Los Alamos
NATIONAL LABORATORY

Event shape analysis for DIS at the EIC

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in collaboration with
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Uncovering New Laws of Nature at the EIC
BNL Workshop
Nov. 20-22, 2024

Outline

- Background and Motivation:
  Why and how we study τ_1^b
- Formalism
- Results and comparison with HERA data
- Summary

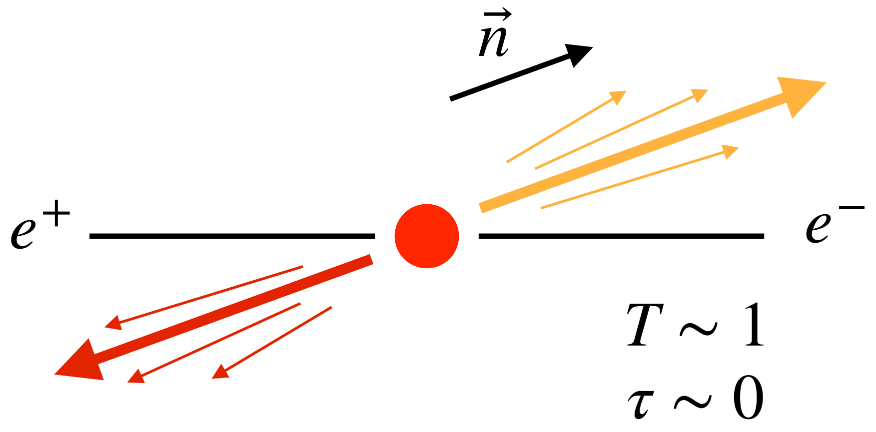
Background and Motivations:

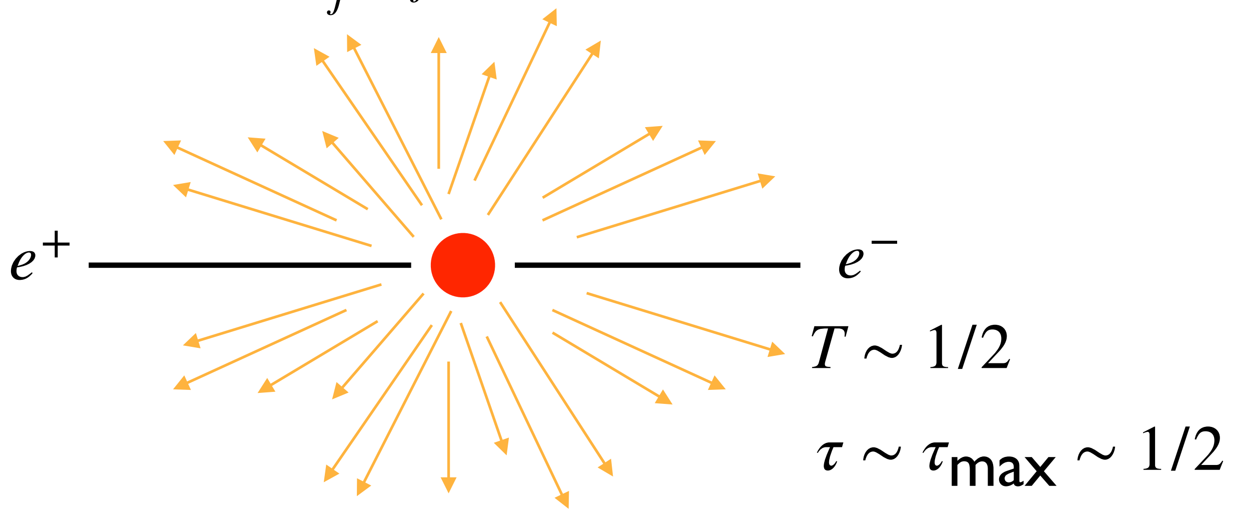
Why do we study τ_1^b event shape in DIS?

Hadronic Event shapes

- **Event shape**: Captures global geometry of events

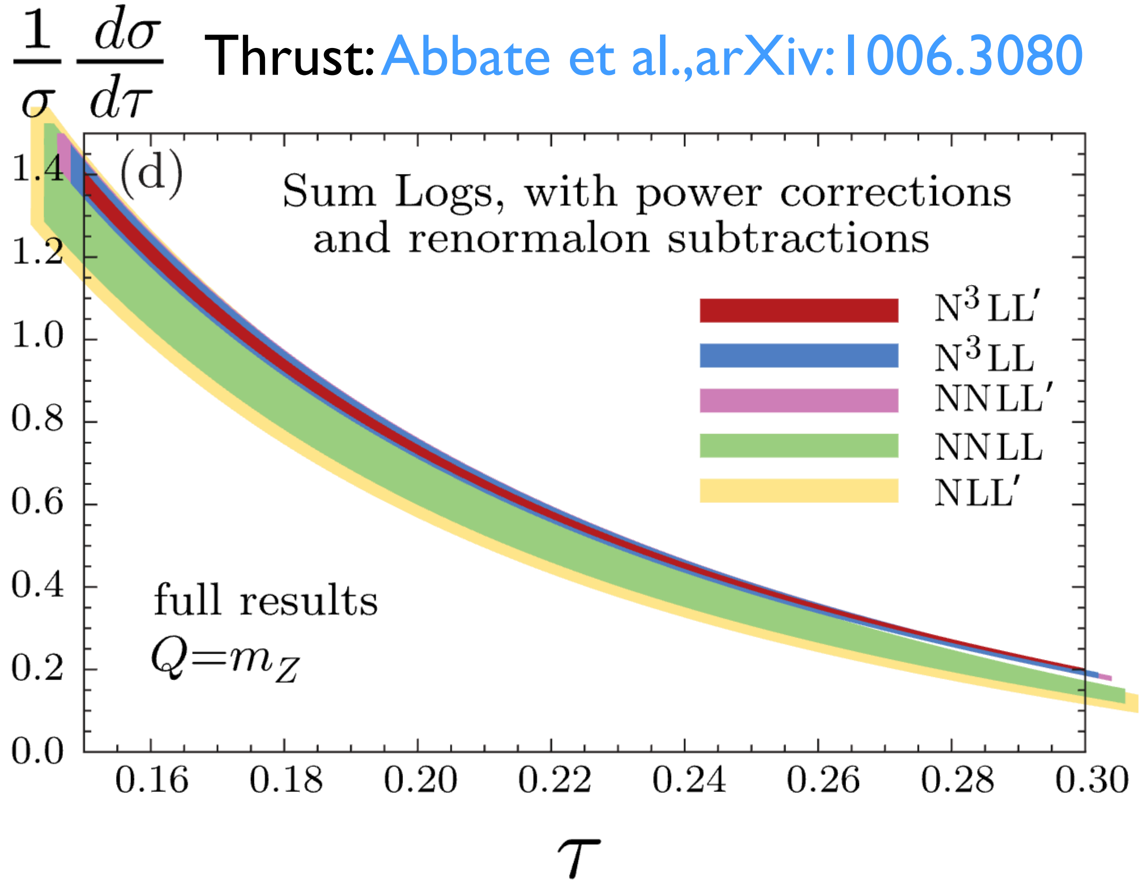
(e.g. e^+e^- thrust) $\tau = 1 - T$ where $T = \max\limits_{\vec{n}} \frac{\sum_j |\vec{p}_j \cdot \vec{n}|}{\sum_j |\vec{p}_j|}$ (Sum over all final states; inclusive)



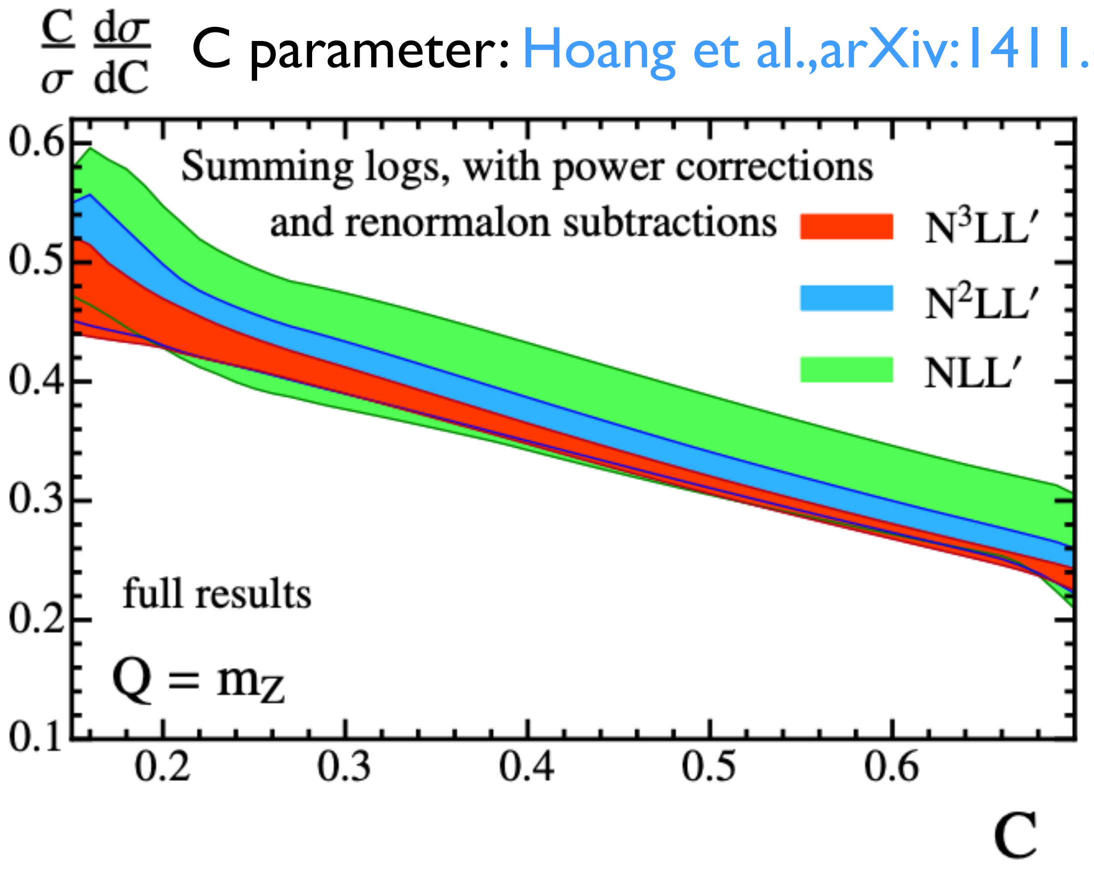


- For e^+e^-, N^{3}LL$'$ resummed event shape distributions with nonperturbative corrections:

Thrust: Abbate et al.,arXiv:1006.3080



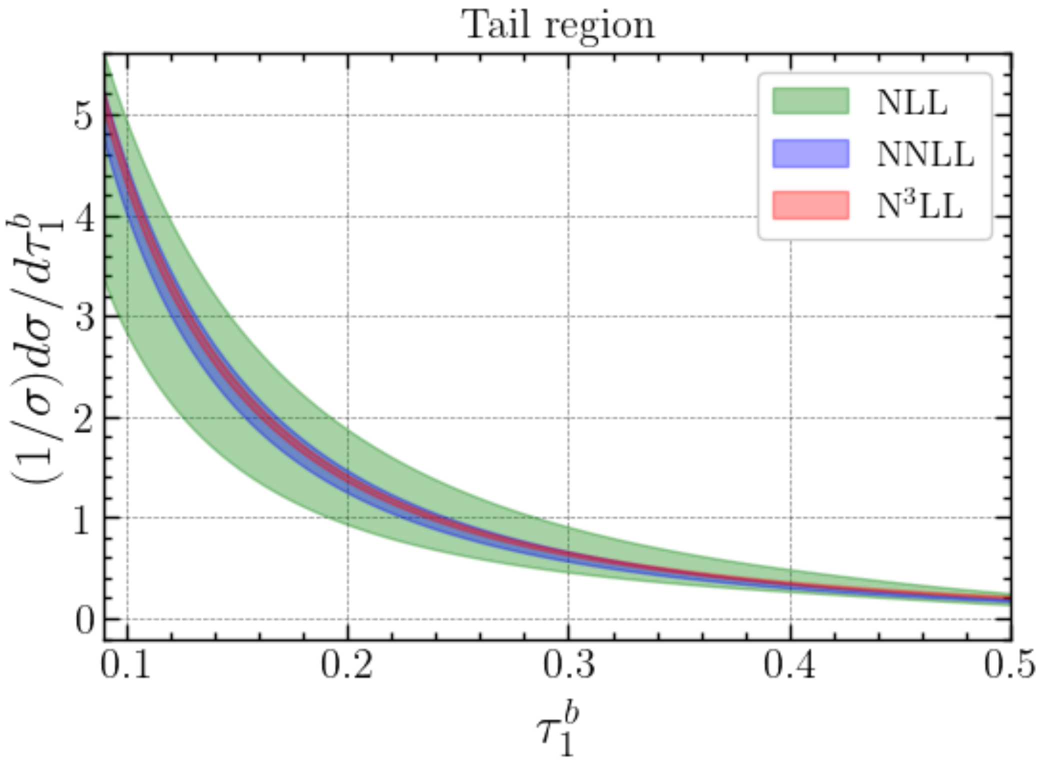
C parameter: Hoang et al.,arXiv:1411.6633



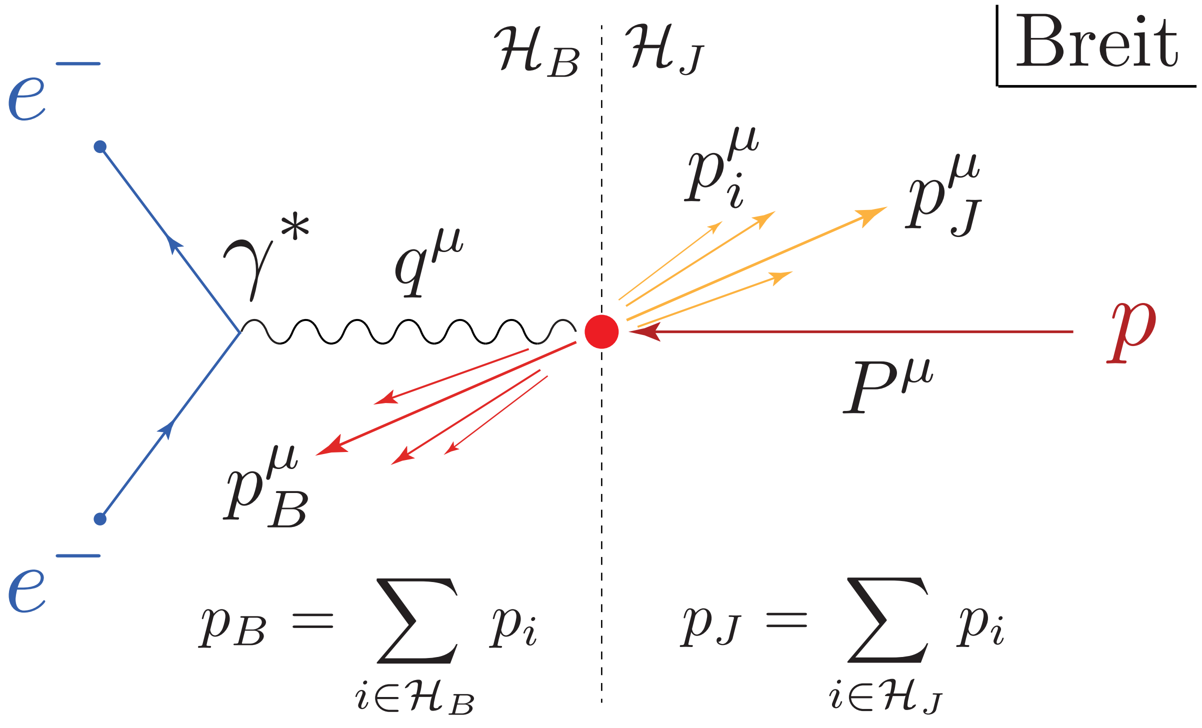
Background and Motivations

- **Objective**: Accurately describe cross sections in DIS (ep) for jet production
- **Observable**: DIS event shape τ_1^b, a special form of N-jettiness.
- **Method**: SCET-1 factorization theorem with N^{3}LL resummation, combined with 2-loop fixed-order QCD corrections
- **Result**: Cross section presented as a distribution in τ_1^b

This framework provides one of the most precise methods to determine α_s and universal nonperturbative constant Ω_1 in DIS.



Kinematics and Definitions



$$p_B = \sum_{i\in\mathcal{H}_B} p_i \qquad p_J = \sum_{i\in\mathcal{H}_J} p_i$$

The momenta in Breit frame are

$$q^\mu \xrightarrow{\text{Breit}} Q\frac{n_z^\mu - \bar{n}_z^\mu}{2} = Q(0,0,0,1)$$

$$P^\mu \xrightarrow{\text{Breit}} \frac{Q}{2}\frac{\bar{n}_z^\mu}{2} = \frac{Q}{2x}(1,0,0,-1)$$

(x is the Bjorken x)

- The general expression for **DIS 1-jettiness**: $\tau_1 = \sum_i \min\left\{\frac{q_B\cdot p_i}{Q_B}, \frac{q_J\cdot p_i}{Q_J}\right\}$

  q_B, q_J: the reference light-like vectors along beam and jet

  Q_B, Q_J: the normalization factors which control the relative importance of q_B and q_J.

- Different versions of DIS 1-jettiness are defined by the specific choice of $q_{B,J}$ and $Q_{B,J}$.

Kinematics and Definitions

- **The formal definition τ_1^b:** $Q_{B,J} = Q^2/2$ (Lorentz invariant and makes τ_1^b dimension less)

$$q_B^\mu = xP^\mu \overset{\text{Breit}}{=} \frac{Q}{2}n_z \qquad q_J^\mu = q^\mu + xP^\mu \overset{\text{Breit}}{=} \frac{Q}{2}\bar{n}_z$$

- Then, $\tau_1^b = \frac{2}{Q^2}\sum_{i\in X}\min\left\{\underset{\underset{\mathscr{H}_B}{\downarrow}}{q_B^b\cdot p_i},\ \underset{\underset{\mathscr{H}_J}{\downarrow}}{q_J^b\cdot p_i}\right\} \overset{\text{Breit}}{=} \frac{1}{Q}\sum_{i\in X}\min\left\{n_z\cdot p_i, \underset{\downarrow}{\bar{n}_z}\cdot p_i\right\}$

The sign of z component of p_i determines $p_i \in \mathscr{H}_{B/J}$

- τ_1^b agrees with the classical DIS thrust τ_Q:

$$\tau_1^b \overset{\overset{\text{Energy-momentum conservation}}{\downarrow}}{\overset{\text{Breit}}{=}} 1 - \frac{2}{Q}\sum_{i\in\mathscr{H}_J}(p_i)_z = \overset{\overset{\text{classical DIS thrust variable } \tau_Q}{\uparrow}}{\tau_Q}$$

arXiv:hep-ph/9912488
Antonelli, Dasgupta, Salam

- Reduces contamination from remnant fragmentation in its measurements, making it highly **desirable for experimental studies.**

- Lorentz invariant, and global observable, so free of NGL
→ **Can be computed with high theoretical accuracy.**

Other DIS 1-jettiness work

- *The DIS 1-jettiness Event Shape at N³LL +* $\mathcal{O}(\alpha_s^2)$

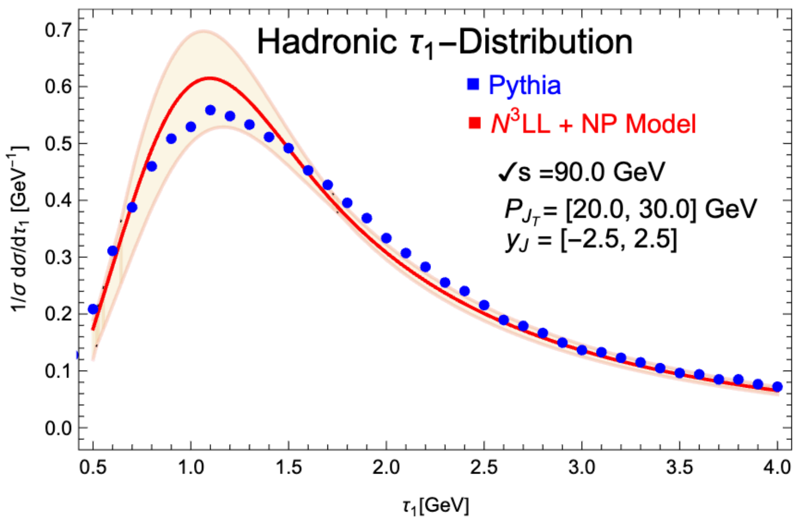
arXiv:2401.01941
Cao, Z. Kang, Liu, Mantry

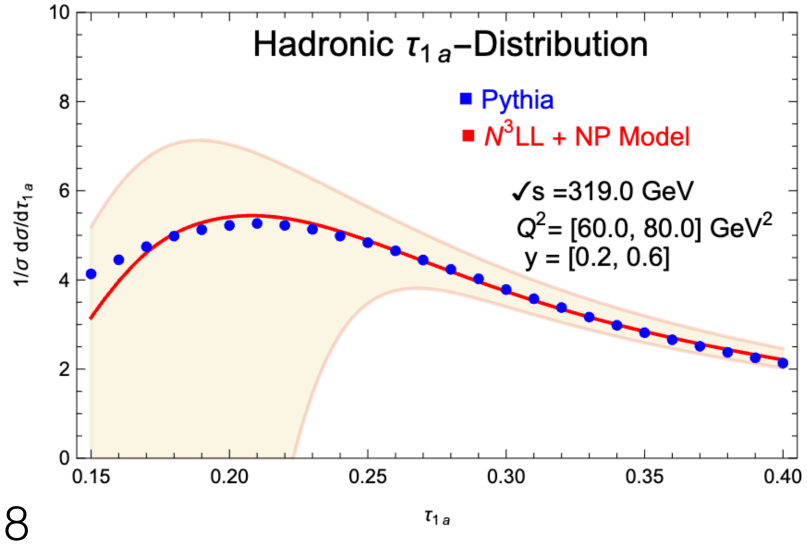
  - Same theoretical accuracy, but different definition of DIS 1-jettiness

1)
$$\tau_1 = \sum_k \min\left\{\frac{2q_B\cdot p_k}{Q_B}, \frac{2q_J\cdot p_k}{Q_J}\right\}$$
$$q_B = xP, \qquad Q_B = x\sqrt{s}.$$
$$Q_J = 2K_{J_T}\cosh y_K, \qquad q_J = (K_{J_T}\cosh y_K, \vec{K}_{J_T}, K_{J_T}\sinh y_K).$$

2)
$$\tau_{1a} = \sum_k \min\left\{\frac{2q_B\cdot p_k}{Q^2}, \frac{2q_J\cdot p_k}{Q^2}\right\}.$$

  - The jet axis is aligned with the jet momentum
 → $\mathbf{p}_\perp^2$ dependence in the beam function to be integrated out, reducing it to an ordinary beam function





Formalism

Formalism

- In this work, we compute the τ_1^b distribution as follows:

$$\sigma(\tau_1^b) = \int dk \left[\sigma_{\mathsf{PT}}^{\mathsf{s}} + \sigma_{\mathsf{PT}}^{\mathsf{ns}}\right]\left(\tau_1^b - \frac{k}{Q}\right)\left[e^{-2\delta(R,\mu_S)(d/dk)} F\left(k - 2\Delta(R,\mu_S)\right)\right]$$

- $\sigma_{\mathsf{PT}}^{\mathsf{s}}$: Singular contribution (Leading Power in SCET)

  Represents two-jet events, combined with all-order log resummation at N^{3}LL level

- $\sigma_{\mathsf{PT}}^{\mathsf{ns}}$: Nonsingular contribution (Power Suppressions)

  Represents multi-jet events, estimated using full-QCD fixed-order up to $\mathcal{O}(\alpha_s^2)$

- $e^{-2\delta(R,\mu_S)(d/dk)} F\left(k - 2\Delta(R,\mu_S)\right)$: Nonperturbative hadronization corrections

  Incorporates the nonperturbative shape function F, and employs R-gap scheme to subtract $\mathcal{O}(\Lambda_{\mathsf{QCD}})$ renormalon ambiguity.

$\sigma^{\mathrm{S}}_{\mathrm{PT}}$: Singular contribution

- The SCET factorization formula for τ_1^b distribution is given by

arXiv:1303.6952
Kang, Lee, Stewart

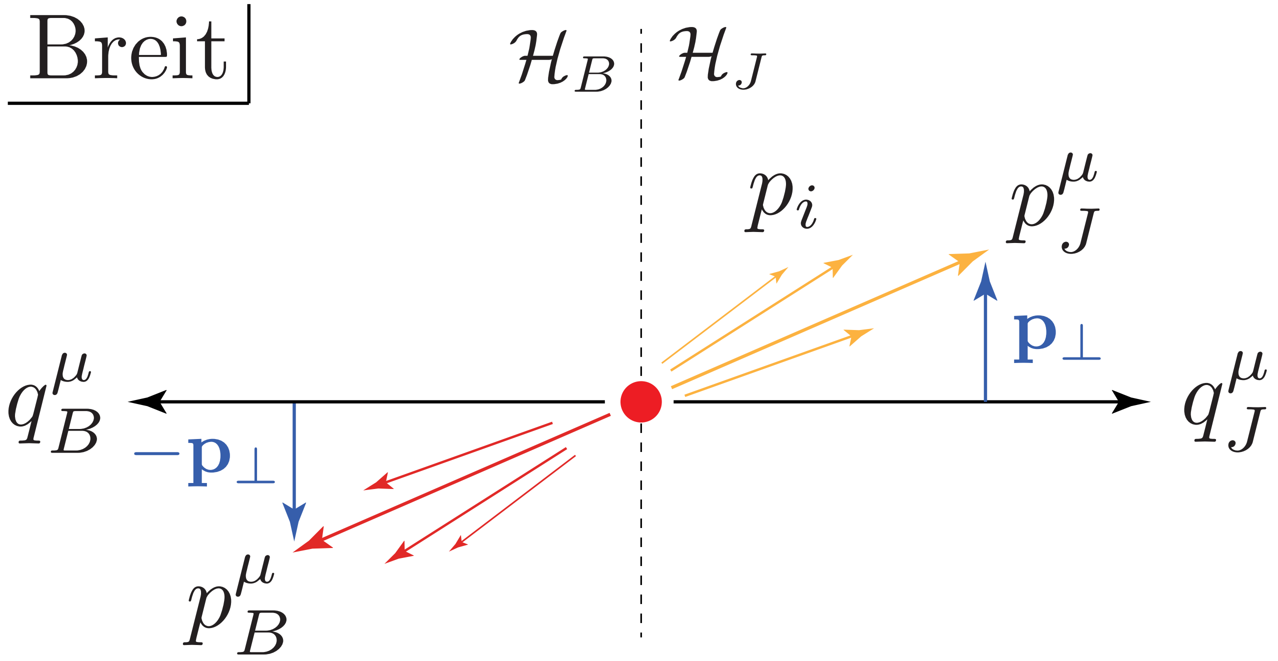
$$\frac{d\sigma}{dx\,dQ^2\,d\tau_1^b} = \frac{d\sigma_0^b}{dx\,dQ^2}\int dt_J\,dt_B\,dk_S\,\overbrace{\delta\left(\tau_1^b - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\right)}^{\text{Measurement function for } \tau_1^b}\overbrace{S(k_S,\mu)}^{\text{Single variable soft function}}$$

$$\times \int d^2\mathbf{p}_\perp \underbrace{J_q(t_J - \mathbf{p}_\perp^2,\mu)}_{\text{Quark jet function}}\Big[\underbrace{H_q^b(y,Q^2,\mu)}_{\text{Hard function}}\underbrace{\mathscr{B}_q(t_B,x,\mathbf{p}_\perp^2,\mu)}_{\text{Quark beam function}} + (q\to\bar{q})\Big],$$

where Born-level cross section $\frac{d\sigma_0^b}{dx\,dQ^2} = \frac{2\pi\alpha_{\mathrm{em}}^2}{Q^4}\left[(1-y)^2+1\right]$ (Note that $Q^2 = sxy$)

- With $t_J \to t_J + \mathbf{p}_\perp^2$, and $t_B \to t_B - \mathbf{p}_\perp^2$, we can confine the $\mathbf{p}_\perp^2$ integration to the beam function only:

$$\hat{B}_q(t_B,x,\mu) = \int d^2\mathbf{p}_\perp \mathscr{B}_q(t_B - \mathbf{p}_\perp^2, x, \mathbf{p}_\perp^2,\mu)$$



$\sigma^{\text{ns}}_{\text{PT}}$: Nonsingular contribution

- Nonsingular contributions from fixed-order full QCD calculations:

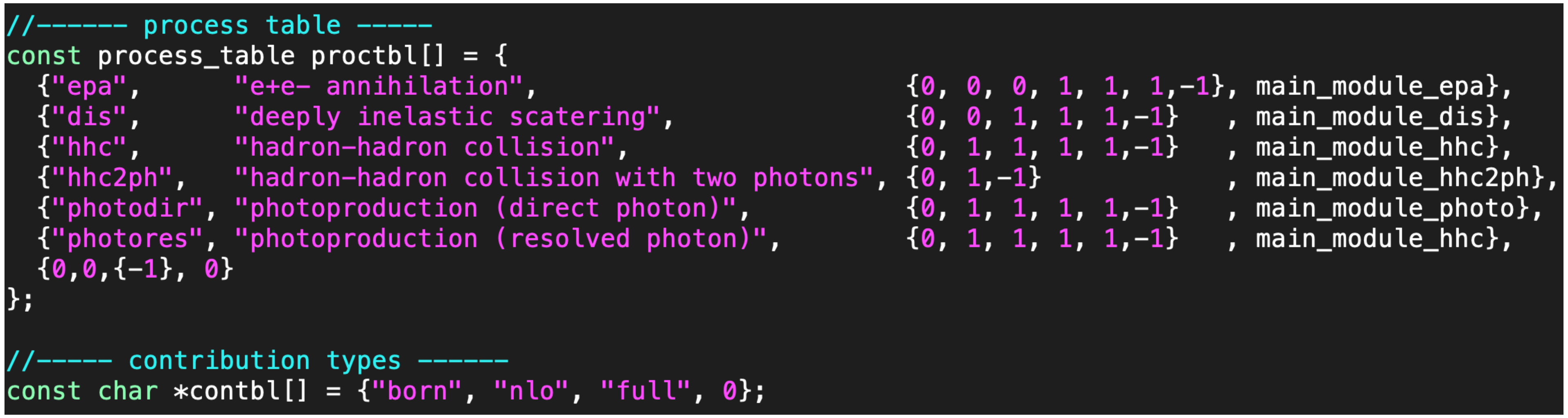
$$\frac{d\sigma_{\text{ns}}}{d\tau_1^b} = \frac{d\sigma_{\text{QCD}}}{d\tau_1^b} - \frac{d\sigma_{\text{s}}}{d\tau_1^b}$$

LO nonsingular:
arXiv:1407.6706
Kang, Lee, Stewart

- NLOJet++ is the C++ program for calculating LO and NLO QCD jet cross sections based on Catani-Seymour dipole subtraction method. (Author: Zoltan Nagy at DESY)

arXiv:hep-ph/9605323
Catani and Seymour

arXiv:hep-ph/0307268
Nagy



```
//------ process table -----
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  {"dis",      "deeply inelastic scatering",               {0, 0, 1, 1, 1,-1}    , main_module_dis},
  {"hhc",      "hadron-hadron collision",                  {0, 1, 1, 1, 1,-1}    , main_module_hhc},
  {"hhc2ph",   "hadron-hadron collision with two photons", {0, 1,-1}             , main_module_hhc2ph},
  {"photodir", "photoproduction (direct photon)",          {0, 1, 1, 1, 1,-1}    , main_module_photo},
  {"photores", "photoproduction (resolved photon)",        {0, 1, 1, 1, 1,-1}    , main_module_hhc},
  {0,0,{-1}, 0}
};

//----- contribution types ------
const char *contbl[] = {"born", "nlo", "full", 0};
```
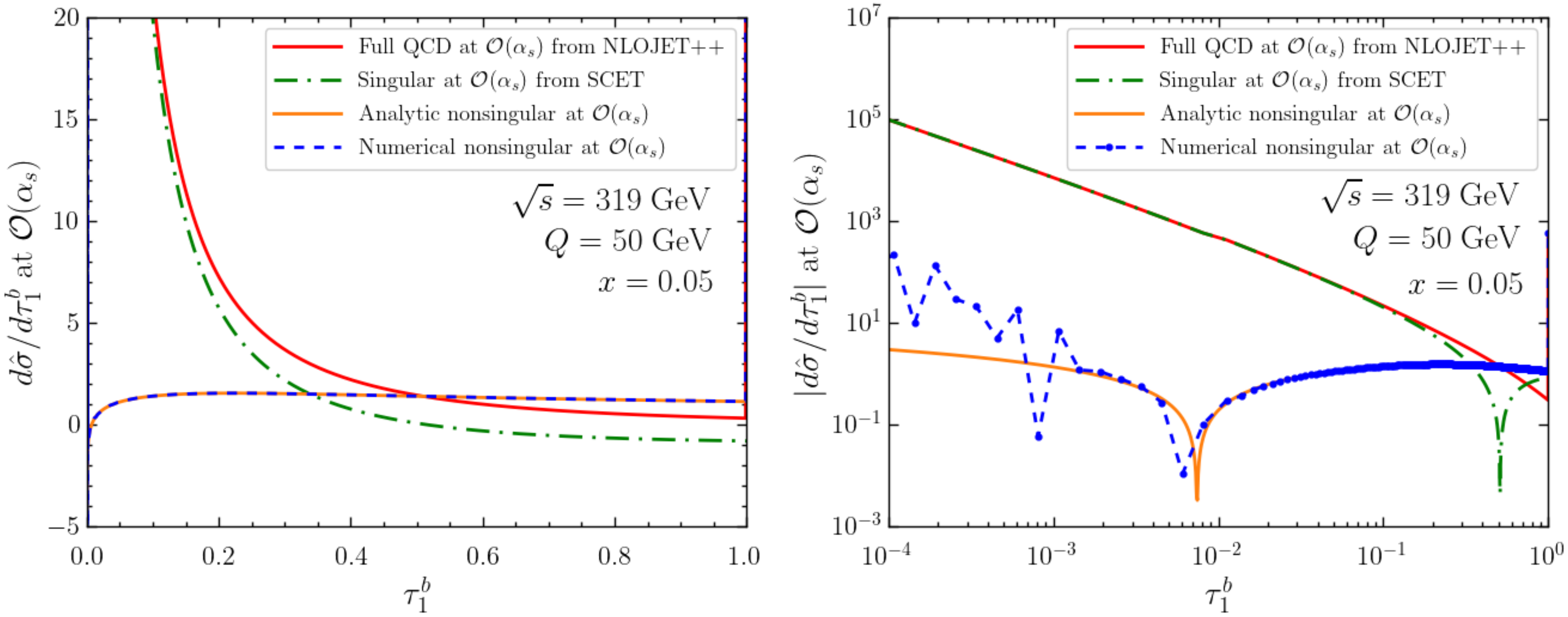
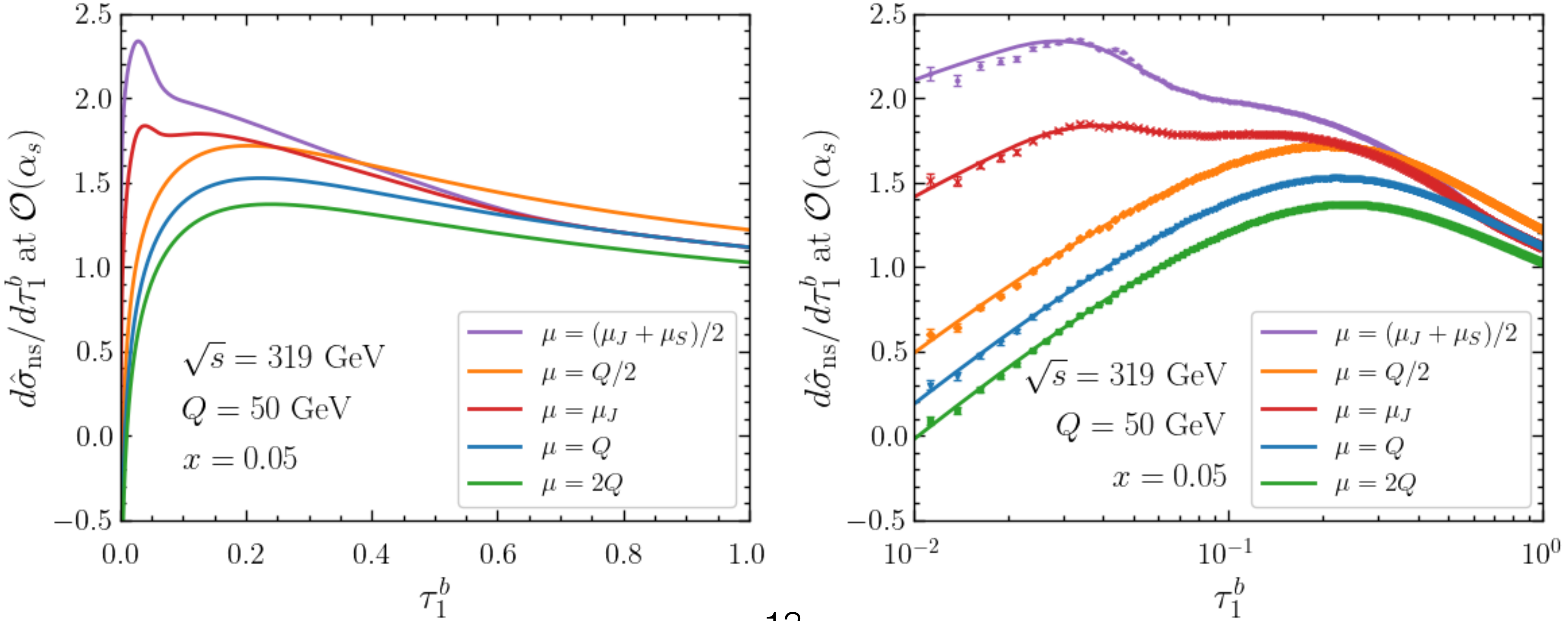

- e^+e^-, ep, pp and photo production processes.

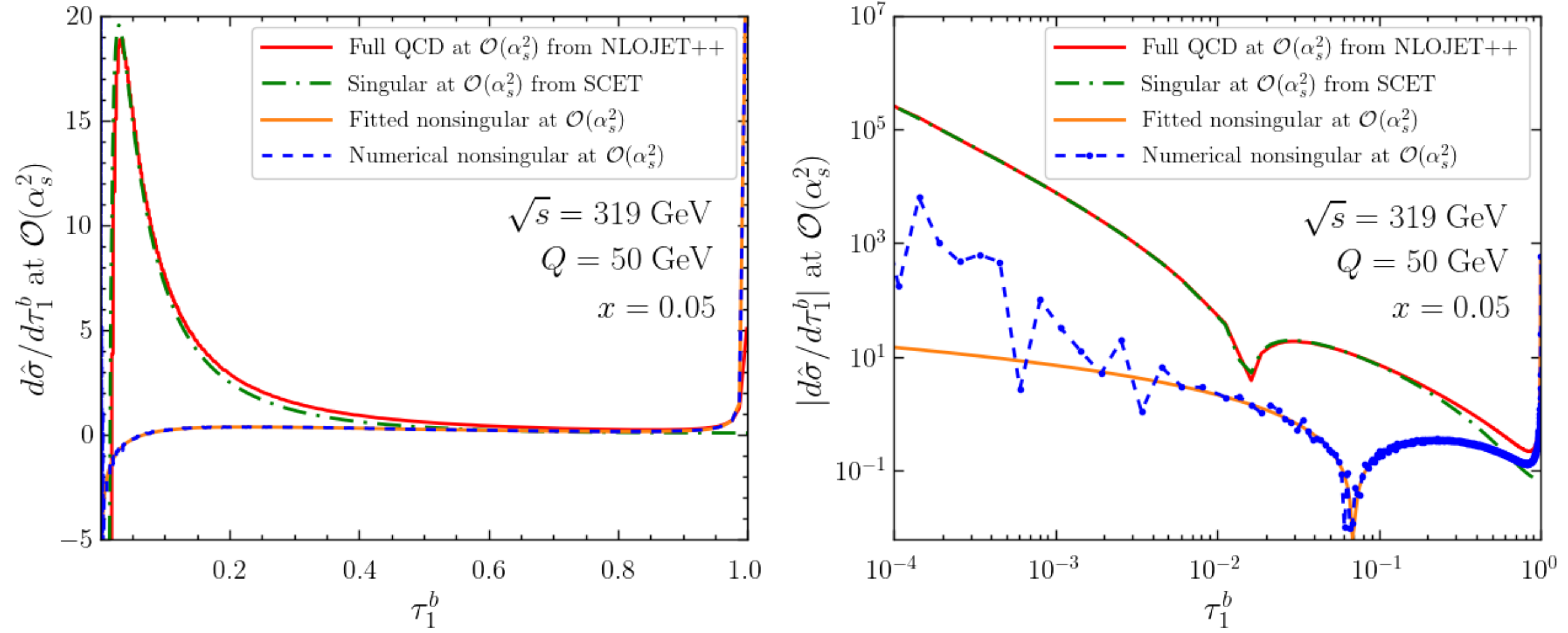
$\sigma^{\mathrm{ns}}_{\mathrm{PT}}$: Nonsingular at LO

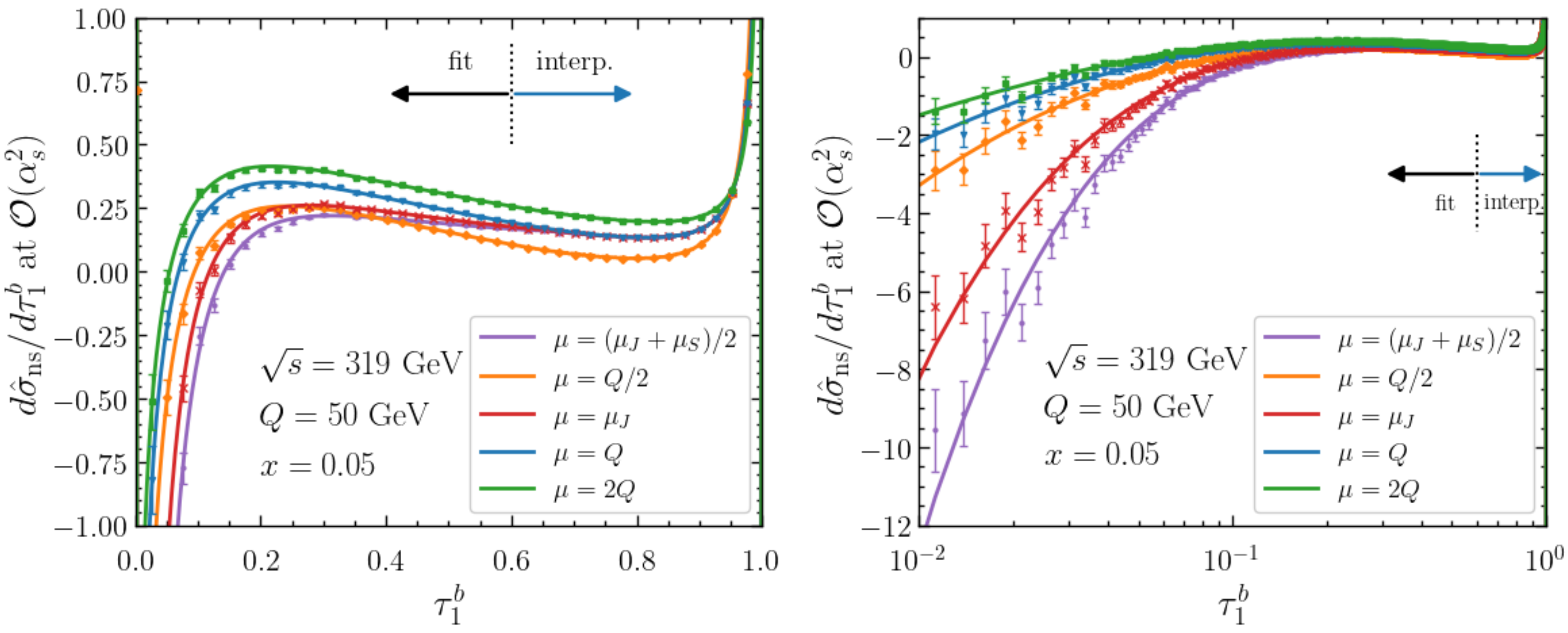
Analytic 1-loop nonsingular
arXiv:1407.6706
Kang, Lee, Stewart





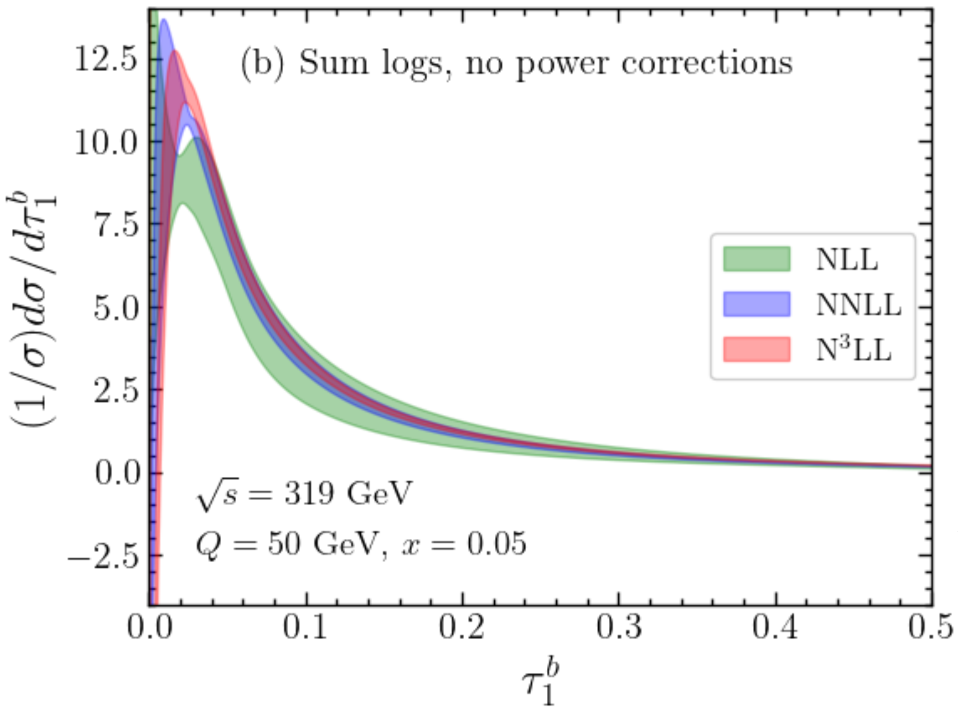
$\sigma_{\mathrm{PT}}^{\mathrm{ns}}$: Nonsingular at NLO





NP corr.: Shape function

$$S(k,\mu) = \int dk'\, S_{\text{pert}}(k-k',\mu) F(k') \quad \longrightarrow \quad \frac{d\sigma}{d\tau_1^b}(\tau_1^b) = \int dk \frac{d\sigma_{\text{pert}}}{d\tau_1^b}\left(\tau_1^b - \frac{k}{Q}\right) F(k)$$

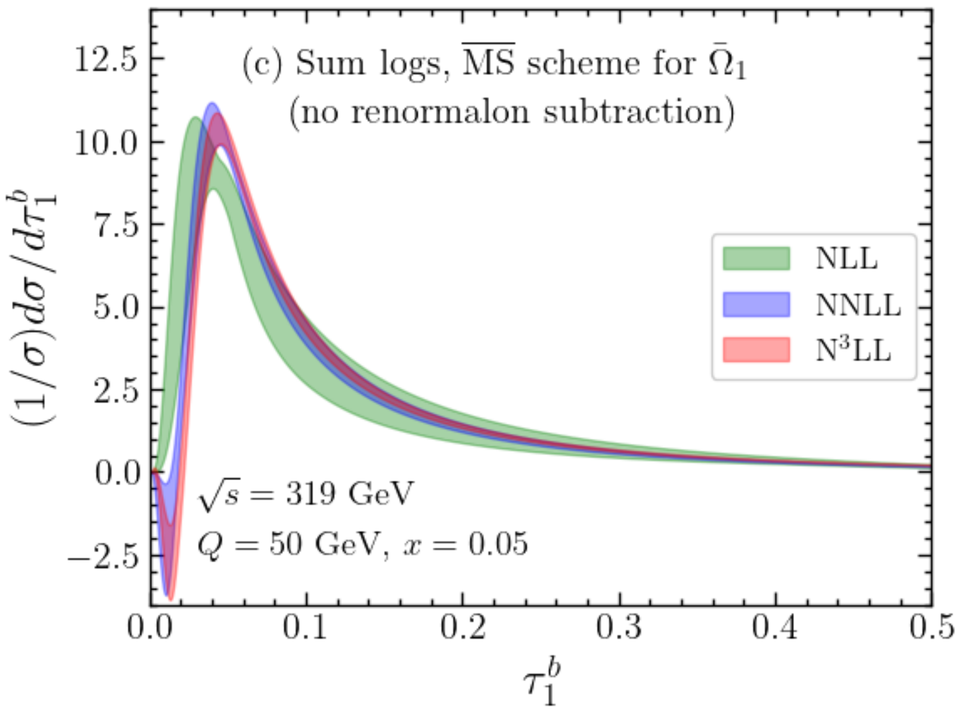


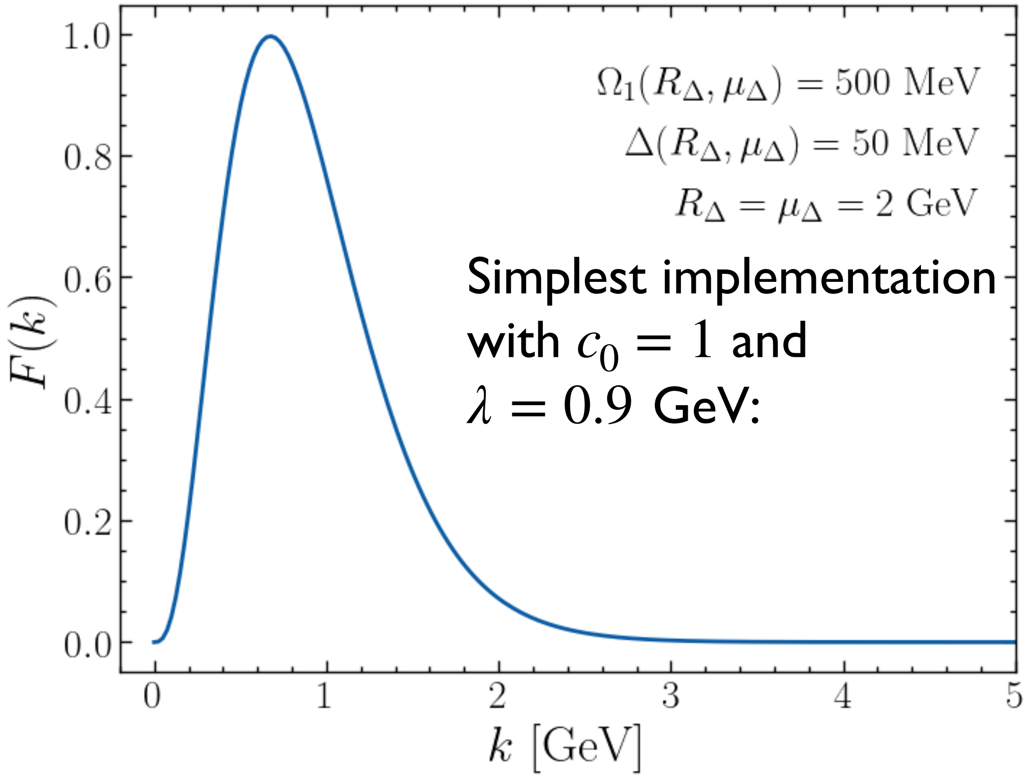
- According to OPE,

$$\frac{d\sigma}{d\tau_1^b}(\tau_1^b) = \left\{ \frac{d\sigma_{\text{pert}}(\tau_1^b)}{d\tau_1^b} - \frac{2\Omega_1}{Q}\frac{d\sigma^2_{\text{pert}}(\tau_1^b)}{d\tau_1^{b2}} \right\} \left[1 + \mathcal{O}(\Lambda_{\text{QCD}}/(\tau_1^b Q))\right]$$

$2\Omega_1$ is the 1st moment of $F(k)$: $\quad 2\Omega_1 = \int dk\, kF(k)$

In the tail region,
$\tau_1^b \to \tau_1^b - 2\Omega_1/Q$
(translation!)

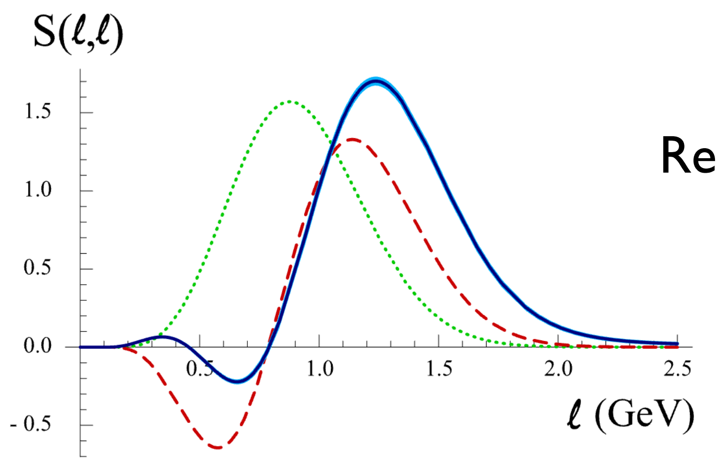




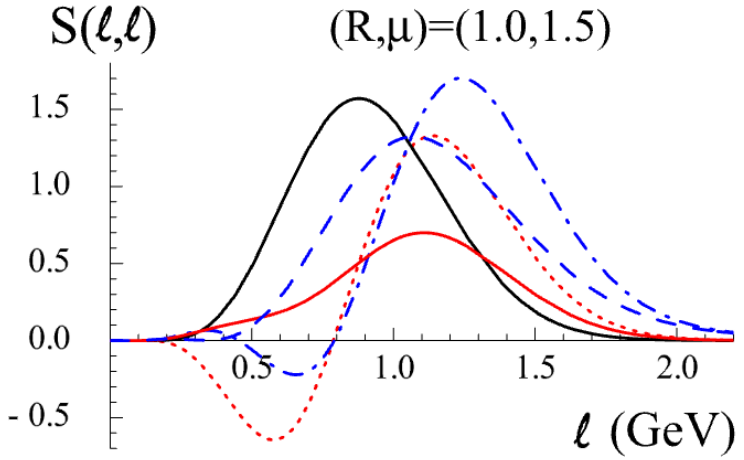
Simplest implementation with $c_0 = 1$ and $\lambda = 0.9$ GeV:

NP corr.: Renormalon ambiguity

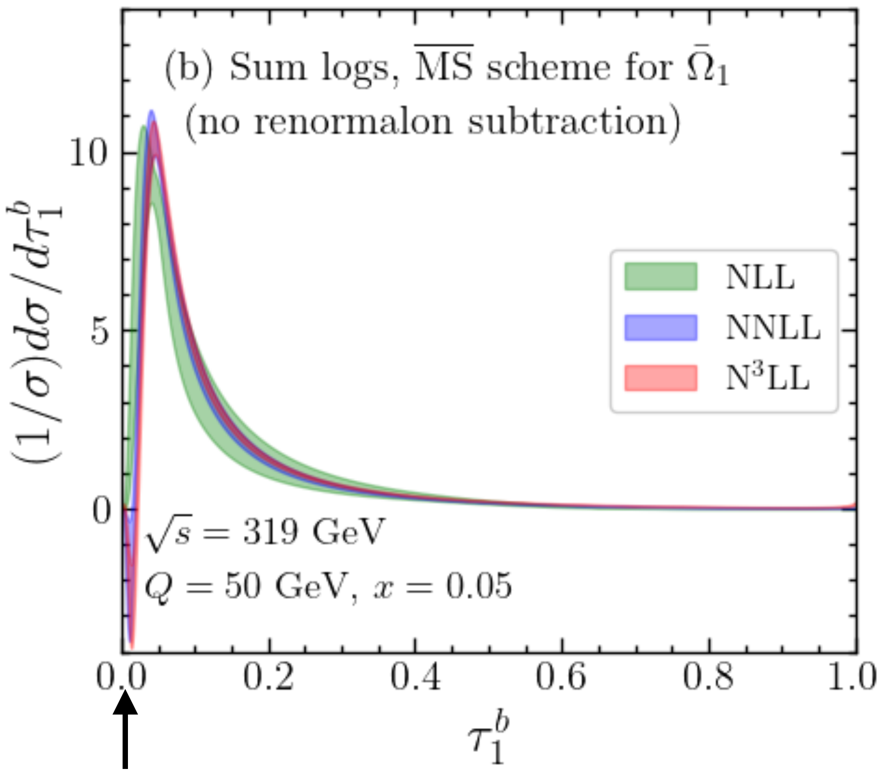
- We employ the R-gap scheme introduced in [arXiv:0806.3852, Hoang, Kluth].



Renormalon subtraction $\longrightarrow$

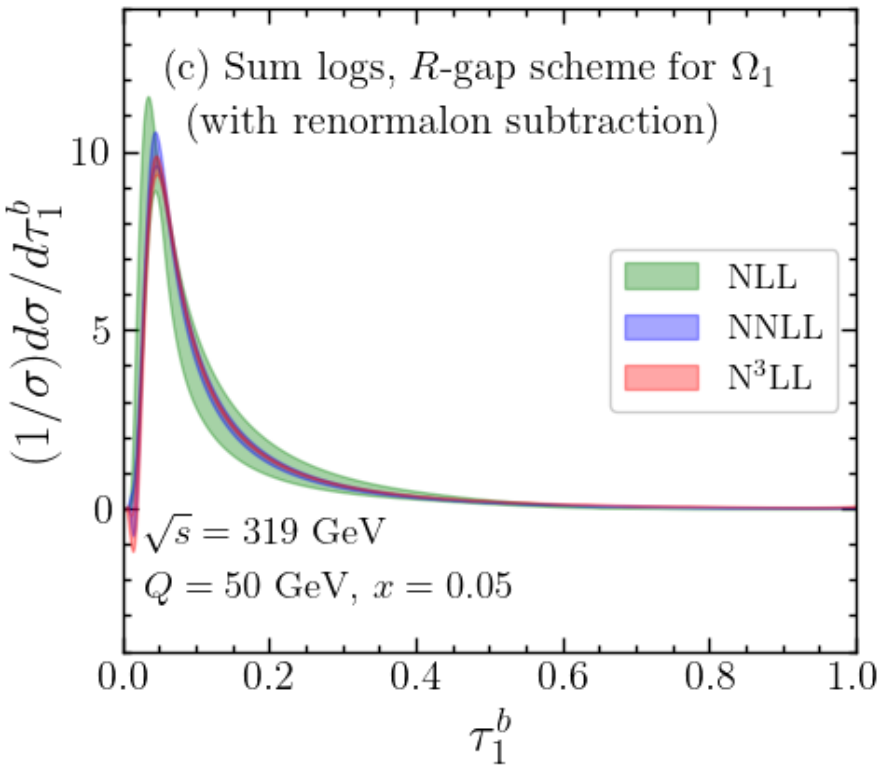


Before renormalon subtraction
(shape function only)



$\mathcal{O}(\Lambda_{\mathrm{QCD}})$ renormalon ambiguity

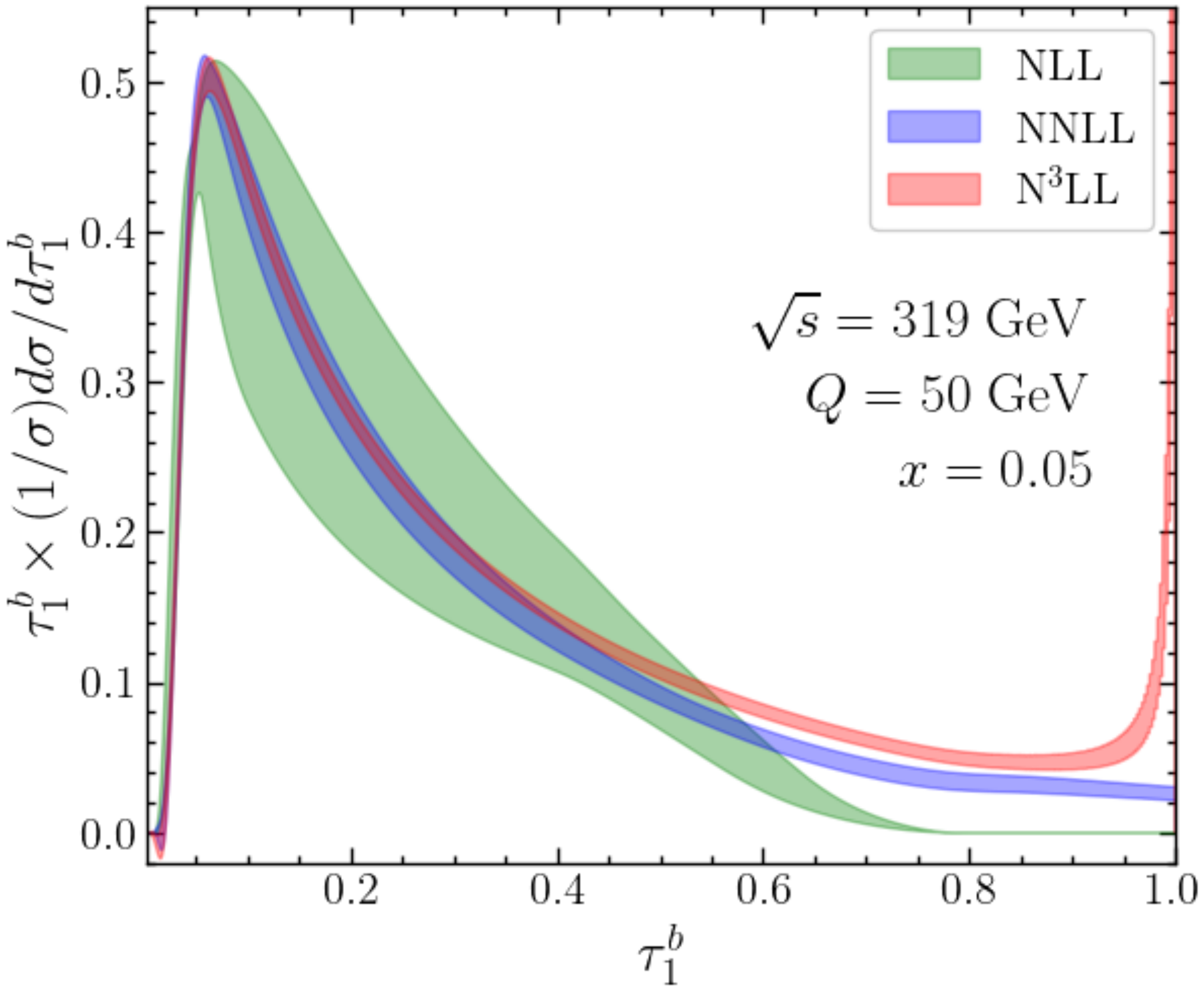
After renormalon subtraction
(R-gap scheme)



$$\left[e^{-2\delta(R,\mu_S)(d/dk)} F\left(k - 2\Delta(R,\mu_S)\right)\right]$$

Results and comparison with HERA data

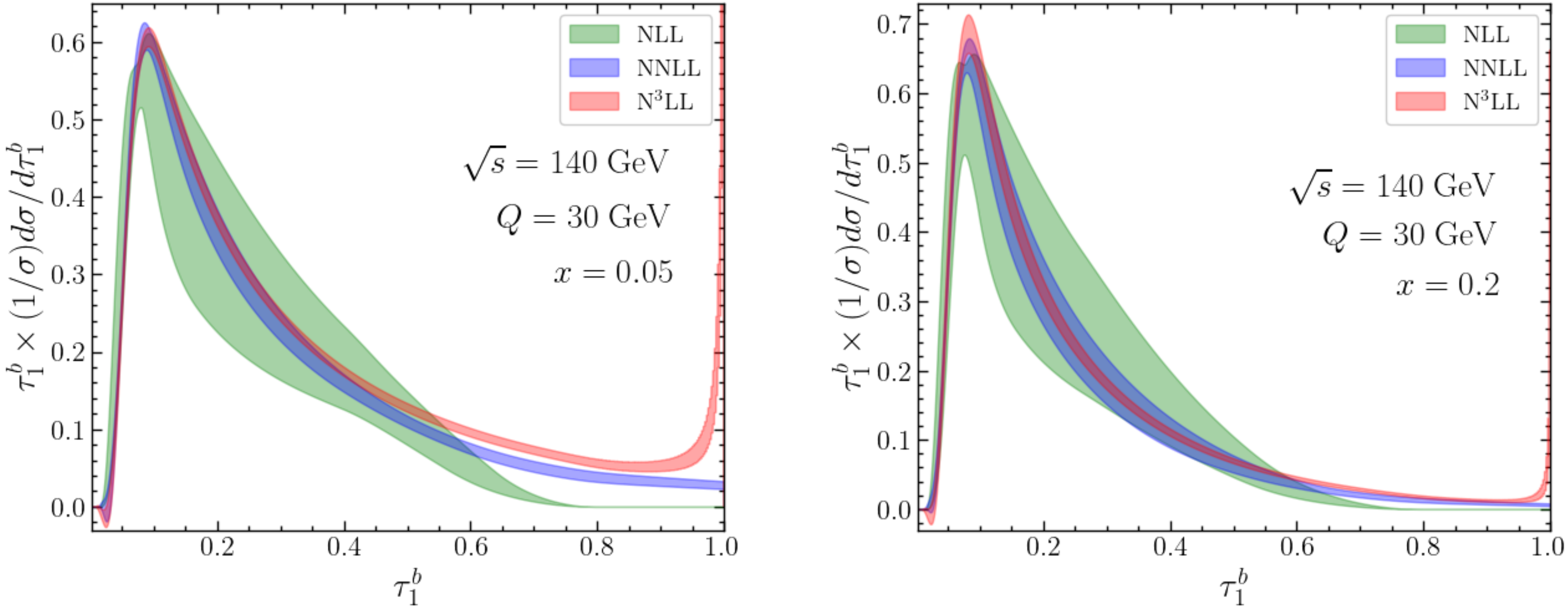
Final N³LL + $\mathcal{O}(\alpha_s^2)$ prediction



- Relevant to HERA setup
- Good perturbative convergence of the distributions, especially in the tail region.
- Can observe a peak as $\tau_1^b \to 1$, which characterizes the events with nearly empty jet hemisphere.

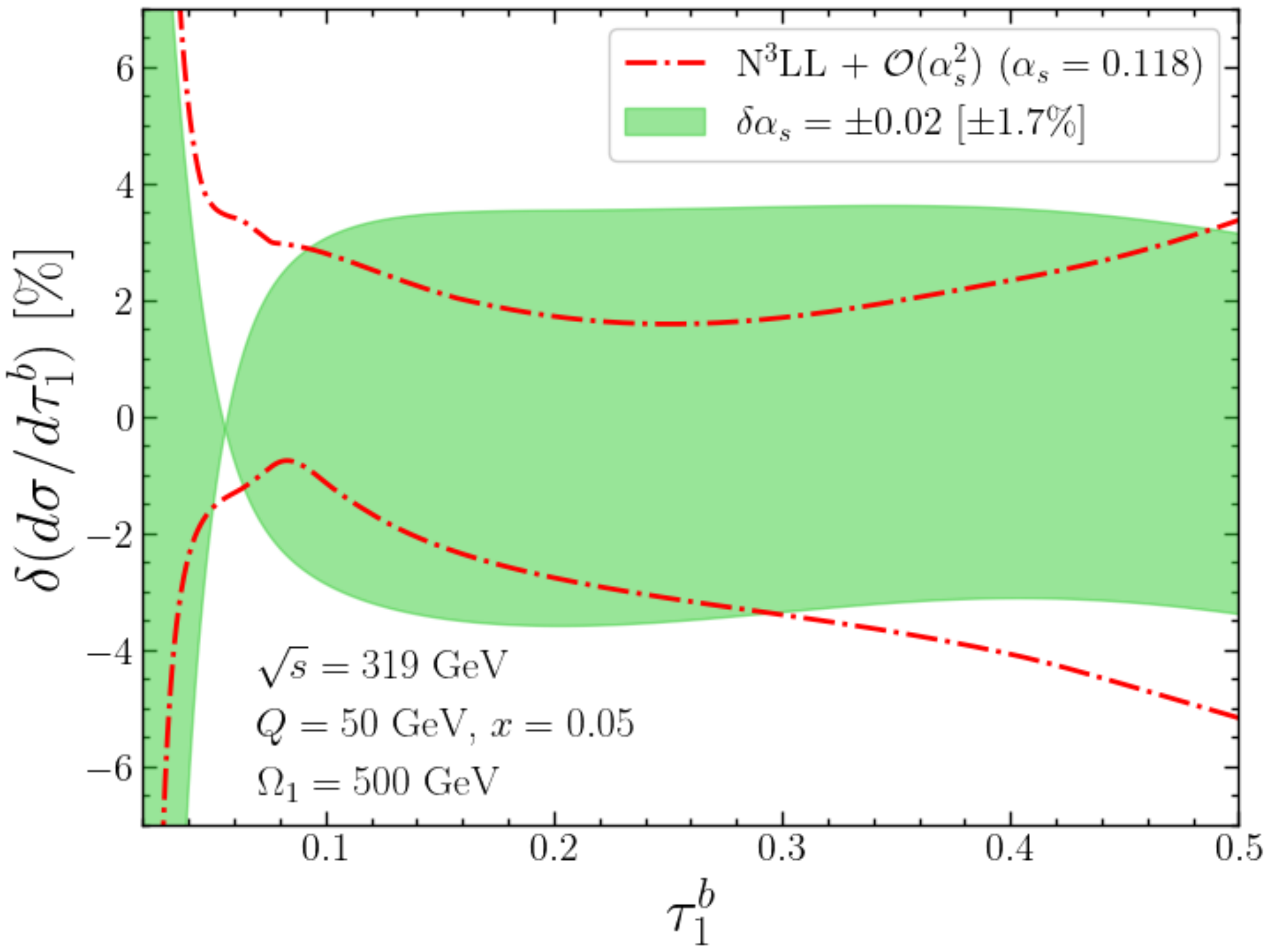
$$\tau_1^b \overset{\text{Breit}}{=} 1 - \frac{2}{Q} \sum_{i \in \mathscr{H}_J} (p_i)_z = \tau_Q$$

Final N³LL + $\mathcal{O}(\alpha_s^2)$ prediction

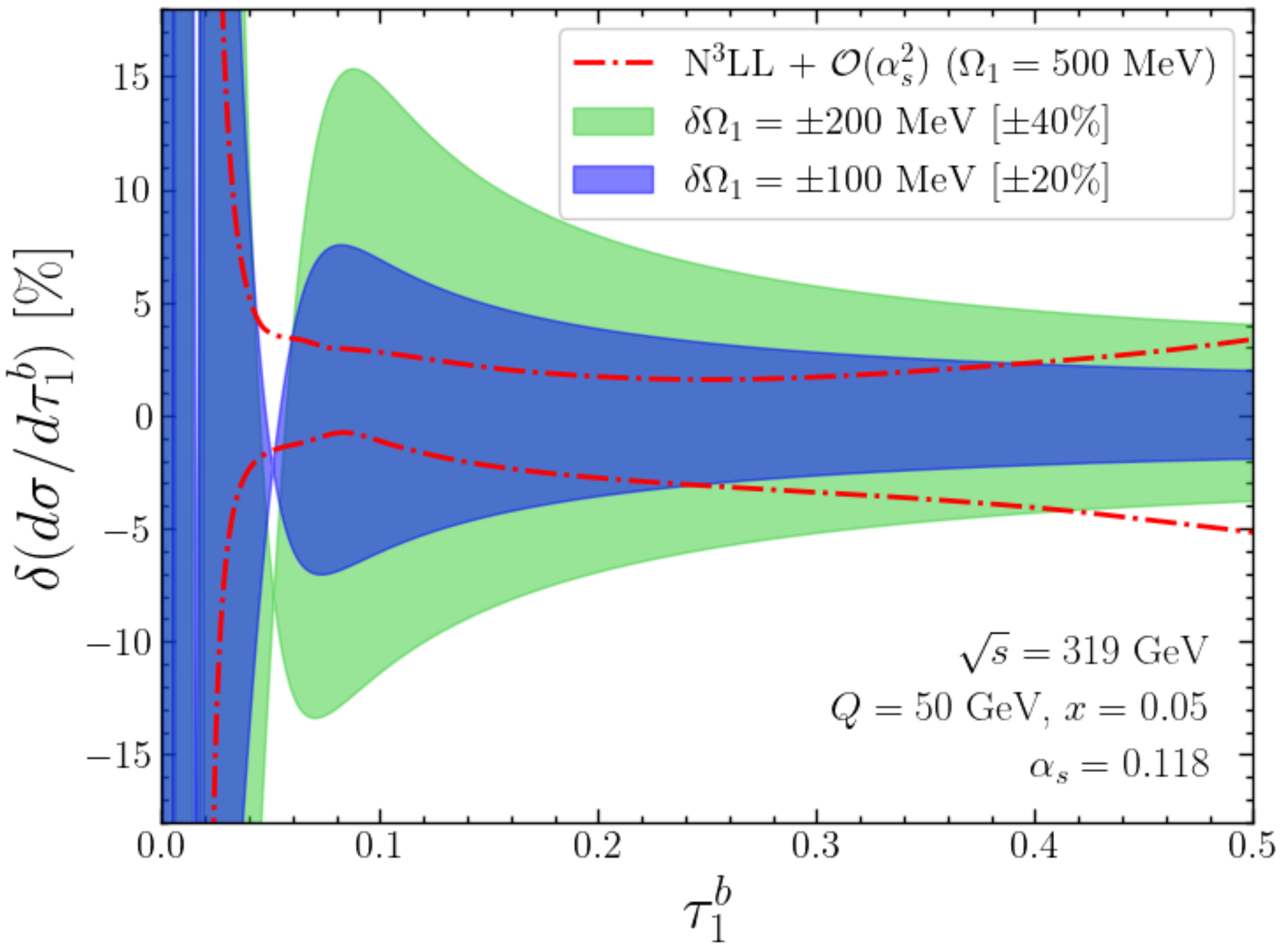


- Relevant to EIC setup
- Good perturbative convergence
- Can observe the peak as $\tau_1^b \to 1$, and this feature is more pronounced at smaller x.

α_s and Ω_1 sensitivities



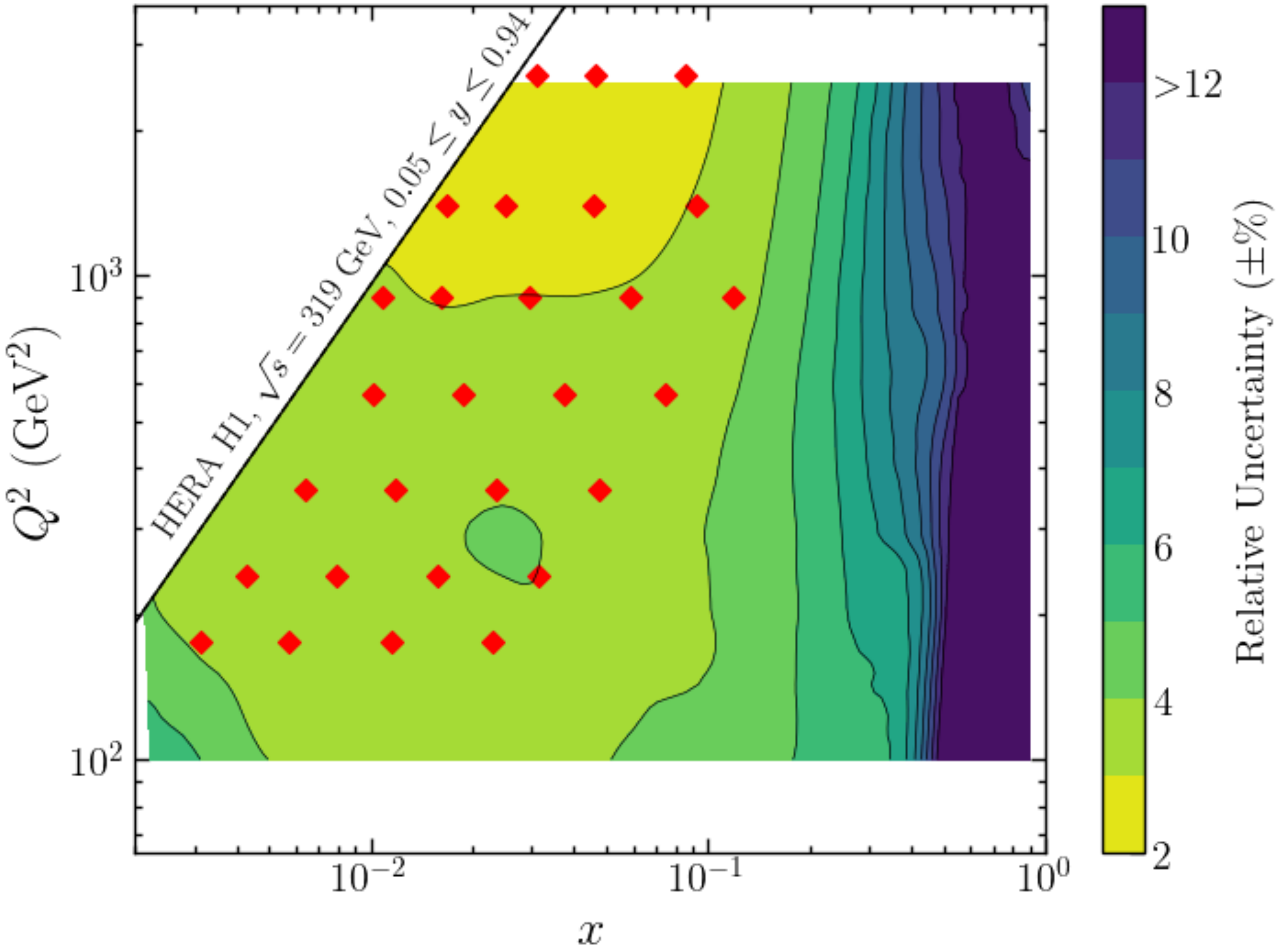
Requires uncertainties below 4% for $\delta\alpha_s = \pm 0.02$



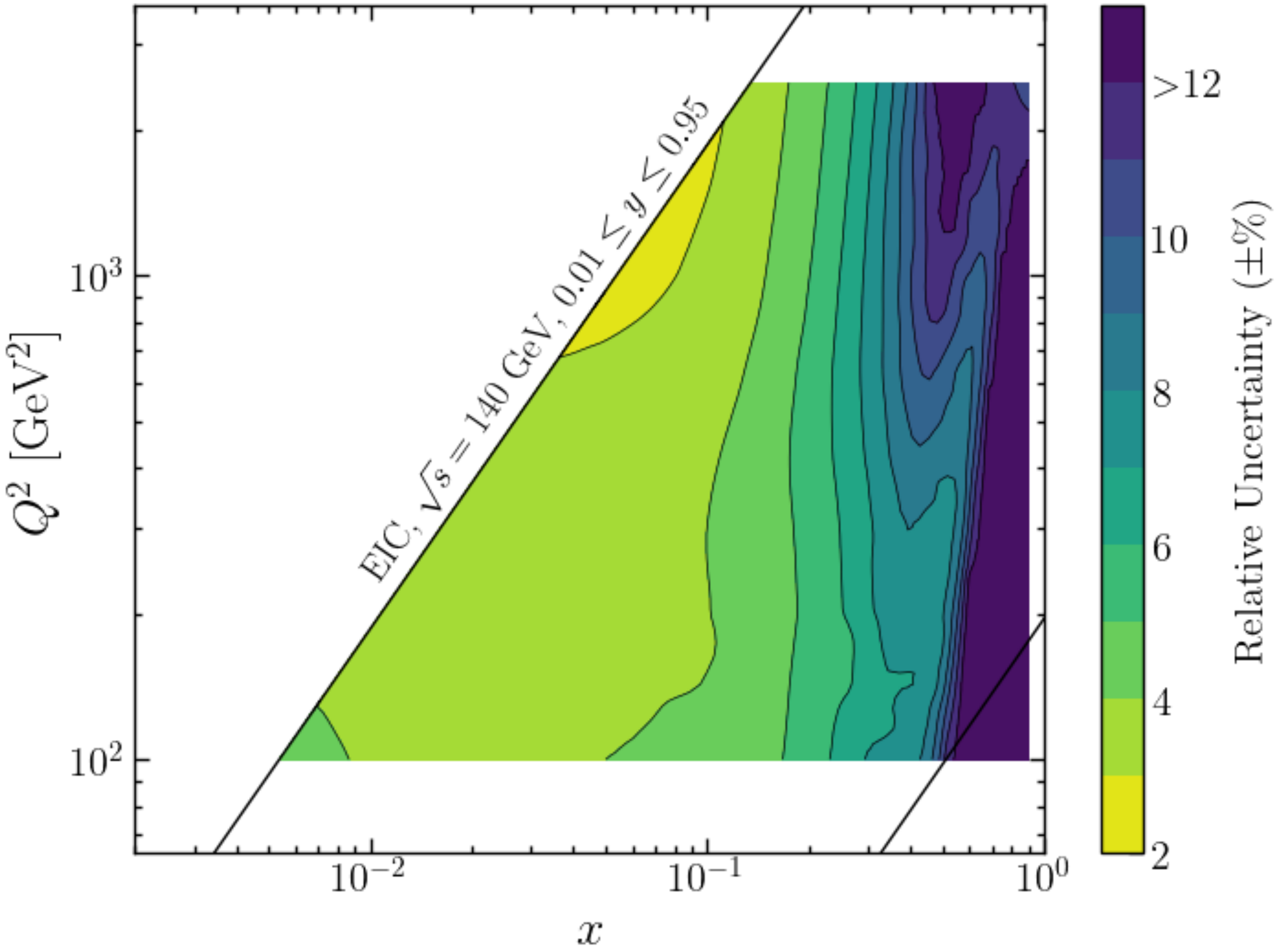
Requires uncertainties below 5% for $\delta\Omega_1 = \pm 100$ MeV

α_s and Ω_1 sensitivities

HERA ($\sqrt{s} = 319$ GeV)



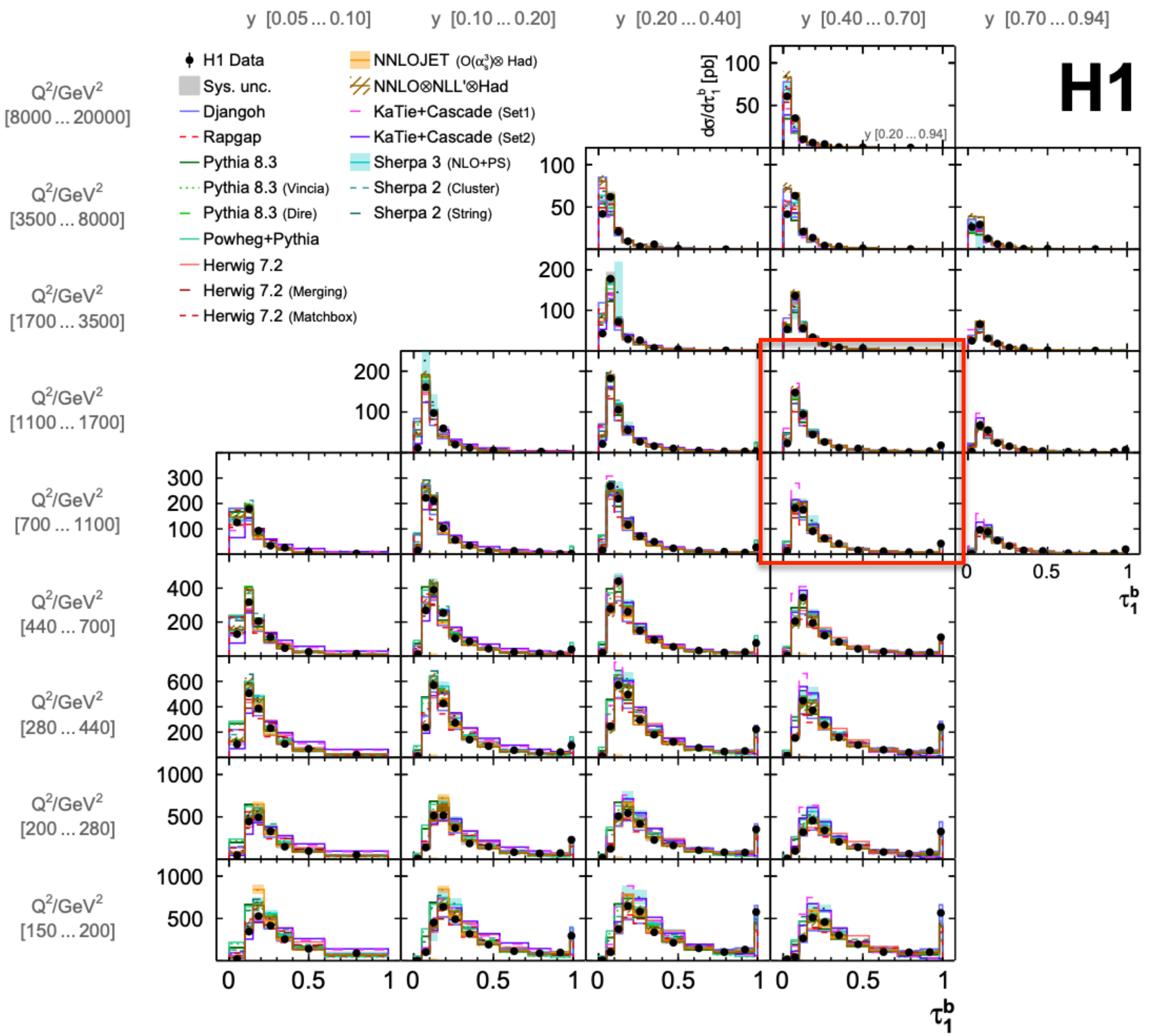
EIC ($\sqrt{s} = 140$ GeV)



- Our predictions exhibit uncertainties below 4% across large range of x and Q.

HERA H1 measurement

- Recently, the H1 collaboration reported the measurement of τ_1^b in DIS based on the data sample collected in 2003-2007 ($\sqrt{s} = 319$ GeV, integrated luminosity of $\mathscr{L} = 351.1$ pb^{-1}.

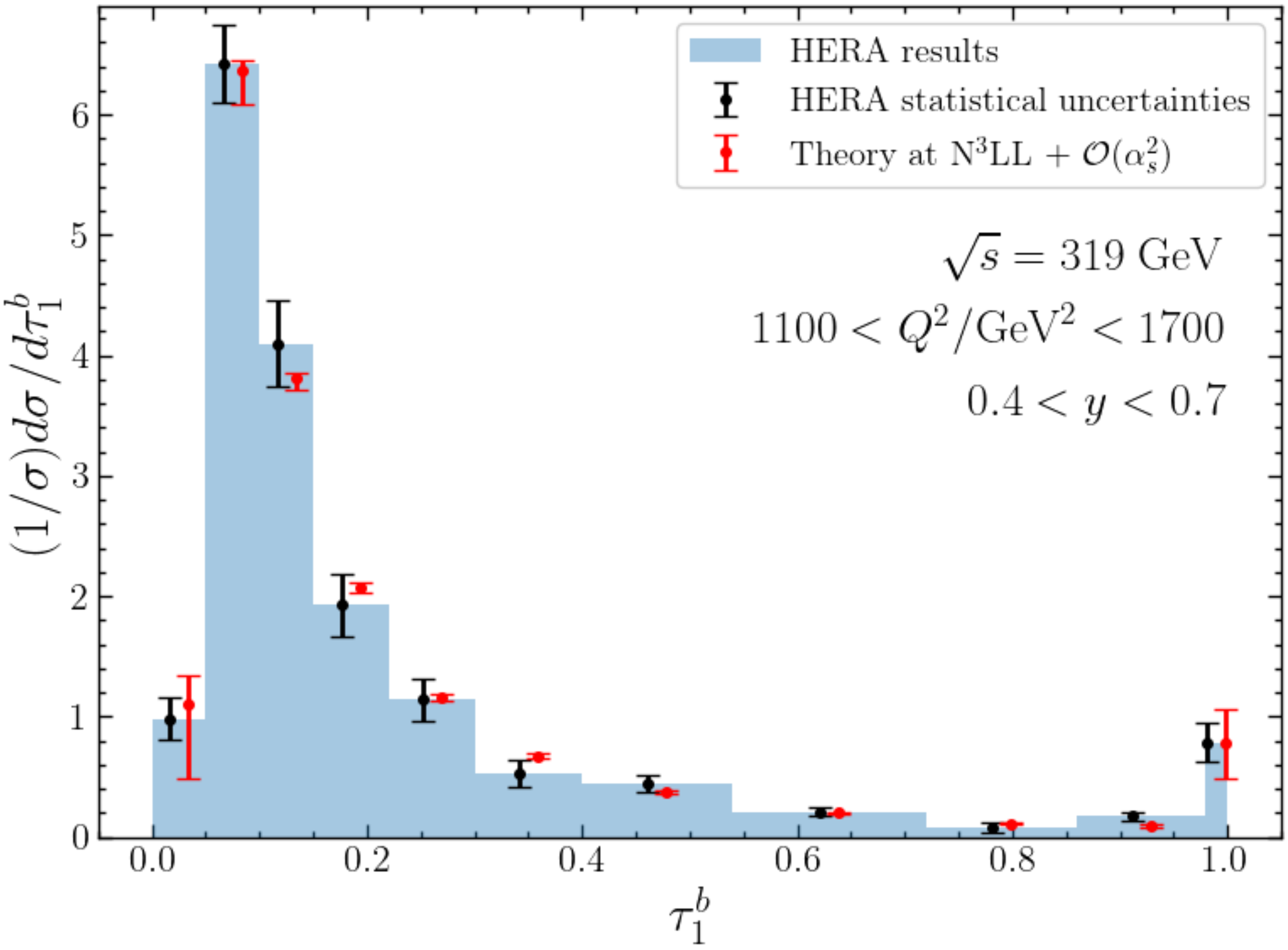


arXiv:2403.10109
H1 Collaboration

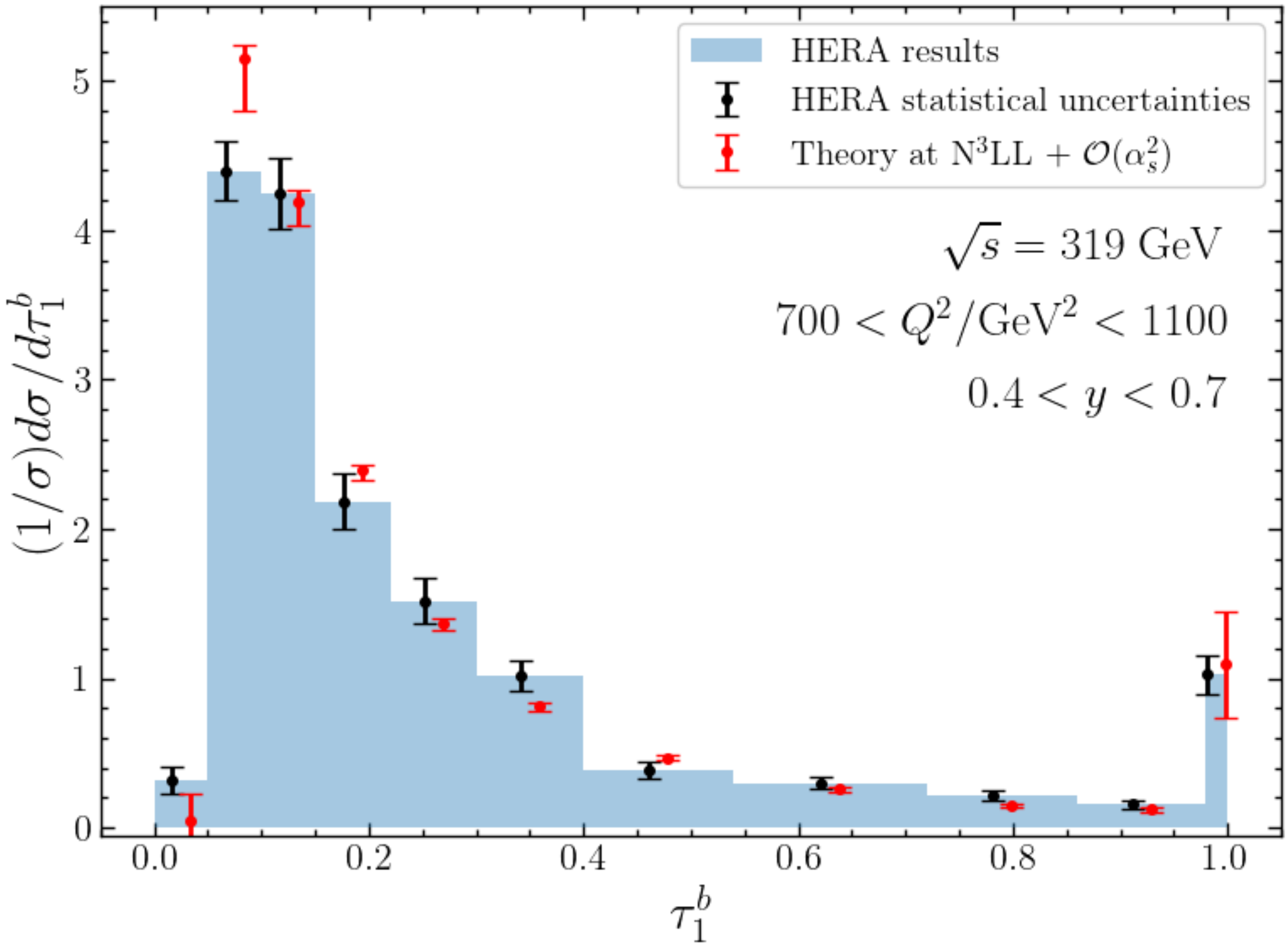
- The distribution in τ_1^b given by
$$\int_{\Delta y} dy \int_{\Delta Q^2} dQ^2 \frac{d\sigma}{dy dQ^2 d\tau_1^b}$$

- We can compare our theory predictions with these measurements (red box).

HERA H1 measurement



HERA H1 measurement



Summary

- τ_1^b is an DIS event shape which has many advantages in experimental measurements, and as a global observable, can be computed with high precision.

- Computed the τ_1^b distributions at N3LL + $\mathcal{O}(\alpha_s^2)$ accuracy, and included power corrections and renormalon subtractions for NP soft physics.

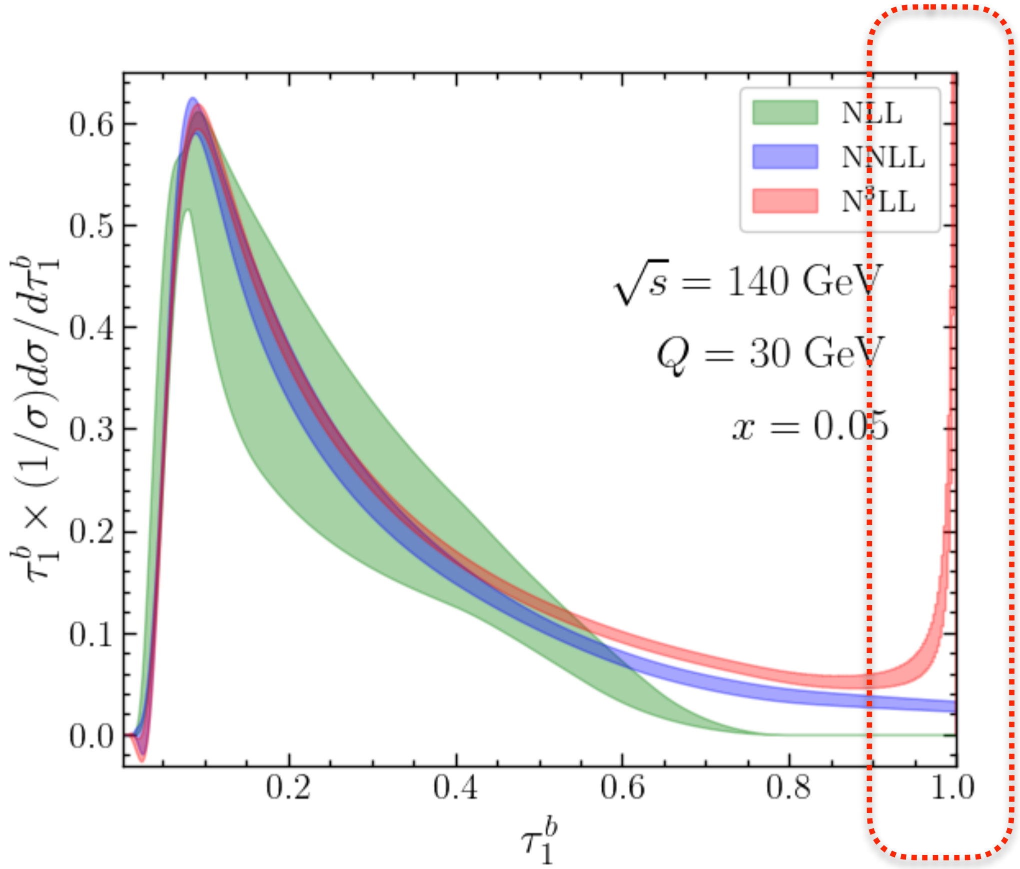
- With the recent HERA measurements as well as the future EIC results, τ_1^b can be used as an independent event shape method for the α_s, Ω_1 determination.
- Additionally, this could work as a quantitative measure of gapped events.

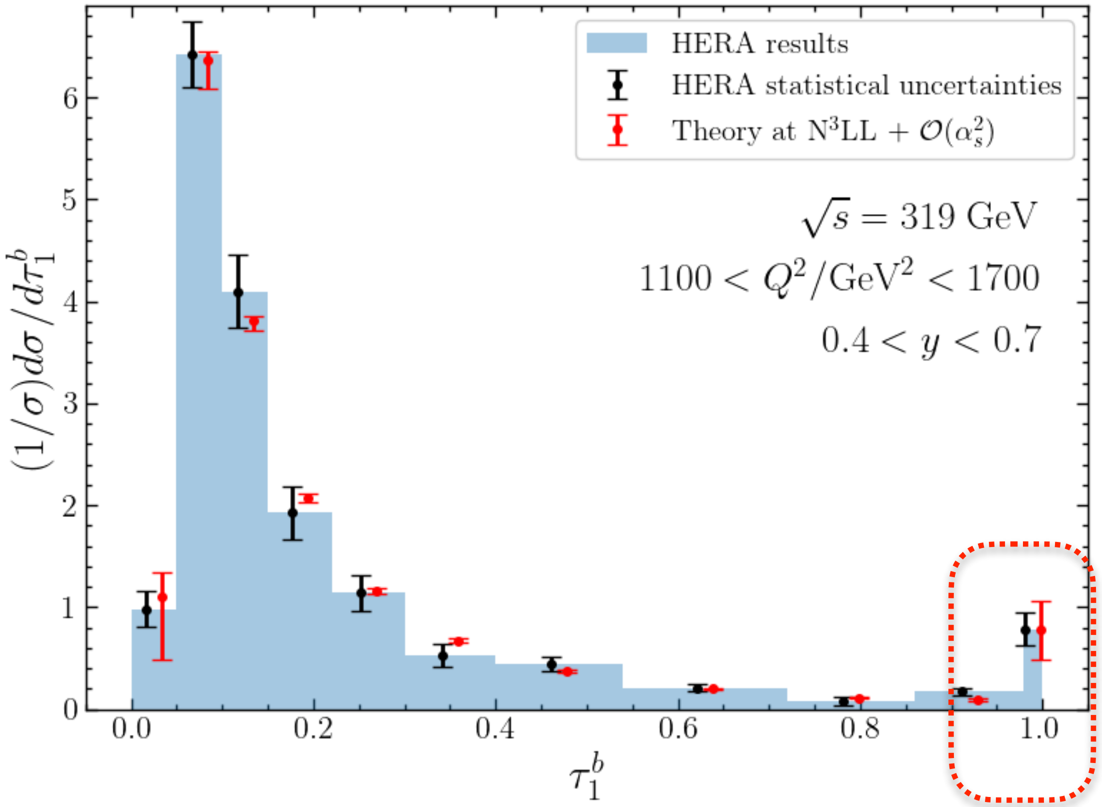
- Sensitive to hadron PDFs, so could also be used as a probe to PDFs.

Thanks!

Backup

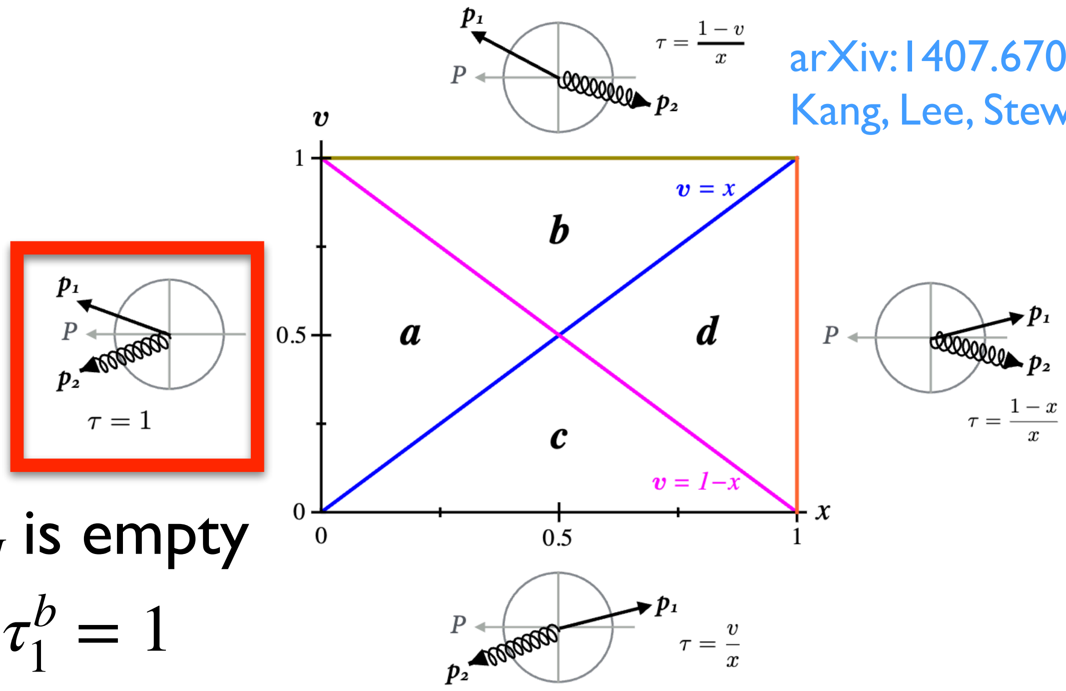
Peak as $\tau_1^b \to 1$





- $\tau_1^b \to 1$ characterizes events where nearly all final-state particles are confined to the beam hemisphere. (Empty jet hemisphere)

- This contribution becomes increasingly significant as $x \to 0$.



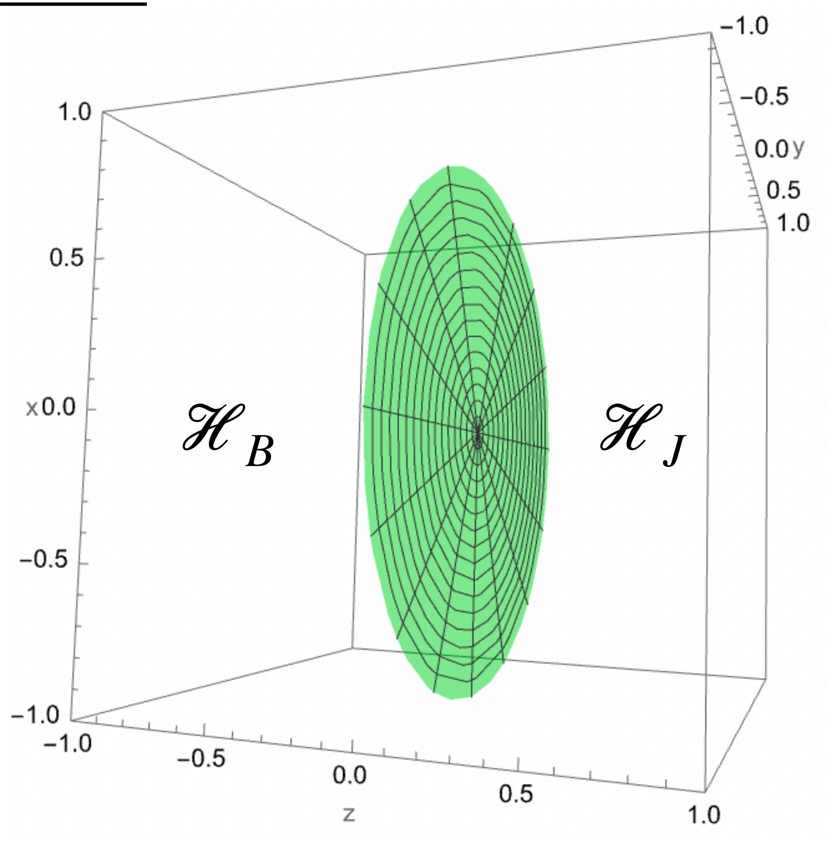
arXiv:1407.6706
Kang, Lee, Stewart

$\mathscr{H}_J$ is empty

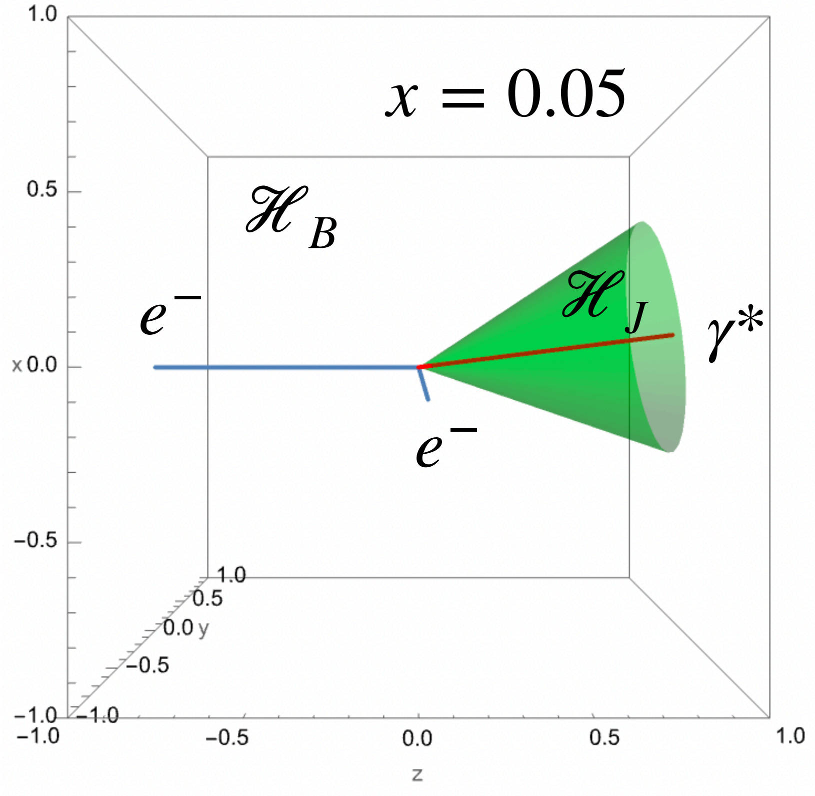
$\tau_1^b = 1$

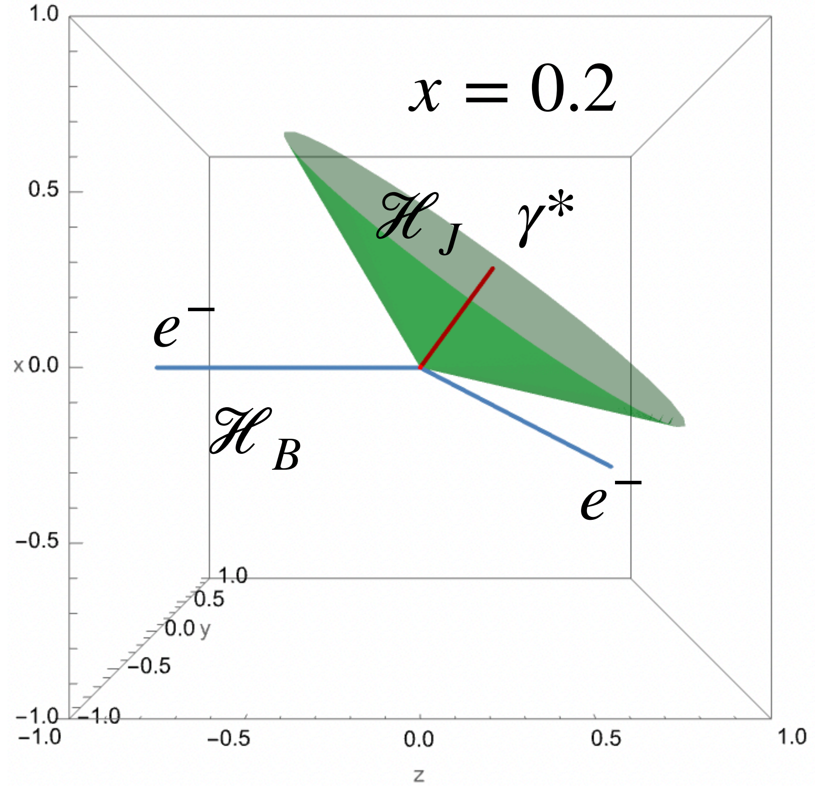
Peaks as $\tau_1^b \to 1$

Breit frame



CM frame





In Breit frame, the separation of the $\mathscr{H}_{B,J}$ is always $z = 0$, regardless of x and Q.

- However, in the CM frame, the $\mathscr{H}_J$ takes on cone-like shape, with its opening angle varying based on x.
- Events with $\tau_1^b \to 1$ provide a quantitative measure of gapped events, where the jet hemisphere is nearly empty.

Nonsingular: $r_c(1)$ test

- We can check the numerical results from NLOJET++ in terms of the cumulant of the nonsingular distribution.

$$\begin{tikzcd} & \frac{d\sigma_{\text{s}}}{d\tau_1^b} = A\delta(\tau_1^b) + \left[B(\tau_1^b)\right]_+ \\ \frac{d\sigma_{\text{total}}}{d\tau_1^b} = A\delta(\tau_1^b) + \left[B(\tau_1^b)\right]_+ + r(\tau_1^b) \arrow[ur] \arrow[dr] & \\ & \frac{d\sigma_{\text{ns}}}{d\tau_1^b} = r(\tau_1^b) \end{tikzcd}$$

Fixed-order total

arXiv:0806.3852
Hoang and Kluth

arXiv:1808.07867
Bell, Hornig, Lee, Talbert

- Integrating the fixed-order total and the singular distribution in τ_1^b from 0 to 1, we have

$$\sigma_{\text{total}} = A + \int_0^1 d\tau_1^b r(\tau_1^b) \quad \text{and} \quad \sigma_{\text{s}} = A \quad \text{where we used} \quad \int_0^1 d\tau_1^b \left[B(\tau_1^b)\right]_+ = 0$$

So, from the known analytic fixed-order results for σ_{total} and σ_{s}, we can determine the cumulant nonsingular distribution.

arXiv: 1005.1481, Botje (QCDNUM)

$$r_c(1) \equiv \int_0^1 d\tau_1^b r(\tau_1^b) = \sigma_{\text{total}} - \sigma_{\text{s}} \qquad \text{(Analytic)}$$

- From the numerical results of NLOJET++, we can access the distribution for $\tau_1^b > 0$.

$$\left.\frac{d\sigma_{\text{total}}}{d\tau_1^b}\right|_{\tau_1^b>0}^{\text{NLOJET++}} = B(\tau_1^b) + r(\tau_1^b)$$

$$\left.\frac{d\sigma_{\text{s}}}{d\tau_1^b}\right|_{\tau_1^b>0} = B(\tau_1^b)$$

Integrating the difference of the two quantities from ϵ to 1, ($\epsilon \to 0$), we obtain

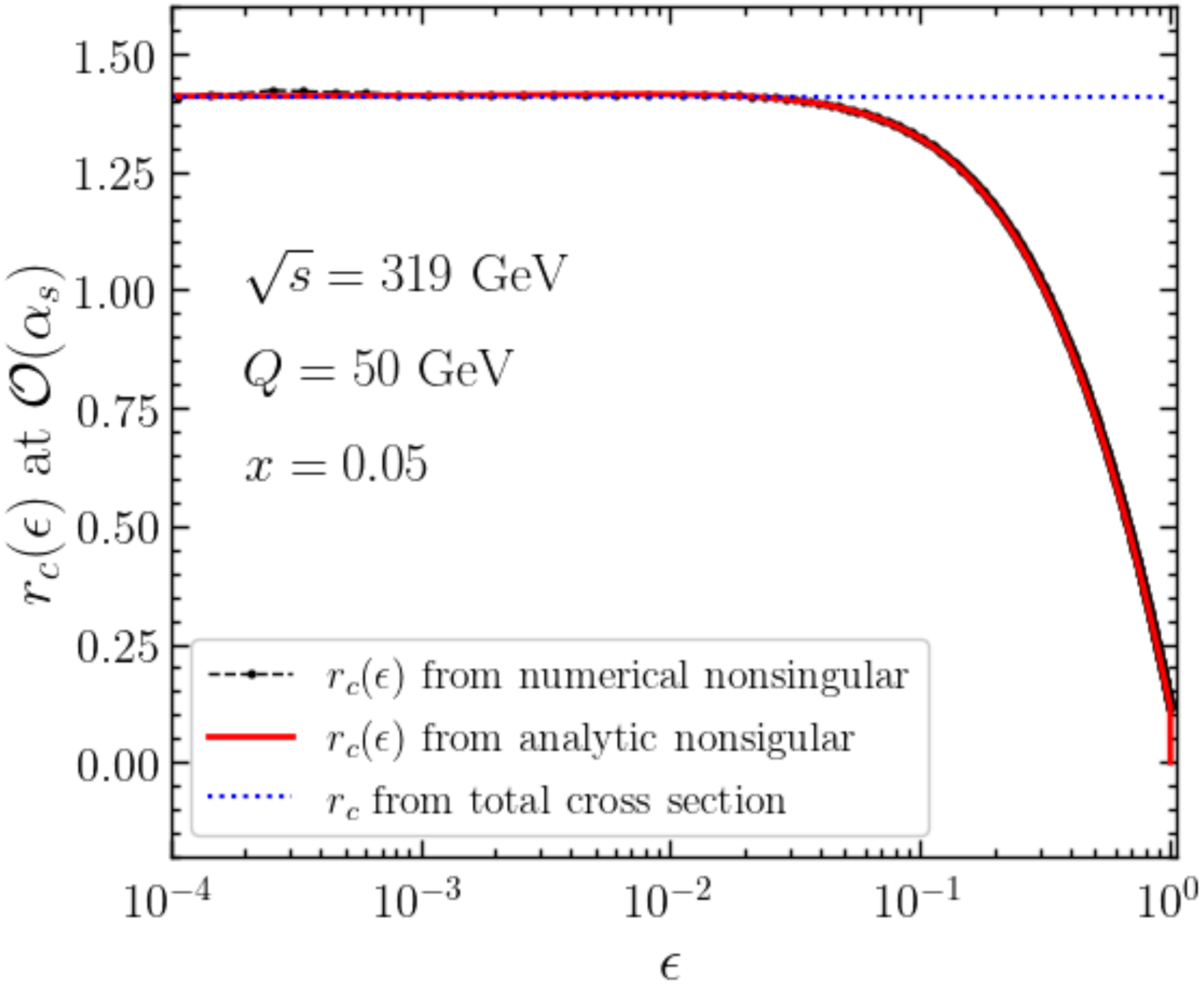
$$\lim_{\epsilon\to 0}\int_\epsilon^1 d\tau_1^b r(\tau_1^b) = \lim_{\epsilon\to 0}\int_\epsilon^1 d\tau_1^b \left[\left.\frac{d\sigma_{\text{total}}}{d\tau_1^b}\right|_{\tau_1^b>0}^{\text{NLOJET++}} - \left.\frac{d\sigma_{\text{s}}}{d\tau_1^b}\right|_{\tau_1^b>0}\right]$$

(Numerical)

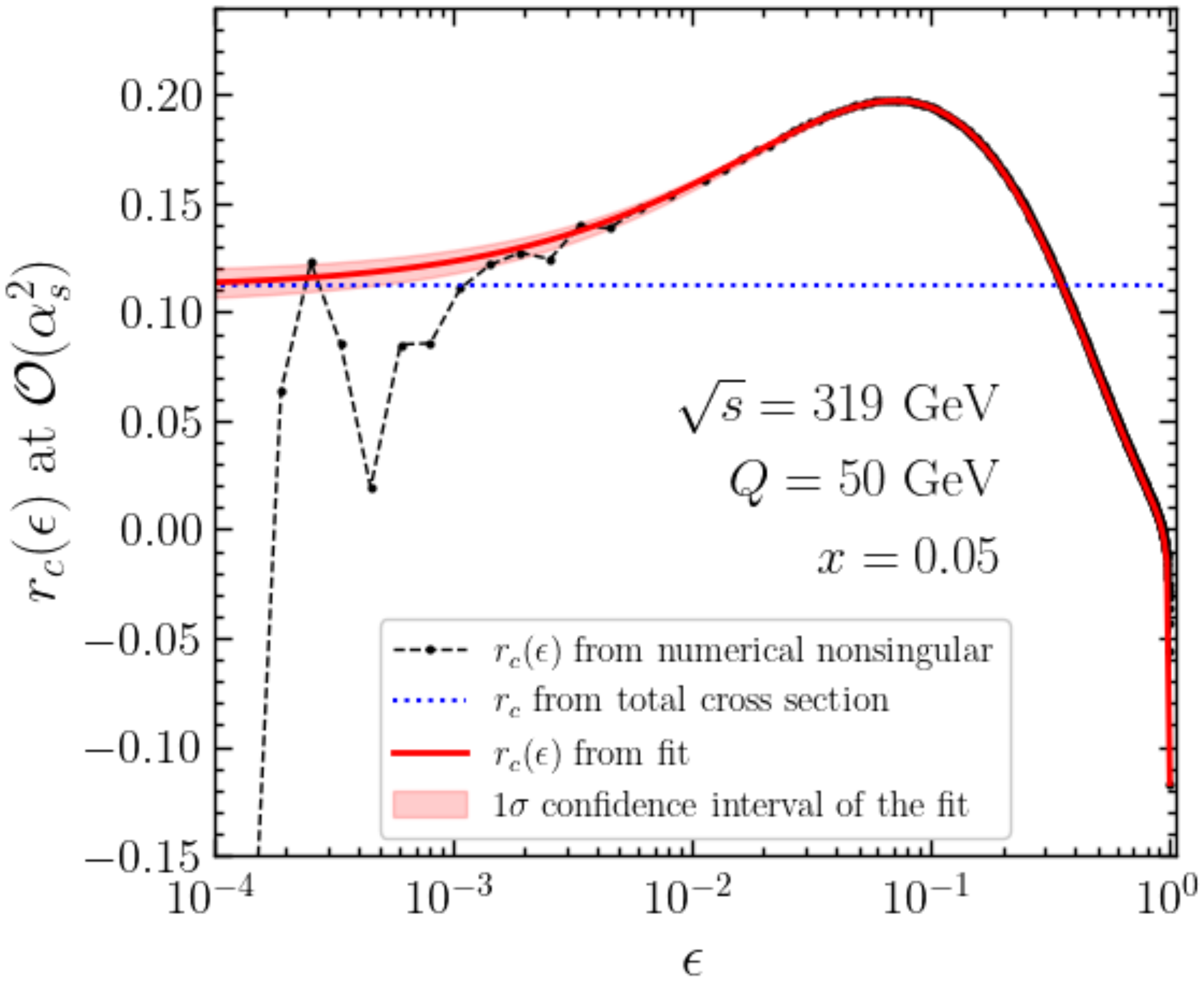
Nonsingular: $r_c(1)$ test

- So, by comparing $r_c(1)$ determined from the two independent ways, we can test the validity of the numerical results from NLOJET++

Integrate the nonsingular from $\tau_1^b = 1$ to some sufficiently small number and see if the results could develop the flat values predicted by the analytic result.



At $\mathcal{O}(\alpha_s)$

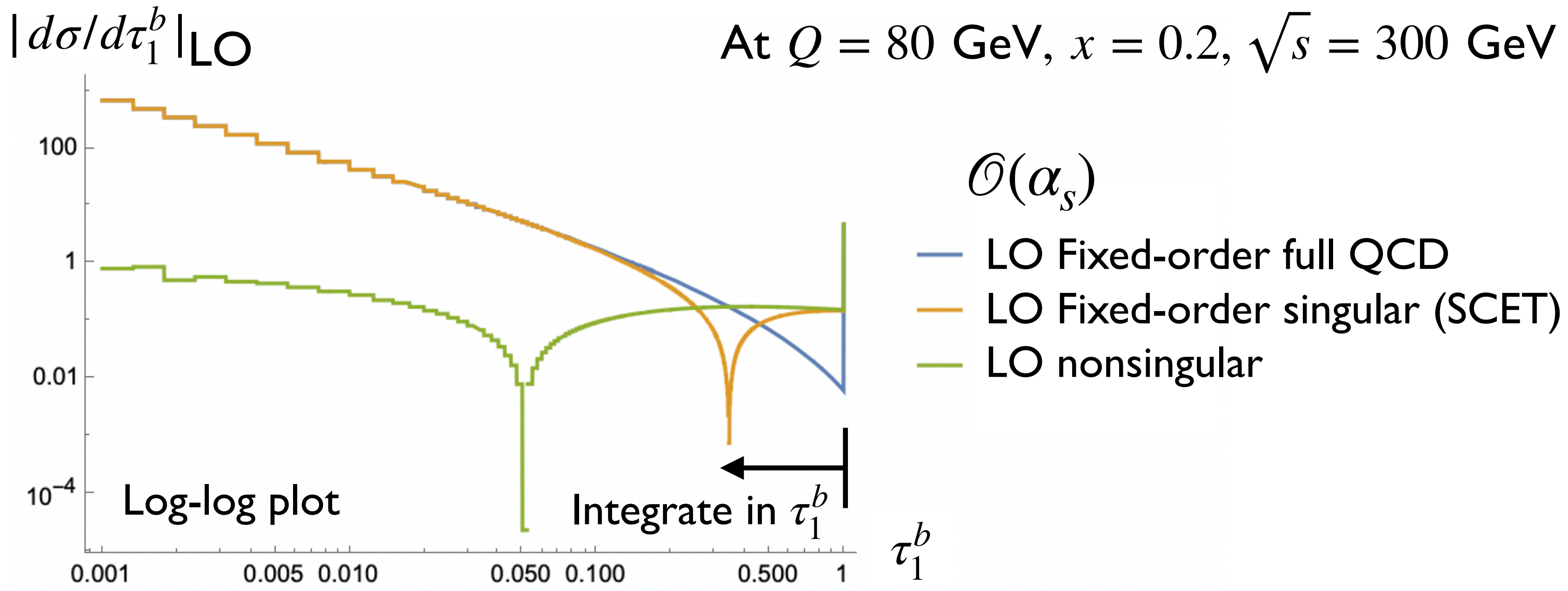


At $\mathcal{O}(\alpha_s^2)$

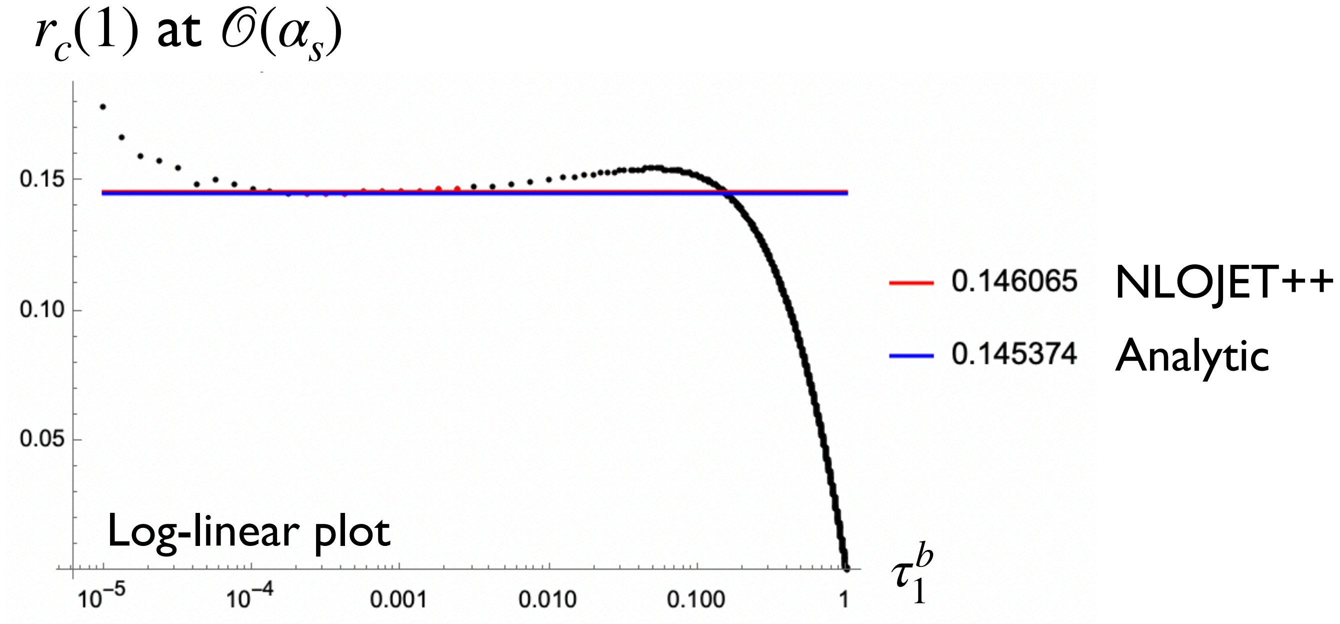
- We can clearly see the flat region from the LO result, and a bit noisier result for NLO, but they both correctly produce the analytic results. (Improvable by collecting more MC events)

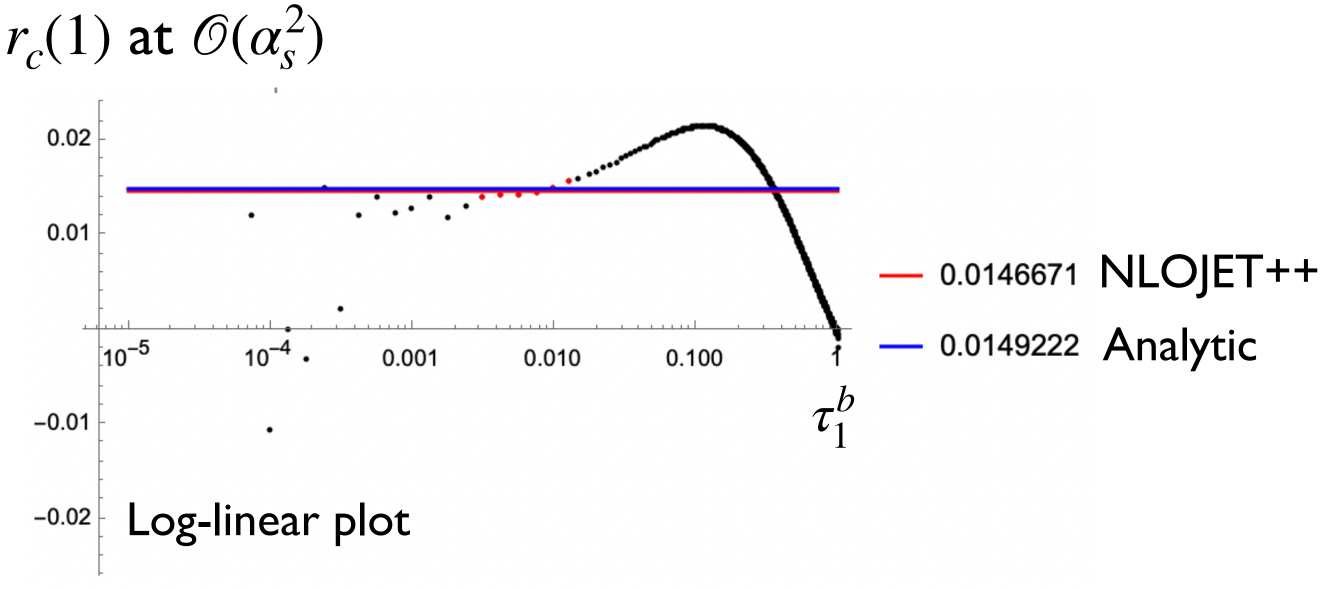
Nonsingular: $r_c(1)$ test

- So, by comparing $r_c(1)$ determined from the two independent ways, we can test the validity of the numerical results from NLOJET++


At $Q = 80$ GeV, $x = 0.2$, $\sqrt{s} = 300$ GeV

Integrate the nonsingular from $\tau_1^b = 1$ to some sufficiently small number and see if the results could develop the flat values predicted by the analytic result.


$r_c(1)$ at $\mathcal{O}(\alpha_s)$


$r_c(1)$ at $\mathcal{O}(\alpha_s^2)$

- We can clearly see the flat region from the LO result, and a bit noisier result for NLO, but they both correctly produce the analytic results. (Improvable by collecting more MC events)

SCET FT for τ_1^b: Resummation

- Once we establish the description for the fixed-order functions, we can implement the resummations of large logs in τ_1^b. By N^{x}LL, we mean the summation of the following logs:

LL $\log \tau_1^b (\alpha_s \log \tau_1^b)^n \sim \mathcal{O}(\alpha_s^{-1})$ NLL $(\alpha_s \log \tau)^n \sim \mathcal{O}(\alpha_s^0)$ $(n \geq 1)$

NNLL $\alpha_s (\alpha_s \log \tau)^n \sim \mathcal{O}(\alpha_s)$ N3LL $\alpha_s^2 (\alpha_s \log \tau)^n \sim \mathcal{O}(\alpha_s^2)$

where the last relations assumed power counting of large logs, $\log \tau_1^b \sim 1/\alpha_s$ when $\tau_1^b \ll 1$.

- The RG equations of the hard, jet, beam, and soft functions are

$$\mu \frac{d}{d\mu} H(Q^2, \mu) = \gamma_H(\mu) H(Q^2, \mu) \qquad \mu \frac{d}{d\mu} G(t, \mu) = \int dt' \gamma_G(t - t', \mu) G(t', \mu) \qquad \mu \frac{d}{d\mu} S(k, \mu) = \int dk' \gamma_S(k - k', \mu) S(k', \mu)$$

for $G = \{J, B\}$

- And the corresponding anomalous dimensions are given by

$$\gamma_H(\mu) = \Gamma_H[\alpha_s(\mu)] \log \frac{Q^2}{\mu^2} + \gamma_H[\alpha_s(\mu)] \qquad \gamma_G(t, \mu) = \Gamma_G[\alpha_s(\mu)] \frac{1}{\mu^2} \left[\frac{\theta(t/\mu^2)}{t/\mu^2} \right]_+ + \gamma_G[\alpha_s(\mu)] \delta(t)$$

$$\gamma_S(k, \mu) = \Gamma_S[\alpha_s(\mu)] \frac{1}{\mu} \left[\frac{\theta(k/\mu)}{k/\mu} \right]_+ + \gamma_S[\alpha_s(\mu)] \delta(k)$$

SCET FT for τ_1^b: Profile function

$$\begin{aligned}\sigma_c(x,Q^2,\tau) = {} & \underline{\frac{e^{\mathcal{K}-\gamma_{\mathrm{E}}\Omega}}{\Gamma(1+\Omega)}\left(\frac{Q}{\mu_H}\right)^{\eta_H}\left(\frac{\xi Q}{\mu_B^2}\right)^{\eta_B}\left(\frac{\xi Q}{\mu_J^2}\right)^{\eta_J}\left(\frac{\xi}{\mu_S}\right)^{2\eta_S}} \\ & \times L_q^b(y,Q^2)\sum_j\int_x^1\frac{dz}{z}f_j(x/z,\mu_B) \\ & \times H(Q^2,\mu_H)\sum_{n_1,n_2,n_3=-1}\underline{J_{n_1}\left[\alpha_s(\mu_J),\frac{\xi Q}{\mu_J^2}\right]\mathcal{J}_{n_2}\left[\alpha_s(\mu_B),z,\frac{\xi Q}{\mu_B^2}\right]S_{n_3}\left[\alpha_s(\mu_S),\frac{\xi}{\mu_S}\right]} \\ & \times\sum_{\ell_1=-1}^{n_1+n_2+1}\sum_{\ell_2=-1}^{\ell_1+n_3+1}\sum_{\ell_3=-1}^{\ell_2+1}V_{\ell_1}^{n_1n_2}V_{\ell_2}^{\ell_1n_3}V_{\ell_3}^{\ell_2}(\Omega)\underline{F_{\ell_3}^{\Omega}(\tau Q/\xi)}+(q\leftrightarrow\bar{q}),\end{aligned}$$

We introduced a scaling parameter ξ to make the arguments of plus distributions have a common convolution variable, so the cross section is independent of ξ.

$$F_{\ell_3}^{\Omega}(\tau Q/\xi)=\int_0^{\frac{\tau Q}{\xi}}dy\left[\frac{\theta(y)\log^{\ell_3}y}{y^{1-\Omega}}\right]_+$$

- The most obvious way to deal with the explicit τ dependence in $F_{\ell_3}^{\Omega}(\tau Q/\xi)$ is to choose $\xi \sim \tau Q$ (but the result is independent of ξ).
- With this choice of ξ, we get to introduce τ-dependent log factors (blue) in the fixed-order functions, this can best be dealt with in terms of the well-established resummation factors (orange).
- The logs in the fixed-order function can be minimized through the following **canonical** scales:

$$\mu_H = Q, \quad \mu_J = \mu_B = \sqrt{\tau_1^b}Q, \quad \mu_S = \tau_1^b Q$$

- We can see that the relative hierarchy between these scales changes w.r.t. the value of τ_1^b.
- In order to properly implement the τ-dependent scales to the SCET FT, we need the profile function.

τ_1^b for DIS and thrust for e^+e^-

[when $x \leq 1/2$, otherwise, $(1-x)/x$]

$$\tau_1^b = \frac{2}{Q^2}\sum_{i\in X} \text{min}\left\{q_B^b\cdot p_i, q_J^b\cdot p_i\right\} \quad \in \quad (0,1)$$

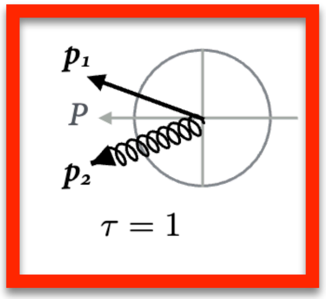
⟵ ∵ No minimization in $q_{J,B}$

$$\tau_{e^+e^-} = 1 - T \quad \text{where} \quad T = \text{max}_{\hat{t}}\frac{\sum_i |\hat{t}\cdot\vec{p}_i|}{\sum_i |\vec{p}_i|} \quad \in \quad (0,1/2)$$

⟵ ∵ Minimization in thrust axis

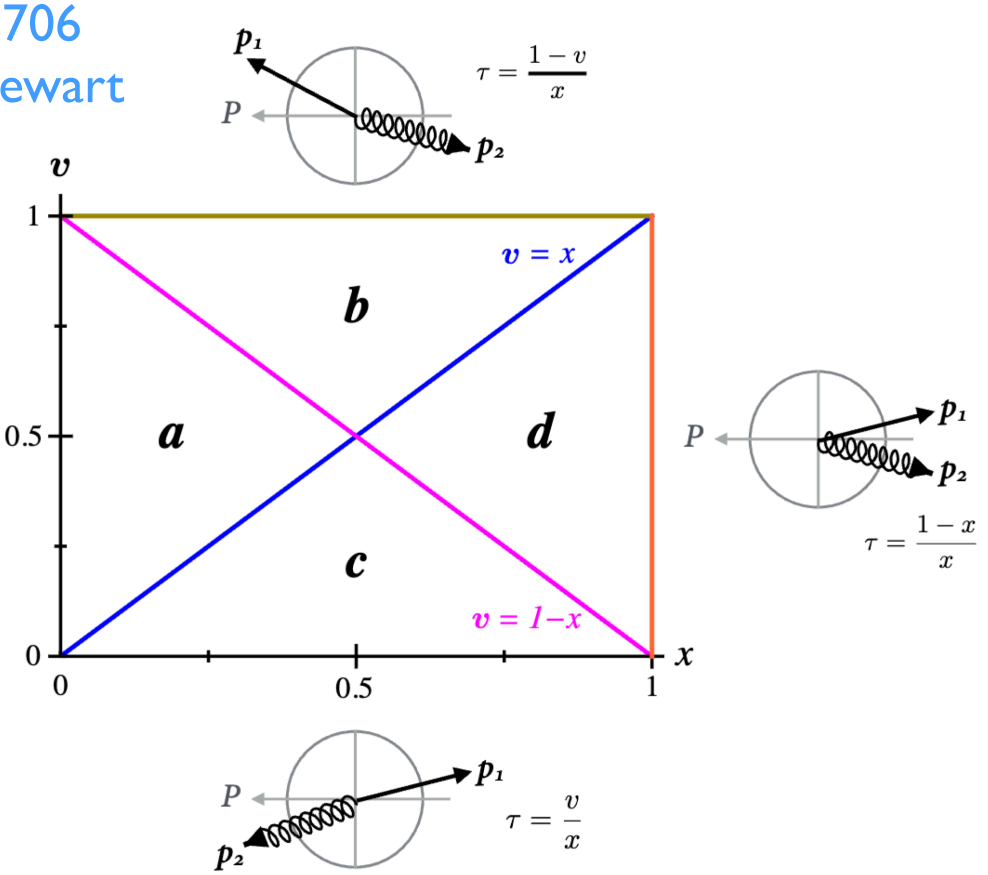
- Characteristic singular behavior as $\tau_1^b \to 1$

arXiv:1407.6706
Kang, Lee, Stewart



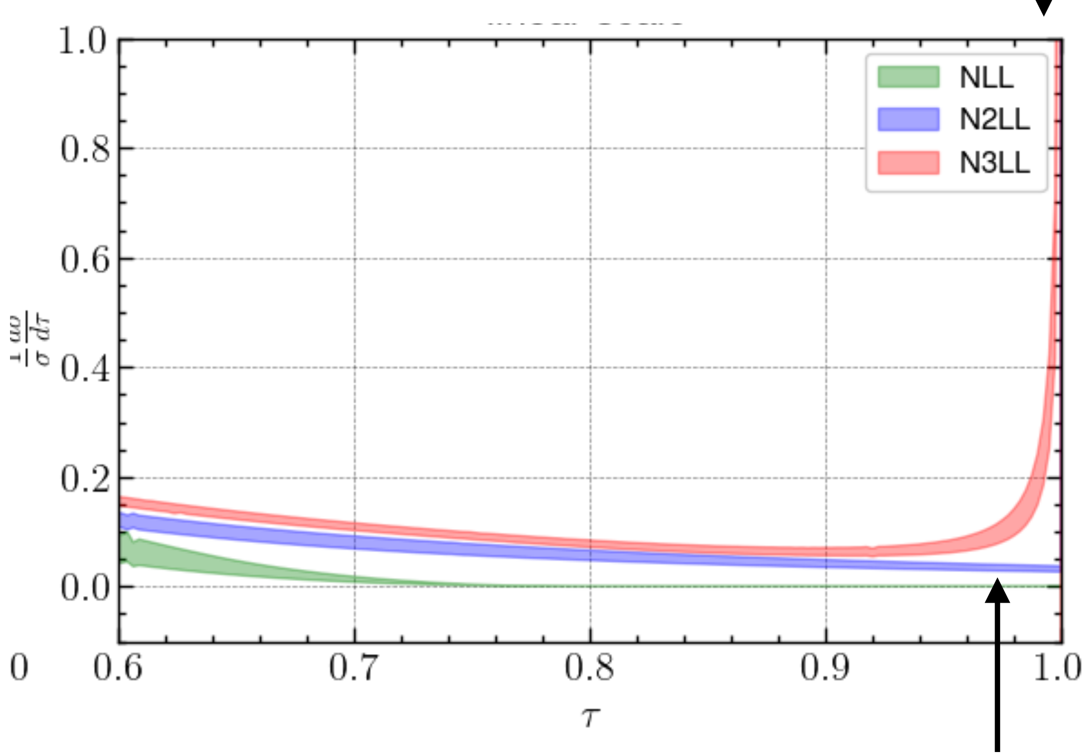
$\mathscr{H}_J$ is empty

$\tau_1^b = 1$



$$\tau_1^b \overset{\text{Breit}}{=} 1 - \frac{2}{Q}\sum_{i\in\mathscr{H}_J}(p_i)_z$$

Smeared at $\mathscr{O}(\alpha_s^2)$



$\delta(1-\tau)$ at $\mathscr{O}(\alpha_s)$

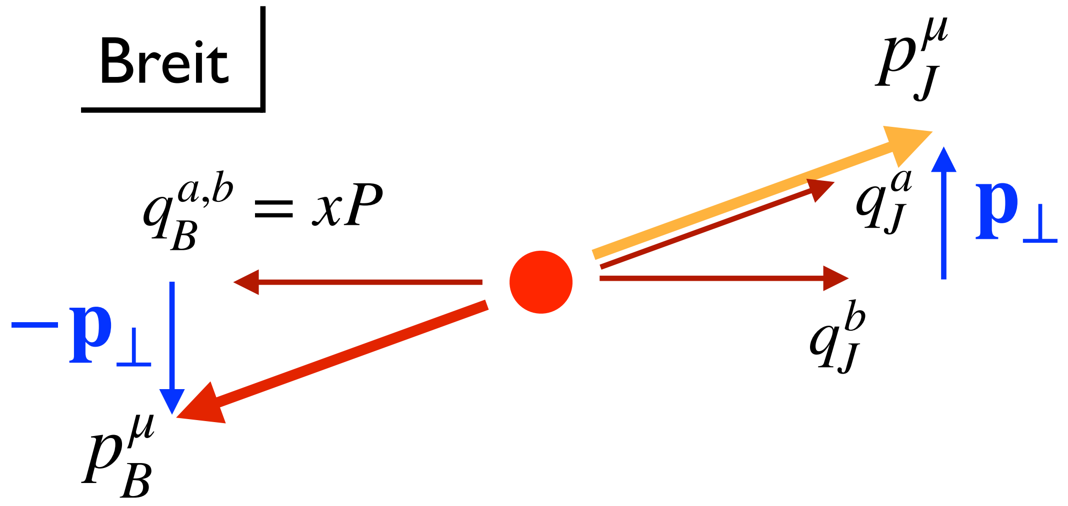
τ_1^b and other DIS 1-jettiness

- Another version of the DIS 1-jettiness is τ_1^a, and the only difference is the definition of q_J:

$$q_J^b = q + xP \quad \longrightarrow \quad q_J^a = K_J = q_J^b + q_J^\perp$$

arXiv:1303.6952
Kang, Lee, Stewart

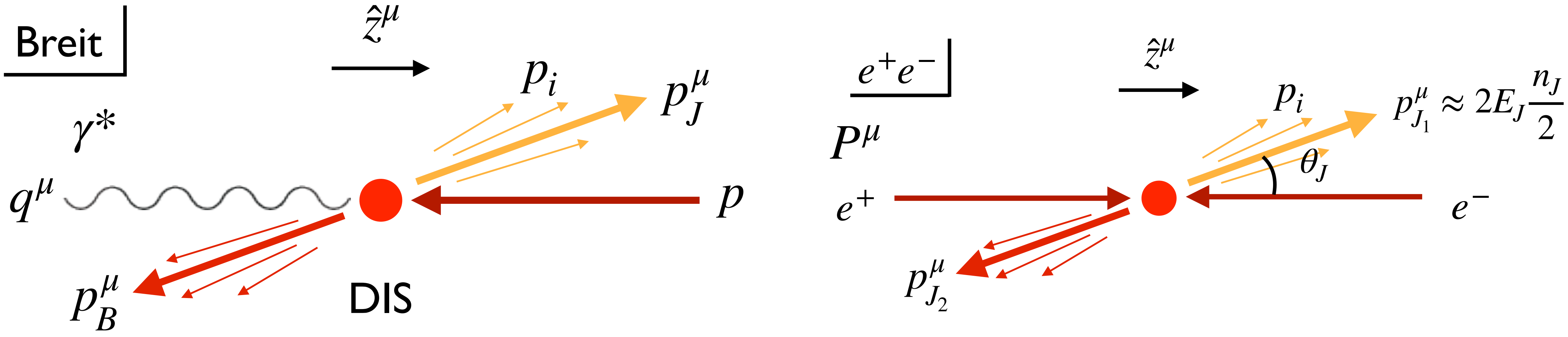
- $q_J^{a\mu}$ is defined to a light-like vector along the jet momentum P_J, whose light-like projection is K_J.



- τ_1^b distribution has $\mathbf{p}_\perp^2$ dependences because jet and beam momenta are not aligned with $q_{J,B}$.
- However, τ_1^a distribution has $\mathbf{p}_\perp^2$ dependence only on the beam momenta, so we can just integrate it to find the ordinary beam function.

- Theoretically, it is more involved to deal with the transverse-momentum dependent beam function, but τ_1^b is Lorentz invariant. (τ_1^a needs a jet algorithm, which refers to a specific frame.)
- Lorentz invariant, and global observable, so free of NGL
  → **Can be computed with high theoretical accuracy.**
- Reduces contamination from remnant fragmentation from its measurements, so makes it **desirable to be measured in experiments.**

τ_1^b and and e^+e^- thrust



- $\tau_1^b \sim 0$ and $\tau_{e^+e^-} \sim 0$ describe event collimated along jet axes, and could be best described by **SCET with high theory precision. (N3LL)**
- One of the nontrivial differences between τ_1^b and $\tau_{e^+e^-}$ is one of the **jet radiations** in e^+e^- should be replaced by **ISR** from the proton for τ_1^b,
In FT for τ_1^b, a jet function is replaced by the beam function.

σ^{S}_{PT}: FT and Fixed-order functions

$$\frac{d\sigma}{dx\,dQ^2\,d\tau_1^b} = \frac{d\sigma_0^b}{dx\,dQ^2}\int dt_J\,dt_B\,dk_S\,\overset{\text{Measurement function for }\tau_1^b}{\overline{\delta\left(\tau_1^b-\frac{t_J}{Q^2}-\frac{t_B}{Q^2}-\frac{k_S}{Q}\right)}}\;\overset{\text{Single variable soft function}}{\overline{S(k_S,\mu)}}$$

$$\times\,\underset{\text{Quark jet function}}{\underline{J_q(t_J,\mu)}}\Big[\underset{\text{Hard function}}{\underline{H_q^b(y,Q^2,\mu)}}\,\underset{\textbf{Projected }\tau_1^b\textbf{ quark beam function}}{\underline{\hat{B}_q(t_B,x,\mu)}}+(q\to\bar{q})\Big],$$

- Our target theory precision is N³LL + $\mathcal{O}(\alpha_s^2)$, so we need 2-loop fixed-order expressions for each parts of FT:

1. Hard function: $qq \to qq$ through neutral currents (γ^*, Z^*): 2-loop

   arXiv:hep-ph/0605068, Idilbi, Ji, Yuan

   arXiv:hep-ph/0607228, Becher, Neubert, Pecjak

   arXiv:1006.3080, Abbate, Fickinger, Hoang, Mateu, Stewart

2. Quark jet function: 2-loop

   arXiv:hep-ph/0607228, Becher, Neubert, Pecjak

3. Soft function: 2-loop

   arXiv:1105.3676, Kelley, Schabinger, Schwartz, Zhu

4. τ_1^b **quark beam function**: $\hat{B}_q(t_B,x,\mu) = \int d^2\mathbf{p}_\perp \mathscr{B}_q(t_B-\mathbf{p}_\perp^2, x, \mathbf{p}_\perp^2,\mu)$

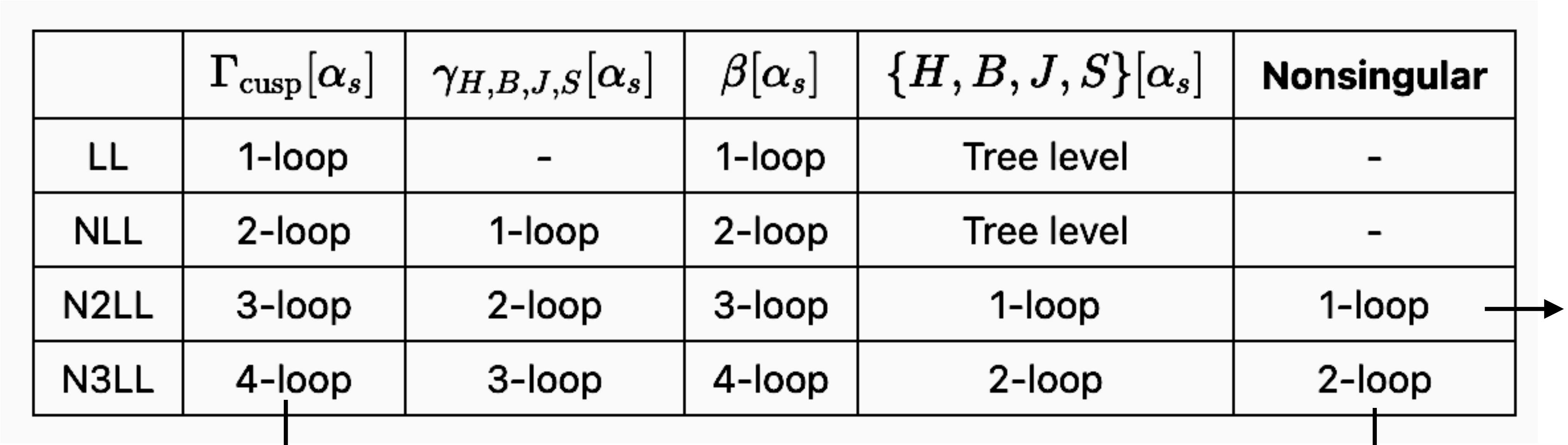
   $\mathscr{B}_q(t,x,\mathbf{k}_\perp^2,\mu)$ is the $k_\perp$-dep. beam function, $\mathscr{B}_i(t,x,\mathbf{k}_\perp^2,\mu) = \mathscr{I}_{ij}(t,x/\xi,\mathbf{k}_\perp^2,\mu)\otimes_\xi \overset{\text{PDF for parton } j}{\overline{f_j(\xi,\mu)}}$

   Known to 2-loop.

   arXiv:1401.5478, Gaunt, Stahlhofen, Tackmann

   arXiv:1409.8281, Gaunt, Stahlhofen

$\sigma^{\mathrm{S}}_{\mathrm{PT}}$: Resummation



	$\Gamma_{\text{cusp}}[\alpha_s]$	$\gamma_{H,B,J,S}[\alpha_s]$	$\beta[\alpha_s]$	$\{H,B,J,S\}[\alpha_s]$	**Nonsingular**
LL	1-loop	-	1-loop	Tree level	-
NLL	2-loop	1-loop	2-loop	Tree level	-
N2LL	3-loop	2-loop	3-loop	1-loop	1-loop
N3LL	4-loop	3-loop	4-loop	2-loop	2-loop

Analytic 1-loop nonsingular
arXiv:1407.6706
Kang, Lee, Stewart

4-loop cusp anomalous dimension

arXiv:1911.10174
Henn, Korchemsky, Mistlberger

Numerical 2-loop
(NLOJet++)

- The resulting FT for the cumulant singular distributions after resummation:

$$\sigma_c(x,Q^2,\tau) = \frac{e^{\mathcal{K}-\gamma_E\Omega}}{\Gamma(1+\Omega)}\left(\frac{Q}{\mu_H}\right)^{\eta_H}\left(\frac{\tau Q^2}{\mu_B^2}\right)^{\eta_B}\left(\frac{\tau Q^2}{\mu_J^2}\right)^{\eta_J}\left(\frac{\tau Q}{\mu_S}\right)^{2\eta_S} \longrightarrow$$

Log resummation

$$\times L_q^b(y,Q^2)\sum_j\int_x^1\frac{dz}{z}f_j(x/z,\mu_B) \longrightarrow$$

Lepton parts and PDFs

$$\times H(Q^2,\mu_H)\sum_{n_1,n_2,n_3=-1} J_{n_1}\left[\alpha_s(\mu_J),\frac{\tau Q^2}{\mu_J^2}\right]\mathscr{J}_{n_2}\left[\alpha_s(\mu_B),z,\frac{\tau Q^2}{\mu_B^2}\right]S_{n_3}\left[\alpha_s(\mu_S),\frac{\tau Q}{\mu_S}\right] \longrightarrow$$

Fixed-order functions (Coefficients of plus distributions)

$$\times\sum_{\ell_1=-1}^{n_1+n_2+1}\sum_{\ell_2=-1}^{\ell_1+n_3+1} V_{\ell_1}^{n_1n_2}V_{\ell_2}^{\ell_1n_3}V_{-1}^{\ell_2}(\Omega)+(q\leftrightarrow\bar{q}). \longrightarrow$$

Convolutions of plus distributions (momentum space formulation)

$\mathscr{K}$ and Ω are the functions of the scales $\mu_{H,B,J,S}$ and the μ_{goal} (arbitrary scale, usually set to be $\mu_{\text{goal}} \sim \mu_{J,B}$)

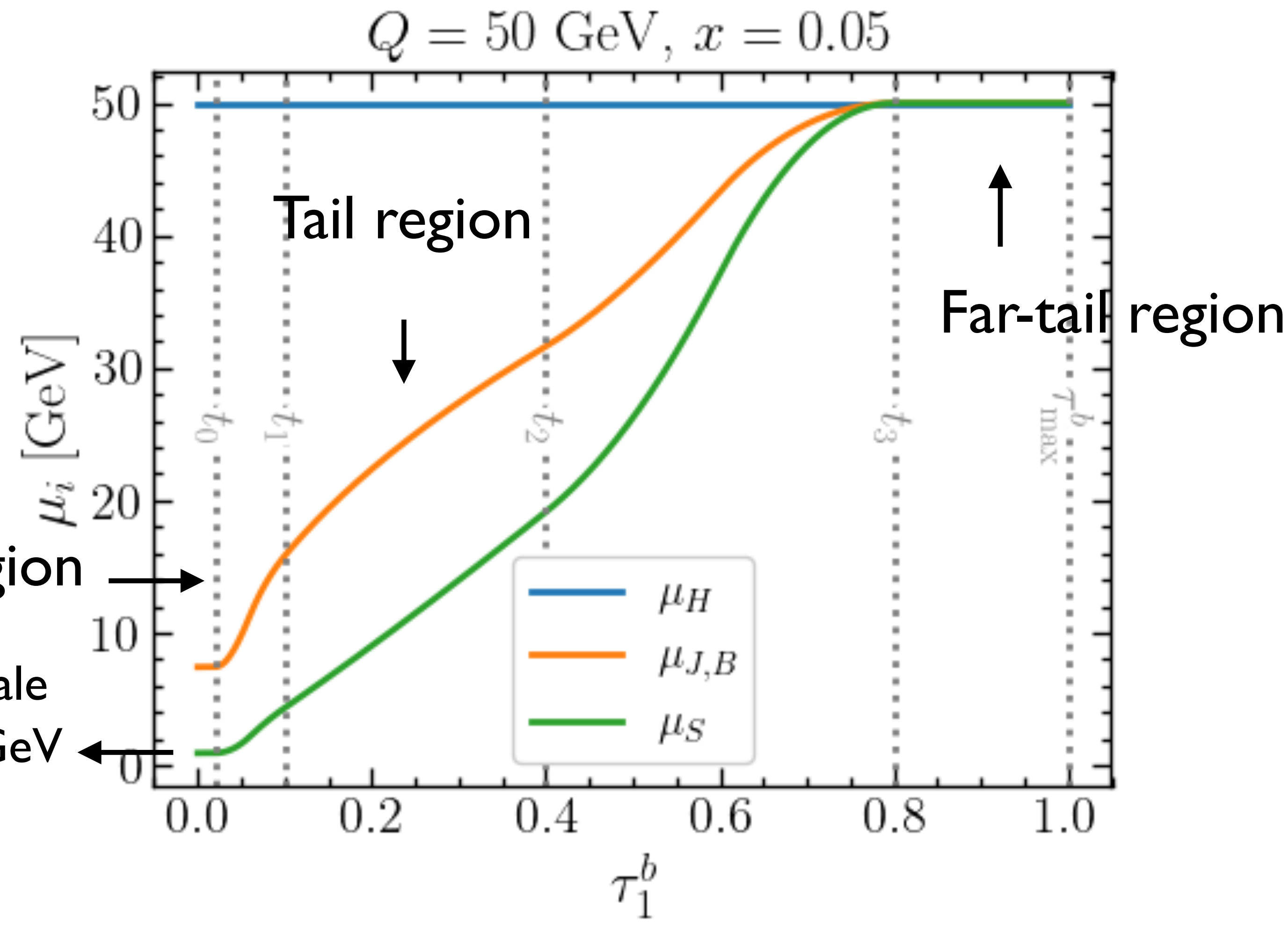
σ^{S}_{PT}: Profile function

- The natural scales for fixed-order functions are $\mu_H = Q, \quad \mu_J = \mu_B = \sqrt{\tau_1^b}Q, \quad \mu_S = \tau_1^b Q.$
- Depending on the values of τ_1^b, we have quite different physical description:

- Peak region: $\tau_1^b \sim 2\Lambda_{\mathrm{QCD}}/Q \ll 1$

- Tail region: $2\Lambda_{\mathrm{QCD}}/Q \ll \tau_1^b \ll 1$

- Far-tail region: $\tau_1^b \sim 1$



Tail region

Far-tail region

Peak region

Frozen scale

$\mu_0 = 1.1$ GeV

$$t_3 = 0.8\tau^b_{\max},$$

$$t_2 = \min\left(\frac{1-\log(x+x_c)}{10}, 0.6t_3\right),$$

$$t_1 = \min\left(\frac{5\ \mathrm{GeV}}{Q}, 0.6t_2\right),$$

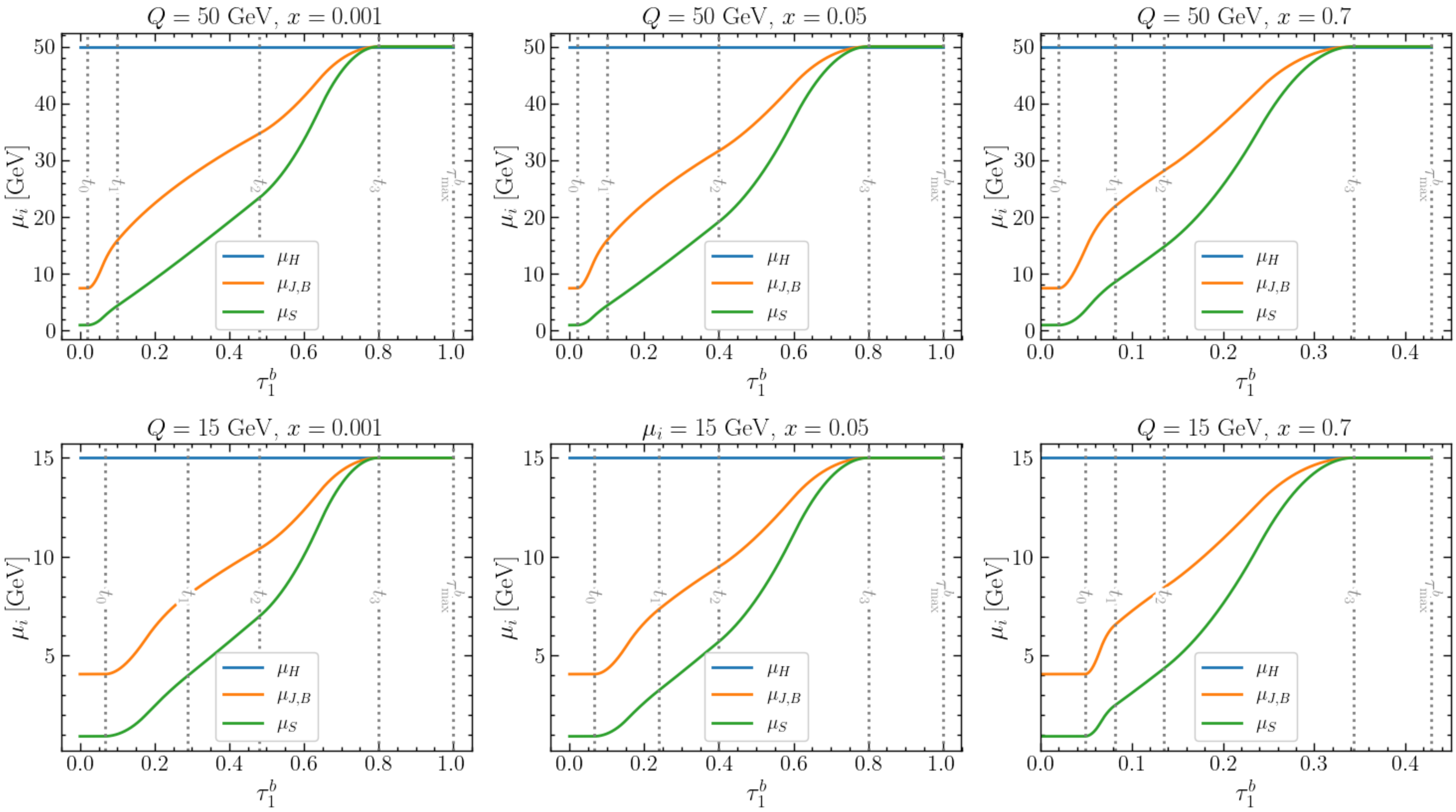
$$t_0 = \min\left(\frac{1\ \mathrm{GeV}}{Q}, 0.6t_1\right),$$

Depend on x

$x_c = 0.0001234$

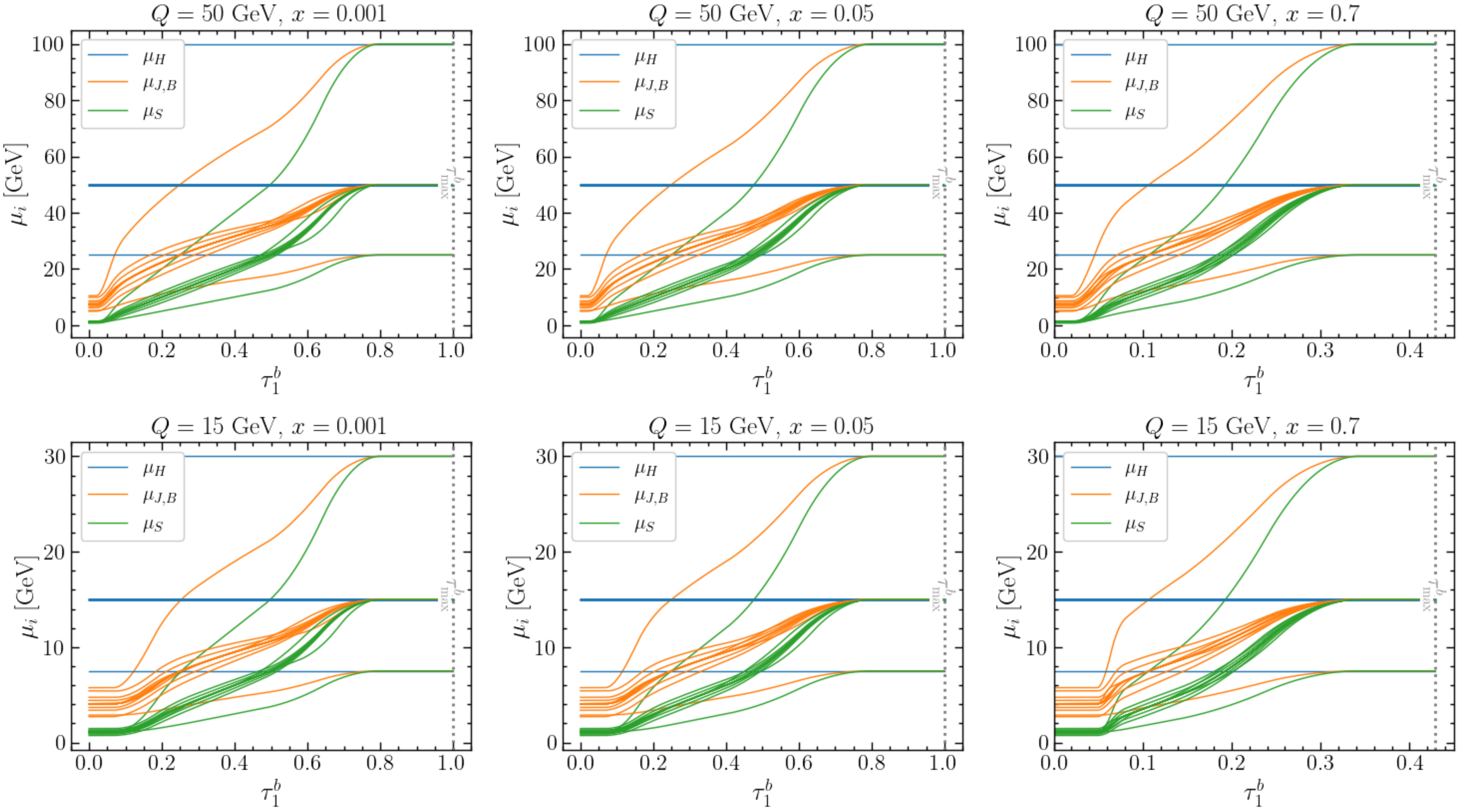
- A characteristic feature of our profile is that it changes w.r.t. Q and x, because the relative importance of the singular and nonsingular changes in x.

$\sigma^{\mathrm{S}}_{\mathrm{PT}}$: Profile function



- The tail region (resummation region) with the canonical scales move w.r.t. Q and x.

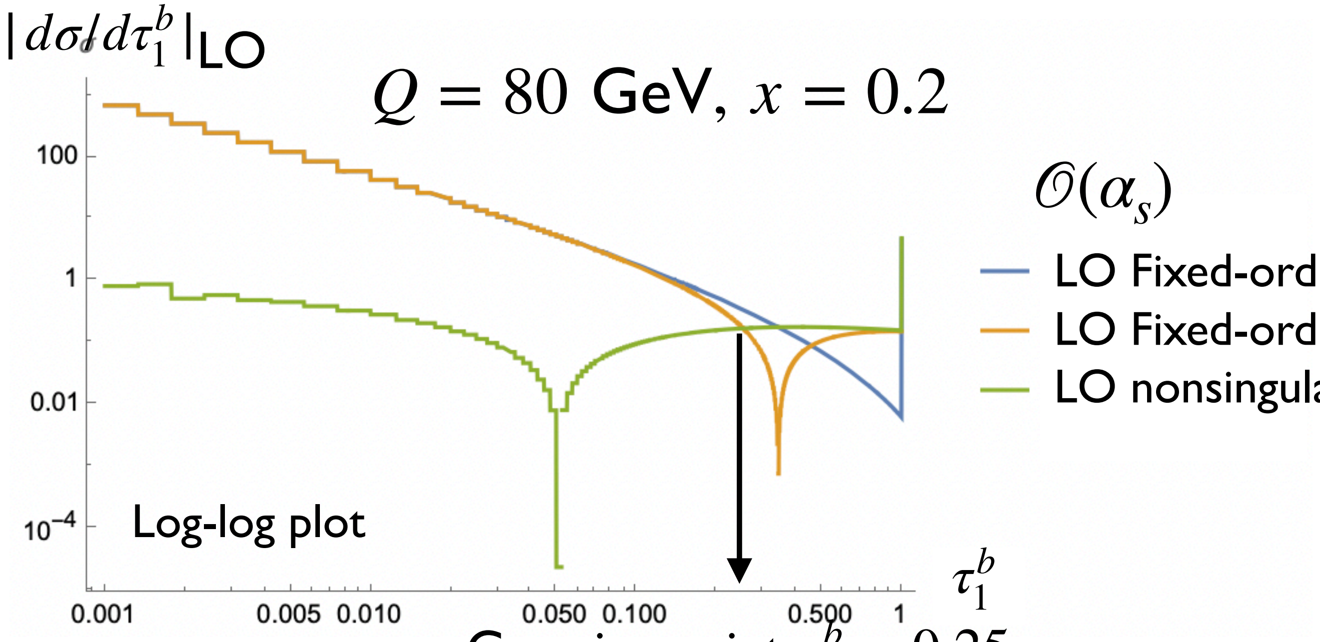
σ^{s}_{PT}: Scale variations



σ^{s}_{PT}: Profile function

Fixed-order singular vs. nonsingular

$Q = 80$ GeV, $x = 0.2$



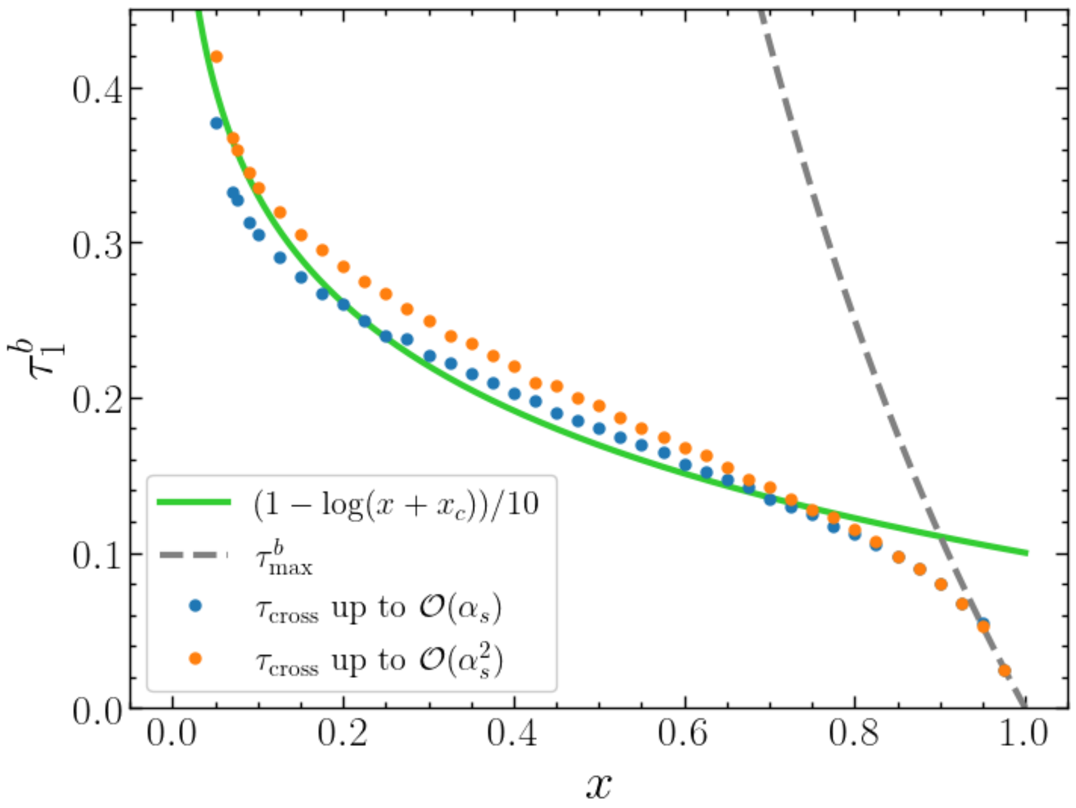
Log-log plot

Crossing point $\tau_1^b \sim 0.25$

$$t_2 = \frac{1 - \log(x + x_c)}{10}$$

$$x_c = 0.0001234$$

- $\tau = t_2$ is set to depend on x, below which the singular contribution gets larger than the nonsingular.
  → resummation matter!

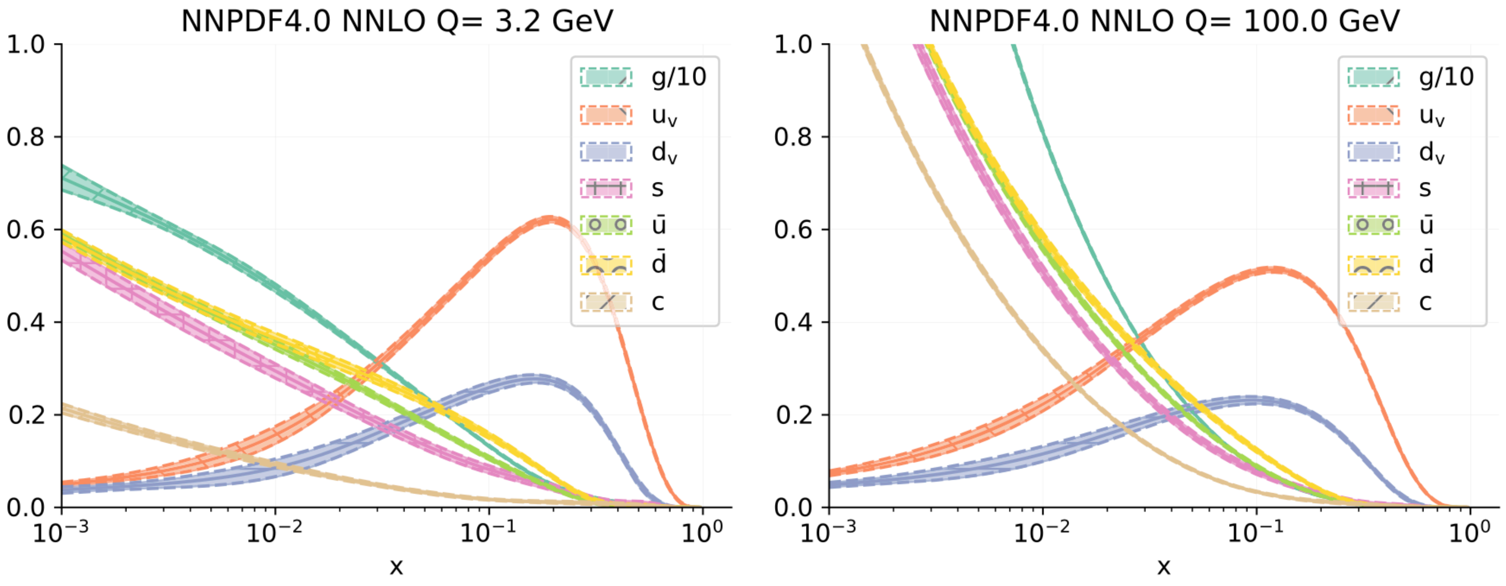


σ^{S}_{PT}: PDF sets

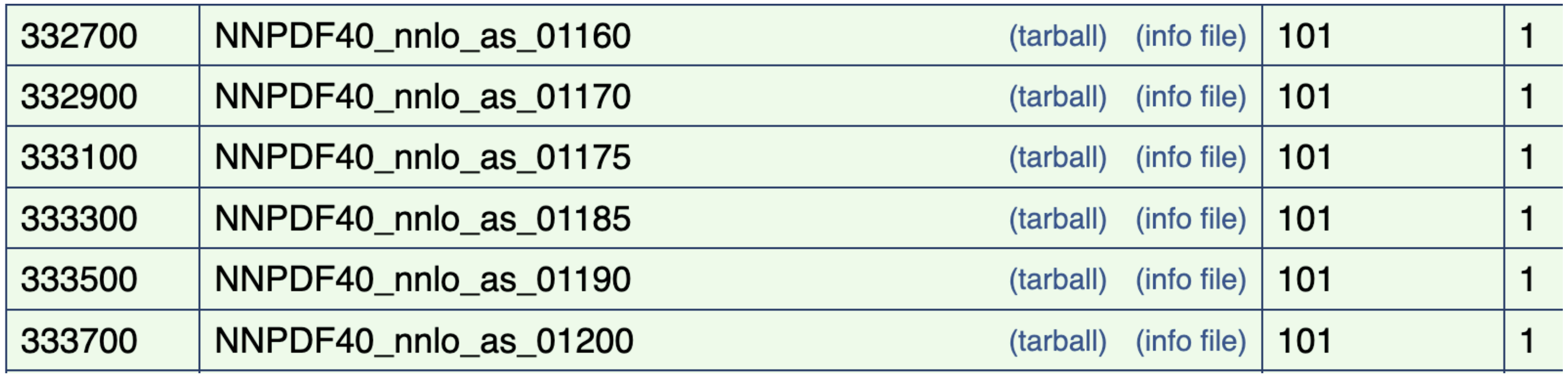
4. τ_1^b **quark beam function**: $\hat{B}_q(t_B, x, \mu) = \int d^2\mathbf{p}_\perp \mathscr{B}_q(t_B - \mathbf{p}_\perp^2, x, \mathbf{p}_\perp^2, \mu)$

PDF for parton j

$\mathscr{B}_q(t, x, \mathbf{k}_\perp^2, \mu)$ is the $k_\perp$-dep. beam function, $\mathscr{B}_i(t, x, \mathbf{k}_\perp^2, \mu) = \mathscr{I}_{ij}(t, x/\xi, \mathbf{k}_\perp^2, \mu) \otimes_\xi \underline{f_j(\xi, \mu)}$



- NNPDF4.0 NNLO PDF set implemented in LHAPDF.
- PDFs are determined w.r.t. α_s value.
- Should change PDFs for different α_s simultaneously.



332700	NNPDF40_nnlo_as_01160	(tarball)	(info file)	101	1
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333100	NNPDF40_nnlo_as_01175	(tarball)	(info file)	101	1
333300	NNPDF40_nnlo_as_01185	(tarball)	(info file)	101	1
333500	NNPDF40_nnlo_as_01190	(tarball)	(info file)	101	1
333700	NNPDF40_nnlo_as_01200	(tarball)	(info file)	101	1