QCD in the Early Universe[1,2]

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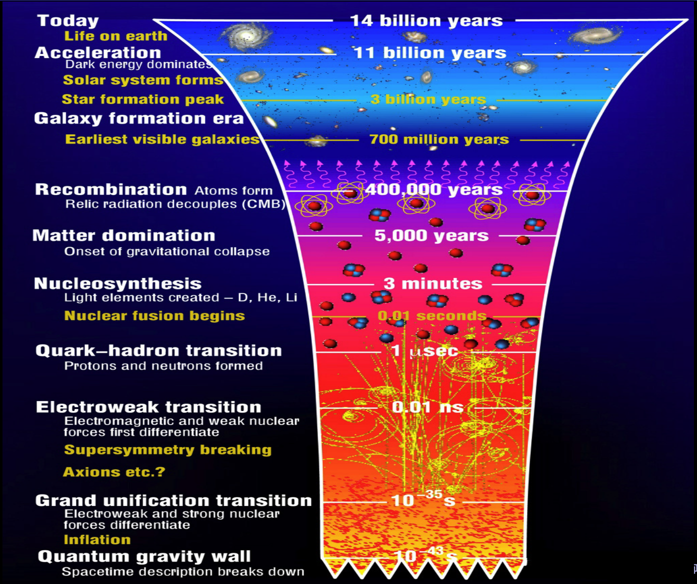
Introduction: Equation of State of the Early Universe

A Brief History of the Universe



Evolution of the Early Universe

Cooling of the Universe: The Insight from the Asymptotic Freedom

Universe cooled down via series of first- or second-order phase transitions (PT) associated with the various spontaneous symmetry breakings (SSBs) of the basic non-Abelian gauge fields, see e.g. the classical textbooks [Linde:1978], [Bailin and Love:2004], [Boyanovsky et al:2006].

Standard Model Predicts Two Phase Transitions

1. The electroweak (EW) PT at $T \sim m_H$ provides masses to elementary particles. LQFT calculations show that for $m_H \geq 67$ GeV this PT is an analytic crossover [Kajantie et al.:1996], [Csikor et al.:1999].
2. At $T < 200$ MeV the SSB of the chiral symmetry of the $SU(3)_c$ color group, the QCD, takes place.

Early Universe was in local thermal equilibrium (LTE) (see slide A1)
$\Rightarrow$ Its evolution can be described by hydrodynamics.

Early Universe Made Simple

Friedman equation: SCM $\Rightarrow g_{\mu\nu}^{FLRW} \rightarrow R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}(R - 2\Lambda) = -8\pi G T_{\mu\nu}$ with $\mu = \nu = 0$

$$ds^2 = g_{\mu\nu}^{FLRW} dx^\mu dx^\nu = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right] \quad (1)$$

$$H^2(t) \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\epsilon - \frac{k}{a^2} + \frac{\Lambda}{3} \quad (2)$$

Perfect fluid: $T^\mu{}_\mu = diag(\epsilon, -p, -p, -p) \Rightarrow$ Fluid equation

$$\dot{\epsilon} + 3(\epsilon + p)H(t) = 0 \quad (3)$$

- [Ornik,Weiner:1987] : Early Universe: $\epsilon \gtrsim 1\text{GeV fm}^{-3} \Rightarrow$ can neglect k and Λ in (2)

$$-\frac{d\epsilon}{3\sqrt{\epsilon}(\epsilon + p)} = \sqrt{\frac{8\pi G}{3}}dt \quad \Rightarrow \quad \dot{\epsilon} + \sqrt{\frac{24\pi G}{3}}(\sqrt{\epsilon}(\epsilon + p(\epsilon)) = 0 \quad (4)$$

- Integration of (4) using barotropic form of EoS $p(\epsilon)$ yields $\epsilon(t)$.
- **Example:** Time-independent speed of sound $p/\epsilon = w$ (see also slide A10):

$$\int \frac{d\epsilon}{\epsilon + p(\epsilon)} = -\log a^3 + const., \ \epsilon \sim a^{-3(1+w)} \sim (t(1+w))^{-2}, \ a(t) \sim t^\alpha, \alpha = \frac{2}{3(1+w)} \quad (5)$$

What is Changing During Phase Transitions: Effective Degrees of Freedom

$$g_{\rm eff}(T) \equiv \frac{\epsilon(T)}{\epsilon_0(T)}\,, \quad \epsilon_0(T) = \frac{\pi^2}{30}T^4 \tag{6}$$

$$h_{\rm eff}(T) \equiv \frac{s(T)}{s_0(T)}\,, \quad s_0(T) = \frac{2\pi^2}{45}T^3 \tag{7}$$

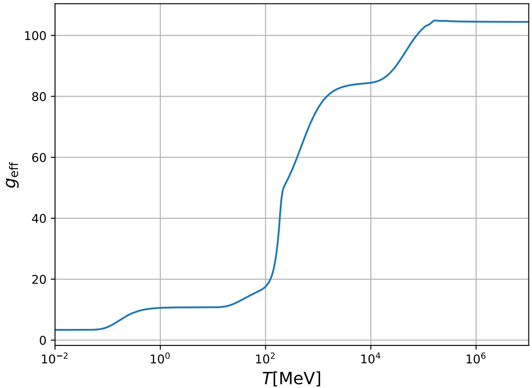
$$g_{\rm eff}^{id}(T) = h_{\rm eff}^{id}(T) = \frac{7}{8}4N_{\rm F} + 3N_{\rm V} + 2N_{\rm V0} + N_{\rm S} \tag{8}$$

- For an adiabatic process

$$s(T) = \frac{\epsilon(T) + p(T)}{T} \tag{9}$$

- EoS in cosmology $p = w\epsilon$

$$w(T) = \frac{sT}{\epsilon} - 1 = \frac{4h_{\rm eff}(T)}{3g_{\rm eff}(T)} - 1 \tag{10}$$



$g_{\rm eff}(T)$ in the SM taking into account interactions between particles, obtained with both perturbative and lattice methods. [Hindmarsh et al.:2020].

Cosmological Parametrization of EoS: from Particle Dust to Stiff Fluid

- Causality: $$\frac{p}{\epsilon} = w(T) = \frac{sT}{\epsilon} - 1 = \frac{4h_{\text{eff}}(T)}{3g_{\text{eff}}(T)} - 1 \leq 1 \;\Rightarrow\; \frac{h_{\text{eff}}(T)}{g_{\text{eff}}(T)} \leq \frac{3}{2} \qquad (11)$$
- The EOS with $w > 1/3$ has $\epsilon - 3p < 0$ and was first discussed in [Zeldovich:1961] where the upper bound of $w = 1$ was identified with the absolutely stiff fluid.
(*Baryons interacting via massive vector particles or free time-like massless scalar field*.)
- At the the stiff fluid limit $c_s \to 1$ the functions h_{eff} and g_{eff} become frozen, i.e. temperature independent.
$$c_s^2(T) = \frac{dp}{d\epsilon} = T\frac{ds}{d\epsilon} + s\frac{dT}{d\epsilon} - 1 = \frac{4}{3}\left[\frac{4h_{\text{eff}}(T) + T\cdot h'_{\text{eff}}(T)}{4g_{\text{eff}}(T) + T\cdot g'_{\text{eff}}(T)}\right] - 1 \qquad (12)$$
- The stiff fluid EoS is frequently used in the description of the neutron stars and in cosmology of the Very Early Universe.
- Can the stiff fluid represent the (non-equilibrium) QCD matter filling the Early Universe?

Energy Density of Ideal Massless Gas in Early Universe

- Simultaneous presence of the EW and the QCD matter in LTE is one of the remarkable differences between the QGP produced in accelerator experiments and the deconfined QCD matter in the Early Universe.
- Including only particles which can be at $T \lesssim T_{\mathrm{EW}} = 160$ GeV considered massless: $g_{\mathrm{eff}}^{\mathrm{EW}} = \frac{7}{8}(12+6)+2 = 17.75$ and $g_{\mathrm{eff}}^{\mathrm{QCD}} = 2\times 8 + \frac{7}{8}(3\times N_F \times 2 \times 2)$.
- At $T \lesssim T_{\mathrm{EW}}$ and for N_F active quark flavors, QCD matter has a factor of $g_{\mathrm{eff}}^{\mathrm{QCD}}/g_{\mathrm{eff}}^{\mathrm{EW}} \simeq 2 \div 4$ more energy and pressure than the EW matter.
- Even for $T \gg T_c^{\mathrm{EW}}$ wih $g_{\mathrm{eff}}^{\mathrm{QCD}} = 79$ and $g_{\mathrm{eff}}^{\mathrm{EW}} = 26.75$ it has a factor of $g_{\mathrm{eff}}^{\mathrm{QCD}}/g_{\mathrm{eff}}^{\mathrm{EW}} \simeq 3$ larger ϵ and p than the EW matter.
- Quarks and gluons make the densest form of ordinary matter in Early Universe below T_{GUT}.

EoS Based on the Fundamental Theory: the SM and GUT

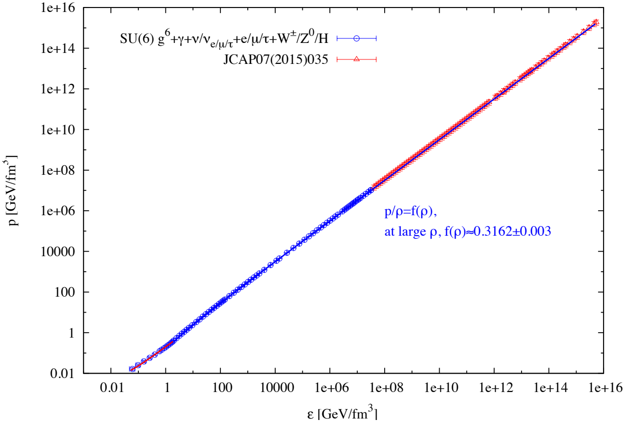


Figure 1: Combined EoS $p(\epsilon), \epsilon \equiv \rho$ of QCD and EW matter, using lattice results of [Borsanyi et al:2016] extended to include other DoFs such as γ, neutrinos, leptons, EW, and Higgs bosons as well as perturbative results from [Laine, Mayer:2015]. Adapted from [Tawfik,Mishustin:2019].

- For details on calculations of the EoS in the SM see slide A3

The EoS parametrization

- GUT EoS (△ in Fig. 1) $p_{\rm GUT} = (0.330 \pm 0.024)\epsilon$ valid for $10^8 \lesssim \epsilon \leq 10^{16}$ GeV·fm^{-3} corresponds to ideal massless gas.
- Combined EoS for QCD and EW eras has two independent contributions $p_1(\epsilon)$ and $p_2(\epsilon)$.

$$p_{\rm SM} = p_1(\epsilon) + p_2(\epsilon)\,, \quad p_1(\epsilon) = b\epsilon\,, \quad p_2(\epsilon) = a + c\epsilon^d \tag{13}$$

$$a = 0.048 \pm 0.016,\ b = 0.316 \pm 0.031,\ c = -0.21 \pm 0.014,\ d = -0.576 \pm 0.034$$

- $a \approx 0 \Rightarrow$ EoS $p_2(\epsilon) \approx c\epsilon^d, c < 0, d < 0 \Rightarrow$ must have $p_2 < 0$! Coincides with generalized Chaplygin EoS often applied as for DE EoS, see e.g. [Kamenshchik et al:2001].
- The trace anomaly $\Theta(T) = T^{\mu}_{\ \mu}/T^4$, where $T^{\mu}_{\ \mu}$ is the energy-momentum tensor

$$\Theta(T) \equiv \frac{\epsilon - 3p}{T^4} = \frac{\epsilon(1-3b) - 3a - 3c\epsilon^d}{T^4} > 0; \quad \epsilon \gtrsim (3-4){\rm GeV \cdot fm^{-3}}\,, \tag{14}$$

For $\epsilon \to \infty$ falls as $\Theta \sim g_{\rm eff}\epsilon^{-3/2}$, cf. Eq. (7).

N.B. $\Theta(T) > 0$ is also satisfied by non-interacting gas of massive particles, see slide A10

Saturated QCD matter in the Early Universe ?

Glass Properties of Saturated QCD matter, CGC–BH Correspondence

- **In condensed matter** glass is a non-equilibrium, disordered state of matter acting like solids on short time scales but liquids on long time scales [Mauro:2014,Sethna:2021].
- Two scales of glasma: $$\tau_{\rm wee} = \frac{1}{k^-} = \frac{2k^+}{k_\perp^2} = \frac{2xP^+}{k_\perp^2} \ll \frac{2P^+}{k_\perp^2} \approx \tau_{\rm valence}\,. \qquad (15)$$
  $\Rightarrow$ Valence modes are static over the time scales of wee modes [Berges et al.:2020].
- Glasses are formed when liquids are cooled too fast to form the crystalline equilibrium state. This leads to an enormous # of possible configurations $N_{\rm gl}(T)$ into which the glasses can freeze $\Rightarrow$ large entropy $S = \ln N_{\rm gl}(T)$, such that $S(T=0) > 0$, [Sethna:1988].
- Correspondence between highly occupied condensates of N weakly interacting gravitons and gluons [Dvali, Venugopalan:2021].
  Both attain a maximal entropy S_{max} when the occupation number f and the coupling α of their constituents satisfy $f = 1/\alpha(Q_S)$, where Q_S represents the point of optimal balance between the kinetic energies of the individual constituents and their potential energies.
- $\exists$ Glasma $T > 0$? If yes, what is its relevance for Early Universe?

Early Universe Occupied by Massless Particles

- Universe represents expanding sphere $V = (4/3)\pi R^3,\ R \sim a(T)$ occupied by weakly interacting massless particles with thermal de Broglie wavelength* $\lambda_{\rm th} = \pi^{2/3}/T$ in LTE.
- Each particle carries the fraction $x = k^+/P^+$ of the total light-cone momentum $P^+ \sim 1/a(T)$ of the matter contained in V.
- LTE $\Rightarrow$ particle momenta are concentrated around $k^+ \approx Q = 2\pi T$, i.e. around $x \approx k^+/P^+ \approx 2\pi T \cdot a(T)$.
- Particle with $\lambda_{\rm th} = \pi^{2/3}/T$ occupies the fraction $F(T)$ of V

$$F(T) \sim \frac{(\lambda_{\rm th}(T))^3}{a^3(T)} = \pi^2(T \cdot a(T))^{-3} \sim T^{-3}(\epsilon)\epsilon^{1/(1+w)} \tag{16}$$

*) Ideal gas at fixed T: $\lambda_{\rm th} \approx \langle \lambda_{\rm B} \rangle = \langle h/p \rangle$. Gas is classical if the average interparticle spacing $r_s \approx (V/N)^{1/3} \gg \lambda_{\rm th}$. For $r_s < \lambda_{\rm th}$ we have a quantum gas.

For massless particles $\lambda_{\rm th} = \pi^{2/3}/T$. For massive particles $\lambda_{\rm th} = \sqrt{2\pi/(mT)}$.

The Glasma Phase of the Early Universe

- **Example:** $T^{-3}(\epsilon) \sim \epsilon^{-3/4}$ & $w = (\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, 1) \Rightarrow F(T) = (const., \epsilon^{-1/12}, \epsilon^{-3/20}, \epsilon^{-1/4})$.
- The QCD saturation scale $R_S = 1/Q_S$ is now corresponds to thermal de Broglie wavelength of massless gluons $\lambda_S = \pi^{2/3}/T_S$, where T_S is the saturation temperature.
- At what moment of expansion is $f = 1/\alpha_s(2\pi T_S)$ satisfied?
- Saturation is relevant for $T \leq T_S$ i.e. when $$\left.\frac{xG_U(x, T_S^2)F(T_S)}{2(N_c^2 - 1)}\right|_{x \approx T_S \cdot a(T_S)} \approx \frac{1}{\alpha_S(T_S)} \quad (17)$$
- But $xG_U(x, T_S^2)$ is unknown ... is ill defined (better talk to Raju).

Yang-Mills Field EoS in Cosmology

- Abelian vector field breaks down the isotropy of the Universe expected by SCM $\Rightarrow$ at least triplet of vector fields is needed.
- For the classical gluon field A_μ^a in the space-time with metric $g_{\mu\nu}$ with the scale-invariant Lagrangian $\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu,a}$

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\lambda)} \partial_\nu A_\lambda - g^{\mu\nu} \mathcal{L} = \frac{1}{4\pi} \left(-F^{\mu\alpha,a} F_\alpha^{\nu,a} + \frac{1}{4} g^{\mu\nu} F_{\alpha\beta}^a F^{\alpha\beta,a} \right) \tag{18}$$

  where $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\nu^b A_\nu^a$ is YM field tensor.
- Early studies: Coupled Einstein-Yang-Mills equations with gauge group SU(2) in the FLRW universes [Galtsov, Volkov:1991].
- Basic features of the Einstein-Yang-Mills homogeneous and isotropic cosmological solutions can be attributed to the conformal nature of the YM field.

$\Rightarrow$ Needs break down of conformal symmetry by some new emergent scale.

Emergent Scales of QCD

- QCD has two emergent scales both originating from the breakdown of translation invariance due to the extended geometry of the object: Λ_{QCD} and Q_S.
- Both violations of conformal invariance can be modeled by appropriate EoS.
- Calculable in the effective action approach [Addazi et al.:2022] which incorporates the vacuum polarisation effects giving rise to the trace anomaly $T^{\mu}_{\ \mu} \neq 0$ already at the perturbative level (analogical to the Coleman-Weinberg symmetry breaking!).

$$T^{\mu}_{\ \mu} = \frac{\beta(\bar{g}^2)}{2\bar{g}^2}\mathcal{F}^2; \;\; \mathcal{A}^a_\mu \equiv g A^a_\mu; \;\; \mathcal{F}^a_{\mu\nu} \equiv g F^a_{\mu\nu} \,, \tag{19}$$

where $\bar{g}$ is the running coupling $\bar{g}^2 = \bar{g}^2(\mathcal{J})$ and $\mathcal{J} = \mathcal{F}^a_{\mu\nu}\mathcal{F}^{\mu\nu a}$. For details see A13.

- Thermal evolution can be computed for the scalar glueball field in the confined phase (work in progress).

Prevalence of Chromomagnetic Field at High T

- At high T, in the asymptotic freedom regime $\beta(\bar{g}^2) < 0$, the chromomagnetic vacuum represents a stable solution:

$$\langle T^{\mu}_{\ \mu} \rangle = \frac{\beta(\bar{g}^2)}{2\bar{g}^2} \langle \mathcal{F}^2 \rangle < 0 \,, \qquad \langle \mathcal{F}^2 \rangle > 0 \,, \qquad \langle \mathbf{B}^2 \rangle > \langle \mathbf{E}^2 \rangle \,. \tag{20}$$

- At low temperatures in the non-perturbative regime, a new metastable chromoelectric solution with $T^{\mu}_{\ \mu} > 0$ is also possible [Addazi et al.:2022] $\Rightarrow$ look for a mechanism which flips the sign of $T^{\mu}_{\ \mu}$

- Two color charges α_s of radius $R = \lambda_{th} = \pi^{2/3}/T$ passing each other at relative distance b create in the center-of-mass frame $B^a_{\perp}$ (Biot–Savarat)

$$B^a_{\perp} = \gamma \alpha_s(T) \frac{b}{R^3} = \gamma \alpha_s(T) \frac{b T^3}{\pi^2} \tag{21}$$

- Effect strongly depends on the temperature $B^a_{\perp} \sim T^3$. The breakdown of superposition principle for YM fields leads to large non-zero contribution when summing over many pairs of particles. $\Rightarrow$ In Early Universe large chromomagnetic fields we prevalent.

Phenomenological Shortcut: Modified Bag Model EoS

$$\epsilon(T) = \sigma T^4 - CT^2 + \mathcal{B}\,, \qquad p(T) = \frac{\sigma}{3} T^4 - DT^2 - \mathcal{B}\,, \qquad \Theta(T) = \frac{3D - C}{T^2} \equiv \frac{A}{T^2} \tag{22}$$

$$\sigma = \frac{\pi^2}{30} g_{\mathrm{eff}}^{\mathrm{QCD}}\,, \quad \sigma(N_F = 0, \ldots, 5) \approx 5, \ldots, 23\,; \quad \mathcal{B}^{1/4} \approx 220\mathrm{MeV} \tag{23}$$

- In pure gauge theory up to $T \approx (2 \div 5) T_c$, the dominant power-like correction to pQCD behavior is $\mathcal{O}(T^{-2})$ rather than $\mathcal{O}(T^{-4})$ [Pisarski:2006].
- Quadratic thermal terms in the deconfined phase can also be obtained from gauge/string duality [Zuo:2014vga].

1. $C = D > 0$: LQCD motivated "fuzzy" bag model EoS [Pisarski:2006, Megias et al.:2007].
2. $C = -D < 0$: Gluonic q-particle EoS with $\mathcal{B}(T) = -CT^2 + \mathcal{B}$ [Schneider, Weise:2001].

- In both cases the trace anomaly $\Theta(T) = (\epsilon - 3p) T^{-4} > 0^*$ $\Rightarrow$ Quadratic terms lead to $dp/d\epsilon < 1/3$ and hence to the softening of the EoS close to the critical temperature T_c.

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*) $\Theta(T) > 0$ is also satisfied by non-interacting gas of massive particles, see slide A10.

Modified Bag Model EoS with Negative Trace Anomaly

- Consider the same EoS but with $\mathcal{B} = 0$ and $\Theta(T) < 0^*$ valid at $T \gg \Lambda_{QCD}$.

$$\Theta(T) = \frac{\epsilon - 3p}{T^4} = \frac{3D - C}{T^2} = \frac{A}{T^2} < 0\,; \quad A < 0 \tag{24}$$

$\Rightarrow$ with decreasing T the fluid moves away from the ideal massless gas EoS.

- The barotropic form of the new EoS reads:

$$p(\epsilon) = \frac{1}{3}\left(\epsilon - \Theta(T)T^4\right) = \frac{1}{3}\left(1 - sgn(A)|A|\,T^2(\epsilon)\right)\,, \quad T^2(\epsilon) \equiv \frac{C + \sqrt{C^2 + 4\sigma\epsilon}}{2\sigma} > 0\,. \tag{25}$$

- With decreasing ϵ speed of sound c_s increases. But it can't exceed the speed of light!

$$c_s^2(\epsilon) = \frac{dp(\epsilon)}{d\epsilon} = \frac{1}{3}\left(1 - \frac{sgn(A)|A|}{\sqrt{C^2 + 4\sigma\epsilon}}\right) \leq 1 \iff \epsilon \geq \epsilon_c = \frac{(A^2 - 4C^2)}{16\sigma} = \frac{3(D - C)(3D + C)}{16\sigma} \tag{26}$$

$\Rightarrow$ For $\epsilon \leq \epsilon_c$ the EoS (25) must be replaced by the stiff matter EoS with $p = \epsilon$.

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*) The EoS with $\epsilon - 3p < 0$ was first discussed by [Zeldovich:1961].

Transition to Stiff Matter: the Pressure Drop

- The formation of stiff fluid needs $(D-C)(3D+2C)>0$ and $3D-C<0$ $\Rightarrow D<0$ & $D<C<-3D$ or.
- EoS ansatz: $D=-m^2$, $C=m^2$

$\Rightarrow$ The transition from the EoS (24) to the absolute stiff matter is accompanied by

$$\Delta p = p_{\rm stiff}(\epsilon_c) - p(\epsilon_c) = \epsilon_c - p(\epsilon_c) = -\frac{3(C-D)^2}{8\sigma} < 0\,. \tag{27}$$

- Pressure drop provides a suitable conditions for primordial black hole (PBH) creation, see e.g. the review [Carr:2020].
- $\Delta p<0$ may lead to the lumpiness of stiff matter.
- The scalar metric fluctuations $\delta g_{\mu\nu}$ influence the values of C and D. $\Rightarrow \Delta p$ in various parts of the Early Universe may have different magnitude.

The Fate of Stiff Matter and Emergence of sQGP

- Saturated matter was important in a period of evolution with $T \gtrsim T_S \gg \Lambda_{\rm QCD}$.
  N.B. Already at $T_S \gtrsim 1$ GeV we have $\alpha_S(2\pi T_S) \lesssim 0.2$, see slide A9.
- At $T \approx T_{EW} \gg \Lambda_{\rm QCD}$ for YM bosons $g_{\rm eff}^{\rm QCD}/g_{\rm eff}^{\rm EW} \simeq 8/3 \quad \Rightarrow$
  The Glasma might have been a prevalent form of matter also during the EW era.
- For expansion times $R_s\alpha_S^{-1} < t < R_s\alpha_S^{-3/2}$ the semiclassical picture of the Glasma as a high occupancy state with $f \sim \alpha_S^{-1}$ gluons breaks down due to their rescattering.
- This is followed by the quantum kinetic $2 \to 3$ process among liberated gluons at $t \sim R_s\alpha_S^{13/5}$ which leads to bottom-up thermalization [Baier:2001] and the emergence of a strongly interacting QGP*. Cosmological expansion is restored from $a(t) \sim t^{1/3} \to t^{1/2}$.

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*) For more on sQGP see slide A 4.

Summary

Take-Home Message

- Breaking of QCD scale invariance by quantum effects leads to the emergence of two scales $\Lambda_{\rm QCD}$ and $Q_S \gg \Lambda_{\rm QCD}$. First associated with confinement and second with saturation.
- Application of saturation to the Early Universe predicts the new state of matter, the precursory of the sQGP, which might have played an essential role over a broad range of temperatures $T_{\rm GUT} \gtrsim T \gtrsim T_{\rm EW} > T_{\rm sQGP}$.
- At high temperatures QCD EFT predicts the prevalence of the chromomagnetic field and therefore $\Theta(T) < 0$ of the QCD matter filling the Early Universe.
- A solvable model modifying the EoS of non-interacting massless gas by the inclusion of the quadratic thermal terms leads to an increase in the fraction of volume occupied by thermal gluons with decreasing temperature $F(T) \sim T^{-3}$ and hence to the saturation.
- The speed of sound c_s in the matter described by this new EoS increases with decreasing energy density leading ultimately to its transformation into stiff matter.

Thank you for your attention!

Backup Slides (A1-A15) - LTE, SCM, sQGP, Glasma, EYM, GUT,...

The Early Universe in Local Thermal Equilibrium

Q: *How do we know that the Early Universe was in the state of LTE?*
A: [Mukhanov: Physical Foundations of Cosmology, CUP, 2005]

- Collision time among the constituents $t_c = 1/(\sigma n v)$
- LTE $\Leftrightarrow$ Local equilibrium must be reached well before expansion becomes relevant.

$\Rightarrow$ At the cosmic time $t_H \sim 1/H(t)$: $t_c \ll t_H$.

- At $T > T_{\mathrm{EW}}$ all (most of) particles of the SM are ultra-relativistic ($k^2 \gg m^2$) and gauge bosons are massless. $\Rightarrow \sigma \approx \mathcal{O}(1)\alpha^2\lambda^2 \sim \alpha^2/k^2 \sim \alpha^2/E^2 \sim \alpha^2/T^2$,
- For $n \sim T^3, v = 1$ and $\alpha \simeq 10^{-1} - 10^{-2}, \alpha = \alpha_{\mathrm{EM}}, \alpha_{\mathrm{W}}, \alpha_{\mathrm{S}}$:

$$t_c \sim \frac{1}{\alpha^2 T} \ll t_H \sim \frac{1}{H} \sim \frac{1}{\sqrt{\epsilon}} \sim \frac{1}{T^2} \, .$$

$\Rightarrow$ For $10^{15} - 10^{17}$ GeV $\gtrsim T \gtrsim T_{\mathrm{EW}}$ LTE in expanding fluid persists.

- For $T_{\mathrm{EW}} > T > T_c^{\mathrm{QCD}} \approx 160$ MeV the LTE continues due to large effective cross-section among the particles forming the QGP medium. For details see slides A4 and A5.

Standard Cosmological Model

Einstein equations of general relativity

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}(R - 2\Lambda) = -8\pi G \mathcal{T}_{\mu\nu} \qquad (28)$$

$g^{\mu\nu}$ – metric tensor, $R_{\mu\nu} = f(g_{\mu\nu}, \partial_\lambda g_{\mu\nu}, \partial^2_{\lambda,\kappa} g_{\mu\nu})$ – Ricci tensor,
$R = R_{\mu\nu} g^{\mu\nu}$ – scalar curvature, Λ, G – cosmological, gravitational constants,
$\mathcal{T}_{\mu\nu}$ – energy-momentum tensor.

Cosmological principle: the Universe is Homogenous and Isotropic

Solution of (28) preserving space homogeneity and isotropy under its time evolution is spacetime of constant curvature $k = \{+1, 0, -1\}$ with FLRW metrics

$$ds^2 = g^{FLRW}_{\mu\nu} dx^\mu dx^\nu = dt^2 - a^2(t)\left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right] \qquad (29)$$

$a(t)$ = scale factor of the Universe – connects co-moving (Lagrange) and physical (Euler) coordinates $\hat{r}(t) = a(t) r$.

Equation of State Based on the Fundamental Theory

- In the SM $$p_B(T) = \lim_{V\to\infty} \frac{T}{V} \ln \mathcal{Z}(T,V)$$

$$\mathcal{Z}(T,V) = \exp\left[\frac{p_B(T)V}{T}\right], \quad p_B(T) = p_E(T) + p_M(T) + p_G(T) \tag{30}$$

- p_B is the "bare" result related to the physical (renormalized) pressure as $p(T) = p_B(T) - p_B(0)$.
- $p_E(T), p_M(T), p_G(T)$ collect the contributions from the momentum scales $k \sim \pi T$, $k \sim gT$, and $k \sim g^2T/\pi$, respectively.
- Couplings of SM are $g \in \{h_t, g_1, g_2, g_3\}$, where h_t is the Yukawa coupling between the top quark and the Higgs boson, and g_1, g_2, g_3 are related to $U_Y(1)$, $\mathrm{SU_L}(2)$ and $\mathrm{SU}_c(3)$ gauge groups, respectively.
- Calculations of the dimensionless function $p(T)/T^4$ and of the trace anomaly $\Theta(T)$ up to $\mathcal{O}(g^5)$ were performed in [Laine,Mayer:2015].

The Plasma of Charged Particles

- Plasma = system of mobile charged particles [Ichimaru:1982].
- Electrically neutral gas (liquid, crystal) at high temperatures T turns into a system of charged particles with the long-range $U(1)$ interaction.
- Plasma interaction parameter

$$\Gamma_{\mathrm{EM}} = \frac{q^2}{r_s k_B T} \sim \frac{U_{int}}{E_{th}}, \qquad r_s = \left(\frac{3V}{4\pi N}\right)^{1/3} \approx 0.62 n^{-1/3}, \tag{31}$$

  q - particle charge
  r_s - average inter-particle distance (Wigner-Seitz radius)
- Strongly-coupled (SC) plasma: $\Gamma_{\mathrm{EM}} > 1$, i.e. when $U_{int} > E_{th} \sim k_B T$, interaction energy prevails over thermal energy of the plasma particles.
- **Example** Table salt – crystalline plasma made of permanently charged Na^+ and $C\ell^-$ ions. $\Gamma_{\mathrm{EM}} \approx 60$ at $T \approx 10^3$K. [Shuryak:2008].

QCD Plasma: The Phenomenologic Approach

The Plasma of Quarks and Gluons

- Generalization $U(1) \to SU(3)_c$ [Thoma:2005], see also [Bannur:2005]

$$\Gamma_{\rm QCD} \simeq 2\frac{C_{q,g}\alpha_{\rm S}}{r_s T}\,, \quad C_q = \frac{N_c^2-1}{2N_c} = \frac{4}{3}\,, \quad C_g = N_c = 3 \qquad (32)$$

N.B. In relativity chromo-electric $\approx$ chromo-magnetic $\Rightarrow$ 2 in (32).

- For ideal massless QCD gas with N_F active quarks and $d_F = g_{\rm eff}^{\rm QCD}$

$$n = d_F\frac{\zeta(3)}{\pi^2}T^3 \approx d_F\left(\frac{T}{2}\right)^3, d_F = 2\times 8 + \tfrac{7}{8}(3\times N_F\times 2\times 2) \qquad (33)$$

$$r_s \simeq 1.24 d_F^{-1/3}T^{-1} \Rightarrow r_s T = f(N_F(T)) \qquad (34)$$

1. $T \approx 200$ MeV: $\alpha_{\rm S} = 0.3{-}0.5$, $N_F = 2$, $d_F = 37 \Rightarrow \Gamma_{\rm QCD} \simeq 2{-}8$.
2. $T \simeq T_{\rm EW}$: $\alpha_{\rm S} = 0.08$, $N_F = 5$, $d_F = 52.5$ and $\Gamma_{\rm QCD} \simeq 0.5{-}1.5$.
3. $T \gg T_{\rm EW}$: $N_F = 6$, $\Gamma_{\rm QCD}(T)$ is solely driven by $\alpha_{\rm S}(T) \sim -\ln T$.

The Weakly Interacting QCD: DGLAP and BFKL

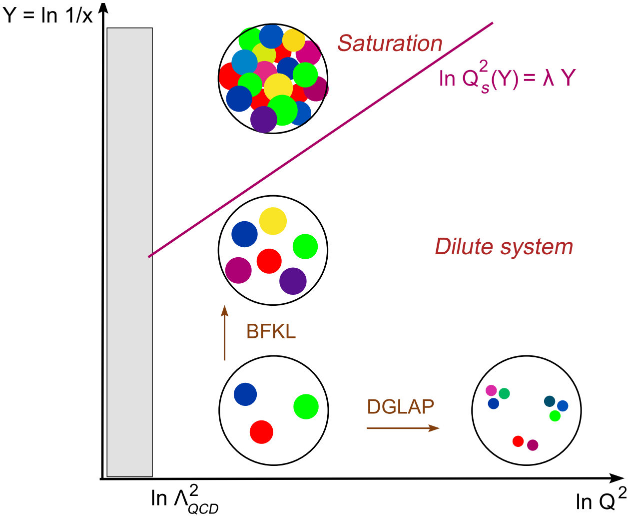

Figure 2: Parton density and size as a function $Y = \ln(1/x)$ and $\ln Q^2$. From [Gelis et al:2010].

Q: What's missing from the previous EoS?

A: The physics of QCD saturation.

- In the Bjorken limit of QCD $Q^2 \to \infty$ (impulse approximation) hadron viewed, in the infinite momentum frame (IMF), represents a large number of gluons and sea quark pairs with very small phase-space density.
- At large, fixed $Q^2 \gg \Lambda^2_{QCD}$ gluon distribution $xG(x, Q^2)$ in a proton rises very fast with decreasing x .

QCD at High Parton Densities and Saturation

[Gribov jr.,Levin,Ryskin:1983, McLerran,Venugopalan:1993]

- Partons "overlap" when $\sigma_{gg} \sim (\alpha_S/Q^2) \times (xG_A(x, Q^2))$ – the probability to find at fixed Q a parton carrying a fraction x of the parent parton momentum – becomes comparable to the geometrical cross section πR_A^2 of the object A which the gluons occupy.

$$Q_S^2(x) = \frac{\alpha_S(Q_S)}{2(N_c^2-1)} \frac{xG_A(x, Q_S^2)}{\pi R_A^2} \sim \frac{1}{x^\lambda} \quad \Rightarrow \quad \ln Q_S^2(x) = \lambda Y \qquad (35)$$

- $Q_S(x) \equiv$ Fixed point of the PDF evolution in $x \Rightarrow$ the emergent "close packing" scale.
- Repulsive gg interactions $\Rightarrow$ occupation number f_g (# of gluons with a given x times the area each gluon fills up divided by the transverse size of the object) saturates at $f_g \sim 1/\alpha_S$.
- The same scaling as for the Higgs condensate, superconductivity, or the inflaton field.
- Saturated gluonic matter is weakly coupled. $\Rightarrow$ weakly interacting means semi-classical. (For details see slide A8).

QCD in the classical regime [Kharzeev:2002]

- Introduce coupling-independent field tensors

$$A^a_\mu \to \mathcal{A}^a_\mu \equiv g_\mathrm{S} A^a_\mu, \quad g_\mathrm{S}^2 = 4\pi\alpha_\mathrm{S}$$

$$F^a_{\mu\nu} \to g_\mathrm{S} F^a_{\mu\nu} \equiv \mathcal{F}^a_{\mu\nu} = \partial_\mu \mathcal{A}^a_\nu - \partial_\nu \mathcal{A}^a_\mu + f^{abc}\mathcal{A}^b_\mu \mathcal{A}^c_\nu \,, \tag{36}$$

- Calculate action of the gluon field

$$S_g = -\frac{1}{4}\int F^a_{\mu\nu} F^{\mu\nu,a} d^4x = -\frac{1}{4g_\mathrm{S}^2}\int \mathcal{F}^{\mu\nu,a}\mathcal{F}^a_{\mu\nu} d^4x \tag{37}$$

- Gluon occupation number $$f_g \sim \frac{S_g}{\hbar} = \frac{1}{\hbar g_\mathrm{S}^2}\rho_4 V_4 \tag{38}$$

  where $\rho_4 \sim \langle \mathcal{F}^{\mu\nu,a}\mathcal{F}^a_{\mu\nu}\rangle$ is four-dimensional gluon condensate density.

- Saturated gluon matter is weakly coupled.
- The limits $g_\mathrm{S}^2 \to 0$ and $\hbar \to 0$ are equivalent! $\Rightarrow$ weakly interacting means semi-classical.

Thermal Evolution of the Strong Coupling Constant $\alpha_s(2\pi T)$

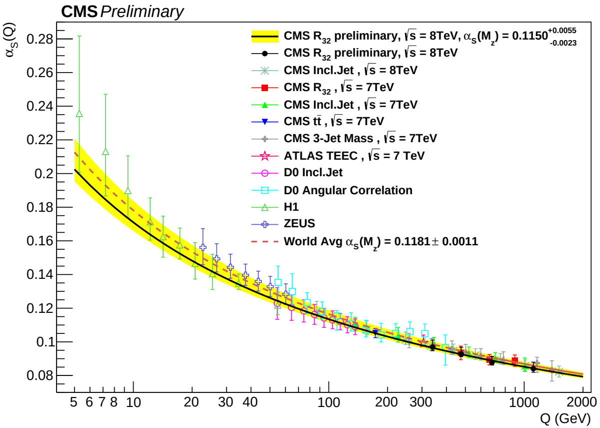

Figure 3: $\alpha_S(Q)$ obtained from MSTW2008 NLO PDF set. [CMS:2017].

- "Temperature" dependence of the running coupling $0.2 \lesssim \alpha_S(T) \lesssim 0.08$ for $T \approx Q/(2\pi) \in (1, 220)$ GeV.

T^{μ}_{μ} of the Ideal Gas of Free Particles in D-Dimensional Space

- Ideal gas of free particles in D-dimensional space expressed in terms of single-particle statistical sum $f(E,T,\mu)$ (see e.g. [Kapusta:1989]):

$$n = \gamma \int f \cdot d^D P\,, \quad f(E,T,\mu) = \frac{1}{\exp[(E-\mu)/T] \pm 1}\,, \quad \gamma \equiv \frac{2s+1}{(2\pi)^D}\,, \tag{39}$$

$$\epsilon = \gamma \int f \cdot E(P) d^D P = \gamma S(D) \int_0^\infty f \cdot E(P) P^{D-1} dP\,, \quad S(D) = \frac{D\pi^{D/2}}{\Gamma(D/2+1)}\,, \tag{40}$$

$$p = -\frac{T}{V} \ln Z = -\gamma T \int \ln f \cdot d^D P = \gamma \frac{S(D)}{D} \int_0^\infty f \cdot \frac{\partial E(P)}{\partial P} P^D dP\,, \tag{41}$$

- (40) and(41) $\Rightarrow$ the sign of $T^{\mu}_{\mu} = \epsilon - Dp$ is given by the sign of

$$\Delta(P,m) \equiv \sqrt{P^2+m^2} - \frac{P^2}{\sqrt{P^2+m^2}}; \quad \Delta(P,m) > 0, \quad \Delta(P,0) = 0\,. \tag{42}$$

Medium with $c_s = const.$ and Ideal Gas of Quasi-particles

- Substitution of (40) and (41) into Equation $p - w\epsilon = 0$ gives

$$\frac{P}{D}\frac{\partial E(P)}{\partial P} = wE(P) \quad \Rightarrow \quad E(P) = \xi P^{wD}, \xi = const. \tag{43}$$

$\Rightarrow$ Medium with $c_s^2 = w = const.$ in $D = 3$ space is equivalent to an ideal gas of quasi-particles with dispersion relation $E(P)$ (43) in D-dimensional space [Trojan:2011ma].

- $wD = 1$ corresponds to massless particles in D-dimensional space with the EoS $p = \epsilon/D$.

Standard Model Couplings in the Hot Big Bang Era

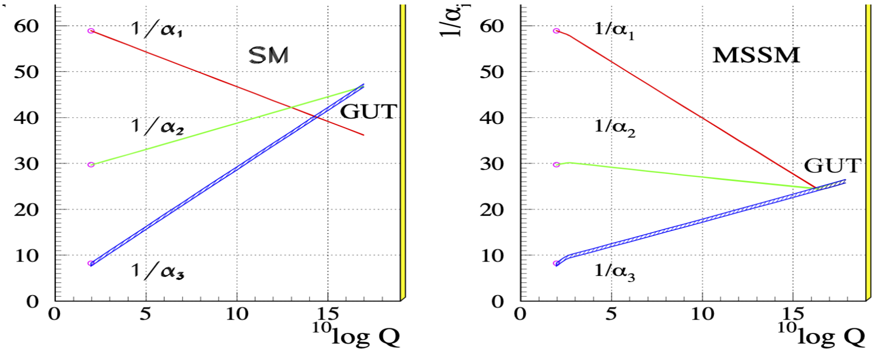

Figure 4: Evolution of the inverse of the three coupling constants $\alpha_1 = \alpha_{EM}, \alpha_2 = \alpha_W, \alpha_3 = \alpha_S$ in the Standard Model $U(1)_Y \times SU(2)_L \times SU(3)_C$ (left) and in its supersymmetric extension MSSM (right).

- Thermodynamics is applicable if the Universe is in global equilibrium.
- Hydrodynamical description needs only local thermal equilibrium (LTE) (see slide A1).

Effective Yang-Mills theory

- The Lagrangian of the effective $SU(N)$ YM theory in Minkowski background,

$$\mathcal{L}_{\rm eff} = -\frac{\mathcal{F}^2}{4\bar{g}^2(\mathcal{F}^2)}\,, \qquad \mathcal{F}^2 \equiv \sum_{a=1}^{N^2-1} \mathcal{F}^a_{\mu\nu}\mathcal{F}^{a\,\mu\nu} = 2\big(\mathbf{B^2}-\mathbf{E^2}\big)\,, \tag{44}$$

is obtained from the classical theory upon rescaling $A^a_\mu \to \mathcal{A}^a_\mu \equiv gA^a_\mu$, $F^a_{\mu\nu} \to gF^a_{\mu\nu} \equiv \mathcal{F}^a_{\mu\nu}$, see A8, and accounting for vacuum polarisation through running gauge coupling $g \to \bar{g}^2(\mathcal{F}^2)$.

- Considering the standard β-function of YM theory,

$$\frac{\mathrm{d}\ln\bar{g}^2}{\mathrm{d}\ln(\mathcal{F}^2/\mu_0^4)} \equiv \frac{1}{2}\beta(\bar{g}^2)\,, \qquad \beta_{1-\rm loop} = -\frac{bN}{48\pi^2}\bar{g}^2_{1-\rm loop} < 0\,, \qquad b = 11\,, \tag{45}$$

$\mathcal{L}_{\rm eff}$ gives rise to the effective EMT:

$$T^\nu{}_\mu = \frac{1}{\bar{g}^2}\left[\frac{\beta(\bar{g}^2)}{2}-1\right]\left(\mathcal{F}^a_{\mu\lambda}\mathcal{F}^{a\,\nu\lambda}-\frac{1}{4}\delta^\nu{}_\mu\mathcal{F}^2\right)+\delta^\nu{}_\mu\frac{\beta(\bar{g}^2)}{8\bar{g}^2}\mathcal{F}^2\,, \qquad T^\mu{}_\mu = \frac{\beta(\bar{g}^2)}{2\bar{g}^2}\mathcal{F}^2 \tag{46}$$

Approaching the Saturation Temperature

- Substitution for the $p+\epsilon$ from Eq.(25) into Eq.(3) and integrating $\delta = \int d\epsilon/(\epsilon+p)$

$$\delta(\epsilon) = \frac{3\left((A-2C)\log\left(A+2\sqrt{C^2+4\sigma\epsilon}-2C\right)-2C\log\left(\sqrt{C^2+4\sigma\epsilon}+C\right)\right)}{2(A-4C)}\,. \qquad (47)$$

- Solving the eq. $\delta(\epsilon)-\delta(\epsilon_c)=\log a^{-3}$, cf. (5), yields $(a(\epsilon))^{-3}=\exp(\delta(\epsilon)-\delta(\epsilon_c))$.
- Substitution $\epsilon\to\sigma T^4-CT^2$ in $(a(\epsilon))^{-3}$ allows us to obtain $F(T)=(T\cdot a(T))^{-3}$
- Solution with $C=0$: $$F(T)=T^{-3}\left(\frac{A+4\sigma T^2}{A(2\sigma+1)}\right)^{3/2}\xrightarrow[T^2\ll A/2]{}T^{-3} \qquad (48)$$
- Solution with $C=A/2$:

$$F(T)=T^{-3}\left(\frac{A+\sqrt{(A-4\sigma T^2)^2}}{A(2\sigma+1)}\right)^{3/2}\xrightarrow[T^2\ll A/(4\sigma)]{}(\sqrt{2\sigma+1}\,T)^{-3} \qquad (49)$$

Approaching the Saturation Temperature

- $\epsilon_c = (A^2\sigma^2 - C^2)/(4\sigma) > 0 \Rightarrow$ General solution: $C \in (-\sigma A, \sigma A)$: $\lim_{T\to 0} a(T) < \infty$.
- The steep increase in $F(T) \sim T^{-3}$, the fraction of volume occupied by the gluons, with decreasing T must be compensated by the increased rate of gluon fusion $\Rightarrow$ saturation.