



UNIVERSITY OF  
BIRMINGHAM

SCHOOL OF  
PHYSICS AND  
ASTRONOMY

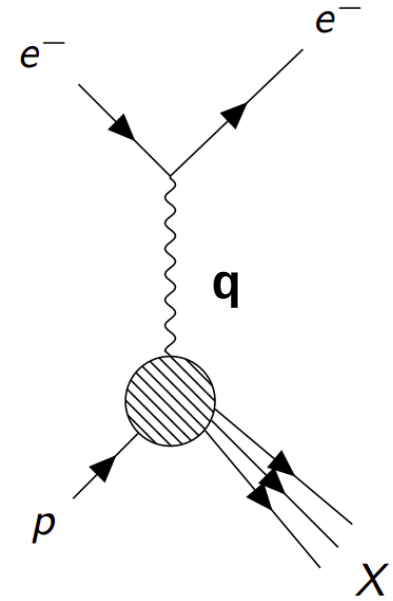
# Kinematic Fitting for Inclusive DIS reconstruction

S. Maple



# Inclusive NC DIS Kinematics

- DIS kinematics can be reconstructed from **two measured quantities**
  - $\vec{D} = \{\mathbf{E}_e, \theta_e, \delta_h, \mathbf{p}_{t,h}\}$
  - Where  $\delta_h$  is  $\mathbf{E} - \mathbf{p}_z$  sum of all particles in the Hadronic Final State:  $\sum E_i(1 - \cos \theta_i)$
  - $\mathbf{P}_{t,h}$  is the transverse momentum of the HFS
- Resolution of conventional reconstruction methods depend on:
  - Event x- $Q^2$
  - Detector acceptance and resolution effects
  - Size of radiative processes



## Electron method

$$Q^2 = 2E_e E'_e (1 + \cos \theta_e)$$

$$y = 1 - \frac{E'_e}{2E_e} (1 - \cos \theta_e)$$

## JB method

$$y = \frac{\delta_h}{2E_e}$$

$$Q^2 = \frac{p_{t,h}^2}{1 - y}$$

## e- $\Sigma$ method

$$Q_{e\Sigma}^2 = Q_e^2 \quad \left| \quad y_\Sigma = \frac{\delta_h}{\delta_h + \delta_e}$$

$$x_{e\Sigma} = \frac{Q_\Sigma^2}{s y_\Sigma} \quad \left| \quad Q_\Sigma^2 = \frac{p_{t,e}^2}{1 - y_\Sigma}$$

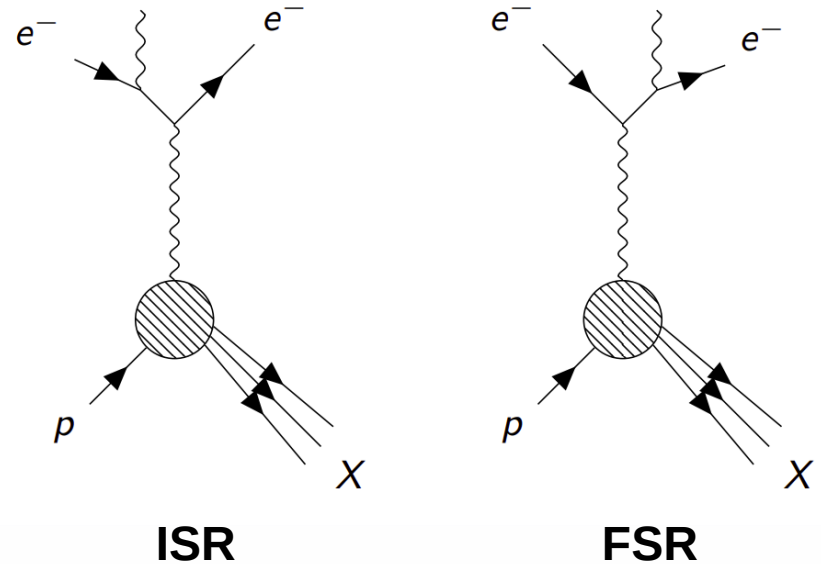
## Double Angle method

$$y_{DA} = \frac{\alpha_h}{\alpha_h + \alpha_e} \quad \left| \quad \alpha_{e/h} = \tan \frac{\theta_{e/h}}{2}$$

$$Q_{DA}^2 = \frac{4E_e^2}{\alpha_e(\alpha_e + \alpha_h)}$$

# Inclusive NC DIS Kinematics with QED radiation

- Presence of **QED radiation changes event kinematics** → Errors in reconstruction when only using two measured quantities
- **FSR not too problematic**: typically collinear to scattered electron → measured together in ECAL
- **ISR more difficult to account for**: reduces electron beam energy, radiated photon typically disappears down beampipe



# Kinematic Fitting for DIS

- Only **need** 2 quantities to obtain  $\mathbf{x}$ ,  $\mathbf{y}$ ,  $Q^2$
- Using measured quantities  $\vec{\mathbf{D}} = \{E_e, \theta_e, \delta_h, p_{t,h}\}$  a kinematic fit can extract additional information:  $\vec{\lambda} = \{\mathbf{x}, \mathbf{y}, E_\gamma\}$ 

$E_\gamma$  is energy of an ISR photon
- For kinematic fit, can use a **likelihood** function based on knowledge of the detector resolutions:

## Likelihood

$$P(\vec{\mathbf{D}} | \vec{\lambda}) \propto \frac{1}{\sqrt{2\pi}\sigma_E} e^{-\frac{(E_e - E_e^\lambda)^2}{2\sigma_E^2}} \frac{1}{\sqrt{2\pi}\sigma_\theta} e^{-\frac{(\theta_e - \theta_e^\lambda)^2}{2\sigma_\theta^2}} \frac{1}{\sqrt{2\pi}\sigma_{\delta_h}} e^{-\frac{(\delta_h - \delta_h^\lambda)^2}{2\sigma_{\delta_h}^2}} \frac{1}{\sqrt{2\pi}\sigma_{P_{T,h}}} e^{-\frac{(P_{T,h} - P_{T,h}^\lambda)^2}{2\sigma_{P_{T,h}}^2}}$$

- Note: above quantities taken to be uncorrelated → Correlations between  $E_e$ ,  $\theta_e$  and  $\delta_h$ ,  $p_{t,h}$  will later need to be taken into account

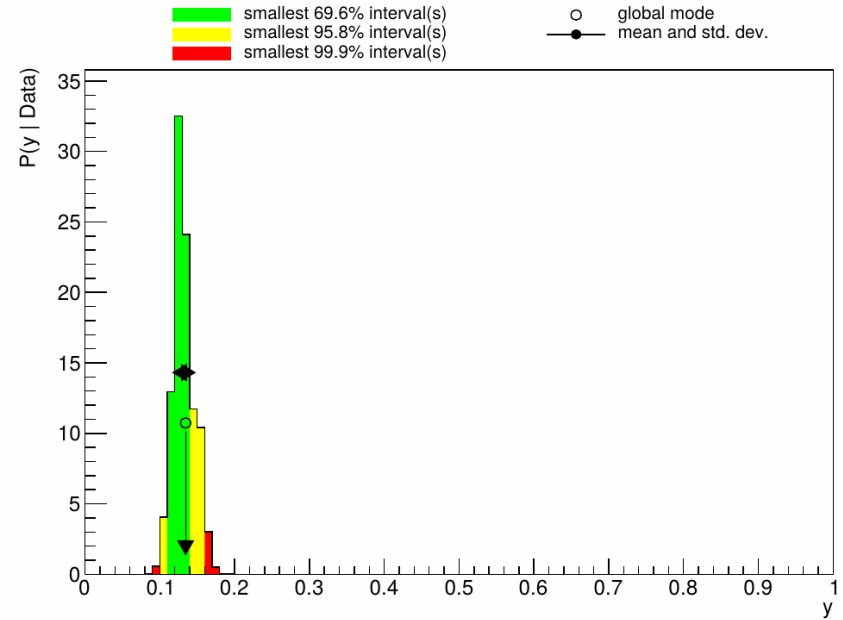
# Kinematic Fitting for DIS – A Bayesian Approach

- A Bayesian method can be applied in which basic features of the DIS cross section are encoded as a **prior**:

## Prior

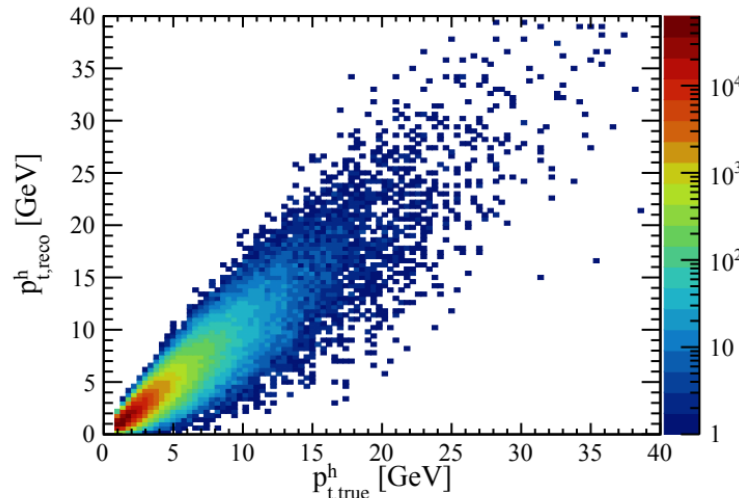
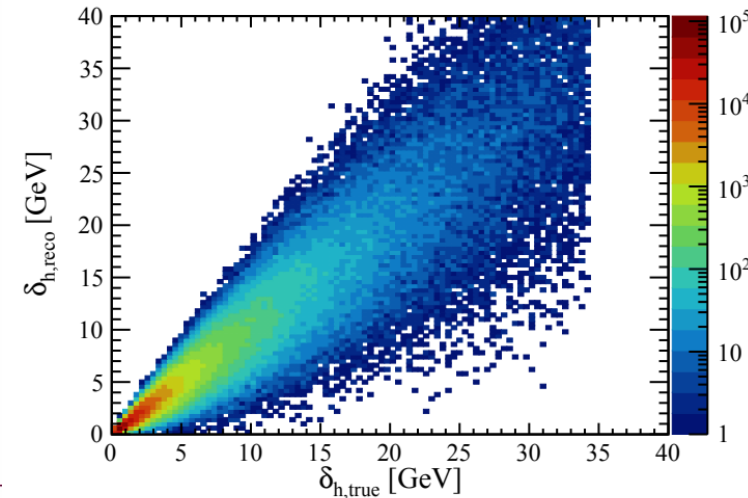
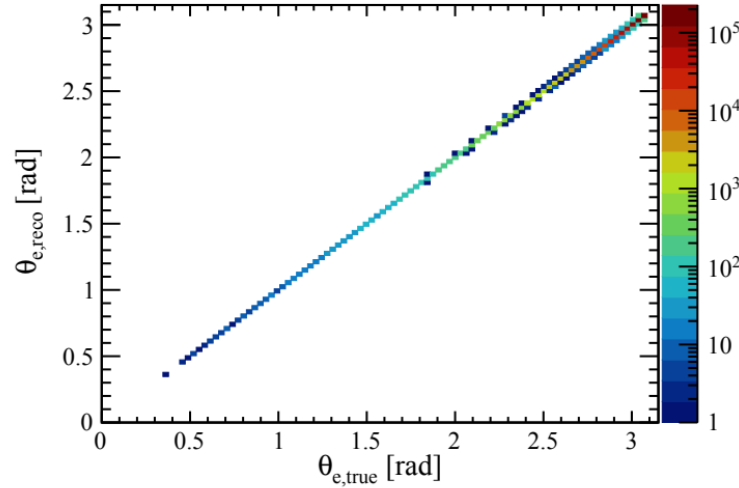
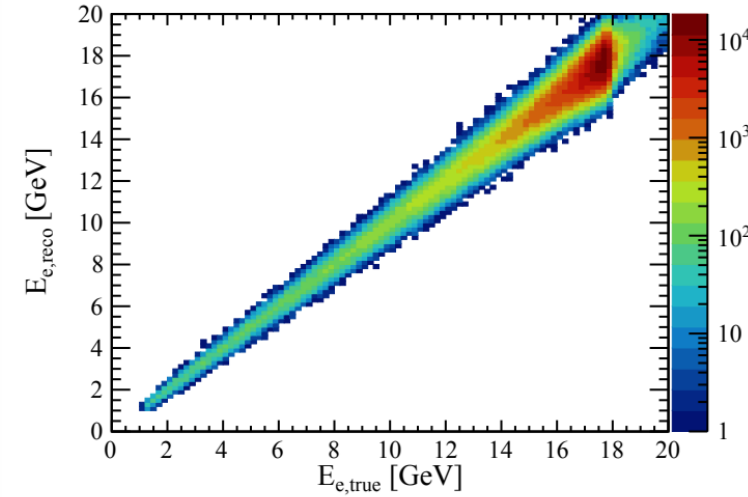
$$P_o(\vec{\lambda}) = \frac{1 + (1 - y)^2 [1 + (1 - E_\gamma/A)^2]}{x^3 y^2 E_\gamma/A}$$

- Use “Bayesian analysis toolkit” to calculate most probable values of set  $\vec{\lambda}$  given measured quantities  $\vec{D}$ 
  - Values for  $x$ ,  $y$ ,  $E_\gamma$  taken from global mode



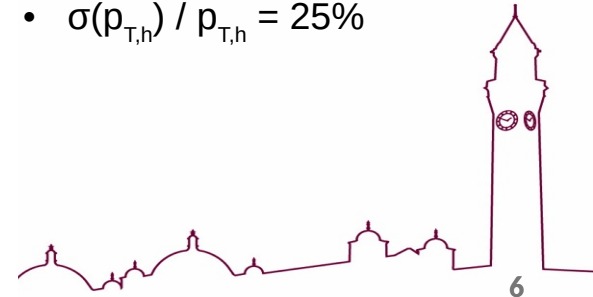
Marginalised  $y$  distribution for a single DIS event

# Smeared EIC pseudodata

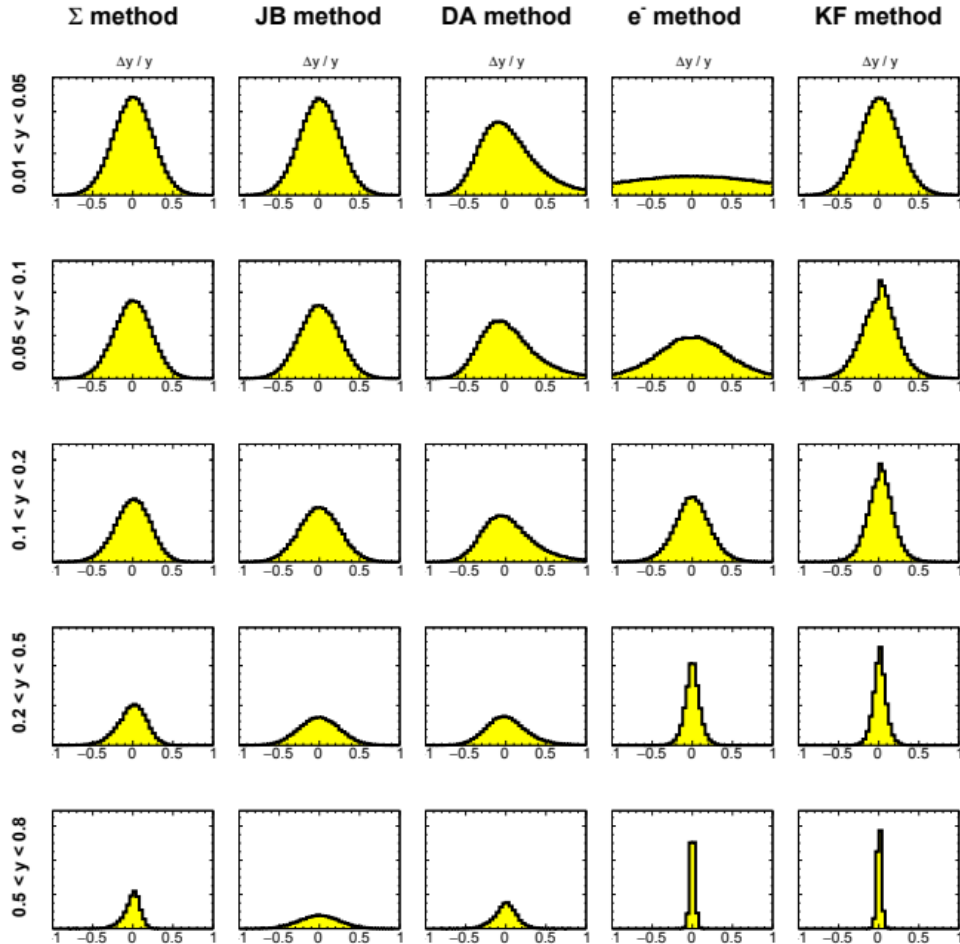


- EIC DIS events generated with Djangoh
  - $18 \times 275$ ,  $Q^2 > 1$
- Smear by estimated resolutions

- $\sigma(\theta_e) = 0.1 \text{ mrad}$
- $\sigma(E_e) / E = 11\% / \sqrt{E} \oplus 2\%$
- $\sigma(\delta_h) / \delta_h = 25\%$
- $\sigma(p_{T,h}) / p_{T,h} = 25\%$



# Smearing EIC pseudodata (No ISR)



- Smearing resolutions used as input for KF

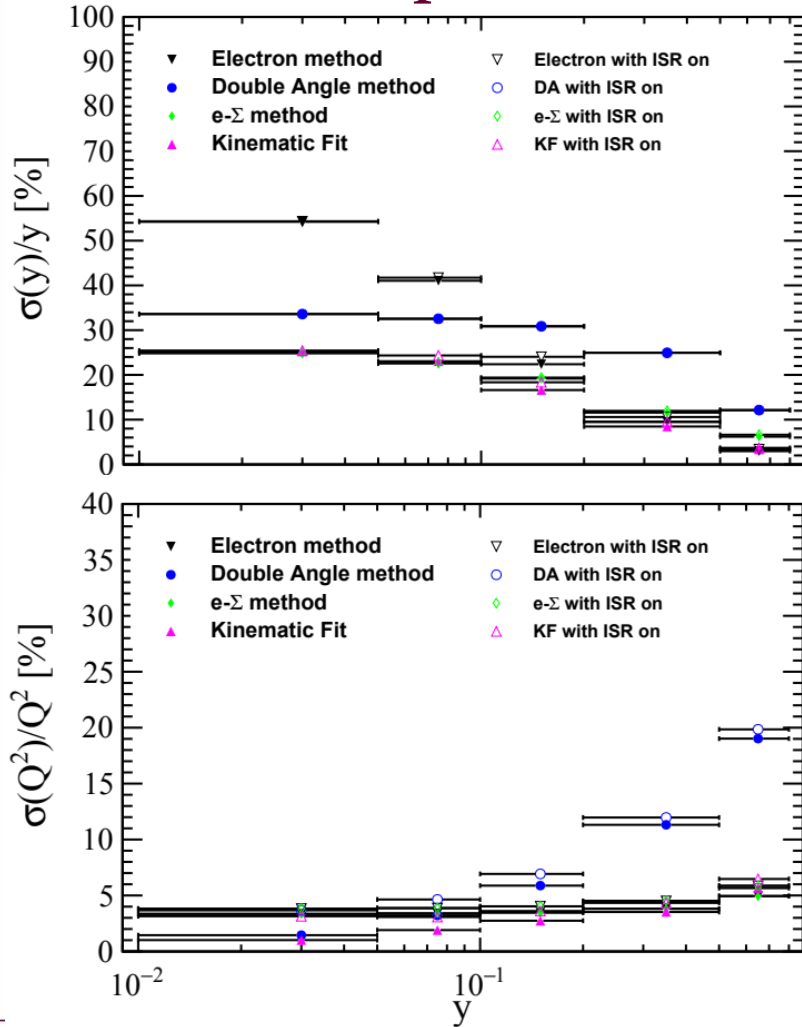
$$P(\vec{D} | \vec{\lambda}) = \frac{1}{\sqrt{2\pi}\sigma_E} \exp\left[-\frac{(E_e - E_e^\lambda)^2}{2\sigma_E^2}\right] \times \frac{1}{\sqrt{2\pi}\sigma_\theta} \exp\left[-\frac{(\theta_e - \theta_e^\lambda)^2}{2\sigma_\theta^2}\right] \times \frac{1}{\sqrt{2\pi}\sigma_{\delta_h}} \exp\left[-\frac{(\delta_h - \delta_h^\lambda)^2}{2\sigma_{\delta_h}^2}\right] \times \frac{1}{\sqrt{2\pi}\sigma_{p_t^h}} \exp\left[-\frac{(p_t^h - p_t^{h\lambda})^2}{2\sigma_{p_t^h}^2}\right]$$

- Stick to using prior 1 from <https://arxiv.org/abs/2206.04897>

$$P_0(\vec{\lambda}) = \frac{1 + (1 - y)^2}{x^3 y^2} \frac{1 + (1 - E_\gamma/E_0)^2}{E_\gamma/E_0}$$

- Compare y resolutions:
  - KF method meets or exceeds conventional

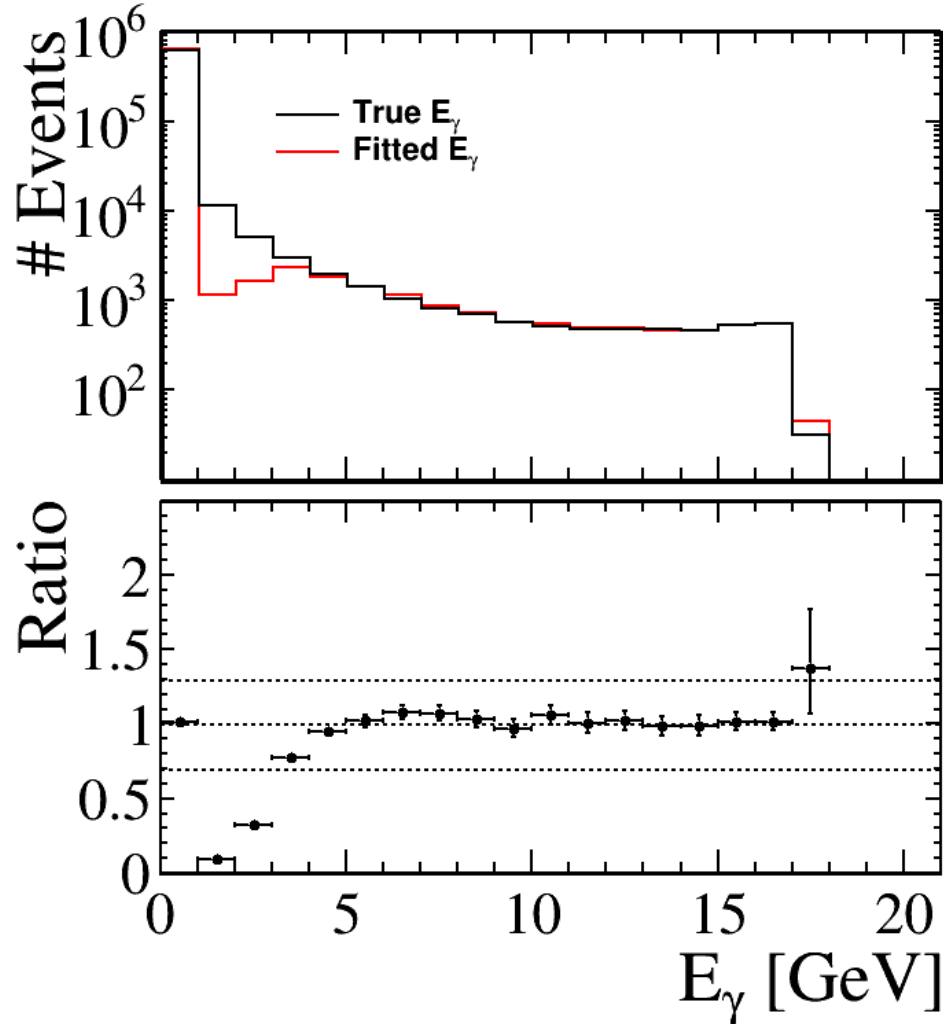
# Smearred EIC pseudodata (W/ ISR)



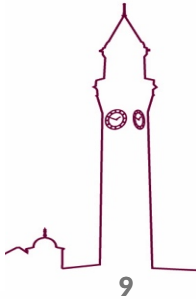
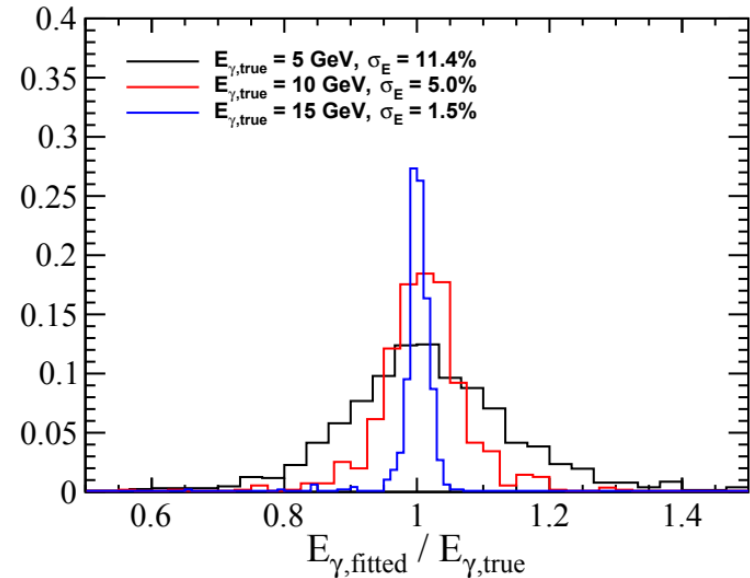
- Compare resolutions: no ISR to with ISR on
  - “Realistic”  $\Sigma_{\text{tot}}$  cut of 31 GeV applied to remove high energy ISR
- Some, but not big, difference between observed resolutions
  - Even for the electron method!



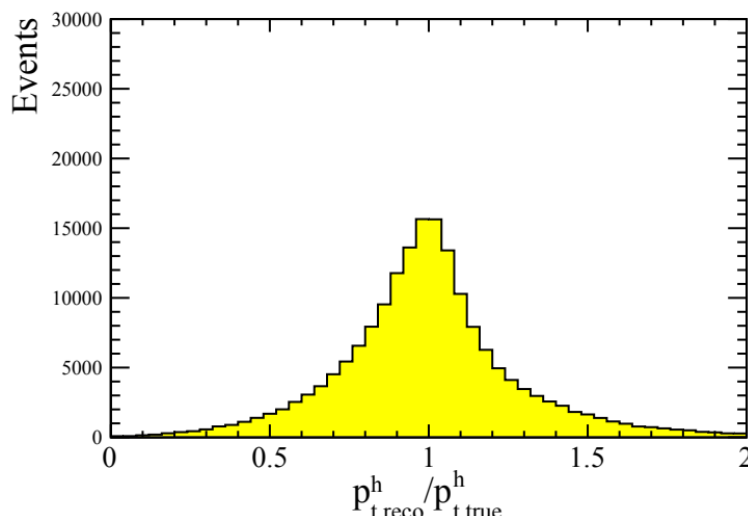
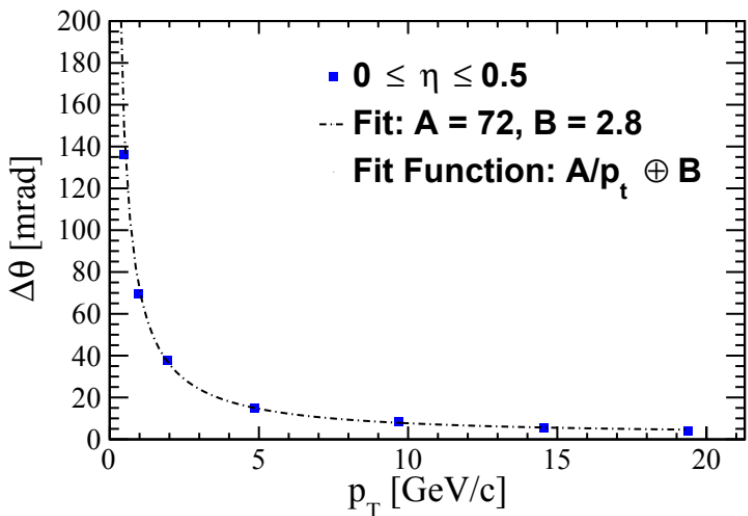
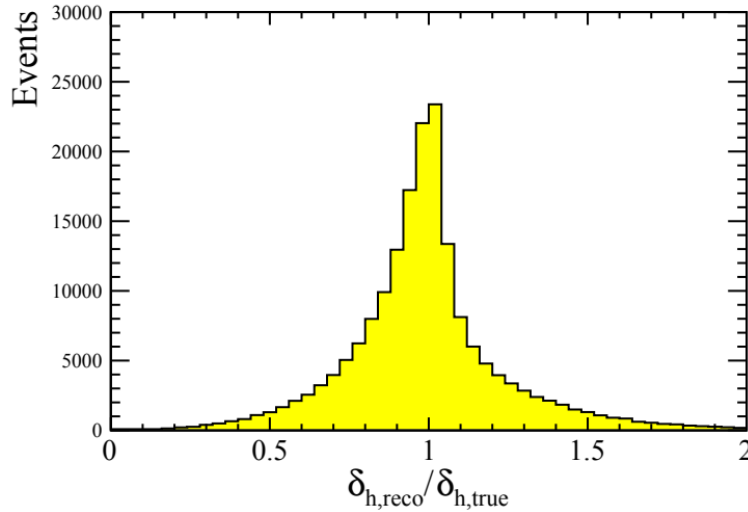
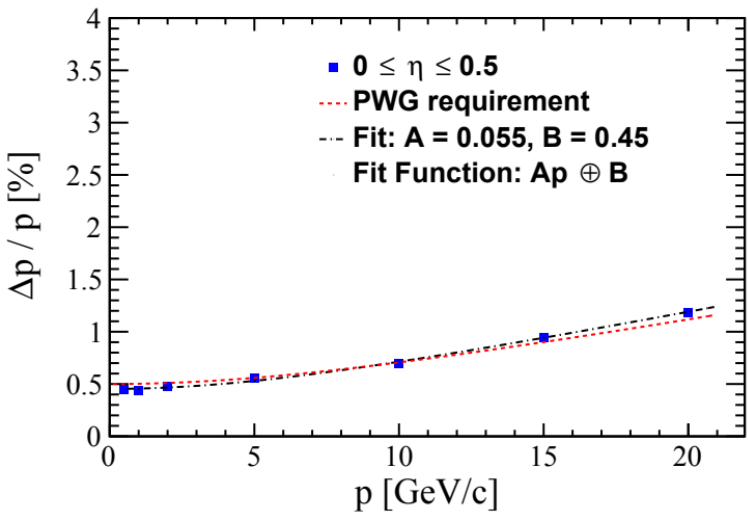
# Smeared EIC pseudodata (W/ ISR)



- Compare true and measured ISR energy distributions
  - Distribution well reproduced for higher  $E_\gamma$
  - Ratio within 30% for  $E_\gamma > 3$  GeV
  - Within 10% for  $E_\gamma > 4$  GeV
- Reasonable resolution



# Fully Simulated ePIC pseudodata (No ISR)



$$\sigma_E = 0.055 \cdot p \oplus 0.45 \text{ in GeV}$$

$$\sigma_\theta = 72/p_t \oplus 2.8 \text{ in mrad}$$

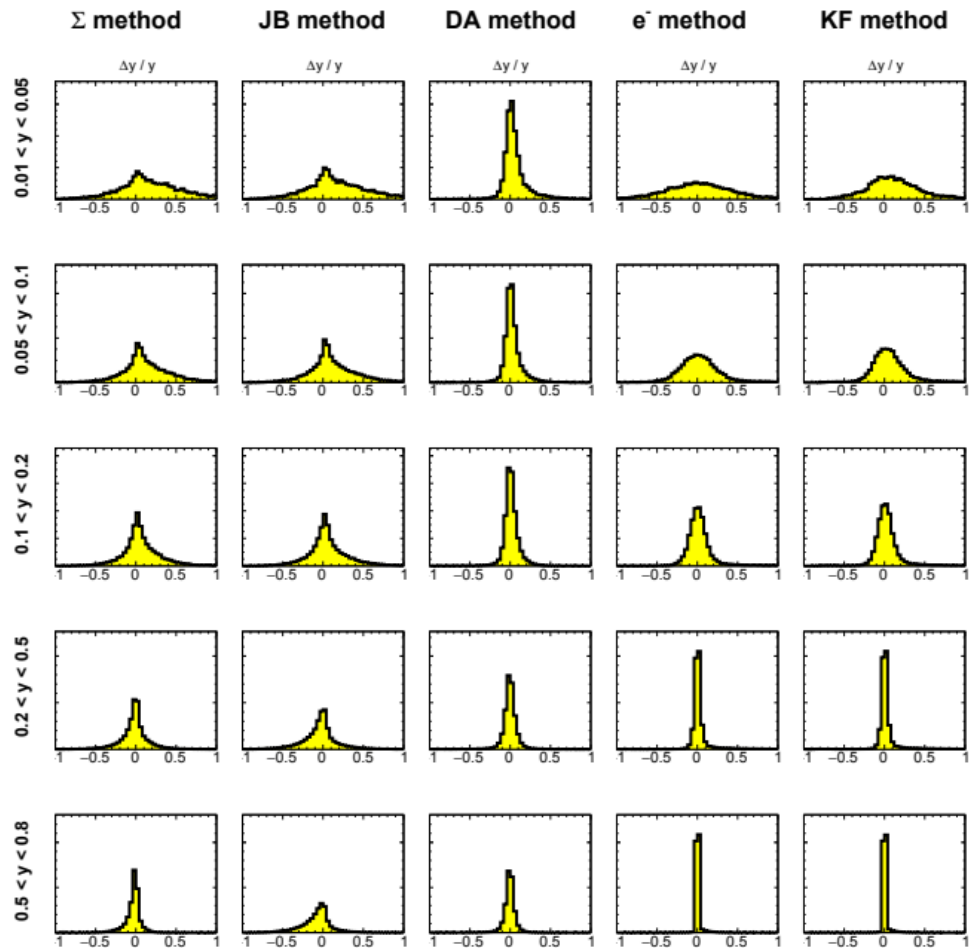
$$\sigma_{\delta_h} = 0.25 \cdot \delta_h \text{ in GeV}$$

$$\sigma_{p_t^h} = 0.25 \cdot p_t^h \text{ in GeV.}$$

- Parametrised ePIC full sim resolutions
  - Pythia8 NCDIS
  - Craterlake 23.12.0
  - $Q^2 > 100 \text{ GeV}^2$
  - Ele from tracking

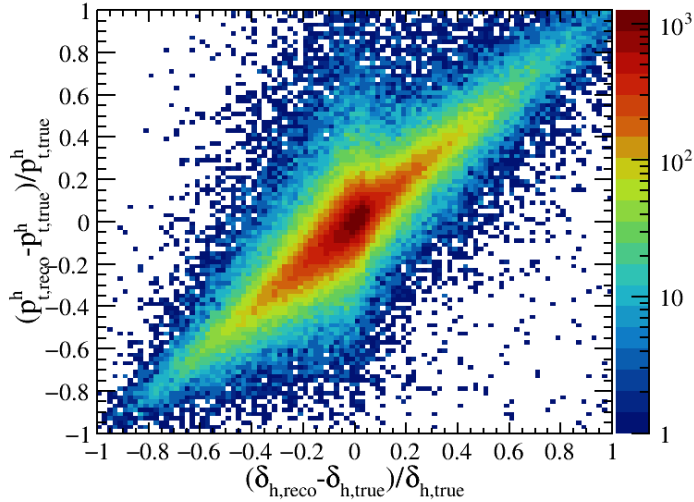


# Fully Simulated ePIC pseudodata (No ISR)



- KF gives **comparable  $y$  resolution to electron method** at high  $y$
- **Loses at low  $y$  to DA method**

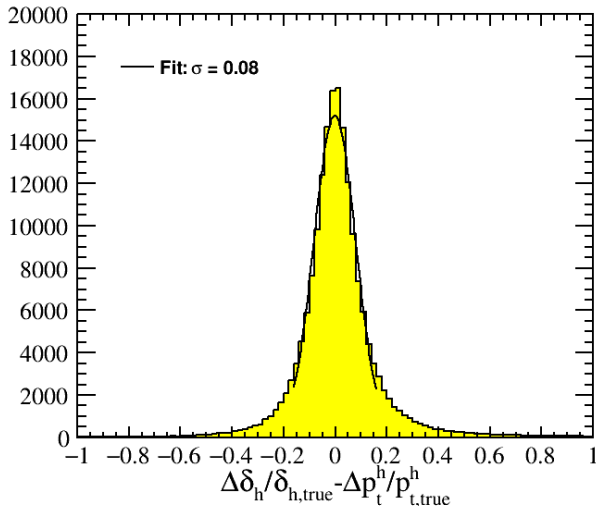
# HFS Correlations



- Correlations in HFS variables mostly due to energy fluctuations in calorimeters
- Introduce extra term that reduces likelihood if  $p_t$  is overestimated and  $\delta$  underestimated or vice versa:

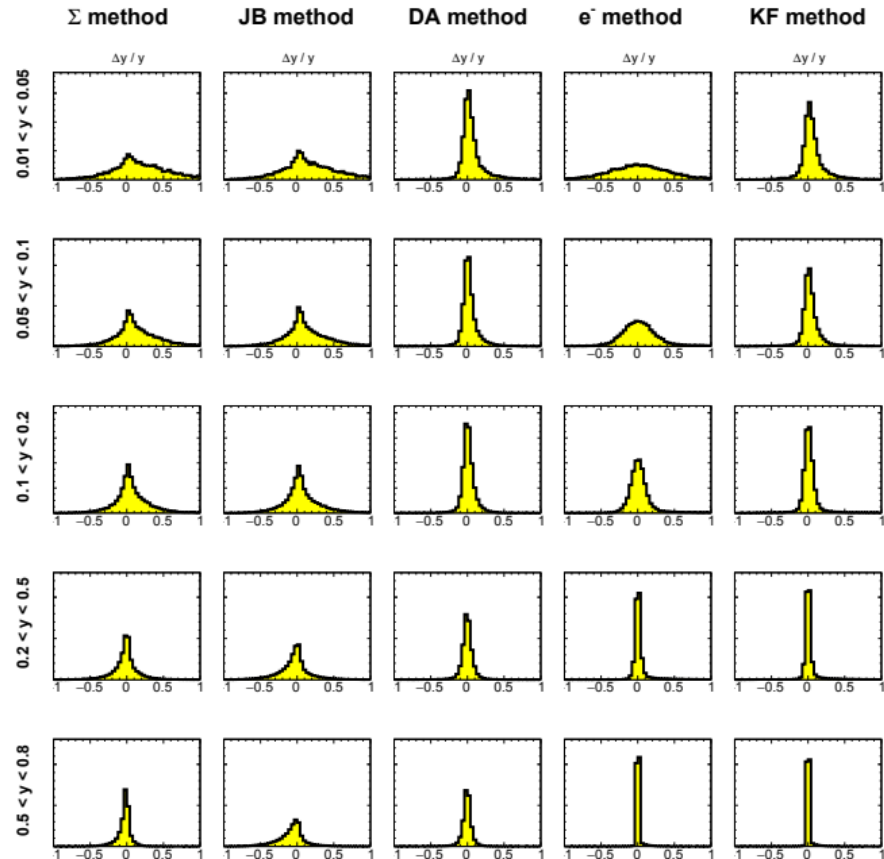
$$P(\vec{D} | \vec{\lambda})_{corr} = P(\vec{D} | \vec{\lambda})_{uncorr} \frac{1}{\sqrt{2\pi}\sigma_{corr}} \cdot \exp - \frac{(c - c^\lambda)^2}{2\sigma_{corr}^2}$$

$$c = \frac{\delta_{h,reco} - \delta_{h,true}}{\delta_{h,true}} - \frac{p_{t,reco}^h - p_{t,true}^h}{p_{t,true}^h}$$



← Correlation width  $\sigma_{corr} \sim 8\%$

# Fully Simulated ePIC pseudodata (No ISR) – HFS Correlation

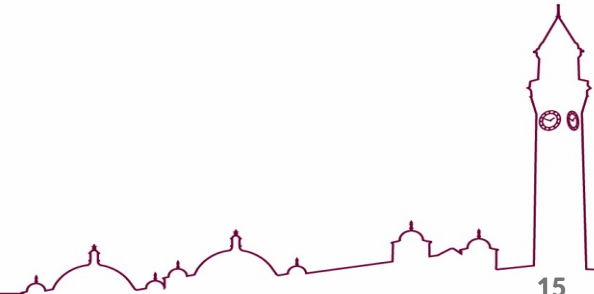


- Performance of KF recovered at low  $y$ !
  - Not yet perfect → but performance comparable to DA method achieved at low  $y$ , while maintaining electron method performance at high  $y$
- Further improvements in likelihood possible for HFS resolutions and correlation parametrisations

# Summary

- Conventional reconstruction methods do not fully exploit information measured in NC events → methods such as kinematic fitting (or DNNs) use all measured quantities simultaneously to give a best estimate of the kinematics
- Kinematic fitting method explored using an informative prior based on features of DIS and bremsstrahlung cross sections:
  - In ideal case (smeared uncorrelated) the KF method matches or exceeds the performance of conventional methods
  - If correlations between HFS quantities are included, as in full ePIC detector simulations, the DA method may exceed the basic (uncorrelated) KF
- Extending the KF method to account for correlations in the HFS recovers this performance → delivers  $y$  resolution comparable to best method for each  $y$  bin
- Can identify ISR with good efficiency and resolution for  $E_\gamma$  greater than a few GeV
  - Possibility to extend measurements down to lower  $Q^2$ , or add to an  $F_L$  extraction

# Backup

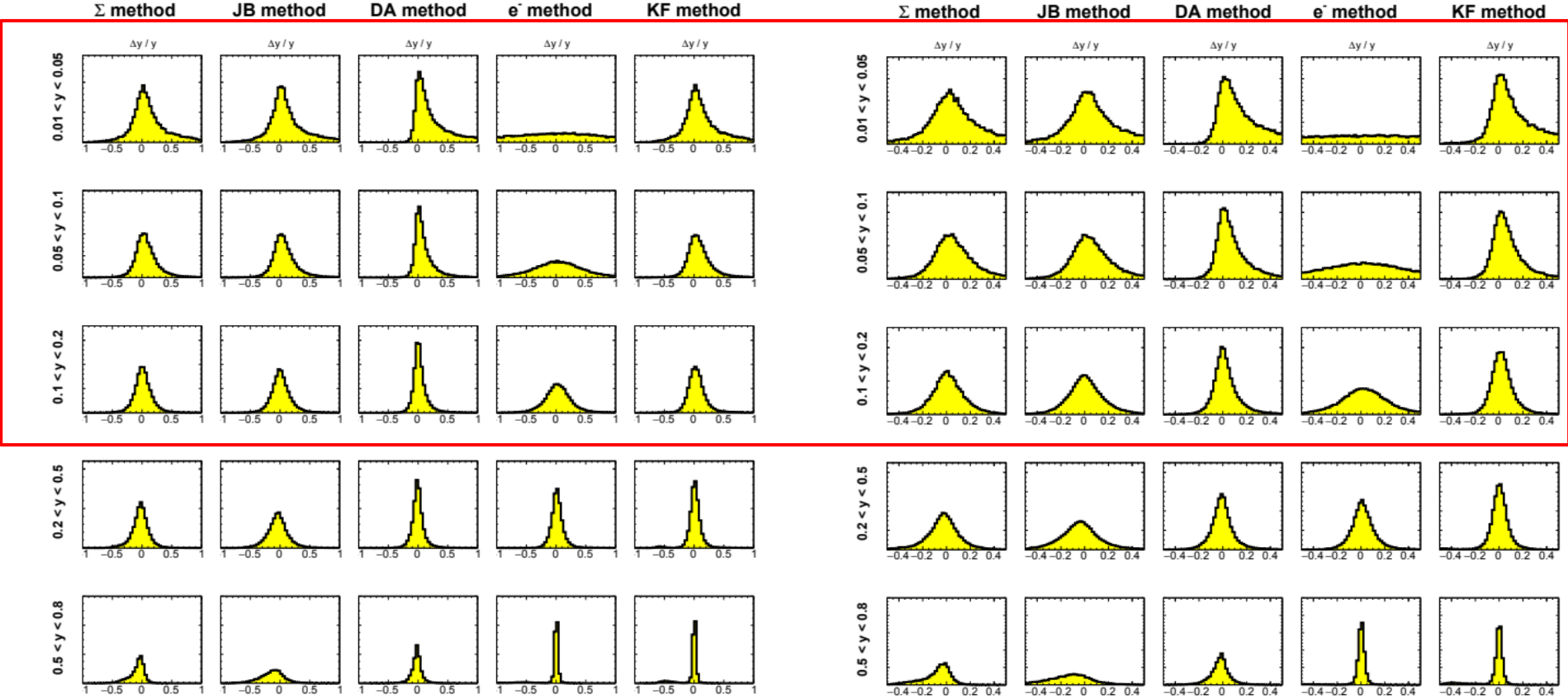


\*Note different x scale

# H1 Resolution on $y$

## No Correlations

## HFS Correlations

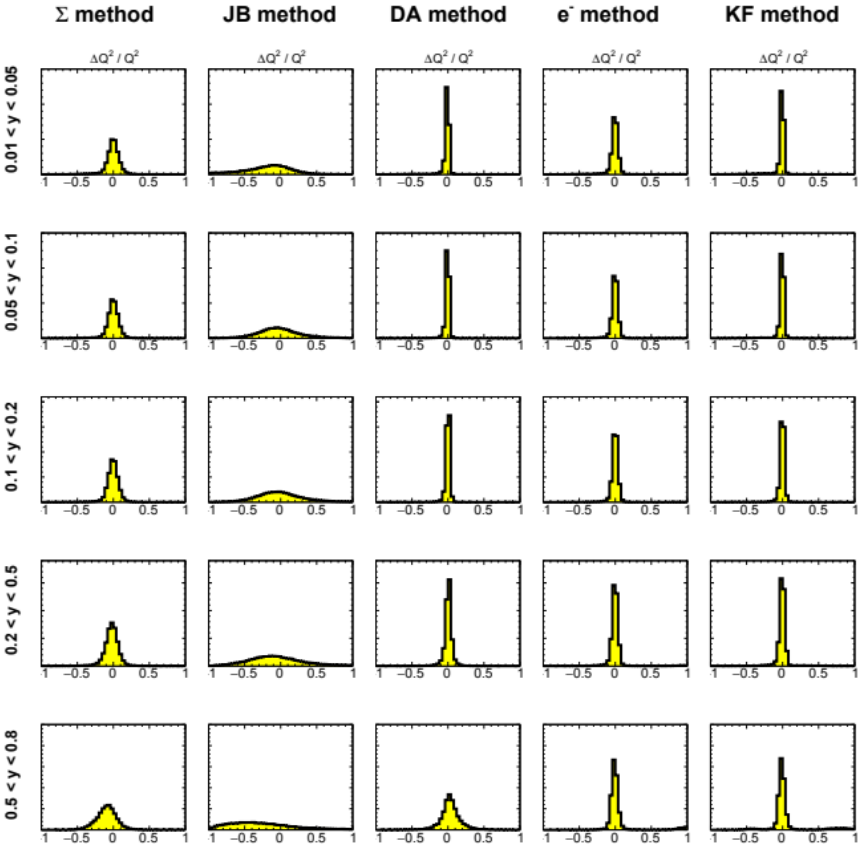




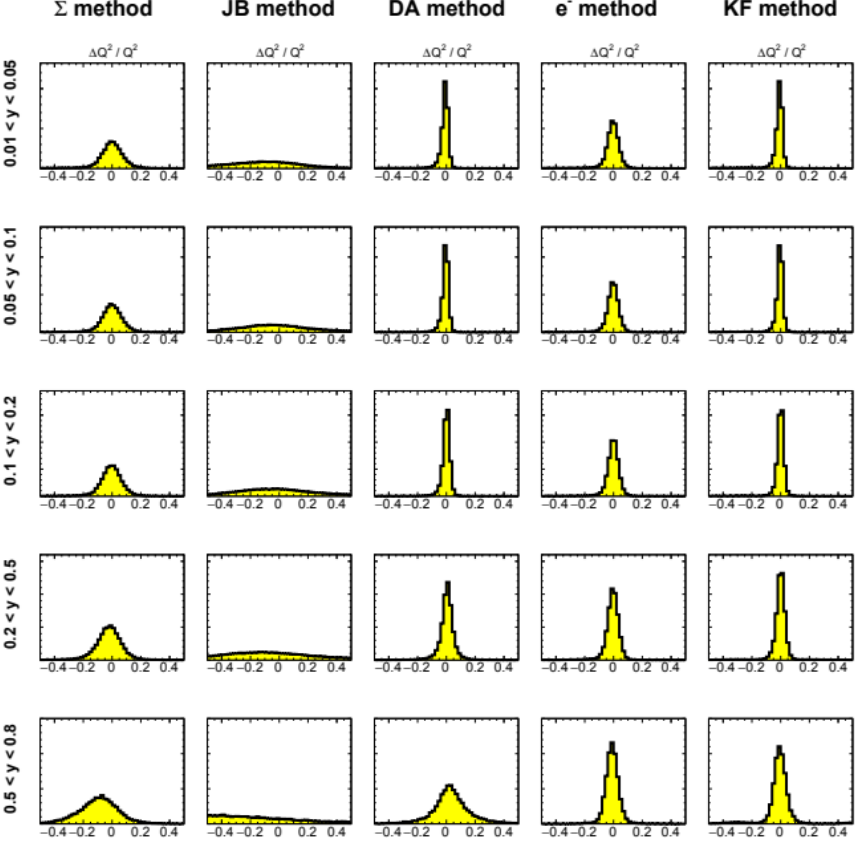
\*Note different x scale

# H1 Resolution on $Q^2$

## No Correlations



## HFS Correlations

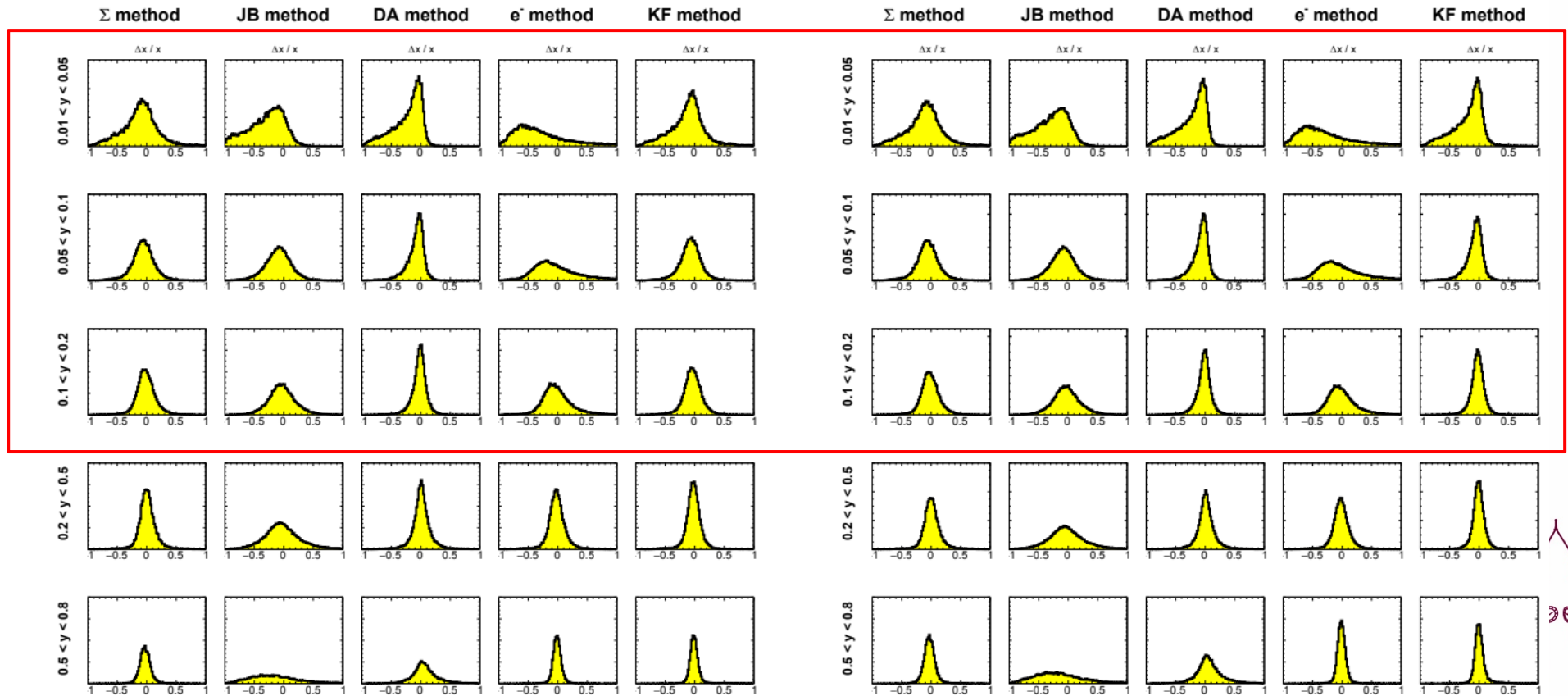


Minimal difference for  $Q^2$

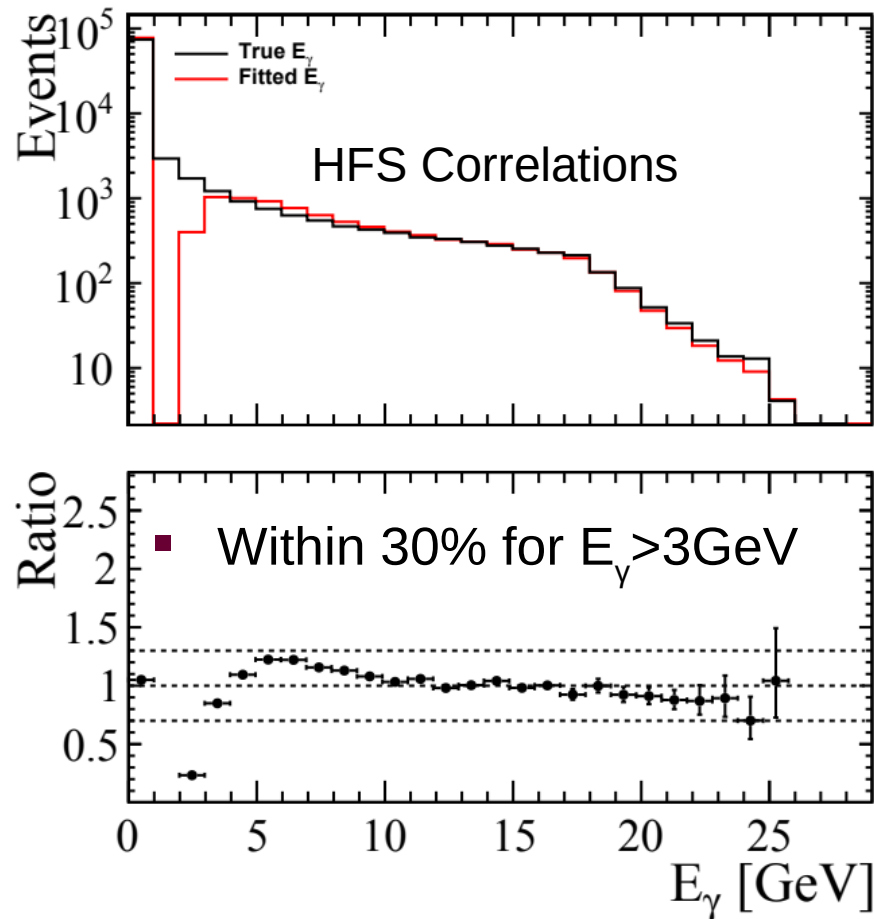
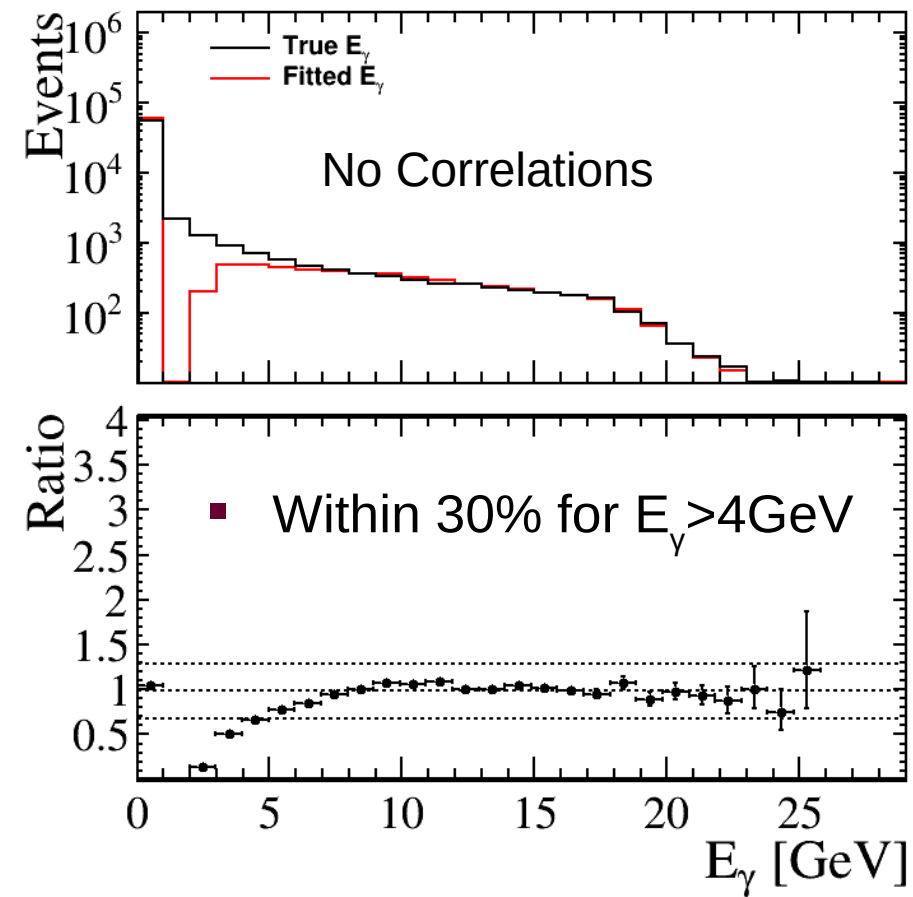
# H1 Resolution on x

No Correlations

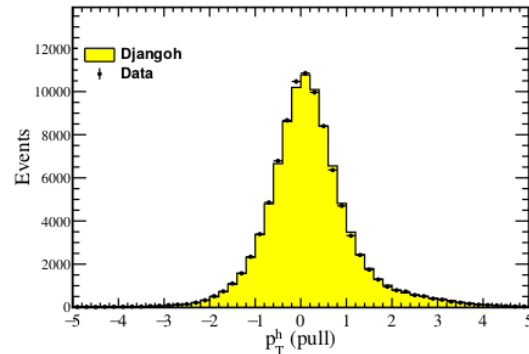
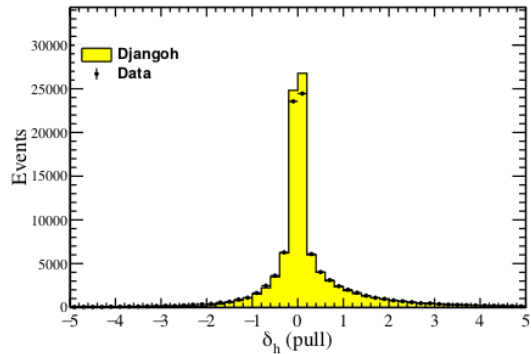
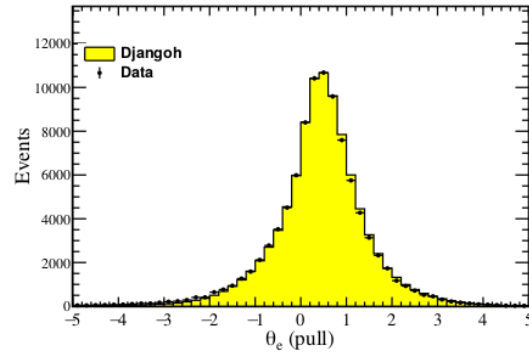
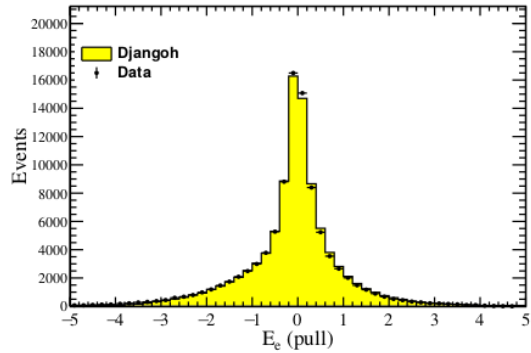
HFS Correlations



# H1 ISR reconstruction



# H1 Data and MC (ISR On)



- KF reconstruction is applied with a likelihood function constructed from the following resolutions:

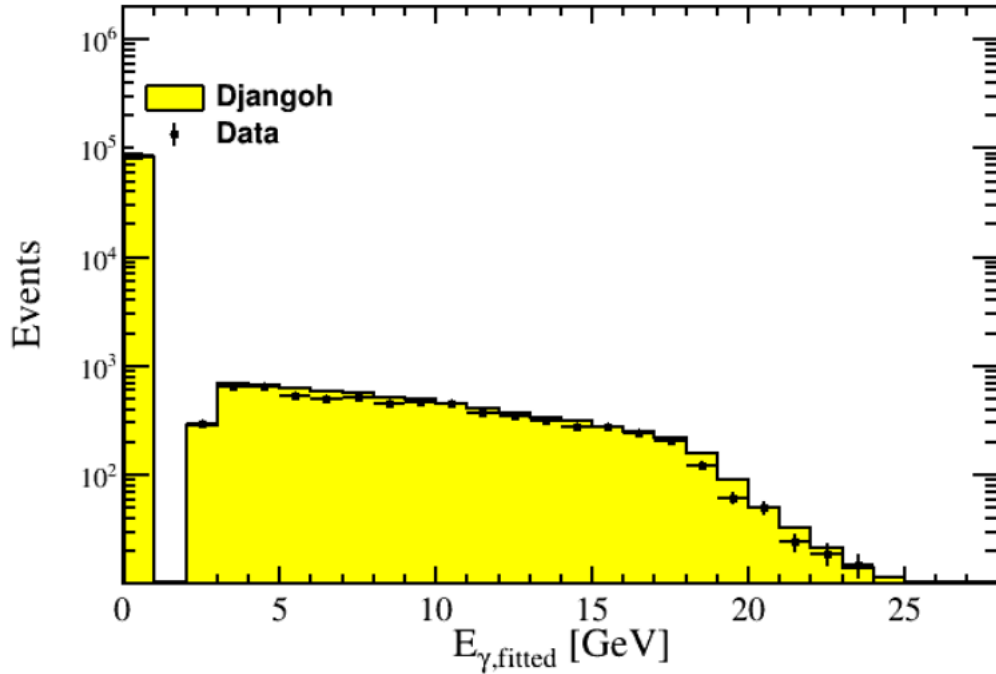
- $\sigma(\theta_e) = 4\text{mrad}$
- $\sigma(E_e) / E = 11\% / \sqrt{E} \oplus 1\%$
- $\sigma(\delta_h) / \delta_h = 13.5\%$
- $\sigma(p_{T,h}) / p_{T,h} = 54\% / \sqrt{p_{T,h}} \oplus 4\%$

- No correlation term included for H1 studies

- Good agreement for pulls from data and Djangoh

$$g = \frac{D_{i,\text{fitted}} - D_{i,\text{reco}}}{RMS_{MC}}$$

# H1 Data and MC (ISR On)



- Good agreement for  $E_{\gamma}$  prediction by data and MC (Djangoh)

# H1 Data and MC (ISR On)

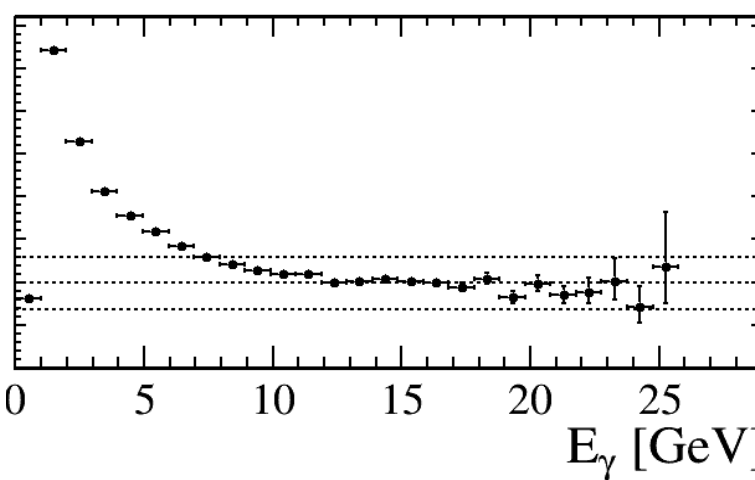
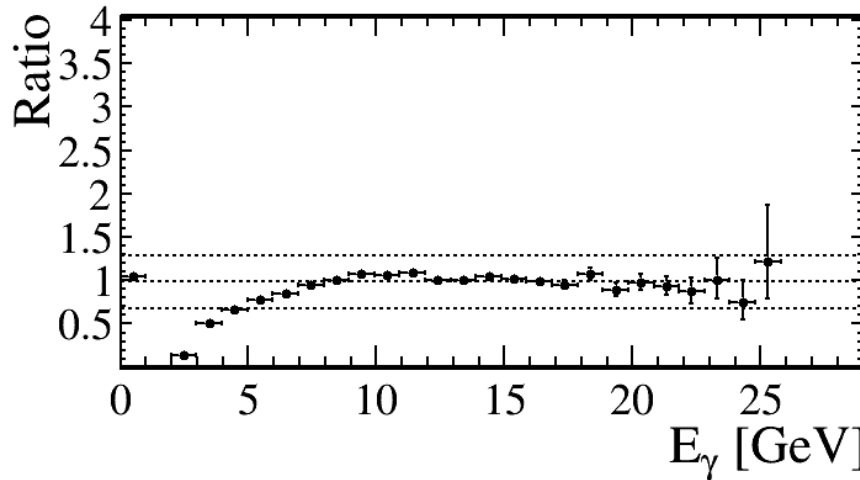
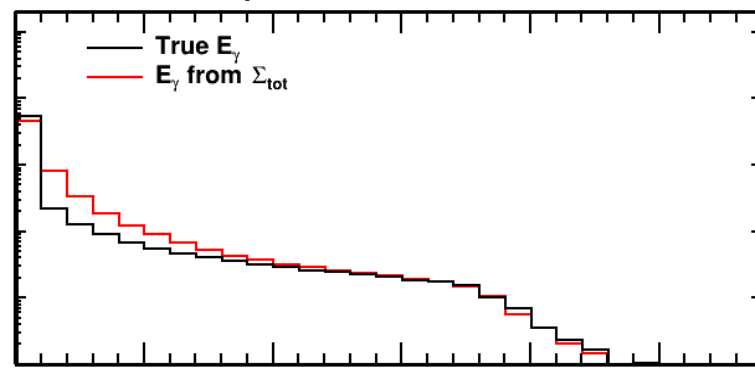
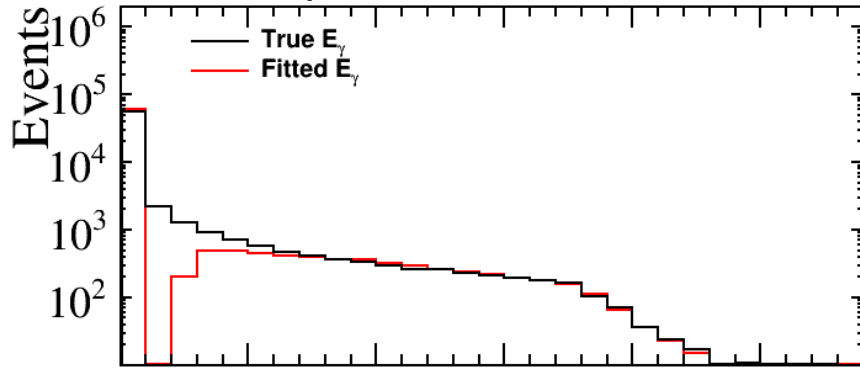
## KF prediction

## $\Sigma$ prediction

KF (w/ prior 1) doesn't typically predict presence of ISR that could be equally explained by a resolution effect

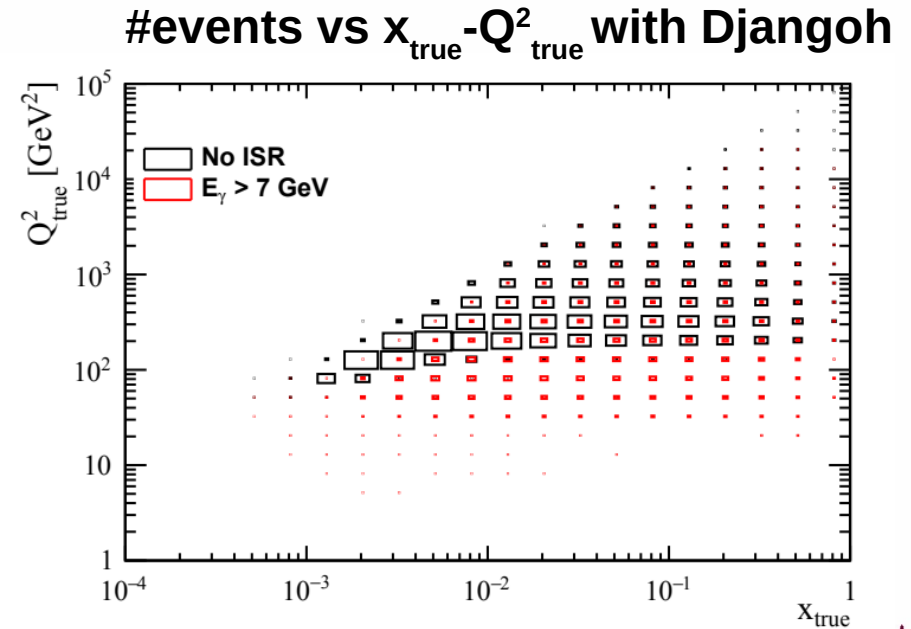
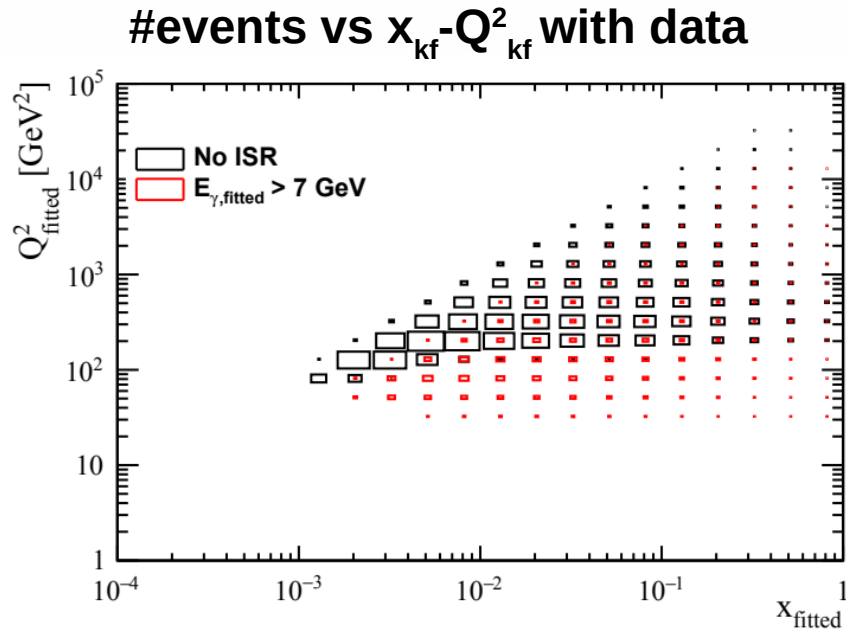
$\Sigma$  approach does not miss ISR events, but overestimates

Ratio within 30% of unity for  $E_\gamma > 4\text{GeV}$  (KF) and  $E_\gamma > 7\text{GeV}$  ( $\Sigma$ )

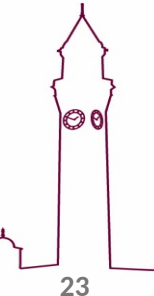


# Why identify ISR?

- ISR lowers the electron beam energy
  - Scattered electrons in low  $Q^2$  events don't enter main detector
    - lower energy electrons are scattered at larger angles that may be within the detector acceptance
    - kinematic reach extended



Note  $x-Q^2$  binning here is arbitrary (not an official H1 binning)



# Truth Smearing correlations

