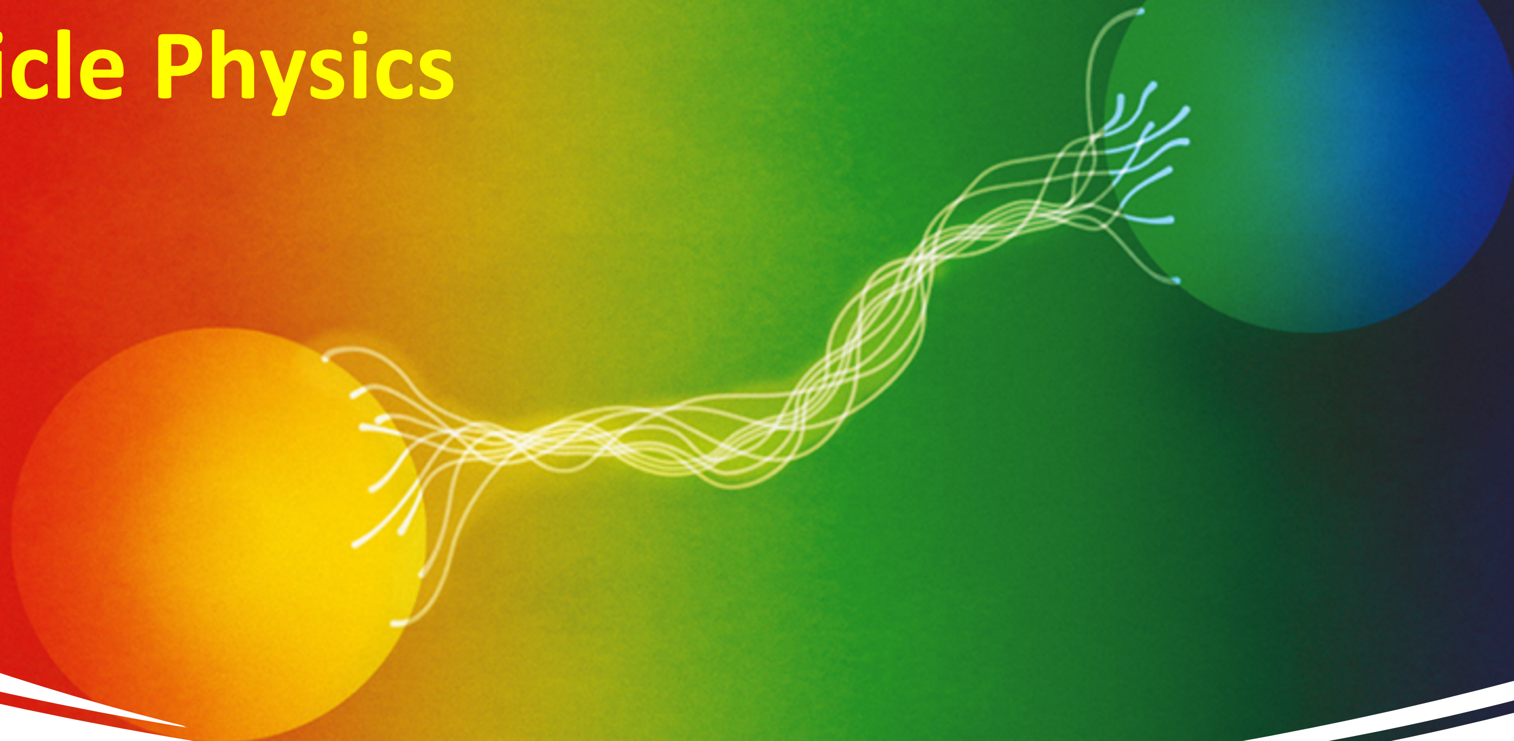
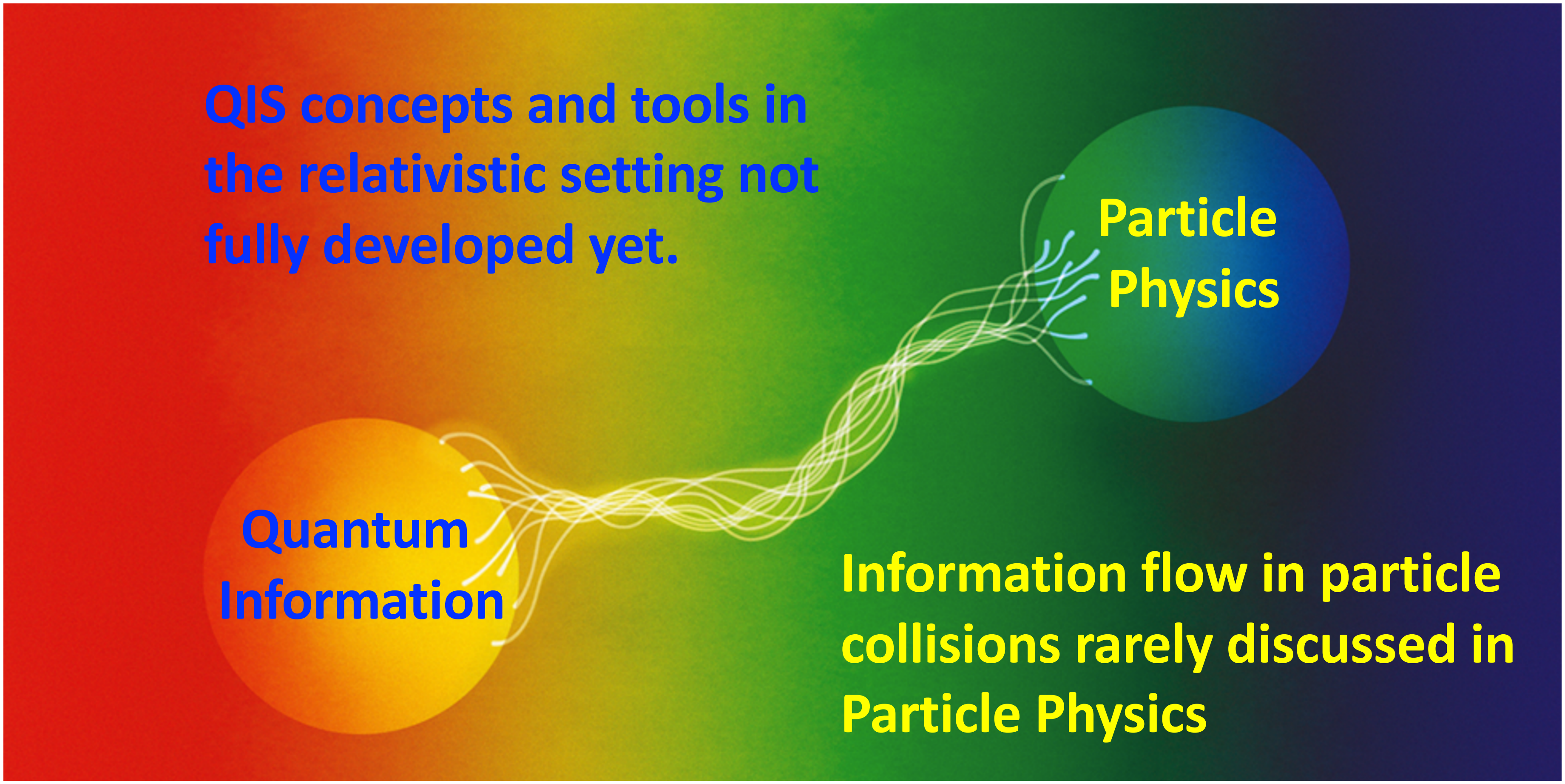
Quantum Information Meets Particle Physics



Ian Low
Argonne/Northwestern
October 22, 2025

Quantum Mechanics + Information Theory + Relativity



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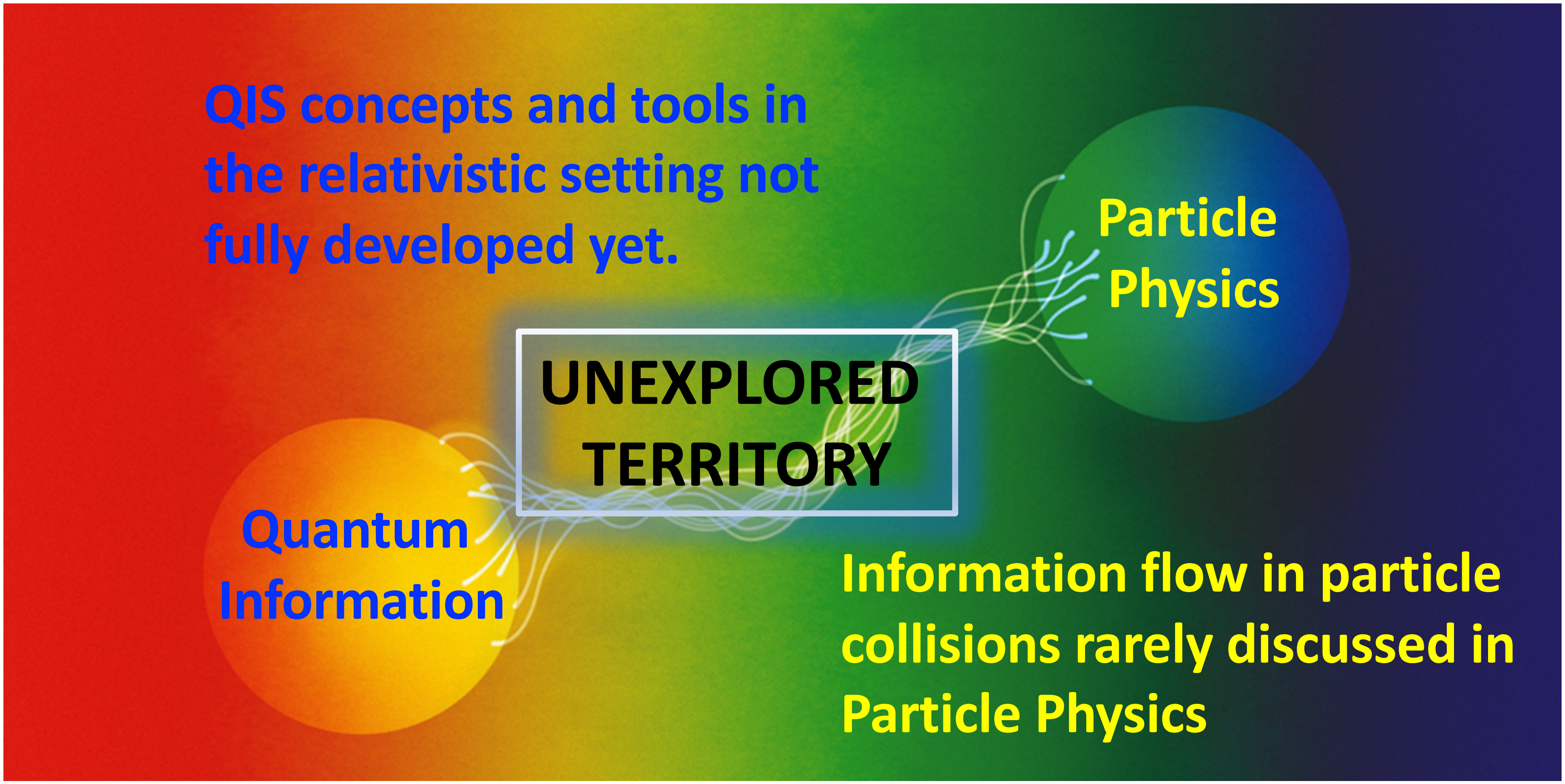

Entanglement is the most prominent feature of Quantum:

- It refers to the situation where a measurement on a subsystem will improve our knowledge on the rest of the system.

- A quantum state of a system is entangled if it cannot be written as a tensor-product state of its subsystems.

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Consider a system of two spin-1/2 particles.

- $|\uparrow\downarrow\rangle \equiv |\uparrow\rangle \otimes |\downarrow\rangle$ is an unentangled state:

  Measurement of one spin would not change the outcome of the other.

- $(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}$ is an entangled state:

  Measurement of the first spin would collapse the state into $|\uparrow\downarrow\rangle$ or $|\downarrow\uparrow\rangle$, which consequently determines the second spin.

John Wheeler famously claimed:

It from bit : "All things physical are information-theoretic in origin"

INFORMATION, PHYSICS, QUANTUM: THE SEARCH FOR LINKS

John Archibald Wheeler * †

Abstract

This report reviews what quantum physics and information theory have to tell us about the age-old question, How come existence? No escape is evident from four

winnowing: **It from bit**. Otherwise put, every **it** — every particle, every field of force, even the spacetime continuum itself — derives its function, its meaning, its very existence entirely — even if in some contexts indirectly — from the apparatus-elicited answers to yes or no questions, binary choices [52], **bits**.

Three topics at the intersection of Quantum Information and Particle Physics:

- An information-theoretic origin of symmetry
- Quantum computational advantage in fundamental forces
- An area law for entanglement entropy in particle scatterings

An Information-theoretic Origin of Symmetry

Symmetry is among the most fundamental principles in physics:

Chen-Ning Yang famously coined the phrase --

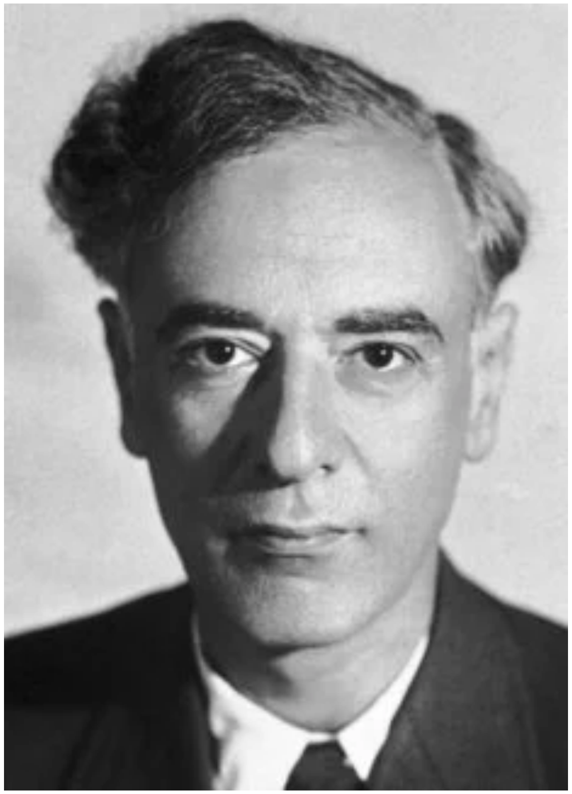
Symmetry dictates Interaction.

- Lorentz invariance → Special Relativity
- General coordinate invariance → General Relativity
- Gauge invariance → QCD and Electroweak theory.

In condensed matter physics, the Landau paradigm:

Phases of matter are represented by their symmetries and whether they are spontaneously broken or not.

- Gapless degrees of freedom → Goldstone modes
- Locus of critical points → Enhanced (emergent) symmetries
- Ginzburg-Landau theory gives a macroscopic description.



But what is the origin of symmetry?

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Can symmetry come from qubit?

Recent efforts to understand the origin of symmetry from the information-theoretic perspective have uncovered intriguing insights:

- Extremization (minimization or maximization) of entanglement entropy in particle interactions lead to enhanced symmetries.
- Examples encompass both non-relativistic (low-energy QCD) and fully relativistic (two-Higgs-doublet models) systems.
- The observation applies to qubits (spin-1/2) and qudits (spin-3/2).

Emergent symmetries in low-energy QCD (not transparent in the QCD Lagrangian):

- Schrodinger symmetry (non-relativistic conformal invariance):

$$
\begin{aligned}
&\text{boosts:} && \vec{x}\,' = \vec{x} + \vec{v}\,t \;,\quad t' = t \;, \\
&\text{scale:} && \vec{x}\,' = \vec{x} + s\vec{x} \;,\quad t' = t + 2st \;, \\
&\text{conformal:} && \vec{x}\,' = \vec{x} - ct\vec{x} \;,\quad t' = t - ct^2 \;,
\end{aligned}
$$

Hagen and Niederer, 1972

- Wigner's SU(4) Spin-flavor symmetries for protons and neutrons

$$
N = \begin{pmatrix} p_\uparrow \\ p_\downarrow \\ n_\uparrow \\ n_\downarrow \end{pmatrix} \qquad N \to \mathcal{U} N \;, \quad \mathcal{U} \in SU(4)
$$

E. P. Wigner (1934)

Let's consider non-relativistic, S-wave scattering of a neutron and a proton:

- Treat them as two qubits -- Alice (neutron) and Bob (proton)

- The S-matrix can be decomposed into ${}^1S_0$ and ${}^3S_1$ channels
  → there are two phase shifts: δ_0 and δ_1 , respectively.

- Rotational invariance and Unitarity then uniquely fix the S-matrix:

$$S = e^{2i\delta_0}\,\frac{(1-\boldsymbol{\sigma}\cdot\boldsymbol{\sigma})}{4} + e^{2i\delta_1}\,\frac{(3+\boldsymbol{\sigma}\cdot\boldsymbol{\sigma})}{4}$$

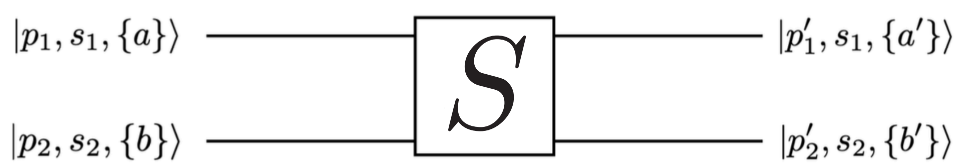
Spin-projector into ${}^1S_0$ channel

Spin-projector into ${}^3S_1$ channel

- In the scattering process the S-matrix acts on the IN-state:

$$|\text{out}\rangle = S\ |\text{in}\rangle$$

- For 2-to-2 scattering of spin-1/2 fermions, the S-matrix can be viewed as a two-qubit quantum logic gate acting on the spin-space:



- Can characterize the ability of the S-matrix to generate entanglement from unentangled initial states.

Many possibilities to quantify entanglement. For bipartite systems:

von Neumann entropy: $$E(\rho) = -\mathrm{Tr}(\rho_1 \ln \rho_1) = -\mathrm{Tr}(\rho_2 \ln \rho_2)$$

Linear entropy: $$E(\rho) = -\mathrm{Tr}(\rho_1(\rho_1 - 1)) = 1 - \mathrm{Tr}\rho_1^2$$

$$\rho = |\psi\rangle\langle\psi| \qquad \rho_{1/2} = \mathrm{Tr}_{2/1}(\rho)$$

The common property is that the entanglement measure vanishes for a product state $|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$, but attains the maximum for maximally entangled states (such as the Bell states.)

- Entanglement is a property of the quantum state. But we are more interested in the ability of a *quantum-mechanical operator* (i.e. the S-matrix) to entangle.

- However, there is a subtlety here, as the amount of entanglement generated by an operator could depend on the initial state.

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- The "entanglement power" deals with this issue is by averaging over the initial states:

$$E(U) = \overline{E(U\,|\psi_1\rangle \otimes |\psi_2\rangle)},$$

For quibts, the average is over the Bloch sphere.

It is a measure of the ability of an operator *U* to generate entanglement on product states.

- A minimally entangling operator has *E*(*U*) = 0, i.e.,

$$|\ \ \rangle \otimes |\ \ \rangle \xrightarrow{U} |\ \ \rangle \otimes |\ \ \rangle$$

It turns out there are two and only two minimally entangling operators, which in the computational basis, $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} , \qquad \mathrm{SWAP} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} .$$

Identity gate: do nothing.
SWAP gate: interchange the qubits.

$$\mathrm{SWAP} \sim -1 \quad \text{as} \quad [\mathrm{SWAP}]^2 = 1$$

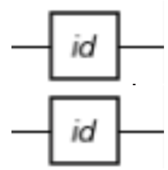
In terms of Pauli matrices,

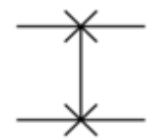
$$\mathrm{SWAP} = (1 + \boldsymbol{\sigma} \cdot \boldsymbol{\sigma})/2 \ , \qquad \boldsymbol{\sigma} \cdot \boldsymbol{\sigma} \equiv \sum_a \boldsymbol{\sigma}^a \otimes \boldsymbol{\sigma}^a \ .$$

Low, Mehen: 2104.10835

Re-write the S-matrix in terms of quantum logic gates,

$$S = \frac{1}{2}\left(e^{2i\delta_1} + e^{2i\delta_0}\right)\mathbf{1} + \frac{1}{2}\left(e^{2i\delta_1} - e^{2i\delta_0}\right)\ \text{SWAP},$$

$$S = \frac{1}{2}\left(e^{2i\delta_1} + e^{2i\delta_0}\right)$$ 

$$+ \ \frac{1}{2}\left(e^{2i\delta_1} - e^{2i\delta_0}\right)$$ 

Conditions for the S-matrix to minimize entanglement:

1. $S = 1$ if $\delta_0 = \delta_1$ ➡ SU(4) spin-flavor symmetry

2. S = SWAP if $|\delta_0 - \delta_1| = \pi/2$ ➡ Schrodinger symmtery

Low, Mehen: 2104.10835

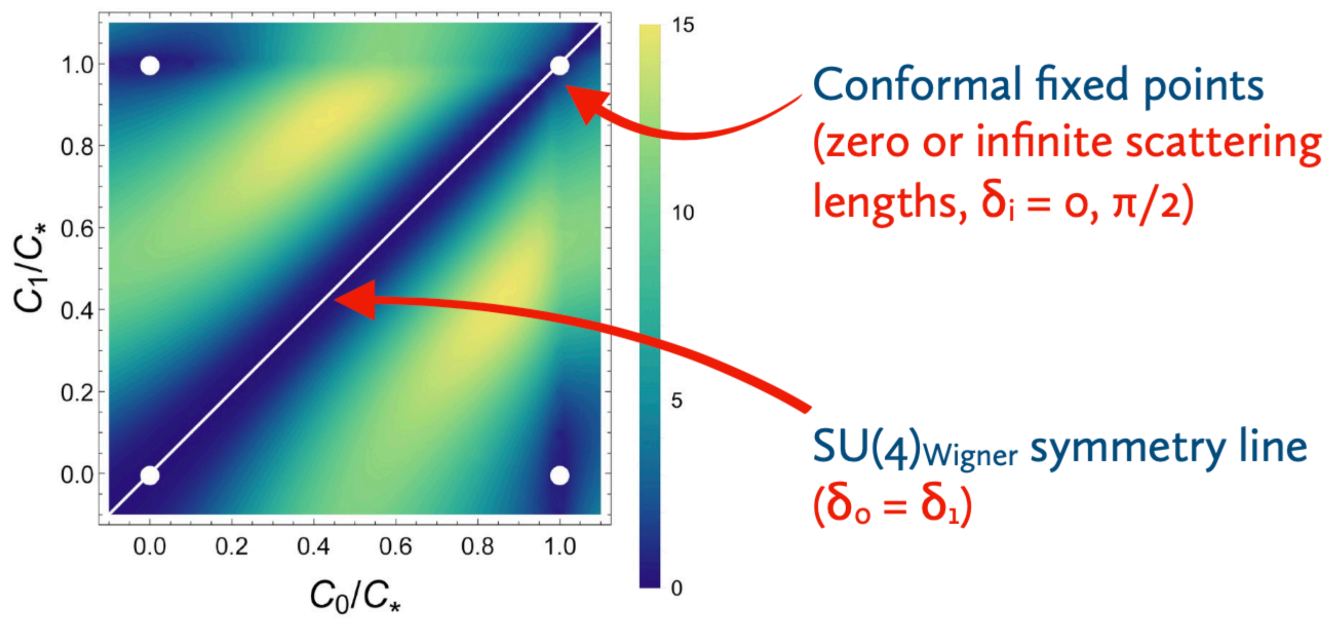
This observation was first made by the Seattle group in 1812.03138:

$$\mathcal{L}_6 = -\frac{1}{2}C_S(N^\dagger N)^2 - \frac{1}{2}C_T(N^\dagger \vec{\sigma} N)^2$$

$$^1S_0: \qquad \bar{C}_0 = (C_S - 3C_T)$$

$$^3S_1: \qquad \bar{C}_1 = (C_S + C_T)$$

$$\mathcal{E}(\hat{\mathbf{S}}) = \frac{1}{6}\ \sin^2\left(2(\delta_1 - \delta_0)\right)$$



Slide by D.B. Kaplan

We have observed similar correlations between entanglement minimization and the appearance of enhanced symmetries in several other systems:

- 2-to-2 scattering of spin-1/2 octet baryons. (Liu, Low, Mehen: 2210.12085)
- 2-to-2 scattering of spin 3/2 decuplet baryons. (Hu, Sone, Guo, Hyodo, Low: 2506.08960)
- Exotic mesons (four-quark bound states) in X(3872) and $T_{cc}(3875)^{+}$. (Hu, Chen, Guo: 2404.05958.)
- 2-to-2 scattering of Higgs bosons in two-Higgs-doublet models. (Carena, Low, Wagner and Xiao: 2307.08112)

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There is also an example of entanglement *maximization* and enhanced symmetries. (Carena, Coloreti, Liu, Littmann, Low, and Wagner: 2505.00873)

We are in pursuit of a new paradigm:

Can symmetry be the outgrowth of more fundamental principles?

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Can symmetry be the outgrowth of more fundamental principles?

- **Several "data points" are very suggestive, but we don't yet have a precise statement on what the "new principle" is.**

- **Can spontaneously broken symmetries be understood/defined in a similar fashion ?**

  → Can we classify phases of matter from the information-theoretic perspective?

Quantum Computational Advantage in Fundamental Forces

Liu, Low, Yin: 2502.17550;
2503.03098;
2509.18251

What separates "Quantum" from "Classical"?

- Much of the hype on Quantum Supremacy in quantum computing relies on identifying and characterizing quantum resources, such as the entanglement.

  For instance, Shor's algorithm utilizes entanglement.

- However, not all quantum resources provide computational advantages over classical algorithms (Gottesman-Knill theorem):

  Certain commonly employed quantum circuits, which include maximally entangled states, can be simulated efficiently using classical algorithms.

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  Certain commonly employed quantum circuits, which include maximally entangled states, can be simulated efficiently using classical algorithms.

- A second layer of “quantumness” is needed to for quantum speedup – the magic (non-stablizerness).

- Magic is an essential ingredient for universal quantum computation. (Bravyi and Kitaev: quant-ph/0403025)

What is the Question?

- Basic forces in nature are known to generate entanglement easily and abundantly.

- What about computational advantages? How well can fundamental interactions generate quantum advantages?

- Is the quantum advantage built into the fundamental interactions in the UV or is it an emergent phenomenon in the IR?

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As a starting point, we consider the ability of QED to generate magic states in 2-to-2 scatterings of electrons and muons, starting from an initial state with zero magic.

- A quantum circuit is a series of unitary “gate operations” on the states. Examples of important “single-qubit” gates are

$$X = \sigma_x = \mathrm{NOT} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

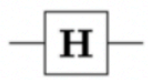
$$Y = \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix},$$

$$Z = \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

$$I^2 = X^2 = Y^2 = Z^2 = -iXYZ = I$$

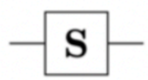
$$ZX = iY = -XZ.$$

Hadamard (H)



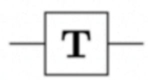
$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Phase (S, P)



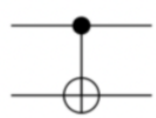
$$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

$\pi/8$ (T)



$$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$

Controlled Not (CNOT, CX)



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The Stabilizer Formalism:

- The Pauli group G_n for n-qubit:

$$G_n = \{\phi\, P_1 \otimes P_2 \otimes \cdots \otimes P_n \mid P_i \in \{I, X, Y, Z\} \text{ and } \phi \in \{\pm 1, \pm i\}\ \}$$

$$I = \sigma^0,\ X = \sigma^1,\ Y = \sigma^2 \text{ and } Z = \sigma^3$$

- A ”Stabilizer” state is an eigenstate of some elements of G_n :

$$g|\psi\rangle = |\psi\rangle\ , \qquad g \in G_n$$

Such *g's* form an abelian subgroup called the “Stabilizer Group.”

The maximal stabilizer group *S* of each stabilizer state is unique!

- For n-qubit, the maximal stabilizer group *S* has 2^n elements but only *n* generators, whose products generate *S*.

- A unitary operation *U* on a stabilizer state is another stabilizer state:

$$U|\psi\rangle = Ug|\psi\rangle = UgU^\dagger U|\psi\rangle$$

whose stabilizer group is $USU^\dagger$.

- Instead of specifying 2^n amplitudes of $U|\psi\rangle$, one can simply specify the *n* generators of $USU^\dagger$.

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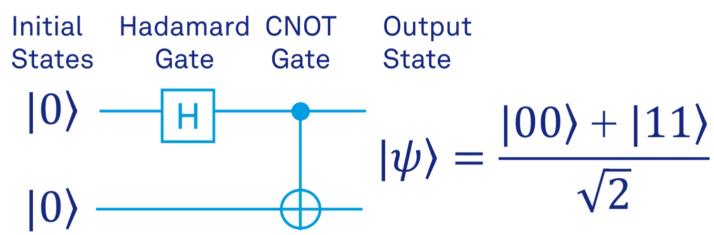
- Instead of specifying 2^n amplitudes of $U|\psi\rangle$, one can simply specify the *n* generators of $USU^\dagger$.

This is the essence of Gottesman-Knill theorem and why the stabilizer states can be simulated efficiently using classical algorithms!

- The stabilizer formalism is particularly powerful when applying to the "Clifford gates":

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \qquad \mathrm{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

- Clifford gates and stabilizer states are heavily utilized in quantum computing, because they generate highly entangled the Bell states:



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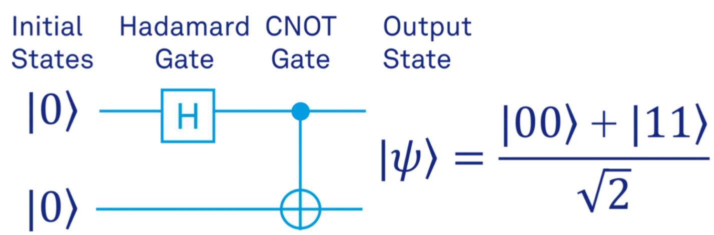


Gottesman-Knill theorem:

Quantum circuits involving Clifford gates and stabilizer states can be simulated efficiently using classical computers.

- However, Clifford gates and stabilizer states are NOT universal – they are not able to approximate all unitary transformations.

Clifford gate + magic states are universal

- However, Clifford gates and stabilizer states are NOT universal – they are not able to approximate all unitary transformations.

Clifford gate + magic states are universal

- Stabilizer states by definition have zero magic. For 2-q system, there are 60 stabilizer states:

$$|\psi\rangle = c_1|\uparrow\uparrow\rangle + c_2|\uparrow\downarrow\rangle + c_3|\downarrow\uparrow\rangle + c_4|\downarrow\downarrow\rangle.$$

Order #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
c_1	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
c_2	0	0	1	0	1	-1	-1	1	i	$-i$	$-i$	i	i	-1	$-i$	1	$-i$	-1	i	1
c_3	0	0	0	1	1	-1	1	-1	i	$-i$	i	$-i$	1	i	-1	$-i$	1	$-i$	-1	i
c_4	0	1	0	0	1	1	-1	-1	-1	-1	1	1	i	$-i$	i	$-i$	$-i$	i	$-i$	i
Order #	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
c_1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	1	1	1
c_2	$-i$	1	0	0	i	1	0	0	1	1	0	0	-1	1	0	0	0	0	0	0
c_3	0	0	1	i	0	0	1	$-i$	0	0	1	-1	0	0	1	1	0	0	0	0
c_4	0	$-i$	i	0	0	i	$-i$	0	0	1	-1	0	0	-1	1	0	1	-1	i	$-i$
Order #	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
c_1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
c_2	1	1	1	1	-1	-1	1	1	$-i$	i	-1	$-i$	-1	i	$-i$	i	1	1	$-i$	i
c_3	1	-1	i	$-i$	-1	1	-1	1	$-i$	i	$-i$	-1	i	-1	1	1	$-i$	i	i	$-i$
c_4	0	0	0	0	-1	1	1	-1	1	1	$-i$	$-i$	i	i	i	$-i$	i	$-i$	-1	-1

- Among them 24 states are maximally entangled!
- Entanglement does not imply computational advantage!

- There are several quantitative measures of non-stabilizerness – the magic – and we will adopt the 2nd order Stabilizer Renyi Entropy (SRE):

$$\mathcal{P}_n = \{P_1 \otimes P_2 \otimes \cdots \otimes P_n\}, \quad P_i \in \{I, X, Y, Z\}$$

$$M_2(|\psi\rangle) = -\log \Xi_2(|\psi\rangle) \, , \qquad \Xi_2(|\psi\rangle) \equiv \sum_{P \in \mathcal{P}_n} \frac{\langle\psi|P|\psi\rangle^4}{4}$$

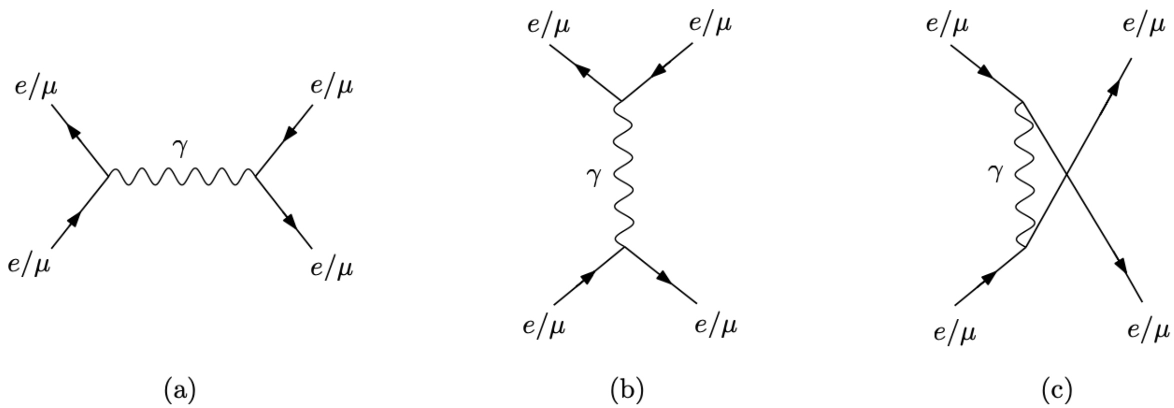
- The SRE is invariant under Clifford gates.

- For a stabilizer state, SRE vanishes.

- For 2-q states, the maximal SRE is

$$M_2 \leq \log \frac{16}{7} \approx 0.827$$

- Our goal is to start from a stabilizer state and compute the final state SRE for QED processes:



- We consider the following scattering processes, in both the non-relativistic and ultra-relativistic limits:

$$e^- e^+ \to \mu^- \mu^+$$

$$e^- \mu^- \to e^- \mu^-$$

$$\mu^- \mu^+ \to e^- e^+$$

Møller scattering $e^- e^- \to e^- e^-$

Bhabha scattering $e^- e^+ \to e^- e^+$

- We include all 60 stabilizer states as the initial states and compute the final state magic as a function of the scattering angle θ.

Low Energy Limit: $e^- e^+ \to \mu^- \mu^+$

- Near the kinematic threshold $\sqrt{s} \geq 2m_\mu$ the amplitude only depends on

$$\lambda = \frac{m_e}{m_\mu} \,, \qquad \lambda \approx 0.005$$ in real world

- We compute the magic as a function of λ :

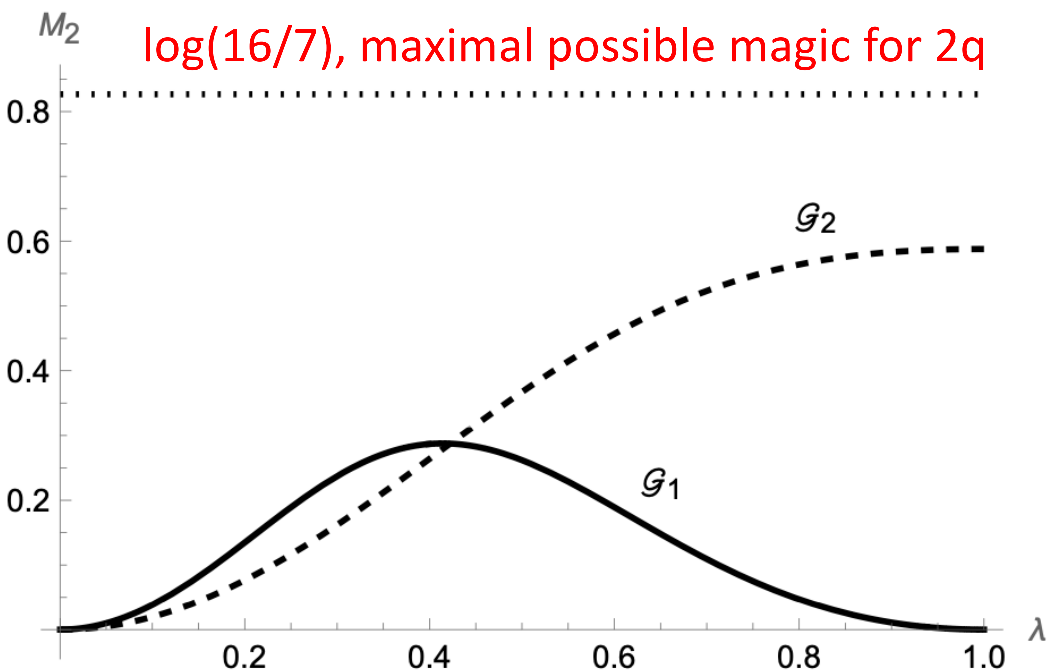
Stabilizer States	Ξ_2
$1, 2, 3, 4, 5, 6, 9, 10, 37, 38, 39, 40, 42, 43, 44, 45, 48, 49, 50$	$\mathcal{F}_1$
$7, 8, 11, 12, 46, 47, 59, 60$	$\mathcal{G}_1$
$13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28,$ $29, 30, 31, 32, 33, 34, 35, 36, 51, 52, 53, 54, 55, 56, 57, 58$	$\mathcal{G}_2$
41	—

$$\mathcal{F}_1 = 1, \qquad \mathcal{G}_1 = \frac{\lambda^8 + 14\lambda^4 + 1}{(\lambda^2+1)^4}, \qquad \mathcal{G}_2 = \frac{\lambda^8 + 28\lambda^4 + 16}{(\lambda^2+2)^4}$$

$$M_2 = \begin{cases} -\log \mathcal{F}_1 = 0 \\ -\log \mathcal{G}_1 \sim 10^{-5} \\ -\log \mathcal{G}_2 \sim 10^{-5} \end{cases}$$

- Using the real world value, the magic produced is practically zero.
- Among the 60 stabilizer initial states, only three different magic is produced.

- We can plot the magic production as a function of λ:

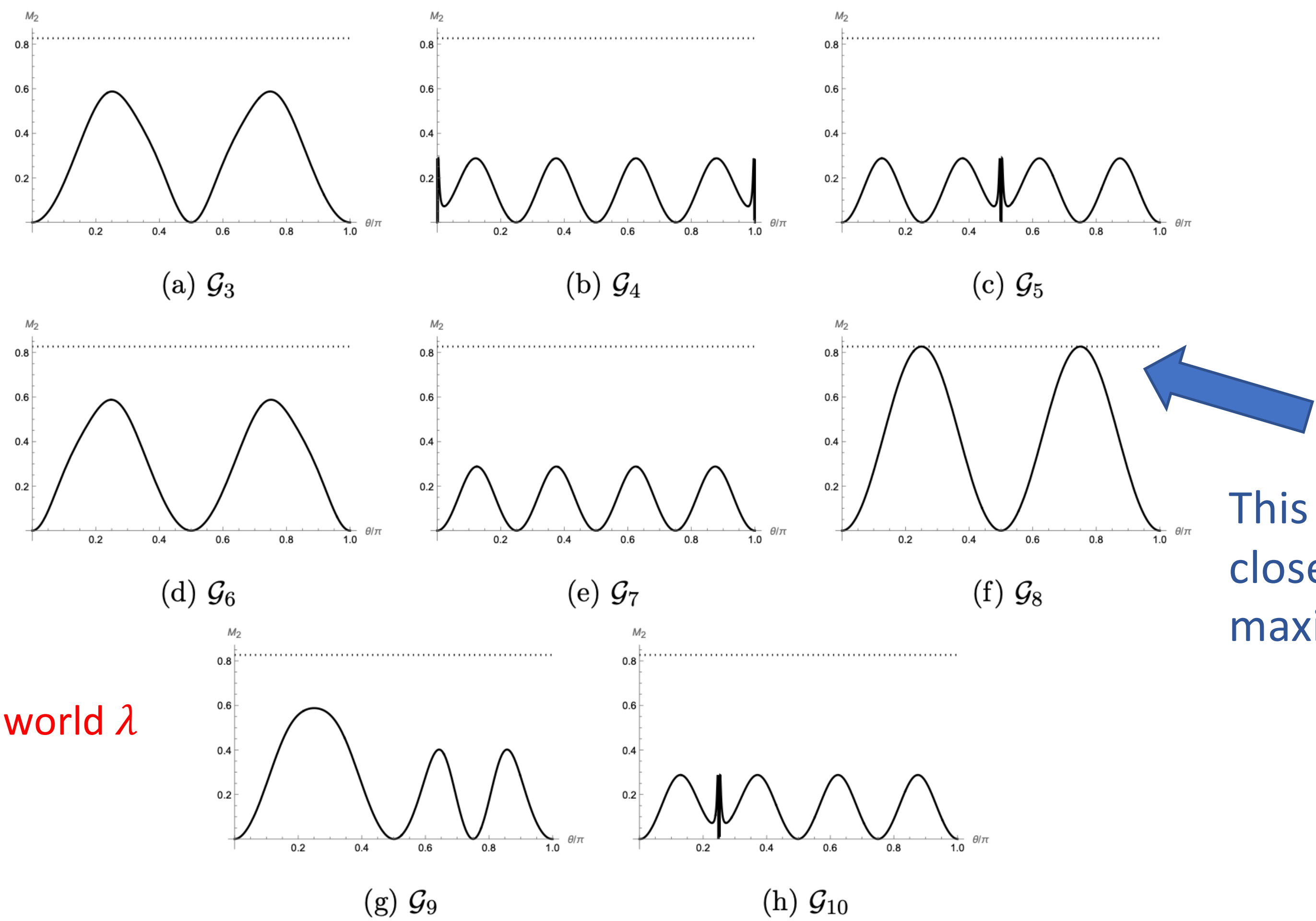


Ξ_2	$(M_2)_{\max}$	$\lambda_{\max}$
$\mathcal{F}_1$	—	—
$\mathcal{G}_1$	$\log(4/3)$	$\sqrt{2}-1$
$\mathcal{G}_2$	$\log(9/5)$	1

These numbers appear repeatedly!

- Even if we allow λ to vary, the largest magic produced is significantly less than the maximum value.
- **These observations persist in most other channels:**

- The most interesting channel is $\mu^- \mu^+ \rightarrow e^- e^+$, which has a much richer structure and the magic production is governed by 8 different patterns:



(a) $\mathcal{G}_3$ (b) $\mathcal{G}_4$ (c) $\mathcal{G}_5$ (d) $\mathcal{G}_6$ (e) $\mathcal{G}_7$ (f) $\mathcal{G}_8$ (g) $\mathcal{G}_9$ (h) $\mathcal{G}_{10}$

This is very close to the maximal magic!

Plotted for real world λ

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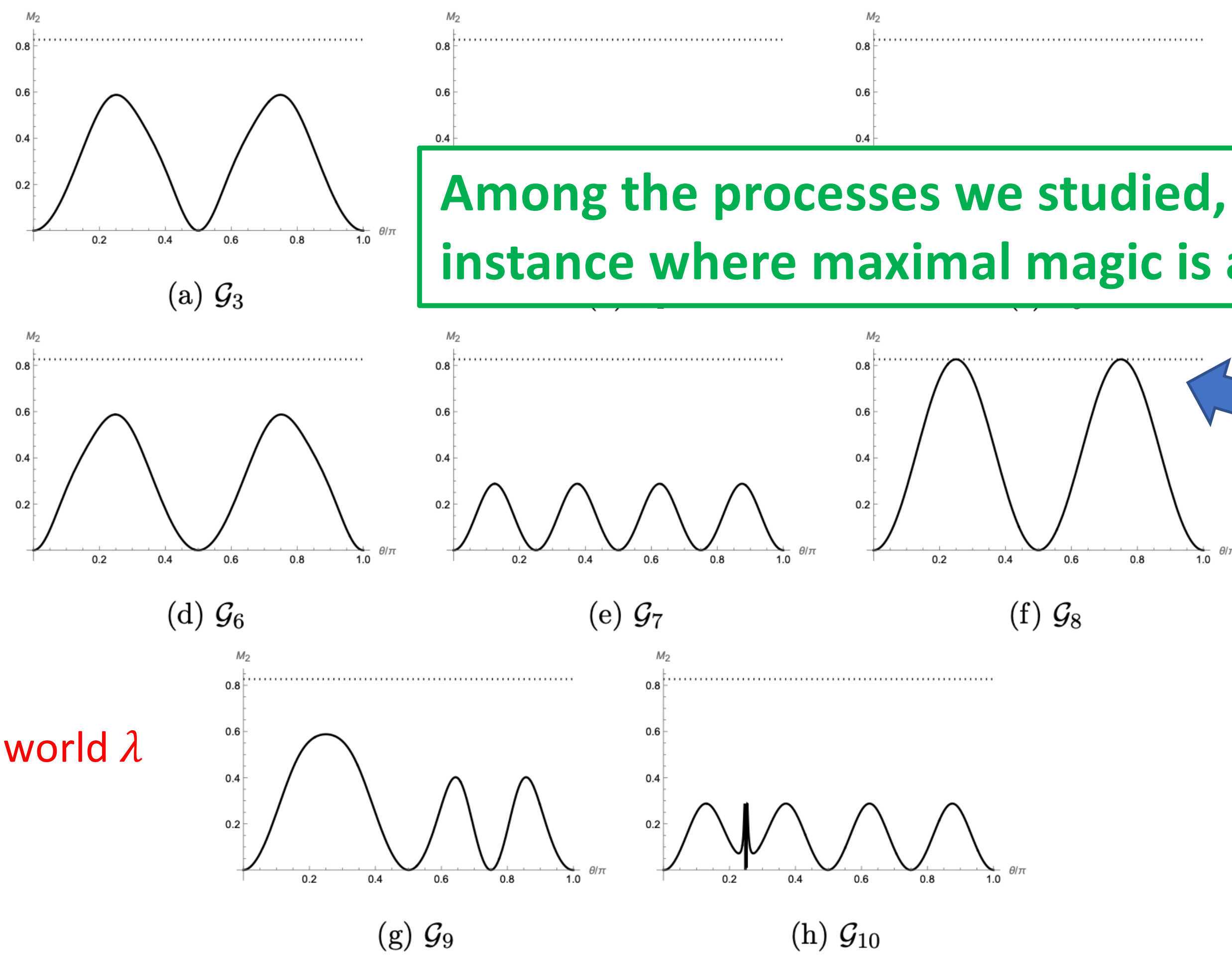


(a) $\mathcal{G}_3$ (d) $\mathcal{G}_6$ (e) $\mathcal{G}_7$ (f) $\mathcal{G}_8$ (g) $\mathcal{G}_9$ (h) $\mathcal{G}_{10}$

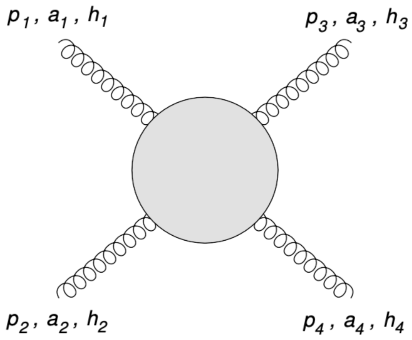
Among the processes we studied, this is the only instance where maximal magic is achieved.

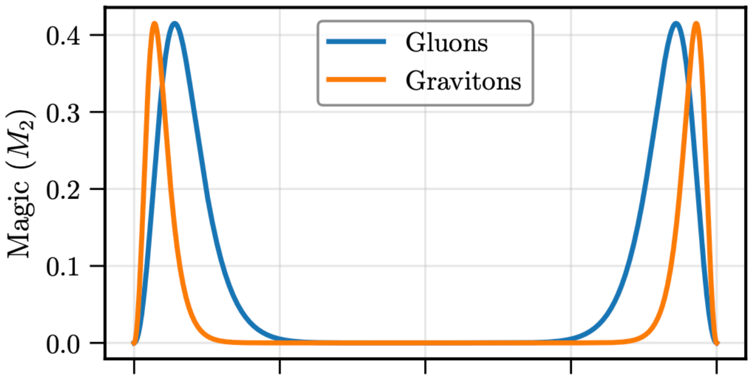
This is very close to the maximal magic!

Plotted for real world λ

- Although capable of producing maximally entangled states abundantly, QED doesn't seem to produce a lot of quantum advantage in terms of magic production.

  The observation seems to extend to other fundamental forces:





Gargalionis et al: 2508.14967

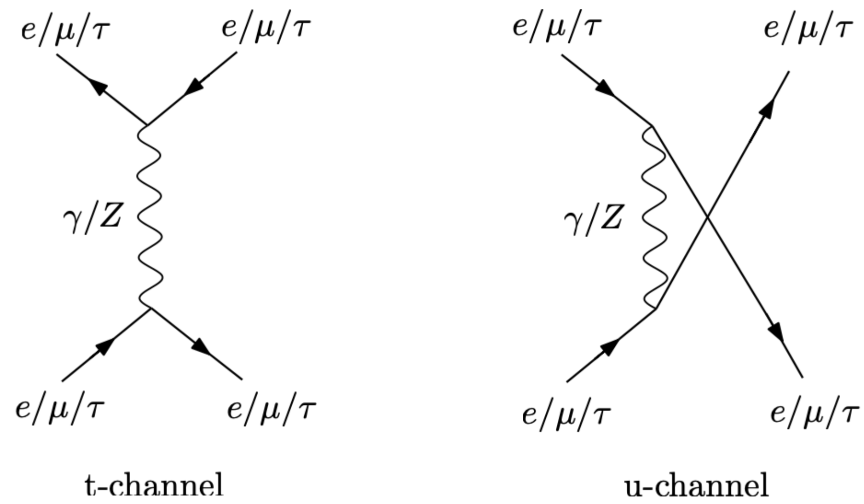
Is Quantum Advantage an emergent property??

- Magic production of all 60 stabilizer states is governed only be a few patterns. Some numbers for the largest magic keep popping up. **Why??**

It is natural to ask what happens when we go beyond QED.

We considered the charged lepton scattering in the SM:

$$\ell^- \ell^- \to \ell^- \ell^-$$



Why this channel:

- No s-channel diagram – free of kinematic thresholds.
- Tree-level magic production only depends on the weak mixing angle.

$$\mathcal{L}_{Z\bar{f}f} = -\frac{e}{2 s_W c_W}\, \bar{f}\gamma^\mu (g_V^\ell - g_A^\ell \gamma^5) f\, Z_\mu \ ,$$

$$g_V^\ell = T_3^f - 2Q_f\, s_W^2 \ , \qquad g_A^\ell = T_3^f \ .$$

- Let's look at the angular distribution first:

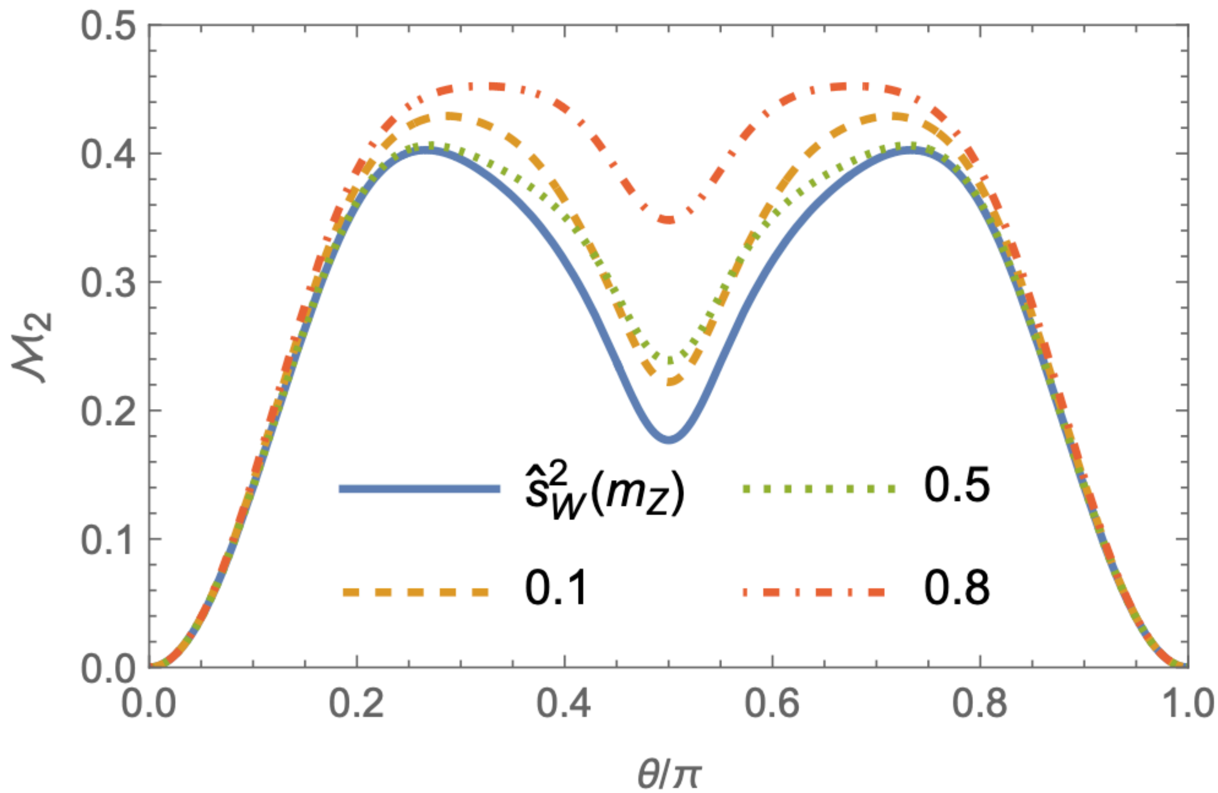
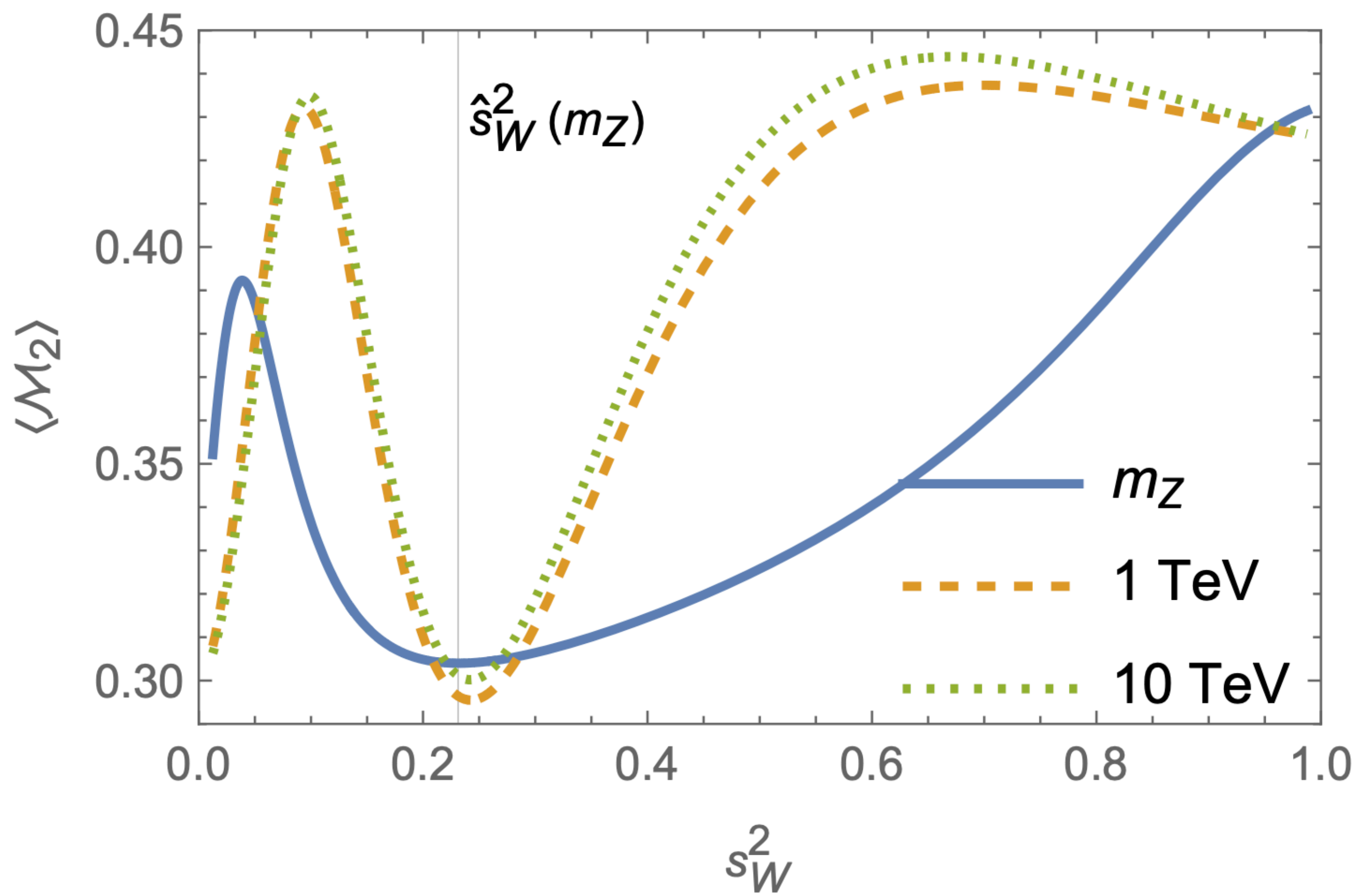


FIG. 2: Angular distribution of magic production $\mathcal{M}_2(\theta)$ for Møller scattering $e^-e^- \to e^-e^-$ at $\sqrt{s} = m_Z$ with $s_W^2 = \widehat{s}_W^2(m_Z), 0.1, 0.5,$ and 0.8.

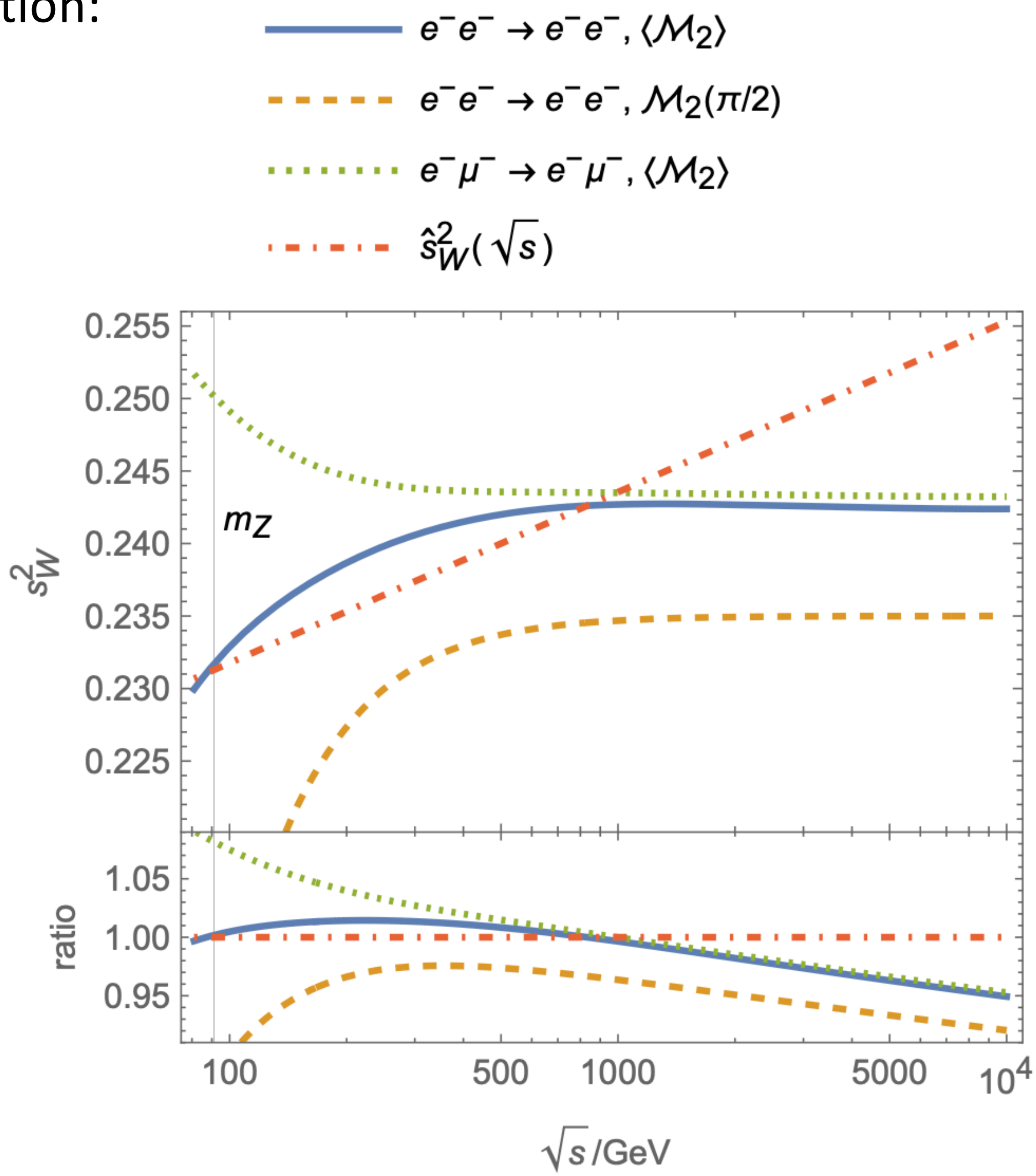
- Average (over solid angle) of magic production as a function of weak mixing angle:

$$\widehat{s}^2_W(m_Z) = 0.23129 \pm 0.00004$$

$$\mathbf{s}^2_W(m_Z) = 0.2317$$



- The Weinberg angle sits at a value which minimizes magic production:



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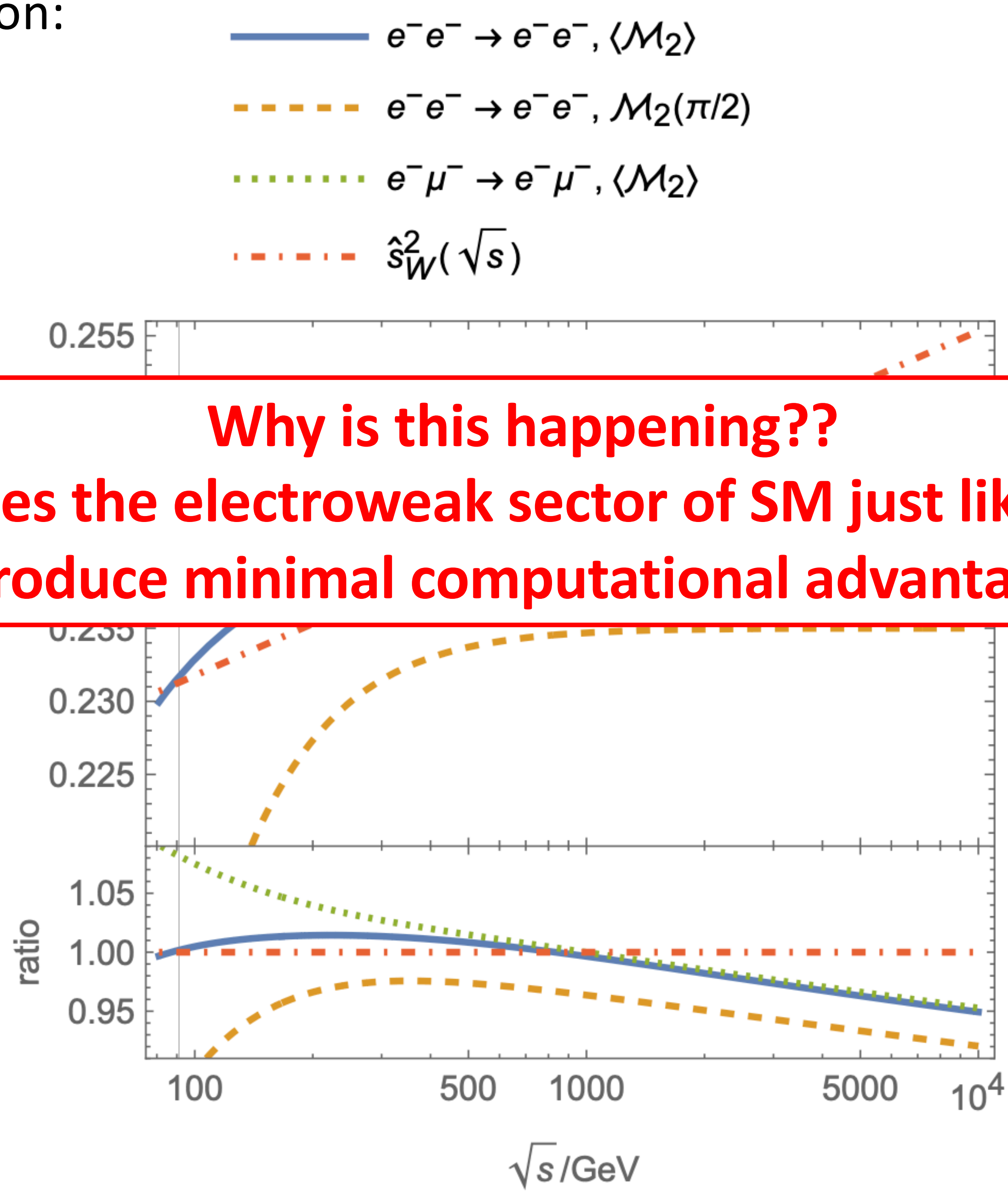


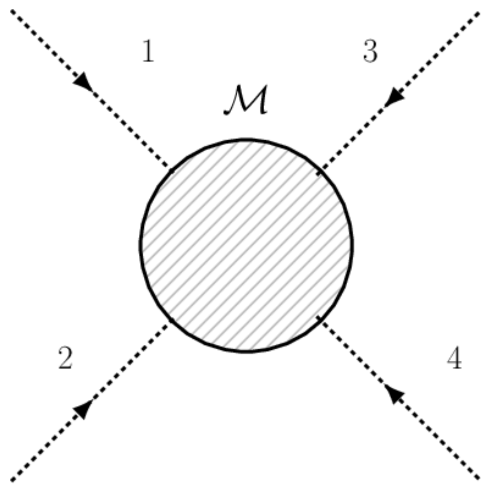
Why is this happening??
Does the electroweak sector of SM just like to produce minimal computational advantage?

An Area Law for Entanglement Entropy in Particle Scatterings

Low and Yin: 2405.08056 and 2410.22414

We are interested in the quantum correlations in 2-to-2 scattering of distinguishable particles in the S-matrix formalism:

A B → A B



$$|\text{out}\rangle \equiv S|\text{in}\rangle \qquad S = 1 + iT$$

$$\langle \{k_\text{f}\}, f_\text{f}|T|\{k_\text{i}\}, f_\text{i}\rangle$$
$$= (2\pi)^4 \delta^4 \left(\sum k_\text{f} - \sum k_\text{i}\right) M_{f_\text{i}, f_\text{f}}(\{k_\text{i}\}; \{k_\text{f}\})$$

We construct the bipartite system as

$$\mathcal{H}_\text{AB} = \mathcal{H}_\text{A} \otimes \mathcal{H}_\text{B}$$

$$\mathcal{H}_{A/B} = \mathcal{H}_\text{kinematic} \otimes \mathcal{H}_\text{flavor}$$

Kinematic = momentum and mass
Flavor = everything non-kinematic (could be spin!)

For now, we assume

- A pure initial state
- No entanglement between the incoming momenta
- No entanglement between momentum and flavor quantum numbers
- Allow possible entanglement among flavors

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In QFT textbooks it is customary to employ momentum eigenstates for the incoming particles.

$$\langle p|q\rangle = (2\pi)^3\, 2E_p\, \delta^3(\vec{p}-\vec{q})$$

But then $\rho = |p\rangle\langle p| \qquad \mathrm{Tr}\ \rho = \langle p|p\rangle \ \propto\ \delta^3(0)$

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One possibility is to introduce finite-volume regularization:

$$\delta^3(0) = \int d^3x \ \longrightarrow \ V$$

We will instead introduce wave packets, which is really how we do the experiment!

$$|\mathrm{in}\rangle = \sum_{i,\bar{i}} \Omega_{i\bar{i}} |\psi_{\mathrm{A}}\rangle \otimes |i\rangle \otimes |\psi_{\mathrm{B}}\rangle \otimes |\bar{i}\rangle$$

$$|\psi_{\mathrm{A/B}}\rangle = \int_p \psi_{\mathrm{A/B}}(p)\,|p\rangle, \qquad \int_p \equiv \int \frac{d^3\vec{p}}{(2\pi)^3\sqrt{2E_p}}$$

$$\langle\psi|\psi\rangle = \int \frac{d^3\vec{p}}{(2\pi)^3}\ |\psi(p)|^2 = 1$$

The initial density matrix is now properly normalized:

$$\rho^{\mathrm{i}} = |\mathrm{in}\rangle\langle\mathrm{in}|$$

$$\mathrm{tr}\rho^{\mathrm{i}} = \langle\mathrm{in}|\mathrm{in}\rangle = \langle\psi_{\mathrm{A}}|\psi_{\mathrm{A}}\rangle\langle\psi_{\mathrm{B}}|\psi_{\mathrm{B}}\rangle = 1$$

We will need an explicit form of wave packet to carry out the calculation.

Finite plane wave limit for wave packets:

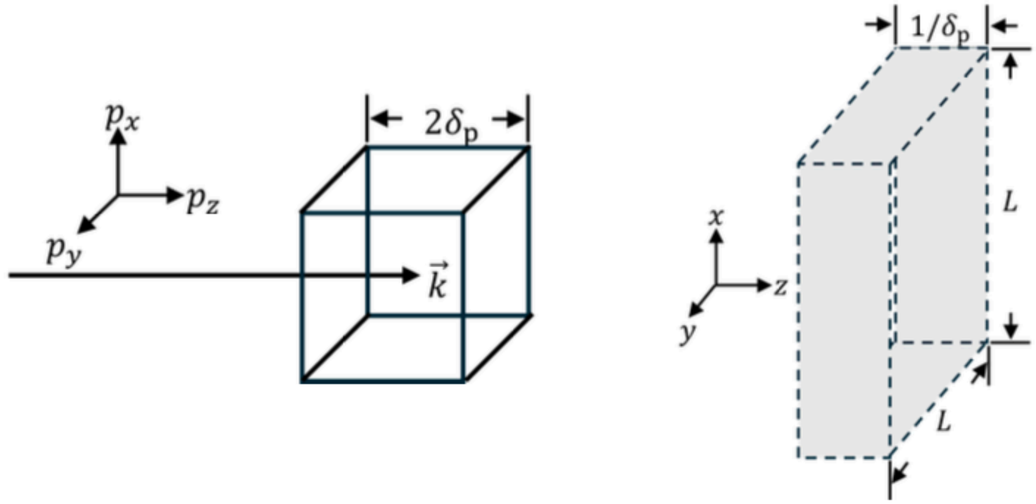
- For any wave packet, we will take the limit that the momentum space wave function is localized about a definitive momentum.

- In the strict limit of momentum eigenstate, the position wave function is a plane wave with an infinite extent

  → this is not how the experiment is conducted.

- Instead we will take a "finite" plane wave limit where the transverse dimension of the wave packet is much larger than the longitudinal dimension.

An example: a wave packet that is approximately uniform in the transverse plane in the position space:



(a) In momentum space (b) In position space

L^2 characterizes the transverse size of the wave packet in position space!

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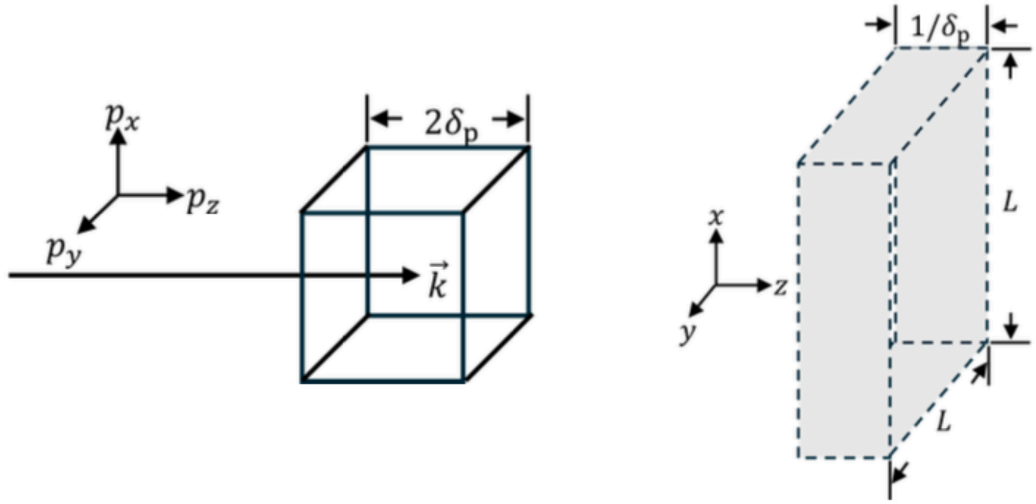

(a) In momentum space (b) In position space

L^2 characterizes the transverse size of the wave packet in position space!

The finite plane wave limit is

$$\delta_{\rm p}/|\vec{k}| \to 0 \ , \qquad \delta_{\rm p} L \gg 1$$

In the position space it looks like a ”square pancake”, as we expect the longitudinal direction to be “Lorentz contracted.”

After carefully set up a wave packet formalism to compute the cross section and entanglement entropy in the finite plane wave,

we are going to compute everything to the leading order in $\delta_{\rm p}/|\vec{k}|$

After expanding around the finite plane wave limit,

$$\mathcal{P}_{\rm el} \ = \ \langle {\rm in}|T^\dagger P_{\rm AB} T|{\rm in}\rangle = \frac{\sigma_{\rm el}}{L^2} + \mathcal{O}({\delta_{\rm p}}^5/|\vec{k}|^5).$$

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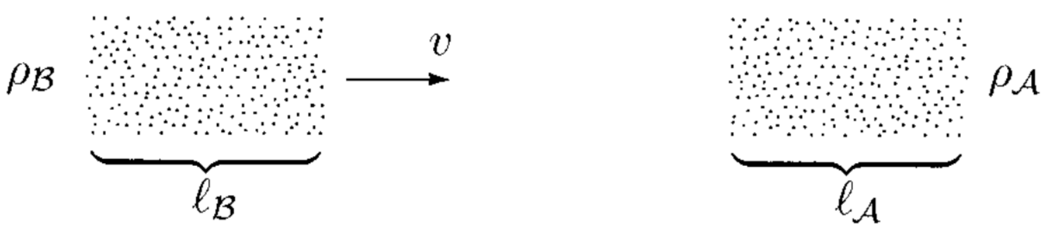
There’s an intuitive understanding of this result. Let’s go back to Chapter 4 in Peskin and Schroeder:



$$\sigma \equiv \frac{\text{Number of scattering events}}{\rho_\mathcal{A}\, \ell_\mathcal{A}\, \rho_\mathcal{B}\, \ell_\mathcal{B}\, A}$$

We are scattering only two particles head on, so

$$\rho_\mathcal{A}\ell_\mathcal{A}A = \rho_\mathcal{B}\ell_\mathcal{B}A = 1\ , \quad A = L^2\ , \quad N_{\rm inel} = \mathcal{P}_{\rm inel}$$

Using this result, when the initial state is unentangled in both momentum and flavors, the entanglement entropy between particle A and particle B is

$$\mathcal{E}_2^{\mathrm{f}} = 2\frac{\sigma_{\mathrm{el}}}{L^2} + \mathcal{O}({\delta_{\mathrm{p}}}^5/|\vec{k}|^5),$$

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$$\mathcal{E}_2^{\rm f} \;=\; 2\frac{\sigma_{\rm el}}{L^2} + \mathcal{O}(\delta_{\rm p}{}^5/|\vec{k}|^5)\,,$$

In plain English:

The entanglement entropy is the cross section in unit of the transverse size of the wave packet.

A few comments are in order:

- To create quantum correlations (entanglement) in the final state, "something" must occur.

  (The "1" in S = 1 +i T can't create entanglement}

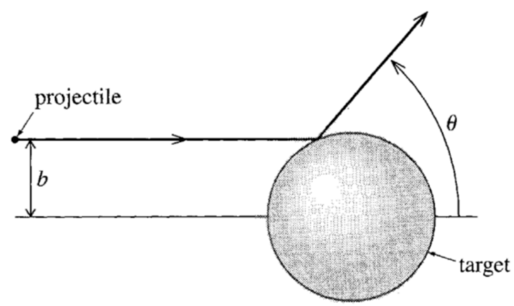
- The physical observable characterizing the probability of "something" happens is cross-section!

- The "non-trivial" outcome is that the entanglement entropy is **linearly** proportional to the cross-section.

- Dimensional analysis dictates the dimensionless ratio (cross-section)/L^2. (In a different regularization scheme, it's less clear what this ratio is.)

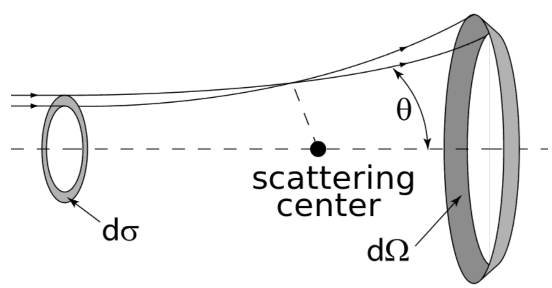
Dual interpretations of the cross section:

- It is an effective area characterizing the strength of interaction when two particles collide:



$$\sigma = \pi r^2$$

- Quantum-mechanically, it is a probability measure of a specific process taking place.



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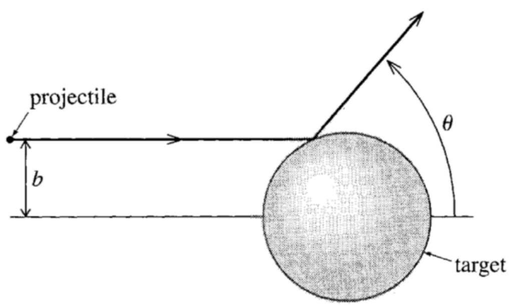

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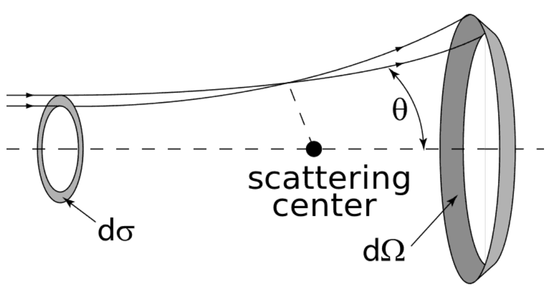

This is an area law: Entropy ~ Area

- It is also natural to wonder if other “area laws” can also be interpreted as some sort of “scattering cross sections”?

The celebrated Bekenstein-Hawking formula for black hole entropy:

$$\begin{aligned} S_{BH} &= \frac{A}{4G_N} \\ &= \frac{\pi R^2}{L_{\mathrm{P}\ell}^2} \end{aligned}$$

$$A = 4\pi R^2; \quad G_N = L_{\mathrm{P}\ell}^2$$

Instead of the surface area of the event horizon, the formula can be written as

$$\mathrm{Entropy} = \frac{\mathrm{Cross-Sectional\ Area\ of\ the\ Black\ Hole}}{\mathrm{Transverse\ Area\ of\ Wave\ Packets}}$$

Is there a calculation one can do?

- **Total and elastic cross sections are known to increase with respect to energy:**

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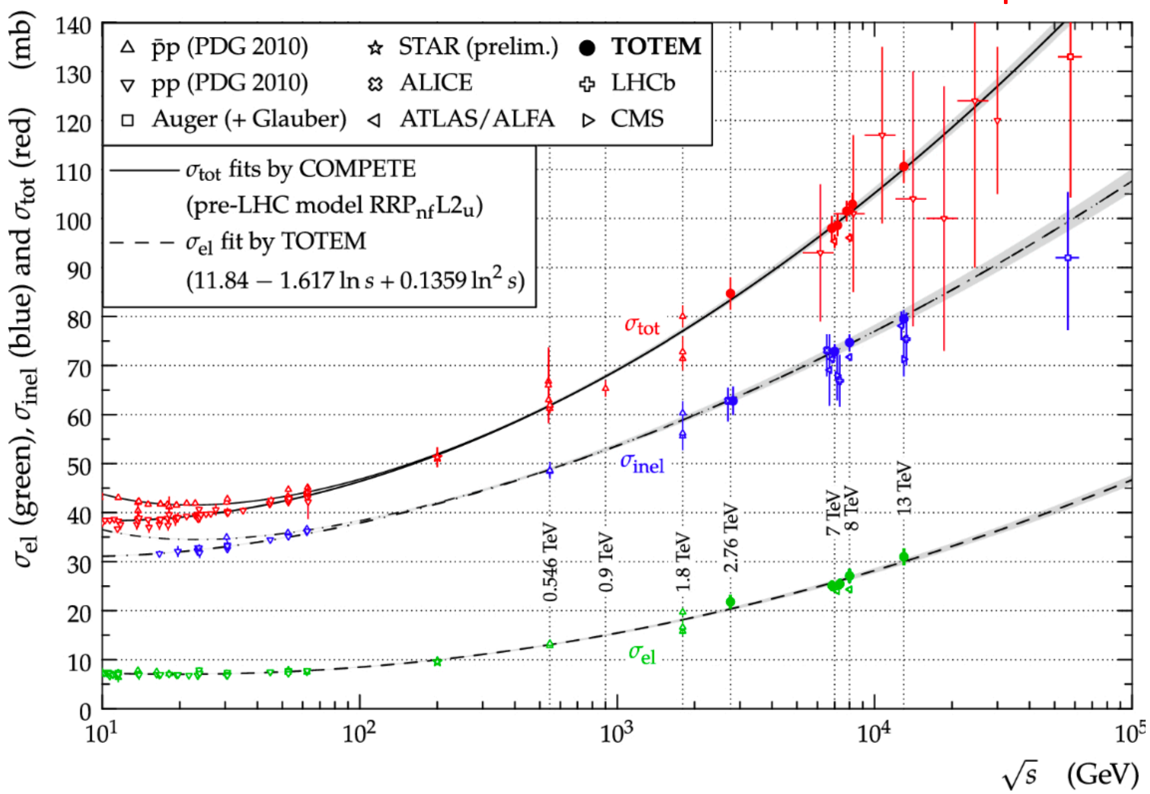

- Froissiart and Martin showed there's a universal bound on the total cross section:

$$\sigma_{\text{tot}} \leq \log^2 s$$

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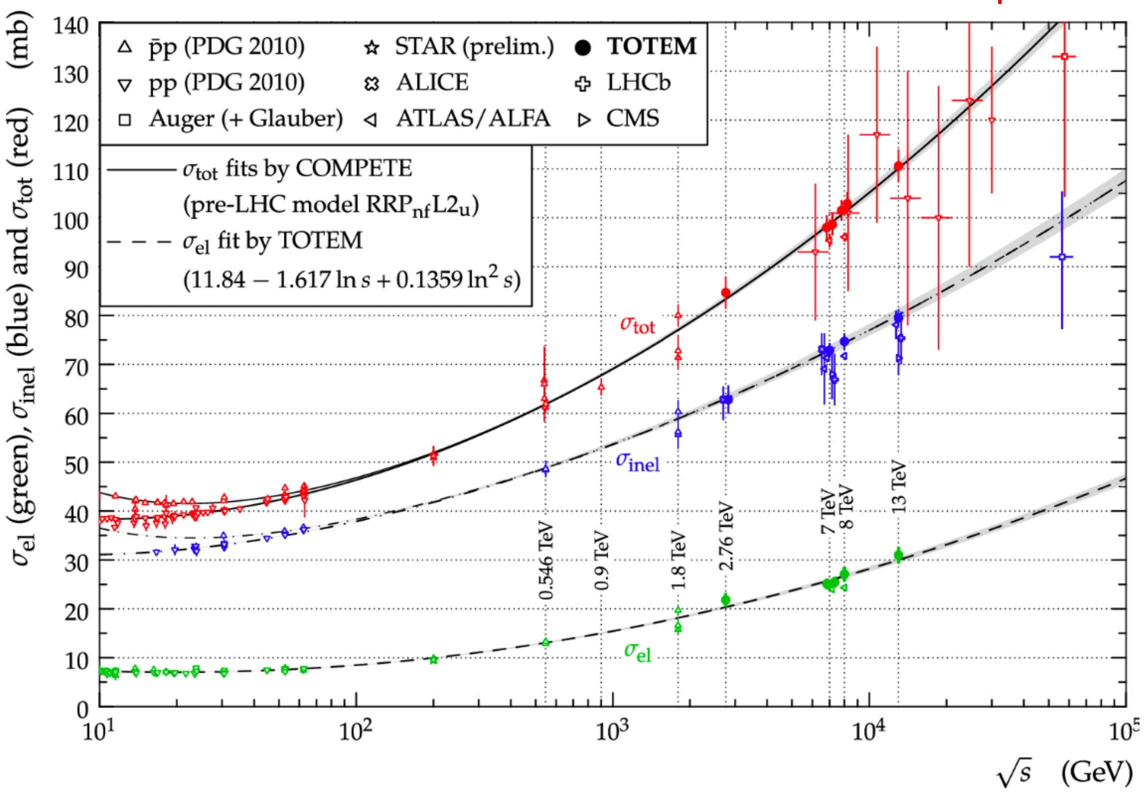

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$$\sigma_{\mathrm{tot}} \leq \log^2 s$$

With entropy and energy, one can define a "temperature"

➡ ***Thermodynamic laws in particle scattering?***

More Questions than answers. The journey has just begun!