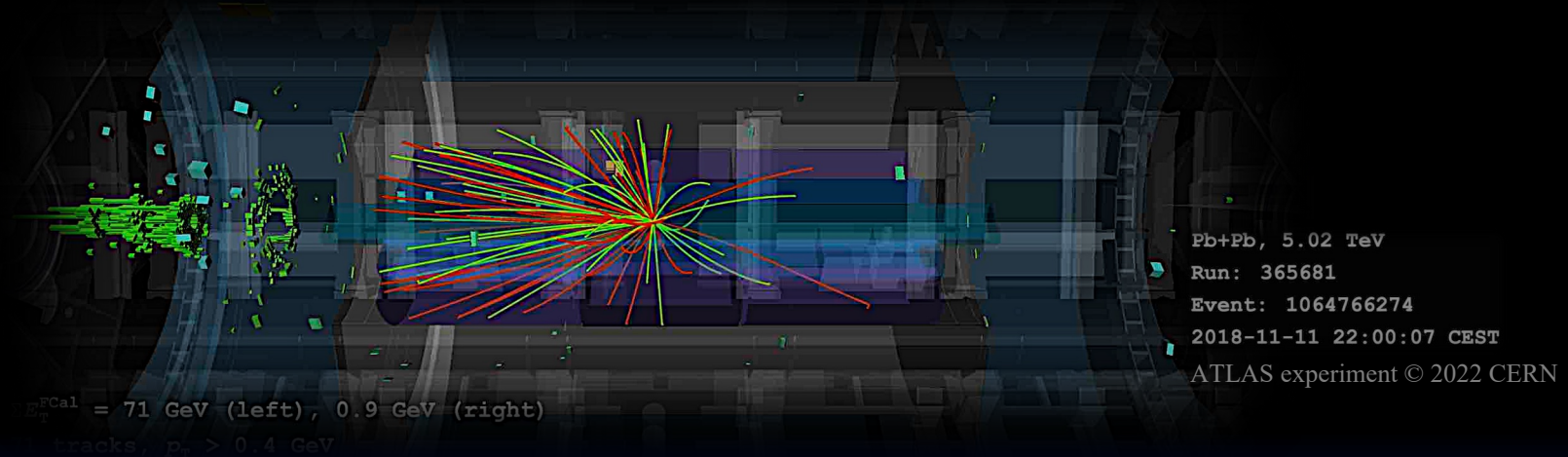


Diffraction and small- x dynamics at the EIC

Stella Schindler

Los Alamos National Laboratory



Uncovering New Laws of Nature at the EIC
Brookhaven National Laboratory
Thursday, November 21, 2024



LA-UR-24-32331



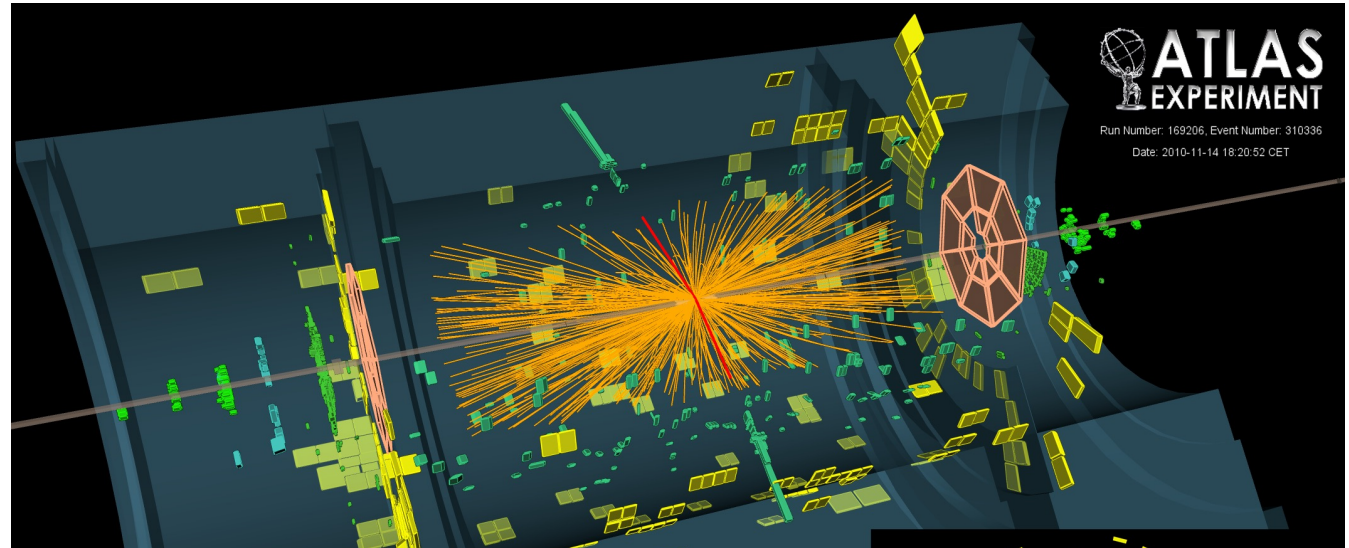
Collaborators

Iain Stewart (MIT)

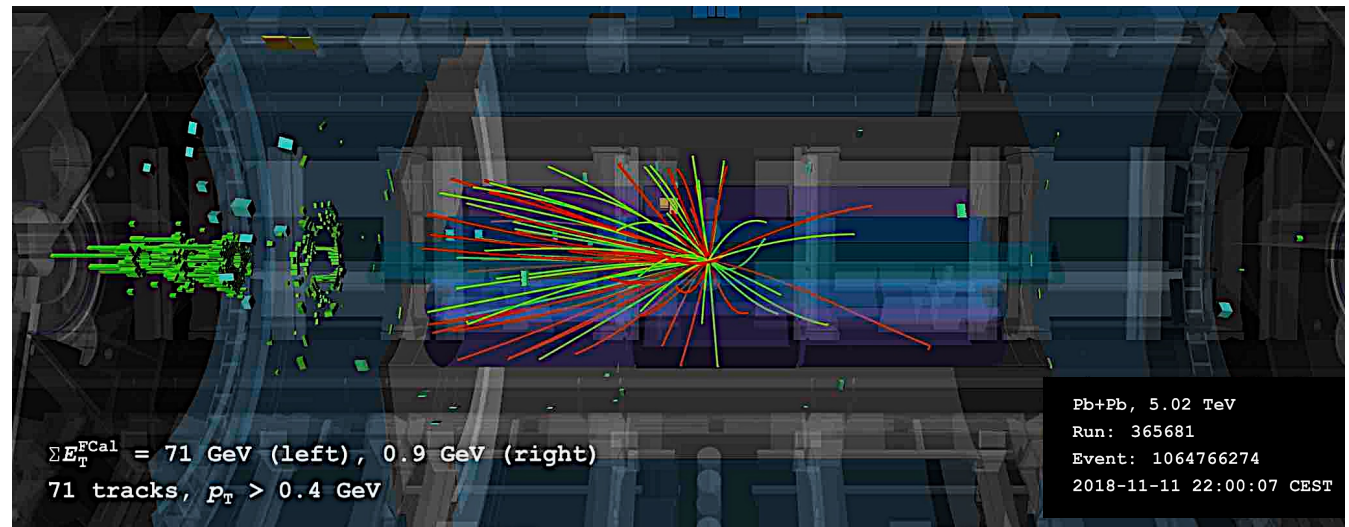
Kyle Lee (MIT)

What is happening in these collisions?

Normal



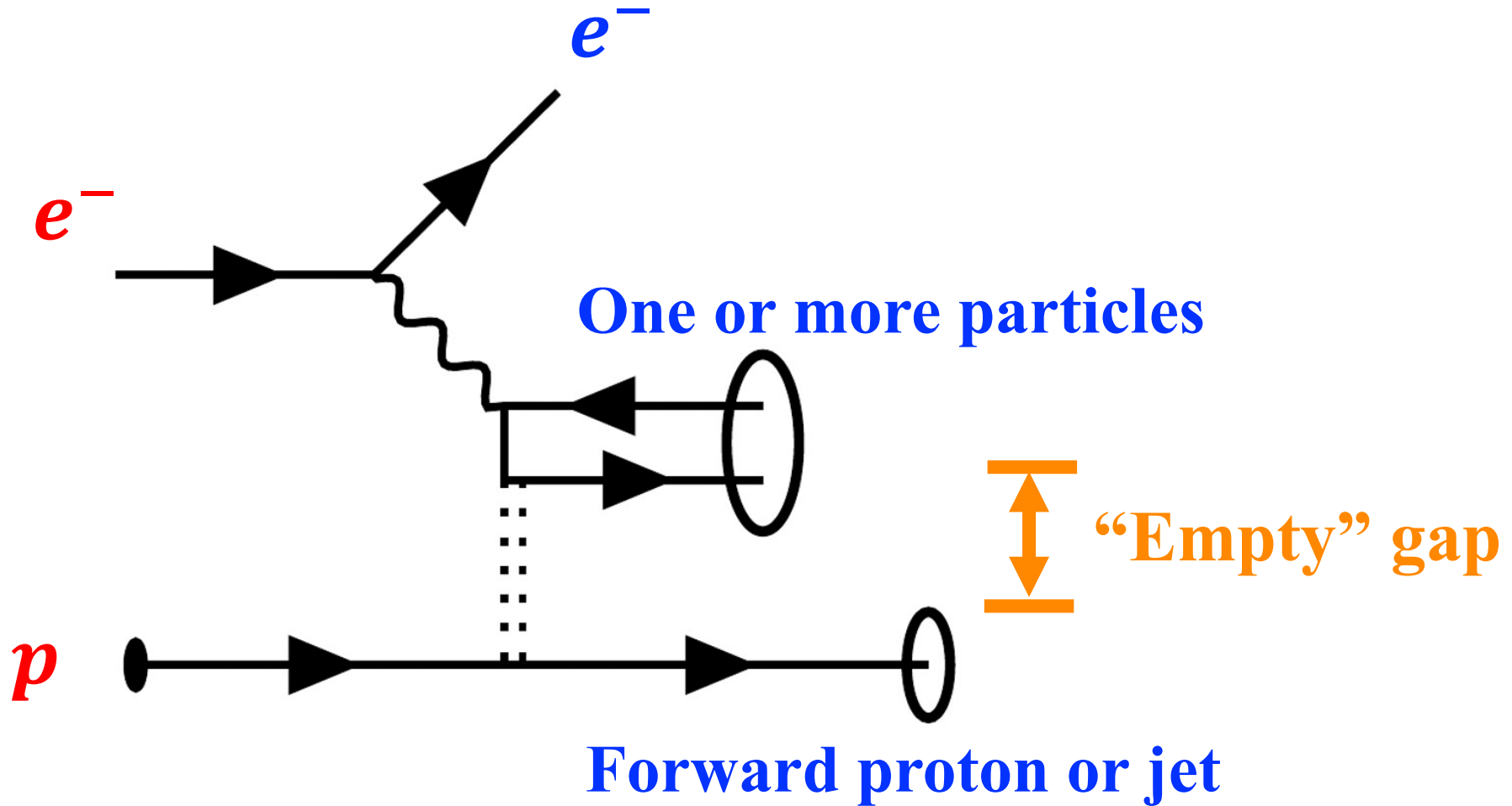
Gapped
(Diffraction)



Diffractive ep collision

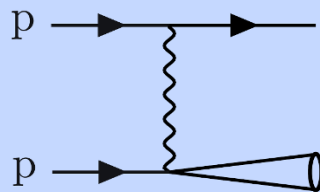
Incoming

Outgoing

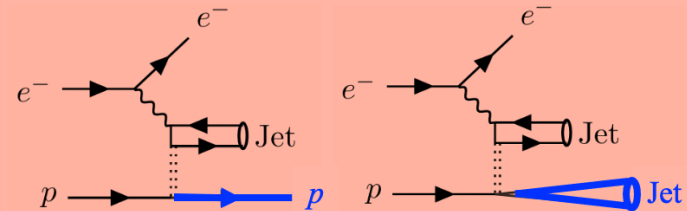


Many other types of diffractive collisions

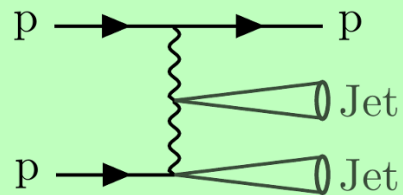
eA , AA , ep , pp collisions



Coherent or incoherent



Single or multi jet/gap



Tagged final states

Heavy mesons
Dijet photoproduction
Etc.

- **10%** of HERA events
- **20%** of EIC events
- **30%** of inelastic LHC events

Current state of diffractive understanding

Collins' factorization of the hard physics in diffraction:

$$F_{2/L}^D = H_{2/L}^{(i)} \otimes f_i^D \quad (\text{Diffractive PDF})$$

Berera/Soper, hep-ph/9509239. Collins, hep-ph/9709499.

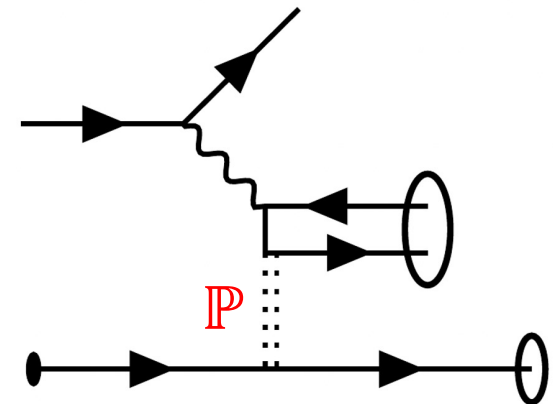
Large Q^2

These papers note that they do not rigorously factorize the forward scattering dynamics (“**Regge factorization**”)

Ingelman-Schlein model for Regge physics:

$$f_i^D = f_{i/\mathbb{P}} \times f_{\mathbb{P}/p} \quad (\text{Pomeron PDFs})$$

Ingelman/Schlein (1984). Frankfurt et al., 2203.12289.



How well can we do without a full factorization? ⁶

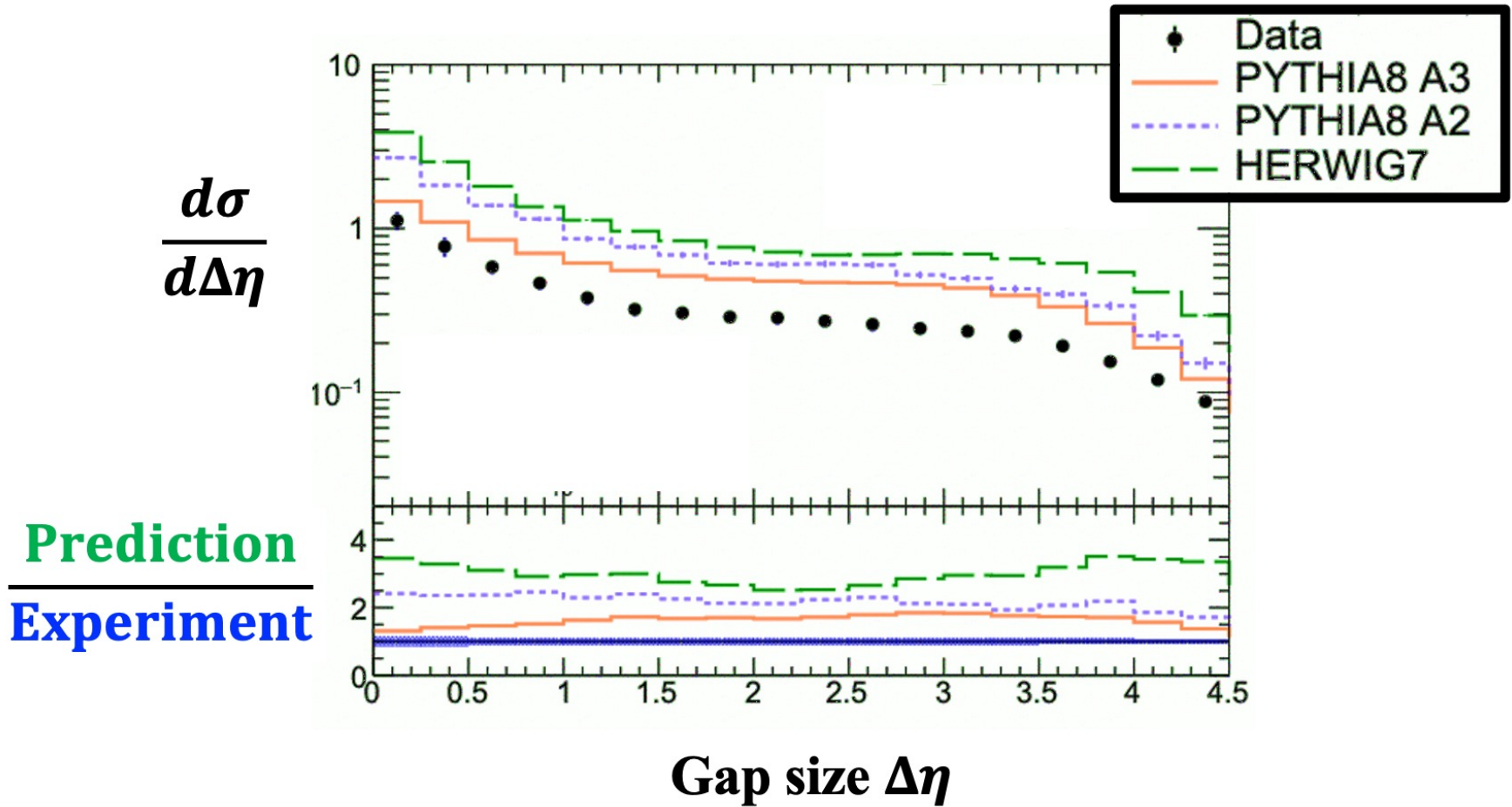
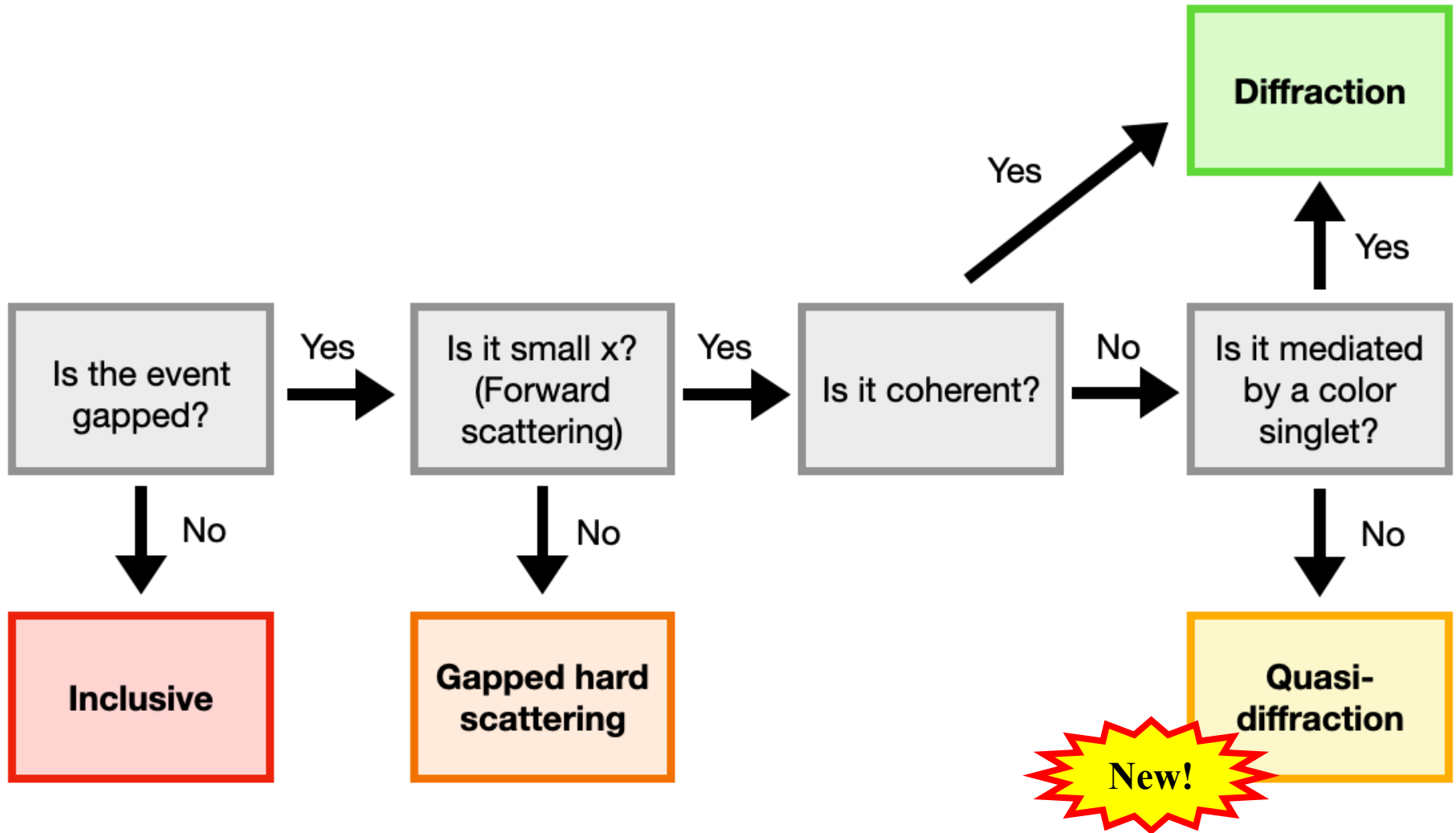


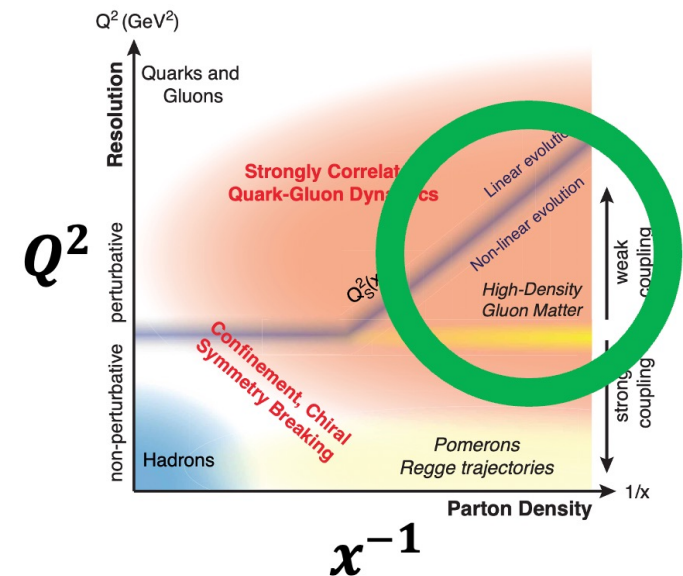
Figure: ATLAS, 1911.00453. $\sqrt{s} = 8 \text{ TeV}$, $0.016 < |t| < 0.43 \text{ GeV}^2$, $-4.0 < \log_{10} \xi < -1.6$

Distinguishing diffraction from backgrounds



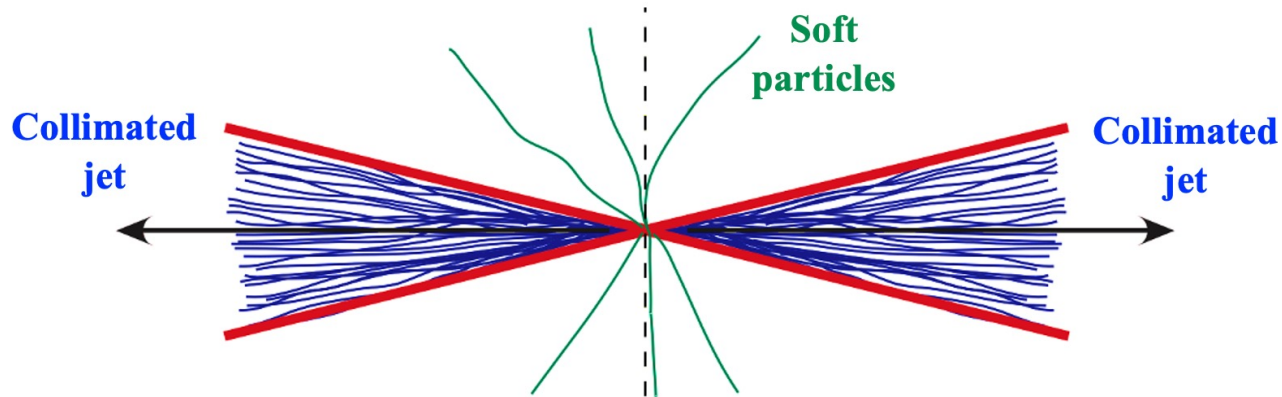
Tool building for small x physics

- 1950s** Pomeron & Reggeon description of high-energy scattering
(Regge, Pomeranchuk, Chew, Frautschi...)
- 1973** Development of QCD
(Gross, Politzer, Wilczek)
- 1977** BFKL equation for small- x evolution
(Balitsky, Fadin, Kuraev, Lipatov)
- 1983** Discussion of saturation
(Gribov, Levin, Ryskin)
- 1986** Nonlinear corrections to DGLAP
(Mueller, Qiu)
- 1994** Color Glass Condensate formalism for saturation regime
(McLerran, Venugopalan)
- 1999** BK/JIMWLK equations, smaller- x evolution
(Balitsky, Kovchegov, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner)
- 2016** A new tool for forward physics: Glauber SCET
(Rothstein, Stewart)



Effective field theory (EFT), schematically

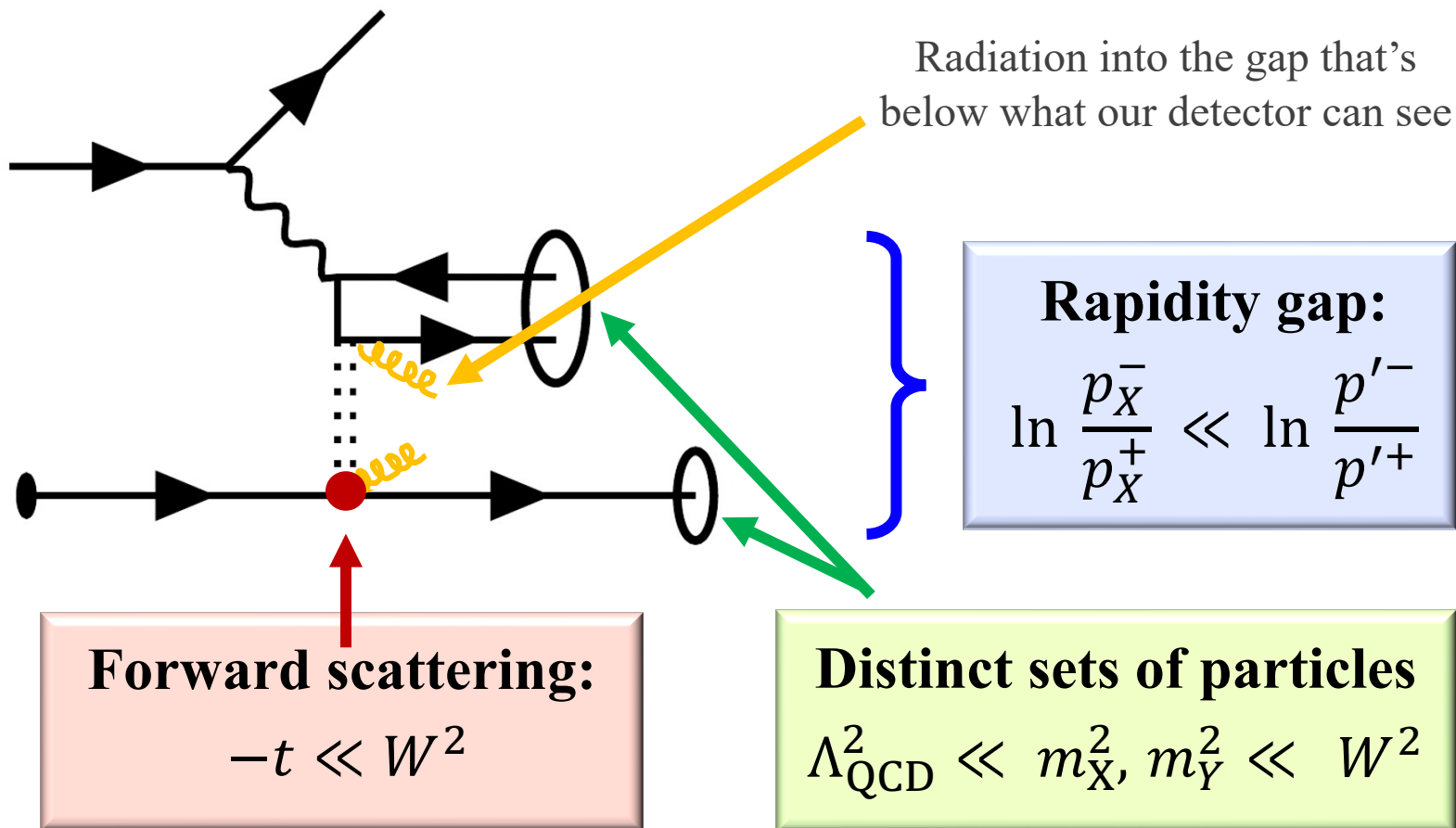
- Collisions generally involve many distinct energy scales
- We can separately calculate the physics at each scale



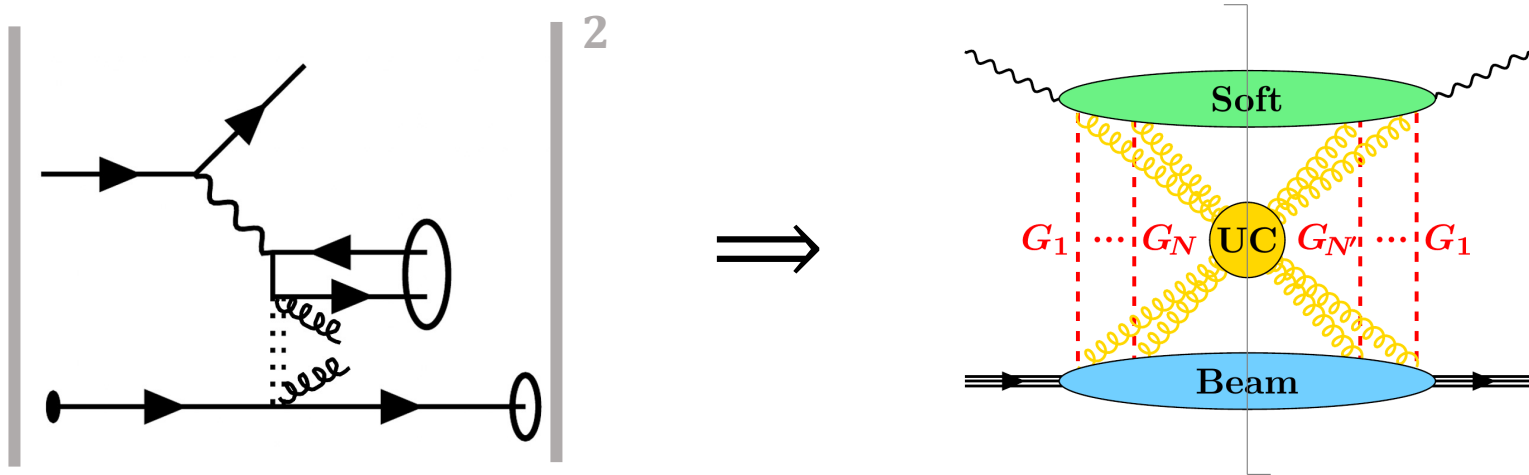
$$\mathcal{L}_{\text{QCD}} \xrightarrow[\text{irrelevant modes}]{\text{Integrate out}} \mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{jet}} + \mathcal{L}_{\text{soft}}$$

$$\langle \mathcal{O} \rangle = \langle \mathcal{O}_{\text{jet}} \rangle \times \langle \mathcal{O}_{\text{soft}} \rangle \times \langle \mathcal{O}_{\text{jet}} \rangle$$

Constraints on (quasi-)diffraction



Factorizing (quasi-)diffraction using SCET



$$F_i^D = B \otimes_{\perp} S_i \otimes_{\pm} U$$

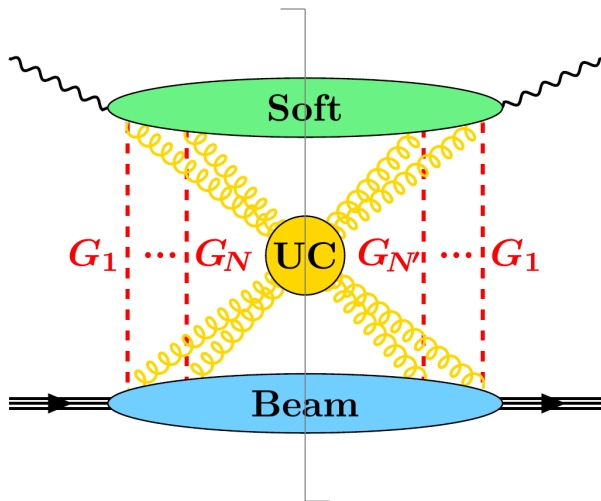
Beam function	Ultrasoft-collinear function	Soft function
Hadronic physics	Radiation into gap	Central region
Hadronic matrix element of SCET operators	Vacuum matrix element of Wilson line operators	Vacuum matrix element of SCET operators

Factorizing (quasi-)diffraction using SCET

$$\mathbf{F}_i^D = \sum_{N, N'=1}^{\infty} \sum_{\{R_X\}} \iint_{(N, N')}^{\perp} \int dp_n^+ dp_s^- dp_g^+ dp_g^- (2\pi)^2 \delta(Qz - p_n^+ - p_g^+) \delta(Q/\beta - p_s^- - p_g^-) \mathbf{B} \mathbf{U} \mathbf{S}$$

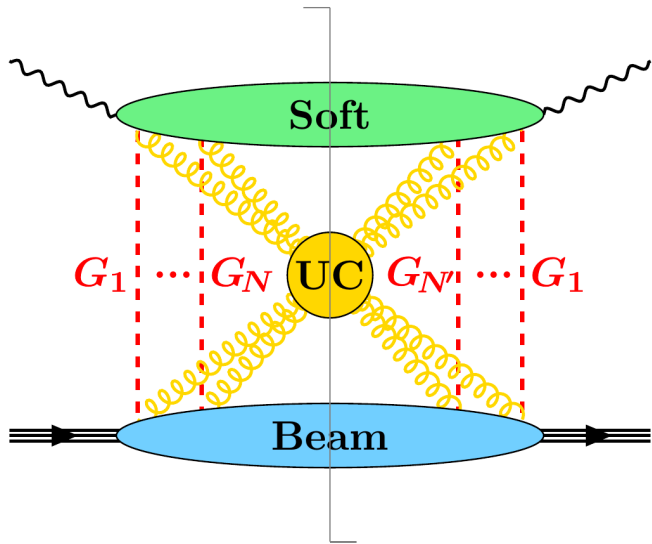
$$\mathbf{B} = \int [d\tilde{v}] e^{\frac{i}{2}v^- p_n^+} \sum_{Y_n} \langle p | \left\{ \prod_{i=1}^{N-1} \mathcal{O}_n^{A_i}(\tilde{v}, \tau_{i\perp}) \bar{\mathcal{O}}_n^{A_N}(\tilde{v}) \right\} P_{NR_A} | Y_n \rangle \langle Y_n | P_{N'R_A} \left\{ \prod_{j=1}^{N'-1} \mathcal{O}_n^{A'_j}(0, \tau'_{j\perp}) \bar{\mathcal{O}}_n^{A'_{N'}}(0) \right\} | p \rangle.$$

$$\mathbf{U} = \frac{N_{\vec{R}}}{4} \int dy^+ dy^- e^{\frac{i}{2}y^+ \bar{p}_g^- + \frac{i}{2}y^- \bar{p}_g^+} \sum_{X'_{uc}} \langle 0 | P_{NR_A} \bar{T} \prod_{i=1}^N \mathbb{U}_{n\bar{n}}^{A_i B_i}(y^+, y^-) P_{NR_B} | X'_{uc} \rangle \langle X'_{uc} | P_{N'R_A} T \prod_{j=1}^{N'} \mathbb{U}_{n\bar{n}}^{A'_j B'_j}(0) P_{N'R_B} | 0 \rangle$$



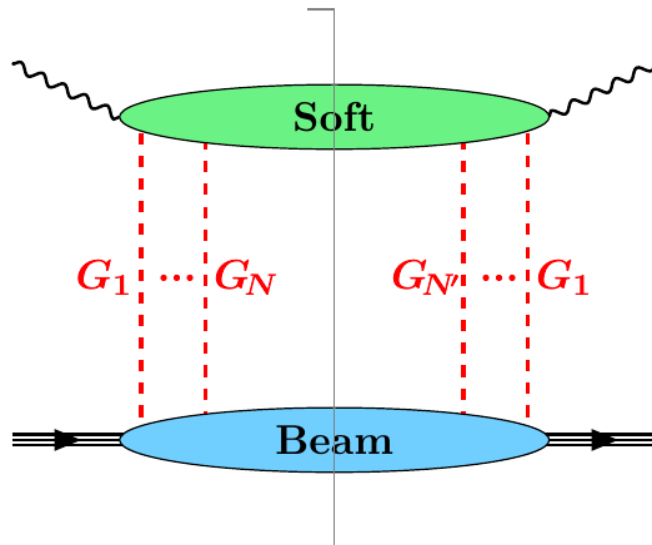
$$\begin{aligned} \mathbf{S} &= \int [d\tilde{y}_1] [d\tilde{y}'_1] d^d z e^{\frac{i}{2}(y_1^+ - y_1'^+) p_s^-} e^{iz \cdot q} \mathcal{P}_{i\mu\nu} \\ &\times \sum_{X_s} \langle 0 | \bar{T} J_s^\mu(z) \left\{ \prod_{i=1}^N \mathcal{O}_s^{B_i}(\tilde{y}_1, -\tau_{i\perp}) \right\} P_{NR_B} | X_s \rangle \\ &\times \langle X_s | P_{N'R_B} T J_s^\nu(0) \left\{ \prod_{j=1}^{N'} \mathcal{O}_s^{B'_j}(\tilde{y}'_1, -\tau'_{j\perp}) \right\} | 0 \rangle \end{aligned}$$

Diffraction vs quasi-diffraction



$$F_i^{\text{quasi}} = B \otimes U \otimes S$$

- Color non-singlet: U nontrivial
- Can radiate into the gap



$$F_i^D = B \otimes S$$

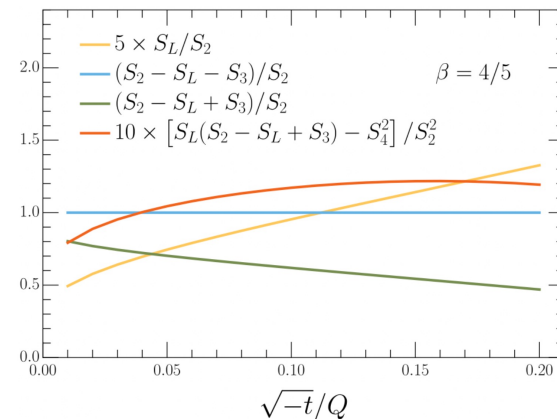
- Color singlet: U is a δ -function
- Nontrivial result: can't radiate into the gap!

Many predictions we can make

E.g., leading F_i^{quasi} ratio predictions for $t \ll Q^2 \ll s$

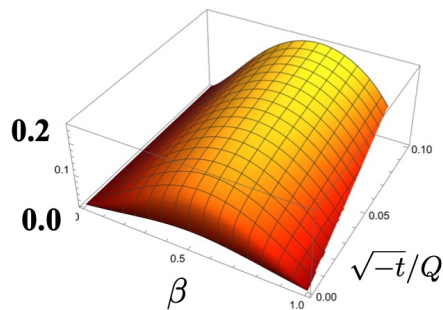
One Glauber: no convolution!

$$\frac{F_i^{\text{quasi}}}{F_2^{\text{quasi}}} = \frac{B \times U \times S_i}{B \times U \times S_2} = \frac{S_i}{S_2}$$

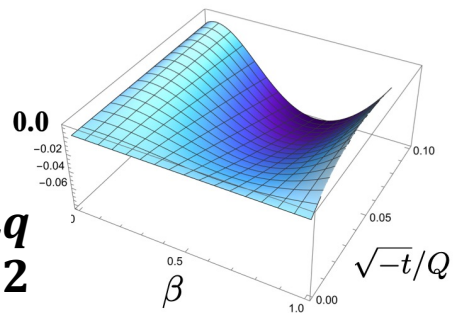


Cross-check: **Positivity bounds** from
Arens et al. hep-ph/9605376

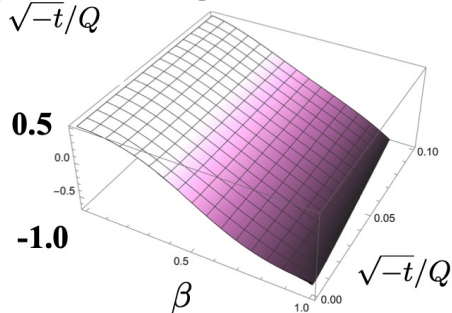
$$F_L^q / F_2^q$$



$$F_4^q / F_2^q$$

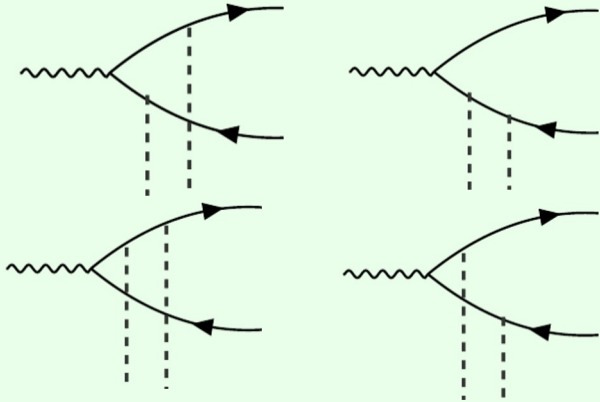


$$F_3^q / F_2^q$$



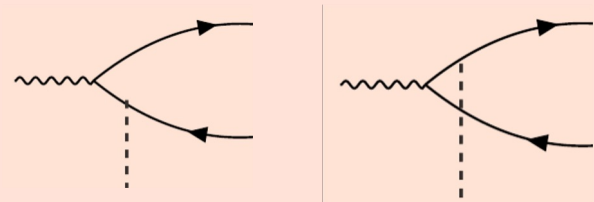
- 4 unpolarized & 4 polarized F_i^D at leading power, **large!**
- $F_{3,4}^D$ are largely ignored in literature

Diffraction (color singlet)



- α_s^2 from two Glaubers
- No suppression from U
(we can show it's trivial!)

Quasi-diffraction (color non-singlet)



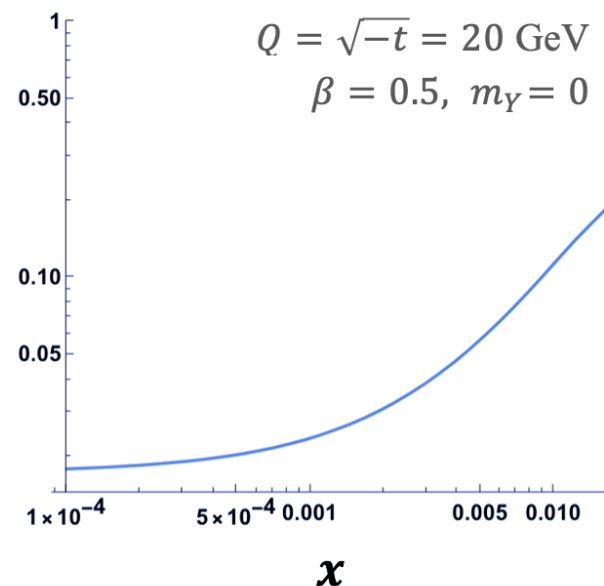
- α_s^1 from one Glauber
- Sudakov suppression
from nontrivial U

Radiation into the gap

Ultrasoft-collinear function:

- Result obtained from e^+e^- hemisphere soft function
- $U \propto \exp \left[-C_A \alpha_s \log^2 \frac{W^2}{Q^2} \right]$
 $\propto \exp \left[-C_A \alpha_s \log^2 \frac{1}{x} \right]$

**Sudakov
suppression**

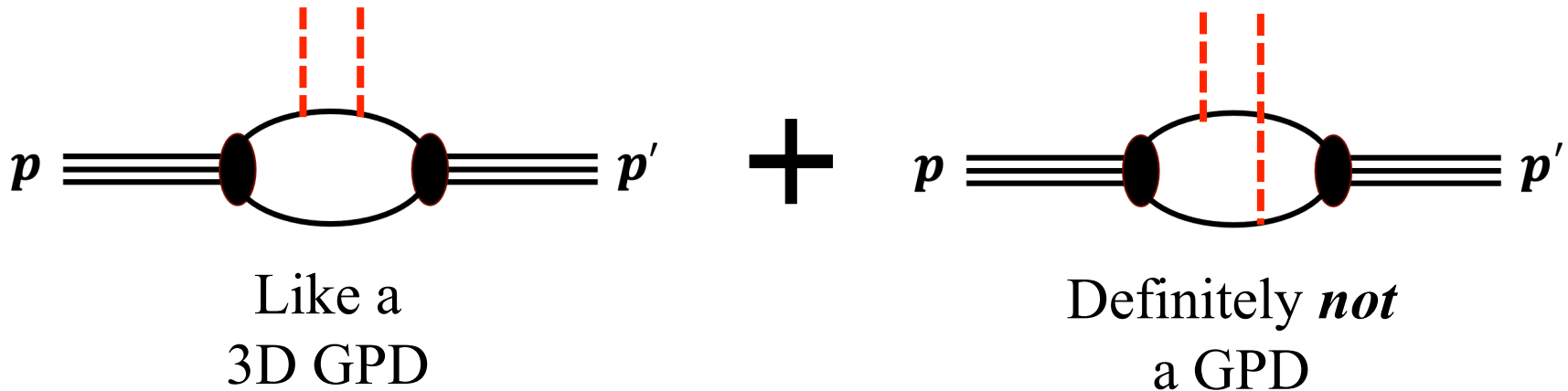


- Can't measure color of incoherent exchange
- Irreducible color-nonsinglet (quasi-diffractive) background!

What type of PDF does diffraction access?

$$\mathbf{B} = \int [d\tilde{v}] e^{\frac{i}{2}v^- p_n^+} \sum_{Y_n} \langle p | \left\{ \prod_{i=1}^{N-1} \mathcal{O}_n^{A_i}(\tilde{v}, \tau_{i\perp}) \bar{\mathcal{O}}_n^{A_N}(\tilde{v}) \right\} P_{NR_A} | Y_n \rangle \langle Y_n | P_{N'R_A} \left\{ \prod_{j=1}^{N'-1} \mathcal{O}_n^{A'_j}(0, \tau'_{j\perp}) \bar{\mathcal{O}}_n^{A'_{N'}}(0) \right\} | p \rangle$$

Many beam function topologies, e.g.

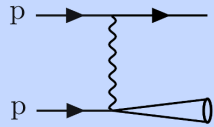


Ongoing work:

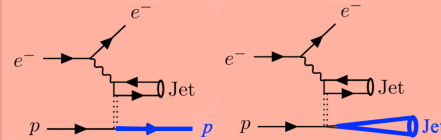
- What is the nature of the diffractive beam function?
- Does it match onto any traditional PDFs, etc., in any limits?

Much more we can do now...

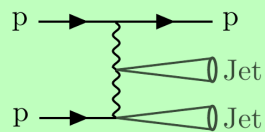
eA, AA, ep, pp collisions



Coherent or incoherent

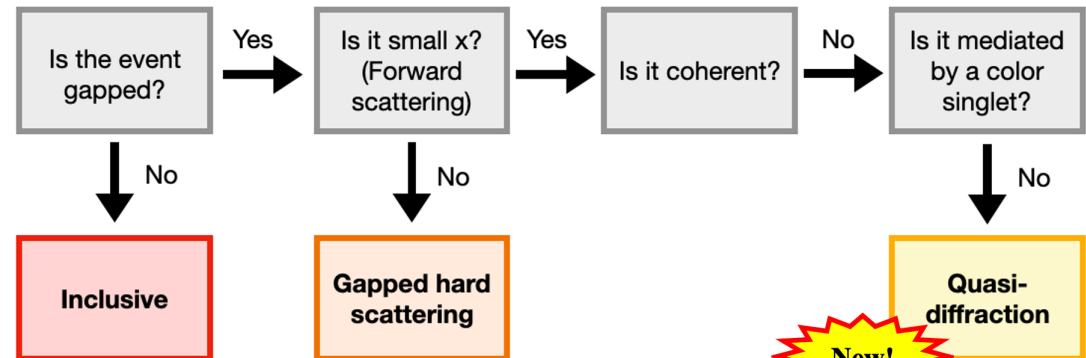


Single or multi jet/gap



Tagged final states

Heavy mesons
Dijet photoproduction
Etc.



SCET: $F_j^D(x, Q^2, \beta, t, m_Y^2) = \mathcal{P}_j^{\mu\nu} \mathbf{S}_{\mu\nu}(Q^2, \beta, t, \tau_{i\perp}) \otimes \mathbf{B}(t, m_Y^2, \tau_{i\perp})$

Collins: $F_{2/L}^D(x, Q^2, \beta, t) = H_{2/L}^{(i)}\left(\frac{\beta}{\zeta}, Q^2\right) \otimes f_i^D\left(\zeta, Q^2, \frac{x}{\beta}, t\right)$

“Diffractive PDF”

Differences:

- SCET $\lambda = Q/\sqrt{s} \ll 1$ vs. Collins $\lambda_t = \sqrt{-t}/Q \ll 1$
- As explained by Collins, his formula does **not** include the Regge factorization for forward physics at bottom vertex
- However, these results should match for λ & $\lambda_t \ll 1$

Regge factorization of dPDF

SCET: $F_j^D(x, Q^2, \beta, t, m_Y^2) = \mathcal{P}_j^{\mu\nu} S_{\mu\nu}(Q^2, \beta, t, \tau_{i\perp}) \otimes B(t, m_Y^2, \tau_{i\perp})$

Collins: $F_{2/L}^D(x, Q^2, \beta, t) = H_{2/L}^{(i)}\left(\frac{\beta}{\zeta}, Q^2\right) \otimes f_i^D\left(\zeta, Q^2, \frac{x}{\beta}, t\right)$

Refactorize dPDF for λ & $\lambda_t \ll 1$, giving $F = H \otimes S_c \otimes B'$:

$$f_i^D\left(\zeta, Q^2, \frac{x}{\beta}, t, m_Y^2\right) = S_c(\zeta, Q, t) \otimes B'(Qz, t, m_Y^2)$$

SCET: $f_i^D \left(\zeta, Q^2, \frac{x}{\beta}, t, m_Y^2 \right) = S_c(\zeta, Q, t) \otimes B'(Qz, t, m_Y^2)$

IS model: $f_i^D \left(\zeta, Q^2, \frac{x}{\beta}, t \right) = f_{i/\mathbb{P}}(\zeta, Q^2) \times f_{\mathbb{P}/p} \left(\frac{x}{\beta}, t \right)$

Differences:

- Convolution vs. multiplication
- Number and nature of arguments
- Transverse momentum dependence vs. longitudinal-type PDFs

1. Four interesting structure functions
2. First all-orders Regge (forward) factorization
3. Study of backgrounds (e.g. color nonsinglet)
4. Experimental implications

Next steps:

- **Precision physics:** Higher order, resummation, ...
- **Underlying physics:** Behavior in saturation regime?
Connection to PDFs/GPDs/etc. in any limit?
- **Other cases:** Hadron colliders, semi-inclusive processes, more jets/gaps, etc.

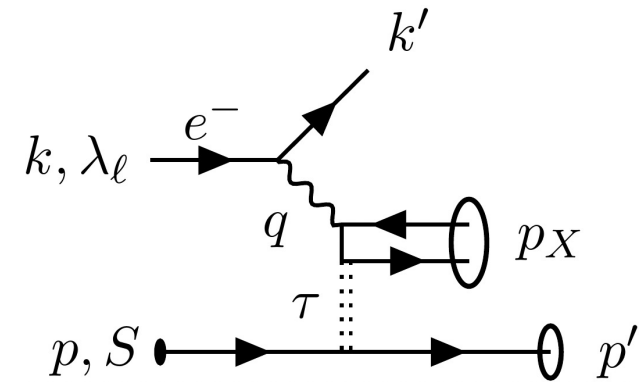
New horizons for HEP theorists?

- **Theory:** New tools for EIC physics can naturally extend to HEP-focused colliders like the LHC
- **Phenomenology:** Improves understanding of Standard Model physics, which enables studies of new physics in the far-forward region
- **Experiment:** Better understanding of diffraction can improve tracking of luminosity, understanding pile-up, building MC generators

Backup slides

Lorentz invariants

	Energy scales	Momentum fractions
Familiar from DIS	$Q^2 = -q^2$ $W^2 = (p + q)^2$ $s = (p + k)^2$	$x = \frac{Q^2}{2p \cdot q}$ $y = \frac{p \cdot q}{p \cdot k}$
Diffraction	$t = \tau^2 < 0$ $m_Y^2 = p'^2 > 0$ $m_X^2 = p_X^2 > 0$	$\beta = \frac{Q^2}{2q \cdot \tau}$ $\bar{x} = \frac{k \cdot \tau}{k \cdot p}$ $z = \frac{p \cdot p'}{p \cdot q}$



Largely unexplored variable \bar{x}

Note that only 7 of these are linearly independent

New kinematic bounds

$$0 < x < 1, \quad 0 < y < 1, \quad 0 < Q^2 < s$$

$$\frac{1}{1 + \frac{-t}{Q^2} + \left(\frac{\bar{x}}{x} - y\right)(1-z)} < \beta < \frac{1}{1 + \frac{-t}{Q^2}}$$

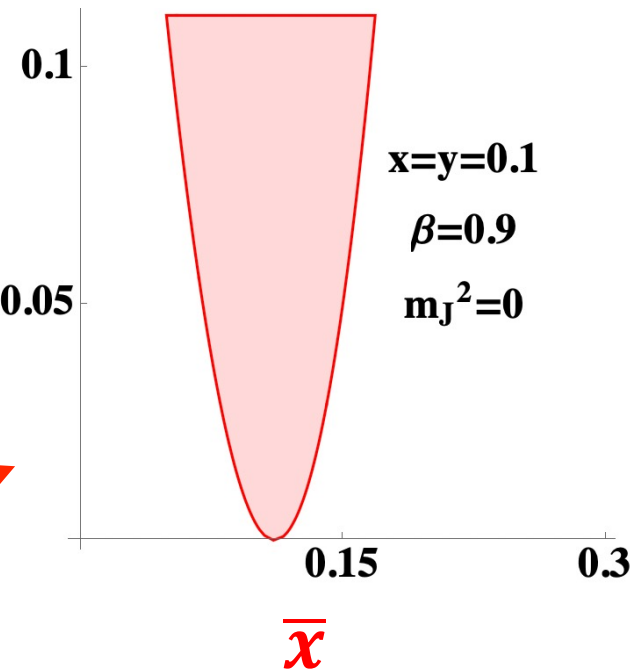
$$\Lambda_{QCD}^2 \lesssim m_Y^2 < \frac{1 - \bar{x}}{\bar{x}}(-t)$$

$$\frac{\bar{x}}{1 - \bar{x}} \frac{m_Y^2}{Q^2} < \frac{-t}{Q^2} < \frac{1 - \beta}{\beta}$$

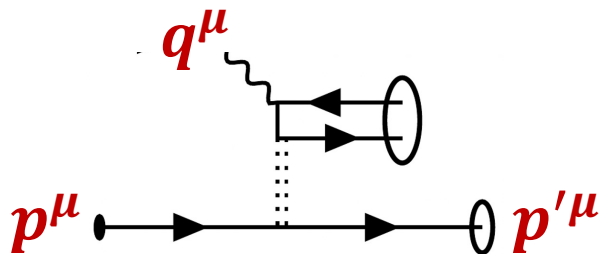
$$-1 < \frac{\beta xyz - 2\beta xz + x - \beta \bar{x}}{2\sqrt{\beta xz(1-y)(\beta xz - x + \beta)}} < 1$$

Etc.

$$\frac{-t}{Q^2}$$



Constructing structure functions



Constraints: $q_\mu W_D^{\mu\nu} = 0$ and $W_D^{\mu\nu} = W_D^{\nu\mu}$

➤ Convenient to build orthonormal basis $q^\mu \perp U^\mu \perp X^\mu$

$$\begin{aligned}
 W_D^{\mu\nu} = & \frac{1}{2x} \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) F_L^D + \frac{1}{2x} \left(U^\mu U^\nu - g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) F_2^D \\
 & + \frac{1}{2x} \left(2X^\mu X^\nu - U^\mu U^\nu + g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) F_3^D + \frac{U^\mu X^\nu + X^\mu U^\nu}{2x} F_4^D
 \end{aligned}$$

Literature often neglects F_3^D & F_4^D !

Coefficients

$$L_{\mu\nu} W_D^{\mu\nu} = \frac{2s}{y} \left[-\frac{y^2}{2} F_L^D + \left(1 - y + \frac{y^2}{2} \right) F_2^D \right. \\ \left. + \left(\frac{2(\mathbf{k} \cdot \mathbf{X})^2 y^2}{Q^2} - 1 + y \right) F_3^D + \frac{2y^2 (\mathbf{k} \cdot \mathbf{X})(\mathbf{k} \cdot \mathbf{U})}{Q^2} F_4^D \right]$$

\bar{x} & y only appear in coefficients, not

$$F_i^D(x, Q^2, \beta, t, m_Y^2)$$

How to miss $F_{3,4}^D$:

- Integrate over \bar{x}
- Assume $p' \parallel p$

Coefficients:

$$\text{➤ } k \cdot X = Q^2 \frac{x - \bar{x}\beta - (2-y)xz\beta}{2N_X xy\beta}$$

$$\text{➤ } k \cdot U = \frac{Q(2-y)}{2y}$$

Auxiliary:

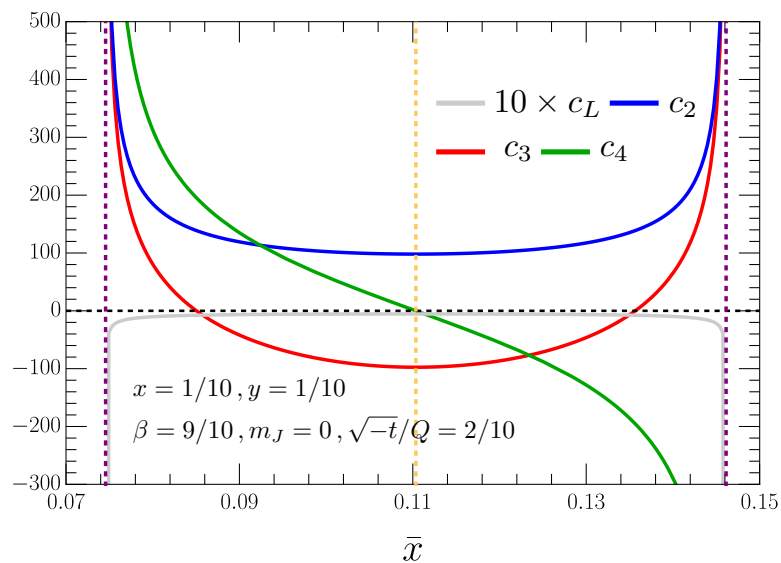
$$\text{➤ } N_X^2 = -t + z^2 Q^2 - \frac{zQ^2}{\beta}$$

$$\text{➤ } z = \frac{x}{Q^2} (m_Y^2 - t)$$

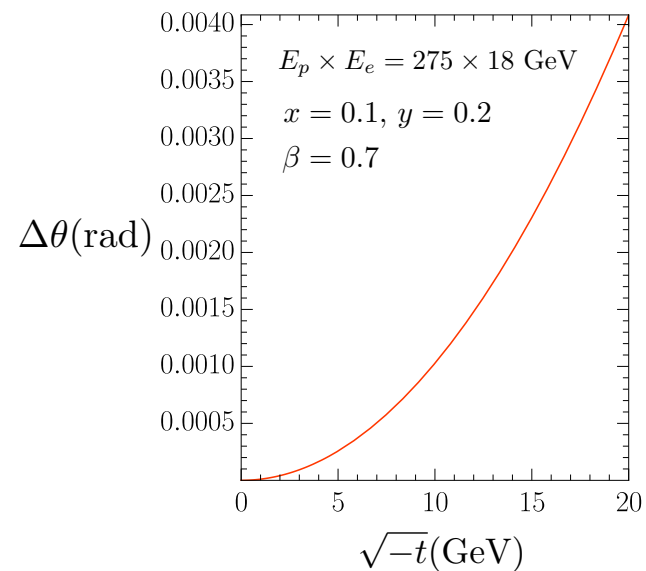
Seeing $F_{3,4}^D$ in experiments

Why did ZEUS not see $F_{3,4}^D$ in hep-ex/0408009 ?

Coefficients c_i of F_i^D



Resolution to resolve four bins of \bar{x}



Coefficients are large, **but** need good angular resolution at small t

Summary of kinematics

	Our work	Most of the literature
Independent invariants	7 (DIS + \bar{x}, t, β, m_Y^2)	6 (No \bar{x})
Unpolarized structure functions	4	2
Polarized structure functions	14 (4 nonzero at leading power)	2

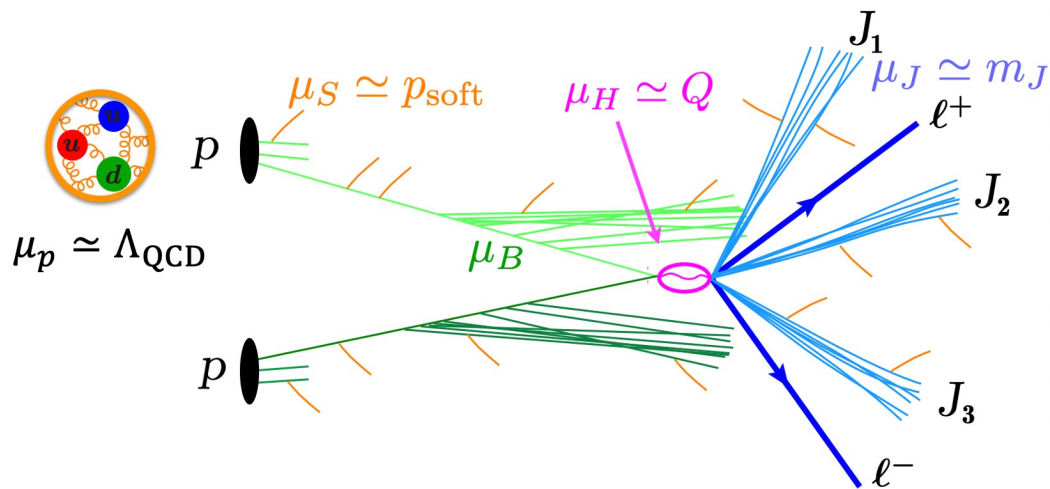
+ **Classification of backgrounds**

+ **New kinematic bounds**

Advantages of EFT

$$\mathcal{L}_{\text{QCD}} \rightarrow \lambda^0 \mathcal{L}_{\text{EFT}}^{(0)} + \lambda^1 \mathcal{L}_{\text{EFT}}^{(1)} + \lambda^2 \mathcal{L}_{\text{EFT}}^{(2)} + \dots$$

- Power expansion, efficient for multi-scale problems
- Calculations are systematically improvable, to arbitrary precision



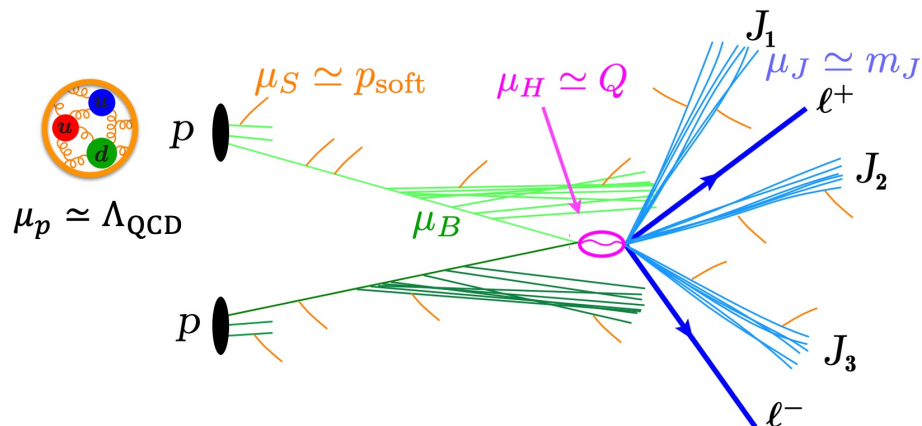
Simplifies study of:

- Factorization
- Perturbative calculations
- Resummation
- Power corrections

SCET Lagrangian

Mode	Momentum in $(+, -, \perp)$
Hard	$(1, 0, 0)$
Collinear	$(1, \lambda^2, \lambda)$ or $(\lambda^2, 1, \lambda)$
Soft	$(\lambda, \lambda, \lambda)$
Ultrasoft	$(\lambda^2, \lambda^2, \lambda^2)$
Glauber	$(\lambda^a, \lambda^b, \lambda)$ for $a + b > 2$

$$p^\pm = p^t \mp p^z$$



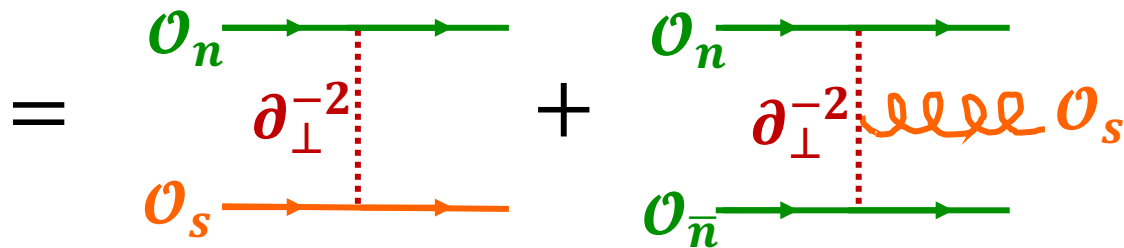
$$\mathcal{L}_{\text{SCET}}^{(0)} = \mathcal{L}_{\text{hard}} + \mathcal{L}_{\text{collinear}} + \mathcal{L}_{\text{soft}} + \mathcal{L}_{\text{ultrasoft}} + \mathcal{L}_{\text{Glauber}}$$

- **Glauber:** talks between different sectors
- **Hard interaction:** also connects sectors, but only occurs *once*

SCET operators

Example:

$\mathcal{L}_{\text{Glauber}}^{(0)}$



$$= i \frac{\not{n} \cdot p}{2(n \cdot p)(\bar{n} \cdot p) + p_{\perp}^2 + i0}$$

$$= ig T^A n_{\mu} \frac{\not{n}}{2}$$

$$= ig T^A \left[n_{\mu} + \frac{\gamma_{\perp}^{\mu} \not{p}_{\perp}}{\bar{n} \cdot p} + \frac{\not{p}'_{\perp} \gamma_{\perp}^{\mu}}{\bar{n} \cdot p'} - \frac{\not{p}'_{\perp} \gamma_{\perp}^{\mu}}{(\bar{n} \cdot p)(\bar{n} \cdot p')} \bar{n}_{\mu} \right] \frac{\not{n}}{2}$$

$$= \frac{ig^2 T^A T^B}{\bar{n} \cdot (p-q)} \left[\gamma_{\perp}^{\mu} \gamma_{\perp}^{\nu} - \frac{\gamma_{\perp}^{\mu} \not{p}_{\perp}}{\bar{n} \cdot p} \bar{n}_{\nu} - \frac{\not{p}'_{\perp} \gamma_{\perp}^{\nu}}{\bar{n} \cdot p'} \bar{n}_{\mu} + \frac{\not{p}'_{\perp} \gamma_{\perp}^{\nu}}{(\bar{n} \cdot p)(\bar{n} \cdot p')} \bar{n}_{\mu} \bar{n}_{\nu} \right] \frac{\not{n}}{2}$$

$$+ \frac{ig^2 T^B T^A}{\bar{n} \cdot (p'+q)} \left[\gamma_{\perp}^{\nu} \gamma_{\perp}^{\mu} - \frac{\gamma_{\perp}^{\nu} \not{p}'_{\perp}}{\bar{n} \cdot p} \bar{n}_{\mu} - \frac{\not{p}_{\perp} \gamma_{\perp}^{\mu}}{\bar{n} \cdot p'} \bar{n}_{\nu} + \frac{\not{p}_{\perp} \gamma_{\perp}^{\mu}}{(\bar{n} \cdot p)(\bar{n} \cdot p')} \bar{n}_{\mu} \bar{n}_{\nu} \right] \frac{\not{n}}{2}$$

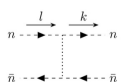
$$= \frac{-i}{(\bar{n} \cdot q)(n \cdot k) + q_{\perp}^2 + i0} (g_{\mu\nu} - (1-\tau) \frac{q_{\mu} q_{\nu}}{(n \cdot q)(n \cdot k) + q_{\perp}^2}) \delta_{ab}$$

$$= g^{abc} n_{\mu} \left\{ (\bar{n} \cdot q_1) g_{\nu\lambda} - \frac{1}{2} \left(1 - \frac{1}{\tau}\right) (\bar{n}_{\lambda} q_{1,\nu} + \bar{n}_{\nu} q_{2,\lambda}) \right\}$$

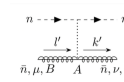
$$= -\frac{ig^2}{2} n_{\mu} \left[f^{abc} f^{cde} (\bar{n}_{\lambda} g_{\nu\rho} - \bar{n}_{\rho} g_{\nu\lambda}) + f^{ade} f^{bce} (\bar{n}_{\nu} g_{\lambda\rho} - \bar{n}_{\lambda} g_{\nu\rho}) \right.$$

$$\left. + f^{ace} f^{bde} (\bar{n}_{\nu} g_{\lambda\rho} - \bar{n}_{\rho} g_{\nu\lambda}) \right]$$

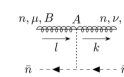
$$= \frac{ig^2}{4} n_{\mu} \bar{n}_{\nu} \bar{n}_{\rho} n_{\lambda} \left(1 - \frac{1}{\alpha}\right) (f^{ace} f^{bde} + f^{ade} f^{bce})$$



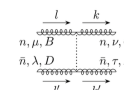
$$= \frac{-8\pi i \alpha_s}{(\bar{l}_{\perp} - \bar{k}_{\perp})^2} \left[\bar{u}_n \frac{\not{n}}{2} T^A u_n \right] \left[\bar{v}_s \frac{\not{n}}{2} T^A v_s \right]$$



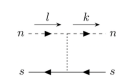
$$= \frac{-8\pi \alpha_s f^{ABC}}{(\bar{l}_{\perp} - \bar{k}_{\perp})^2} \left[\bar{u}_n \frac{\not{n}}{2} T^A u_n \right] \left[(\bar{n} \cdot k) g_{\perp}^{\mu\nu} - \bar{n}^{\mu} l_{\perp}^{\nu} - \bar{n}^{\nu} k_{\perp}^{\mu} + \frac{(l'_{\perp} \cdot k'_{\perp}) \bar{n}^{\mu} n^{\nu}}{n \cdot k'} \right]$$



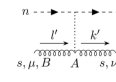
$$= \frac{-8\pi \alpha_s f^{ABC}}{(\bar{l}_{\perp} - \bar{k}_{\perp})^2} \left[(\bar{n} \cdot k) g_{\perp}^{\mu\nu} - \bar{n}^{\mu} l_{\perp}^{\nu} - \bar{n}^{\nu} k_{\perp}^{\mu} + \frac{(l_{\perp} \cdot k_{\perp}) \bar{n}^{\mu} \bar{n}^{\nu}}{\bar{n} \cdot k} \right] \left[\bar{v}_s \frac{\not{n}}{2} T^A v_s \right]$$



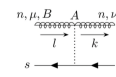
$$= \frac{8\pi i \alpha_s f^{ABC} f^{ADE}}{(\bar{l}_{\perp} - \bar{k}_{\perp})^2} \left[(\bar{n} \cdot k) g_{\perp}^{\mu\nu} - \bar{n}^{\mu} l_{\perp}^{\nu} - \bar{n}^{\nu} k_{\perp}^{\mu} + \frac{(l_{\perp} \cdot k_{\perp}) \bar{n}^{\mu} \bar{n}^{\nu}}{\bar{n} \cdot k} \right] \times \left[(n \cdot k') g_{\perp}^{\lambda\tau} - n^{\lambda} l_{\perp}^{\tau} - n^{\tau} k_{\perp}^{\lambda} + \frac{(l'_{\perp} \cdot k'_{\perp}) n^{\lambda} n^{\tau}}{n \cdot k'} \right]$$



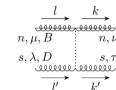
$$= \frac{-8\pi i \alpha_s}{(\bar{l}_{\perp} - \bar{k}_{\perp})^2} \left[\bar{u}_n \frac{\not{n}}{2} T^A u_n \right] \left[\bar{v}_s \frac{\not{n}}{2} T^A v_s \right]$$



$$= \frac{-8\pi \alpha_s f^{ABC}}{(\bar{l}_{\perp} - \bar{k}_{\perp})^2} \left[\bar{u}_n \frac{\not{n}}{2} T^A u_n \right] \left[(n \cdot k') g_{\perp}^{\mu\nu} - n^{\mu} l_{\perp}^{\nu} - n^{\nu} k_{\perp}^{\mu} + \frac{(l'_{\perp} \cdot k'_{\perp}) n^{\mu} n^{\nu}}{n \cdot k'} \right]$$



$$= \frac{-8\pi \alpha_s f^{ABC}}{(\bar{l}_{\perp} - \bar{k}_{\perp})^2} \left[(\bar{n} \cdot k) g_{\perp}^{\mu\nu} - \bar{n}^{\mu} l_{\perp}^{\nu} - \bar{n}^{\nu} k_{\perp}^{\mu} + \frac{(l_{\perp} \cdot k_{\perp}) \bar{n}^{\mu} \bar{n}^{\nu}}{\bar{n} \cdot k} \right] \left[\bar{v}_s \frac{\not{n}}{2} T^A v_s \right]$$



$$= \frac{8\pi i \alpha_s f^{ABC} f^{ADE}}{(\bar{l}_{\perp} - \bar{k}_{\perp})^2} \left[(\bar{n} \cdot k) g_{\perp}^{\mu\nu} - \bar{n}^{\mu} l_{\perp}^{\nu} - \bar{n}^{\nu} k_{\perp}^{\mu} + \frac{(l_{\perp} \cdot k_{\perp}) \bar{n}^{\mu} \bar{n}^{\nu}}{(\bar{n} \cdot k)} \right] \times \left[(n \cdot k') g_{\perp}^{\lambda\tau} - n^{\lambda} l_{\perp}^{\tau} - n^{\tau} k_{\perp}^{\lambda} + \frac{(l'_{\perp} \cdot k'_{\perp}) n^{\lambda} n^{\tau}}{n \cdot k'} \right]$$

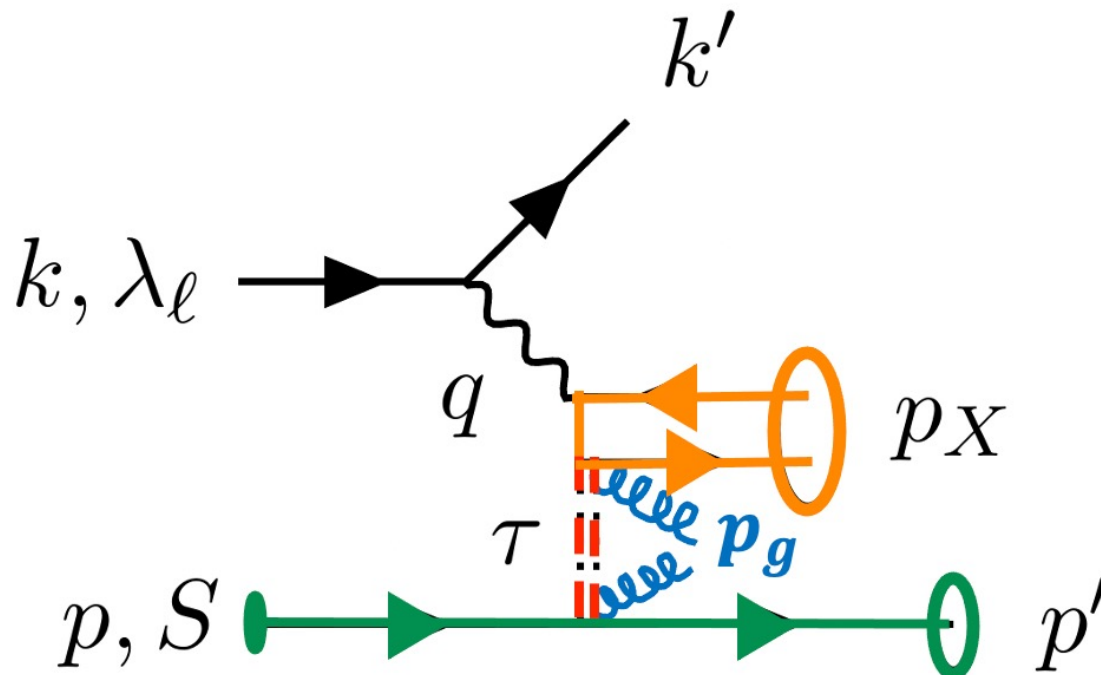
The upshot of EFT: Helpful for organizing calculations for multi-scale problems

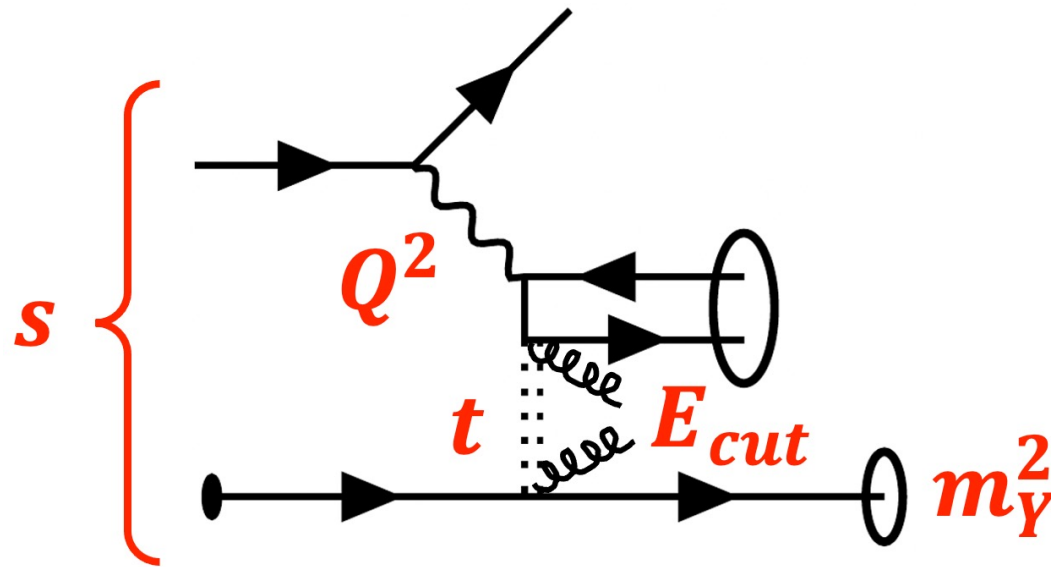
SCET modes in diffraction

Mode	$(+, -, \perp)$
Collinear	$\sqrt{s}(\lambda^3, \lambda^{-1}, \lambda)$
Soft	$\sqrt{s}(\lambda, \lambda, \lambda)$
Glauber	$\sqrt{s}(\lambda^3, \lambda, \lambda)$
Ultrasoft-collinear	$\sqrt{s}(\lambda^3, \lambda, \lambda^2)$

$$\lambda = \frac{Q}{\sqrt{s}} \ll 1$$

Ultrasoft in CM frame
(Here: Breit frame)





Required:

$$\lambda = \frac{Q}{\sqrt{s}} \ll 1$$

Optional: $\lambda_t = \frac{\sqrt{-t}}{Q}$, $\lambda_g = \frac{E_{cut}}{Q}$, $\rho = \frac{m_Y}{\sqrt{-t}}$, $\lambda_\Lambda = \frac{\Lambda_{QCD}}{Q}$

Subject to bounds, can have $\lambda_i \ll 1$, $\lambda_i \sim 1$, or $\lambda_i \gg 1$