Quarkonium/Open Heavy Flavor productions at collider energies in Small-x formalism

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Subjects

e^+e^- and Electron-Proton (ep) scattering provide clean test of the QCD dynamics; Fragmentation functions and hadron structure encoded in PDFs etc.

- Proton-Proton (pp) collision

$\hookrightarrow$ Elementary production mechanisms

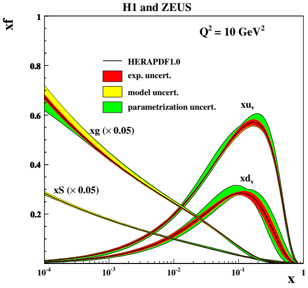
- Proton-Nucleus (pA) collision

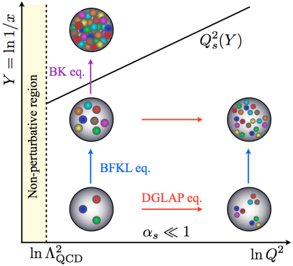
$\hookrightarrow$ Energy loss, Shadowing, Saturation

- Nucleus-Nucleus (AA) collision

$\hookrightarrow$ Energy loss, bound state dissociation in QGP, Regeneration

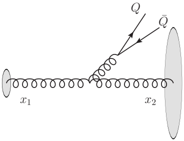
Gluon saturation : Motivation

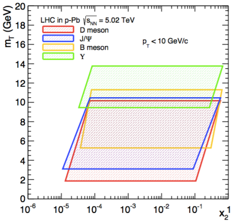




- Q-evolution : linear DGLAP equation which resums $(\alpha_s \ln Q^2)^n$.
- x-evolution : linear BFKL equation which resums $(\alpha_s \ln 1/x)^n$ but breaks unitarity $\Rightarrow$ NONLINEAR BK equation or JIMWLK equation
- The saturation scale $Q^2_{s,A}(x) = \frac{\alpha_s N_c}{S_{A\perp}} xG_A(x) \sim A^{1/3}\left(\frac{1}{x}\right)^{0.3}$
- $Q > Q_s$: dilute regime, collinear factorization is applicable.
- $Q < Q_s$: dense regime, alternative approach is required $\Rightarrow$ Small-x/Color-Glass-Condensate framework.

Overview





- Consider Heavy Quark Pair production in hadron-hadron (pp) and hadron-nucleus (pA) collisions

$$\mathrm{p} + \mathrm{p}/A \to Q + \bar{Q} + X$$

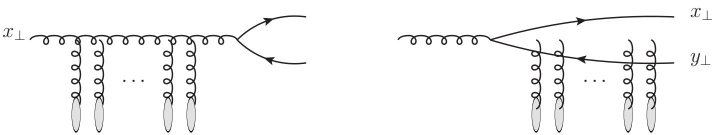
- RHIC ($\sqrt{s_{NN}}$=200GeV) forward and LHC ($\sqrt{s_{NN}}$=5.02TeV, 7TeV, $\cdots$) in the mid–forward rapidities ⇒ Probe Small-x gluons!
- Observables : Quarkonium (J/ψ, ψ', Υ, ...), Open heavy flavor (D, B), Decay lepton ($D, B \to e, \mu$)
- Compare theoretical calculations in Small-x Saturation/CGC framework with rich data.

Talk Plan

1. Introduction
   - $Q\bar{Q}$ production in CGC/Small-x framework
   - Quantum evolution
2. Quarkonium production
   - CGC+CEM
   - CGC+NRQCD
   - TMD and Saturation
3. Open heavy flavor production
   - Single meson production
   - Single lepton production from semileptonic decay
   - Two particle correlation
4. Summary

$Q\bar{Q}$ production in the CGC framework

[Blaizot, Gelis and Venugopalan (2004)][Kovchegov and Tuchin (2006).][Fujii, Gelis and Venugopalan (2006)][Qiu, Sun, Xiao and Yuan(2014)]



- The scattering contributions are coherent as a whole.
- Proton side : Leading twist, Nucleus side : All orders resummation $\alpha_s^2 A^{1/3} \sim \mathcal{O}(1) \Rightarrow$ Multiple scattering
- Background gauge field at large x should be obtained by solving classical Yang-Mills equation.

$$[D_\mu, F^{\mu\nu}] = J^\nu$$

with $J^\nu = g\delta^{\nu+}\delta(x^-)\rho_p(x_\perp) + g\delta^{\nu-}\delta(x^+)\rho_A(x_\perp)$

$Q\bar{Q}$ production in the CGC framework

The $q\bar{q}$ production amplitude from the background gauge fields of $\mathcal{O}(\rho_p^1 \rho_A^\infty)$ can be described as

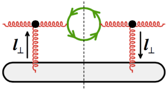
$$M_{s_1 s_2;ij}(q,p) =$$
$$\frac{g^2}{(2\pi)^4}\int d^2k_\perp d^2k_{1\perp}\frac{\rho_p(k_{1\perp})}{k_{1\perp}^2}\int d^2x_\perp d^2y_\perp e^{ik_\perp\cdot x_\perp}e^{i(P_\perp - k_\perp - k_{1\perp})\cdot y_\perp}$$
$$\times \bar{u}_{s_1,i}(q)\left[T_g(k_{1\perp})t^b W^{ba}(x_\perp) + T_{q\bar{q}}(k_{1\perp},k_\perp)U(x_\perp)t^a U^\dagger(y_\perp)\right]v_{s_2,j}(p)$$

At high scattering energies, Eikonal approximation is valid. The interaction between incident quark (gluon) and the dense gluons in the target nucleus can be expressed as the Wilson line in the fundamental (adjoint) representation:

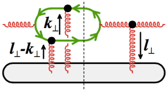
$$U(x_\perp) = \mathcal{P}_+ \exp\left[ig\int_{-\infty}^{+\infty} dz^+ A_A^-(z^+,x_\perp)\cdot t\right],$$
$$W(x_\perp) = \mathcal{P}_+ \exp\left[ig\int_{-\infty}^{+\infty} dz^+ A_A^-(z^+,x_\perp)\cdot T\right]$$

$Q\bar{Q}$ production in the CGC framework

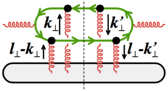
$$\phi_A^{g,g}(\boldsymbol{l}_\perp) =$$



$$\phi_A^{q\bar{q},g}(\boldsymbol{l}_\perp;\boldsymbol{k}_\perp) =$$



$$\phi_A^{q\bar{q},q\bar{q}}(\boldsymbol{l}_\perp;\boldsymbol{k}_\perp,\boldsymbol{k}'_\perp) =$$



$$\frac{d\sigma_{q\bar{q}}}{d^2q_\perp d^2p_\perp dy_q dy_p} = \frac{\alpha_s}{(2\pi)^6 C_F}\int \frac{d^2k_{1\perp}}{(2\pi)^2}\frac{\varphi_{p,Y_p}(k_{1\perp})}{k_{1\perp}^2 k_{2\perp}^2}$$

$$\times \underbrace{\left[\int \frac{d^2k_\perp d^2k'_\perp}{(2\pi)^4}\Xi^{q\bar{q},q\bar{q}}\phi_{A,Y_A}^{q\bar{q},q\bar{q}} + \int \frac{d^2k_\perp}{(2\pi)^2}\Xi^{q\bar{q},g}\phi_{A,Y_A}^{q\bar{q},g} + \Xi^{g,g}\phi_{A,Y_A}^{g,g}\right]}_{\Longrightarrow \int \frac{d^2k_\perp}{(2\pi)^2}\Xi(k_{1\perp},k_{2\perp},k_\perp)\phi_{A,Y_A}^{q\bar{q},g}(k_{2\perp},k_\perp)}$$

Multipoint function

The unintegrated gluon distribution function

$$\varphi_{p,Y_p}(k_\perp) = \frac{(2\pi)^2 \alpha_s S_\perp}{k_\perp^2} \int d^2 x_\perp e^{ik_\perp \cdot x_\perp} \langle \rho_p(0) \rho_p(x_\perp) \rangle_{Y_p}.$$

and the higher multi-point Wilson line correlator

$$\phi^{q\bar{q},g}_{A,Y_A}(k_{2\perp}, k_\perp) \approx S_\perp \frac{N_c k_{2\perp}^2}{4} F_{Y_A}(k_{2\perp} - k_\perp) F_{Y_A}(k_\perp)$$

Fourier transform of the fundamental dipole amplitude is defined as

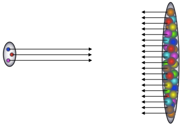
$$F_Y(k_\perp) = \int d^2 x_\perp e^{-ik_\perp \cdot x_\perp} \frac{1}{N_c} \left\langle \mathrm{Tr} \left[U(x_\perp) U^\dagger(0_\perp) \right] \right\rangle_Y .$$

Forward production : Hybrid formalism

At forward rapidity where $x_1 \sim 1$, the phase space of the gluon distribution in projectile proton shrinks. By taking the limit $k_{1\perp} \to 0$ in the hard scattering parts and replacing the unintegrated gluon distribution function of proton with the collinear gluon distribution function (PDF), Hybrid-formula ($x_1 \sim 1$ and $x_2 \ll 1$) can be written as

$$\frac{d\sigma_{q\bar{q}}}{d^2p_{q\perp}d^2p_{\bar{q}\perp}dy_q dy_{\bar{q}}} = \frac{N_c \alpha_s S_\perp^A}{64\pi^2 C_F} \int_{k_\perp} \Xi_{\rm coll}(k_{2\perp}, k_\perp) x_1 G(x_1, \mu) \times F_{x_2}(k_{2\perp} - k_\perp) F_{x_2}(k_\perp)$$

with $p_{q\perp} + p_{\bar{q}\perp} = k_{2\perp}$ and $x_1 G(x_1, \mu) \equiv C \int^{\mu^2} dk_\perp^2 \varphi_{\mathrm{p}, Y_p}(k_\perp)$



Rapidity evolution

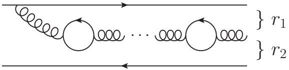
- Balitsky-Kovchegov equation

$$\frac{dT_Y(r)}{dY} = \mathcal{K} \otimes \left[\underbrace{T_Y(r_1) + T_Y(r - r_1) - T_Y(r)}_{\text{BFKL}} - T_Y(r_1)T_Y(r - r_1) \right]$$

- The running coupling kernel in Balitsky's prescription:

$$\mathcal{K}(r_\perp, r_{1\perp}) = \frac{\alpha_s(r^2)N}{2\pi^2} \left[\frac{1}{r_1^2}\left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1\right) + \frac{r^2}{r_1^2 r_2^2} + \frac{1}{r_2^2}\left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1\right)\right]$$

with $\alpha_s(r^2) = \left[\frac{9}{4\pi}\ln\left(\frac{4C^2}{r^2\Lambda^2} + a_{\text{cutoff}}\right)\right]^{-1}$



- NLO BK equation can be used. See [Lappi, Mäntysaari (2015,2016)]

Initial condition of Dipole amplitude

- GBW model [Golec-Biernat, Wusthoff (1998)]

$$S_Y(r_\perp) = 1 - T_Y(r_\perp) = \exp\left[-\frac{Q_s^2 r_\perp^2}{4}\right]$$

Not realistic model: high $k_\perp$ contribution is strongly suppressed.

- MV model [McLerran, Venugopalan (1994)]

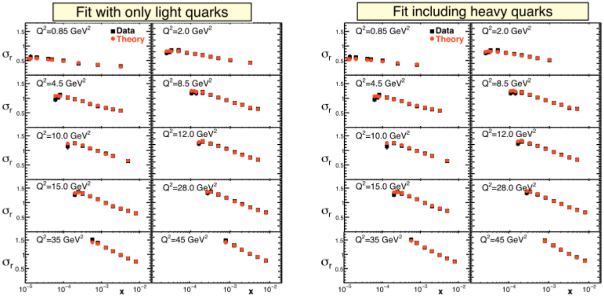
$$S_Y(r_\perp) = \exp\left[-\frac{r_\perp^2 Q_{s,0}^2}{4}\ln\left(\frac{1}{|r_\perp|\Lambda} + e\right)\right]$$

- MV^γ and MV^e models [AAMQS(2010)][Lappi-Mäntysaari(2013)]

$$S_Y(r_\perp) = \exp\left[-\frac{\left(x_\perp^2 Q_{s,0}^2\right)^\gamma}{4}\ln\left(\frac{1}{|r_\perp|\Lambda} + e_c \cdot e\right)\right]$$

Constraint from DIS at HERA

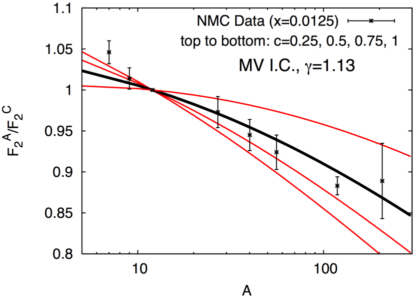
[AAMQS(2010)][Lappi-Mäntysaari(2013)]



set	$Q^2_{s0,\mathrm{p}}/\mathrm{GeV}^2$	γ	$\alpha_s(r \to \infty)$	e_c	$\chi^2/\mathrm{d.o.f.}$
MV	0.2	1	0.5	1	–
MV^γ	0.1597	1.118	1.0	1	1.12
MV^e	0.06	1	0.7	18.9	1.15

IC for the rcBK for nuclei

The initial condition of the rcBK equation for heavy nuclei is poorly understood due to lack of numerous data of eA scattering. A fit to the available NMC (New Muon Collab.) data (fixed target eA) on the nuclear structure functions $F_{2,A}(x, Q^2)$ gives valuable information in this respect.



- Set the initial saturation at $x = 0.01$ as $Q^2_{s,A}(x_2) = cA^{1/3}Q^2_{s0}$
- For MV$^\gamma$ model, $c = 0.5$ is favored from the fit of data. [Dusling, Gelis, Lappi, Venugopalan (2009)]

Extrapolation at $x \geq 0.01$

- From QCD scaling low, [Fujii, Gelis, Venugopalan][Fujii, KW]

$$\phi_{p,Y}(k_\perp) = \phi_{p,Y_0}(k_\perp) \left(\frac{1-x}{1-x_0}\right)^4 \left(\frac{x_0}{x}\right)^{0.15}$$

- Fitting from collinear PDF [Ma, Venugopalan (2014)]

$$F_Y(k_\perp) \overset{x \geq x_0}{=} a(x) F_{Y_0}(k_\perp)$$

where (e.g.)

$$a(x) = \frac{xG_{\text{CTEQ}}(x, Q_0^2)}{xG^{\text{Dipole}}(x, Q_0^2)}$$

with the identity

$$xG^{\text{Dipole}}(x, Q_0^2) = \frac{1}{4\pi^3} \int_0^{Q_0^2} dk_\perp^2 \phi_{Y,p}^{g,g}(k_\perp)$$

R_p is determined by fitting of collinear gluon PDF.

These extrapolations cause systematic uncertainty.

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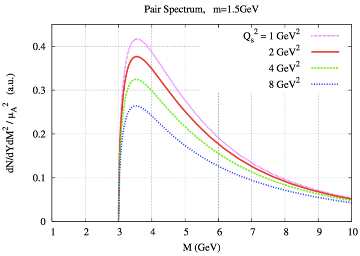
Effective Factorization

- The typical interaction time when proton scatters off nucleus is $\tau_{int} \sim R_A/c$.
- A $q\bar{q}$ pair is produced over $\tau_P \approx \frac{1}{2m_q}\frac{E_g}{2m_q}$
- The formation time of J/ψ is $\tau_F \approx \frac{2}{M_{\psi'}-M_\psi}\frac{E_g}{M_\psi}$
- At very high energies, the coherent length of the $q\bar{q}$ pair production is much longer than the size of the target nucleus : $\tau_F \gg \tau_P \gg \tau_{int}$
- If $Q_{sA} \sim mv \sim Mv/2 \to$ v-expansion is not ensured.
- In the very forward rapidity region, owing to Lorentz time dilation

$$\frac{1}{mv}\frac{p_{\parallel}}{M} \gg \frac{1}{p_\perp} \sim \frac{1}{Q_{sA}} \quad \text{or} \quad y \gg \ln\frac{2mv}{p_\perp} \sim \ln\frac{Mv}{Q_{sA}}$$

The hadronization is effectively frozen when the $Q\bar{Q}$ passes through the nucleus. $\Longrightarrow$ CEM and NRQCD can work.

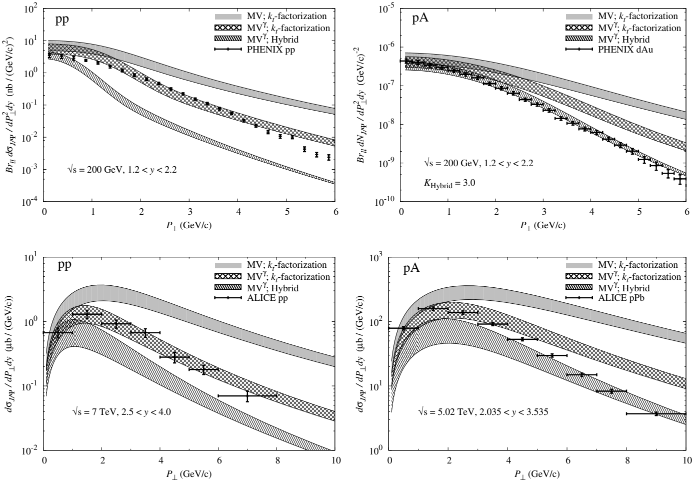
Conventional Color Evaporation Model



$$\frac{d\sigma_\psi}{d^2p_\perp dy} = F_\psi \int_{(2m_{c,b})^2}^{(2M_{D,B})^2} dM^2 \frac{d\sigma_{q\bar{q}}}{dM^2 d^2p_\perp dy}$$

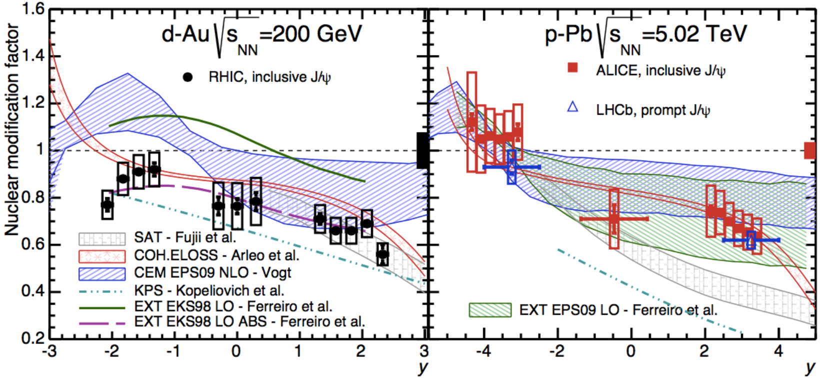
- The produced $q\bar{q}$ pair is going to be bound into a quarkonium ψ with the probability $F_{q\bar{q}\to\psi}$. $F_{q\bar{q}\to\psi}$ is empirical factor.
- Color octet $Q\bar{Q}$ mainly convert into J/ψ and Υ etc.

Forward J/ψ production in CGC + CEM



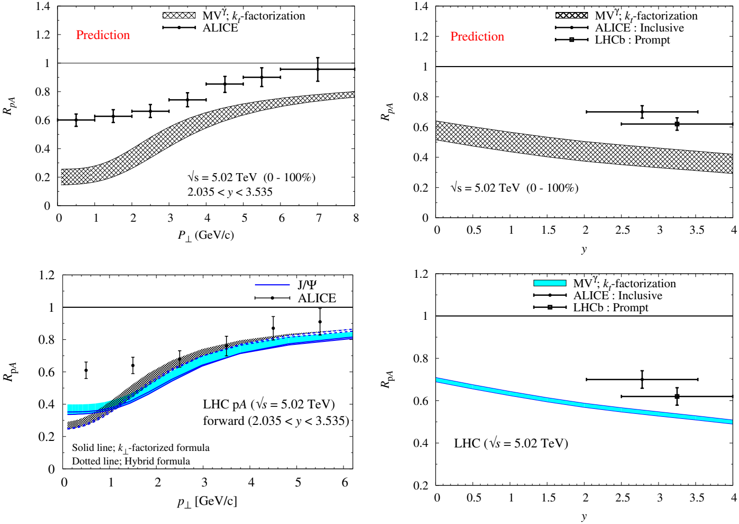
Comments: Uncertainties with the IC for the rcBK and the framework are reduced in pA collisions at LHC

Forward J/ψ production in CGC + CEM



- $Q^2_{sA} = (4-6)Q^2_{s0}$ is chosen for our prediction
- R_{pA} at RHIC $\sim$ R_{pA} at LHC (???)

Forward J/ψ production in CGC + CEM



Comments: $Q_{sA}^2 = (4-6)Q_{s0}^2 \implies Q_{sA}^2 = (2-3)Q_{s0}^2$

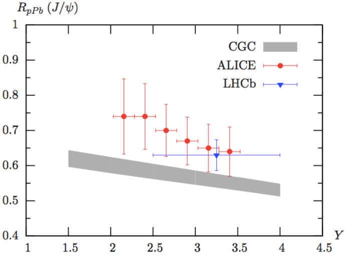
Improvement within CEM

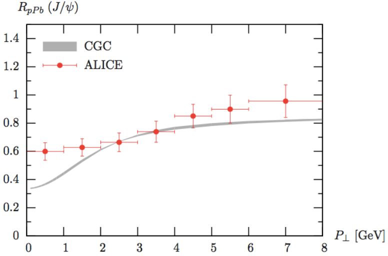
[Ducloué, Lappi, Mäntysaari (2015)] [1]

The initial condition for the rcBK equation for proton is MV^e mode, while the initial condition for the rcBK eq. for target nucleus is set to be

$$S_{x_g=x_0}(r_\perp; b_\perp) = \exp\left[-AT_A(b_\perp)\frac{\sigma_0}{2}\frac{r_\perp^2 Q_{s0}^2}{4}\ln\left(\frac{1}{|r_\perp|\Lambda} + e_c \cdot e\right)\right]$$

where $T_A(b_\perp)$ is the Woods-Saxon distribution and $\frac{\sigma_0}{2}$ is the effective proton transverse area determined by DIS data fitting[2].





[1] Phys. Rev. D **91**, 114005 (2015)

CGC + NRQCD

Non-Relativistic QCD (NRQCD) factorization approach provides more sophisticated treatment for describing bound state formation. The quark velocity v-expansion in NRQCD is effectively ensured at forward rapidity.

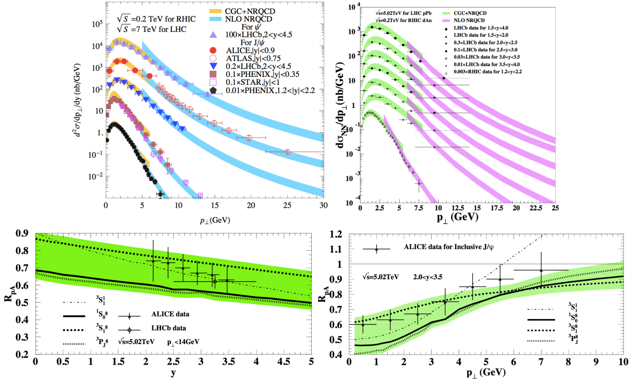
- NRQCD factorization : double expansion in α_s, v

$$d\sigma^H_{pA} = \sum_\kappa d\hat{\sigma}^\kappa_{pA} \times \underbrace{\left\langle \mathcal{O}^H_\kappa \right\rangle}_{\text{LDMEs}}$$

- The projection operators are explicitly put for $Q\bar{Q}$ in final state.
- Important LDMEs : $^3S_1^{[1]}, \quad \underbrace{^1S_0^{[8]}, \quad ^3S_1^{[8]}, \quad ^3P_0^{[8]}}_{\mathcal{O}(v^4)}$

CGC framework + NRQCD

[Ma, Venugopalan (2014)] [Ma, Venugopalan, Zhang (2015)]

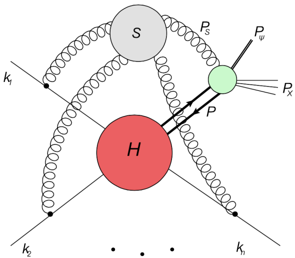


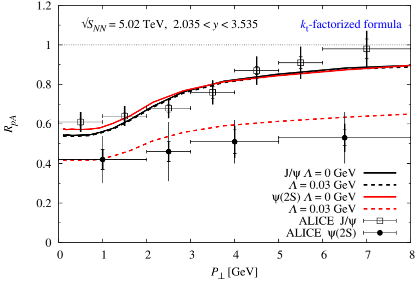
- The initial condition to the rcBK eq. for the nucleus : $Q^2_{s0,A} = 2 \times Q^2_{s0}$.
- The contribution of CS channel is enhanced in pA but relatively small contribution.

  (10% in pp, 15% – 20% in pA at small-$p_\perp$)

$\psi(2S)$ suppression

[Ma, Venugopalan, KW, Zhang in preparation]





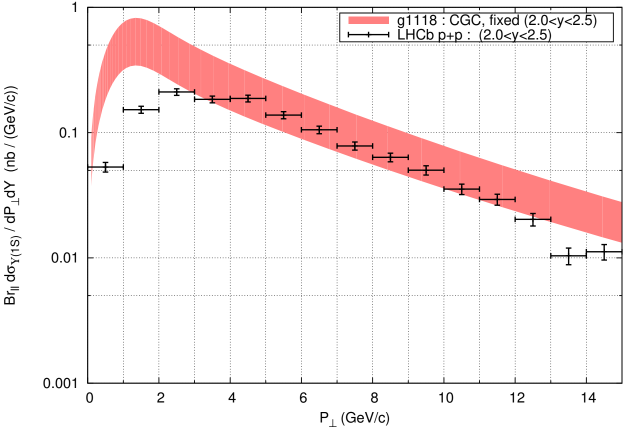
Improved CEM [Ma, Vogt (2016)]

$$\frac{d\sigma_\psi}{d^2p_\perp dy} = F_\psi \int_{(m_\psi)^2}^{(2M_D-\Lambda)^2} dM^2 \frac{d\sigma_{q\bar{q}}}{dM^2 d^2p_\perp dy}$$

- Final state interaction encoded in Λ is important for $\psi(2S)$ production.

Υ production - A puzzle -

[Fujii, KW (2013)][Ducloué, Lappi, Mäntysaari(2015)]



- LHCb data indicate large mean $P_\perp$
- Should be $P_\perp \sim Q_s \sim 1$ GeV.....??

Sudakov resummation in the small-x formalism

Consider Higgs, heavy quarkonium, dijet/dihadron productions in the high scattering energy.

- Two kinds of hard scales : $s \gg M^2 \gg p_\perp^2$
  - $s \gg p_\perp^2$: Small-x resummation is important.

$$\frac{\alpha_s N_c}{2\pi^2} \ln \frac{1}{x_g} \sim \mathcal{O}(1) \Longrightarrow \text{BFKL or BK/JIMWLK evolution}$$

  - $M^2 \gg p_\perp^2$: Sudakov resummation is important.

$$\frac{\alpha_s N_c}{2\pi} \ln^2 \frac{M^2}{p_\perp^2} \sim \mathcal{O}(1) \Longrightarrow \text{CSS evolution}$$

Unified understanding both TMD framework and small-x/CGC framework is a topical issue. [Mueller, Xiao, Yuan (2013)] [Kotko, Kutak, Marquet et al. (2015)] [KW, Xiao (2015)] [Balitsky, Tarasov (2015,2016)] [Zhou(2016)]

Improved small-x formalism: TMD + Saturation

[KW and Xiao (2015)]
Sudakov resummation effect provides further transverse momentum dependence on top of the BK resummation.

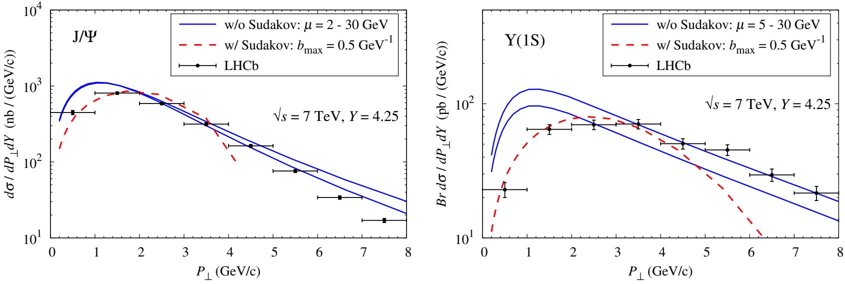
$$\frac{d\sigma_{q\bar{q}}}{d^2q_{q\perp}d^2q_{\bar{q}\perp}dy_q dy_{\bar{q}}} = \frac{\alpha_s^2\overline{S}_\perp}{16\pi^2 C_F}\int d^2l_\perp d^2k_\perp \frac{\Xi_{\rm coll}(k_{2\perp},k_\perp - zl_\perp)}{k_{2\perp}^2} \times F_{\rm TMD}(l_\perp) F_{Y_g}(k_\perp)\, F_{Y_g}(k_{2\perp}-k_\perp+l_\perp)$$

where the TMD gluon distribution is given by

$$F_{\rm TMD}(M,l_\perp) = \int \frac{d^2b_\perp}{(2\pi)^2} e^{-ib_\perp\cdot l_\perp} e^{-S_{\rm Sud}(M,b_\perp)} x_1 G\left(x_1, \mu = \frac{c_0}{b_\perp}\right).$$

Set $S_{\rm NP} \Longrightarrow \sqrt{S_{\rm NP}}$ because Hybrid formalism is used while $S_{\rm NP}$ is fitted by means of collinear factorization.

Υ production revisited



- CSS formalism breaks down around $P_\perp \sim M$.
- Factorization scale $\mu = \frac{c_0}{b_\perp}$ as a conventional choice in TMD.
- Y-term is not considered in this calculation.

Short summary I

	Framewrok	rcBK	IC	Centrality dep.	$Q\bar{Q} \to \psi$
FW	LO k_t, Hybrid	✓	MV$^\gamma$	No	CEM
DLM	LO Hybrid	✓	MV^e	Glauber	CEM
MZV	LO k_t	✓	MV	No	NRQCD

- Theoretical uncertainties are not small but can be reduced in R_{pA}.
- The small saturation scale $Q^2_{s,A} = (2-3)Q^2_{s,0}$ is needed, otherwise we have strong suppression of J/ψ R_{pA} at LHC at forward rapidity.
- So, $Q^2_{s,A} = (2-3)Q^2_{s,0}$ is consistent with other approaches? (e.g. MC-Glauber)
- Proton's fluctuation might be important? [Mäntysaari, Schenke (2016)]
- Sudakov effect is important for Υ production. $\Longrightarrow$ R_{pA} should be changed.

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4 Summary

Single open heavy flavor production

Consider $Q\bar{Q}$ production again

$$\mathrm{p} + \mathrm{p/A} \to Q(q) + \bar{Q}(p) + X$$

Single quark production cross section can be cast into

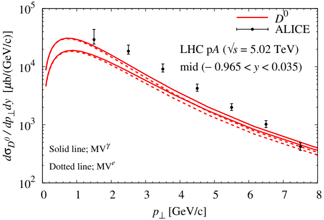
$$\frac{d\sigma_Q}{d^2q_\perp dy_q} = \int d^2p_\perp dy_p \frac{d\sigma_{Q\bar{Q}}}{d^2q_\perp dy_q d^2p_\perp dy_p}$$

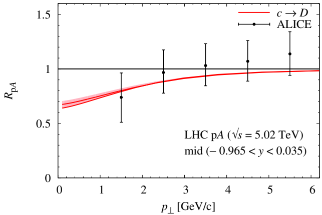
From single heavy quark to open heavy flavor

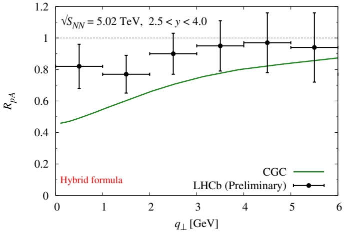
$$\frac{d\sigma_h}{d^2p_{h\perp} dy} = Br(Q \to h) \int \frac{dz}{z^2} D_{h/Q}(z) \frac{d\sigma_Q}{d^2q_\perp dy_q}$$

where $y_q = y_h = y$ and $p_{h\perp} = zq_\perp$.

D meson production at LHC







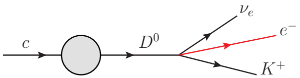
- $Q^2_{sA} = 3Q^2_{s0}$ is used.

Single lepton production

In the RHIC and LHC experiments, the decay leptons are also detected for the study of heavy quark production. Importantly, the muon detection is the important observable relevant to the heavy flavor production at forward rapidity in the RHIC and LHC experiment setup.
Consider single lepton production from semileptonic decay in pp/pA at mid (forward) rapidity : e.g.

$$\mathrm{p} + \mathrm{p}/A \to c + X$$

$$c \to D^0 \to K^+ + e^-(\mu^-) + \nu_e$$



- Bare quark goes through both fragmentation process and decay process.
- The produced lepton e^- reflects the information of the gluon saturation?

From meson to lepton via semileptonic decay

$$\frac{d\sigma_l}{d^2p_{l\perp}dy_l} = \int dp_{h\perp}p_{h\perp}dy_h\,\mathcal{F}(p_l,p_h)\,\frac{d\sigma_h}{d^2p_{h\perp}dy_h}.$$

- $\mathcal{F}(p_h,p_l)$ is the probability for the lepton to be produced in the decay of the heavy meson.

$$\mathcal{F}(p_h,p_l) = \int d\phi \frac{M_h}{4\pi(p_h\cdot p_l)} f\left(\frac{p_h\cdot p_l}{M_h}\right)$$

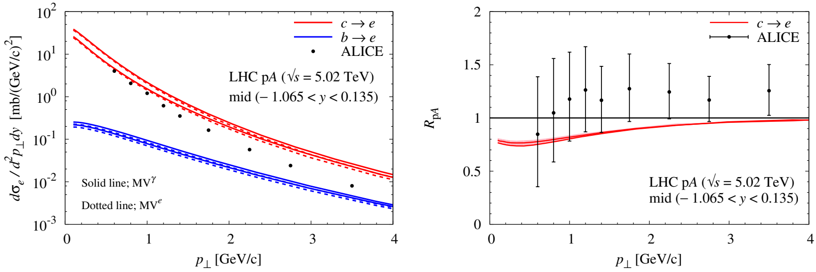
- $f(E_l)$ is the distribution of the lepton with energy E_l in the heavy-meson rest frame.

$$f(E_l) = \omega\frac{E_l^2(M_h^2 - M_X^2 - 2M_hE_l)^2}{M_h - 2E_l}.$$

  The normalization factor $\omega = 96/[(1-8t^2+8t^6-t^8-24t^4\ln t)M_h^6]$ with $t = M_X/M_h$. $M_X = M_K = 0.497$ GeV in the D decay ($M_h = M_D = 1.86$ GeV).

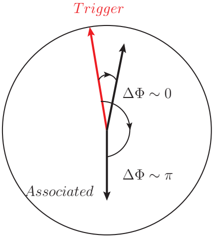
Charm decay lepton

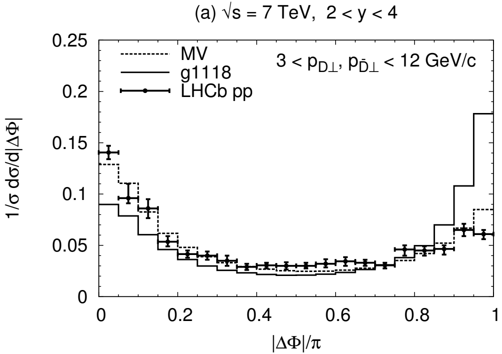
[Fujii and KW (2015)]



- The leptons from the b-decays are much suppressed as compared to those from the c decays in this momentum region $p_\perp \lesssim 4$ GeV.
- $R^l_{pA} > R^D_{pA} > R^{J/\psi}_{pA}$

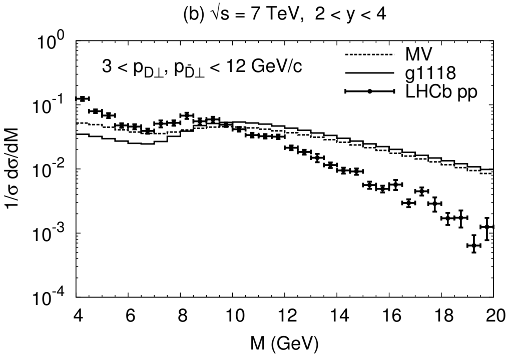
Forward $D\bar{D}$ production





- $|P_{tot}| = |P_{D\perp} + P_{\bar{D}\perp}| \sim Q_s \lesssim |P_{rel}|, |P_{D\perp}|, |P_{D\perp}|$ at away side.
- No strong peak at away side ⇒ Saturation effect!

Forward $D\bar{D}$ production



- Sudakov factor might be needed? ⇒ Yes, I think so. (Future work)
- Large double logarithmic correction like $\#\alpha_s \ln^2 \frac{M^2}{q_\perp}$

Short summary II

- R_{pA} of D production in small-x/CGC results is in good agreement with data of LHC at mid rapidity.
- Strong suppression at forward rapidity as well as J/ψ. Let us wait for more precise data from LHCb.
- Two particle correlation at forward rapidity is unique observable. Sudakov effect could be significant.

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Summary

- Heavy quark pair production provides unique observables to test the small-x/CGC framework. There are numerous data from RHIC and LHC available for comparison. Future EIC could provide more clean test.
- The saturation/CGC formalism reproduces R_{pA} for single heavy flavor production and Quarkonium production within the uncertainties.
- [Question] The initial saturation scale for nuclei is small?
- The Sudakov effect is essential for Υ production. It would be important for $D\bar{D}$ correlation.
- More importantly, full NLO calculations are desired to complete.

$Q\bar{Q}$ production cross section from the CGC

At classical level, it is required to average the squared amplitude over the distributions of the classical color sources ρ_p and ρ_A

$$\frac{d\hat{\sigma}_{q\bar{q}}}{d^2q_\perp d^2p_\perp dy_q dy_p} = \int \frac{d^2b_\perp}{[2(2\pi)^3]^2} \int \mathcal{D}\rho_p \mathcal{D}\rho_A W_p[\rho_p] W_A[\rho_A] \times |M_{s_1 s_2;ij}(q,p)|^2$$

with W_p and W_A are the weight functionals of ρ_p and ρ_A.

- Rapidity dependence of operator $\langle \mathcal{O} \rangle = \int \mathcal{D}\rho_p \mathcal{O} W_p[\rho_p]$ is embodied in the weight functionals which obey the JIMWLK equation

CGC framework + NRQCD

[Kang, Ma, Venugopalan (2013)]

Color singlet channel

$$\frac{d\sigma^{\rm CS}_{q\bar{q}}}{d^2P_\perp dy} \propto$$

$$\int \frac{d^2k_{1\perp}d^2k_\perp d^2k'_\perp}{(2\pi)^6}\frac{\varphi_{\rm p,Y_p}(k_{1\perp})}{k^2_{1\perp}}\frac{1}{2J+1}\sum_{J_z}\mathcal{F}^{J_z}_{q\bar{q}}(P,k_{1\perp},k_\perp)\mathcal{F}^{J_z\dagger}_{q\bar{q}}(P,k_{1\perp},k'_\perp)$$

$$\times\int d^2x_\perp d^2x'_\perp d^2y_\perp d^2y'_\perp e^{i(k_\perp\cdot x_\perp-k'_\perp\cdot x'_\perp)}e^{i(k_{2\perp}-k_\perp)\cdot y_\perp}e^{-i(k_{2\perp}-k'_\perp)\cdot y'_\perp}$$

$$\times\underbrace{\frac{1}{N_c}\langle {\rm tr}[U(x_\perp)t^aU^\dagger(y_\perp)]{\rm tr}[U(y'_\perp)t^aU^\dagger(x'_\perp)]\rangle_{Y_A}}_{\approx\frac{1}{2}\left[Q_{Y_A}(x_\perp,y_\perp;y'_\perp,x'_\perp)-S_{Y_A}(x_\perp,y_\perp)S_{Y_A}(y'_\perp,x'_\perp)\right]}\,.$$

The quadrupole amplitude

$$Q_{Y_A}(x_\perp,y_\perp;y'_\perp,x'_\perp)\equiv\frac{1}{N_c}{\rm tr}\langle U(x_\perp)U^\dagger(x'_\perp)U(y'_\perp)U^\dagger(y_\perp)\rangle_{Y_A}.$$

CGC framework + NRQCD

Color Octet state

$$\frac{d\sigma_{q\bar{q}}^{\mathrm{CO}}}{d^2P_\perp dy} = \frac{\alpha_s S_\perp}{(2\pi)^3(N_c^2-1)} \int \frac{d^2k_{1\perp} d^2k_\perp}{(2\pi)^4} \frac{\varphi_{\mathrm{p},Y_p}(k_{1\perp})}{k_{1\perp}^2}$$
$$\times F_{Y_A}(k_{2\perp} - k_\perp) F_{Y_A}(k_\perp) \Xi^{\mathrm{CO}}$$

- Of particular importance is that the nuclear effect is the same both in CEM and the color octet state in NRQCD with the large-N_c approximation.

Conventional TMD framework

One loop calculations in NRQCD factorization framework [Sun, Yuan, Yuan (2012)]

$$\left.\frac{d\sigma}{d^2P_\perp dy}\right|_{|P_\perp|\ll M} = \int \frac{d^2b_\perp}{(2\pi)^2} e^{iP_\perp\cdot b_\perp} W(M,b_\perp,x_1,x_2) + (\text{Y}-\text{term})$$

with

$$W(M,b_\perp,x_1,x_2) = e^{-S_{sud}(M,b_\perp)} W(M,b_\perp,C_1,C_2)$$

$$W(M,b_\perp,C_1,C_2) = \sigma_0 \frac{M^2}{s} \int \frac{dx}{x}\frac{dx'}{x'} C_{gg}\left(\frac{x_1}{x}\right) C_{gg}\left(\frac{x_2}{x'}\right) G(x_1,\mu)G(x_2,\mu)$$

- TMD factorization [Collins-Soper-Sterman (1985)] [Collins (2011)]
- Y-term is power suppressed by $P_\perp/M$ at low $P_\perp$
- $\mu = \frac{c_0}{b_\perp}$ with $c_0 = 2e^{-\gamma_E} \simeq 1$

Collins-Soper-Sterman (CSS) formalism

Split the Sudakov factor into two parts

$$\boxed{S_{\text{Sud}}(M,b) = S_{\text{perp}}(M,b_\star) + S_{\text{NP}}(M,b)} \quad \text{with } b_\star = \frac{b}{\sqrt{1+(b/b_{\text{max}})^2}}$$

- Perturbative : $b_\star \sim b \ll b_{max}$

$$S_{\text{perp}}(M,b) = \int_{c_0/b^2}^{M^2} \frac{d\mu^2}{\mu^2}\left[A\ln\left(\frac{M^2}{\mu^2}\right)+B\right]$$

$A = \sum_{i=1} A^{(i)}\left(\frac{\alpha_s}{\pi}\right)^i$, $B = \sum_{i=1} B^{(i)}\left(\frac{\alpha_s}{\pi}\right)^i$. $A^{(1)} = C_A$ and $B^{(1)} = -(b_0 + \delta_{8c}/2)N_c$ with $b_0 = \left(\frac{11}{6}N_c - \frac{n_f}{3}\right)\frac{1}{N_c}$.

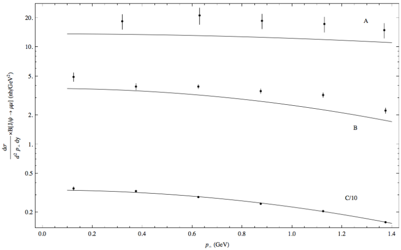
- Non-perturbative : $b > b_{max}$ [Sun, Yuan, Yuan (2012)]

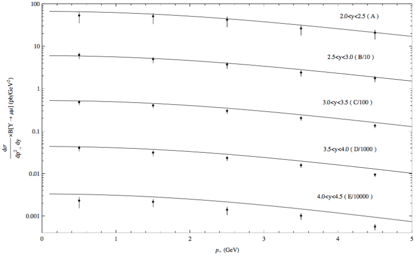
$$S_{\text{NP}}(M,b) = \exp\left[b^2\left(-g_1 - g_2\ln\left(\frac{M}{2Q_0}\right) - g_1g_3\ln(100x_1x_2)\right)\right]$$

$g_1 = 0.03$, $g_2 = 0.87$, $g_1g_3 = -0.17$ with $Q_0 = 1.6$ GeV, $b_{\text{max}} = 0.5$ GeV.

Lesson from conventional TMD calculation

[Sun, Yuan, Yuan (2012)]





- Left fig: J/ψ at LHC forward, RHIC mid and forward. Right fig: Υ at LHC forward.
- The agreement between theoretical results and experimental data for J/ψ production is not as good as that for Υ production

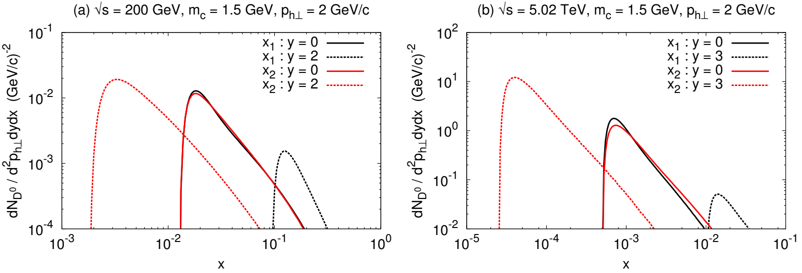
Single open heavy flavor production

The fragmentation function is Kartvelishvili's form

$$D_{h/Q}(z) = (\alpha + 1)(\alpha + 2)z^{\alpha}(1 - z)$$

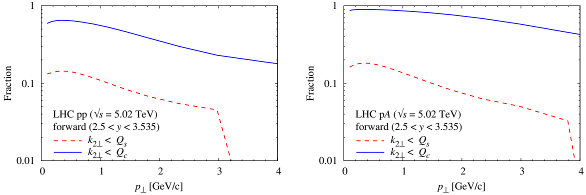
with the parameter $\alpha = 3.5$ (13.5) for D (B) meson. Or Peterson's form FF is often used.

- LO kinematics tells $x_{1,2} = \frac{1}{\sqrt{s}}\left(\sqrt{m^2 + q_\perp^2}e^{\pm y_q} + \sqrt{m^2 + p_\perp^2}e^{\pm y_p}\right)$
- By integrating out the phase space of anti-quark, $x_{1,2}$ should vary.



Fractional contributions

Q. The leptons carry information on the saturation?



- The deep saturation regime : $k_{2\perp} < Q_s$
- The geometrical scaling line : $k_{2\perp} < Q_c(x) = Q_s^2(x)/\Lambda_{\rm QCD}$
  [Iancu-Itakura-McLerran(2002)][Mueller-Triantafyllopoulos(2002)]