Theory status of the muon anomalous magnetic moment in the Fall of 2025

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Brookhaven National Laboratory
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Thinking about Soni and Sharon

Soni and Sharon in Africa, 2004

Outline

Introduction

Muon g-2 Theory Initiative

Hadronic Vacuum Polarization (HVP)

Hadronic Light-by-light (HLbL) scattering

QED and EW

Summary

The muon anomaly a_μ

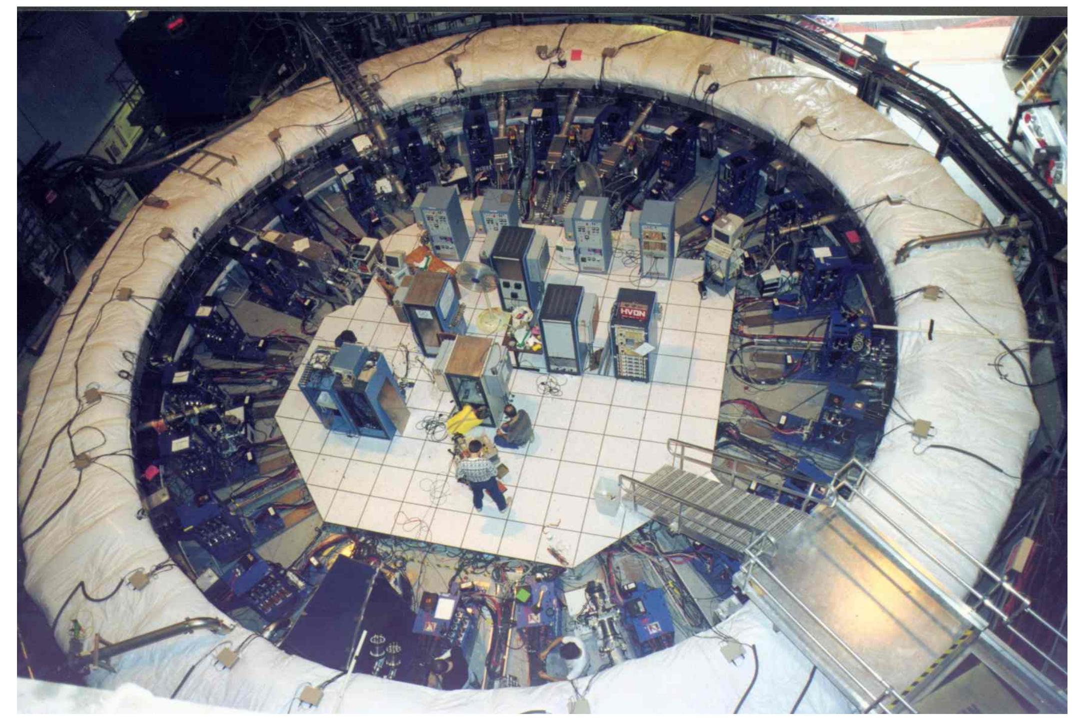
The muon is a spin ½ elementary particle with

$$\mu = g\,\frac{e}{2\,m}\,\mathrm{S} \qquad (\hbar = \boldsymbol{c} = \mathbf{1})$$

$$a_\mu = \frac{g-2}{2}$$

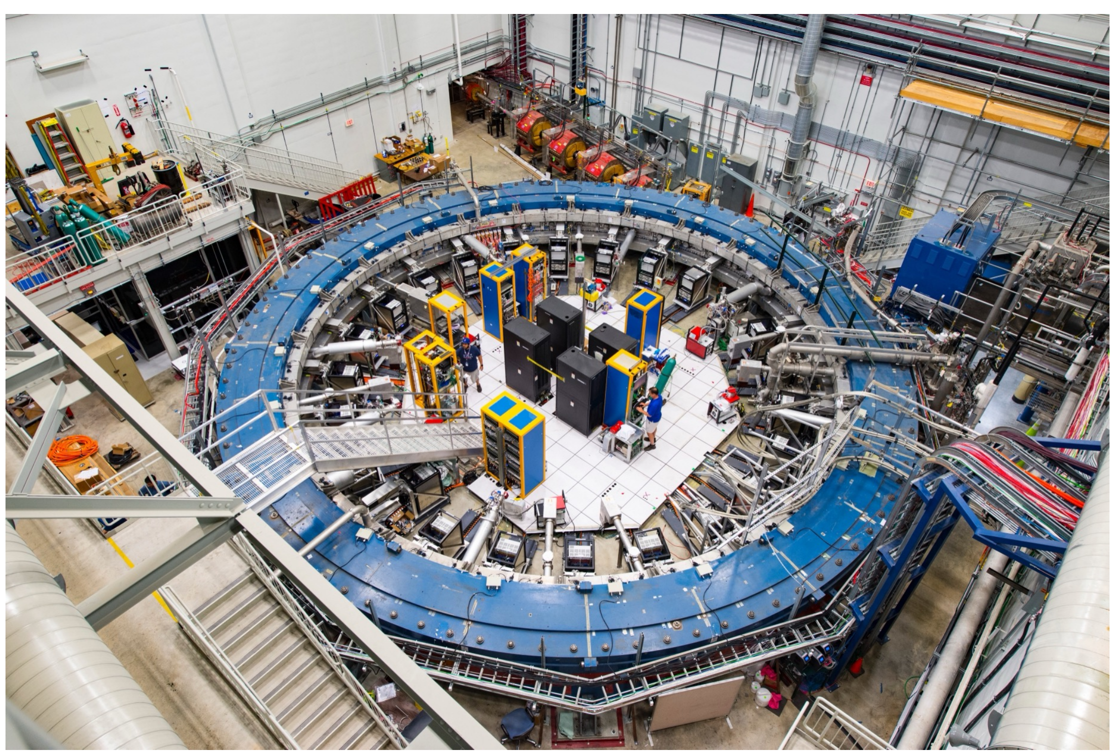
Muon *g*-2 experiments

CERN (1959-1979) (7 ppm)



BNL E821(1997-2001)

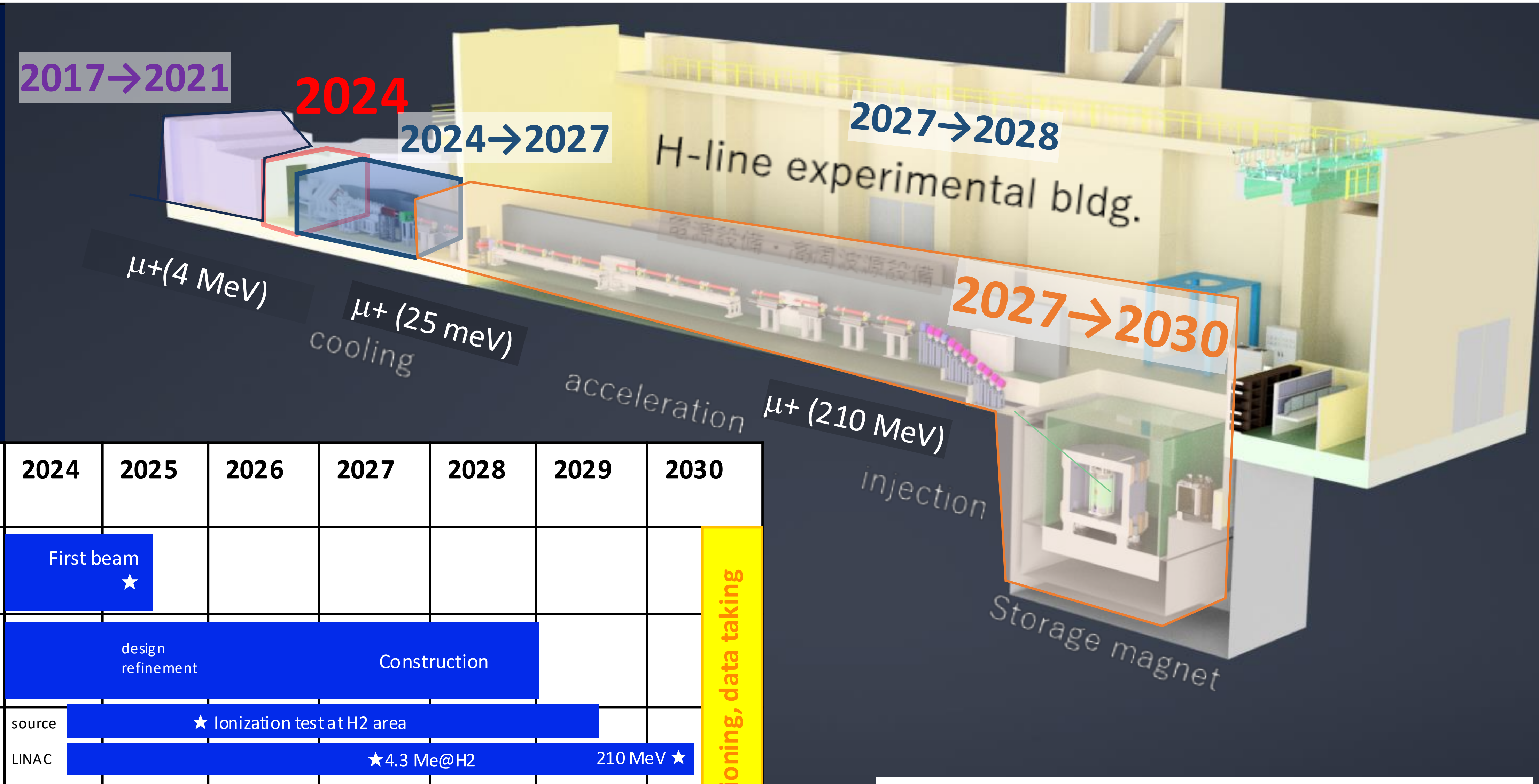
(0.54 ppm)



FNAL E989 (2018-2025) (124 ppb)

(*c.f.* electron: 0.11 ppb)

Muon g-2/EDM : intended schedule



		2024	2025	2026	2027	2028	2029	2030
Beamline		First beam ★						Commissioning, data taking
Bldg. & facility		design refinement			Construction			
Source, LINAC, storage	source		★ Ionization test at H2 area					
	LINAC			★4.3 Me@H2			210 MeV ★	
	Storage				procurement		Installation ★	
Detector		positron tracker magnetic field monitors					Installation ★	

Strong supports from the community is necessary to complete remaining construction.

History

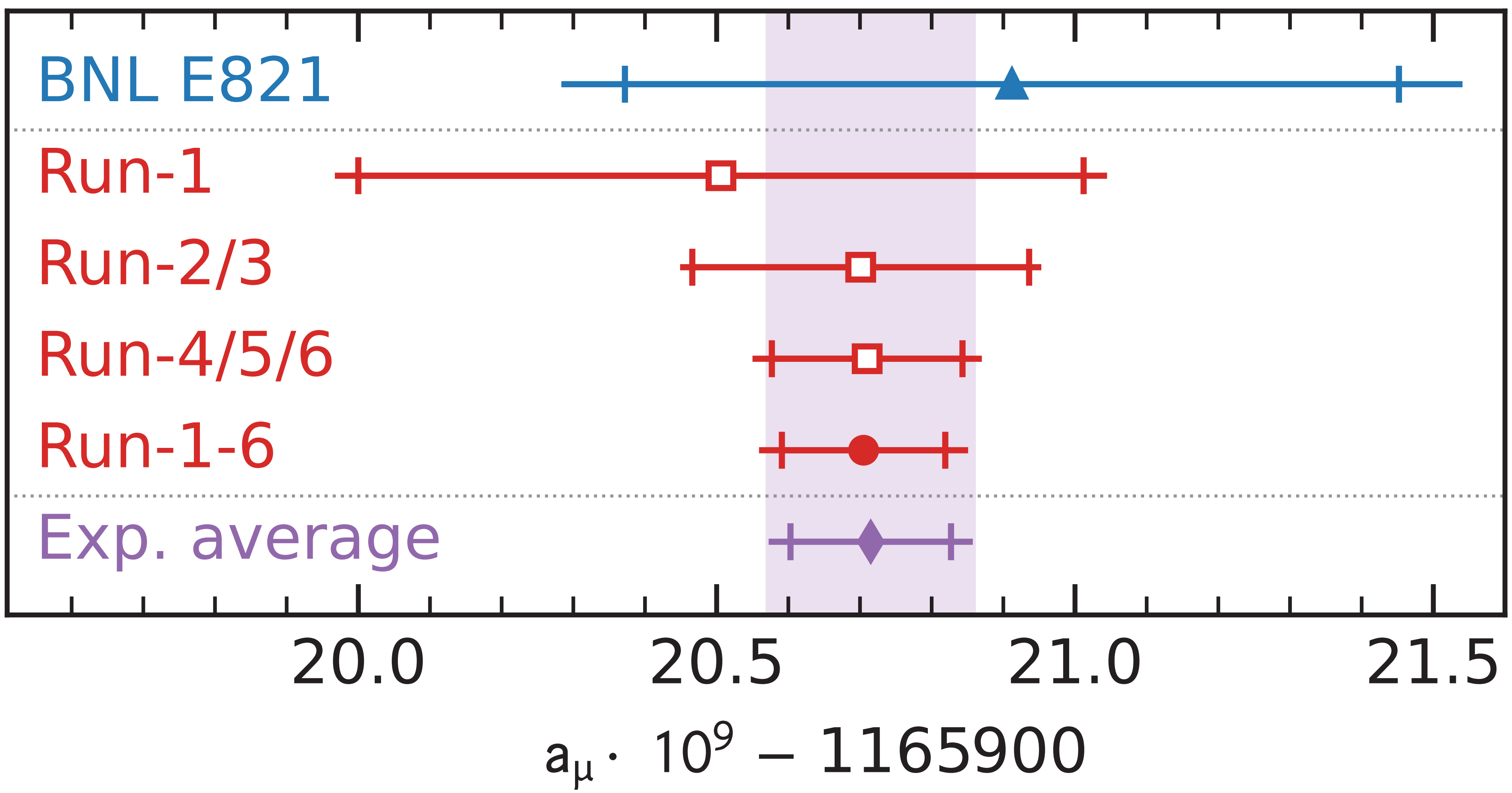
- 2009 proposal
- 2015 TDR
- 2016 IPNS focused review
- 2016 SAC (priority #3)
- 2019 KEK-IPNS stage-2, KEK-IMSS stage-2
- 2024 MEXT funding (partial construction)
- 2025 MEXT funding (partial construction)

[T. Mibe, Orsay 2025]

BNL+FNAL Average

Muon g-2 Collaboration (2025)

PHYSICAL REVIEW LETTERS **135,** 101802 (2025)

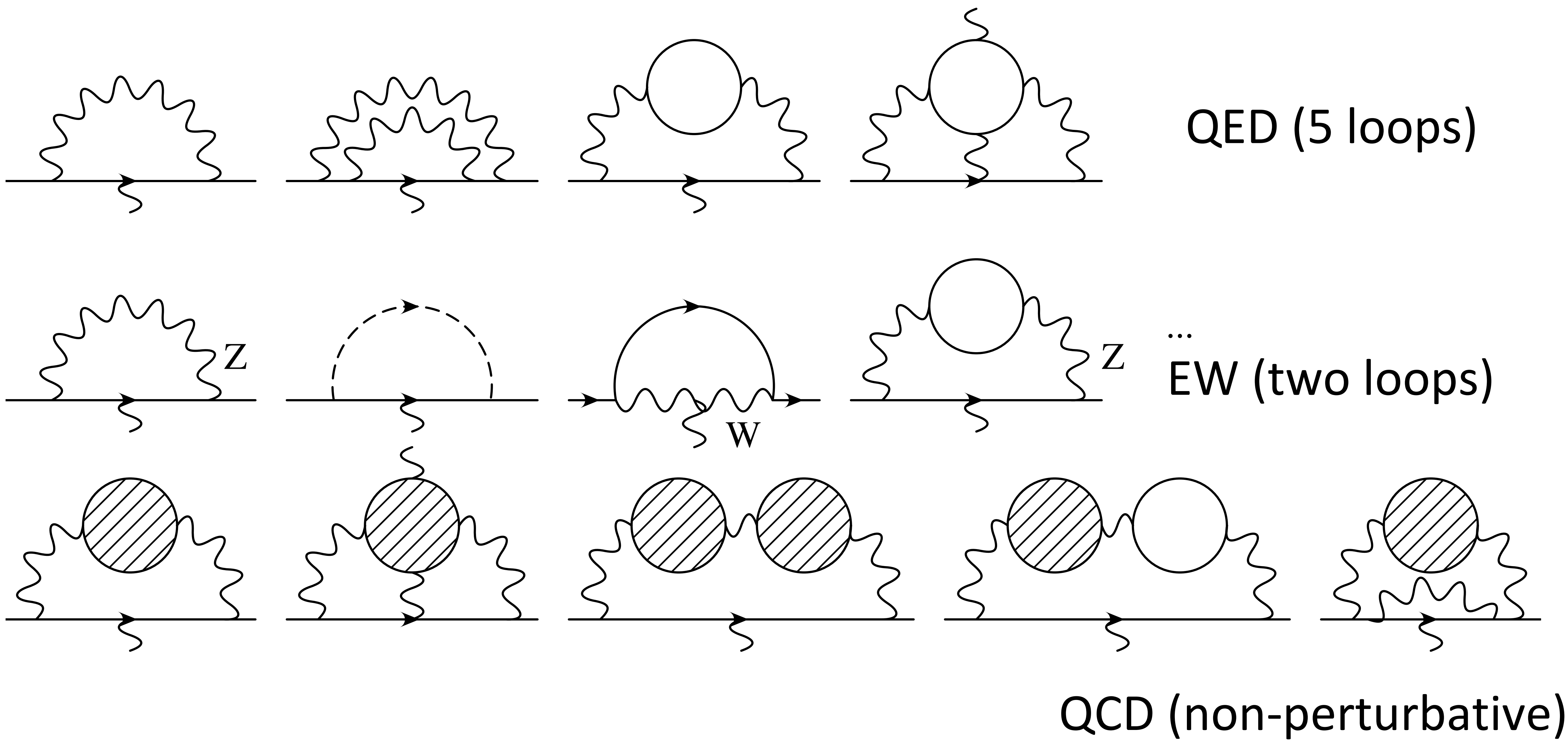


$$a_\mu(\text{exp}) = 1\,165\,920\,715(145) \times 10^{-12} \;\; (124 \text{ ppb})$$

Standard Model Theory: QED+EW+QCD

$$\langle \mu(\vec{p}')|J_\nu(0)|\mu(\vec{p})\rangle = -e\bar{u}(\vec{p}')\left(F_1(q^2)\gamma_\nu + i\frac{F_2(q^2)}{4m}[\gamma_\nu,\gamma_\rho]q_\rho\right)u(\vec{p})$$

$$a_\mu \equiv (g-2)/2 = F_2(0) \qquad (q = p' - p)$$



Perturbative expansion in α

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Muon g-2 Theory Initiative

Started in June 2017

First white paper in 2020

Physics Reports 887 (2020) 1–166

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The anomalous magnetic moment of the muon in the Standard Model

T. Aoyama [1,2,3], N. Asmussen [4], M. Benayoun [5], J. Bijnens [6], T. Blum [7,8], M. Bruno [9], I. Caprini [10], C.M. Carloni Calame [11], M. Cè [9,12,13], G. Colangelo [14,*], F. Curciarello [15,16], H. Czyż [17], I. Danilkin [12], M. Davier [18,*], C.T.H. Davies [19], M. Della Morte [20], S.I. Eidelman [21,22,*], A.X. El-Khadra [23,24,*], A. Gérardin [25], D. Giusti [26,27], M. Golterman [28], Steven Gottlieb [29], V. Gülpers [30], F. Hagelstein [14], M. Hayakawa [31,2], G. Herdoíza [32], D.W. Hertzog [33], A. Hoecker [34], M. Hoferichter [14,35,*], B.-L. Hoid [36], R.J. Hudspith [12,13], F. Ignatov [21], T. Izubuchi [37,8], F. Jegerlehner [38], L. Jin [7,8], A. Keshavarzi [39], T. Kinoshita [40,41], B. Kubis [36], A. Kupich [21], A. Kupść [42,43], L. Laub [14], C. Lehner [26,37,*], L. Lellouch [25], I. Logashenko [21], B. Malaescu [5], K. Maltman [44,45], M.K. Marinković [46,47], P. Masjuan [48,49], A.S. Meyer [37], H.B. Meyer [12,13], T. Mibe [1,*], K. Miura [12,13,3], S.E. Müller [50], M. Nio [2,51], D. Nomura [52,53], A. Nyffeler [12,*], V. Pascalutsa [12], M. Passera [54], E. Perez del Rio [55], S. Peris [48,49], A. Portelli [30], M. Procura [56], C.F. Redmer [12], B.L. Roberts [57,*], P. Sánchez-Puertas [49], S. Serednyakov [21], B. Shwartz [21], S. Simula [27], D. Stöckinger [58], H. Stöckinger-Kim [58], P. Stoffer [59], T. Teubner [60,*], R. Van de Water [24], M. Vanderhaeghen [12,13], G. Venanzoni [61], G. von Hippel [12], H. Wittig [12,13], Z. Zhang [18], M.N. Achasov [21], A. Bashir [62], N. Cardoso [47], B. Chakraborty [63], E.-H. Chao [12], J. Charles [25], A. Crivellin [64,65], O. Deineka [12], A. Denig [12,13], C. DeTar [66], C.A. Dominguez [67], A.E. Dorokhov [68], V.P. Druzhinin [21], G. Eichmann [69,47], M. Fael [70], C.S. Fischer [71], E. Gámiz [72], Z. Gelzer [23], J.R. Green [9], S. Guellati-Khelifa [73], D. Hatton [19], N. Hermansson-Truedsson [14], S. Holz [36], B. Hörz [74], M. Knecht [25], J. Koponen [1], A.S. Kronfeld [24], J. Laiho [75], S. Leupold [42], P.B. Mackenzie [24], W.J. Marciano [37], C. McNeile [76], D. Mohler [12,13], J. Monnard [14], E.T. Neil [77], A.V. Nesterenko [68], K. Ottnad [12], V. Pauk [12], A.E. Radzhabov [78], E. de Rafael [25], K. Raya [79], A. Risch [12], A. Rodríguez-Sánchez [6], P. Roig [80], T. San José [12,13], E.P. Solodov [21], R. Sugar [81], K. Yu. Todyshev [21], A. Vainshtein [82], A. Vaquero Avilés-Casco [66], E. Weil [71], J. Wilhelm [12], R. Williams [71], A.S. Zhevlakov [78]

Muon g-2 Theory Initiative White Paper 2020 Summary

Contribution	Section	Equation	Value x 10^{11}
Experimental average (E821+E989)		[updated]	116 592 061(41)
HVP LO (e^+e^-)	Sec. 2.3.7	Eq. (2.33)	6931(40)
HVP NLO (e^+e^-)	Sec. 2.3.8	Eq. (2.34)	-98.3(7)
HVP NNLO (e^+e^-)	Sec. 2.3.8	Eq. (2.35)	12.4(1)
HVP LO (lattice, *udsc*)	Sec. 3.5.1	Eq. (3.49)	7116(184)
HLbL (phenomenology)	Sec. 4.9.4	Eq. (4.92)	92(19)
HLbL NLO (phenomenology)	Sec. 4.8	Eq. (4.91)	2(1)
HLbL (lattice, *uds*)	Sec. 5.7	Eq. (5.49)	79(35)
HLbL (phenomenology + lattice)	Sec. 8	Eq. (8.10)	90(17)
QED	Sec. 6.5	Eq. (6.30)	116 584 718.931(104)
Electroweak	Sec. 7.4	Eq. (7.16)	153.6(1.0)
HVP (e^+e^-, LO + NLO + NNLO)	Sec. 8	Eq. (8.5)	6845(40)
HLbL (phenomenology + lattice + NLO)	Sec. 8	Eq. (8.11)	92(18)
Total SM Value	Sec. 8	Eq. (8.12)	116 591 810(43)
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	Sec. 8	[updated]	251(59)

Discrepancy between WP20 and experiment is almost 6.0 standard deviations

Muon g-2 Theory Initiative White Paper 2 (2025)

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Review article

The anomalous magnetic moment of the muon in the Standard Model: an update

R. Aliberti [1,2], T. Aoyama [3], E. Balzani [4,5], A. Bashir [6,7], G. Benton [8,9], J. Bijnens [10], V. Biloshytskyi [1,2], T. Blum [11,12], D. Boito [13], M. Bruno [14,15], E. Budassi [16,17], S. Burri [18], L. Cappiello [19], C.M. Carloni Calame [17], M. Cè [14,15], V. Cirigliano [20,21], D.A. Clarke [22], G. Colangelo [18,*], L. Cotrozzi [23], M. Cottini [18], I. Danilkin [1,2], M. Davier [24,*], M. Della Morte [25], A. Denig [1,2,26,27], C. DeTar [22], V. Druzhinin [28], G. Eichmann [29], A.X. El-Khadra [8,9,*], E. Estrada [30], X. Feng [31,32,33], C.S. Fischer [34,35], R. Frezzotti [36], G. Gagliardi [37], A. Gérardin [38], M. Ghilardi [16,17], D. Giusti [39,40], M. Golterman [41], S. Gonzàlez-Solís [42,43], S. Gottlieb [44], R. Gruber [45], A. Guevara [46], V. Gülpers [47], A. Gurgone [48,49], F. Hagelstein [1,2], M. Hayakawa [50,51], N. Hermansson-Truedsson [10,47], A. Hoecker [52], M. Hoferichter [18,*], B.-L. Hoid [1,2], S. Holz [18], R.J. Hudspith [53], F. Ignatov [23], L. Jin [11], N. Kalntis [18], G. Kanwar [47], A. Keshavarzi [54], J. Komijani [45], J. Koponen [1,2], S. Kuberski [55], B. Kubis [56], A. Kupich [28], A. Kupść [57,58], S. Lahert [22], S. Laporta [4,5], C. Lehner [40,*], M. Lellmann [1], L. Lellouch [38,*], T. Leplumey [59,60], J. Leutgeb [61], T. Lin [31], Q. Liu [62], I. Logashenko [28], C.Y. London [13], G. López Castro [30], J. Lüdtke [63], A. Lusiani [49,64], A. Lutz [24], J. Mager [61], B. Malaescu [65], K. Maltman [66,67], M.K. Marinković [45], J. Márquez [30], P. Masjuan [68,69], H.B. Meyer [1,2,26,27], T. Mibe [70,*], N. Miller [26,27], A. Miramontes [71,72], A. Miranda [68], G. Montagna [16,17], S.E. Müller [73], E.T. Neil [74], A.V. Nesterenko [28], O. Nicrosini [17], M. Nio [51,75], D. Nomura [76], J. Paltrinieri [23], L. Parato [45], J. Parrino [40], V. Pascalutsa [1,2], M. Passera [5,77], S. Peris [68,69], P. Petit Rosàs [23], F. Piccinini [17,78], R.N. Pilato [23], L. Polat [65,24], A. Portelli [47], D. Portillo-Sánchez [30], M. Procura [63], L. Punzi [49,64], K. Raya [7], A. Rebhan [61], C.F. Redmer [1,2], B.L. Roberts [79,*], A. Rodríguez-Sánchez [72], P. Roig [30,72], J. Ruiz de Elvira [80], P. Sánchez-Puertas [81], A. Signer [82,60], J.W. Sitison [74], D. Stamen [56], D. Stöckinger [83], H. Stöckinger-Kim [83], P. Stoffer [60,82], Y. Sue [70], P. Tavella [45], T. Teubner [23,*], J.-N. Toelstede [60,82], G. Toledo [84], W.J. Torres Bobadilla [23], J.T. Tsang [55], F.P. Ucci [16,17], Y. Ulrich [23], R.S. Van de Water [85], G. Venanzoni [23,49], S. Volkov [86], G. von Hippel [1,2], G. Wang [38], U. Wenger [18], H. Wittig [1,2,26,27,*], A. Wright [23], E. Zaid [23], M. Zanke [56], Z. Zhang [24], M. Zillinger [18], C. Alexandrou [87,88], A. Altherr [45], M. Anderson [1,2], C. Aubin [89], S. Bacchio [88], P. Beltrame [23], A. Beltran [1,2], P. Boyle [90], I. Campos Plasencia [91], I. Caprini [92],

B. Chakraborty [93], G. Chanturia [56], A. Crivellin [60], A. Czarnecki [94], L.- Y. Dai [95], T. Dave [23], L. Del Debbio [47], K. Demory [24], D. Djukanovic [26,27], T. Draper [96], A. Driutti [49], M. Endo [97], F. Erben [55], K. Ferraby [23], J. Finkenrath [55], L. Flower [23], A. Francis [98], E. Gámiz [99], J. Gogniat [18], A.V. Grebe [85], S. Gündogdu [82,60], M.T. Hansen [47], S. Hashimoto [97], H. Hayashii [100], D.W. Hertzog [20], L.A. Heuser [56], L. Hostetler [44], X.T. Hou [101], G.S. Huang [102], T. Iijima [103,50], K. Inami [50], A. Jüttner [55,93], R. Kitano [104], M. Knecht [38], S. Kollatzsch [82,60], A.S. Kronfeld [85], T. Lenz [1,2], G. Levati [18], Q.M. Li [101], Y.P. Liao [101], J. Libby [105], K.F. Liu [96], V. Lubicz [37], M.T. Lynch [8,9], A.T. Lytle [8,9], J.L. Ma [101], K. Miura [97], K. Möhling [83], J. Muskalla [1,2], F. Noël [18], K. Ottnad [1,2], P. Paradisi [4,5], C.T. Peterson [106], A. Pich [72], S. Pitelis [1], S. Plura [1,2], A. Price [107], D. Radic [82,60], A. Radzhabov [28], A. Risch [108], S. Romiti [18], S. Sahoo [109], F. Sannino [25,19], H. Schäfer [56], Y. Schelhaas [1,2], S.I. Serednyakov [28], O. Shekhovtsova [110,111], J.N. Simone [85], S. Simula [112], E.P. Solodov [28], F.M. Stokes [67], M. Vanderhaeghen [1,2], A. Vaquero [113,114], N. Vestergaard [23], W.P. Wang [1,2], Z. Wąs [115], K. Yamashita [116], Y.B. Yang [117], T. Yoshioka [118], C.Z. Yuan [101], A.S. Zhevlakov [28]

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Introduction

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Hadronic Vacuum Polarization (HVP)

Hadronic Light-by-light (HLbL) scattering

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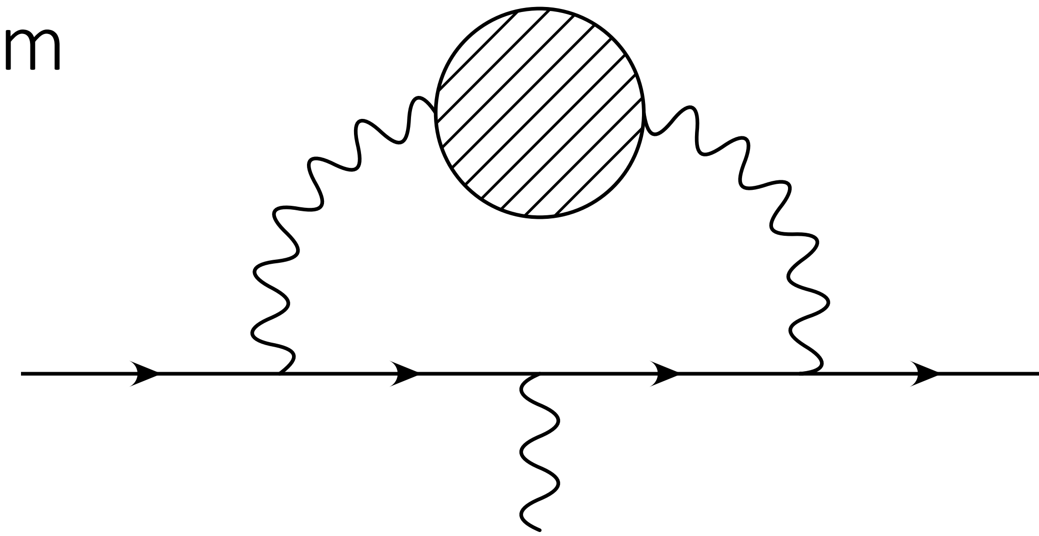
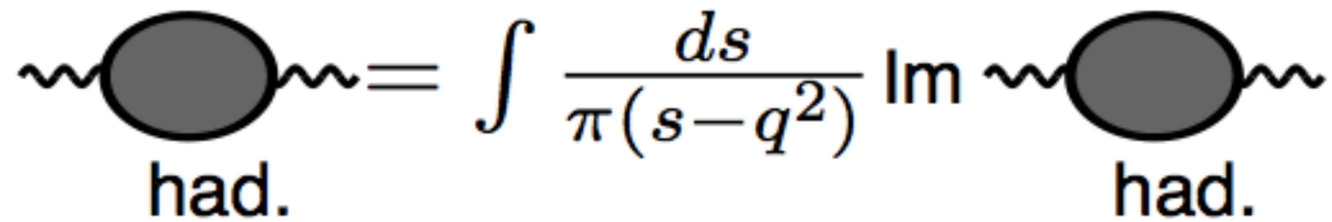
Summary

Hadronic vacuum polarization (HVP): data driven based on e^+e^- → hadrons (R-ratio)

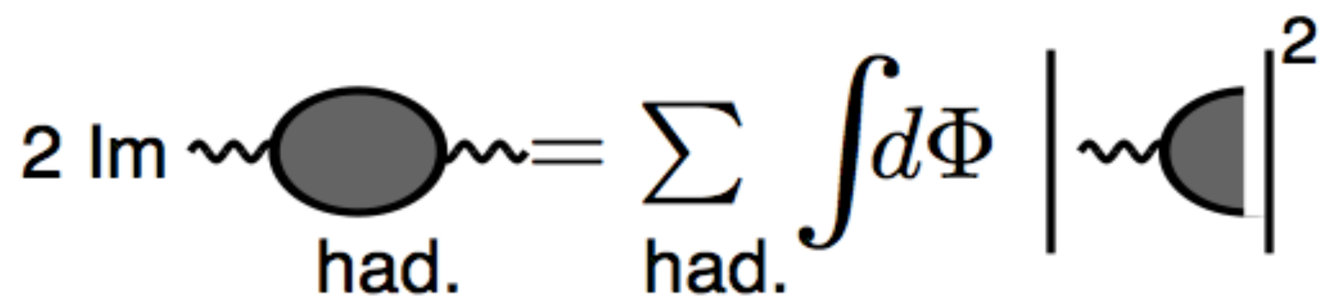
- Need the magnetic part, $F_2(0)$, of the diagram
- “Blob” is hard to calculate! (QED part easy)
- Blob represents the hadrons
- Get it from experimental cross section for e^+e^- → hadrons (two pions,+…)



Dispersion relation



Optical theorem



[Credit: T. Teubner]

$$a_\mu^{\text{HVP, LO}} = \frac{\alpha^2}{3\pi^2}\int_{M_\pi^2}^{\infty} \frac{K(s)}{s} R(s)\, ds$$

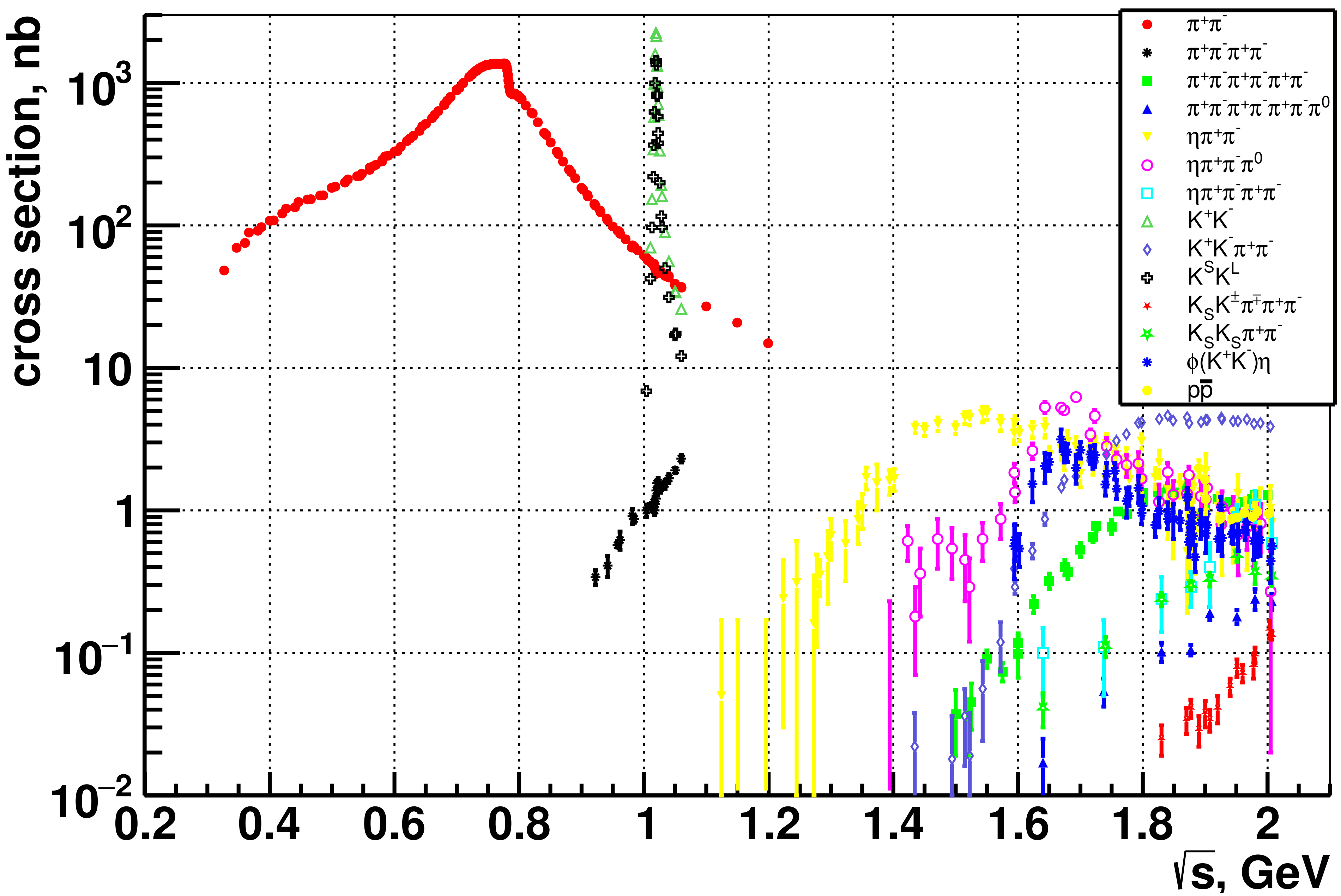
$$R(s) = \frac{\sigma^0(e^+e^- \to \text{hadrons}(+\gamma))}{\sigma_{\text{pt}}}, \quad \sigma_{\text{pt}} = \frac{4\pi\alpha^2}{3s}$$

HVP: Data Driven

- WP20 prediction for $a_\mu^{\mathrm{HVP,LO}} = 693.1(4)\times 10^{-10}$ (reminder: current error on experiment is 1.45 in these units)
- 6 standard deviation discrepancy with experiment
- Situation is very different for WP25 (for essentially two reasons)…

HVP: Data Driven

(CMD-3 Phys. Rev. Lett. 132, 231903 (2024), arXiv:2309.12910 [hep-ex])



HVP: Data Driven

CMD-3 largest statistics ever for rho region: $34\times10^6\ \pi^+\pi^-$ events

Many standard deviations higher than average of previous results

No resolution

PHYS. REV. D 109, 112002 (2024)

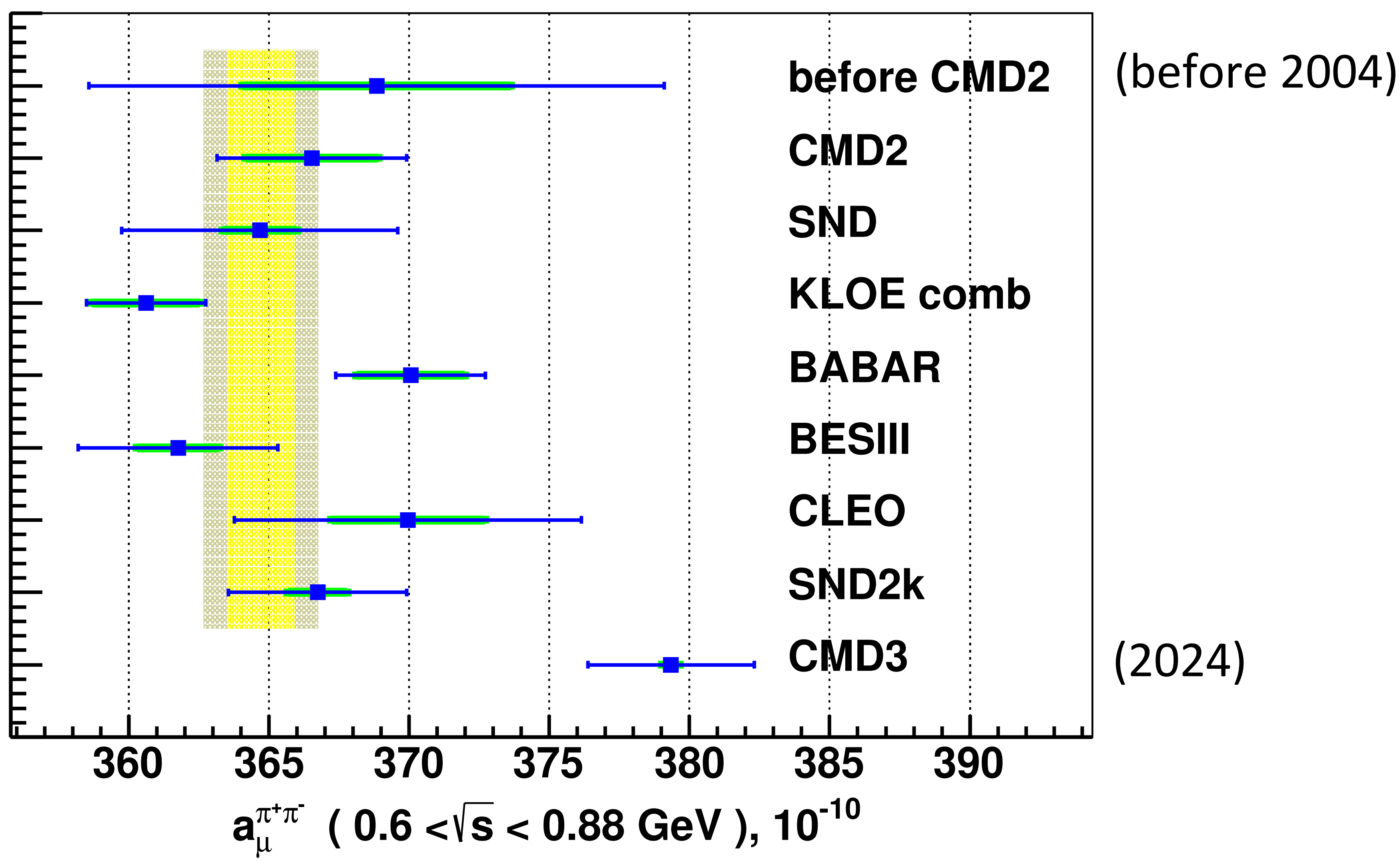


FIG. 36. The $\pi^+\pi^-(\gamma)$ contribution to the $a_\mu^{\mathrm{had,LO}}$ from the energy range $0.6 < \sqrt{s} < 0.88$ GeV obtained from the CMD-3 data and the results of the other experiments.

HVP: Data Driven

- New CMD-3 result is significantly higher than WP20 average
- Experiments differ substantially outside quoted uncertainties, can **not** be resolved by inflating errors/combining various experiments as before
- Until resolved, no new DD average in WP25 (but extensive discussion!)
- Possible paths to a resolution:
  - Improved (higher order) radiative corrections and MC generator(s)
  - New data from BaBar, Belle II, BESIII, CMD-3, KLOE, SND
- Also renewed attention on tau decay (and isospin corrections)

HVP: Lattice

- WP20 lattice average: $a_\mu^{\mathrm{HVP,LO}} = 711.6(18.4)\times 10^{-10}$ (2.6% err), **not** used in WP20 average

- WP25

  - Many independent groups contributing new, improved results

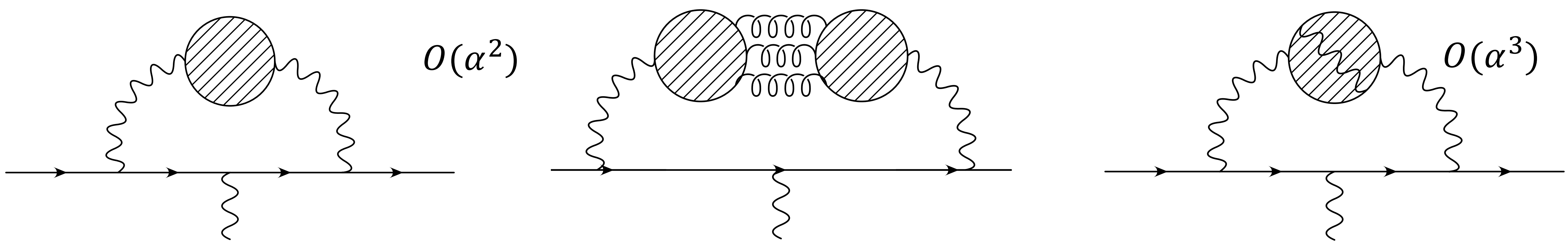
  - Many blinded analyses (key to avoid confirmation bias), especially those for total and or long distance (LD) contribution

  - Window analysis (aids/strengthens combining/cross-checking/comparing results, including with DD ones)

  - Sub-percent precision on the total

HVP contribution from Lattice QCD

Quark-connected | disconnected | Isospin breaking corrections



$$\Pi^{\mu\nu}(q) = \int d^4x e^{iqx} \langle j^\mu(x) j^\nu(0) \rangle$$

$$= \Pi(q^2)(-q^\mu q^\nu + q^2 \delta^{\mu\nu}),$$

$$j^\mu(x) = \sum_i Q_i \bar{\psi}_i(x) \gamma^\mu \psi_i(x).$$

$$a_\mu^{\rm HVP} = 4\alpha^2 \int_0^\infty dq^2 f(q^2) \hat{\Pi}(q^2),$$

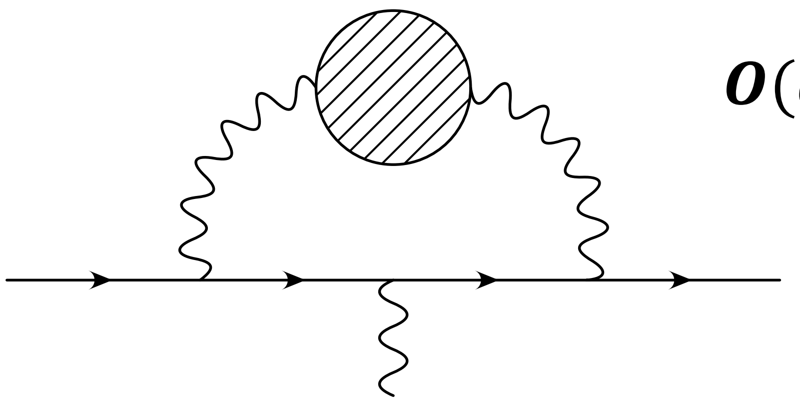
$$f(q^2) = \frac{m_\mu^2 q^2 Z^3 (1 - q^2 Z)}{1 + m_\mu^2 q^2 Z^2},$$

$$Z = -\frac{q^2 - \sqrt{q^4 + 4m_\mu^2 q^2}}{2m_\mu^2 q^2}.$$

- The EM current $\boldsymbol{j^\mu}$ on the lattice can be conserved or local
- They have different O(a^2) corrections
- Local: finite renormalization, Z_V

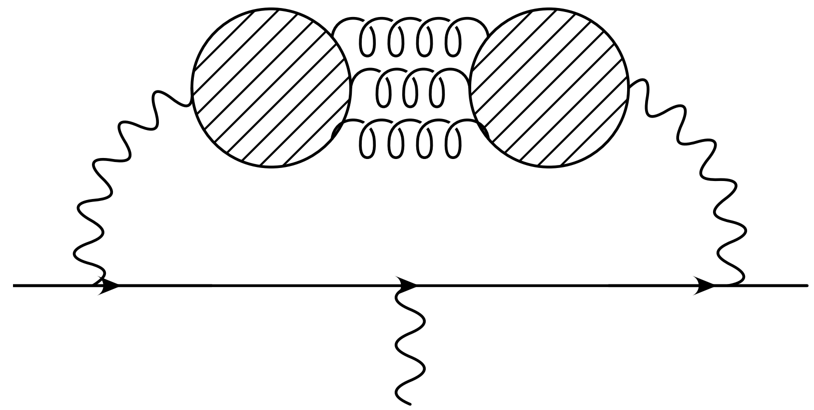
Hadronic vacuum polarization contribution from Lattice QCD

Quark-connected

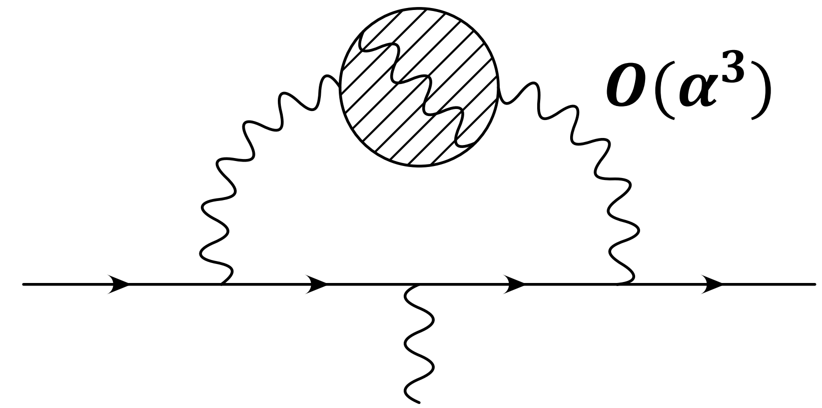


$\boldsymbol{O(\alpha^2)}$

disconnected



Isospin breaking corrections



$\boldsymbol{O(\alpha^3)}$

Blob (directly from quarks and gluons):

$$\Pi(q^2) - \Pi(0) = \sum_t \left(\frac{\cos qt - 1}{q^2} + \frac{1}{2} t^2 \right) C(t),$$

$$C(t) = \frac{1}{3} \sum_{\vec{x},i} \langle j^i(\vec{x}, t) j^i(0) \rangle,$$

$$w(t) = 4\alpha^2 \int_0^\infty d\omega^2 f(\omega^2) \left[\frac{\cos \omega t - 1}{\omega^2} + \frac{t^2}{2} \right],$$

Time momentum representation

[Bernecker, Meyer 2011]

- Interchange order of FT and loop integral over $q^2 (\omega^2)$
- “double subtraction”: $t^2/2$ removes $\Pi(0)$ and -1 the leading FV correction

$$a_\mu^{\mathrm{HVP}}(T) = \sum_{t=-T/2}^{T/2} w(t) C(t) = 2 \sum_{t=0}^{T/2} w(t) C(t)$$

HVP: Lattice Windows

Window method (introduced in RBC/UKQCD 2018)

We also consider a window method. Following Meyer-Bernecker 2011 and smearing over t to define the continuum limit we write

$$a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$

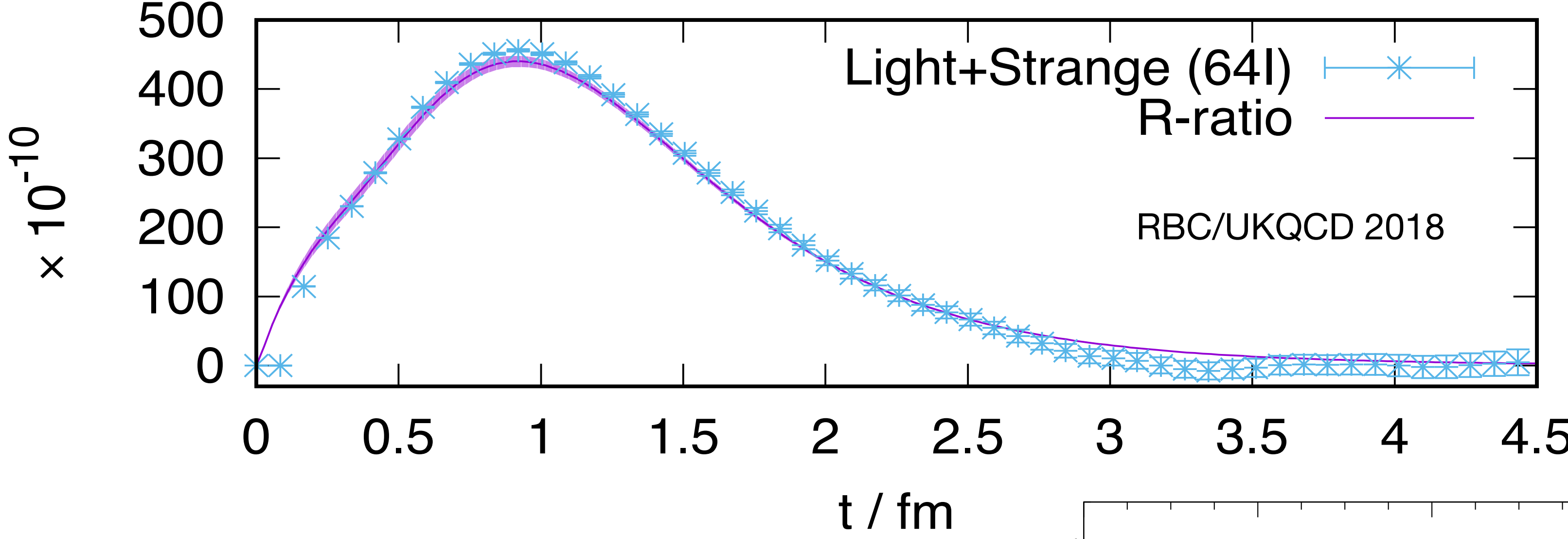
with

$$\begin{aligned}
a_\mu^{\mathrm{SD}} &= \sum_t C(t) w_t [1 - \Theta(t, t_0, \Delta)]\,, \\
a_\mu^{\mathrm{W}} &= \sum_t C(t) w_t [\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)]\,, \\
a_\mu^{\mathrm{LD}} &= \sum_t C(t) w_t \Theta(t, t_1, \Delta)\,, \\
\Theta(t, t', \Delta) &= [1 + \tanh[(t - t')/\Delta]]/2\,.
\end{aligned}$$

All contributions are well-defined individually and can be computed from lattice or R-ratio via $C(t) = \frac{1}{12\pi^2}\int_0^\infty d(\sqrt{s}) R(s) s e^{-\sqrt{s}t}$ with $R(s) = \frac{3s}{4\pi\alpha^2}\sigma(s, e^+e^- \to \mathrm{had})$.

a_μ^{W} has small statistical and systematic errors on lattice!

[slide: C. Lehner]



Summand $w(t)\ C(t)$

Windows in Euclidean and time-like regions:

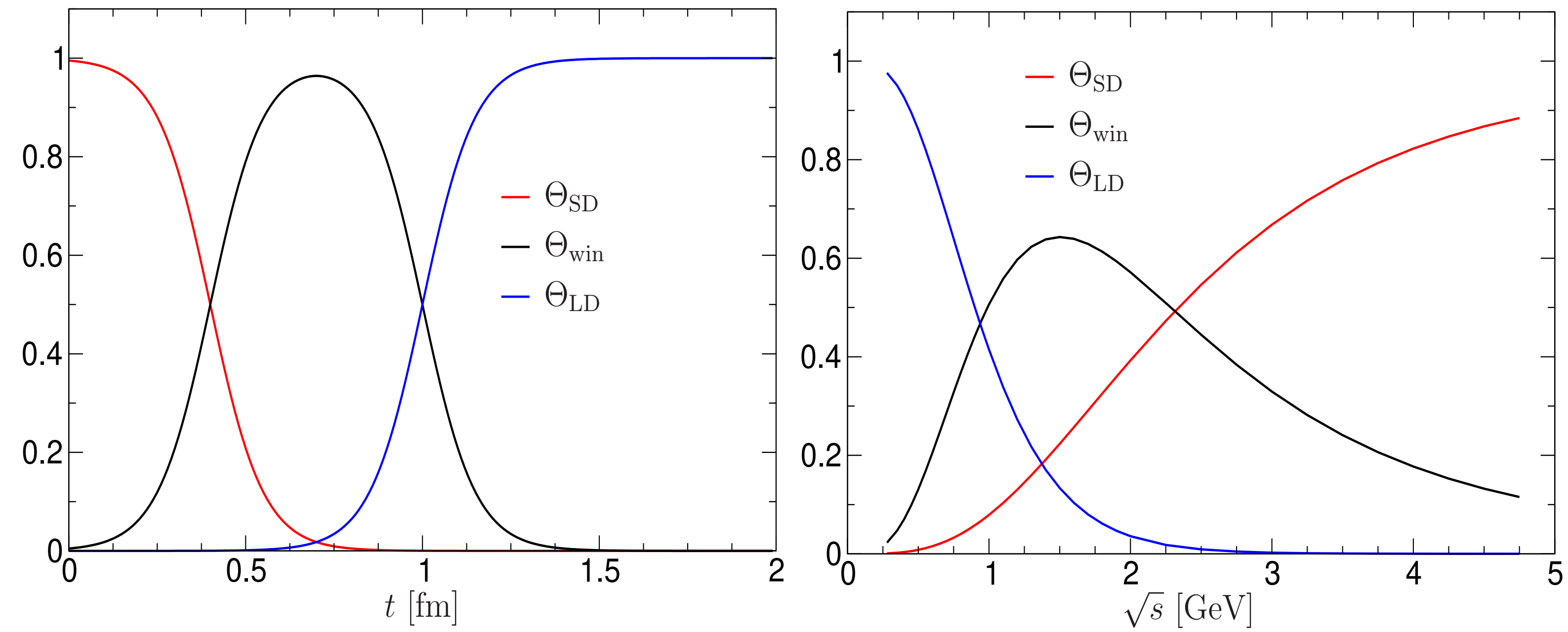


Fig.: G. Colangelo, PWA12/ATHOS7 2021

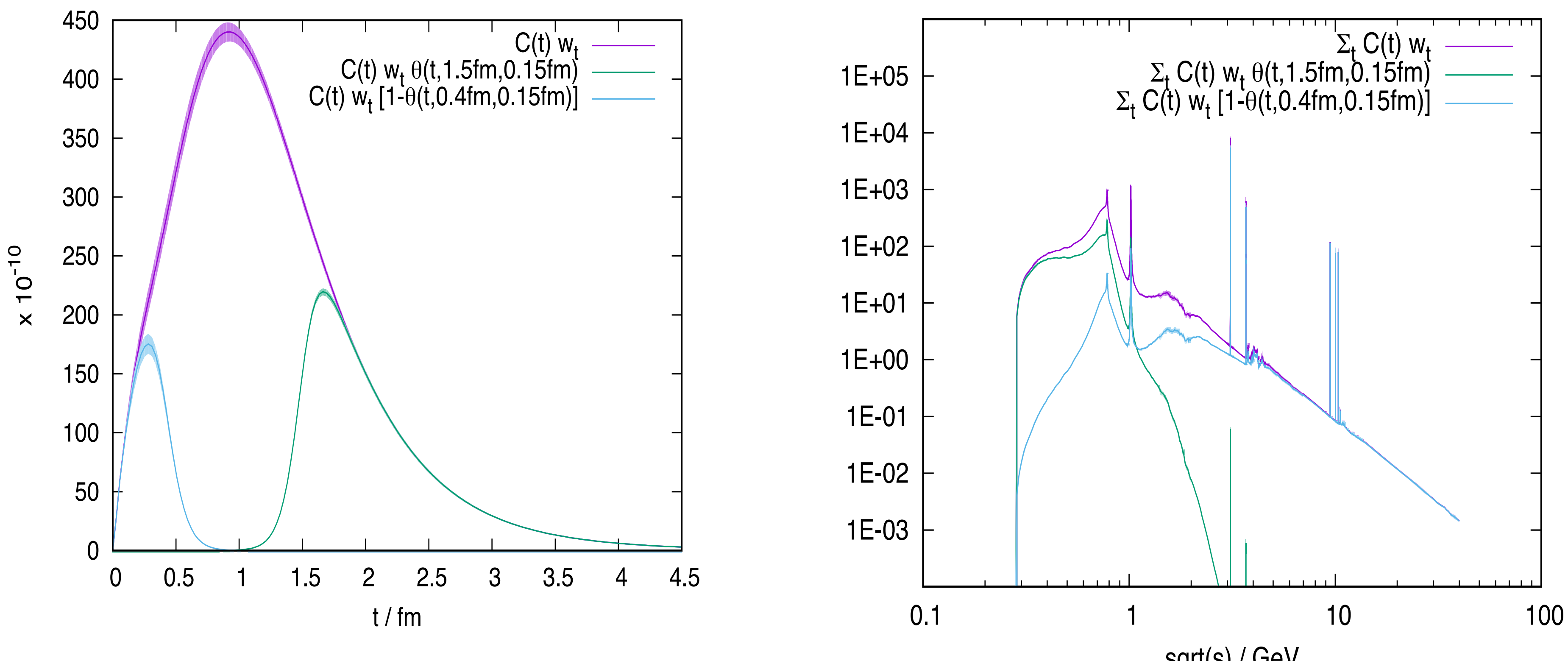
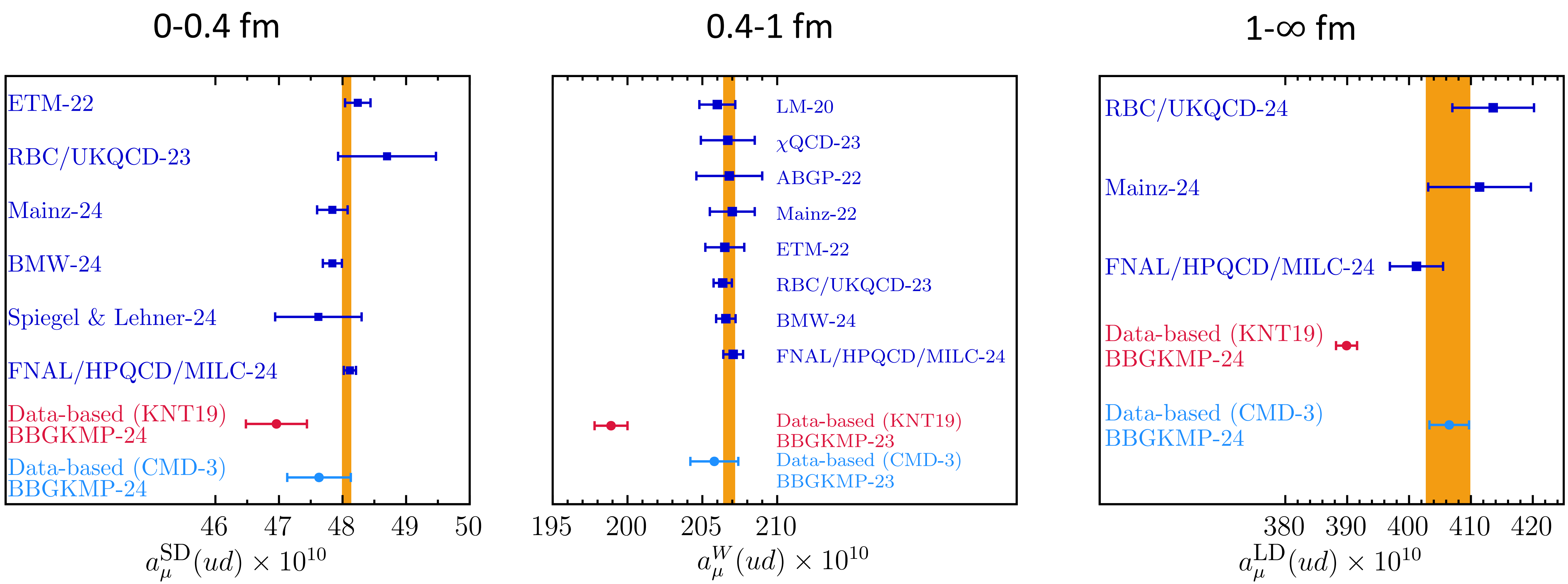


Fig.: RBC/UKQCD 2018

HVP: Lattice Windows

Light quark, connected, isospin symmetric SD, W, LD windows



very precise

all 3 blinded analyses

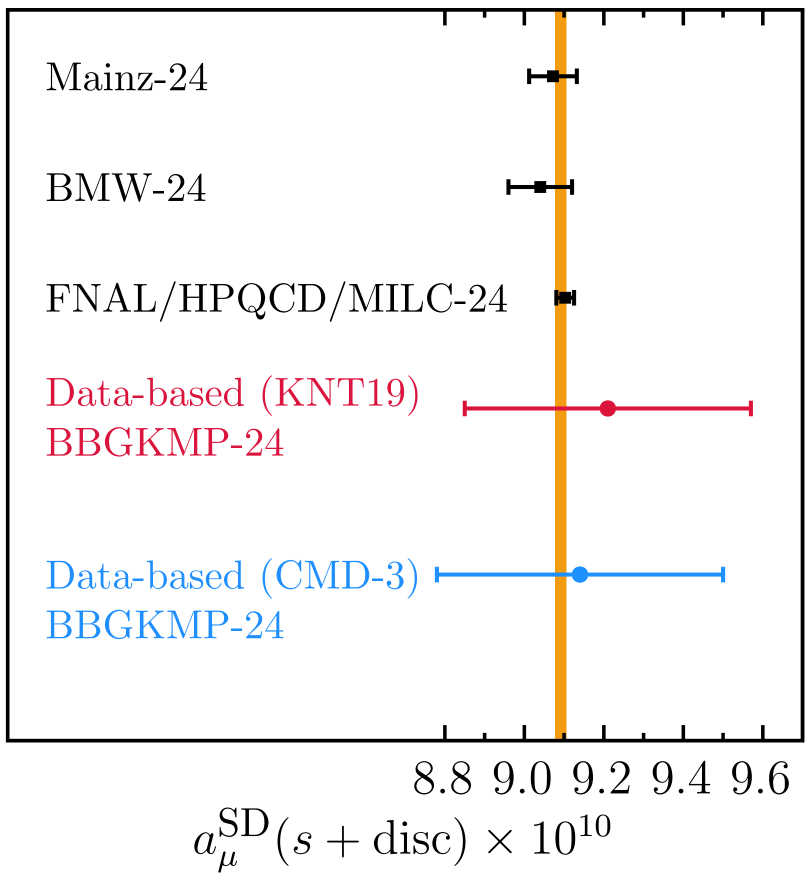
Data Driven (w/o CMD-3) consistently and significantly below lattice averages

Many standard deviations discrepant with old data driven dispersive results

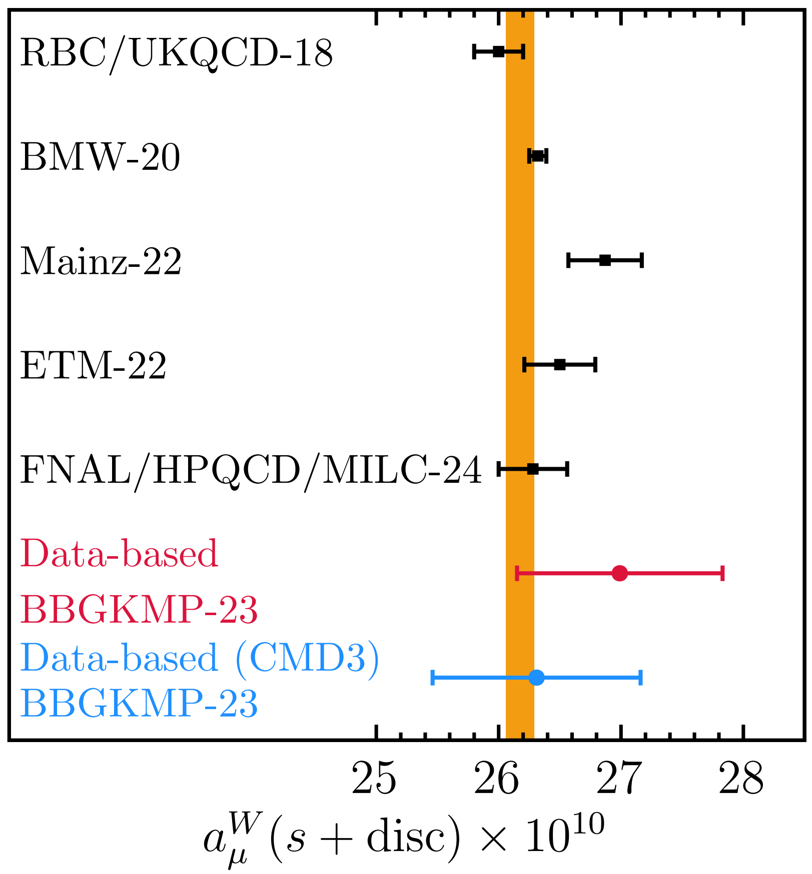
HVP: Lattice Windows

strange + disconnected, isospin symmetric SD, W, LD windows

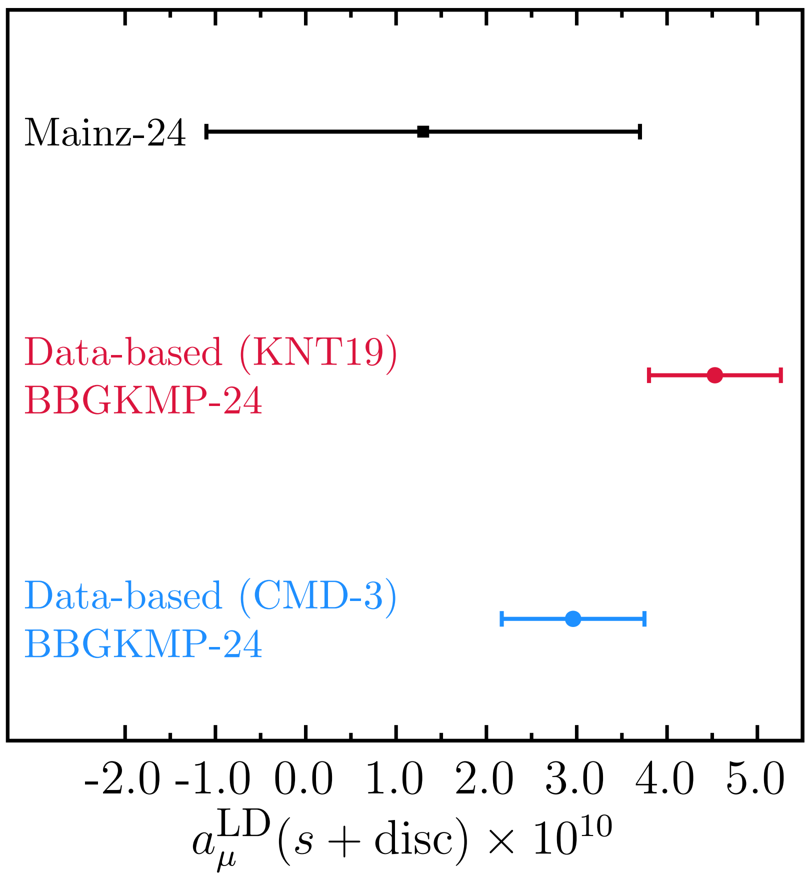
0-0.4 fm



0.4-1 fm



1-∞ fm



blinded analysis by Mainz

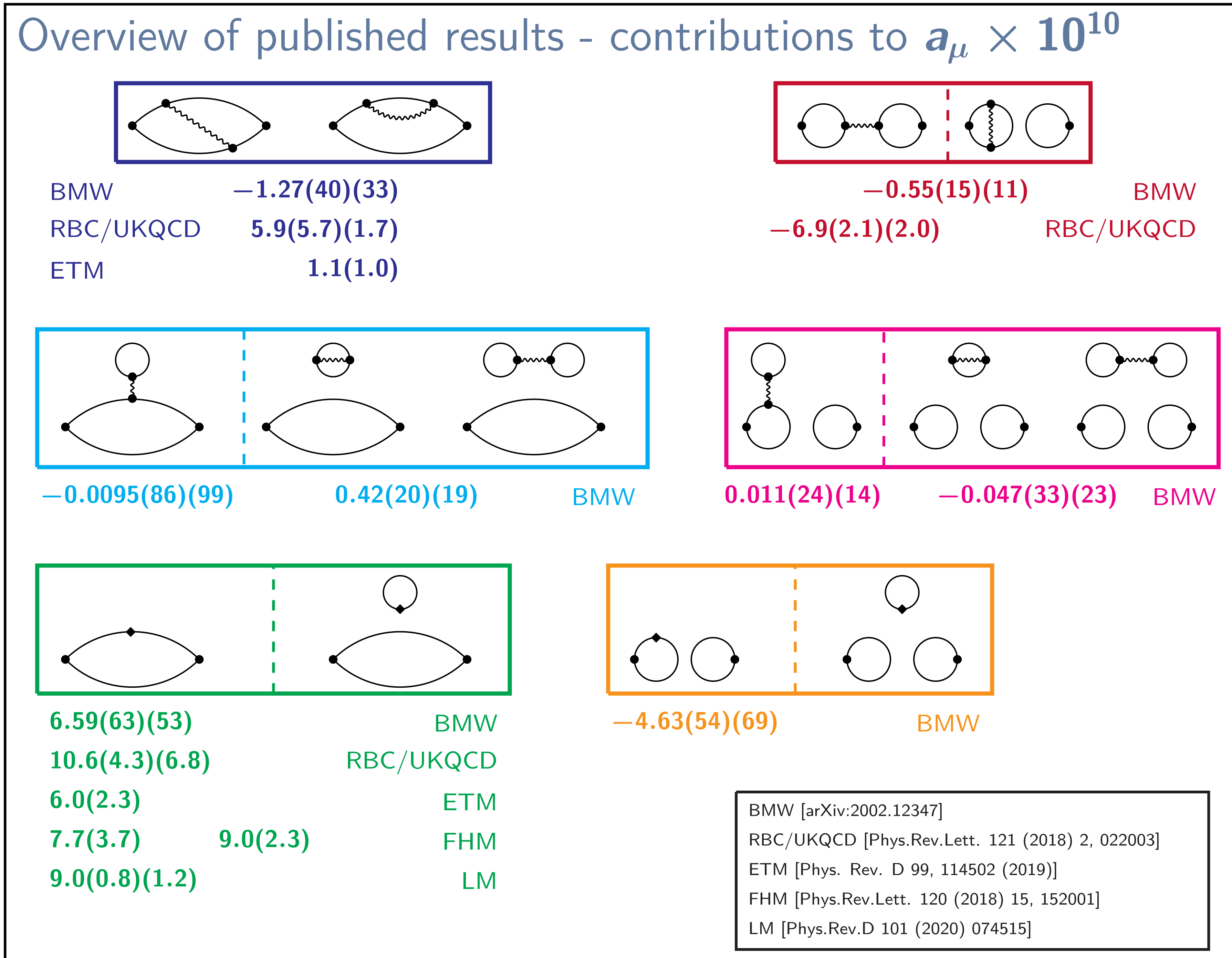
HVP: Lattice isospin-breaking corrections

V. Gülpers @ Lattice HVP workshop (2020)

(some numbers out of date)

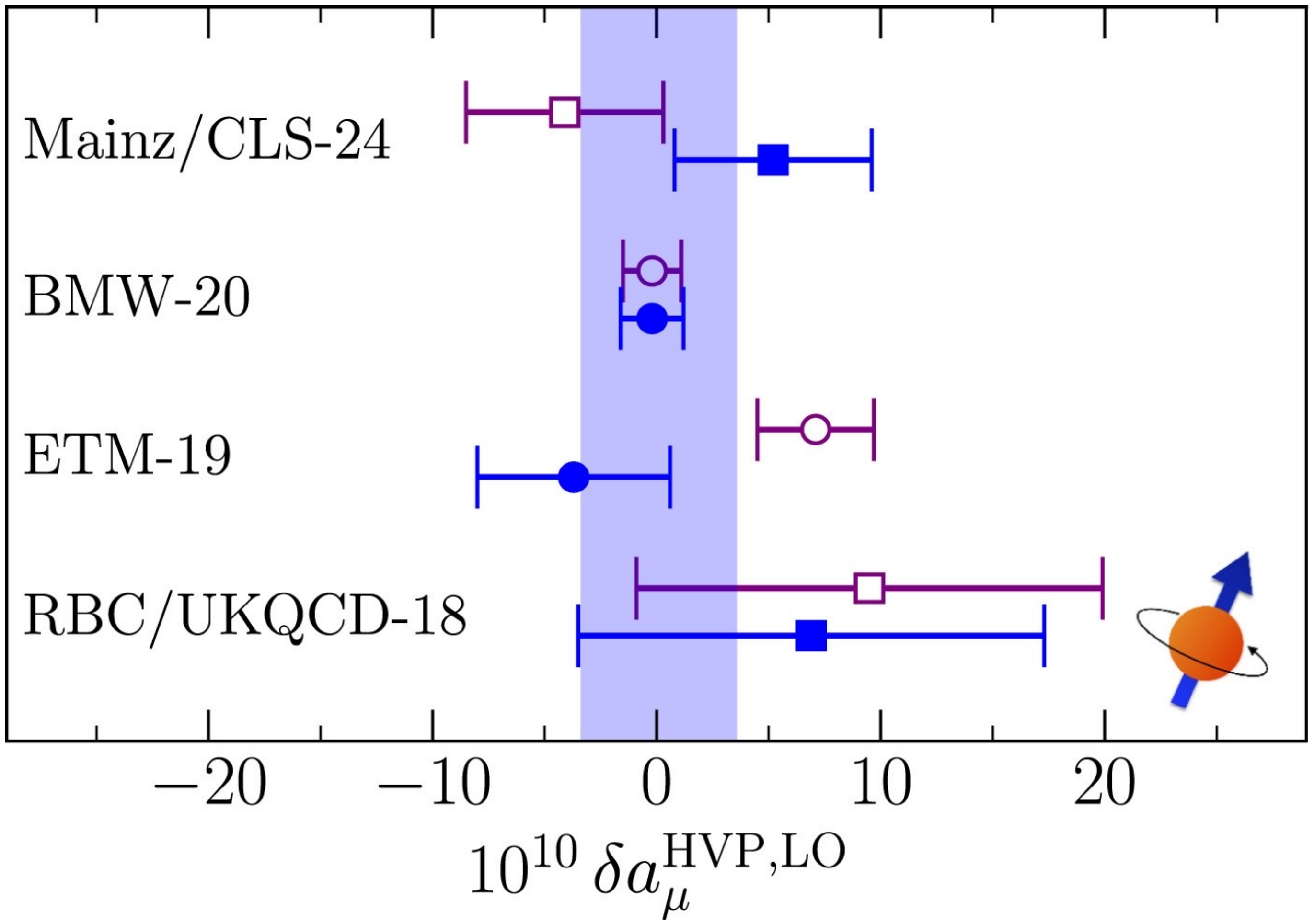
Many contributions that largely cancel

Now five collaborations but only BMW has calculated all diagrams



HVP: Lattice isospin-breaking corrections

- BMW20 complete calculation (corrected in BMW-DMZ 24, WP25) (not shown)
- Mainz 24 estimates missing contributions: increased error
- RBC/UKQCD and ETM 19 corrected for missing QED sea, disconnected diagrams using BMW20
- Correct to same scheme
- Total is very small



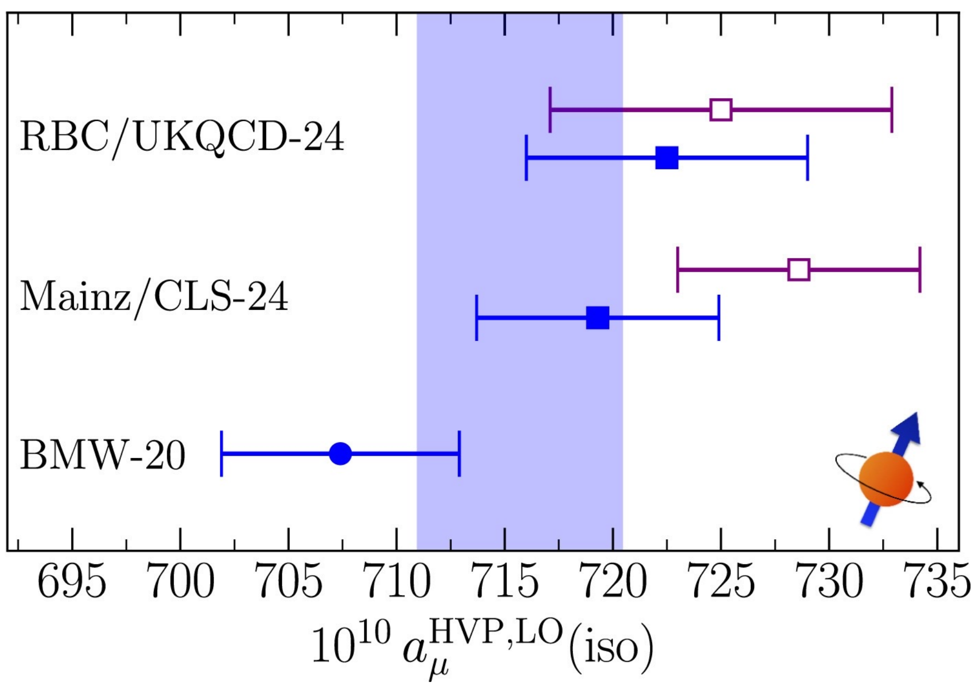
HVP: Lattice Total

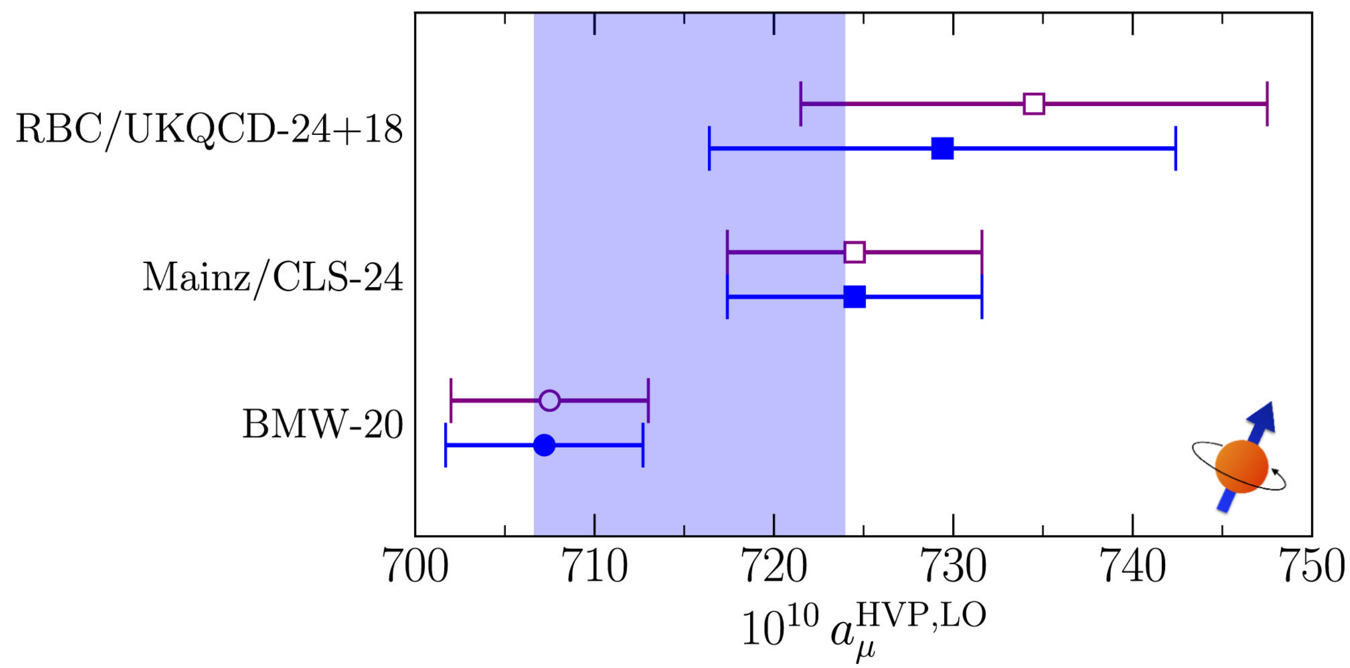
Five different averaging procedures used in WP25

- Sum of window averages + IB correction avg.
- Sum of flavor averages (a) + IB correction avg.
- Sum of flavor averages (b) + IB correction avg.
- Sum of individual, published isospin symmetric totals + IB correction avg.
- Average of individual, published totals

All are quite consistent. WP25 quotes the window average, has largest number of independent results and smallest error

HVP: Lattice Total



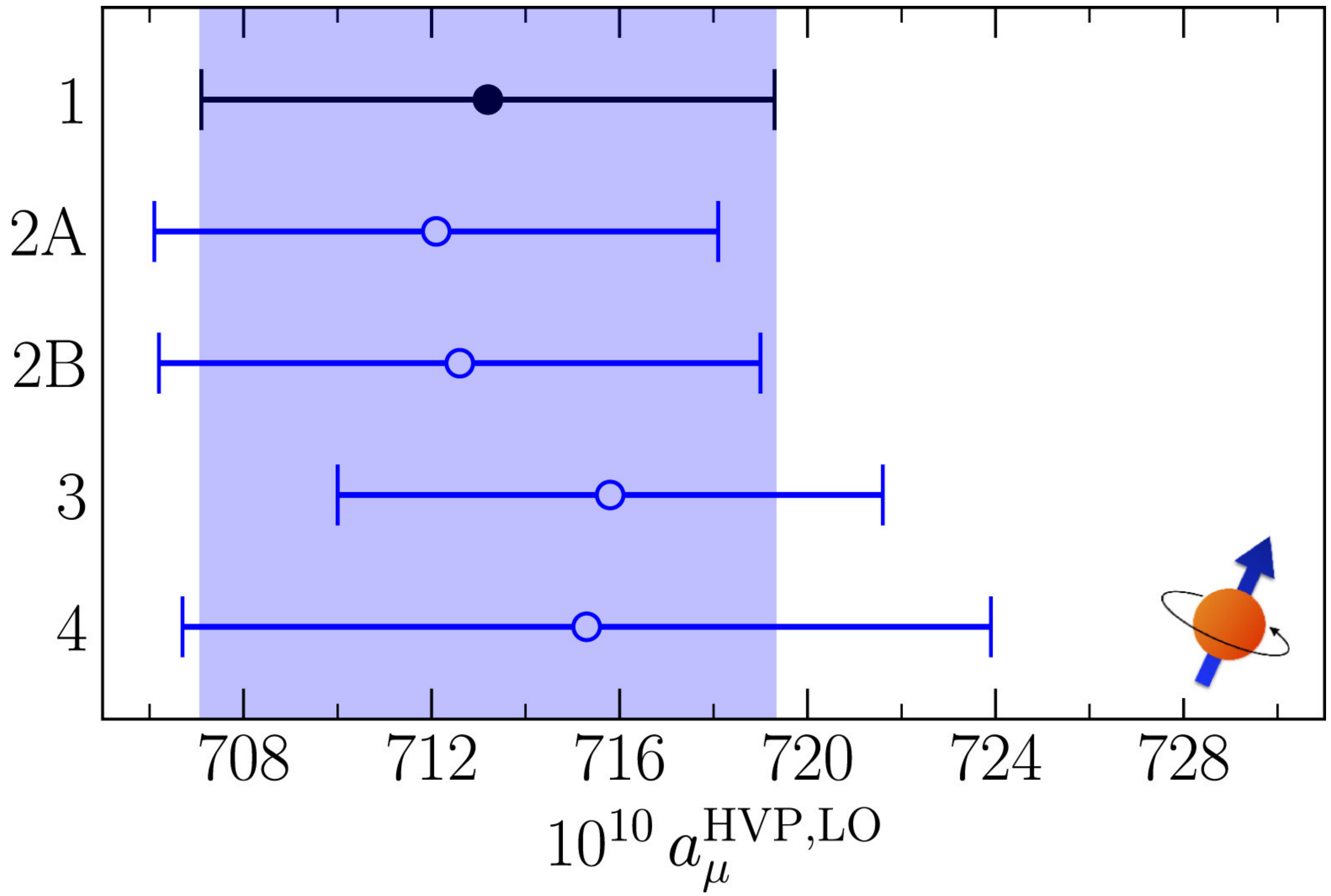


Isospin symmetric total
(IB corrections to be added)

Published totals

Large shift from WP 20 average, 693.1(4), based solely on data driven result

HVP: Lattice Total

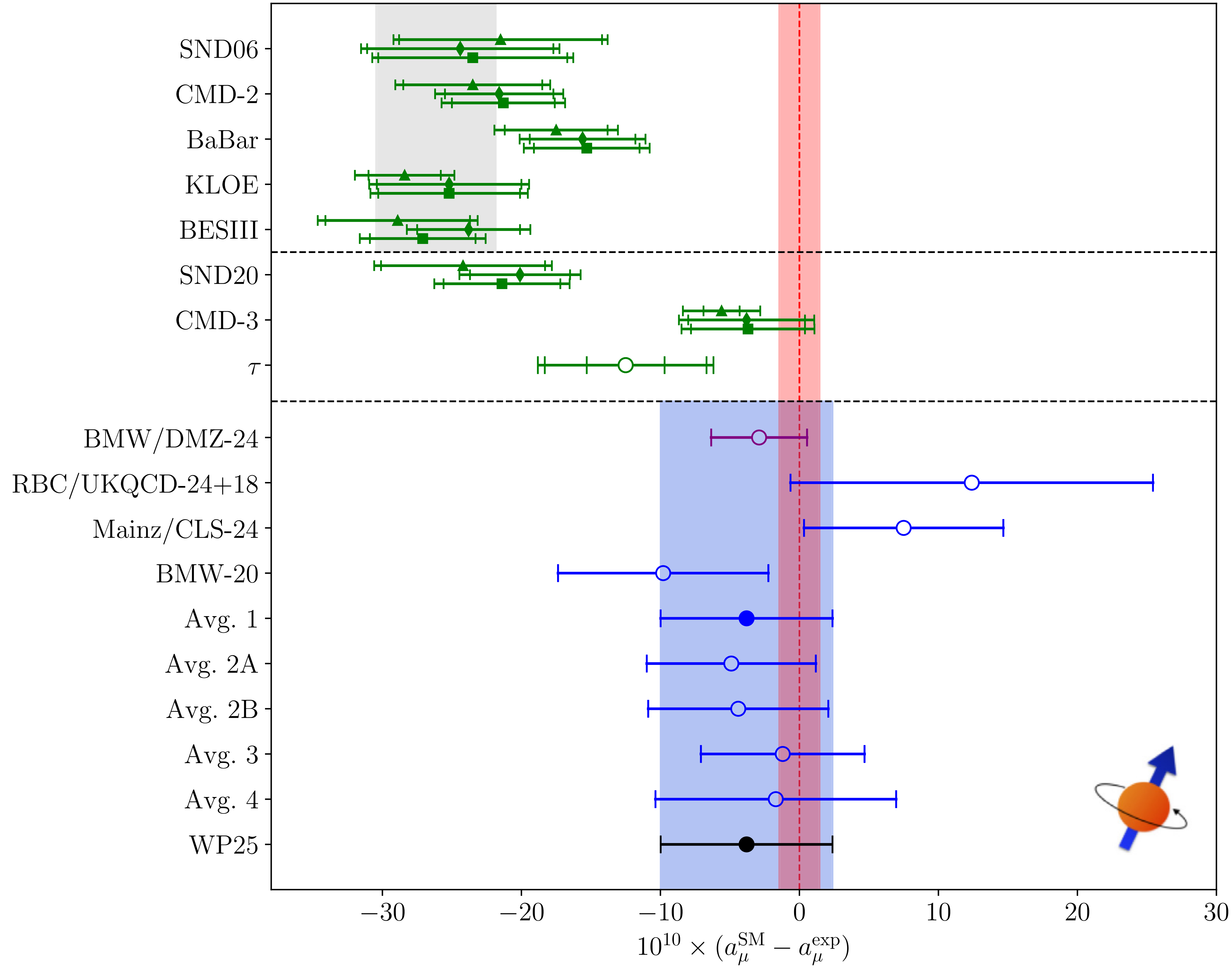


$$a_\mu^{\mathrm{HVP,\ LO}}\Big|_{\mathrm{Avg.\ 1}}^{\mathrm{lat}} = 713.2(6.1) \times 10^{-10} \qquad (3.31)$$

Large shift from WP 20 average, 693.1(4), based solely on data driven result

HVP: Data Driven and Lattice comparison

R. Aliberti, T. Aoyama, E. Balzani et al. — Physics Reports 1143 (2025) 1–158



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Hadronic Vacuum Polarization (HVP)

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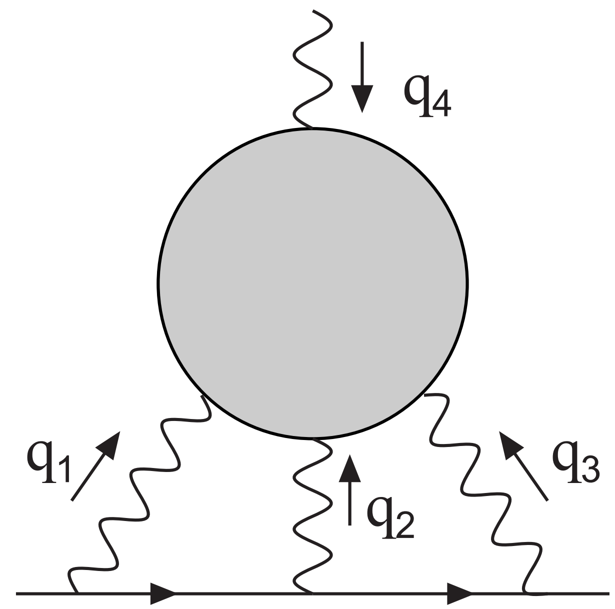
HLbL: Data Driven



- Data Driven dispersive analysis
- pQCD
- Models (hQCD, DSE, BSE,…)

HLbL: Data Driven

- WP20 a_μ^{HLbL}= 92(19) $\times 10^{-11}$
- WP25 a_μ^{HLbL}= 103.3(8.8) $\times 10^{-11}$
- Many improvements (theory and experimental inputs)
- WP25 prediction consistent with WP20 with significantly smaller uncertainty
- Future improvements:
  - Theory: better handling of singularity cancellations
  - Schwinger sum rule in analogy to the HVP contribution
  - More, improved experimental data

HLbL Lattice

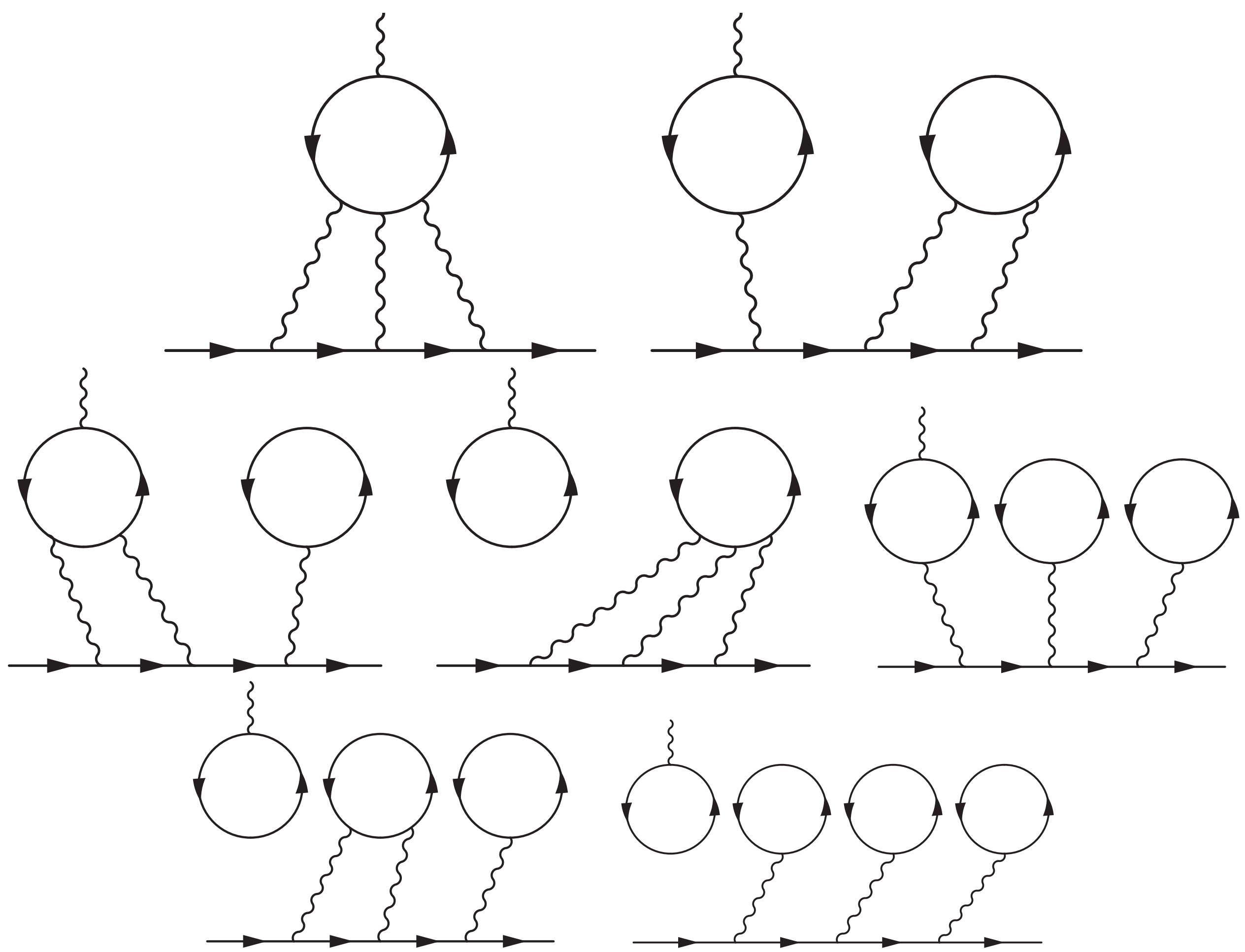


FIG. 2. Diagrams contributing to the muon anomaly.

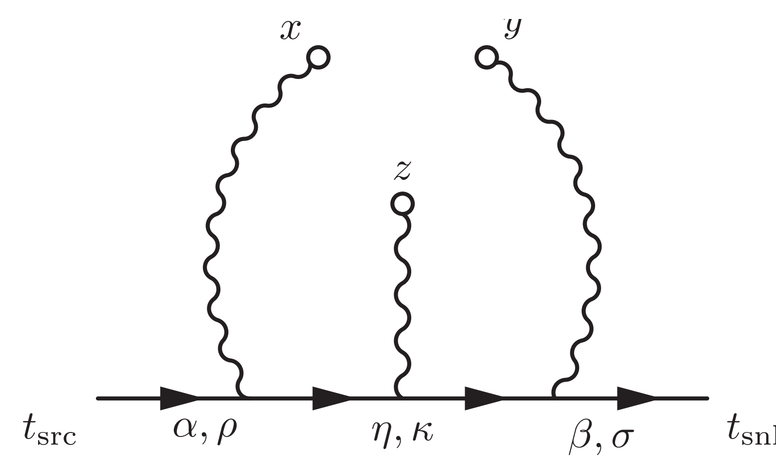


FIG. 3. Diagrammatic representation of the QED weighting function defined in Eq. (9), following Ref. [46].

[RBC/UKQCD (2023)]

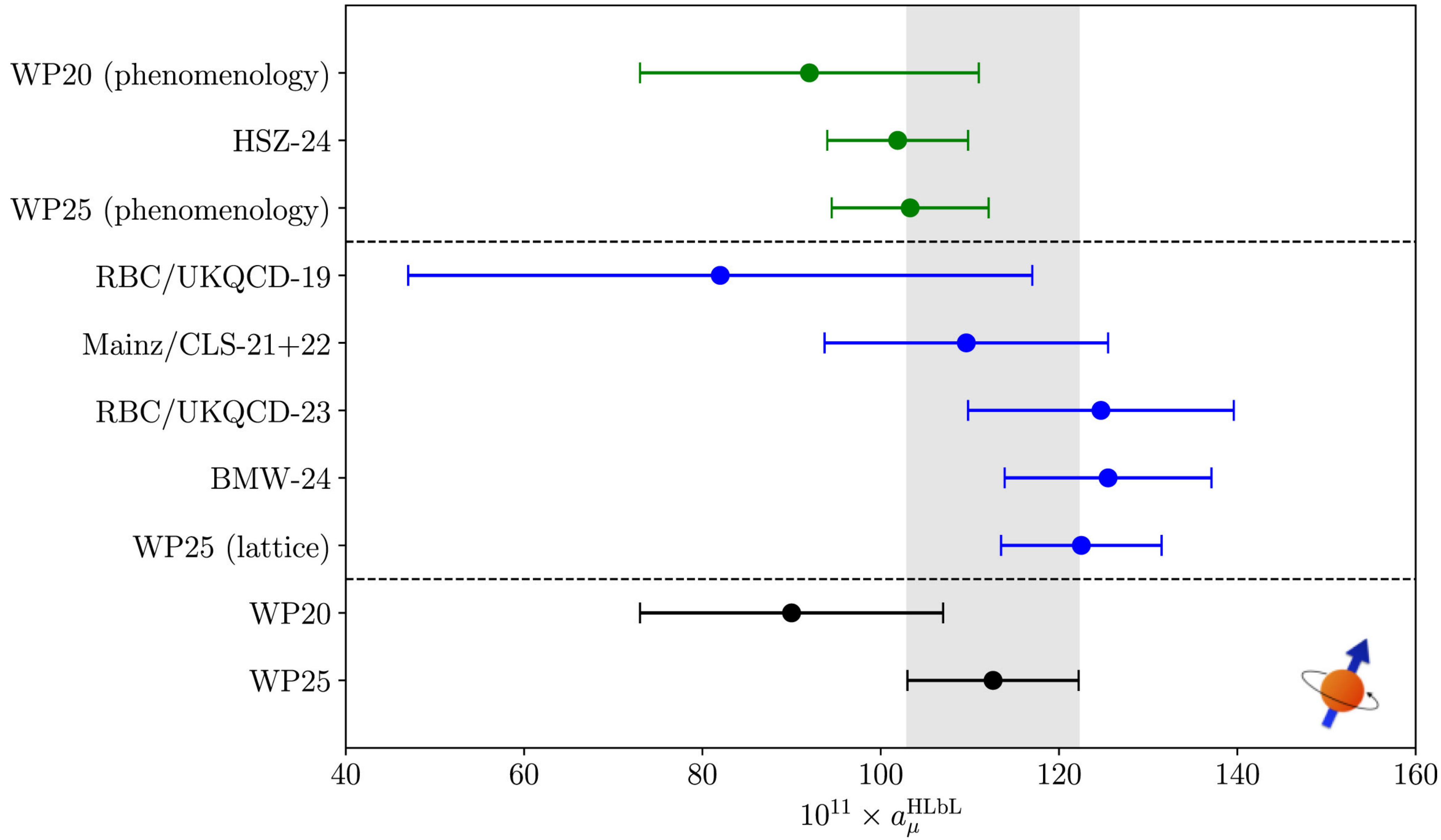
HLbL Lattice

- WP20: a_μ^{HLbL}= 82(35) $\times 10^{-11}$ from RBC/UKQCD only (QED$_L$)
- WP25: three results in average: Mainz, RBC/UKQCD, BMW (QED$_\infty$)
- WP25: a_μ^{HLbL}= 122.5(9.0) $\times 10^{-11}$
- Error reduced dramatically and central value shifted up (within about 1 standard deviation)

HLbL: Combined Average of Data Driven and Lattice

- WP20 prediction: $a_\mu^{\mathrm{HLbL}} = 90\ (17)\times 10^{-11}$
- WP25 prediction: $a_\mu^{\mathrm{HLbL}} = 112.6(9.6)\times 10^{-11}$ (1.4× error inflation factor)
- Central value increased (within WP20 error)
- Error is significantly smaller

HLbL: Combined Average of Data Driven and Lattice



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QED and Electroweak

5 std. dev. tension in α determinations increases error slightly for QED

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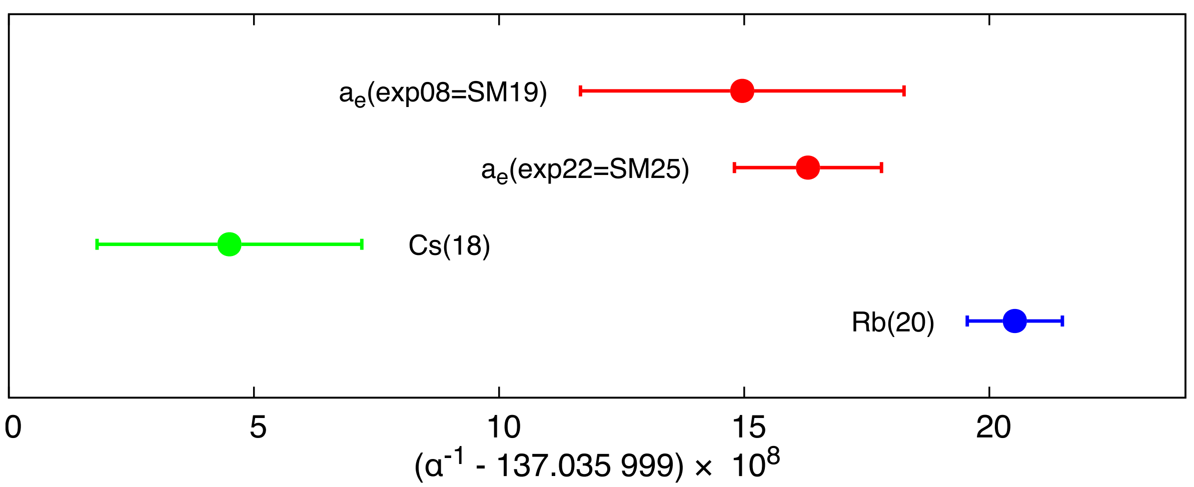

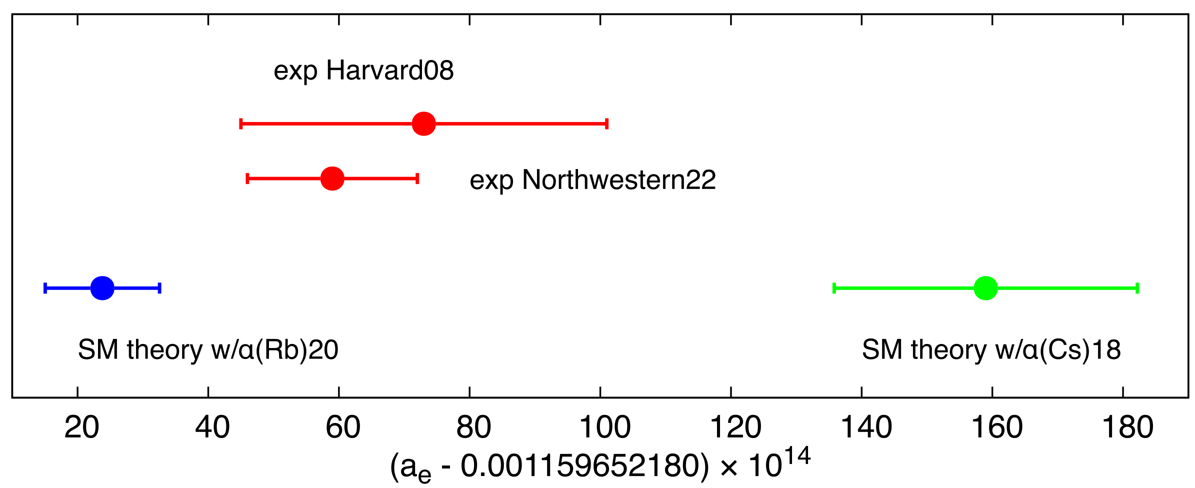

$$a_\mu^{\mathrm{QED}}\big[\alpha(\mathrm{Cs})\big] = 116\,584\,718.926(23)(7)(17)(6)(100)[104] \times 10^{-11}\,,$$
$$a_\mu^{\mathrm{QED}}\big[\alpha(a_e)\big] = 116\,584\,718.825(13)(7)(17)(6)(100)[103] \times 10^{-11}\,,$$
$$a_\mu^{\mathrm{QED}}\big[\alpha(\mathrm{Rb})\big] = 116\,584\,718.789(8)(7)(17)(6)(100)[102] \times 10^{-11}\,, \quad (7.26)$$

where the uncertainties from left to right arise from α, the τ-lepton mass, the QED eighth-order term, the QED tenth-order term, the estimated QED twelfth-order term, and the total combined uncertainties [1]. The difference between the largest and the smallest values of a_μ^{QED} is 0.137×10^{-11}, which is of the same order of magnitude as the estimated QED twelfth-order term. In view of Eq. (7.26), we use

$$a_\mu^{\mathrm{QED}} = 116\,584\,718.8(2) \times 10^{-11}\,, \quad (7.27)$$

QED and Electroweak

EW error significantly reduced due to improved 2-loop calculation with hadronic contributions

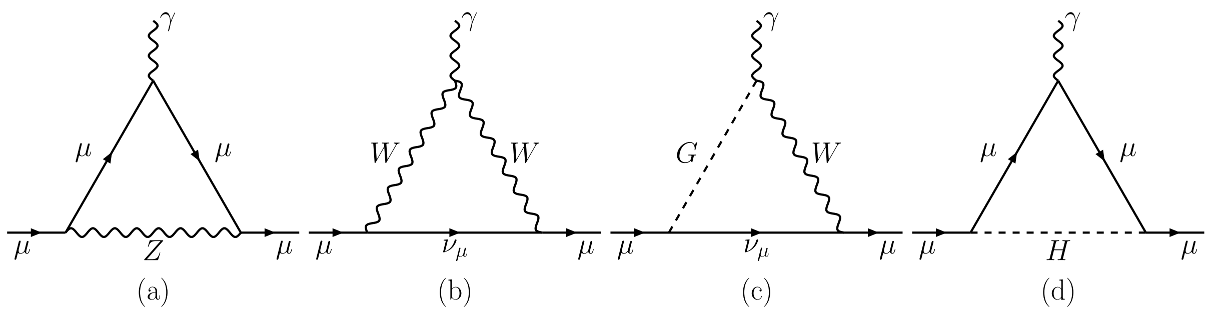

Fig. 79. One-loop Feynman diagrams contributing to a_μ^{EW}.
Source: Figures taken from Ref. [1].

$$a_\mu^{\mathrm{EW}(1)} = \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2}\left[\frac{5}{3} + \frac{1}{3}(1-4s_{\mathrm{W}}^2)^2\right] = 194.79(1) \times 10^{-11}$$

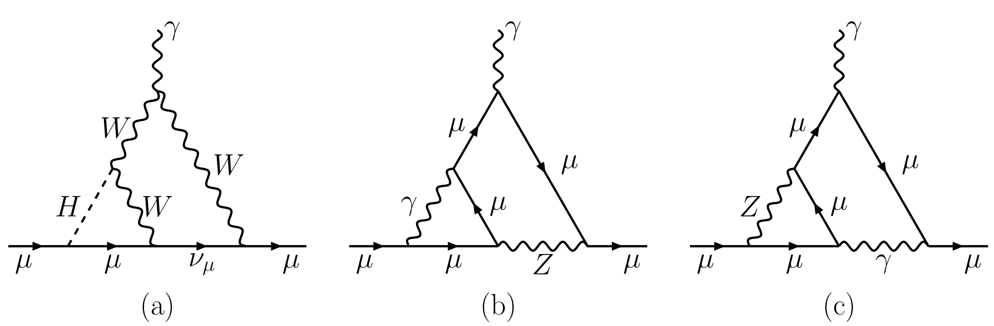

Fig. 80. Sample bosonic two-loop Feynman diagrams contributing to a_μ^{EW}.
Source: Figures taken from Ref. [1].

$$a_\mu^{\mathrm{EW}} = 154.4(4) \times 10^{-11}$$

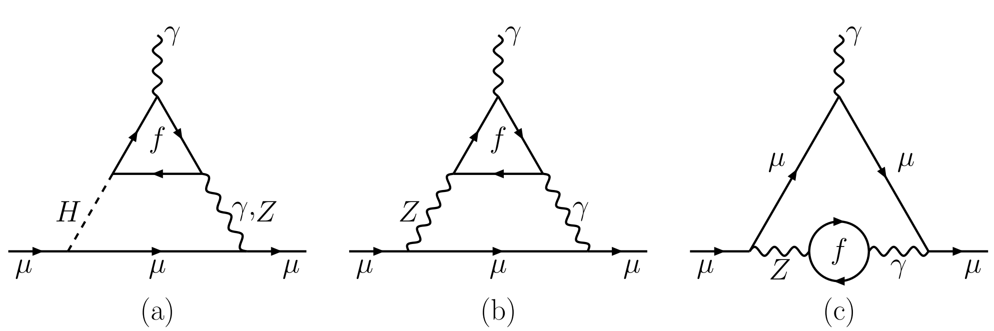

Fig. 81. Sample fermionic two-loop Feynman diagrams contributing to a_μ^{EW}.
Source: Figures taken from Ref. [1].

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Muon g-2 Theory Initiative White Paper 2025 Summary

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Table 33
Comparison of the key results from this work (WP25), as given in Table 1, to the corresponding numbers from WP20 [1] (in units of 10^{-11}). Note that the "HLbL (lattice)" result from WP20 has been adapted to include the charm-loop contribution. The entry "HVP (LO + NLO + NNLO)" derives from HVP LO (lattice) [WP25] and HVP LO (e^+e^-) [WP20], respectively. The asterisk indicates that the LO HVP value from WP20 was based on e^+e^- data only, while in Table 5 we also include the current status for τ-based evaluations.

Contribution	WP25	WP20
HVP LO (lattice)	7132(61)	7116(184)
HVP LO (e^+e^-, τ)	Table 5	6931(40)*
HVP NLO (e^+e^-)	−99.6(1.3)	−98.3(7)
HVP NNLO (e^+e^-)	12.4(1)	12.4(1)
HLbL (phenomenology)	103.3(8.8)	92(19)
HLbL NLO (phenomenology)	2.6(6)	2(1)
HLbL (lattice)	122.5(9.0)	82(35)
HLbL (phenomenology + lattice)	112.6(9.6)	90(17)
QED	116 584 718.8(2)	116 584 718.931(104)
EW	154.4(4)	153.6(1.0)
HVP (LO + NLO + NNLO)	7045(61)	6845(40)
HLbL (phenomenology + lattice + NLO)	115.5(9.9)	92(18)
Total SM Value	116 592 033(62)	116 591 810(43)

World average: $a_\mu(\text{exp}) = 1\,165\,920\,715(145) \times 10^{-12}$ (124 ppb)

Discrepancy between WP25 and experiment is less than 1 standard deviation

Summary

- Fermilab E989 hits its mark, new world average with 124 ppb error
- Muon g-2 Theory Initiative WP 2025: significant changes from WP 2020 for the Standard Model Theory value
- CMD 3 experiment high compared to previous (WP 2020) results, under intense scrutiny, no resolution yet, not in the 2025 average
- Lattice HVP total contribution now with sub-percent errors, enters total SM value as sole HVP determination, dominates SM error
- Continued effort to reduce theory error to level of experiment. Prospects are good.
- SM value is very compatible with experiment

Future Plans

- Theory goal: 1-2 ppm to match experiment, so permille precision for the HVP contribution
  - Lattice: LD window, IB, disconnected diagrams
  - Resolve discrepancies in data driven results
  - New data for the dispersive approach
- Will need sustained effort by lattice and dispersive communities. Theory initiative continues…

Acknowledgements

- TB partially supported by US Department of Energy (DOE)
- Aubin, *et al*., RBC/UKQCD computations done on
  - ALCF at Argonne (MIRA, Aurora), OLCF at Oakridge (Summit, Frontier), Booster (Julich), LUMI-g (Kajaani), Juwels (Julich)
  - Bridges2 at PSC, Expanse at SDSC, and Stampede2 and Frontera at TACC under XSEDE and ACCESS (US NSF)