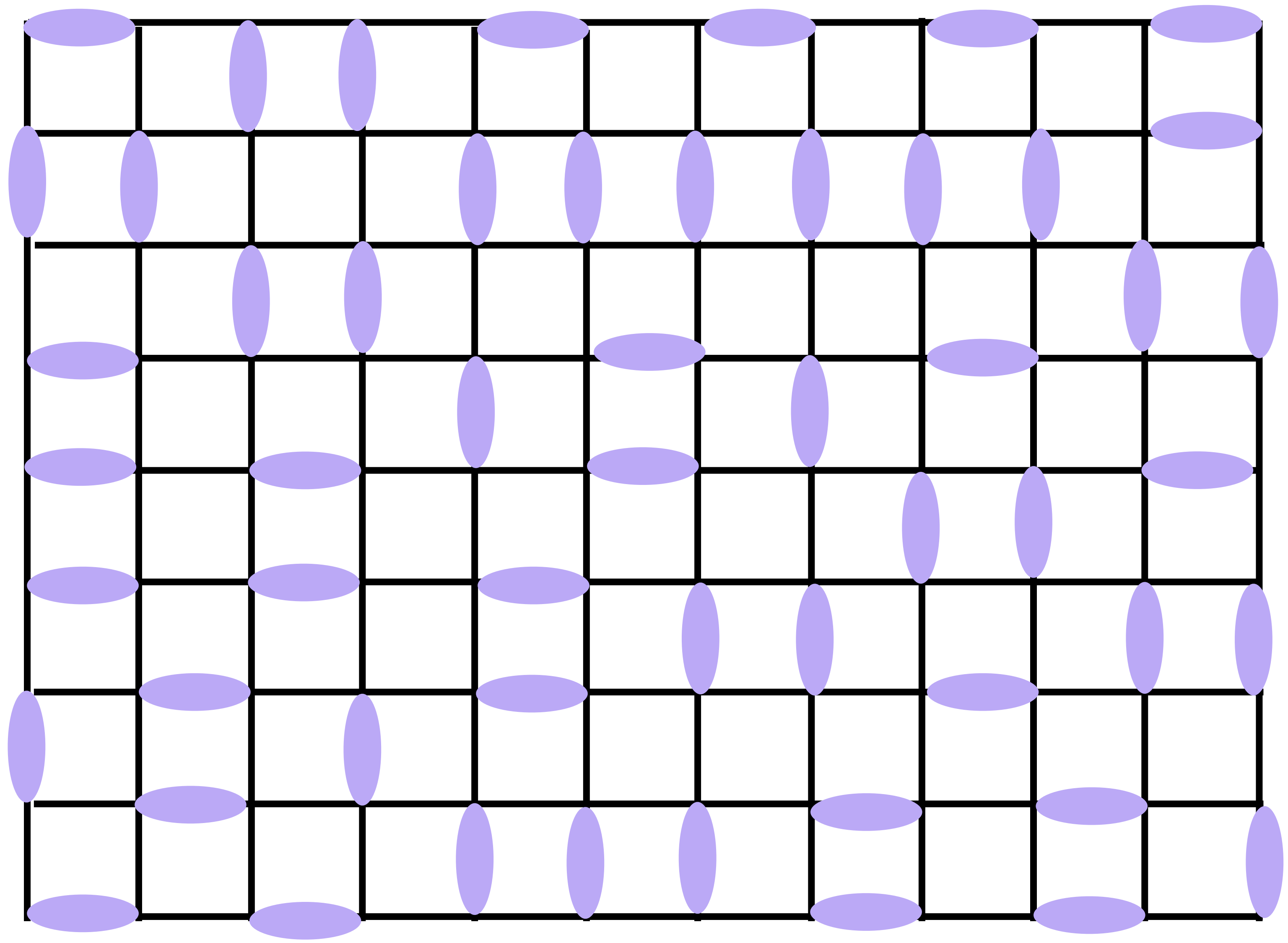
Dimers on the square lattice



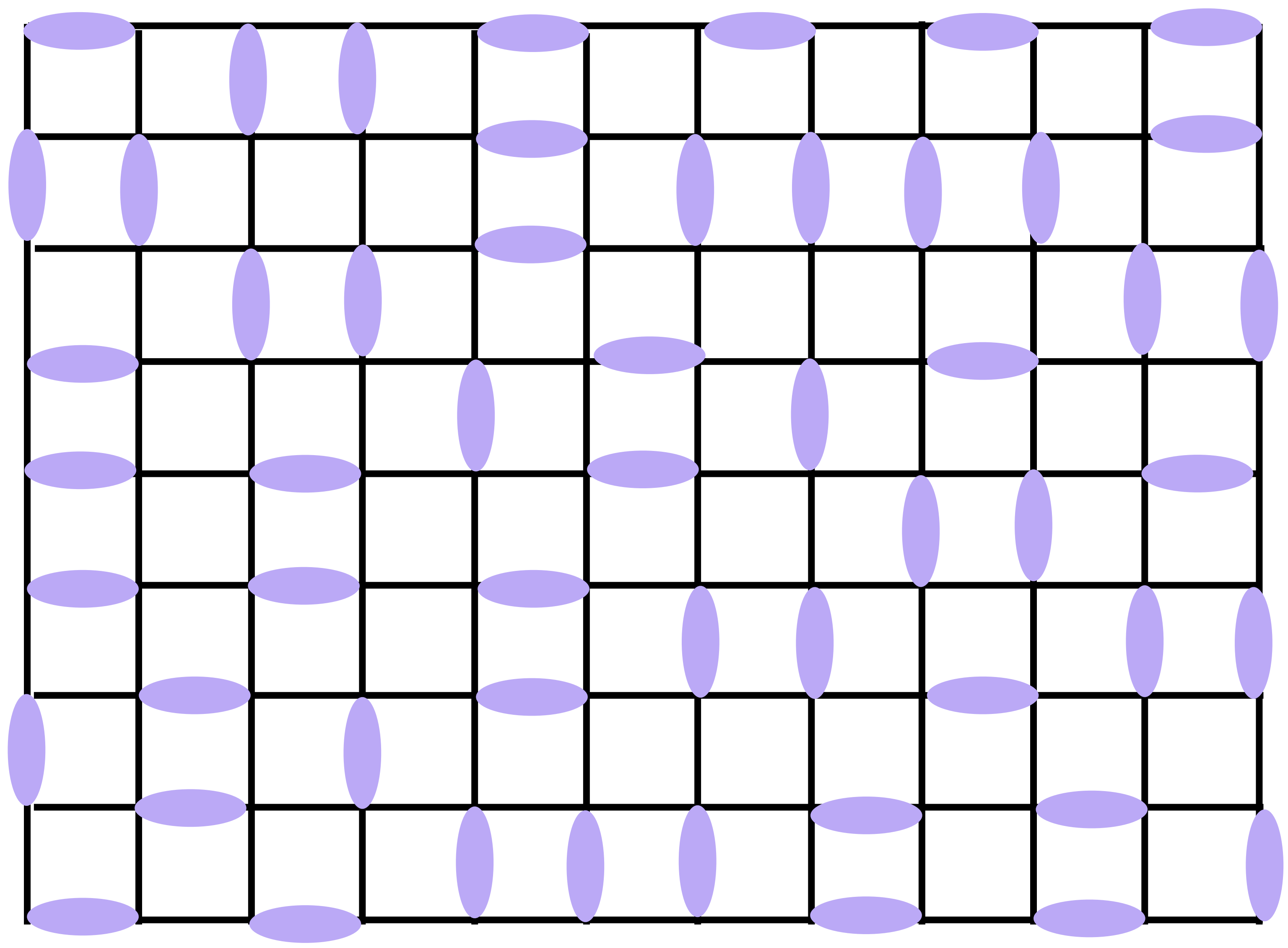
Dimers on the square lattice



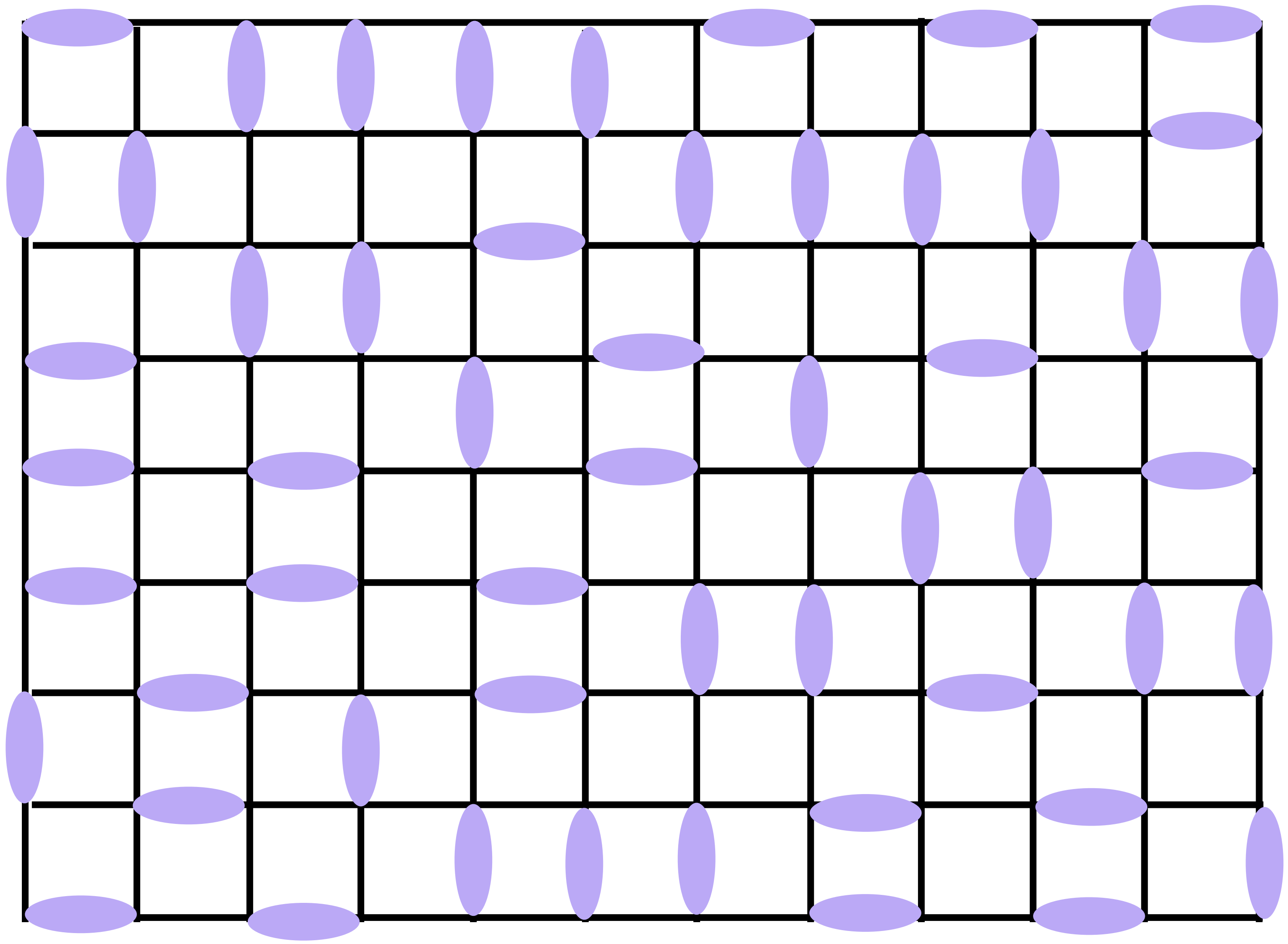
Dimers on the square lattice



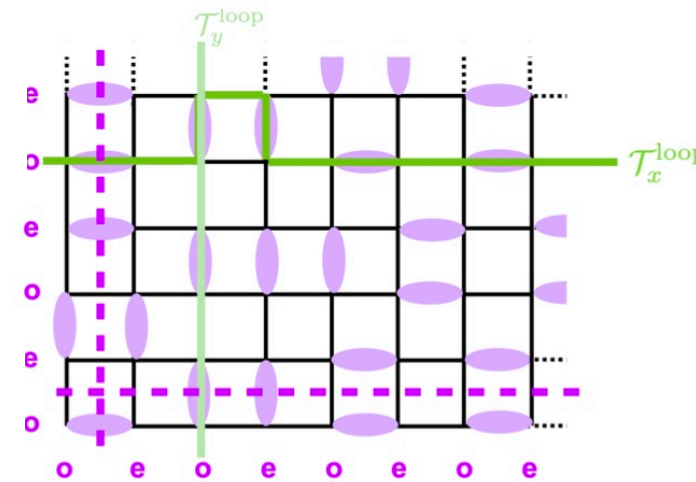
Dimers on the square lattice



Dimers on the square lattice



Quantum dimer model with interlayer coupling



winding numbers $W_\alpha = N_o - N_e$

$\alpha = x, y$

- **quantum dimer model on the square lattice**

$$\mathcal{H} = \sum_{\square} -t\,\hat{T}_\square + v\,\hat{V}_\square$$

$$= \sum_{\square} -t\left(|\square\rangle\langle\square| + |\square\rangle\langle\square|\right) + v\left(|\square\rangle\langle\square| + |\square\rangle\langle\square|\right)$$

- **work at Rokhsar-Kivelson (RK) point:** $t = v$
  - ▶ **spectrum is positive-semidefinite:** $\mathcal{H}|\Psi\rangle = E_n|\Psi\rangle \quad E_n \geq 0$
  - ▶ **ground state:** $|\Psi\rangle = \sum_D |D\rangle$

$S_A \sim$ **sub-volume**

⟹ **quantum dimer model on square lattice with ground states in critical U(1) spin liquid phase**

⟹ **ground state degeneracy depends on lattice dimensions (Lx,Ly:** $-\frac{L_{x(y)}}{2} \leq W_{x(y)} \leq +\frac{L_{x(y)}}{2}$

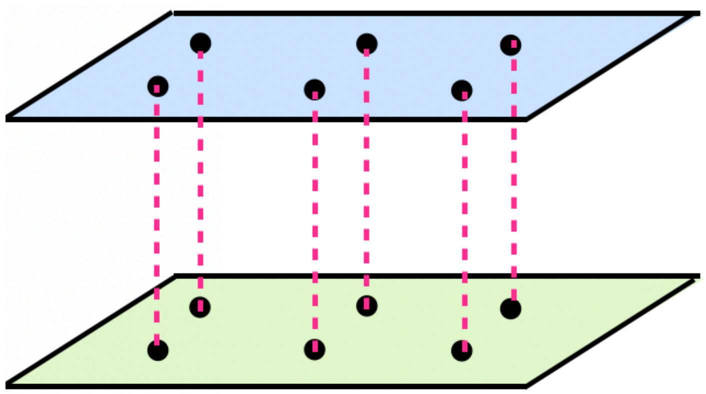
$$N_{sec}^{\square} = (L_x + 1)\cdot(L_y + 1)$$

Quantum dimer model with interlayer coupling

- **quantum dimer model on the square lattice**

$$\mathcal{H} = \sum_{\square} -t\hat{T}_{\square} + v\hat{V}_{\square}$$

$$= \sum_{\square} -t\left(|{\parallel}\rangle\langle{=}| + |{=}\rangle\langle{\parallel}|\right) + v\left(|{\parallel}\rangle\langle{\parallel}| + |{=}\rangle\langle{=}|\right)$$



square lattice bilayer

$$\Longrightarrow \quad \mathcal{H} = \mathcal{H}_1 \otimes I + I \otimes \mathcal{H}_2 + \mathcal{H}_{12}$$

$$\mathcal{H}_1 = \sum_{\square} -t\left(|{\parallel}\rangle\langle{=}| + |{=}\rangle\langle{\parallel}|\right) + v\left(|{\parallel}\rangle\langle{\parallel}| + |{=}\rangle\langle{=}|\right)$$

$$\mathcal{H}_2 = -\mathcal{H}_1$$

$$\mathcal{H}_{12} = \frac{\lambda}{N_D^{\square}}\left(\sum_{\ell_{\mathrm{h}}} |{\leftrightarrow},{\leftrightarrow}\rangle\langle{\leftrightarrow},{\leftrightarrow}| + \sum_{\ell_{\mathrm{v}}} |{\updownarrow},{\updownarrow}\rangle\langle{\updownarrow},{\updownarrow}|\right)$$

$$= \frac{\lambda}{N_D^{\square}} \sum_{\ell} n_\ell \otimes n_\ell$$

Quantum dimer model with interlayer coupling

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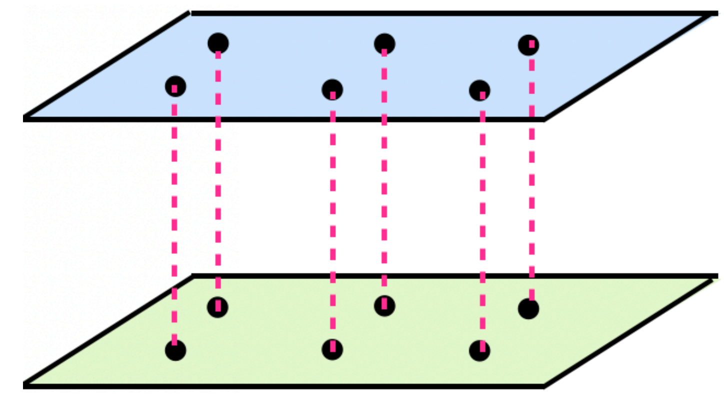
$$\mathcal{H} = \sum_{\square} -t\hat{T}_{\square} + v\hat{V}_{\square}$$

$$= \sum_{\square} -t\left(|\,\|\,\rangle\langle\,=\,| + |\,=\,\rangle\langle\,\|\,|\right) + v\left(|\,\|\,\rangle\langle\,\|\,| + |\,=\,\rangle\langle\,=\,|\right)$$



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$$= \frac{\lambda}{N_D^{\square}}\sum_{\ell} n_\ell \otimes n_\ell$$

Einstein-Podolsky-Rosen (EPR) scar states

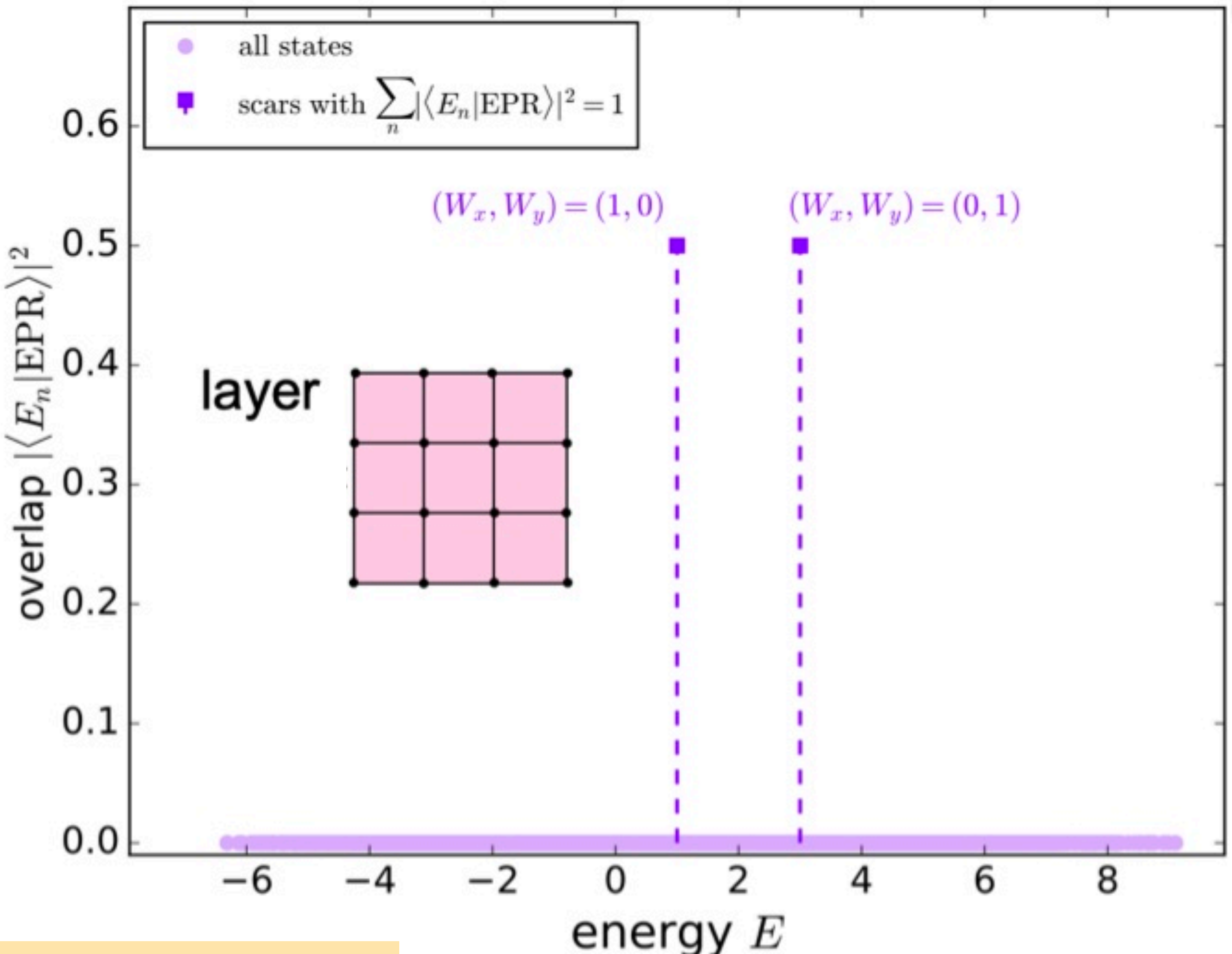
$$|\mathrm{EPR}\rangle = \sum_{(w_x,w_y)} \sum_{D_{w_x,w_y}} \left|D_{w_x,w_y}\right\rangle \otimes \left|D_{w_x,w_y}\right\rangle$$

$$= \sum_{(w_x,w_y)} |\mathrm{RK}\rangle \otimes |\mathrm{RK}\rangle$$

$$= \sum_{(w_x,w_y)} |\mathrm{EPR}\rangle_{(w_x,w_y)}$$

perfectly correlated: dimer configurations identical in both layers

Quantum dimer model with interlayer coupling



Einstein-Podolsky-Rosen (EPR) scar states

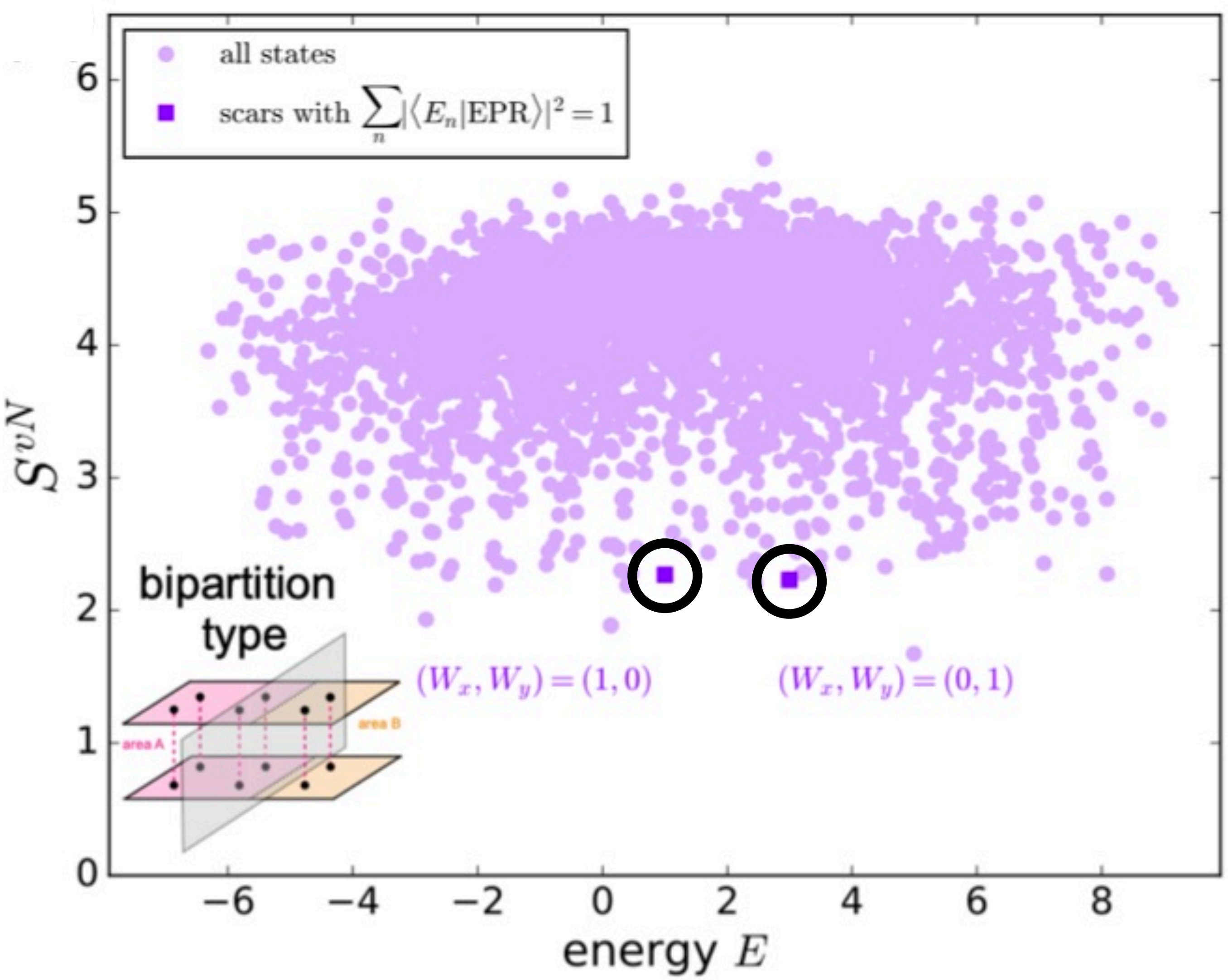
$$|\mathrm{EPR}\rangle = \sum_{(w_x,w_y)} \sum_{D_{w_x,w_y}} |D_{w_x,w_y}\rangle \otimes |D_{w_x,w_y}\rangle = \sum_{(w_x,w_y)} |\mathrm{RK}\rangle \otimes |\mathrm{RK}\rangle = \sum_{(w_x,w_y)} |\mathrm{EPR}\rangle_{(w_x,w_y)}$$

in general tower of

$N_{sec}^{\square} = (L_x+1)\cdot(L_y+1)$ **scar states**

tower of 2 critical U(1) scar states

Quantum dimer model with interlayer coupling



Einstein-Podolsky-Rosen (EPR) scar states

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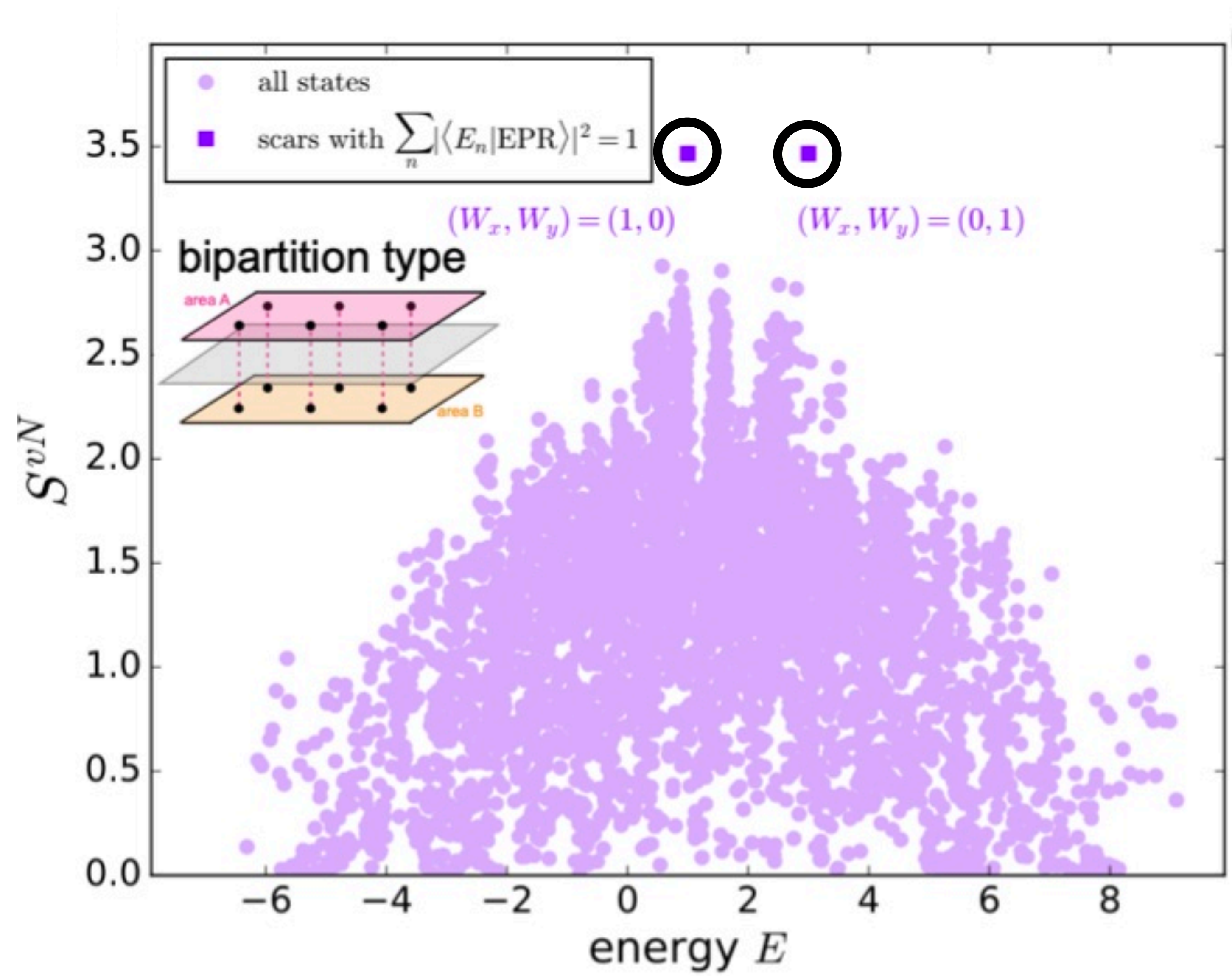
in general tower of

$N^{\square}_{sec} = (L_x + 1) \cdot (L_y + 1)$ **scar states**

tower of 2 critical U(1) scar states with simple entanglement structure

Quantum dimer model with interlayer coupling



note that entanglement depends on choice of bipartition

Einstein-Podolsky-Rosen (EPR) scar states

$$|\mathrm{EPR}\rangle = \sum_{(w_x,w_y)} \sum_{D_{w_x,w_y}} |D_{w_x,w_y}\rangle \otimes |D_{w_x,w_y}\rangle$$
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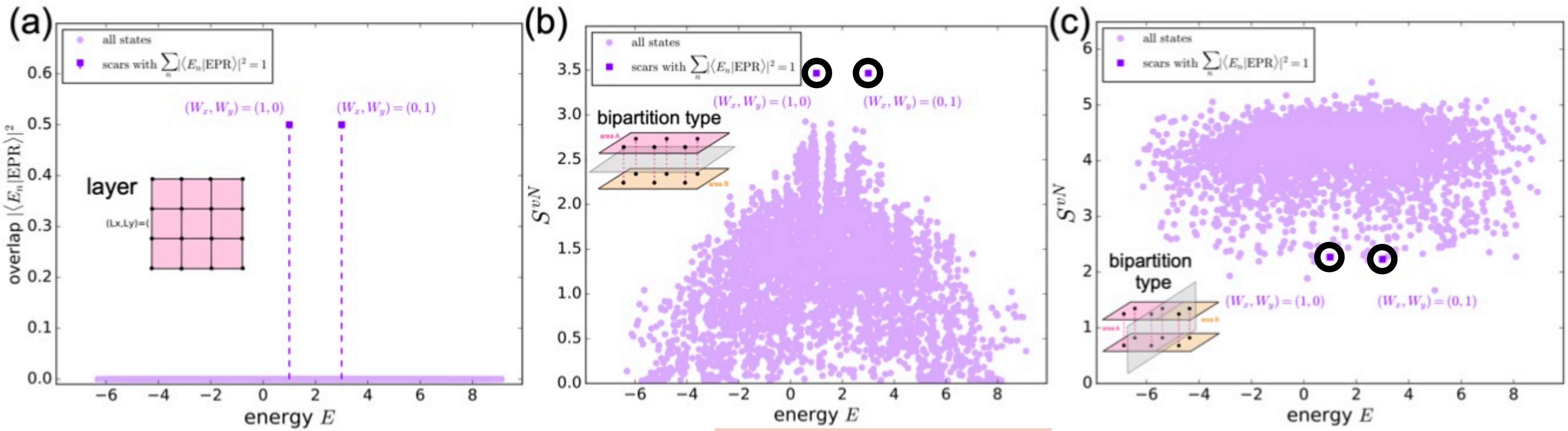
in general tower of

$N^{\square}_{sec} = (L_x+1)\cdot(L_y+1)$ **scar states**

tower of 2 critical U(1) scar states with simple entanglement structure

Quantum dimer model with interlayer coupling



note that entanglement depends on choice of bipartition

Einstein-Podolsky-Rosen (EPR) scar states

$$|\mathrm{EPR}\rangle = \sum_{(w_x,w_y)} \sum_{D_{w_x,w_y}} |D_{w_x,w_y}\rangle \otimes |D_{w_x,w_y}\rangle$$
$$= \sum_{(w_x,w_y)} |\mathrm{RK}\rangle \otimes |\mathrm{RK}\rangle$$
$$= \sum_{(w_x,w_y)} |\mathrm{EPR}\rangle_{(w_x,w_y)}$$

in general tower of

$N_{sec}^{\square} = (L_x+1)\cdot(L_y+1)$ **scar states**

tower of 2 critical U(1) scar states with simple entanglement structure

Bose-Hubbard model with interlayer coupling

- **Bose-Hubbard model** $$\mathcal{H} = \sum_{\langle i,j \rangle} -t_{ij} \left(b_i^\dagger b_j + h.c. \right) + U \sum_i n_i \left(n_i - 1 \right)$$

  bosonic occupation operators $n_i = 0, 1, 2, 3, \ldots$

  $$[\mathcal{H}, n_i] = 0$$

Bose-Hubbard model with interlayer coupling

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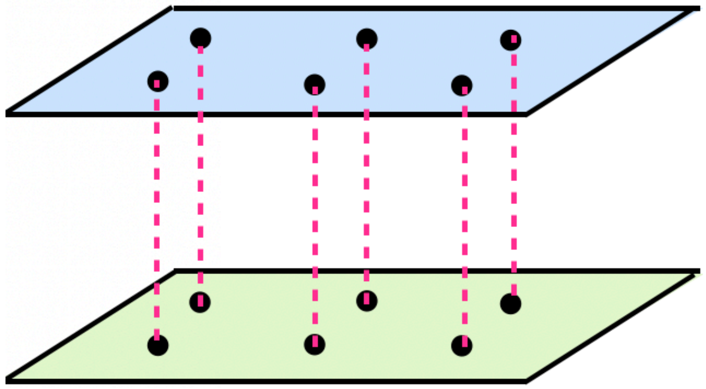
$$\Longrightarrow \quad \mathcal{H}_1 = \sum_{\langle i,j \rangle} -t_{ij} \left(b_i^\dagger b_j + h.c. \right) + U \sum_i n_i \left(n_i - 1 \right)$$

$$\mathcal{H}_2 = -\mathcal{H}_1$$

$$\mathcal{H}_{12} = \lambda \sum_i \left(n_i - n_{\tilde{i}} \right)^2$$



square lattice bilayer

Bose-Hubbard model with interlayer coupling

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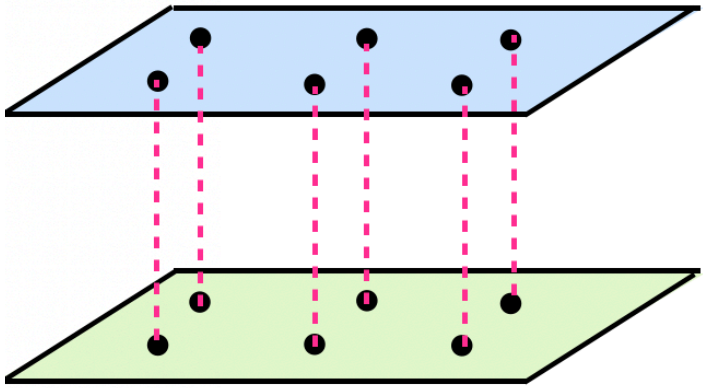


square lattice bilayer

- **Einstein-Podolsky-Rosen (EPR) scar states**

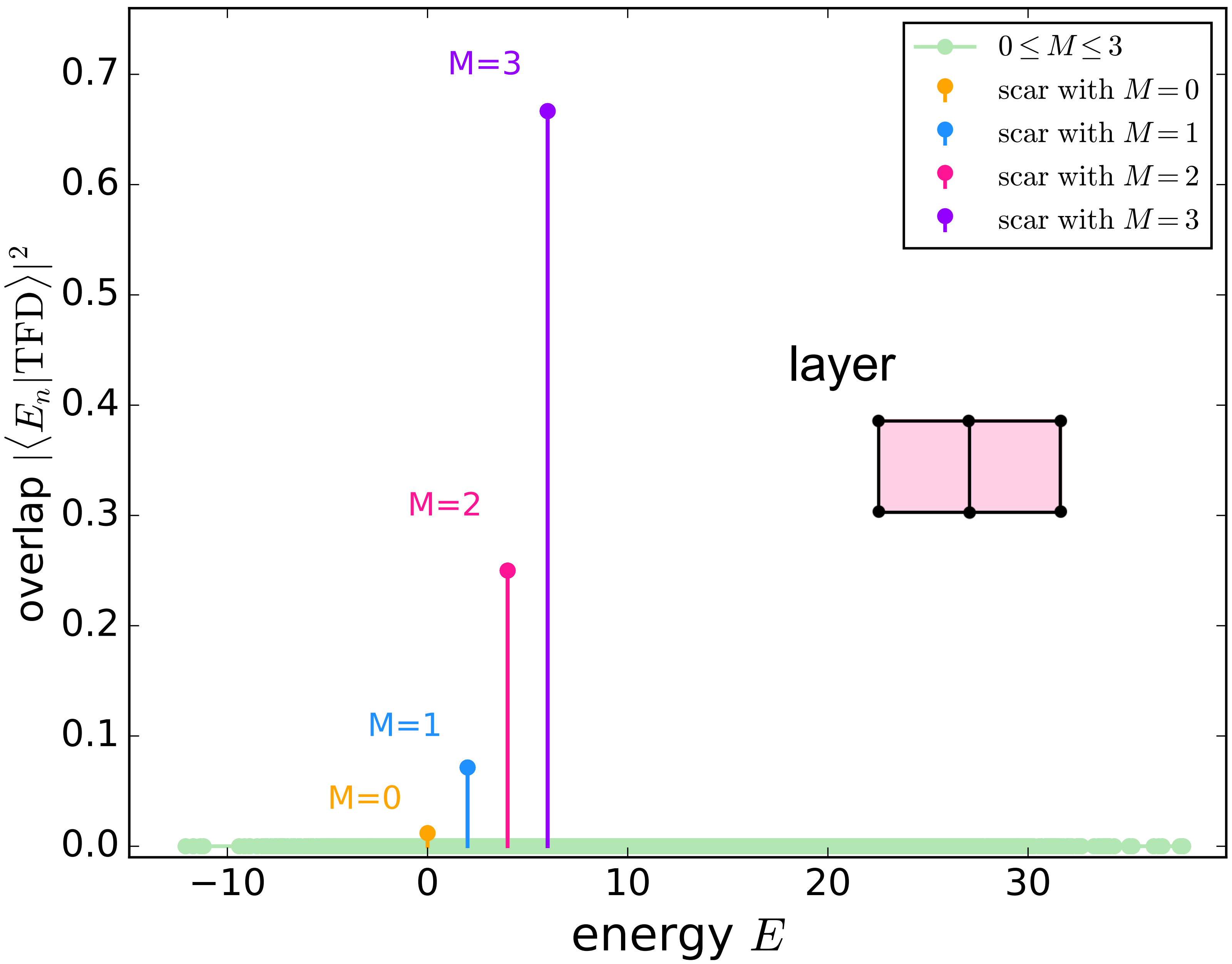
$$|\mathrm{EPR}\rangle = \sum_{M=0}^{M_{\max}} c_M |\mathrm{EPR}\rangle_M$$

sector M bosons

$$|\mathrm{EPR}\rangle_M = \mathcal{P}_M \left[\bigotimes_{i \le N} \left(\frac{1}{\sqrt{\alpha_{BH}}} \sum_{n=0}^{M} |n, n\rangle \right)_{i,\tilde{i}} \right]$$

"bosonic configurations identical in both layers"

Bose-Hubbard model with interlayer coupling



$t_{ij} \in [0.9, 1.1]$

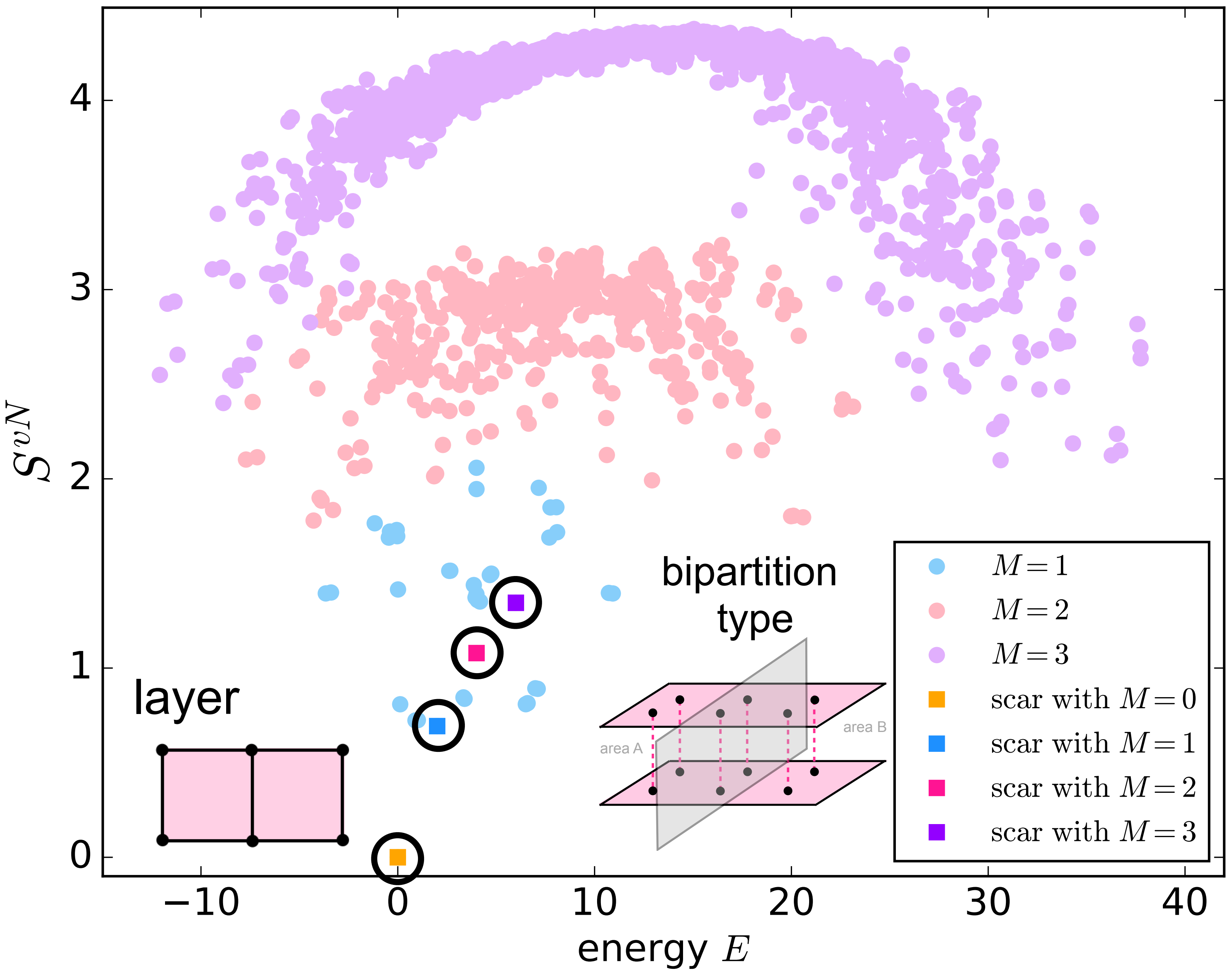
$U = 1.0$

$\lambda = 1.0$

$$|\mathrm{EPR}\rangle = \sum_{M=0}^{M_{\max}=3} c_M |\mathrm{EPR}\rangle_M$$

tower of 4 scar states

Bose-Hubbard model with interlayer coupling



$t_{ij} \in [0.9, 1.1]$

$U = 1.0$

$\lambda = 1.0$

$$|\mathrm{EPR}\rangle = \sum_{M=0}^{M_{\max}=3} c_M |\mathrm{EPR}\rangle_M$$

scar states with simple entanglement structure

Bose-Hubbard model with interlayer coupling

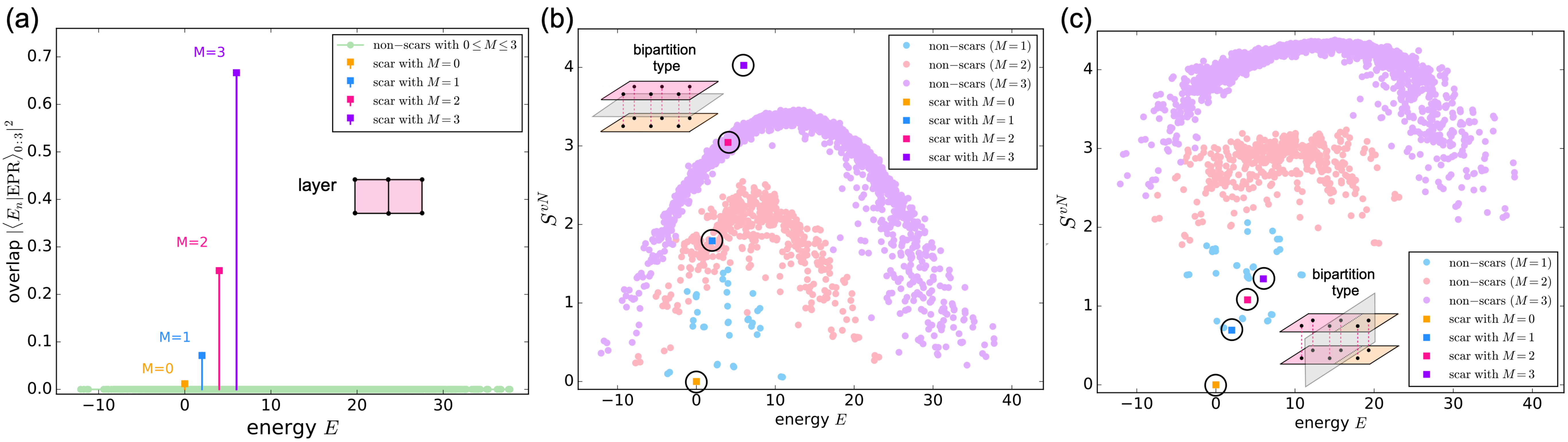
$$\mathcal{H}_1 = \sum_{\langle i,j \rangle} -t_{ij} \left(b_i^\dagger b_j + h.c. \right) + U \sum_i n_i \left(n_i - 1 \right)$$

$$\mathcal{H}_2 = -\mathcal{H}_1$$

$$\mathcal{H}_{12} = \lambda \sum_i \left(n_i - n_{\tilde{i}} \right)^2$$

$$\Longrightarrow \mathcal{H} = \mathcal{H}_1 + \mathcal{H}_2 + \mathcal{H}_{12}$$

$$t_{ij} \in [0.9, 1.1] \qquad U = 1.0 \qquad \lambda = 1.0$$



$$|\mathrm{EPR}\rangle = \sum_{M=0}^{M_{\max}=3} c_M |\mathrm{EPR}\rangle_M$$

scar states with simple entanglement structure

Bilayer triangular lattice Heisenberg model

$$\mathcal{H}_1 = \sum_{\langle ij \rangle} J_{ij}\, \vec{S}_i \cdot \vec{S}_j \qquad \mathcal{H}_2 = -\mathcal{H}_1$$

$$\mathcal{H}_{12} = \lambda \sum_i \vec{S}_i \otimes \vec{S}_{\tilde{i}}$$

$$\Longrightarrow \quad \mathcal{H} = \mathcal{H}_1 \otimes \mathbb{I} + \mathbb{I} \otimes \mathcal{H}_2 + \mathcal{H}_{12}$$

$$\mathcal{H} \longrightarrow \mathcal{H} + S^2_{\text{tot}} + S^2_{\text{tot}}$$

$$J_{ij} \in [0.9, 1.1] \qquad \lambda = 1.0$$

SU(2) symmetry

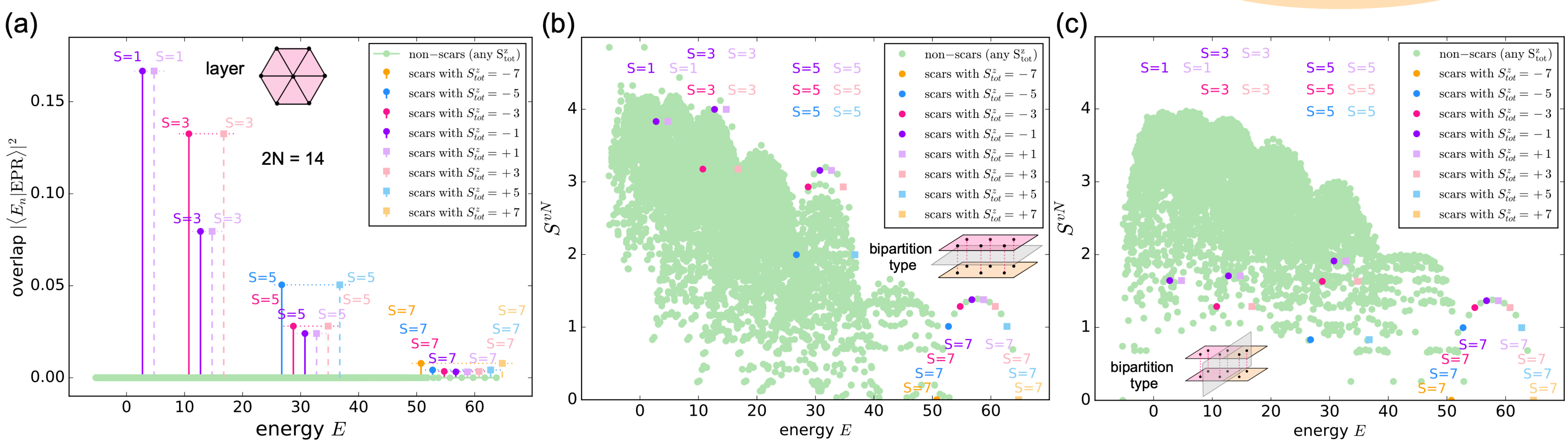
$$S^z_{\text{tot}} = S^{z,1}_{\text{tot}} + S^{z,2}_{\text{tot}} = \sum_i S^z_i \otimes \mathbb{I} + \mathbb{I} \otimes S^z_i$$

$$S^2_{\text{tot}} = \left(\sum_i \vec{S}_i \otimes \mathbb{I} + \mathbb{I} \otimes \vec{S}_i \right)^2$$

Bilayer Triangular Lattice Heisenberg Model

S^z_{tot}	-7	-5	-3	-1	$+1$	$+3$	$+5$	$+7$
total spin S_{tot}	7	5,7	3,5,7	1,3,5,7	1,3,5,7	3,5,7	5,7	7
N_{scars}	1	2	3	4	4	3	2	1

20 scar states



Summary

Quantum many-body scar states

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  - **billiard Bunimovich stadium, Rydberg experiment at Harvard**
  - **PXP model and its experimental realization**

- **2D bilayer systems of various degrees of freedom:**

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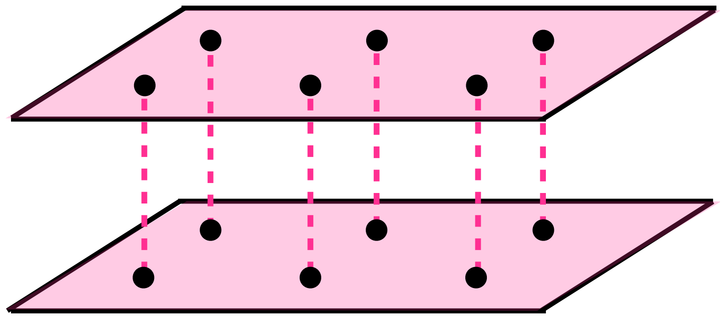

2D bilayer system

Wildeboer et al., PRB 106 (2022)

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  - **How far are the applications for quantum many-body scars (quantum information)?**
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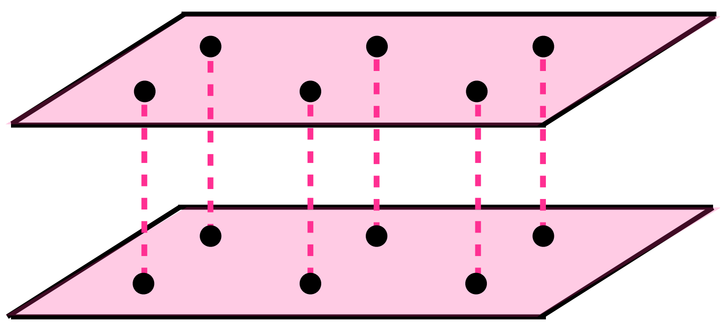

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Thank you for your attention!