Black Hole Mechanics, aka Thermodynamics

Presentation to the Astrophysics Journal Club

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Introduction

I gave an Astro JC presentation during August, 2021 on Hawking Radiation and the Unruh Effect!

See references first

GR: Extraction of Black Hole Energy (Classical)

- Penrose (1969) process to extract rotational energy from the ergosphere of a Kerr BH
- Christodoulou (1971) limit of 29% on the mass of an extreme Kerr BH that can be extracted
- Hawking (1971?) theorized that the total area of BH event horizons does not decrease. This was recently noticed observationally in an analysis of LIGO event GW150914; it related the areas of the event horizons of the BHs before and after the amalgamation
- Bekenstein (1973) introduced the similarity of BH area and entropy.

Four Laws of Thermodynamics and of Black-Hole Mechanics

The latter were formulated as such by Bardeen, Carter, and Hawking in 1973.

Zeroth Law

Classical Thermodynamics

If two systems are each in thermal equilibrium with a third, they are also in thermal equilibrium with each other.

Black-Hole Mechanics

The horizon has constant surface gravity for a stationary black hole. This is a theoretical statement based on differential geometry.

First Law

W

Classical Thermodynamics

In a process without transfer of matter, the change in [internal energy](), ΔU, of a [thermodynamic system]() is equal to the energy gained as heat, ***H,*** *less the work , done by the system on its surroundings:*

$\Delta U = \mathbf{H} - W = T\,\Delta S - P\,\Delta V$

Black-Hole Mechanics

For perturbations of stationary black holes, the change of energy is related to change of area, angular momentum, and electric charge by

$dE = (\kappa/8\pi)dA + \Omega\, dJ + \Phi\, dQ,$

where E is the [energy](), κ is the [surface gravity](), A is the horizon area, Ω is the [angular velocity](), J is the [angular momentum](), Φ is the [electrostatic potential]() and Q is the [electric charge](). Though the first term on the right is analogous to the classical thermodynamic term with the product of temperature and entropy, these factors are ***not*** *the temperature and entropy of the black hole. In fact, the effective temperature of a classical black hole is absolute zero.*

Second Law

Classical Thermodynamics

Heat does not spontaneously flow from a colder body to a hotter body. The law observes that, when the system is isolated from the outside world and from forces, there is a definite thermodynamic quantity, its entropy, that increases as the constraints are removed, eventually reaching a maximum value at thermodynamic equilibrium.

Black-Hole Mechanics

Hawking: Classically, the horizon area A is, assuming the weak energy condition, a non-decreasing function of time:

dA / dt >= 0
An equivalent statement is that if two black holes coalesce, the area of the final event horizon is greater than the sum of the areas of the two original black holes.

This "law" was superseded by Hawking's discovery that black holes radiate, which causes both the black hole's mass and the area of its horizon to decrease over time.

Third Law

Classical Thermodynamics

As the temperature of a system approaches absolute zero, all processes cease and the entropy of the system approaches a minimum value. This law of thermodynamics is a statistical law of nature regarding entropy and the impossibility of reaching absolute zero *of temperature. This law provides an absolute reference point for the determination of entropy. The entropy determined relative to this point is the absolute entropy.*

Black-Hole Mechanics

It is not possible to form a black hole with vanishing surface gravity. That is,

$\kappa = 0$ *cannot be achieved.*

Additional Notes

Classical Thermodynamics

The first and second laws prohibit two kinds of perpetual motion machines, respectively: the perpetual motion machine of the first kind which produces work with no energy input, and the perpetual motion machine of the second kind which spontaneously converts thermal energy into mechanical work.

Classical Black-Hole Mechanics

Are there BH equivalents to the above?

The close mathematical analogy of the zeroth, first, and second laws of thermodynamics to corresponding laws of classical black hole mechanics is broken by the Planck-Nernst form of the third law of thermodynamics, which states that $S \rightarrow 0$ (or a "universal constant") as $T \rightarrow 0$. Thus temperature cannot be related to the surface gravity of a black hole, making inconsistent a link between entropy and the area of black the black hole horizon.

Consequences

Hawking:

Given black hole radiation, $\kappa/2\pi$ truly is the physical temperature of a black hole, not merely a quantity playing a role mathematically analogous to temperature in the laws of black hole mechanics.

Bekenstein:

When common entropy goes down a BH, the common entropy in the BH exterior plus the BH entropy never decreases. Thus, a generalized second law that links the ordinary entropy with black hole mechanics is plausible.

Entropy Bounds

1. Bekenstein proposed a universal bound on the entropy-to-energy ratio of bounded matter, given by

$S/E \leq 2\pi R$

where R denotes the “circumscribing radius” of the body. Two key questions one can ask about this bound are: (1) Does it hold in nature? (2) Is it needed for the validity of the GSL?

2. An alternative entropy bound has been proposed: It has been suggested that the entropy contained within a region whose boundary has area A must satisfy [69], [70], [71]

$S \leq A/4.$

This proposal is closely related to the “holographic principle”, which, roughly speaking, states that the physics in any spatial region can be fully described in terms of the degrees of freedom associated with the boundary of that region.

BH Entropy Calculations-I

By analogy with thermodynamics, BH entropy in Schwarzschild case:

BH event horizon area $A = 4\pi(2GM/c^2)^2$, so

$$d(Mc^2) = c^6 / (32\pi MG^2) \times dA \equiv T\, dS.$$

Then $S = (c^3 k_B / 4 G \hbar) A$ with $T = \hbar c^3 / 8\pi GM k_B$

Note well: many authors use geometrized units with $c = G = \hbar = k_B = 1$

BH Entropy Calculations-II

If $S = (c^3 k_B / 4 G \hbar) A$ truly represents the entropy of a BH, then an accounting of the quantum degrees of freedom of the BH will be necessary.

Various approaches depend involve:

Euclidean quantum gravity,

entanglement entropy across the BH horizon,

the ordinary entropy of the BH thermal atmosphere,

Sakharov's theory of induced gravity,

the framework of quantum geometry, and

most successfully, string theory

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BH Entropy Calculations-III

The approach involving the ordinary entropy of the BH thermal atmosphere fails because the entropy density would scale as T^3 and would diverge as the horizon is approached, regulating in a new type of ultraviolet catastrophe. “Curing” this divergence would require a cutoff frequency, which, if related to the Planck length, results in an entropy proportional to the horizon area A. The entanglement approach entails a similar consideration.

Remarkably, for certain classes of extremal and nearly extremal black holes, the ordinary entropy of the weak coupling states agrees exactly with the expression for A/4 for the corresponding classical BH states. An effort but Carlip holds out the possibility of providing a direct, general explanation of the remarkable agreement between the string theory state counting results and the classical formula for the entropy of a BH.

Issues

- BH information “paradox”—An initial pure state of correlated matter within and outside of a BH, will upon Hawking evaporation, lead to a final mixed state if the correlation is not restored. It is now generally believed that information is preserved in black-hole evaporation, based in part on reasoning by Page.
- Degrees of freedom for BH entropy—Inside, on, or outside the horizon?

Selected References - I

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Bardeen, J. M.; Carter, B.; Hawking, S. W. (1973). "The four laws of black hole mechanics". *Communications in Mathematical Physics*. **31** (2): 161–170. *This paper is very mathematical except for its last section, which codifies the four laws.—LF*

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R. Wald, "The Thermodynamics of Black Holes", arXiv:gr-qc/9912119v2, Sept. 2000. *This paper has very mathematical sections, but is an excellent review of the overall issue with voluminous references. My presentation is largely based upon this paper.*

Selected References - II

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Appendix: Extraction of Black Hole Energy (Quantum Mechanical)

- Hawking (1974) showed that quantum effects lead to radiation with a thermal spectrum by BHs. The temperature is
- $T = \hbar c^3 / 8\pi GM k_B = 6 \times 10^{-8}$ K $(M_\odot / M)$ =
- 6×10^{-8} K for a one $M_\odot$ BH and 6×10^{-15} K for a $10^7 M_\odot$ BH << 2.7 K
- 2.7 K for a $2.2 \times 10^{-8} M_\odot$ BH and
- BH Radiation then leads to a decrease in mass M with a photon luminosity P and "evaporation time" τ_{ev}
- $P = \hbar c^6 / 15360\,\pi G^2 M^2$
- Time $\tau_{ev} = 5120\,\pi G^2 M^3 / \hbar c^4 = 2.1 \times 10^{67}$ years $(M/M_\odot)^3$ =
- Much older than universe for $M = M_\odot$
- Universe age for $M = 7.8 \times 10^{-20} M_\odot = 1.55 \times 10^{11}$ kg << $M_\oplus = 6 \times 10^{24}$ kg