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Spin-dependent hadronization in Pythia

Albi Kerbizi Lund University and INFN Trieste



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Outline
Introduction
Implementation of spin effects for DIS

Collins and 2h asymmetries

Implementation of spin effects for e⁺e⁻

Collins asymmetries

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StringSpinner: spin in the Pythia generator

 PYTHIA 8 standard tool in particle physics, capable of simulating several processes: DIS, e+e-, pp, pA, .. detailed and precise simulations, many physics ingredients lacking of spin effects in hadronization

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□ Spin important for studies of the nucleon structure and hadronization

Collins effect \rightarrow Collins asymmetries in SIDISand $e^+e^- \rightarrow$ transversity PDF and Collins FFinterference FF \rightarrow dihadron asymmetries in SIDIS, Artru-Collins asymmetries in e^+e^- polarizing FF $\rightarrow \Lambda$ transverse polarization, e.g. in $e^+e^ G_1^{\perp}$ FF \rightarrow beam spin asymmetries in SIDIS, ... $\rightarrow e(x)$...jet func. \rightarrow ..

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 \Box Spin effects implemented in Pythia for DIS and e^+e^- by StringSpinner

public package AK, L. Lönnblad, CPC 272 (2022) 108234; CPC 292 (2023) 108886 can be downloaded from gitlab https://gitlab.com/albikerbizi/stringspinner sample main program for DIS

 e^+e^- not available yet from gitlab, will be soon

Implementation of spin effects for DIS

Let Pythia generate the process in the standard way and change behaviour through the UserHooks class, e.g. in hadronization → parton showers OFF

 \square Allow for target polarization \rightarrow parametrizations of transversity PDFs h_1^q for $q = u_v, d_v$

Alternatively, let the user chose the polarization of each quark

Evaluate the polarization of the fragmenting quark

Apply rules of the string+³P₀ model in hadronization



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AK, L. Lönnblad, CPC **272** (2022) 108234; CPC **292** (2023) 108886

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Produced hadrons accessible from the standard Pythia event record

Analyze → Collins asymmetry Dihadron asymmetry Beam spin asymmetry etc.

Results from simulations of SIDIS @ COMPASS and HERMES kinematics



Pythia parameters

StringZ:aLund StringZ:bLund StringPT:sigma StringPT:enhancedFraction StringPT:enhancedWidth StringFlav:probStoUD StringFlav:mesonUDvector StringFlav:mesonSvector default (0.68) default (0.98) default (0.335 GeV) 0.00.0 GeV/cdefault (0.217) default (0.5) default (0.55)

String+³P₀ parameters CPC **292** (2023) 108886

$$\begin{array}{cccc} Re(\mu) & 0.42 & GeV/c^2 \\ Im(\mu) & 0.76 & GeV/c^2 \\ f_L & 0.93 & L \mbox{ pol. VM} \rightarrow \mbox{ large Collins effect} \\ \theta_{LT} & 0.0 \end{array}$$

 e^+e^- annihilation requires small f_L (T pol VMs) and $\theta_{LT}~\neq~0$

Transverse spin effects: Collins asymmetries





PLB 717 (2012) 376

Comparison with COMPASS 160 GeV muons off protons

AK, L. Lönnblad, CPC 292 (2023) 108886

Transverse spin effects: Collins asymmetries





PLB693 (2010) 11

Comparison with HERMES 27 GeV electrons off protons

AK, L. Lönnblad, CPC 292 (2023) 108886

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Transverse spin effects: dihadron asymmetries





PLB 736 (2014) 124

Comparison with COMPASS 160 GeV muons off protons

AK, L. Lönnblad, CPC 292 (2023) 108886

Recipe for the simulation of e⁺e⁻ annihilation

AK, X. Artru, PRD 109 (2024) 5, 05402

Recursive recipe for e^+e^-



Steps:

- 1. Hard scattering
- 2. Joint spin density matrix
- 3. Hadron emission from a
- 4. Update density matrix
- 5. Hadron emission from \bar{q}
- 6. Exit condition



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Set up the scattering $e^+e^- \rightarrow q\overline{q}$ in the c.m.s generate the quark flavors and kinematics using differential cross section





Steps:

1. Hard scattering

2. Joint spin density matrix

- 3. Hadron emission from q
- 4. Update density matrix
- 5. Hadron emission from \bar{q}
- 6. Exit condition

 \Box Set up the joint spin density matrix of the $q\overline{q}$ pair

$$\begin{split} \rho(q,\overline{q}) &= \begin{array}{cc} C_{\alpha\beta}^{q\overline{q}} & \sigma_{q}^{\alpha} \otimes \sigma_{\overline{q}}^{\beta} \\ & \text{correlation} \\ & \text{coefficients} \end{array} \begin{array}{c} \text{Pauli matrices} \\ \text{along QHF and AHF} \\ & \alpha &= 0, x_q, y_q, z_q \\ & \beta &= 0, x_{\overline{q}}, y_{\overline{q}}, z_{\overline{q}} \end{array} \end{split}$$

For γ^* exchange

$$\rho(q,\overline{q}) \propto \mathbf{1}_{q} \otimes \mathbf{1}_{\overline{q}} - \sigma_{q}^{z} \otimes \sigma_{\overline{q}}^{z} + \frac{\sin^{2}\theta}{1 + \cos^{2}\theta} \left[\sigma_{q}^{x} \otimes \sigma_{\overline{q}}^{x} + \sigma_{q}^{y} \otimes \sigma_{\overline{q}}^{y}\right]$$

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Steps:

- 1. Hard scattering
- 2. Joint spin density matrix

3. Hadron emission from q

- 4. Update density matrix
- 5. Hadron emission from \bar{q}
- 6. Exit condition

□ Emit the first hadron using the splitting function (emission probability density)

$$\frac{dP(q \rightarrow h + q'; q\bar{q})}{dZ_{+}Z_{+}^{-1}d^{2}p_{T}} = Tr_{q'\bar{q}}T_{q',h,q}\rho(q,\bar{q}) T_{q',h,q}^{\dagger} = F_{q',h,q}(Z_{+}, \mathbf{p}_{T}; \mathbf{k}_{T}, C^{q\bar{q}})$$

$$T_{q',h,q} \equiv T_{q',h,q} \otimes 1_{\bar{q}}$$
in the QHF

□ For VM emission need also to handle the polarized decay → backup

Recursive recipe for e^+e^-



Steps:

- 1. Hard scattering
- 2. Joint spin density matrix
- 3. Hadron emission from q
- 4. Update density matrix
- 5. Hadron emission from \bar{q}

6. Exit condition

Evaluate the spin density matrix $\rho(q'\bar{q})$

$$\rho(q', \overline{q}) = \mathbf{T}_{q',h,q} \ \rho(q, \overline{q}) \ \mathbf{T}_{q',h,q}^{\dagger}$$

includes the information on the emission of h



Steps:

- 1. Hard scattering
- 2. Joint spin density matrix
- 3. Hadron emission from q
- 4. Update density matrix
- 5. Hadron emission from \overline{q}

6. Exit condition

Depend on the azimuthal angle h

 \Box Emit a hadron from the \overline{q} side using the splitting function

$$\frac{dP(\overline{q} \rightarrow H + \overline{q}'; q'\overline{q})}{dZ_{-}Z_{-}^{-1}d^{2}P_{T}} = Tr_{q'\overline{q}'}T_{\overline{q}',H,\overline{q}} \rho(q',\overline{q}) T_{\overline{q}',H,\overline{q}}^{\dagger} = F_{\overline{q}',H,\overline{q}}(Z_{-},P_{T};\overline{k}_{T},C^{q'\overline{q}})$$

Expressed in the AHF

conditional probability of emitting H, having emitted h \rightarrow correlations between the transverse momenta

[Collins NPB, 304:794-804, 1988, Knowles NPB, 310:571-588, 1988]

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Steps:

- 1. Hard scattering
- 2. Joint spin density matrix
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- 4. Update density matrix
- 5. Hadron emission from \bar{q}
- 6. Exit condition

Iterate until the exit condition is called and the last quark pair is hadronized more details in PRD 109 (2024) 5, 054029

Simulations of e⁺e⁻ annihilation with spin effects using Pythia 8.3 + StringSpinner

AK, L. Lönnblad, A. Martin, Phys. Rev. D 110 (2024) 7, 074029

 $\Box \ \sqrt{s} = 10.6 \text{ GeV}, \gamma^* \text{ exchange, quarks produced u, d, s}$ consistent with BELLE and BABAR data

Free parameters	
spin-less hadronization	as in standard Pythia 8.3
complex mass μ	as in AK, Lonnblad, CPC 292 (2023) 108886
$f_{L} = 0.12$	~ T pol. VMs
$\theta_{\rm LT} = -0.65$	interference between T and L pol. of VMs

found to give a satisfactory agreement with $e^+e^-\mbox{ data},$ ok also for SIDIS

Compare with Collins asymmetries

The Collins asymmetries in e^+e^- for back-to-back h_1h_2



 ϕ_2

Boer et al., NPB504, 345 (1997). Boer, NPB, 806:23–67, 2009 Anselmino et al., PRD 92, 114023 (2015) D'Alesio et al., JHEP 10 (2021) 078

$$N_{h_1h_2} \propto 1 + \frac{\langle \sin^2 \theta \rangle}{\langle 1 + \cos^2 \theta \rangle} A_{12} \cos(\phi_1 + \phi_2)$$

e

 P_1

P_{1T}

 ϕ_1

Collins asymmetry

$$A_{12} = \frac{\sum_{q} e_{q}^{2} H_{1q}^{\perp h_{1}} H_{1\overline{q}}^{\perp h_{2}}}{\sum_{q} e_{q}^{2} D_{1q}^{h_{1}} D_{1\overline{q}}^{h_{2}}}$$

□ Measured asymmetry

$$A_{12}^{UL(UC)} \simeq A_{12}^{U} - A_{12}^{L(C)}$$
 U unlike sign pair e.g. $\pi^{+}\pi^{-} + \pi^{-}\pi^{+}$
L like sign pair e.g. $\pi^{+}\pi^{+} + \pi^{-}\pi^{-}$
C charged pair e.g. $\pi^{+}\pi^{-} + \pi^{-}\pi^{+} + \pi^{+}\pi^{+} + \pi^{-}\pi^{-}$

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The Collins asymmetries in e^+e^- for back-to-back h_1h_2

□ Thrust axis method Boer et al., NPB504, 345 (1997). Boer, NPB, 806:23-67, 2009 Anselmino et al., PRD 92, 114023 (2015) D'Alesio et al., JHEP 10 (2021) 078 . P_2 P_2 P_2 P_1 P_2 P_2 P_1 P_1 P_1 P_2 P_1 P_1 P_1 P_1 P_1 P_1 P_2 P_1 P_1 P_1 P_2 P_1 P_1 P_1 P_2 P_2 P_1 P_1 P_2 P_2 P_2 P_2 P_1 P_1 P_2 $P_$

$$N_{h_1h_2} \propto 1 + \frac{\langle \sin^2 \theta \rangle}{\langle 1 + \cos^2 \theta \rangle} A_{12} \cos(\phi_1 + \phi_2)$$

Collins asymmetry $A_{12} = \frac{\sum_{q} e_q^2 \ H_{1q}^{\perp h_1} \ H_{1\overline{q}}^{\perp h_2}}{\sum_{q} e_q^2 \ D_{1q}^{h_1} \ D_{1\overline{q}}^{h_2}}$

Measured asymmetry

 $A_{12}^{UL(UC)} \simeq A_{12}^{U} - A_{12}^{L(C)}$

$$N_{h_1h_2} \propto 1 + \frac{\langle \sin^2 \theta_2 \rangle}{\langle 1 + \cos^2 \theta_2 \rangle} A_0 \cos(2\phi_0)$$

Collins asymmetry

$$A_0 = \frac{\sum_q e_q^2 \ wH_{1q}^{\perp h_1} \otimes H_{1\overline{q}}^{\perp h_2}}{\sum_q e_q^2 \ D_{1q}^{h_1} \otimes D_{1\overline{q}}^{h_2}}$$

□ Measured asymmetry

$$\begin{array}{lll} U & \text{unlike sign pair} & \text{e.g. } \pi^{+}\pi^{-} + \pi^{-}\pi^{+} & A_{0}^{\text{UL}(\text{UC})} \simeq A_{0}^{\text{U}} - A_{0}^{\text{L}(\text{C})} \\ \text{L} & \text{like sign pair} & \text{e.g. } \pi^{+}\pi^{+} + \pi^{-}\pi^{-} \\ \text{C} & \text{charged pair} & \text{e.g. } \pi^{+}\pi^{-} + \pi^{-}\pi^{+} + \pi^{+}\pi^{+} + \pi^{-}\pi^{-} \end{array}$$

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 ϕ_0

Comparison with the A₁₂ asymmetry

A_{12} asymmetry for charged $\pi \pi$ pairs



 $T > 0.8, z > 0.2, Q_T < 3.5 \text{ GeV}$

A_{12} asymmetry for charged $\pi \pi$ pairs



StringSpinner reproduces trend and size

$\begin{array}{l} A_{12}^{UL} \text{ asymmetry for charged } \pi \ \pi \ \text{pairs} \\ P_{T1} \times P_{T2} \text{ - dependence w.r.t thrust} \end{array}$



Comparison with the A₀ asymmetry

A_0 asymmetry for $\pi \pi$ pairs:



Cuts:

 $T > 0.8, z > 0.2, Q_T < 3.5 GeV$

A_0 asymmetry for $\pi \pi$ pairs:



 $T > 0.8, z > 0.2, Q_T < 3.5 GeV$

Trend reproduced by string+ ${}^{3}P_{0}$ somewhat lower values in the last z_{2} bin

A_0 asymmetry for $\pi \pi$ pairs:



somewhat lower values in the last z₂ bin

A_0 asymmetry for $\pi \pi$ pairs: P_{T0} dependence



 $T > 0.8, z > 0.15, Q_T < 3.5 GeV$

Cuts:

A_0 asymmetry for $\pi \pi$ pairs: P_{T0} dependence



Cuts: $T > 0.8, z > 0.15, Q_T < 3.5 GeV$

Transverse-momentum dependence reproduced by string+³P₀!

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Comparison with phenomenology

Comparison with phenomenology



Collins analysing power $a_p = -\frac{p_\perp}{zM_h} \frac{H_{1q}^{h\perp}}{D_{1q}^h}$

TS-TO15 Martin, Bradamante, Barone PRD 91 (2015) 1, 014034 point-by-point extraction from Belle A^{UL}₁₂

TO-CA24 Boglione et al., PLB 854 (2024) 138712 fit of SIDIS, e^+e^- and pp

TO24n Boglione, Flore (2024, private comm.) like in TO-CA24 + Collins asymm. COMPASS 2022 data on d

Comparison with phenomenology



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string+³P₀ → rising trend with z for fav. and unfav. A guide for the choice of the parametrizations of Collins FFs used in phenomenology?

Conclusions

□ Spin effects implemented in Pythia 8.3 for DIS and e^+e^- StringSpinner → string+³P₀ in Pythia

Encouraging results on transverse spin effects

More developments of the string+³P₀ model foreseen baryon production, parton-showers, ..

□ The work on the systematic implementing spin effects relevant for the Electron Ion Collider

Backup

The recursive recipe for simulating e^+e^- annihilation: VM emission



For a vector meson h=VM

$$\rightarrow \eta(q) = \mathbf{T}_{q',h=VM,q}^{a\prime\dagger} \,\eta(q') \,\mathbf{T}_{q',h=VM,q}^{a} \mathcal{D}_{a'a'} \,\eta(q') = \mathbf{1}_{q'} \text{ and } \eta(\bar{q}) = \mathbf{1}_{\bar{q}}$$

Steps:

i) Emission probability density (summing over decay information, i.e. $D_{a'a} = \delta_{a'a}$) $\frac{dP(q \rightarrow h = VM + q'; q\bar{q})}{dM^2 dZ_+ Z_+^{-1} d^2 p_T} = Tr_{q'\bar{q}} T_{q',h,q}^a \rho(q,\bar{q}) T_{q',h,q}^{a\dagger} = F_{q',h,q}(M^2, Z_+, p_T; k_T, C^{q\bar{q}})$ ii) Calculate the spin density matrix of h=VM, and decay the meson $\rho_{aa'}(h) = Tr_{q'\bar{q}} T_{q',h,q}^a \rho(q,\bar{q}) T_{q',h,q}^{a'\dagger}$ iii) Decay the meson $p \rightarrow p_1 p_2$.. $dN(p_1, p_2..)/d\Omega \propto M_{dec.}^a(p \rightarrow p_1 p_2..) \rho_{aa'}(h)M_{dec.}^{\dagger a'}(p \rightarrow p_1 p_2..)$

iv) Build the decay matrix $D_{a'a}(p_1, p_2, ...) = M_{dec.}^{\dagger a'}(p \rightarrow p_1 p_2...) M_{dec.}^a(p \rightarrow p_1 p_2...)$

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quark splitting $q \rightarrow h + q'$

Relevant variables:		
$\mathbf{k}_{\mathrm{T}} = \mathbf{p}_{\mathrm{T}} + \mathbf{k}_{\mathrm{T}}'$		
$Z_{+} = p^{+}/k^{+}$		
$\varepsilon_h^2 = M^2 + p_T^2$		

Transverse vectors defined w.r.t. string axis

Quark splitting amplitude in the string+³P₀ model $T_{q',h,q} \propto C_{q',h,q} D_h(M^2) \left(\frac{1-Z_+}{\epsilon_h^2}\right)^{\frac{2}{2}} \exp\left[-\frac{\mathbf{b_L}\epsilon_h^2}{2Z_+}\right] N_a^{-\frac{1}{2}}(\epsilon_h^2) e^{-\frac{\mathbf{b_T}k'_T^2}{2}}$ Γ_{h,s_h} $[\boldsymbol{\mu} + \sigma_{z}\boldsymbol{\sigma}_{T} \cdot \mathbf{k'}_{T}]$ ³P_o mechanism Coupling flavor transverse mass [μ complex mass momentum e.g. longitudinal momentum paramter] Free param. Lund (w.r.t string axis) $\Gamma_{h=PS} = \sigma_z$ Free param. string+³P₀

AK, X. Artru, PRD 109 (2024) 5, 05402

A_{12} asymmetry for charged $\pi \pi$ pairs



A_0 asymmetry for charged pions



A₀ asymmetry essential observable included in phenomenological fits

A_{12} asymmetry for charged KK pairs



A_{12}^{UC} much smaller than A_{12}^{UL} at large z reproduced by string+³P₀

A_{12} asymmetry for charged $\pi \pi$ pairs



 $T > 0.8, z > 0.2, Q_T < 3.5 \text{ GeV}$

 $T > 0.8, z > 0.15, Q_T < 3.5 \text{ GeV}, \alpha_0 < \pi/4$

StringSpinner lower than BABAR BABAR and BELLE data different, unlike StringSpinner PRD 90, 052003 (2014)

Cuts: