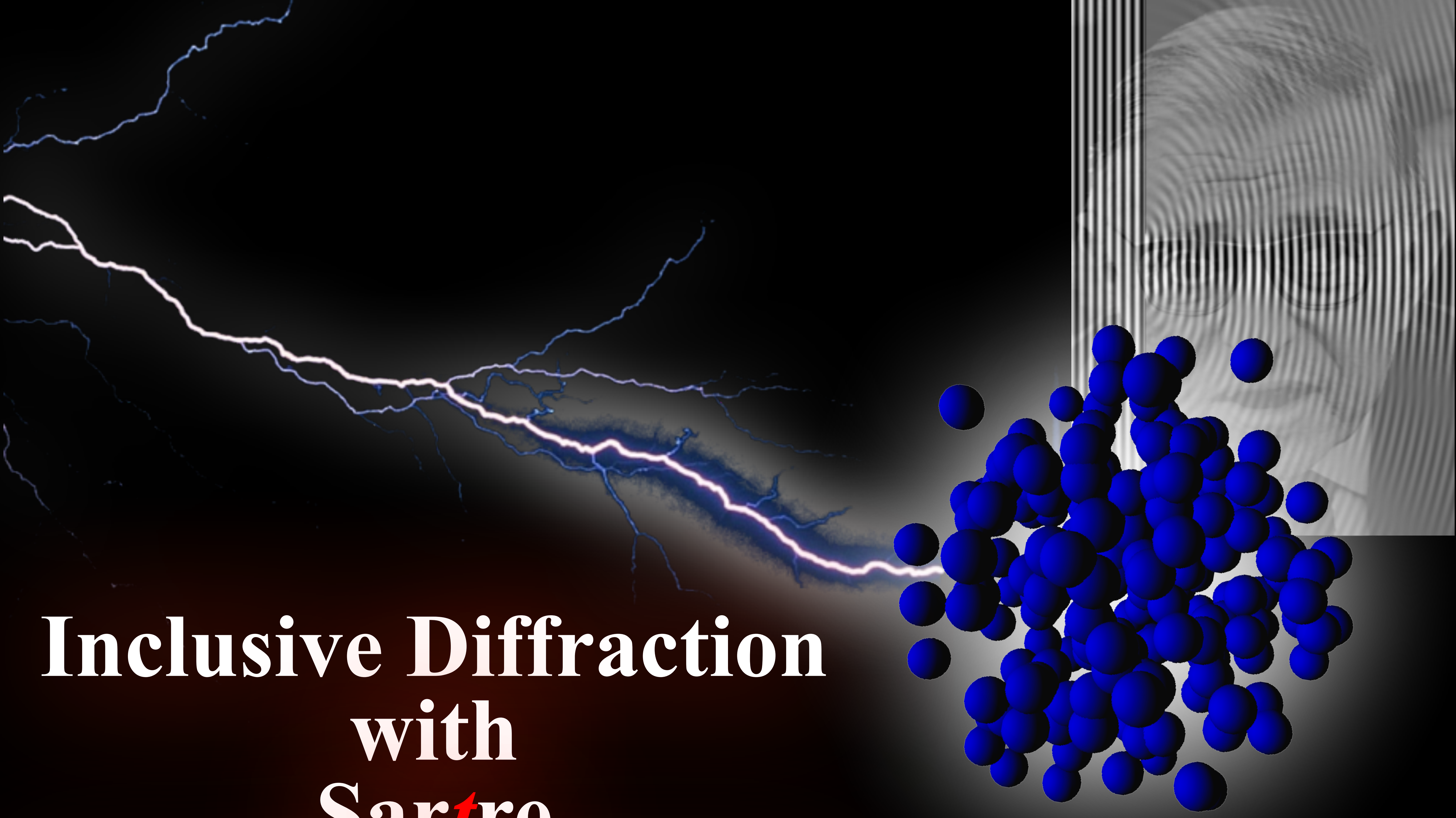


Inclusive Diffraction with Sar*t*re

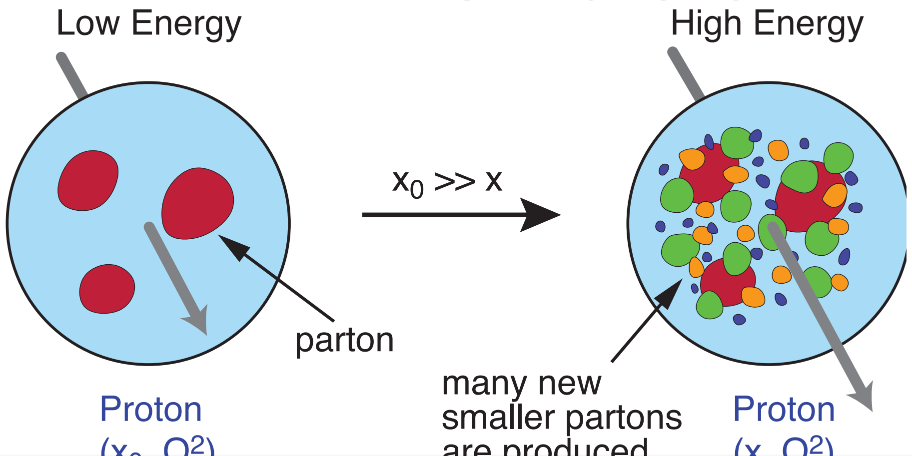
Exclusive, Diffraction, & Tagging Meeting

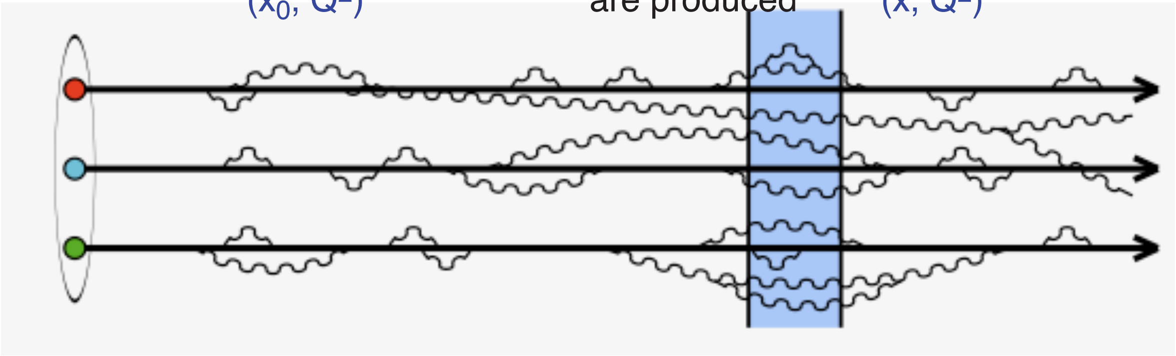
BNL, December 16 2024

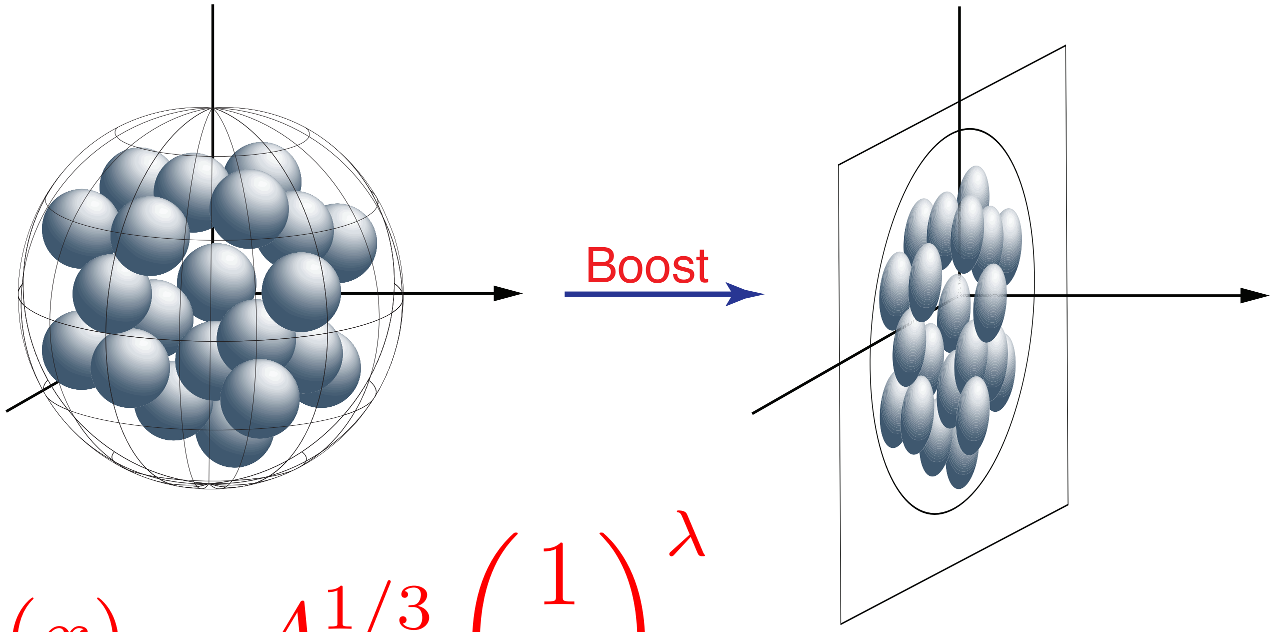
Tobias Toll

Indian Institute of Technology Delhi

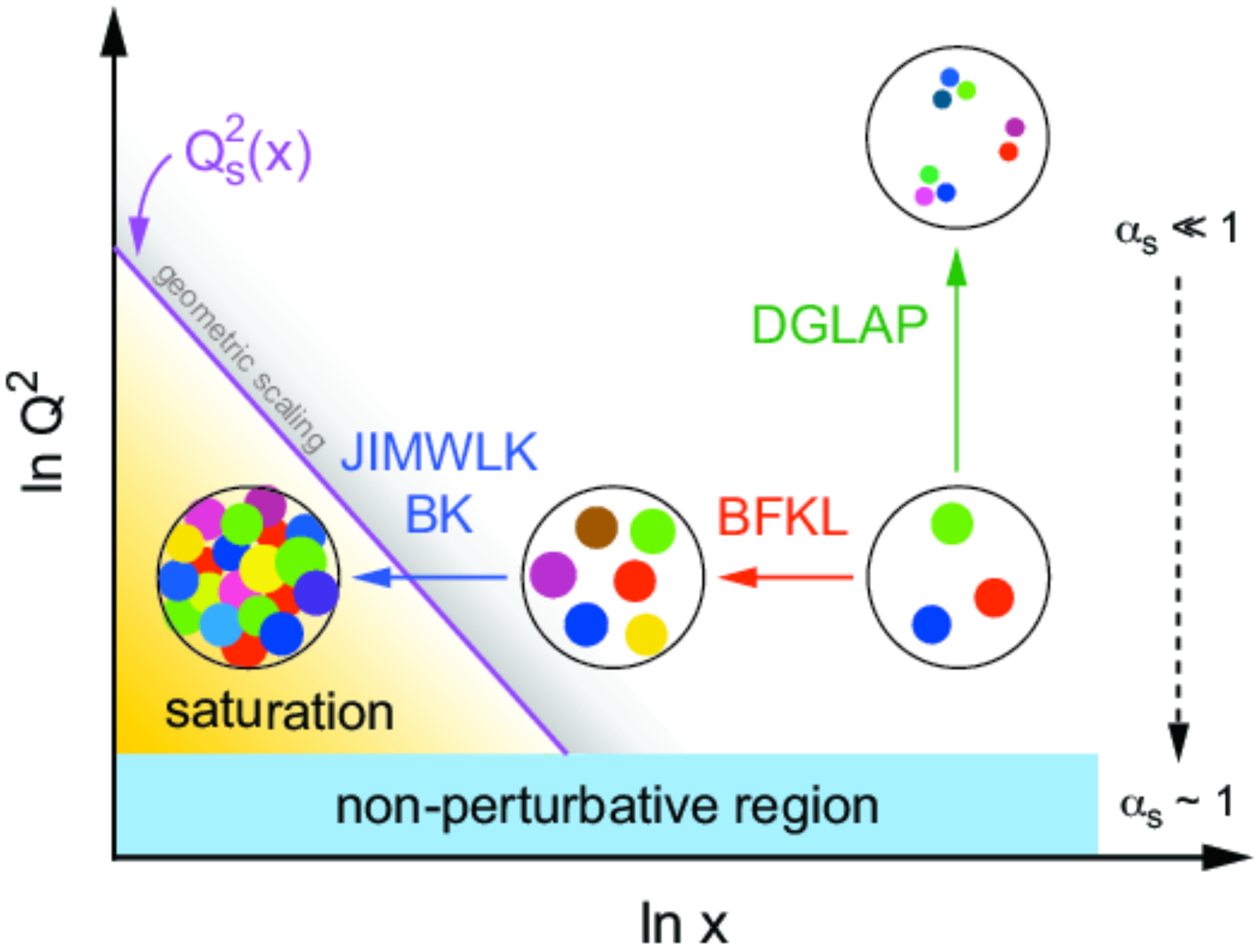
Saturation at EIC

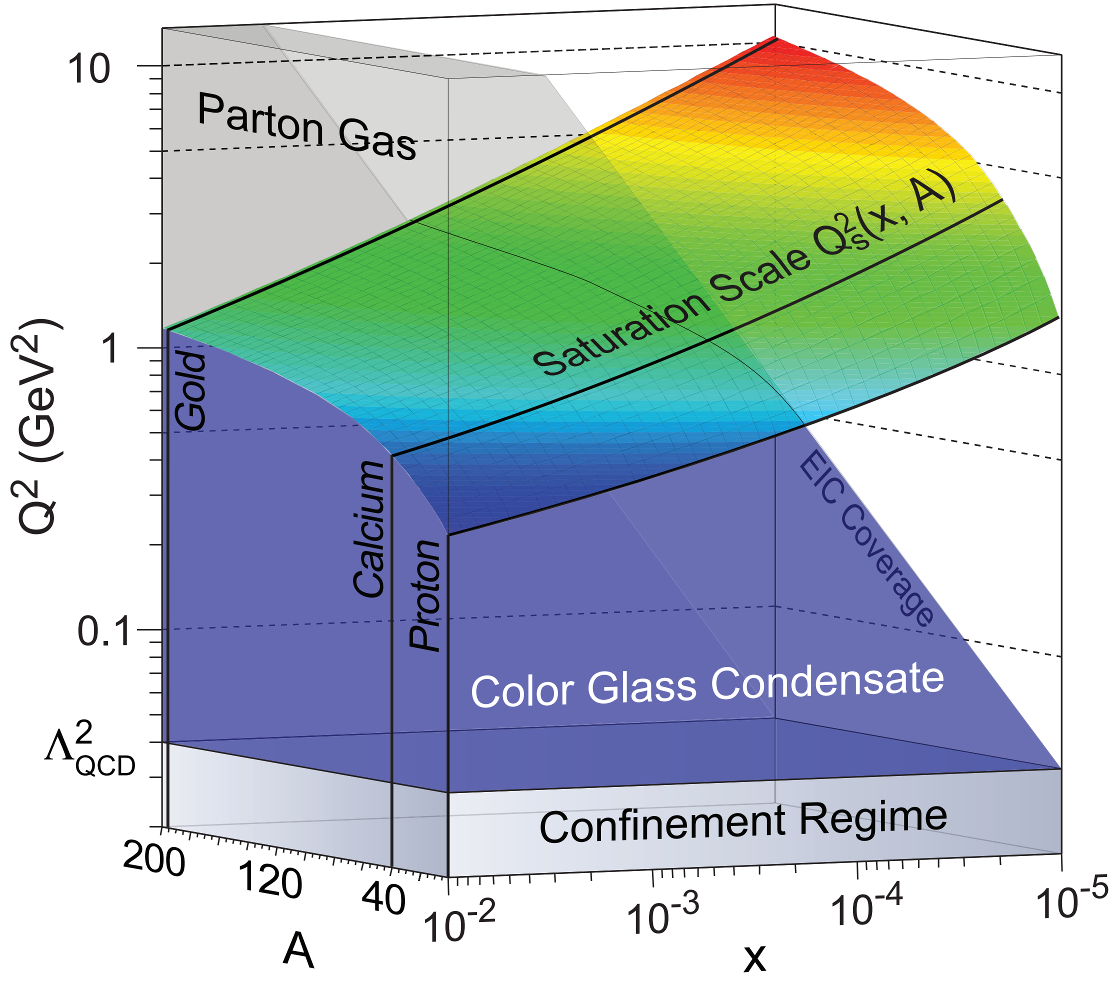






$$Q_s^2(x) \sim A^{1/3}\left(\frac{1}{x}\right)^{\lambda}$$

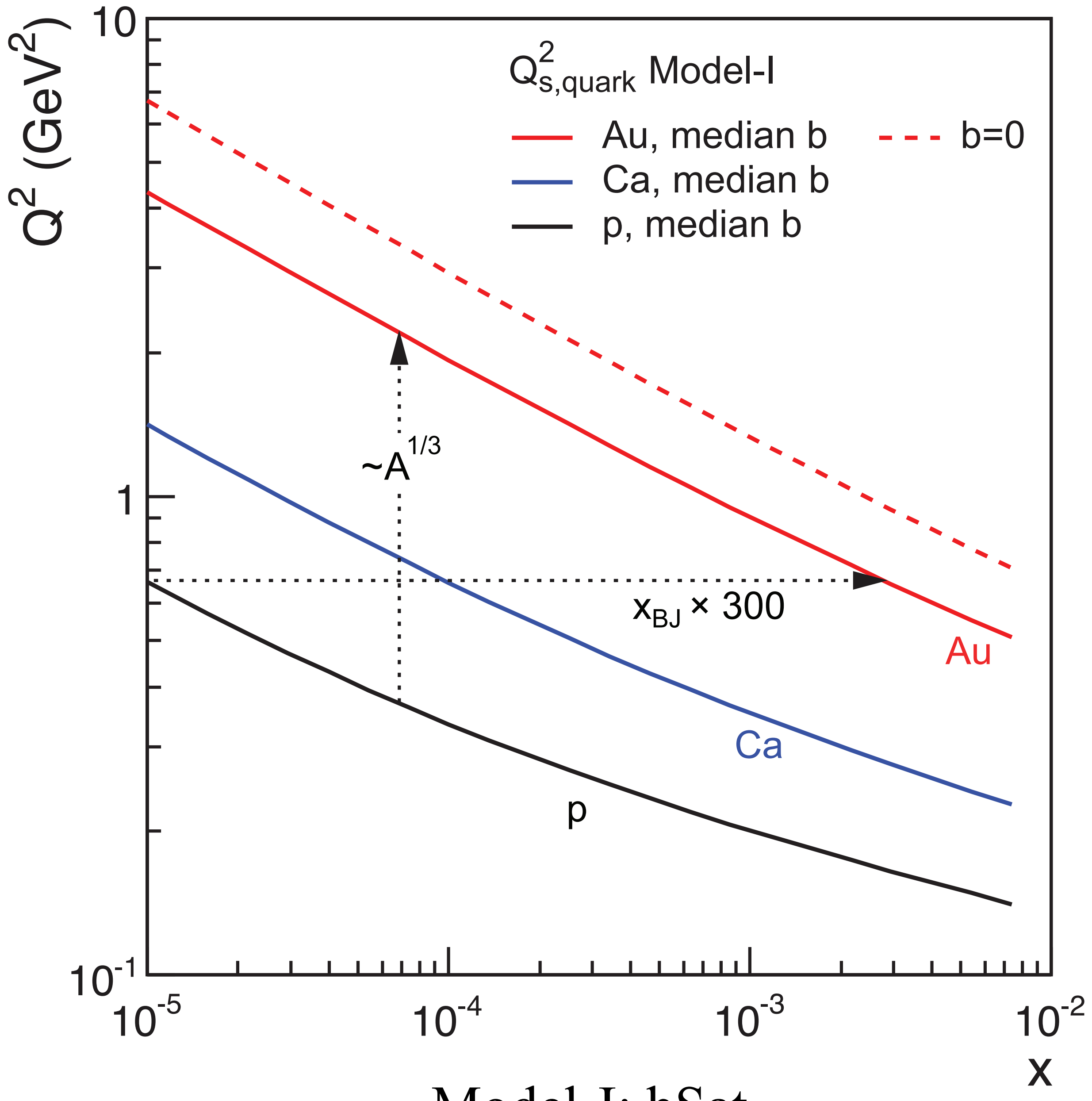




Saturation at EIC

Pocket formula: $Q_s^2(x) \sim A^{1/3}\left(\frac{1}{x}\right)^{\lambda} \sim \left(\frac{A}{x}\right)^{1/3}$

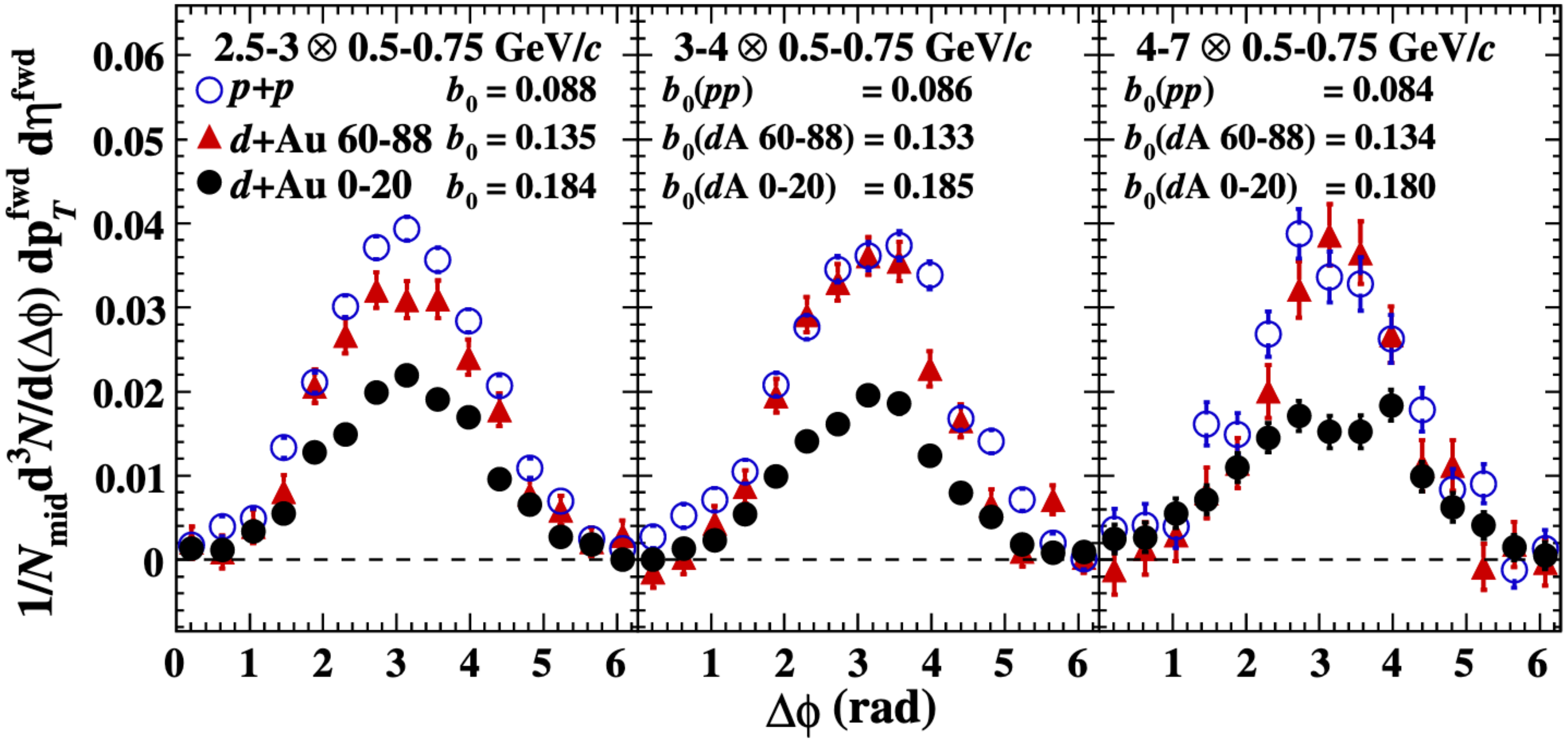
Gold: A=197, x 197 times smaller!



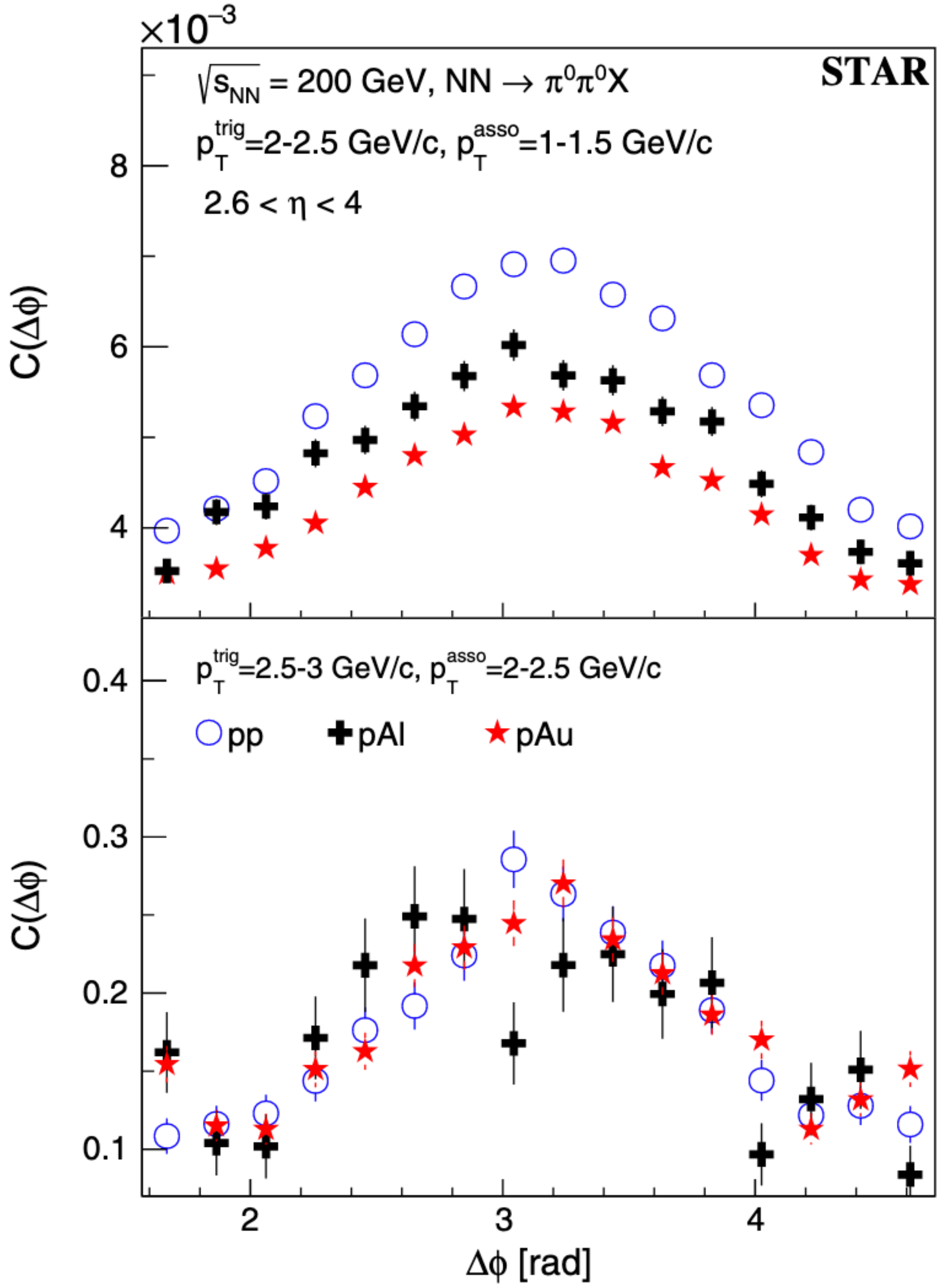
Model-I: bSat

π^0-π^0 forward correlation in *pp* and *d*A at RHIC

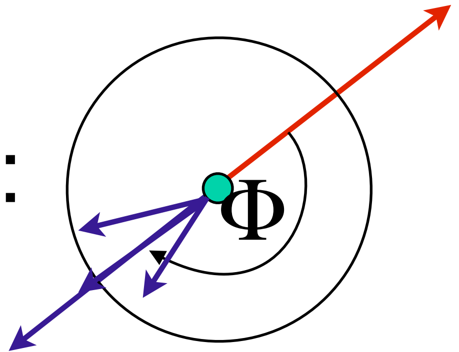
PHENIX, Phys.Rev.Lett. 107 (2011), 172301



STAR, Phys.Rev.Lett. 129 (2022) 9, 092501

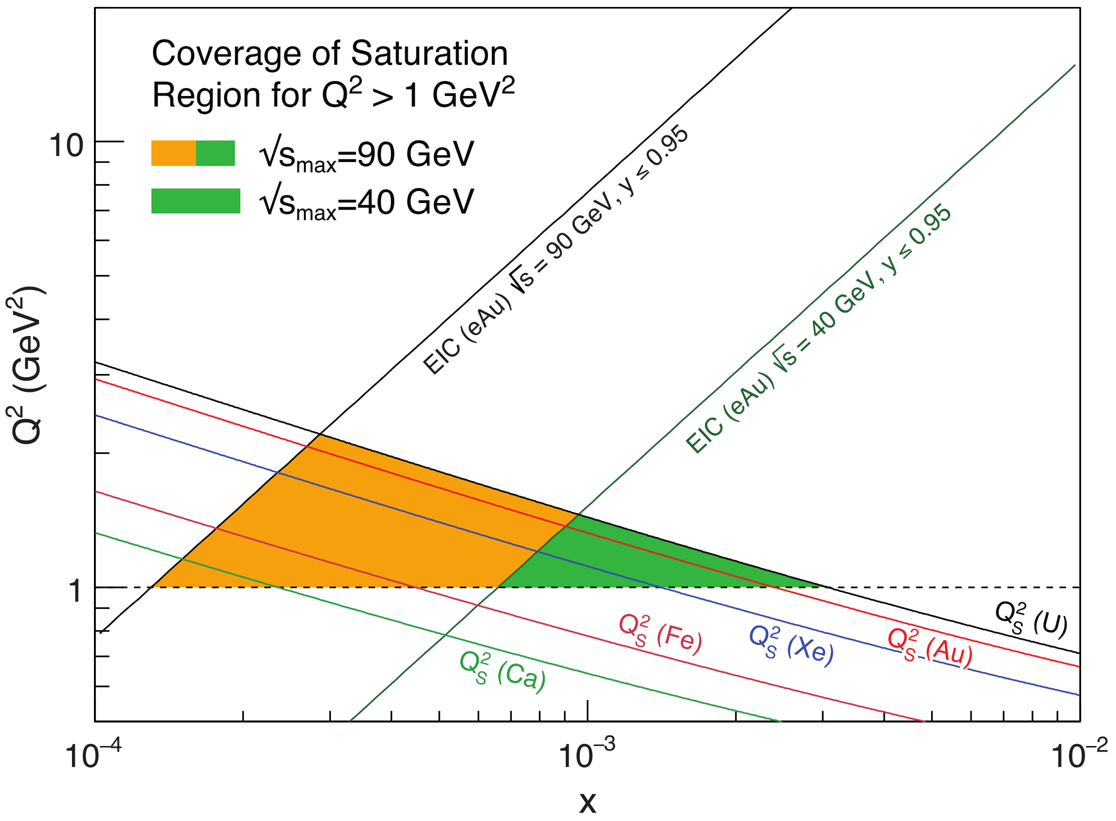


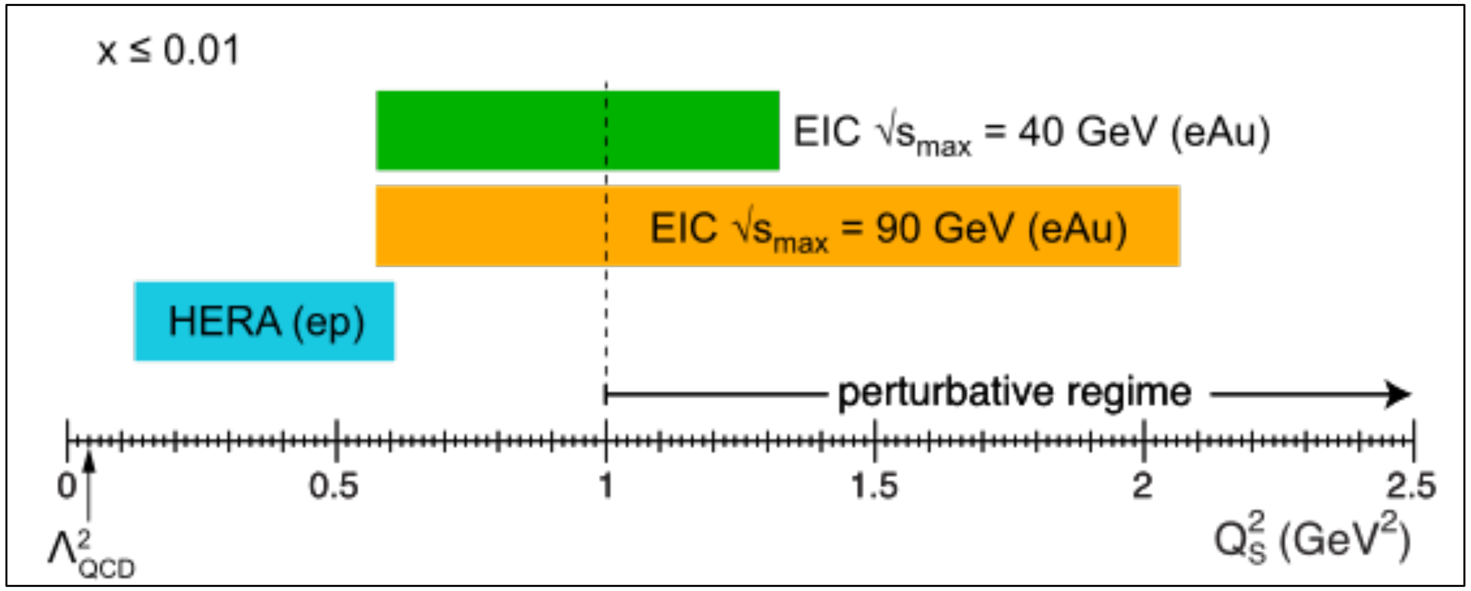
beam-view:

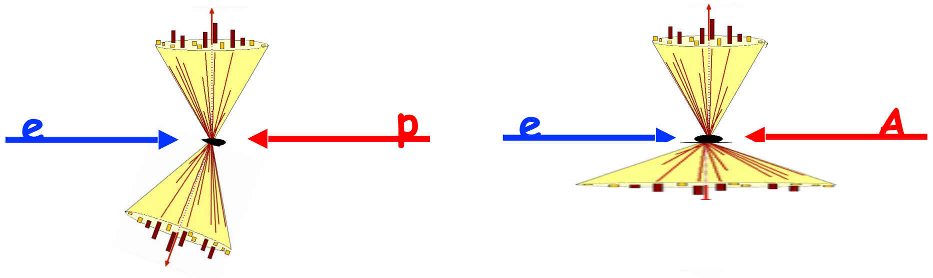


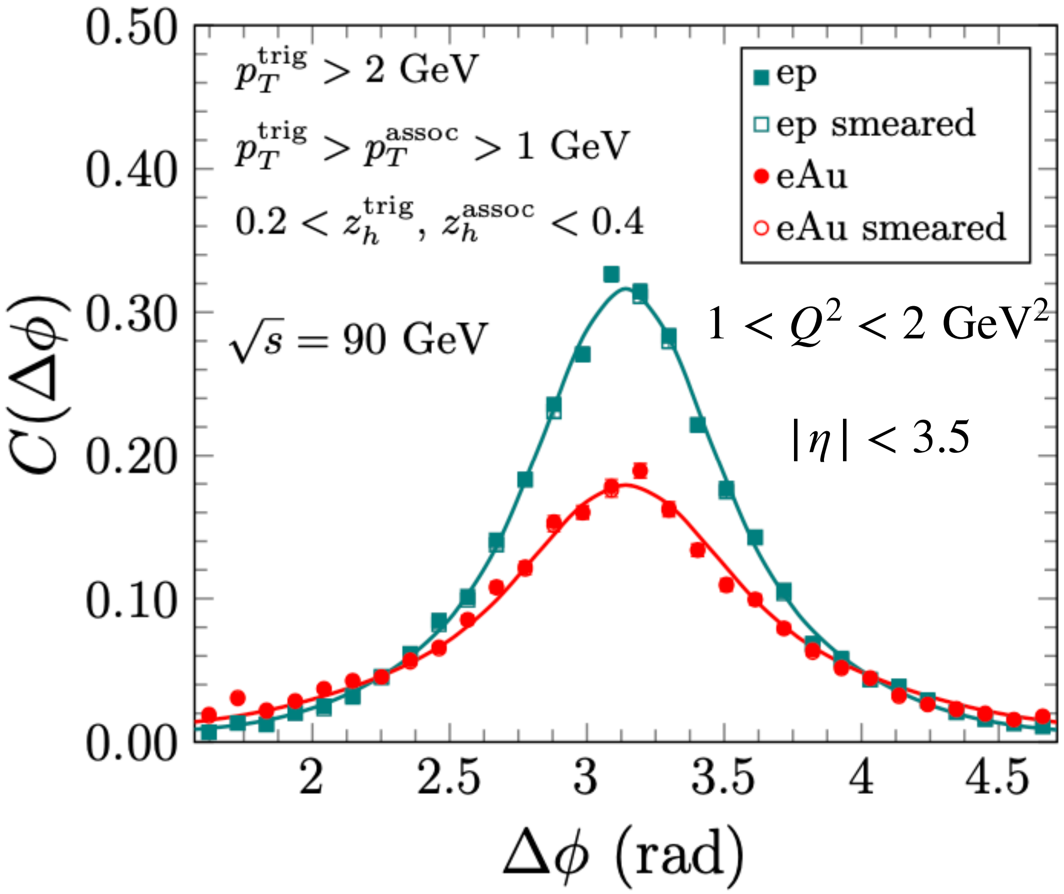
Striking broadening of away side peak in central *p*A and *d*A compared to *pp* and peripheral *d*A!

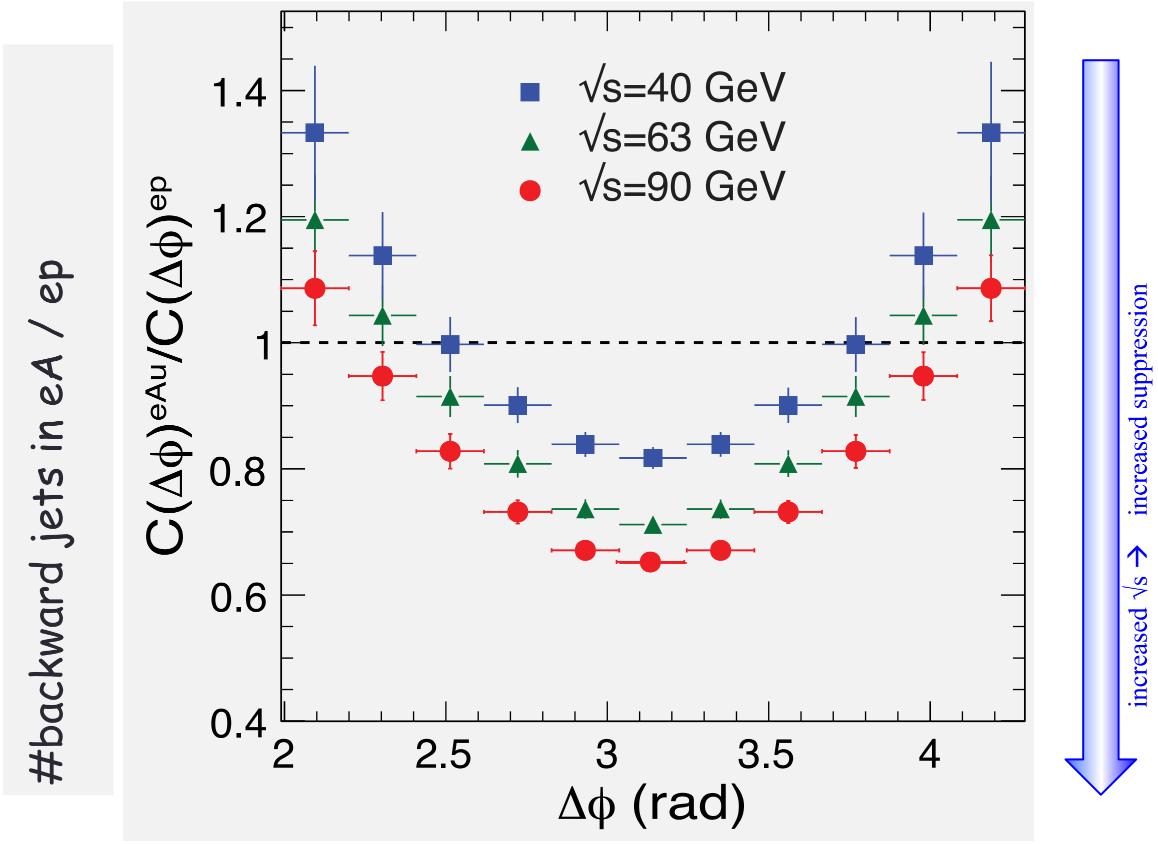
Away peak disappearance at EIC



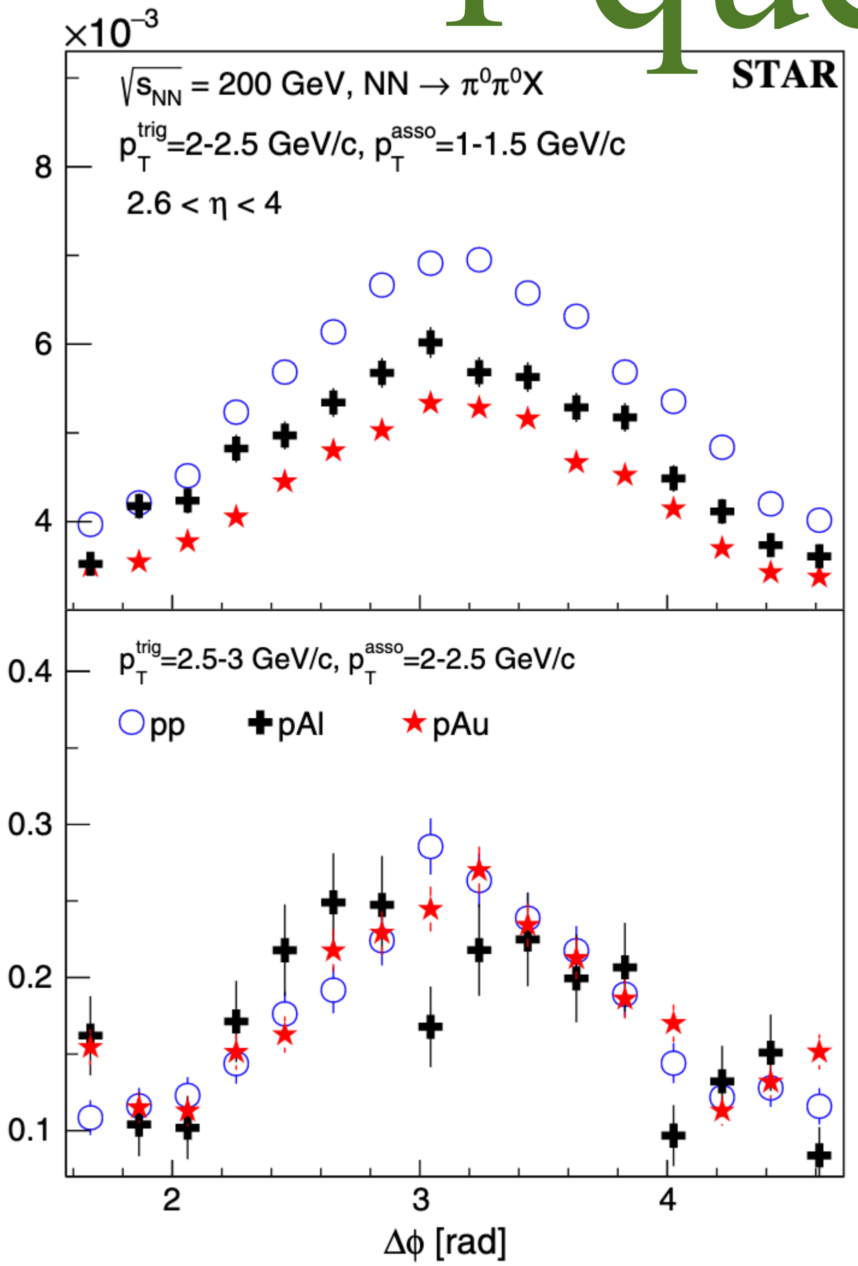






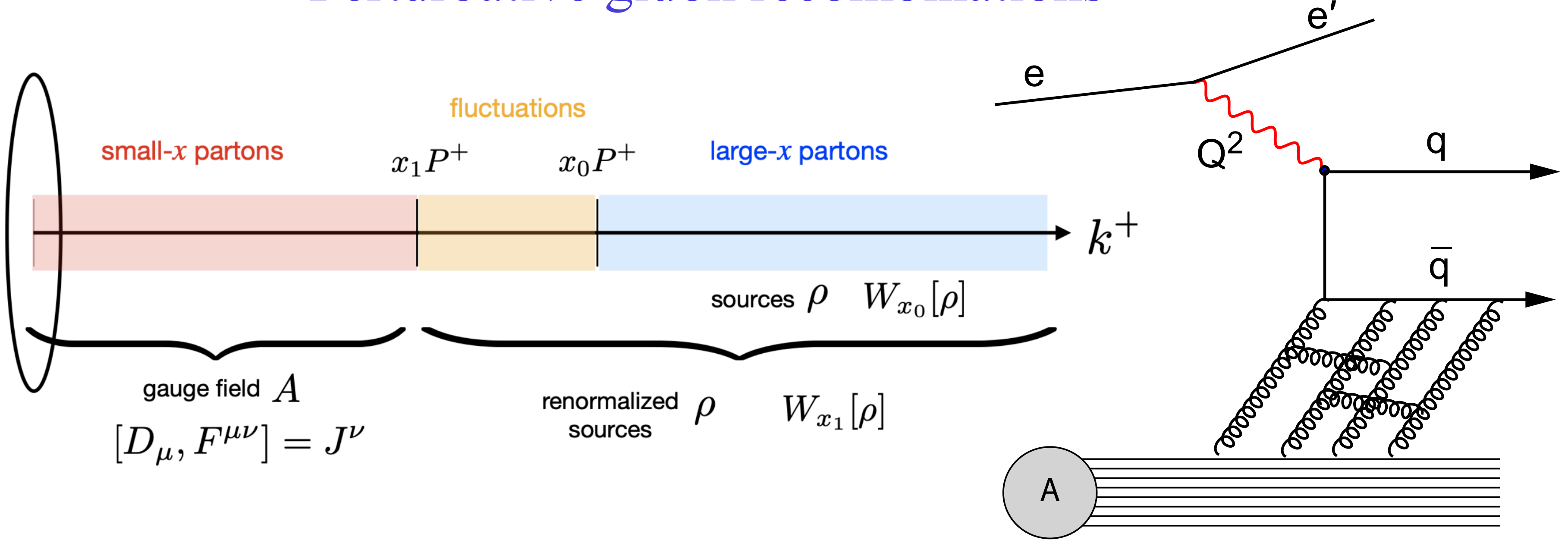


1 question, 2 answers



Universe 7 (2021) 8, 312, arXiv:2108.08254

Color Glass Condensate with JIMWLK evolution:
Perturbative gluon recombinations



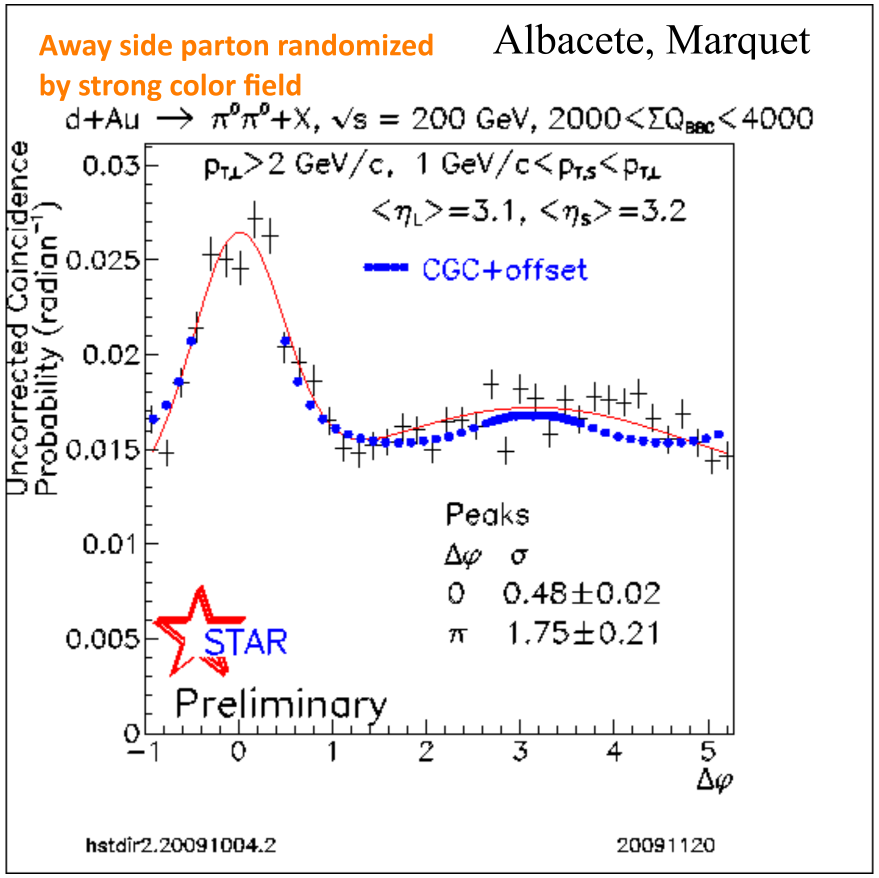
Leading Twist Shadowing:
Suppression in Nuclear PDF + multiple scattering + perturbative DGLAP evolution

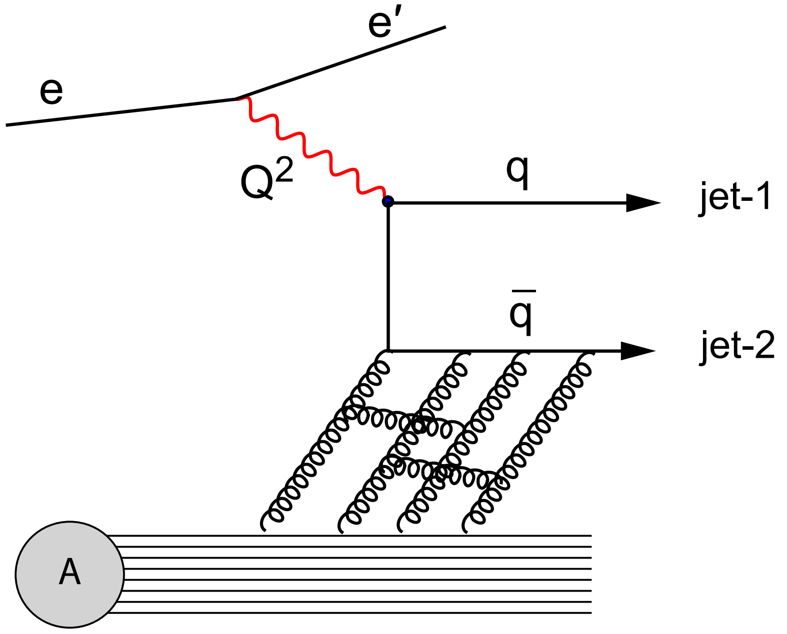
$$xf_{j/A}(x,\mu^2) = Af_{j/N}(x,\mu^2) - \frac{2\sigma_2^j f_{j/N}(x,\mu^2)}{[\sigma_{\rm soft}^j(x)]^2}\int d^2b\left(e^{-\frac{1}{2}\sigma^j_{\rm soft}(x)T_A(b)} - 1 + \frac{\sigma^j_{\rm soft}(x)}{2}T_A(b)\right)$$

Frankfurt, Guzey, Strikman 2012

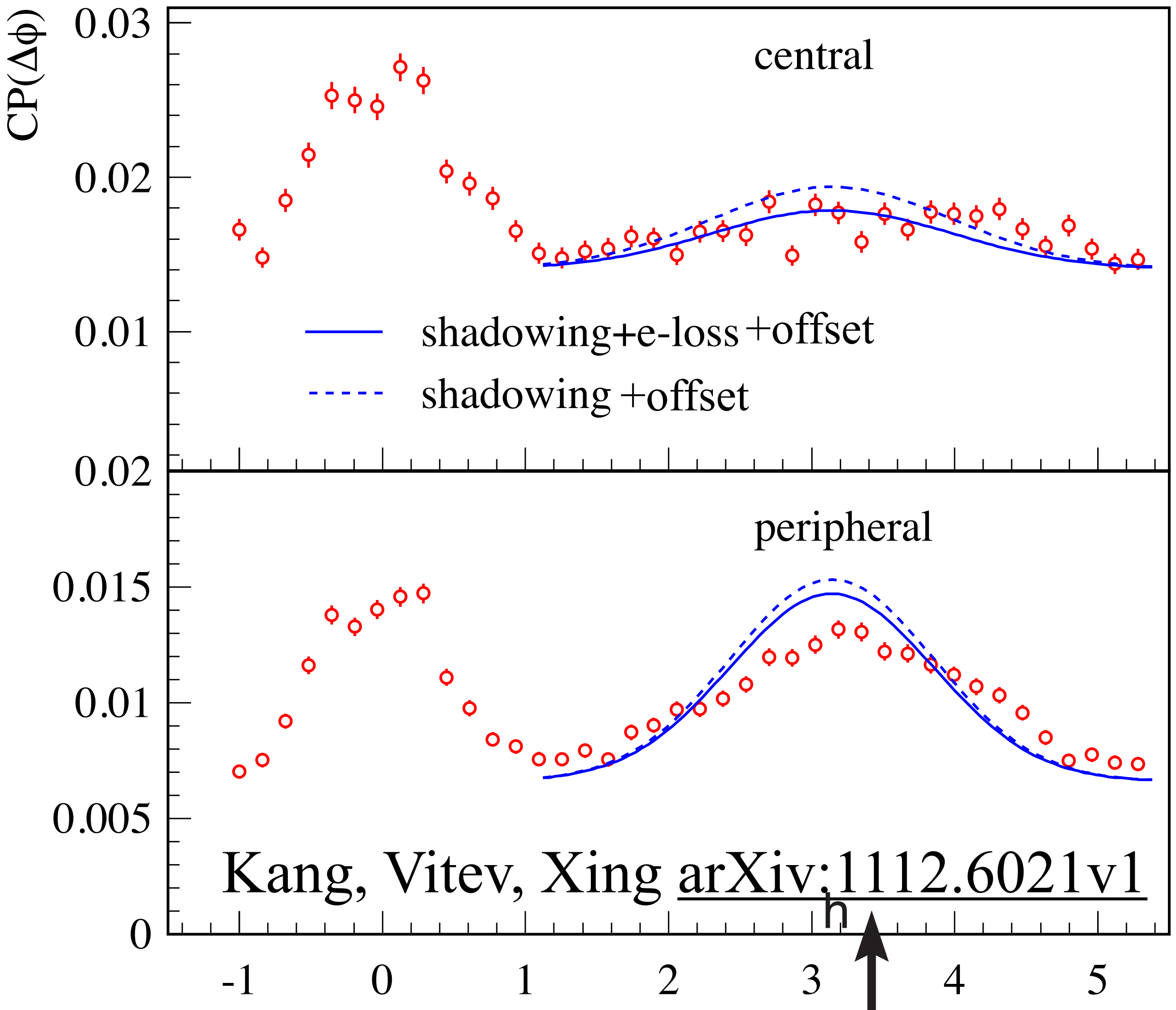
1 question, 2 answers

Saturation Model



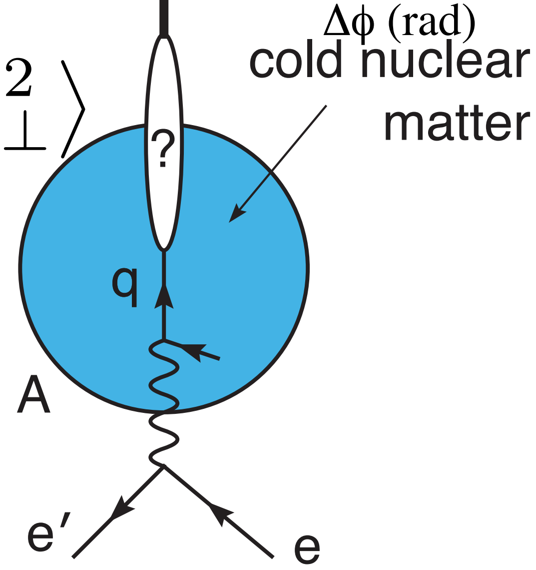


Shadowing Model



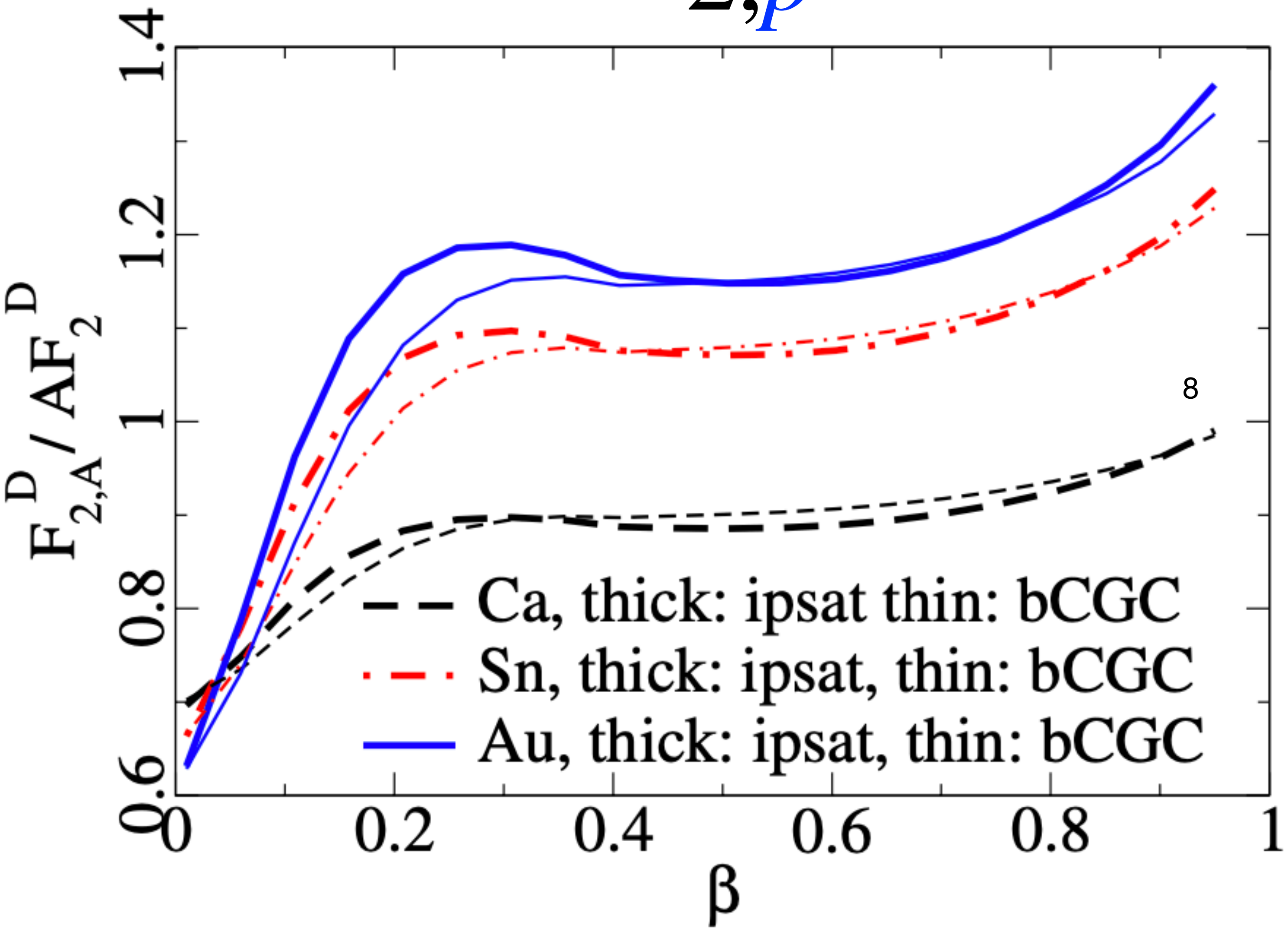
Kang, Vitev, Xing arXiv:1112.6021v1

$$\left\langle q_\perp^2 \right\rangle_{dAu} = \left\langle q_\perp^2 \right\rangle_{pp} + \Delta \left\langle q_\perp^2 \right\rangle$$

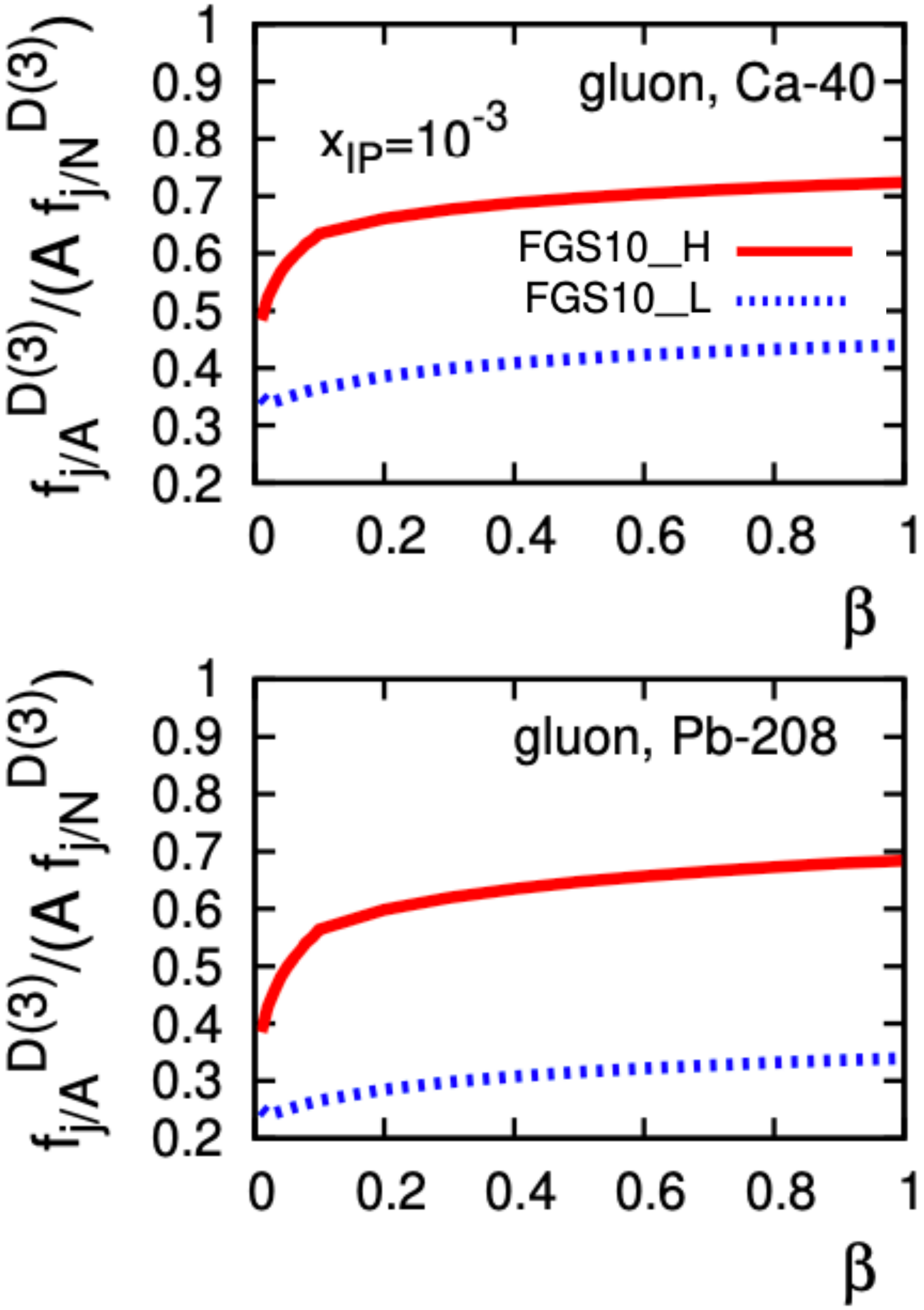


Inclusive Diffraction:
2 very different answers!

$$\frac{F^{D}_{2,A}}{A F^{D}_{2,p}}$$

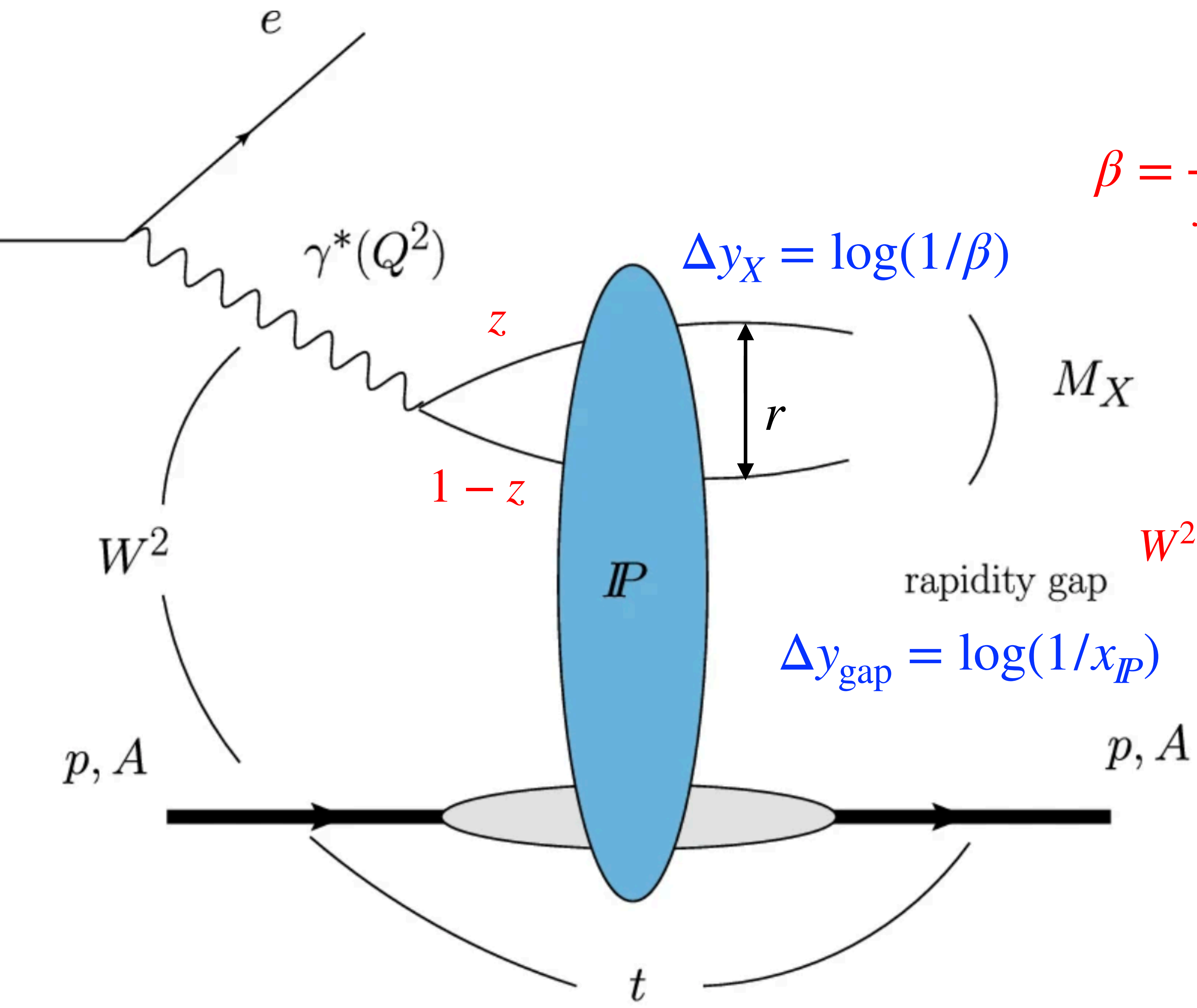


Kowalski, Lappi, Marquet, Venugopalan, Phys. Rev. C 78, 045201, arXiv:0805.4071



Frankfurt, Guzey, Strikman Nuclei, Phys. Rept. 512 (2012) 255–393.
arXiv: 1106.2091

Inclusive Diffraction at small x



$$\Delta y_X = \log(1/\beta)$$

$$\Delta y_{\text{gap}} = \log(1/x_{I\!P})$$

$$\beta = \frac{x}{x_{I\!P}}, \qquad x \leq \beta, x_{I\!P} \leq 1$$

$$M_X^2 = \frac{1-\beta}{\beta} Q^2$$

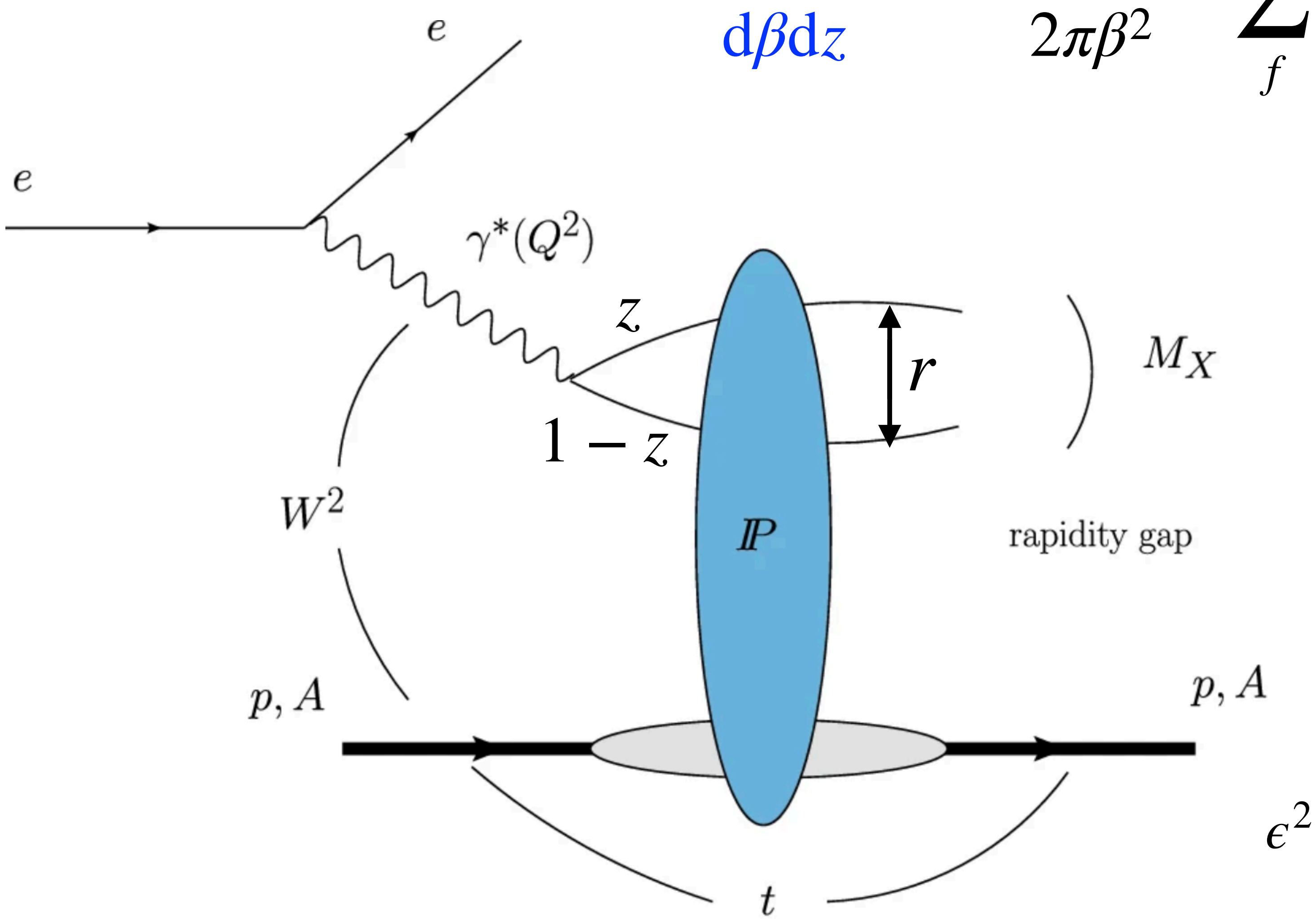
$$W^2 = (q+p)^2 = \frac{M_X^2}{x_{I\!P}} + \frac{1 - x_{I\!P}}{x_{I\!P}} Q^2$$

Inclusive Diffraction at small x

$$\frac{\mathrm{d}^4\sigma^{ep}_{T,L}}{\mathrm{dQ^2dW^2d}\beta\mathrm{dz}} = \frac{\mathrm{d}N_{\gamma T,L}}{\mathrm{d}Q^2\mathrm{d}W^2}\frac{\mathrm{d}^2\sigma^{\gamma^* p}_{T,L}}{\mathrm{d}\beta\mathrm{d}z}$$

$$\frac{d^2\sigma^{\gamma^* p}_{q\bar{q},T}}{\mathrm{d}\beta\mathrm{d}z} = \frac{N_C Q^2\alpha_{\mathrm{EM}}}{8\pi\beta^2}\sum_f e_f^2 z(1-z)\left[\epsilon^2(z^2+(1-z)^2)\Phi_1 + m_f^2\Phi_0\right]$$

$$\frac{d^2\sigma^{\gamma^* p}_{q\bar{q},L}}{\mathrm{d}\beta\mathrm{d}z} = \frac{N_C Q^4\alpha_{\mathrm{EM}}}{2\pi\beta^2}\sum_f e_f^2 z^3(1-z)^3\Phi_0]$$



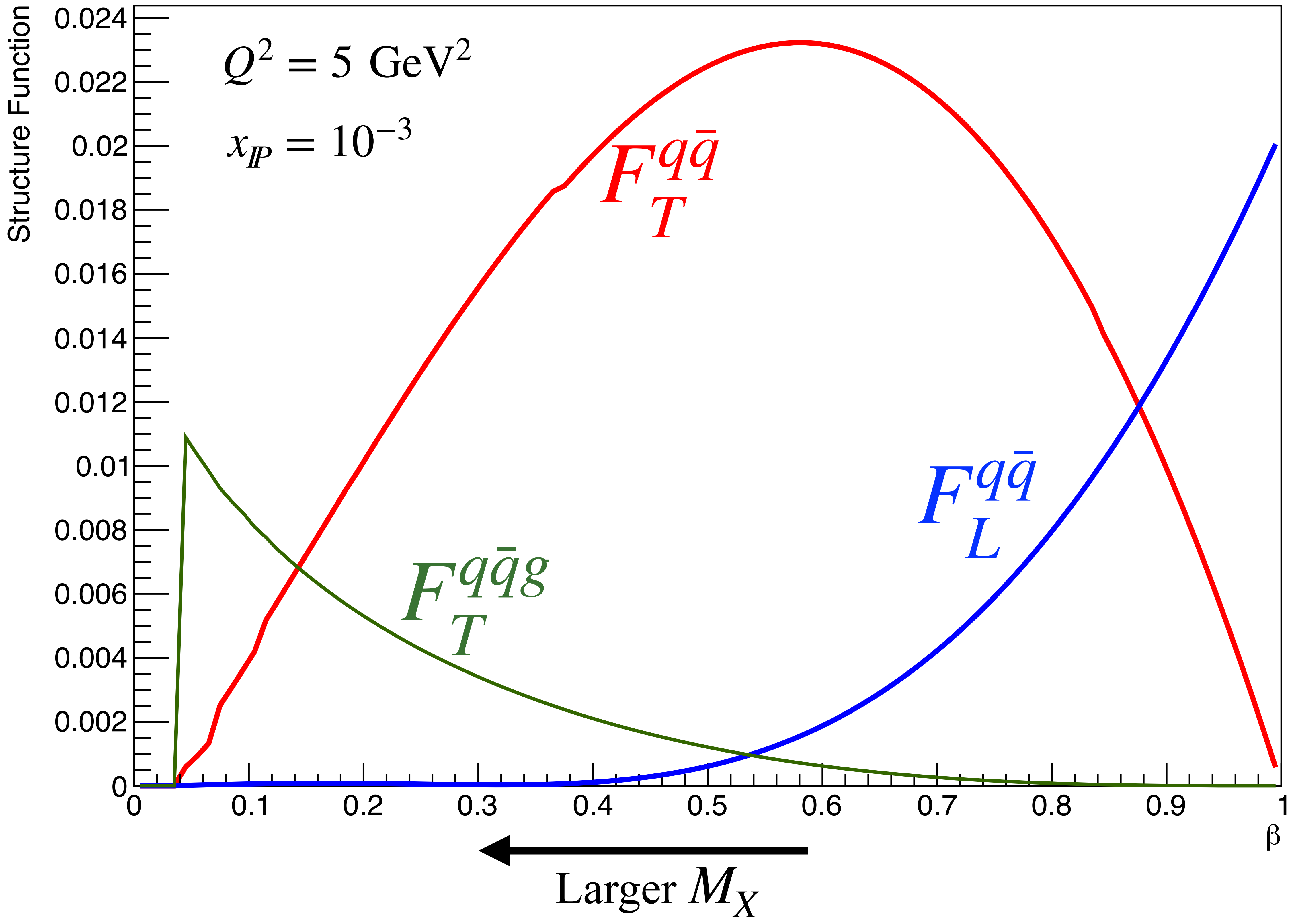
$$\Phi_{0,1} = \int \mathrm{d}b\,|\,\mathscr{A}_{0,1}\,|^2$$

$$\mathscr{A}_{0,1} = \int_0^\infty r\mathrm{d}rK_{0,1}(\epsilon r)J_{0,1}(kr)\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2b}$$

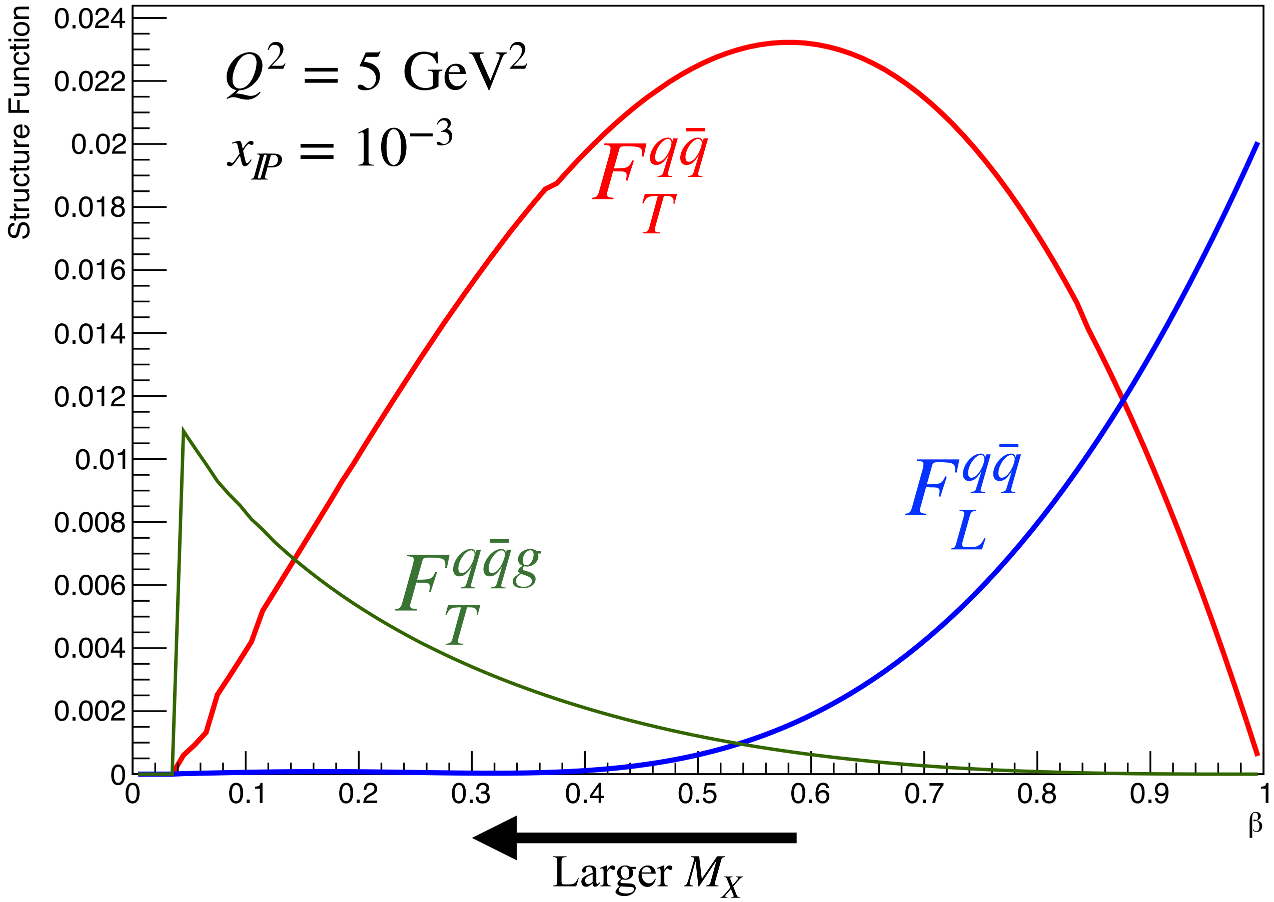
$$\epsilon^2 = z(1-z)Q^2 + m_f^2 \qquad k^2 = z(1-z)M_X^2 - m_f^2$$

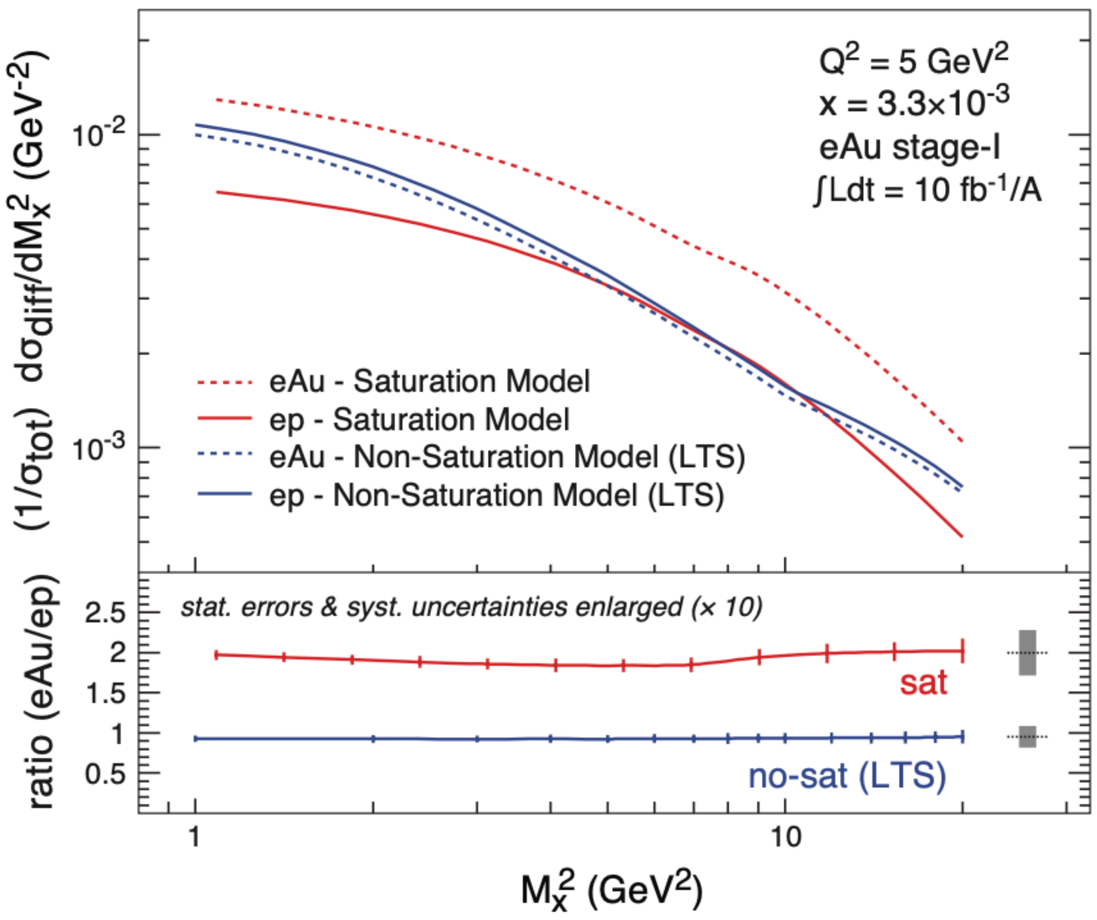
Kowalski, Lappi, Marquet, Venugopalan, Phys. Rev. C 78, 045201, arXiv:0805.4071

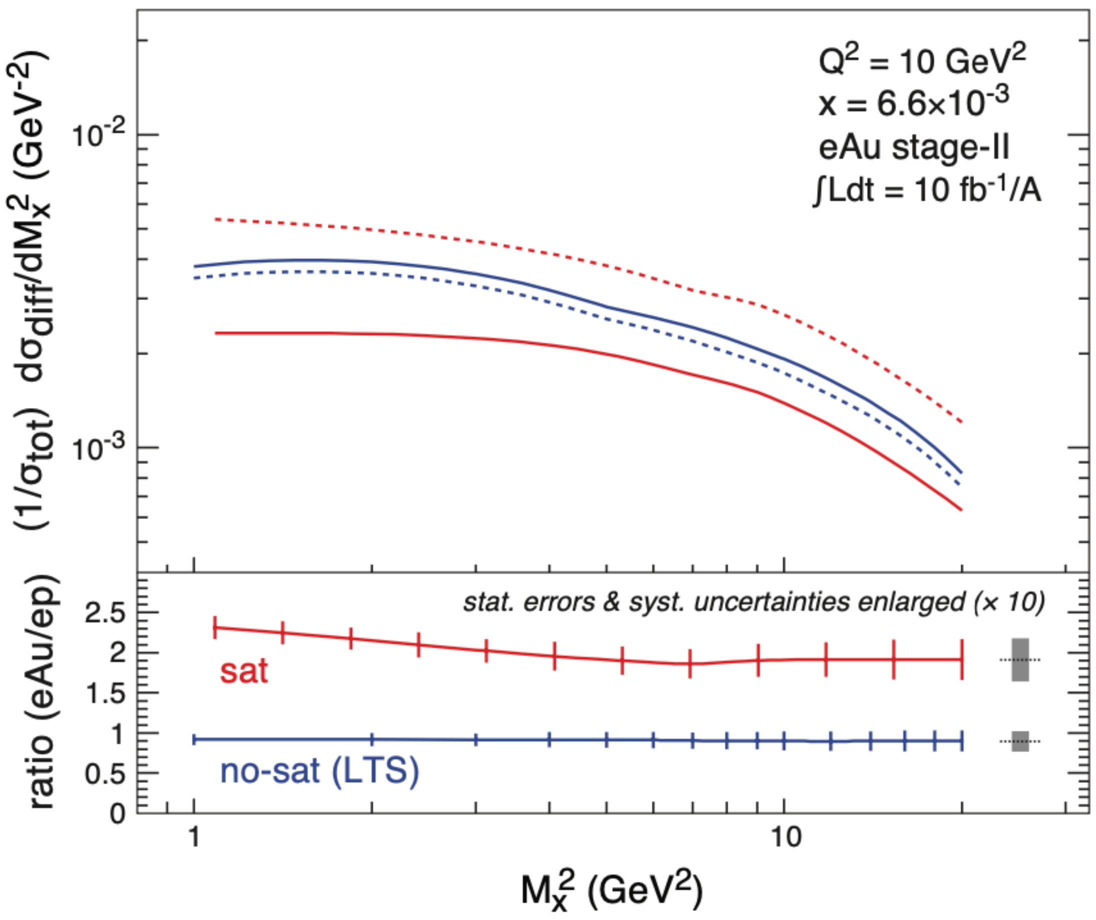
Inclusive Diffraction at small x



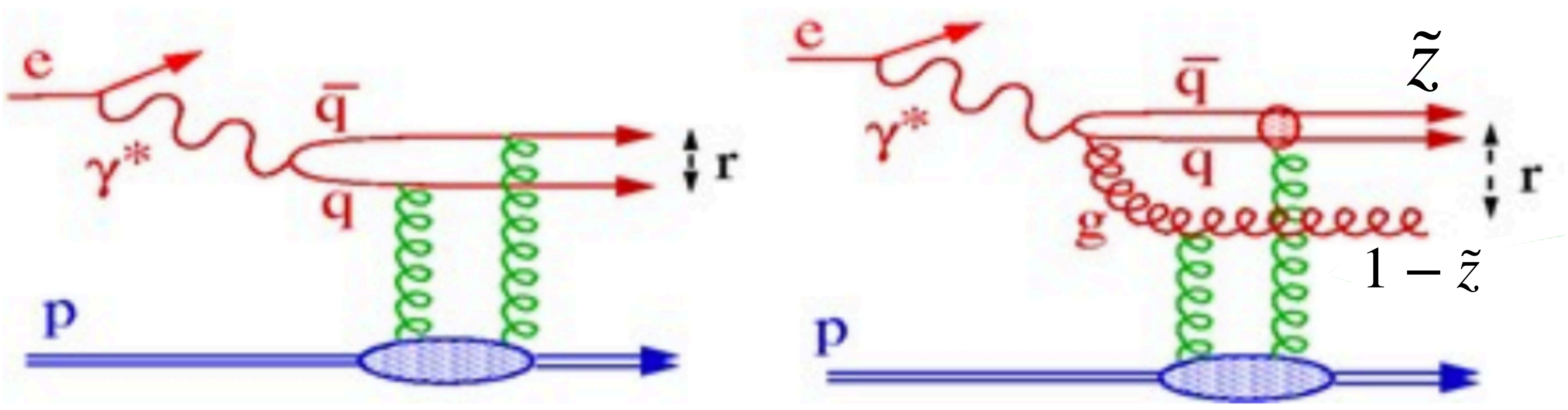
Inclusive Diffraction at small x







Inclusive Diffraction at small x



Marta Ruspa, DIS2004

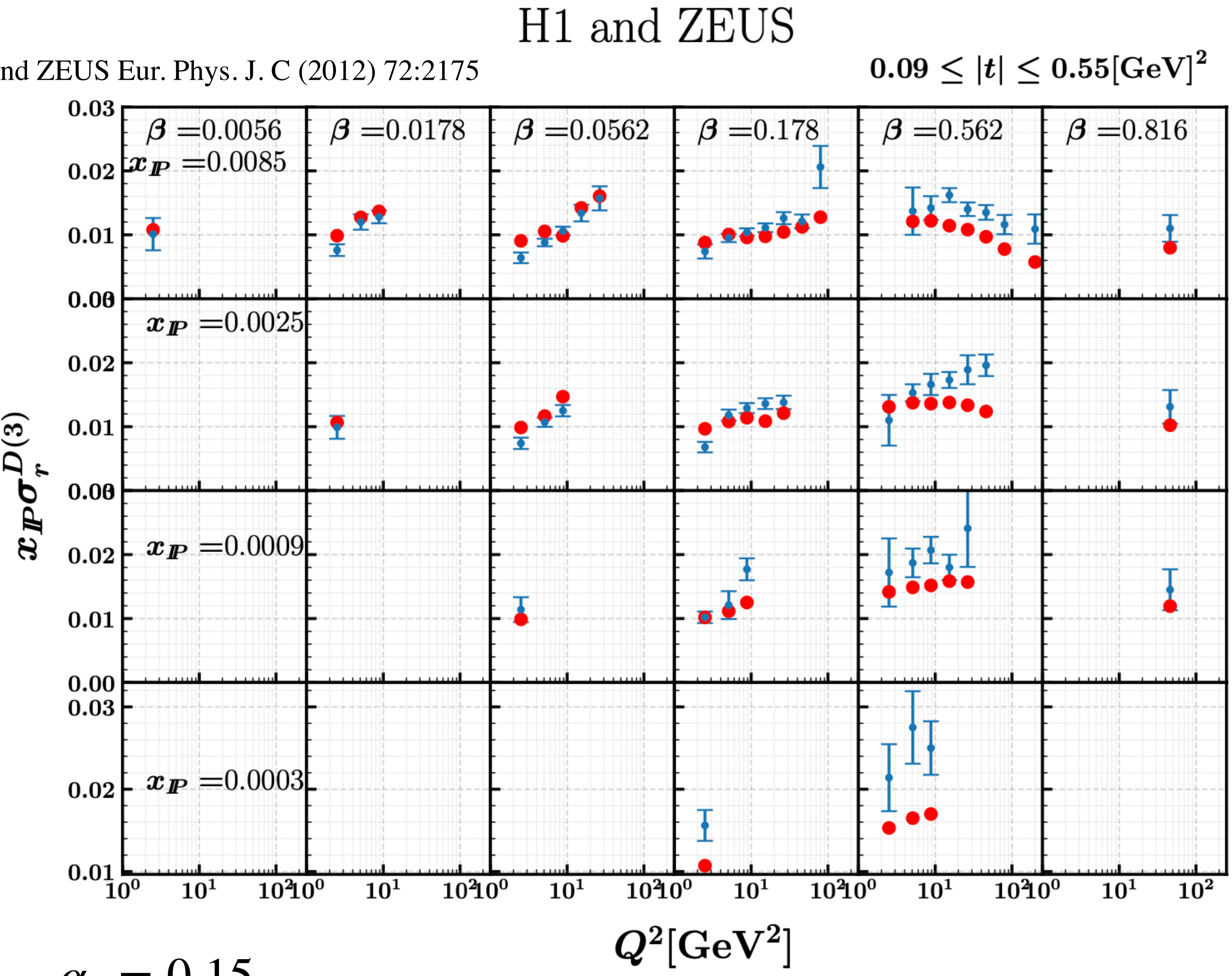
$$\frac{d^2\sigma^{\gamma^* p}_{q\bar q g,T}}{\mathrm{d}\beta\,\mathrm{d}\tilde z} = \frac{\alpha_S\alpha_{\mathrm{EM}}}{2\pi^2 Q^2}\left[\left(1-\frac{\beta}{\tilde z}\right)^2 + \left(\frac{\beta}{\tilde z}\right)^2\right]\sum_f e_f^2\,\Phi_{q\bar q g}$$

$$\Phi_{q\bar q g} = \int \mathrm{d}^2\vec b \int_0^{Q^2} \mathrm{d}\kappa^2\kappa^4 \ln\frac{Q^2}{\kappa^2}\,|\mathcal{A}_{q\bar q g}|^2$$

$$\mathcal{A}_{q\bar q g} = \int_0^\infty r\,\mathrm{d}r\,K_2(\sqrt{\tilde z}\kappa r)J_2(\sqrt{1-\tilde z}\kappa r)\frac{\mathrm{d}\sigma_{gg}}{\mathrm{d}^2 b} \qquad \frac{\mathrm{d}\sigma_{gg}}{\mathrm{d}^2 b} = 2\left[1-\left(1-\frac{1}{2}\frac{\mathrm{d}\sigma_{q\bar q}}{\mathrm{d}^2 b}\right)^2\right]$$

Inclusive Diffraction at small x

H1 and ZEUS Eur. Phys. J. C (2012) 72:2175



$\alpha_S = 0.15$

Work by Jaswant Singh

Inclusive Diffraction in Sar*t*re

Two Event Generators: $q\bar{q}$ and $q\bar{q}g$ final states.

$q\bar{q}$-generator: Generate β, Q^2, W^2, and z from differential cross-section.

$q\bar{q}g$-generator: $\tilde{z}$ instead of z

$$\frac{\mathrm{d}^4\sigma^{ep}_{T,L}}{\mathrm{d}Q^2\mathrm{d}W^2\mathrm{d}\beta\mathrm{d}z} = \frac{\mathrm{d}N_{\gamma T,L}}{\mathrm{d}Q^2\mathrm{d}W^2}\frac{\mathrm{d}^2\sigma^{\gamma^*p}_{T,L}}{\mathrm{d}\beta\mathrm{d}z}$$

Calculate an exclusive final state from these variables.

Create 4D in (Q^2, W^2, β, z)lookup tables for cross-sections, one for each quark flavour

=>12 for each initial state (using 4 flavours)

$$\frac{d^2\sigma^{\gamma^*p}_{q\bar{q},T}}{\mathrm{d}\beta\mathrm{d}z} = \frac{N_C Q^2\alpha_{\mathrm{EM}}}{8\pi\beta^2}\sum_f e_f^2 z(1-z)\left[\epsilon^2(z^2+(1-z)^2)\Phi_1+m_f^2\Phi_0\right]$$

$$\frac{d^2\sigma^{\gamma^*p}_{q\bar{q},L}}{\mathrm{d}\beta\mathrm{d}z} = \frac{N_C Q^4\alpha_{\mathrm{EM}}}{2\pi\beta^2}\sum_f e_f^2 z^3(1-z)^3\Phi_0]$$

$$\frac{d^2\sigma^{\gamma^*p}_{q\bar{q}g,T}}{\mathrm{d}\beta\mathrm{d}\tilde{z}} = \frac{\alpha_S\alpha_{\mathrm{EM}}}{2\pi^2Q^2}\left[\left(1-\frac{\beta}{\tilde{z}}\right)^2+\left(\frac{\beta}{\tilde{z}}\right)^2\right]\sum_f e_f^2\Phi_{q\bar{q}g}$$

Inclusive Diffraction in Sar*t*re

Two Event Generators: $q\bar{q}$ and $q\bar{q}g$ final states.

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$$\frac{\mathrm{d}^4\sigma^{ep}_{T,L}}{\mathrm{d}Q^2\mathrm{d}W^2\mathrm{d}\beta\mathrm{d}z} = \frac{\mathrm{d}N_{\gamma T,L}}{\mathrm{d}Q^2\mathrm{d}W^2}\frac{\mathrm{d}^2\sigma^{\gamma^* p}_{T,L}}{\mathrm{d}\beta\mathrm{d}z}$$

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$$\frac{d^2\sigma^{\gamma^* p}_{q\bar{q},T}}{\mathrm{d}\beta\mathrm{d}z} = \frac{N_C Q^2\alpha_{\mathrm{EM}}}{8\pi\beta^2}\sum_f e_f^2 z(1-z)\left[\epsilon^2(z^2+(1-z)^2)\Phi_1 + m_f^2\Phi_0\right] \qquad \Phi_{0,1} = \int \mathrm{d}b\,|\mathcal{A}_{0,1}|^2$$

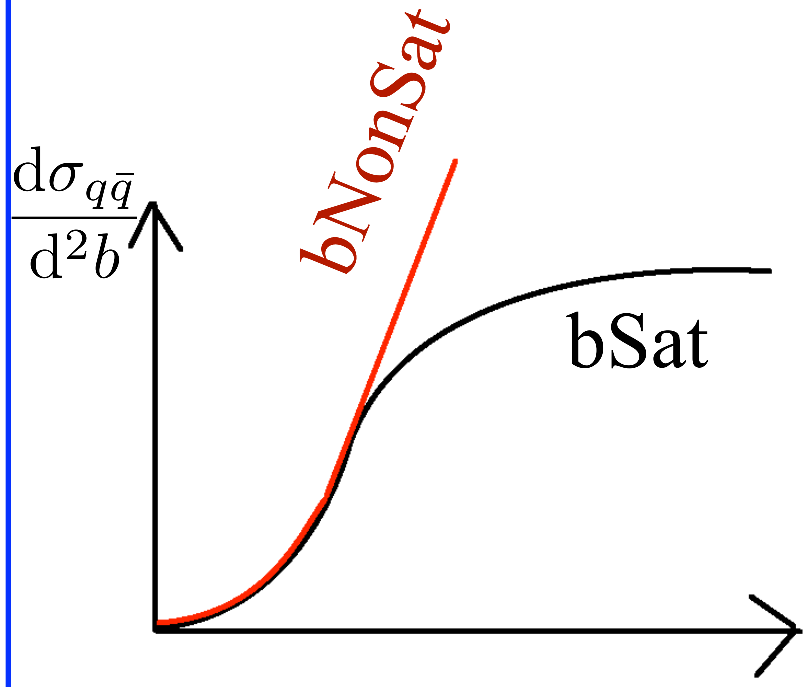
$$\frac{d^2\sigma^{\gamma^* p}_{q\bar{q},L}}{\mathrm{d}\beta\mathrm{d}z} = \frac{N_C Q^4\alpha_{\mathrm{EM}}}{2\pi\beta^2}\sum_f e_f^2 z^3(1-z)^3\Phi_0] \qquad \mathcal{A}_{0,1} = \int_0^\infty r\mathrm{d}r K_{0,1}(\epsilon r)J_{0,1}(kr)\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2 b}$$

$$\frac{d^2\sigma^{\gamma^* p}_{q\bar{q}g,T}}{\mathrm{d}\beta\mathrm{d}\tilde{z}} = \frac{\alpha_S\alpha_{\mathrm{EM}}}{2\pi^2 Q^2}\left[\left(1-\frac{\beta}{\tilde{z}}\right)^2 + \left(\frac{\beta}{\tilde{z}}\right)^2\right]\sum_f e_f^2\Phi_{q\bar{q}g}$$

Inclusive Diffraction in Sar*t*re

$$\mu^2 = \frac{C}{r^2} + \mu_0^2$$

$$T_p(b) = \frac{1}{2\pi B_G} e^{-\frac{b^2}{2B_G}}$$



$$\frac{\mathrm{d}\sigma_{q\bar{q}}^{\mathrm{bNonSat}}}{\mathrm{d}^2 b} = \frac{\pi^2}{N_C} r^2 \alpha_S(\mu^2) x g(x, \mu^2) T(b)$$

$$\frac{\mathrm{d}\sigma_{q\bar{q}}^{\mathrm{bSat}}}{\mathrm{d}^2 b} = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_C} r^2 \alpha_S(\mu^2) x g(x, \mu^2) T(b)\right)\right]$$

H. Kowalski, L. Motyka, G. Watt, Phys.Rev.D 74 (2006) 074016, arXiv: hep-ph/0606272

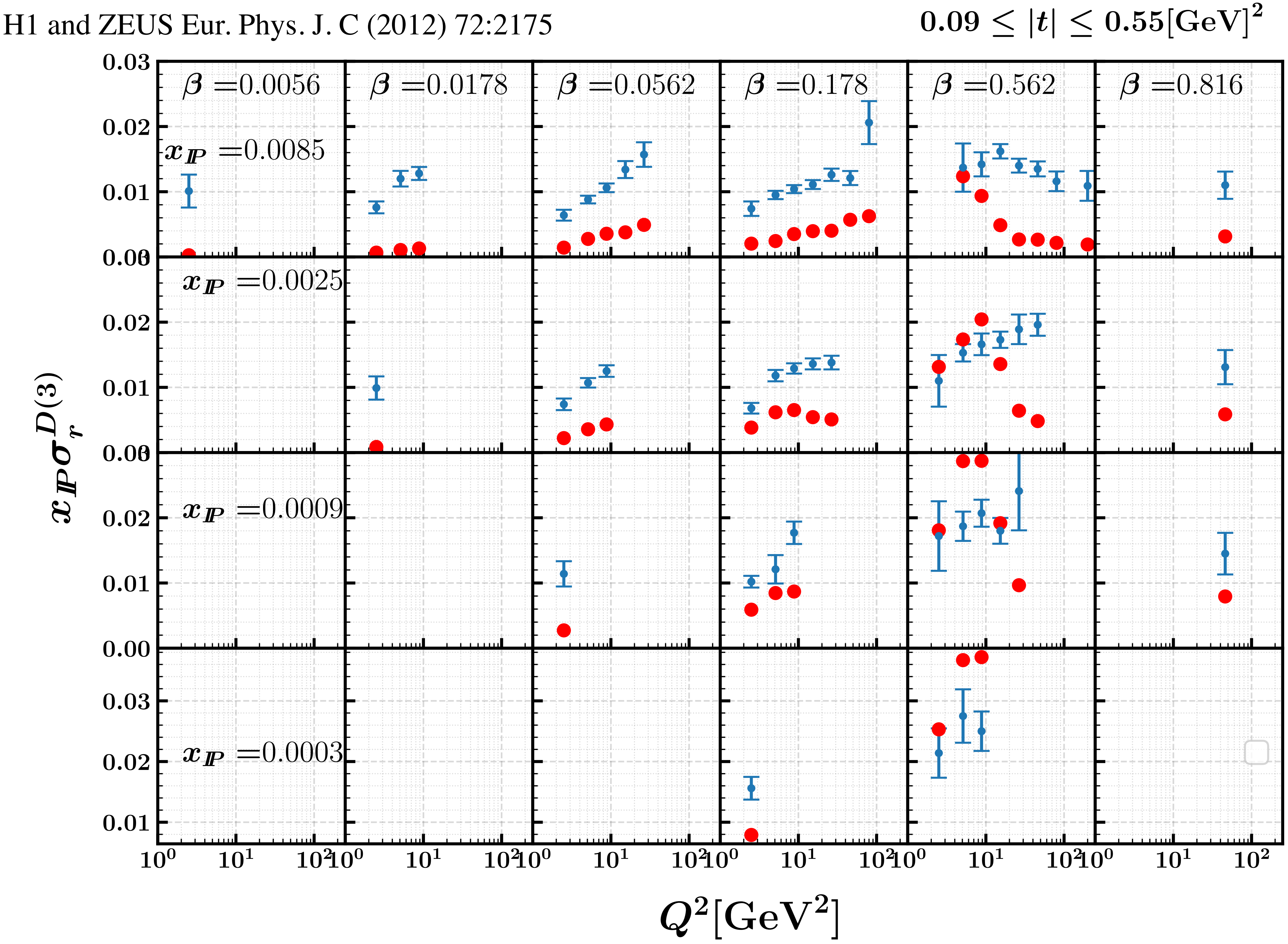
$$\frac{d^2\sigma_{q\bar{q},L}^{\gamma^* p}}{\mathrm{d}\beta \mathrm{d}z} = \frac{N_C Q^4 \alpha_{\mathrm{EM}}}{2\pi\beta^2} \sum_f e_f^2 z^3 (1-z)^3 \Phi_0]$$

$$\mathscr{A}_{0,1} = \int_0^\infty r \mathrm{d}r K_{0,1}(\epsilon r) J_{0,1}(kr) \frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2 b}$$

$$\frac{d^2\sigma_{q\bar{q}g,T}^{\gamma^* p}}{\mathrm{d}\beta \mathrm{d}\tilde{z}} = \frac{\alpha_S \alpha_{\mathrm{EM}}}{2\pi^2 Q^2} \left[\left(1 - \frac{\beta}{\tilde{z}}\right)^2 + \left(\frac{\beta}{\tilde{z}}\right)^2\right] \sum_f e_f^2 \Phi_{q\bar{q}g}$$

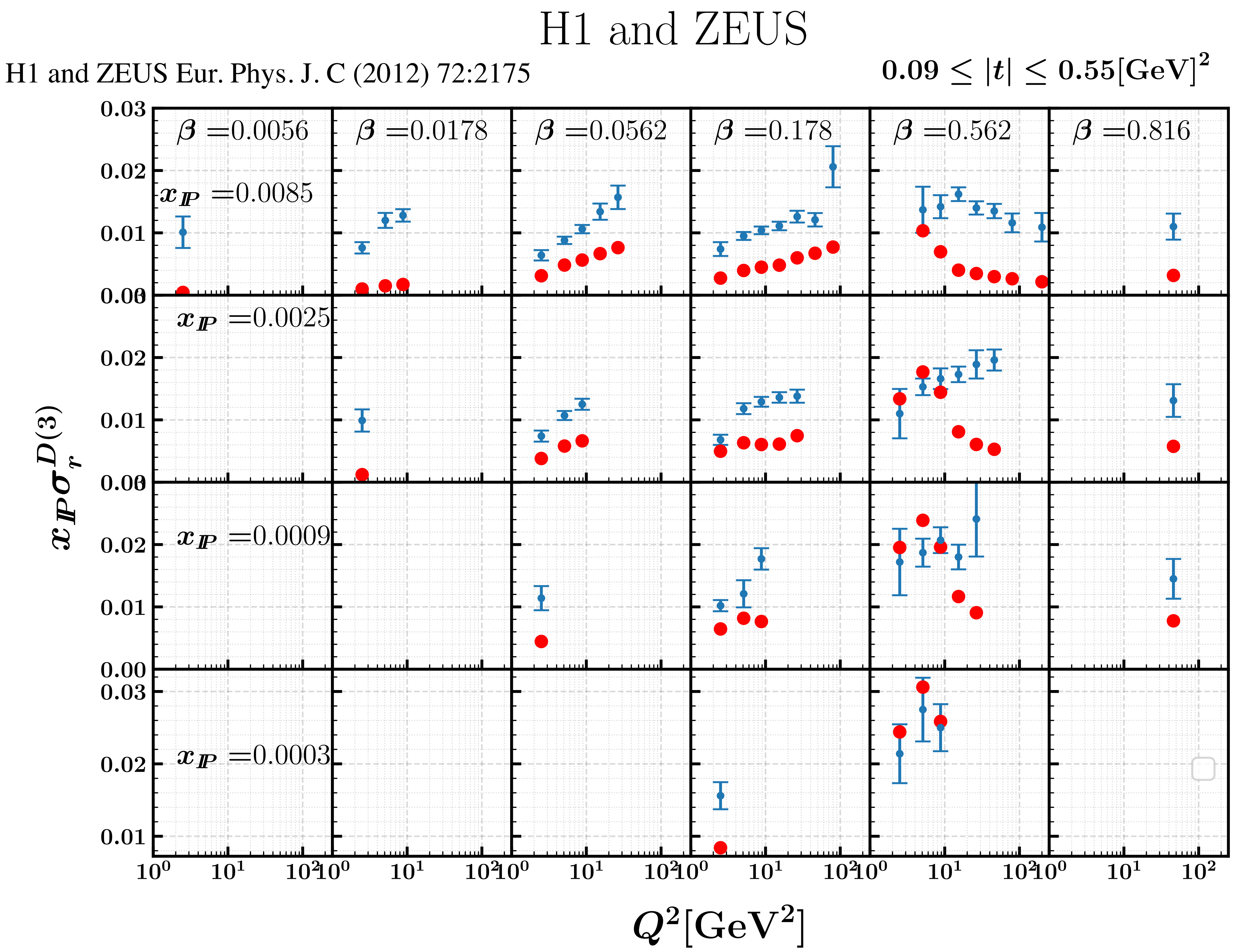
Cross-section Lookup Tables

H1 and ZEUS



Result from tables with 10x10x10x10 bins

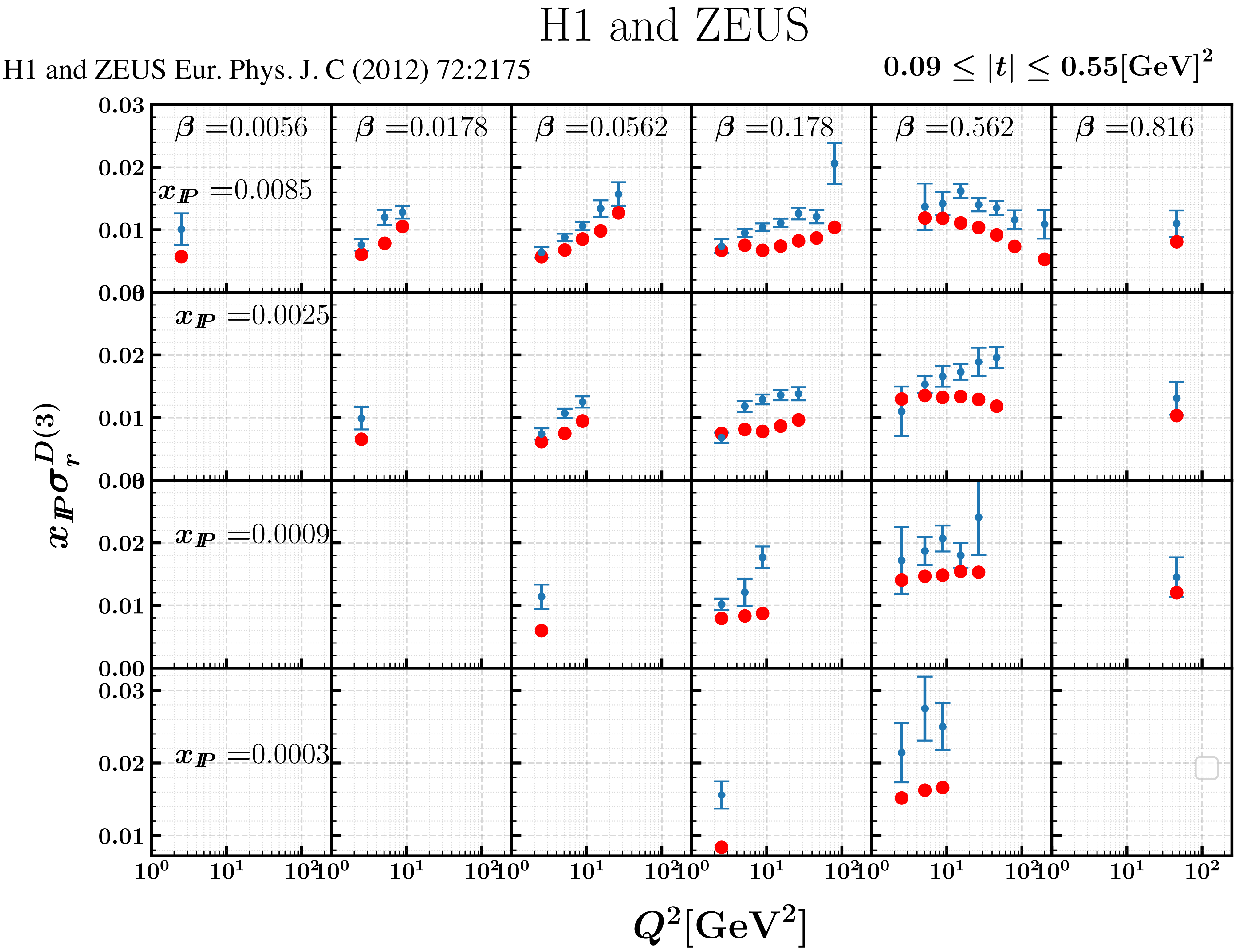
Cross-section Lookup Tables



H1 and ZEUS Eur. Phys. J. C (2012) 72:2175

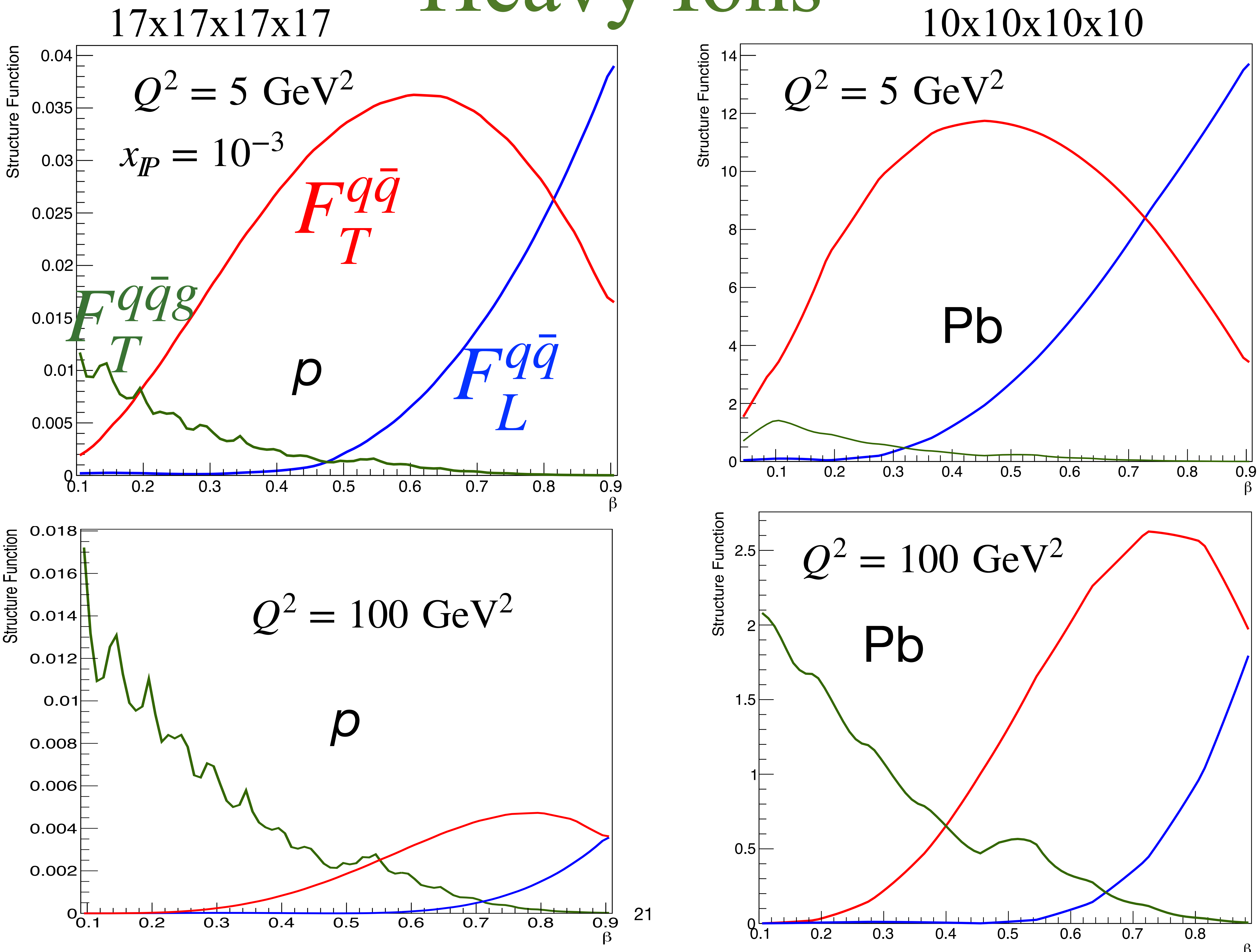
Result from tables with 17x17x17x17 bins

Cross-section Lookup Tables



Result from integrals

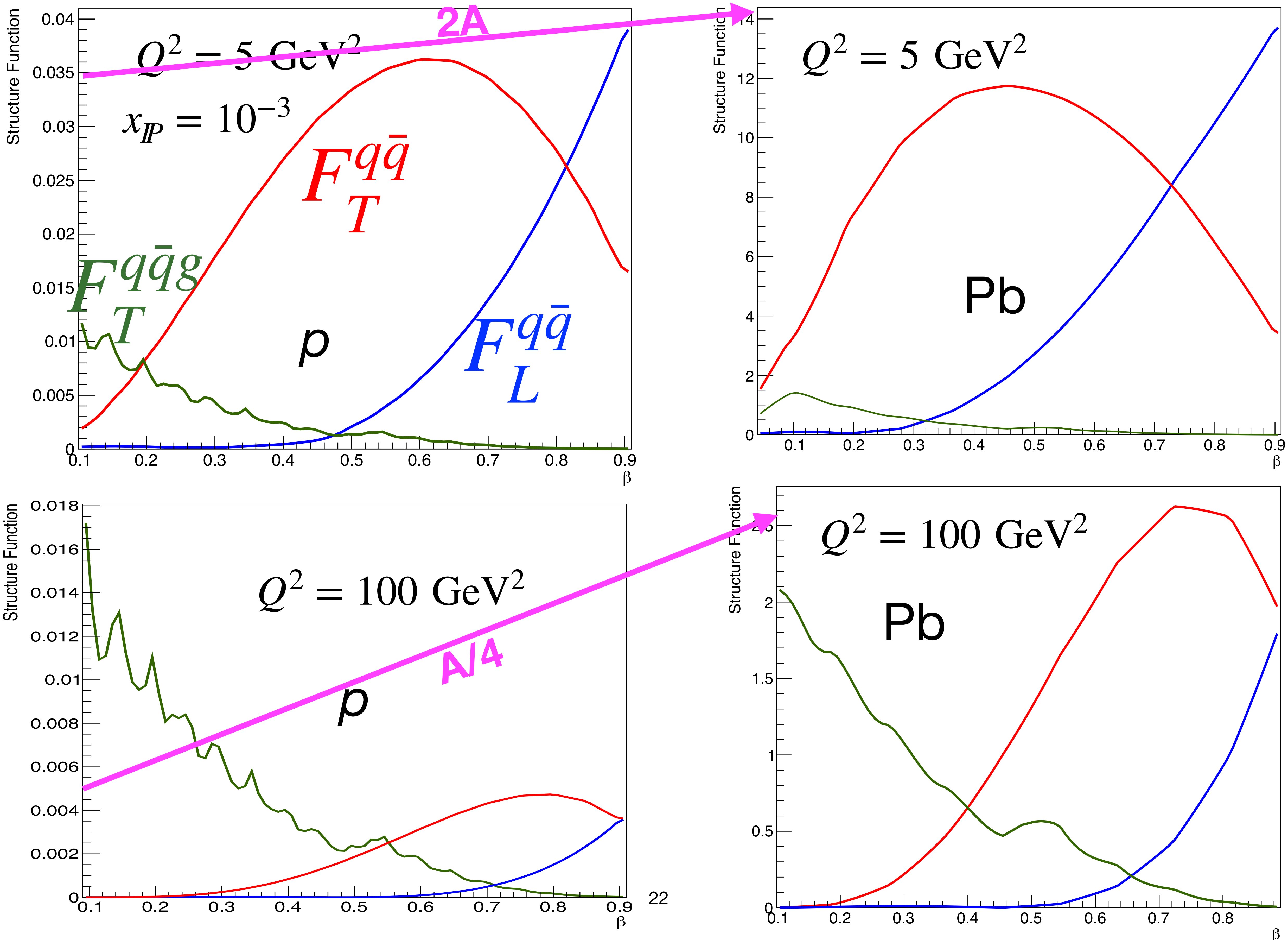
Heavy Ions


17x17x17x17
10x10x10x10
Structure Function
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
$Q^2 = 5\ \mathrm{GeV}^2$
$x_{I\!P} = 10^{-3}$
$F_T^{q\bar{q}}$
$F_T^{q\bar{q}g}$
$F_L^{q\bar{q}}$
p
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
β
Structure Function
14
12
10
8
6
4
2
0
$Q^2 = 5\ \mathrm{GeV}^2$
Pb
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
β
Structure Function
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
$Q^2 = 100\ \mathrm{GeV}^2$
p
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
β
Structure Function
2.5
2
1.5
1
0.5
0
$Q^2 = 100\ \mathrm{GeV}^2$
Pb
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
β

Heavy Ions

17x17x17x17

10x10x10x10

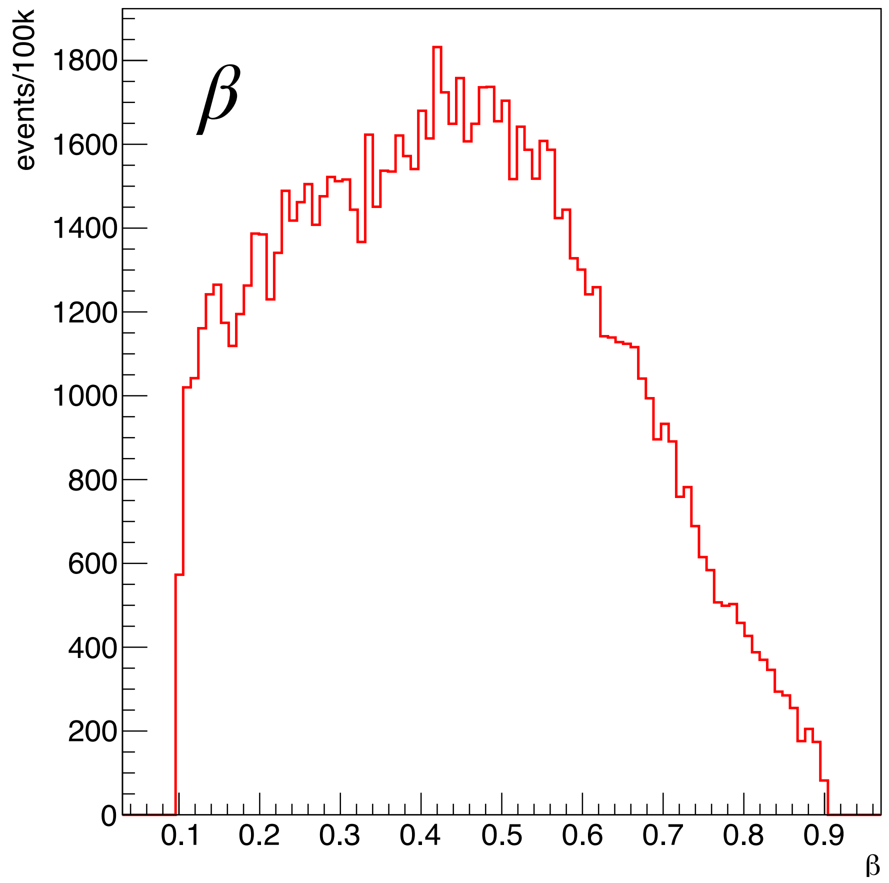


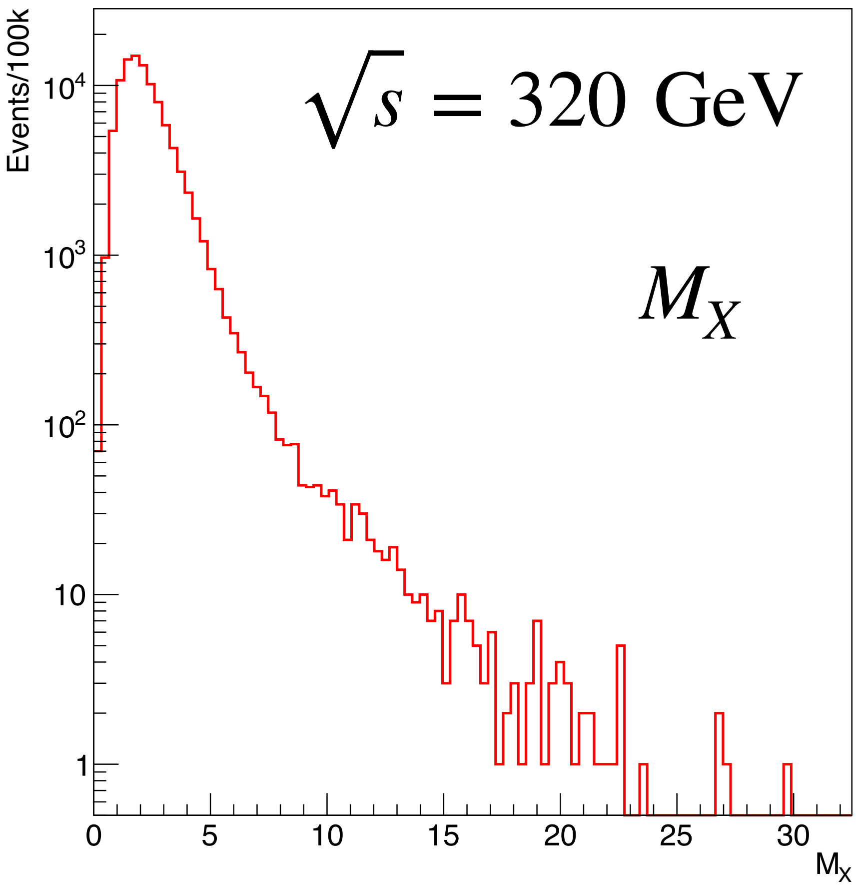
Event Generation Sar*t*re

$2 \leq Q^2 \leq 201\ \mathrm{GeV}^2$

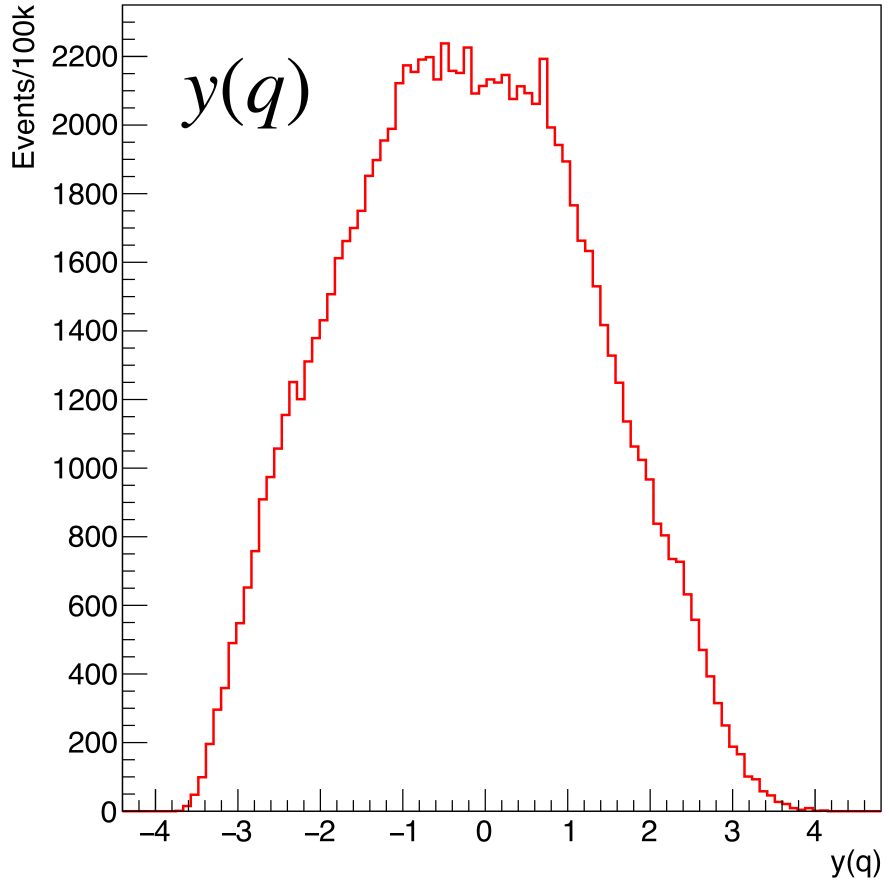
$0.1 \leq \beta \leq 0.9$

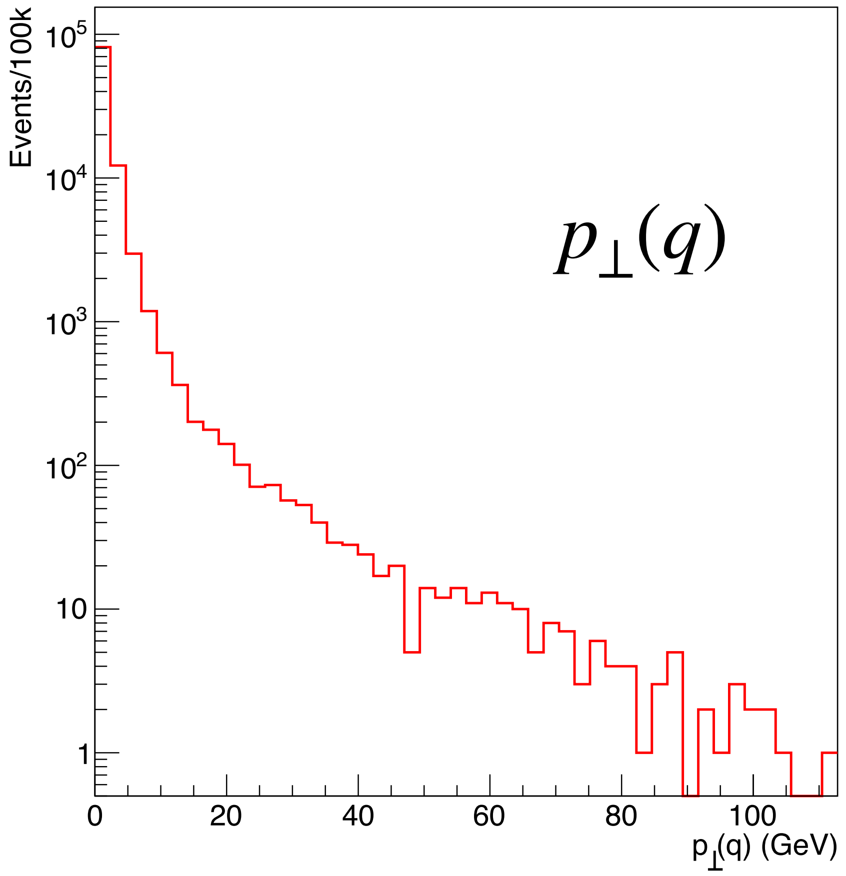
$20 \leq W \leq 240\ \mathrm{GeV}$





ep



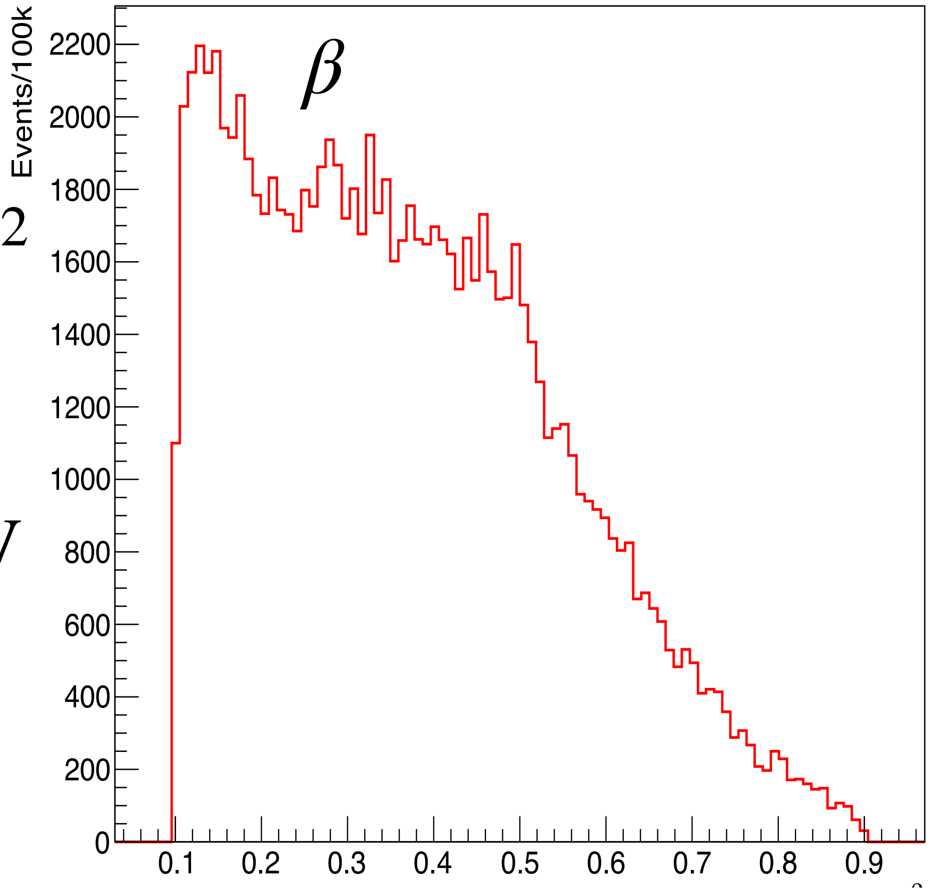


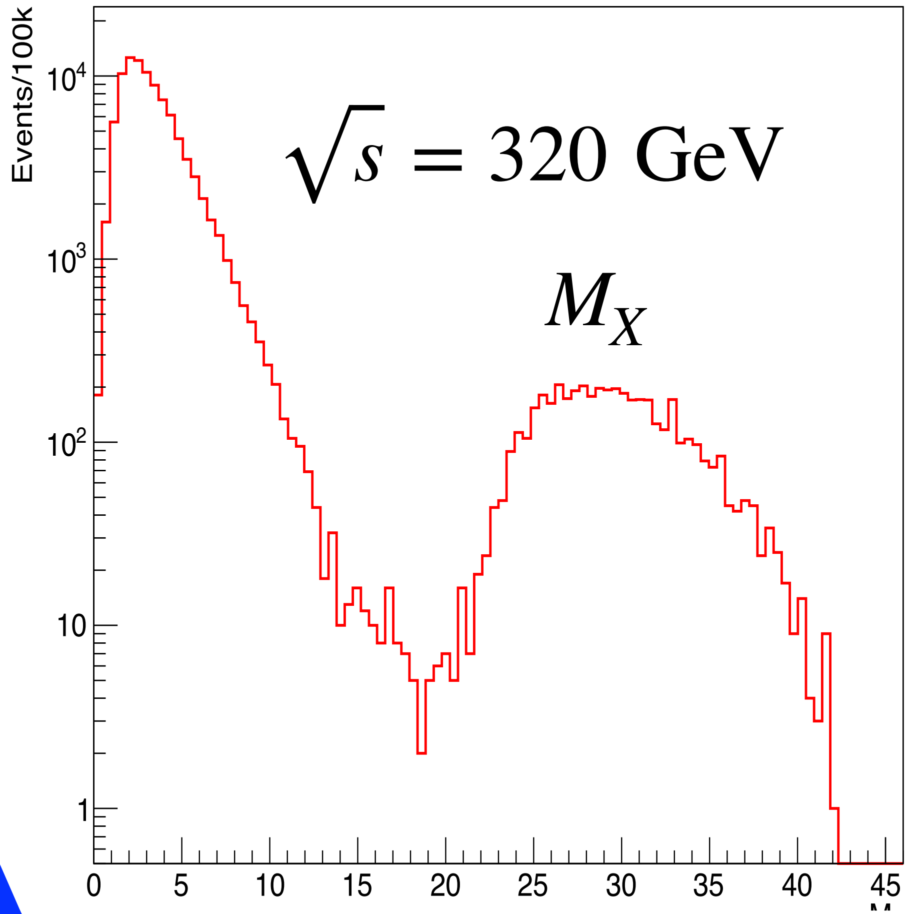
Event Generation Sartre

$2 \le Q^2 \le 201\ \mathrm{GeV}^2$

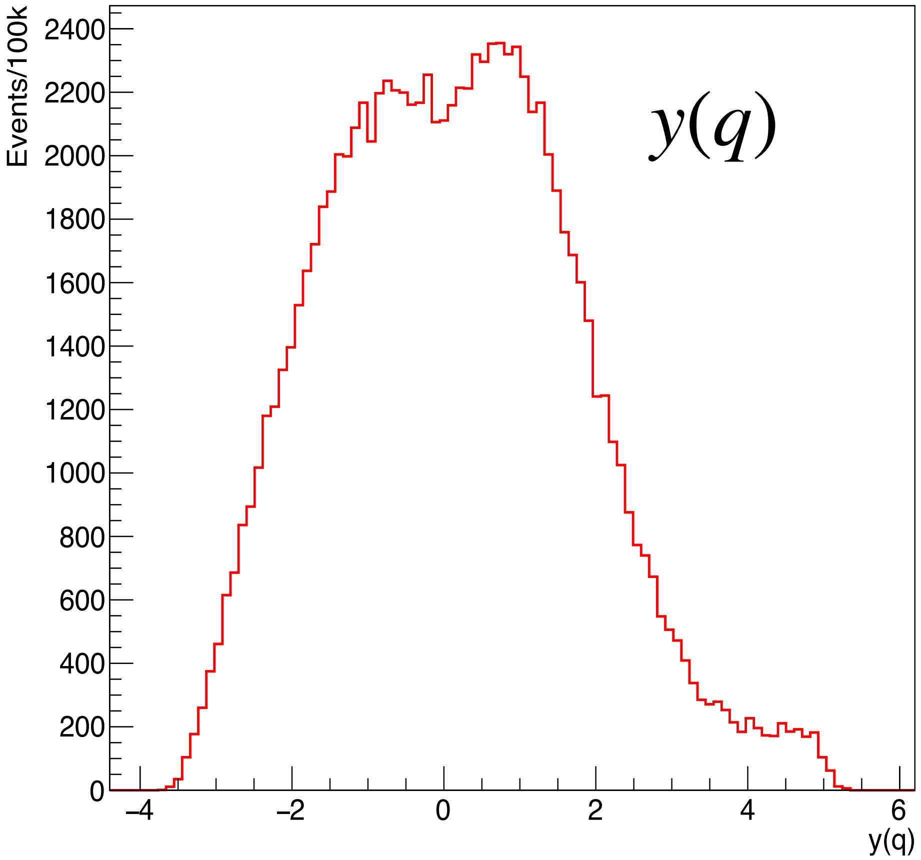
$0.1 \le \beta \le 0.9$

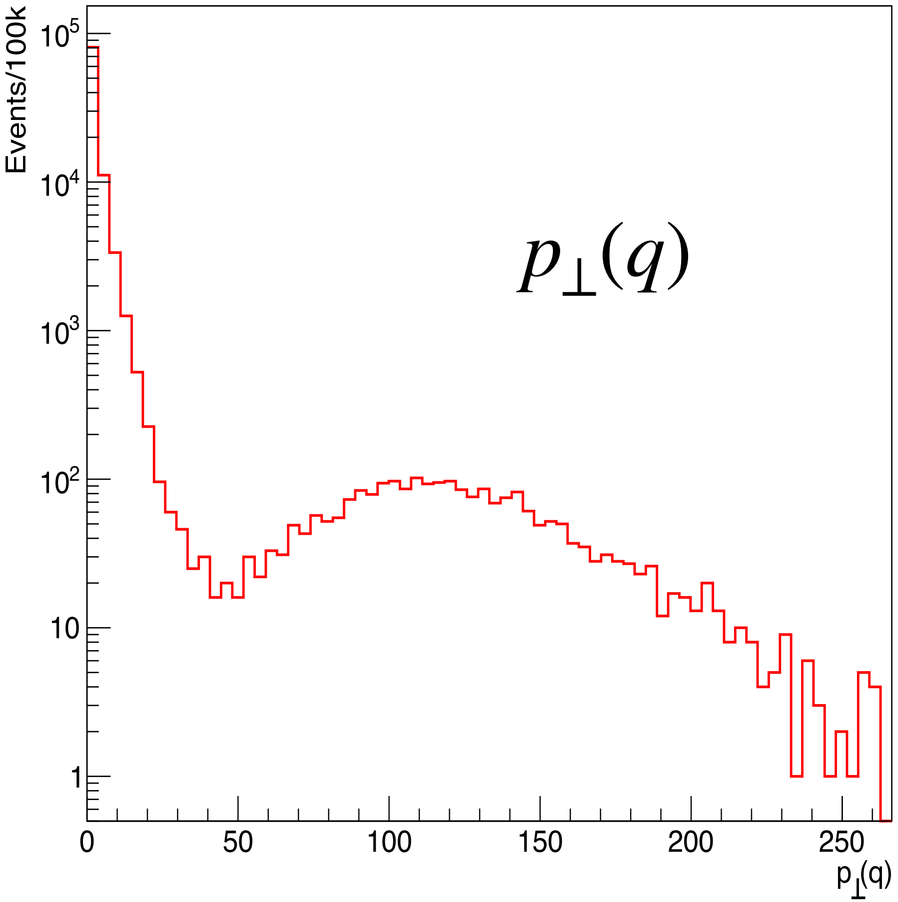
$20 \le W \le 240\ \mathrm{GeV}$





eA



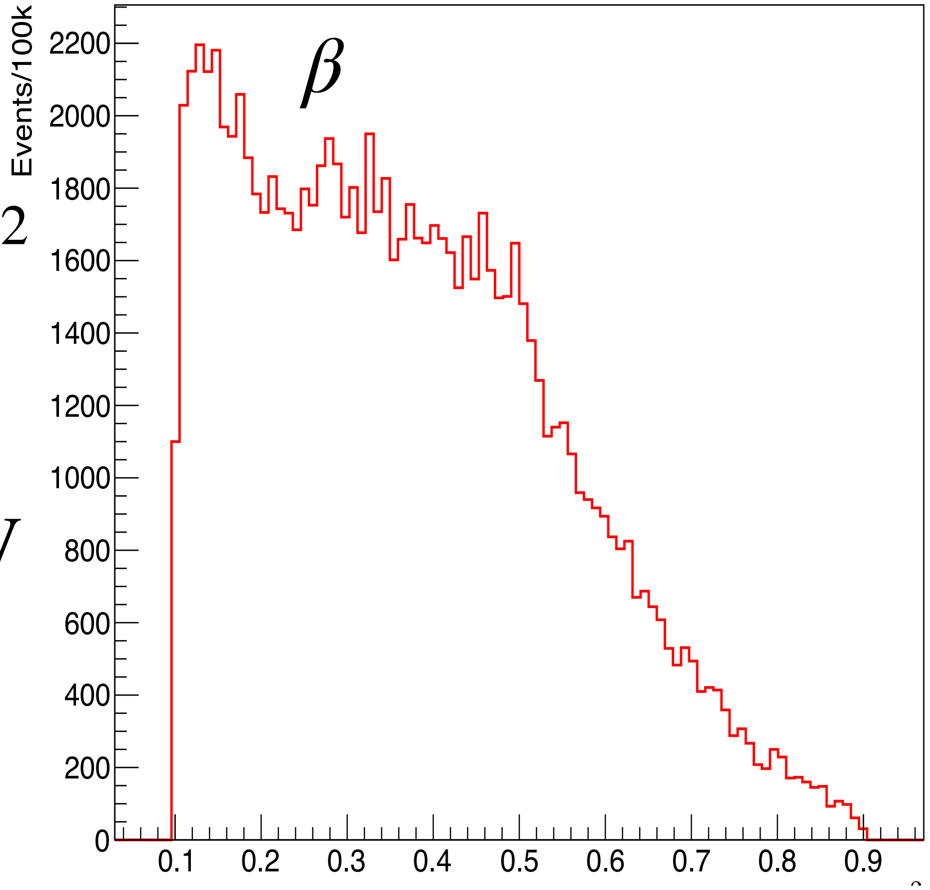


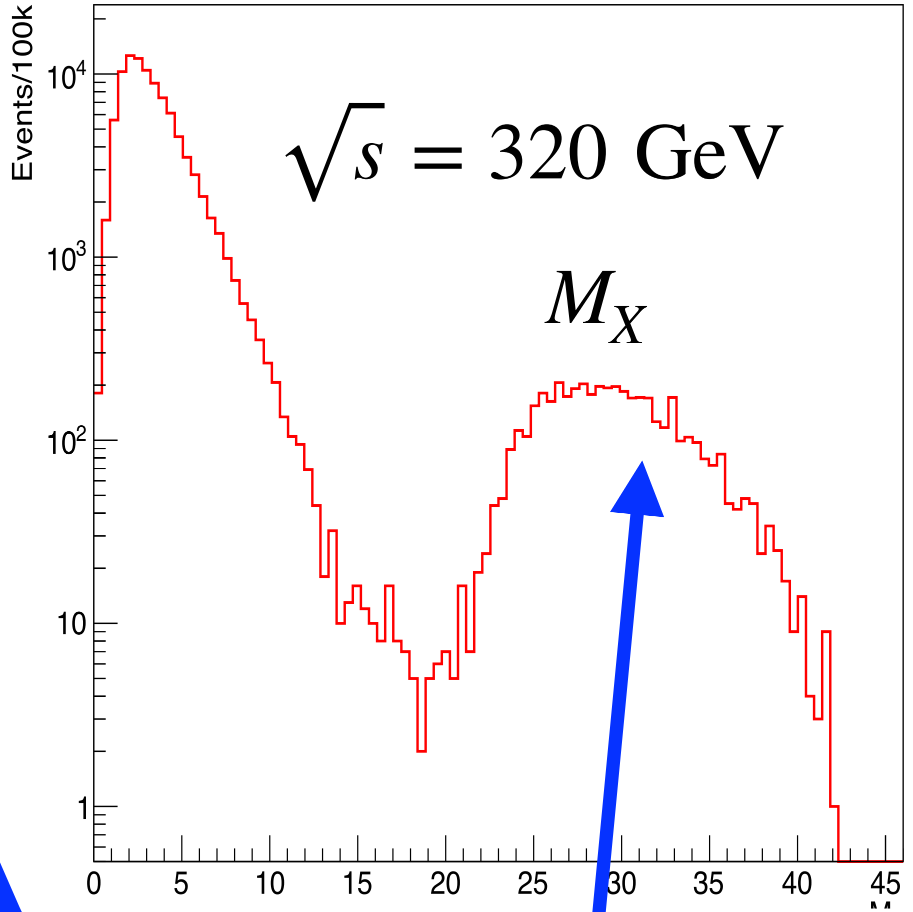
Event Generation Sar*t*re

$2 \leq Q^2 \leq 201\ \mathrm{GeV}^2$

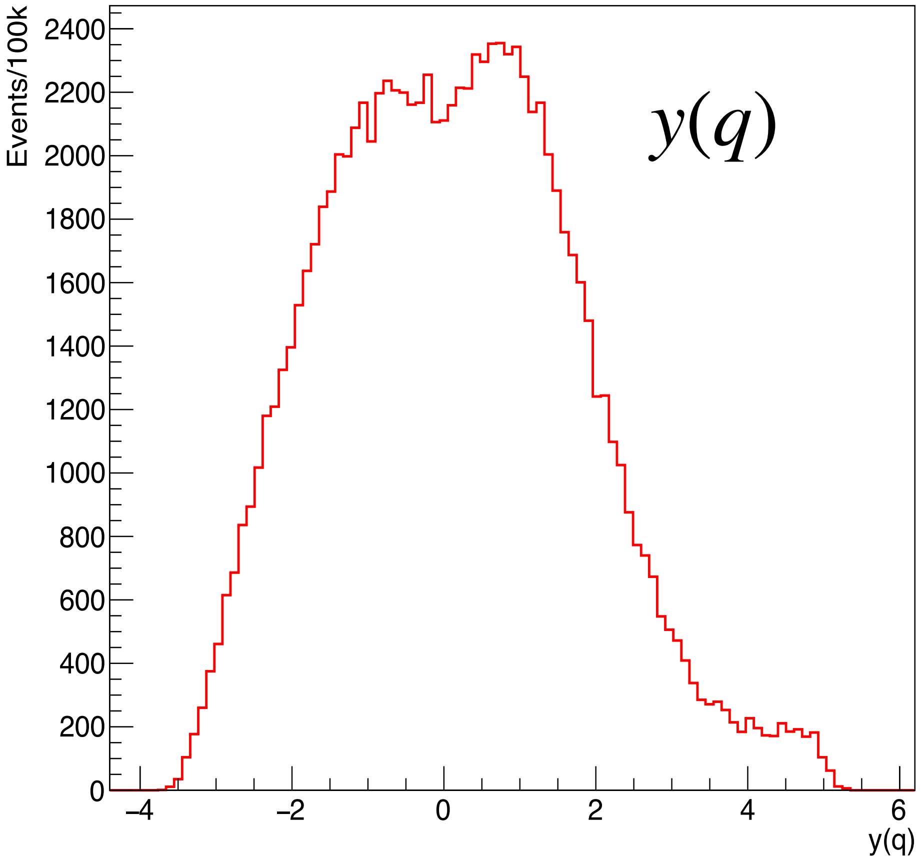
$0.1 \leq \beta \leq 0.9$

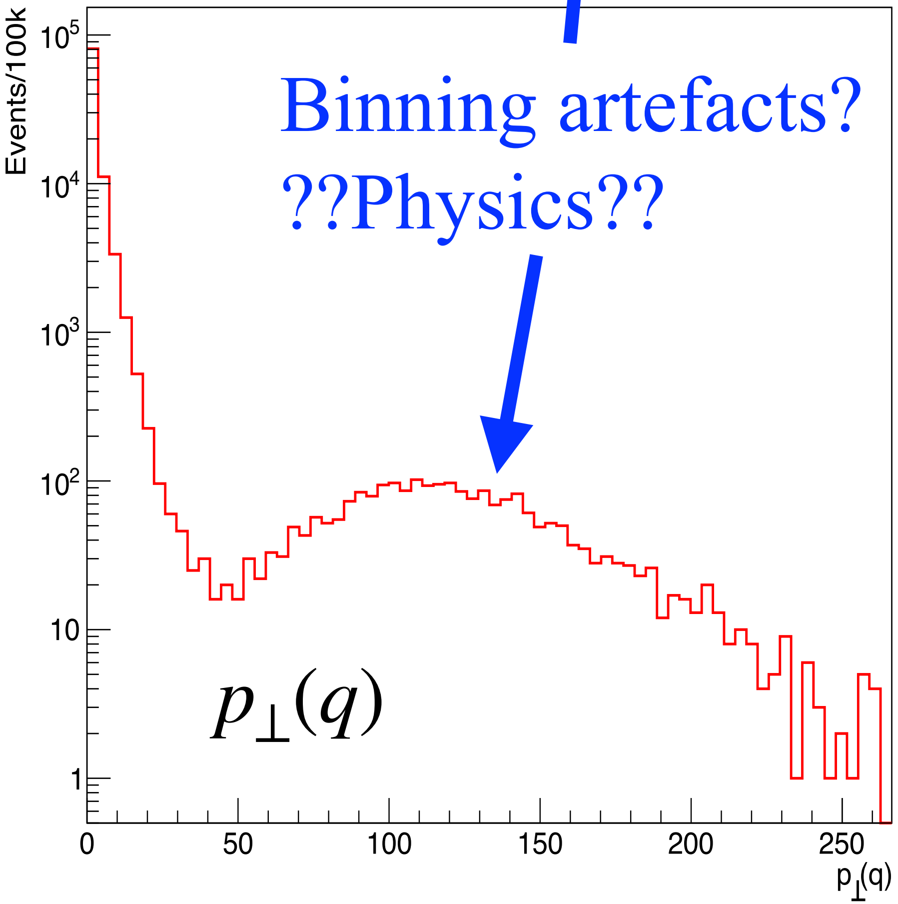
$20 \leq W \leq 240\ \mathrm{GeV}$





*e*A





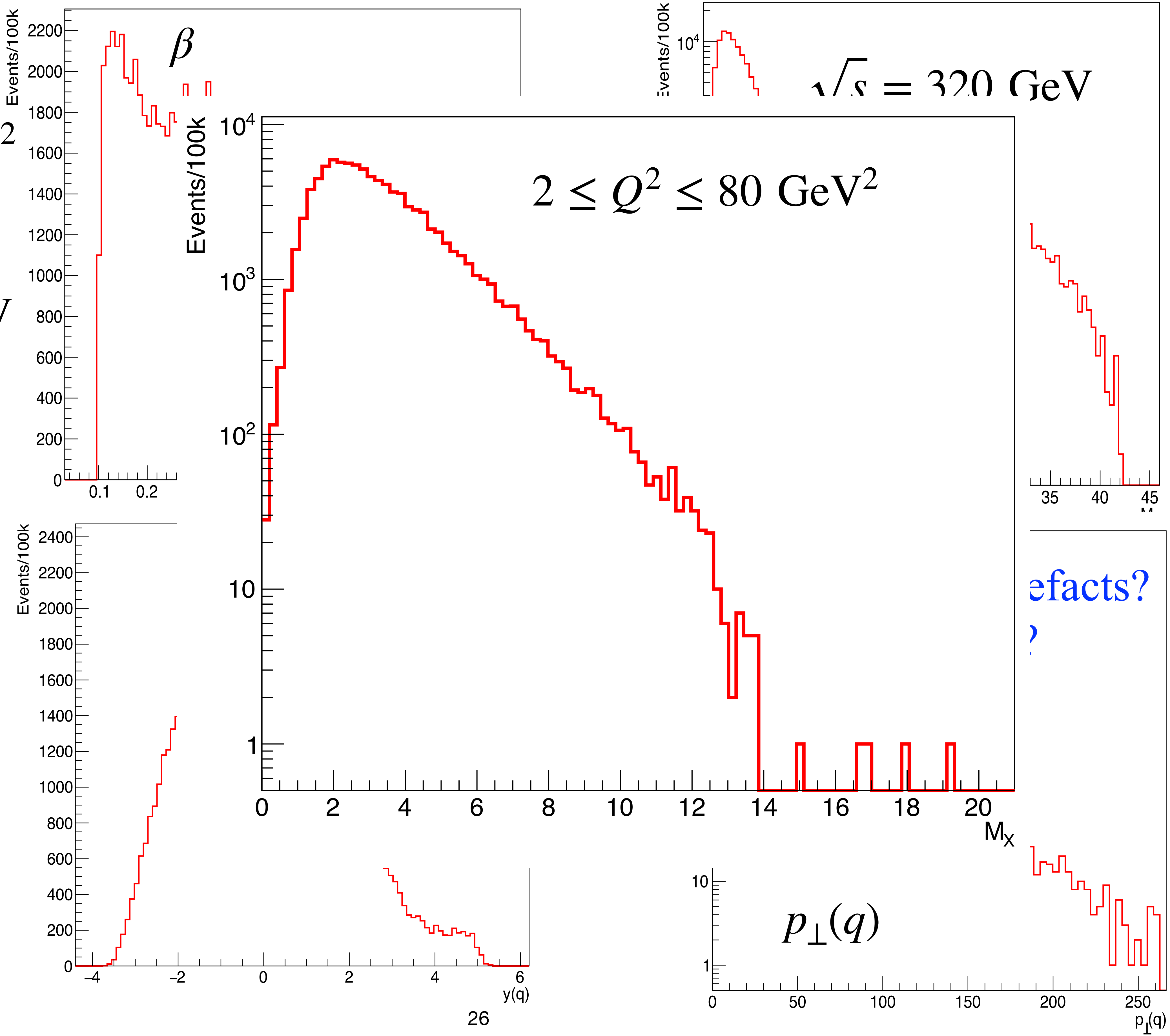
Binning artefacts?
??Physics??

Event Generation Sar*t*re

$2 \leq Q^2 \leq 201\ \mathrm{GeV}^2$

$0.1 \leq \beta \leq 0.9$

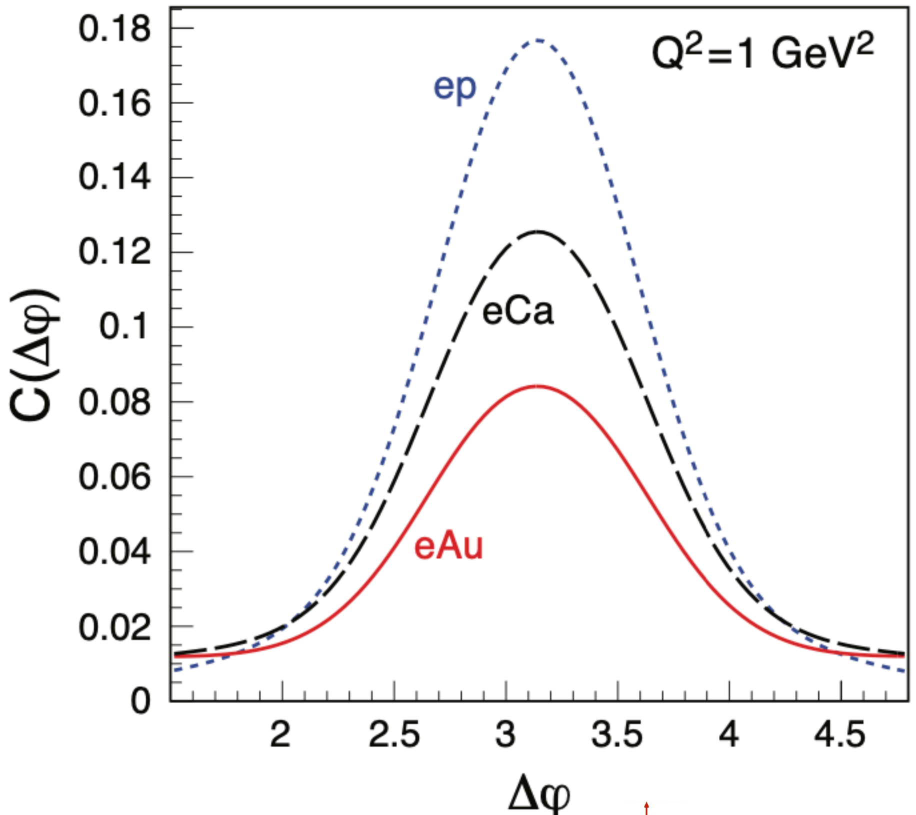
$20 \leq W \leq 240\ \mathrm{GeV}$

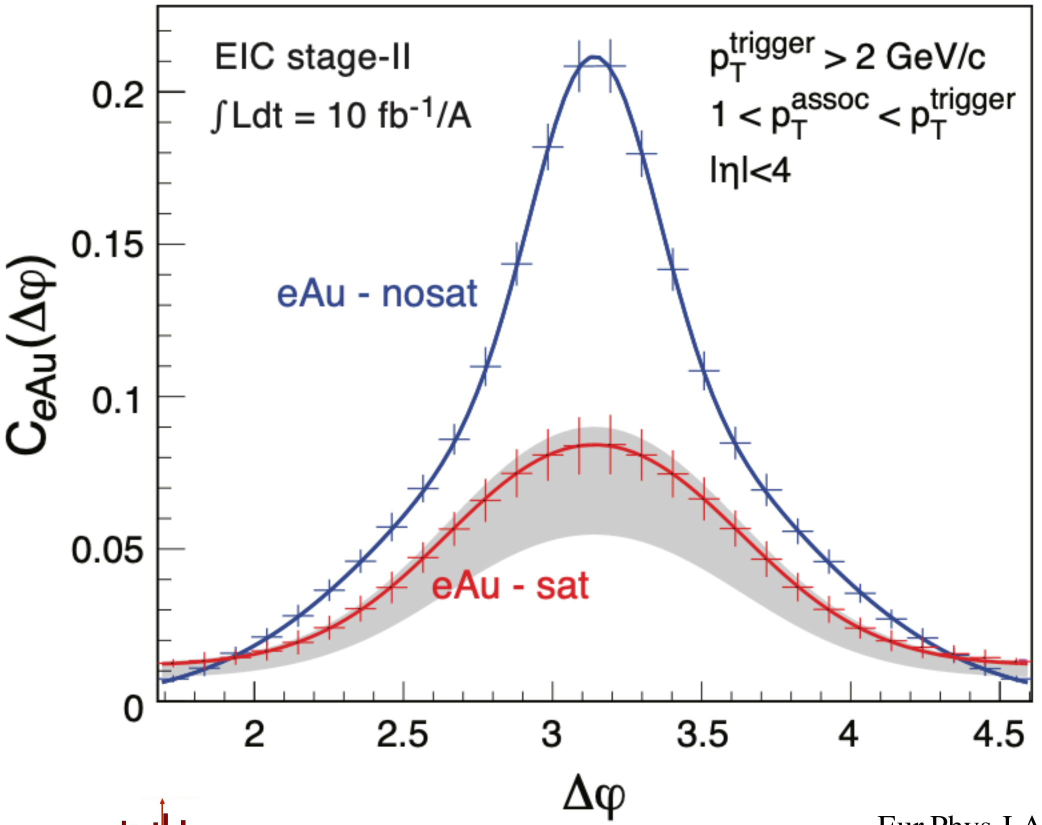


TO DO 1:
Saturation in the Final State

TO DO: Saturation in the Final State

Disappearance of the away peak in DIS:





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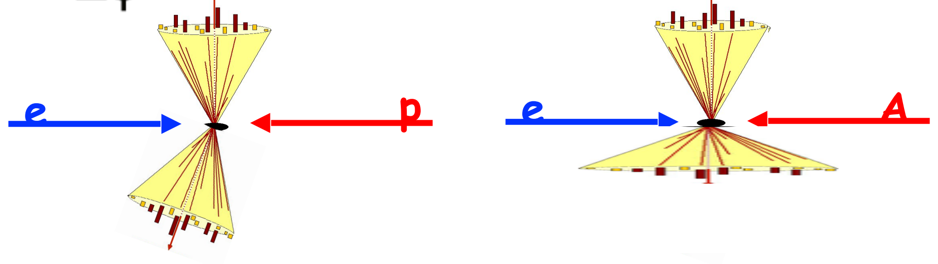

This effect should be even stronger in Inclusive Diffraction,
At least 2-gluon exchange!

TO DO: Saturation in the Final State

$$\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2 b}(x_{I\!P}, r, b) = 2\left[1 - \exp\left(-\frac{\Omega(x_{I\!P}, r, b)}{2}\right)\right] =$$

$$= \Omega - \frac{\Omega^2}{4} + \frac{\Omega^3}{24} - \ldots = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{2}{n!}\left(\frac{\Omega}{2}\right)^n$$

2 gluon exchange

4 gluon exchange

6 gluon exchange

n gluon exchange

At twist n, there are 2n gluons interacting with the dipole, each with transverse momentum $q_{\perp,i}$ such that:

$$\vec{\Delta} = \sum_n \omega_n \sum_{i=1}^{2n} \vec{q}_{\perp,i} \qquad |\vec{\Delta}| = \sqrt{-t}$$

The quark goes through a random walk with $2n$ steps

TO DO: Saturation in the Final State

$$\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2 b}(x_{I\!P}, r, b) = 2\left[1 - \exp\left(-\frac{\Omega(x_{I\!P}, r, b)}{2}\right)\right] =$$

$$= \Omega - \frac{\Omega^2}{4} + \frac{\Omega^3}{24} - \ldots = \sum_{n=1}^{\infty} (-1)^{n-1}\frac{2}{n\,!}\left(\frac{\Omega}{2}\right)^n$$

2 gluon exchange

4 gluon exchange

6 gluon exchange

n gluon exchange

Truncate expansion such that $\left|2\left(1 - e^{-\Omega/2}\right) - \sum_{n=1}^{T}(-1)^{n-1}\frac{2}{n\,!}\left(\frac{\Omega}{2}\right)^n\right| < \epsilon$

$$\left|2e^{-\Omega/2}\gamma_N(n+1, -\Omega/2)\right| < \epsilon$$

γ_N is the normalised lower incomplete gamma-function

$$\gamma_N(n+1, -\Omega/2) = \frac{-1}{n\,!}\int_{-\Omega/2}^{0} t^n e^{-t}\mathrm{d}t$$

TO DO: Saturation in the Final State

$$\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2 b}(x_{I\!P}, r, b) = 2\left[1 - \exp\left(-\frac{\Omega(x_{I\!P}, r, b)}{2}\right)\right] =$$

$$= \Omega - \frac{\Omega^2}{4} + \frac{\Omega^3}{24} - \ldots = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{2}{n\,!} \left(\frac{\Omega}{2}\right)^n$$

2 gluon exchange

4 gluon exchange

6 gluon exchange

n gluon exchange

$\left|2e^{-\Omega/2}\gamma_N(n+1, -\Omega/2)\right| < \epsilon$ Increase n until condition is met.

Let $\Omega = \Omega(x_{I\!P}, \hat{r}, \hat{b})$, with $\hat{r}$ giving the average value of:

$q\bar{q} : rK_{0,1}(\epsilon r)J_{0,1}(\epsilon r)\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2 b}(x_{I\!P}, r, b)$ $\quad q\bar{q}g : rK_2(\sqrt{\tilde{z}}\kappa r)J_n(\sqrt{1-\tilde{z}}\kappa r)\frac{\mathrm{d}\tilde{\sigma}_{q\bar{q}}}{\mathrm{d}^2 b}(x_{I\!P}, r, b)$

and $\hat{b} = \sqrt{\pi B_G/2}$.

TO DO: Saturation in the Final State

$$\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2 b}(x_{I\!P}, r, b) = 2\left[1 - \exp\left(-\frac{\Omega(x_{I\!P}, r, b)}{2}\right)\right] =$$

$$= \Omega - \frac{\Omega^2}{4} + \frac{\Omega^3}{24} - \ldots = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{2}{n\,!} \left(\frac{\Omega}{2}\right)^n$$

2 gluon exchange

4 gluon exchange

6 gluon exchange

n gluon exchange

$\left| 2e^{-\Omega/2} \gamma_N(n+1, -\Omega/2) \right| < \epsilon$ Increase n until condition is met.

With $\Omega = \Omega(x_{I\!P}, \hat{r}, \hat{b})$

Let the quark and anti quark (and gluon) collide with with $2n$ gluons keeping

$(q + \bar{q})^2 = M_X^2$ $\quad \vec{\Delta} = \sum_{i=1}^{2n} \vec{q}_{\perp,i}$ $\quad \vec{q}_{\perp i}$ distributed by a Gaussian with width Q_S.

TO DO 2:
The t-dependence

TO DO: The *t*-dependence

The Correct Way:

Cyrille Marquet Phys.Rev.D 76 (2007), 094017

$$\frac{\mathrm{d}\sigma_{\lambda}^{\gamma^* p\to Xp}}{\mathrm{d}\beta \mathrm{d}t} = \frac{Q^2}{16\beta^2}\sum_f \int \mathrm{d}z z(1-z) \int \mathrm{d}^2\vec{b}_1 \int \mathrm{d}^2\vec{b}_2 \int \mathrm{d}^2\vec{r}_1 \int \mathrm{d}^2\vec{r}_2$$

$$e^{i(\vec{b}_2-\vec{b}_1)\cdot\vec{\Delta}} \frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\vec{b}_1}(r_1, b_1, x) \frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\vec{b}_2}(r_2, b_2, x)$$

$$\Theta(\vec{k}^2) e^{i(\vec{r}_2-\vec{r}_2)\cdot\vec{k}} \phi_{\lambda}^{f}(z, \vec{r}_1, \vec{r}_2, Q^2) \qquad (43)$$

with $\lambda = T,\ L$ and

$$\phi_T^f(z, \vec{r}_1, \vec{r}_2, Q^2) = \frac{\alpha_{\mathrm{EM}} N_C}{2\pi^2} e_f^2 \Bigg((z^2 + (1-z)^2)\epsilon^2 \frac{\vec{r}_1 \cdot \vec{r}_2}{r_1 r_2} K_1(\epsilon r_1) K_1(\epsilon_f r_2)$$

$$+ m_f^2 K_0(\epsilon r_1) K_0(\epsilon r_2) \Bigg) \qquad (44)$$

$$\phi_L^f(z, \vec{r}_1, \vec{r}_2, Q^2) = \frac{\alpha_{\mathrm{EM}} N_C}{2\pi^2} e_f^2 4Q^2 z^2 (1-z)^2 K_0(\epsilon r_1) K_0(\epsilon r_2) \qquad (45)$$

4 extra integrals (2 angles, r_2, and b_2)!

5D Lookup tables!

Possible, but unwieldy

TO DO: The *t*-dependence

The Good Enough Way:

Use available Sar*t*re calculation of exclusive coherent $\frac{\mathrm{d}\sigma_{VM}}{\mathrm{d}t}(Q^2, W^2, t)$ with $VM = \gamma, \rho, \phi, J/\psi, \Upsilon\ldots$ and interpolate/extrapolate the M_x dependence from respective vector-meson mass.

This will yield an approximate t-dependence for a given point (Q^2, W^2, β)

Inclusive Diffraction with Sar*t*re Current Status

Current version on SVN:

Can generate events with both $q\bar{q}$ and $q\bar{q}g$ final states in *ep* and *e*A

To Do (short term):

Implement saturation effects in final state

Create full tables for several initial state species

Thorough testing

To Do (intermediate term):

Implement *t*-dependence

To Do (long term):

Incoherent Diffraction?