Wave dark matter

Lam Hui 許林
Columbia University

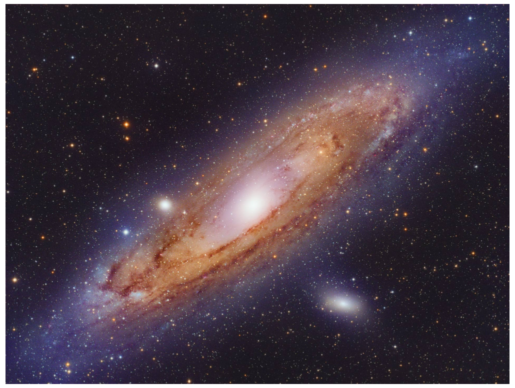
Work done with

Ostriker, Tremaine, Witten 1610.08297
Li, Bryan 1810.01915
Kabat, Li, Santoni, Wong 1904.12803
Joyce, Landry, Li 2004.01188
Li, Yavetz 2011.11416, 2109.06125
Law, Santoni, Sun, Tomaselli, Trincherini 2208.06408
Podo in preparation
Eberhardt 2506.02400

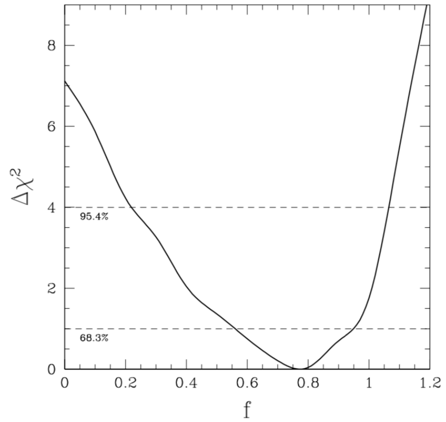
ARAA review 2101.11735

Rich evidence for dark matter - from its gravitational effects

- Dynamical measurements.

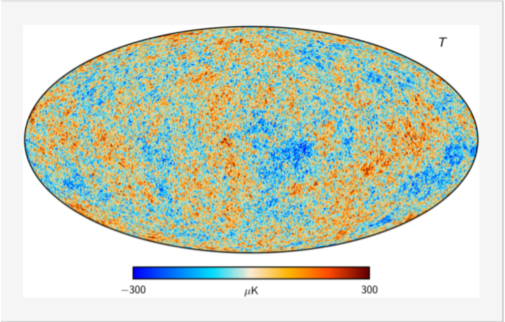


- Gravitational lensing measurements.

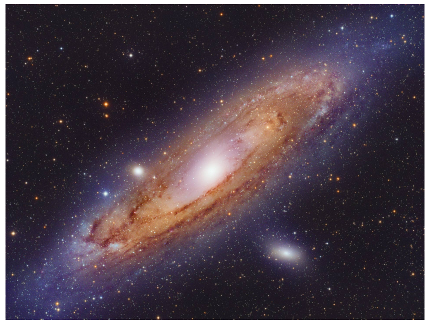


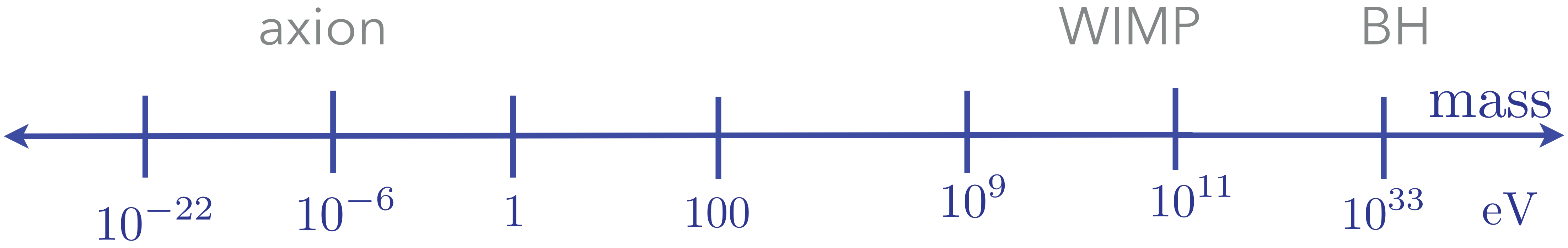
Hoekstra, Yee, Gladders

- Growth of perturbations.



We have rich evidence for the existence of DM, but remain ignorant about its basic properties e.g. mass:





axion WIMP BH

mass

10^{-22} 10^{-6} 1 100 10^{9} 10^{11} 10^{33} eV

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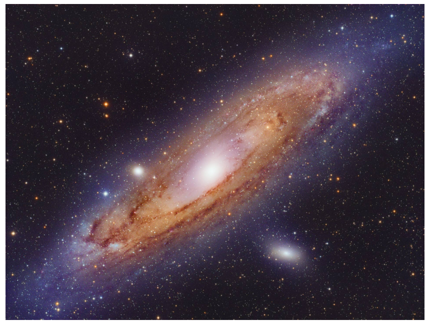

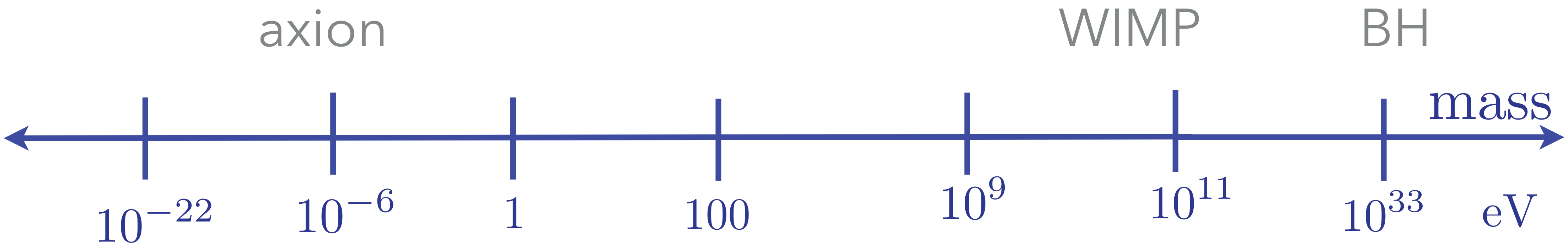

What we do know: mass density in solar neighborhood is $0.3\ \mathrm{GeV/cm^3}$

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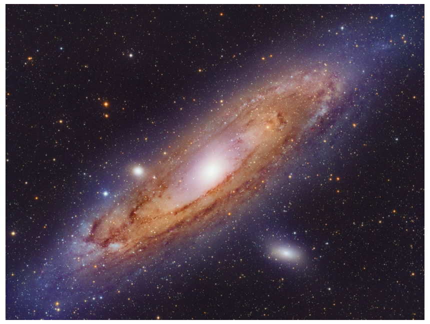

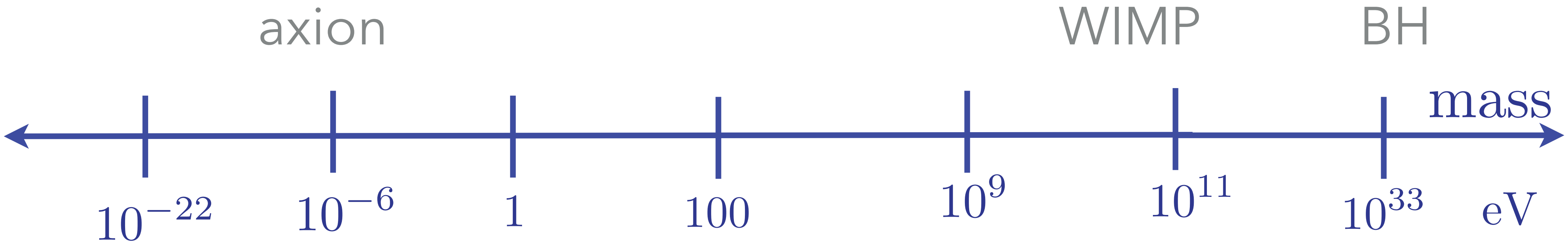

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(1/mv)

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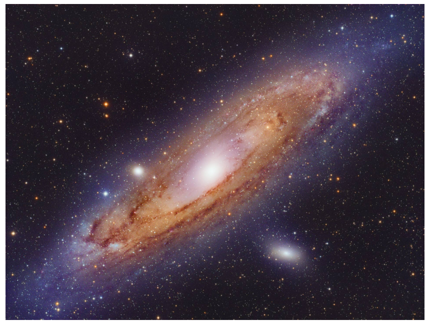

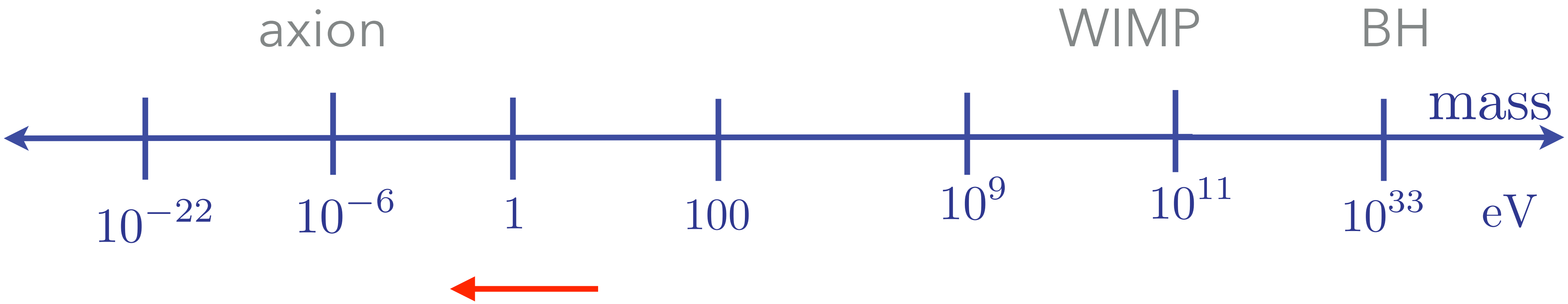

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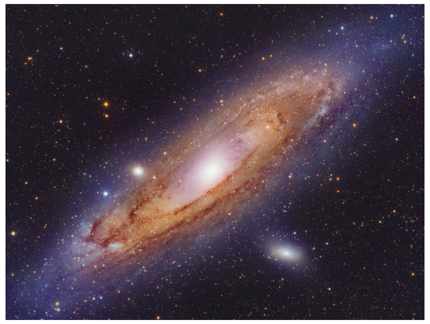

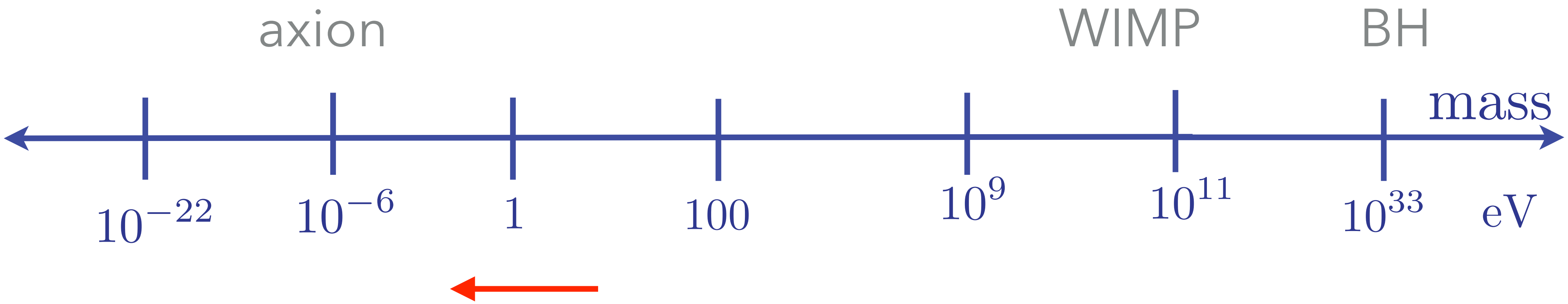

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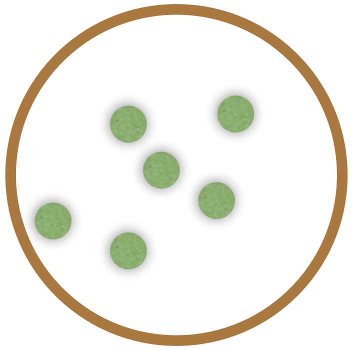
Question: at what mass is the interparticle separation < de Broglie wavelength? (1/mv)

wave regime $m < 30\,\mathrm{eV}$

$$1/mv \sim 10^{-3}\,\mathrm{cm} \quad \text{for} \quad m = 10\,\mathrm{eV}$$

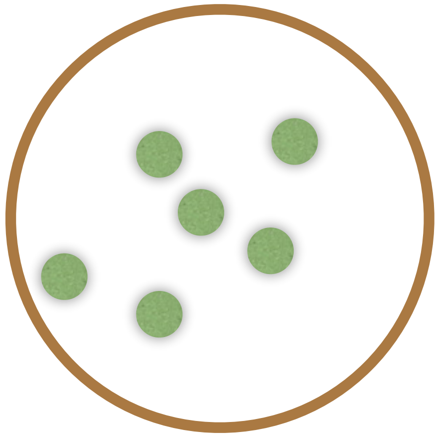
$$10^{4}\,\mathrm{cm} \quad \text{for} \quad m = 10^{-6}\,\mathrm{eV}$$

$$100\,\mathrm{pc} \quad \text{for} \quad m = 10^{-22}\,\mathrm{eV}$$



bosonic

Let’s discuss:



Particle physics motivations

Wave dynamics and phenomenology

Astrophysical implications (ultra-light DM)

Experimental implications (light DM)

$$1/mv \sim 10^{-3}\,\text{cm} \quad \text{for} \quad m = 10\,\text{eV}$$

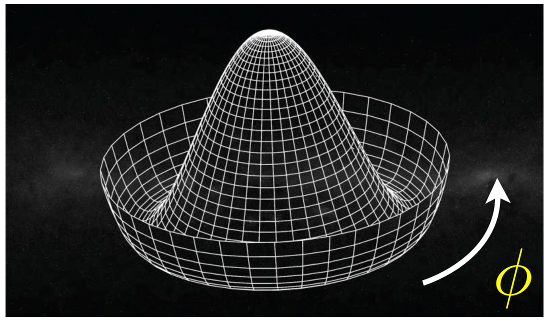
$$10^{4}\,\text{cm} \quad \text{for} \quad m = 10^{-6}\,\text{eV}$$ QCD axion

$$100\,\text{pc} \quad \text{for} \quad m = 10^{-22}\,\text{eV}$$ Fuzzy DM (Hu, Barkana, Gruzinov)

Particle physics motivations

- A natural candidate for a light (scalar) particle is a pseudo-Nambu-Goldstone boson.

  A well known example is the QCD axion (Peccei, Quinn; Weinberg; Wilczek; Kim; Shifman, Vainshtein, Zakharov, Zhitnitsky; Dine, Fischler, Srednicki; Preskill, Wise, Wilczek; Abbott, Sikivie).



  There are also many axion-like-particles in string theory (Svrcek, Witten; Arvanitaki et al.)

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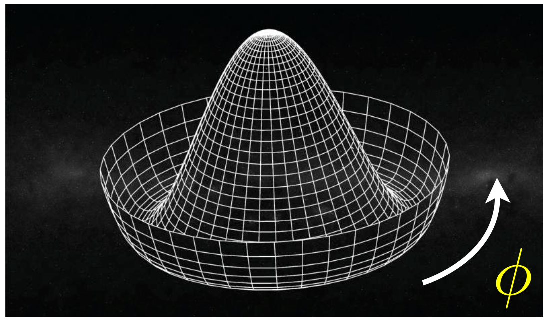

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Footnote on ultra -light version

mass m ← $10^{-22}\,\mathrm{eV}$ →

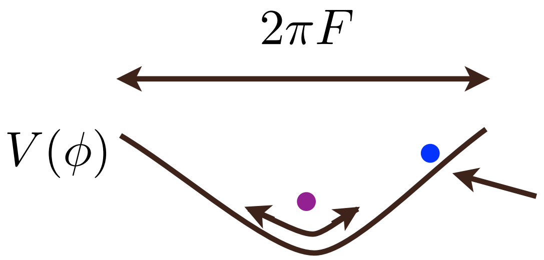
Fuzzy dark matter (FDM)

- Consider an angular field (a pseudo Nambu-Goldstone) of periodicity $2\pi F$ i.e. an axion-like field with a potential from non-perturbative effects (not QCD axion).

$$\mathcal{L} \sim -\frac{1}{2}(\partial\phi)^2 - \Lambda^4(1 - \cos[\phi/F]) \qquad m \sim \Lambda^2/F$$

(candidates: Arvanitaki et al. Svrcek, Witten)

- Relic abundance matches dark matter abundance (mis-alignment mechanism).



$\phi \sim F$ at early times until $H \sim m$

$$\Omega_{\mathrm{matter}} \sim 0.1 \left(\frac{F}{10^{17}\,\mathrm{GeV}}\right)^2 \left(\frac{m}{10^{-22}\,\mathrm{eV}}\right)^{1/2}$$

(Preskill, Wise, Wilczek; Abbot, Sikivie; Dine, Fischler, with constant m)

(low scale inflation)

Dynamics of wave dark matter:

- Ignoring self-interactions $\longrightarrow$ $-\Box\phi + m^2\phi = 0$ Klein Gordon equation

In the non-relativistic limit , useful to define: $\phi = \frac{1}{\sqrt{2m}}\left[\psi e^{-imt} + \psi^* e^{imt}\right]$

$$\ddot{\phi} \propto -m^2\psi e^{-imt} - im\dot{\psi}e^{-imt} + \cancel{\ddot{\psi}e^{-imt}} + \text{c. c.}$$

$$i\dot{\psi} = \left[-\frac{\nabla^2}{2m} + m\Phi_{\text{grav.}}\right]\psi$$ Schrodinger equation

- $\Phi_{\text{grav.}}$ is the gravitational potential, determined by:

$$\text{Poisson eq. :}\ \ \nabla^2\Phi_{\text{grav.}} = 4\pi G\rho = 4\pi G m|\psi|^2$$

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- An alternative viewpoint: ψ as a (classical) fluid. $\psi = \sqrt{\rho/m}\, e^{i\theta}$ i.e. $\rho = m\,|\psi|^2$

mass conservation $\dot{\rho} + \nabla\cdot\rho v = 0$ where $v = \frac{1}{m}\nabla\theta$

Euler equation $\dot{v} + v\cdot\nabla v = -\nabla\Phi_{\text{grav.}} + \frac{1}{2m^2}\nabla\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right)$

superfluid
(see also Berezhiani, Khoury; Fan; Alexander, Cormack)

Feynman Lectures Vol. 3

Classical or quantum?
Think about number-phase commutator.

21–4The meaning of the wave function

When Schrödinger first discovered his equation he discovered the conservation law of Eq. (21.8) as a consequence of his equation. But he imagined incorrectly that P was the electric charge density of the electron and that $\boldsymbol{J}$ was the electric current density, so he thought that the electrons interacted with the electromagnetic field through these charges and currents. When he solved his equations for the hydrogen atom and calculated ψ, he wasn't calculating the probability of anything—there were no amplitudes at that time—the interpretation was completely different. The atomic nucleus was stationary but there were currents moving around; the charges P and currents $\boldsymbol{J}$ would generate electromagnetic fields and the thing would radiate light. He soon found on doing a number of problems that it didn't work out quite right. It was at this point that Born made an essential contribution to our ideas regarding quantum mechanics. It was Born who correctly (as far as we know) interpreted the ψ of the Schrödinger equation in terms of a probability amplitude—that very difficult idea that the square of the amplitude is not the charge density but is only the probability per unit volume of finding an electron there, and that when you do find the electron some place the entire charge is there. That whole idea is due to Born.

The wave function $\psi(\boldsymbol{r})$ for an electron in an atom does not, then, describe a smeared-out electron with a smooth charge density. The electron is either here, or there, or somewhere else, but wherever it is, it is a point charge. On the other hand, think of a situation in which there are an enormous number of particles in exactly the same state, a very large number of them with exactly the same wave function. Then what? One of them is here and one of them is there, and the probability of finding any one of them at a given place is proportional to $\psi\psi^*$. But since there are so many particles, if I look in any volume $dx\,dy\,dz$ I will generally find a number close to $\psi\psi^*\,dx\,dy\,dz$. So in a situation in which ψ is the wave function for each of an enormous number of particles which are all in the same state, $\psi\psi^*$ *can* be interpreted as the density of particles. If, under these circumstances, each particle carries the same charge q, we can, in fact, go further and interpret $\psi^*\psi$ as the density of *electricity*. Normally, $\psi\psi^*$ is given the dimensions of a probability density, then ψ should be multiplied by q to give the dimensions of a charge density. For our present purposes we can put this constant factor into ψ, and take $\psi\psi^*$ itself as the electric charge density. With this understanding, $\boldsymbol{J}$ (the current of probability I have calculated) becomes directly the electric current density.

Long history of scalar field as dark matter:

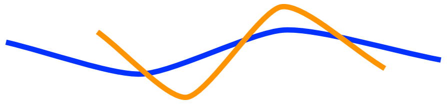
Baldeschi, Ruffini, Gelmini; Turner; Press, Ryden, Spergel; Sin; Hu, Barkana, Gruzinov; Peebles; Goodman; Lesgourgues, Arbey, Salati; Amendola, Barbieri; Chavanis; Suarez, Matos; Matos, Guzman, Urena-Lopez …

Dark matter as superfluid:

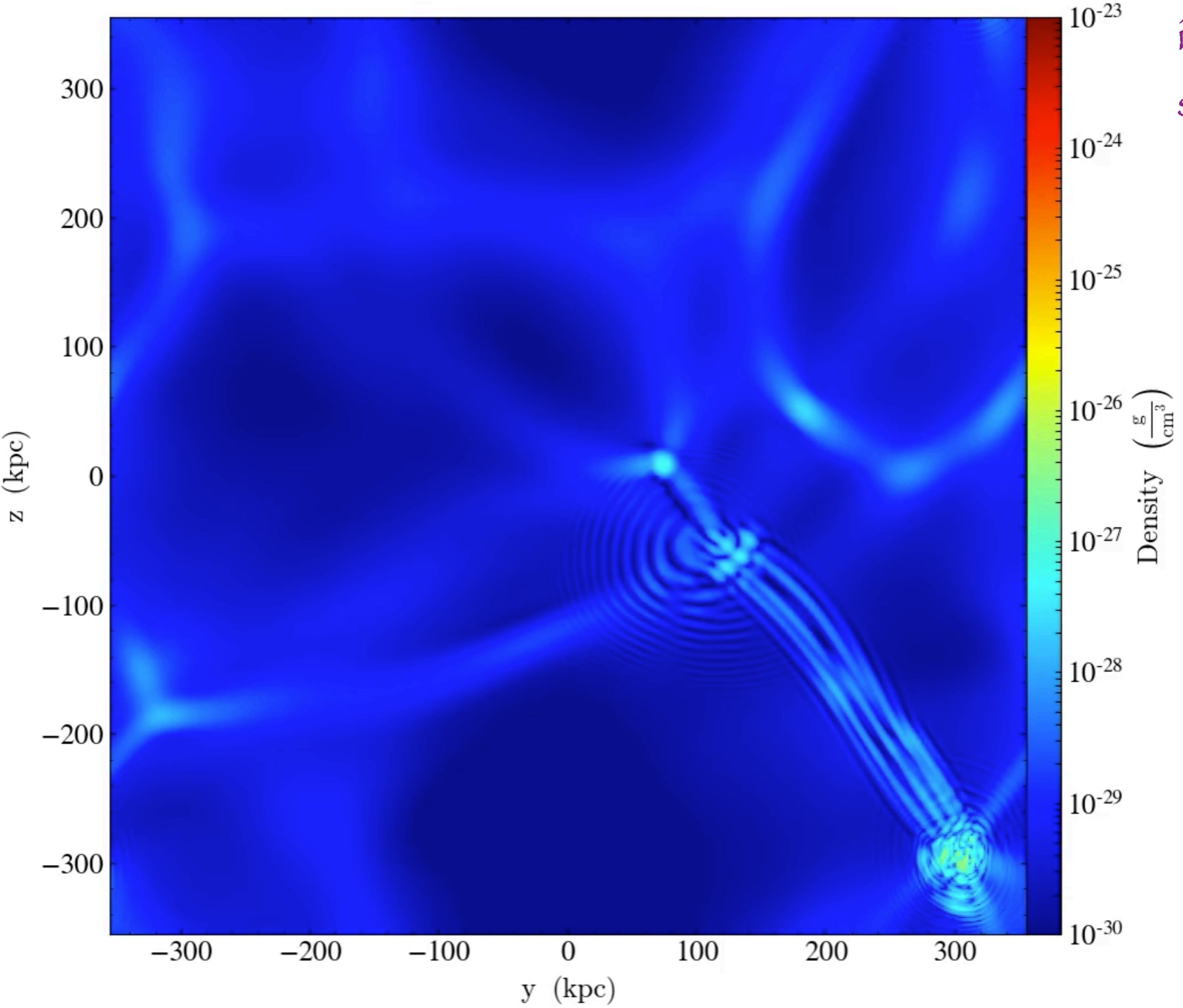
Rindler-Daller, Shapiro; Berezhiani, Khoury; Fan; Alexander, Cormack; Alexander, Gleyzer, McDonough, Toomey; Ferreira, Franzmann, Khoury, Brandenberger …

Wave effects in a cosmological simulation

- $\psi(x,t) = \sum_n a_n \psi_n(x) e^{-iE_n t}$



- $\rho = m|\psi|^2$



Note the irony: interference enhances small scale power.

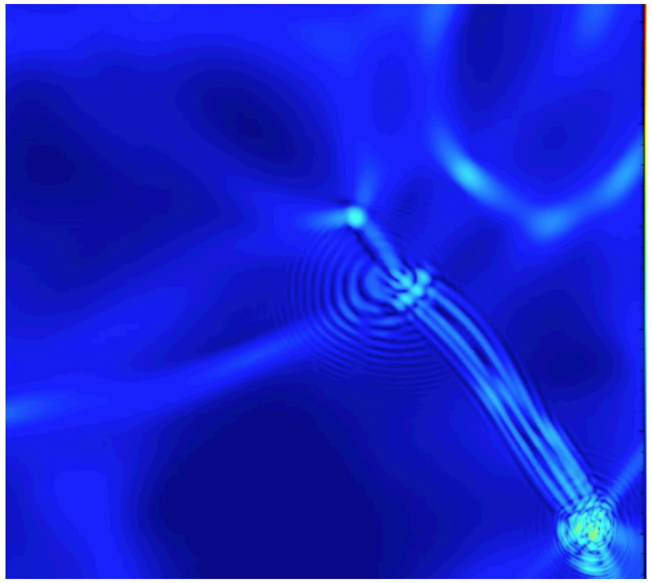
Li, LH, Bryan

See Schive, Chiueh, Broadhurst; Veltmaat, Niemeyer; Schwabe, Niemeyer, Engels; Mocz et al.; Nori, Baldi; Kendall, Easther

Wave effects from light/ultra-light DM:

- Lyman-alpha forest
- solitonic halo core
- dynamical friction
- evaporation of sub-halos by tunneling
- detection by pulsar timing array
- black hole hair/superradiance
- subhalo mass function
- luminosity/mass function
- Interference substructure and vortices
- gravitational lensing
- scattering of tidal streams, stellar heating
- soliton oscillations
- direct detection

$$\lambda_{\rm dB} \sim 120\,{\rm pc}\left(\frac{10^{-21}\,{\rm eV}}{m}\right)\left(\frac{100\,{\rm km/s}}{v}\right)$$



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Caustic fringes applied to tidal shells

Heating constraints

Vortices

Wave interference for axion detection

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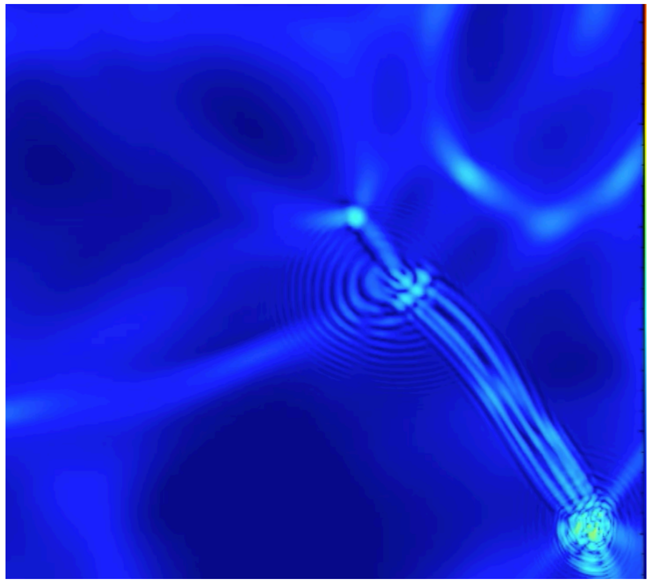
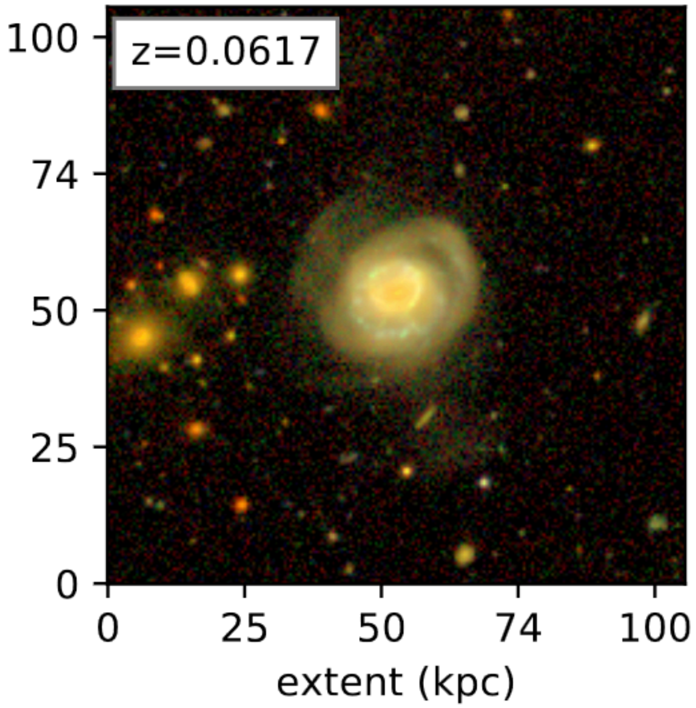
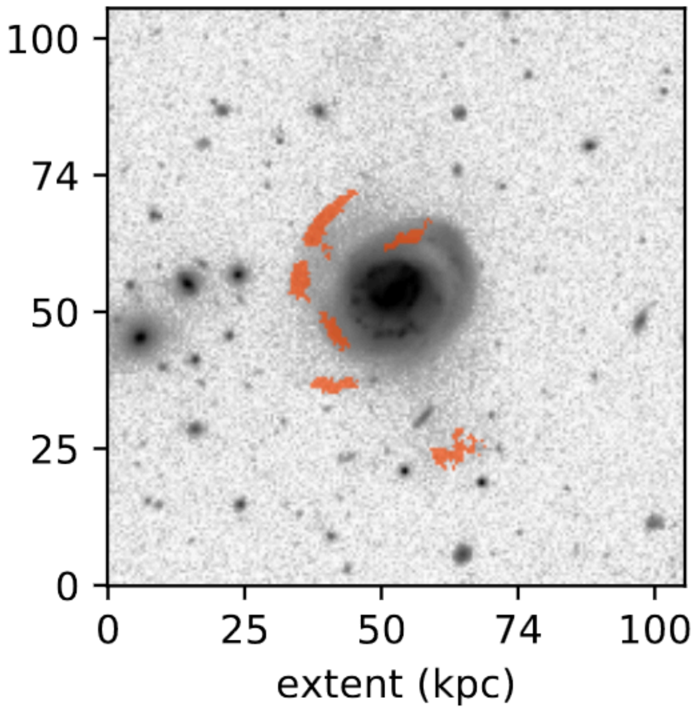


Consider tidal shells:

Data:

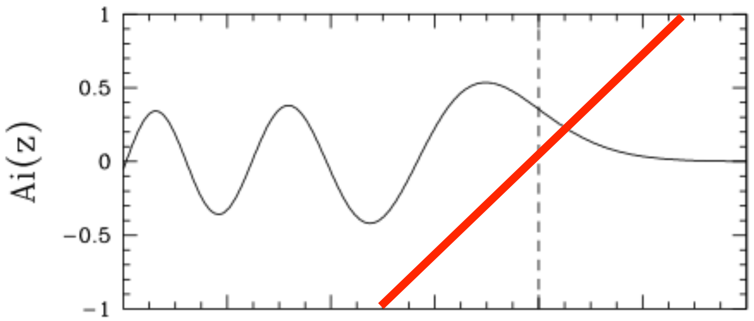




Kado-Fong et al. HSC Subaru

Theory:

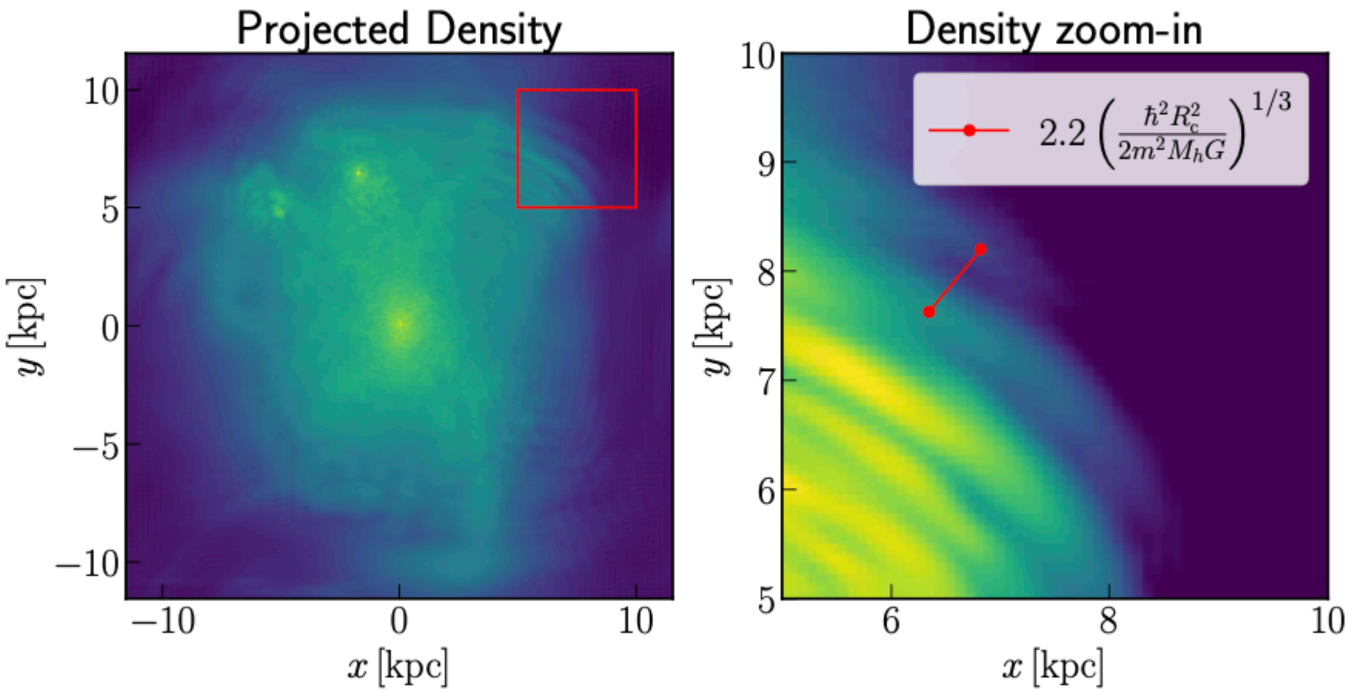
Airy function



$$\Delta x_{\text{fringe}} \sim \left(m^2 \times \text{ acc.}\right)^{-1/3}$$

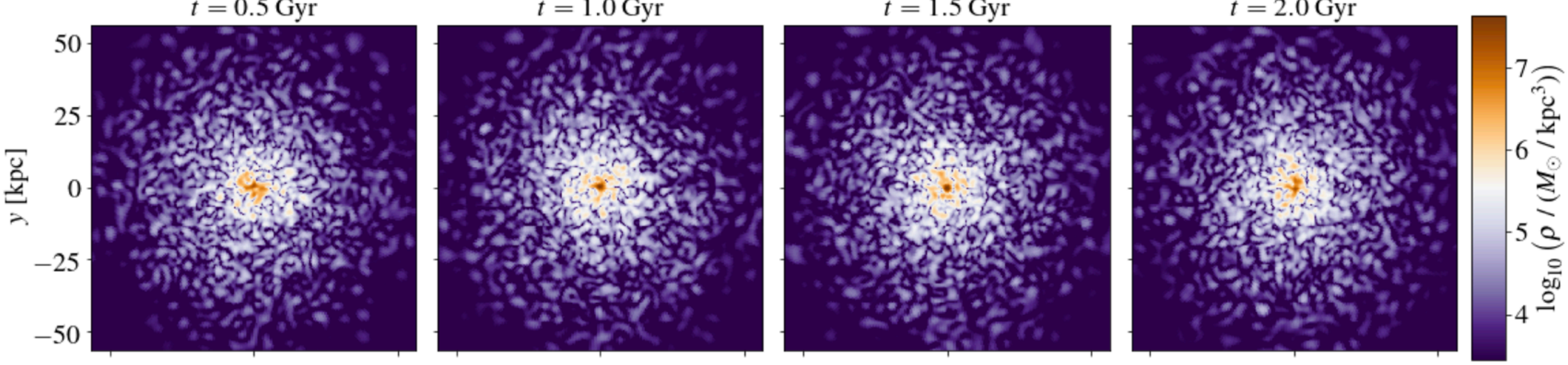
Projected Density

Density zoom-in



Caustic fringes **Eberhardt, LH :** could probe mass up to 10^{-19} eV

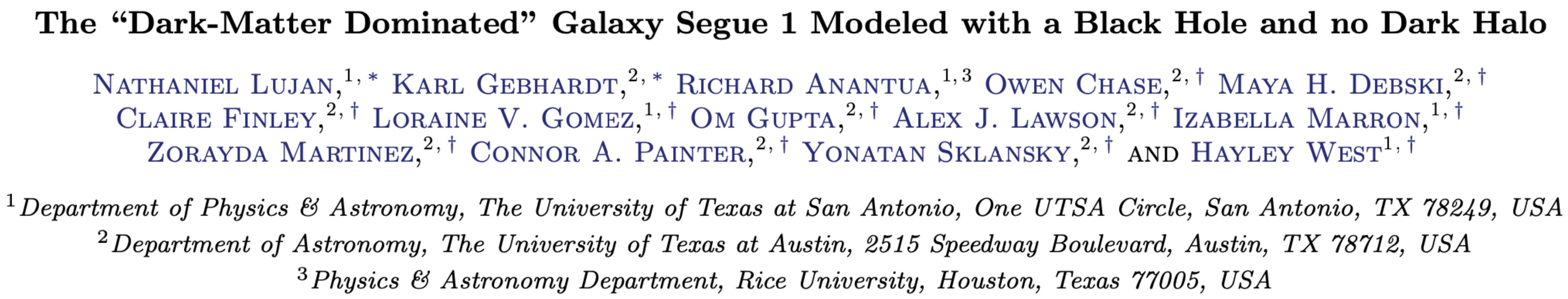
Example of current constraints: heating of stellar distributions



Yavetz, Li, LH

Time dependent wave interference injects energy into stellar orbits

- Dalal, Kravtsov ruled out $m < 10^{-19}\,\mathrm{eV}$ based on ultrafaint dwarfs e.g. Segue 1
- But keep in mind alternative interpretation:



The "Dark-Matter Dominated" Galaxy Segue 1 Modeled with a Black Hole and no Dark Halo

Nathaniel Lujan,[1,*] Karl Gebhardt,[2,*] Richard Anantua,[1,3] Owen Chase,[2,†] Maya H. Debski,[2,†]
Claire Finley,[2,†] Loraine V. Gomez,[1,†] Om Gupta,[2,†] Alex J. Lawson,[2,†] Izabella Marron,[1,†]
Zorayda Martinez,[2,†] Connor A. Painter,[2,†] Yonatan Sklansky,[2,†] and Hayley West[1,†]

[1] *Department of Physics & Astronomy, The University of Texas at San Antonio, One UTSA Circle, San Antonio, TX 78249, USA*
[2] *Department of Astronomy, The University of Texas at Austin, 2515 Speedway Boulevard, Austin, TX 78712, USA*
[3] *Physics & Astronomy Department, Rice University, Houston, Texas 77005, USA*

Vortices

- Consider again fluid formulation: $\psi = \sqrt{\rho/m}\, e^{i\theta}$

$$\dot{\rho} + \nabla \cdot \rho v = 0 \quad \text{where} \quad v = \frac{1}{m}\nabla\theta$$

$$\dot{v} + v \cdot \nabla v = -\nabla\Phi_{\text{grav.}} + \frac{1}{2m^2}\nabla\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right)$$

- Naively, vorticity cannot exist, because the velocity field is a gradient flow. In addition, one might think Kelvin's theorem should hold i.e. no vorticity is generated if there's no vorticity to begin with.

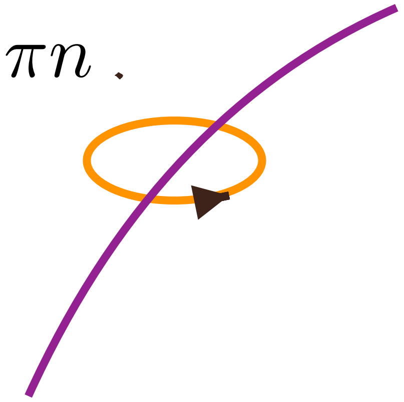
- The loophole: where $\rho = 0$. Note: such complete destructive interference can only occur in the late universe when O(1) fluctuations are present. No vortices in the early universe.

- See condensed matter literature.

Structure of a vortex

- Generically, in 3D, the set of points where both the real & the imaginary parts of the wavefunction vanish fall on a line i.e. a line/string defect.

- The phase of the wavefunction must wraps around the line by $2\pi n$.
  Thus, a vortex: $\oint \vec{v} \cdot d\vec{\ell} = 2\pi n/m$

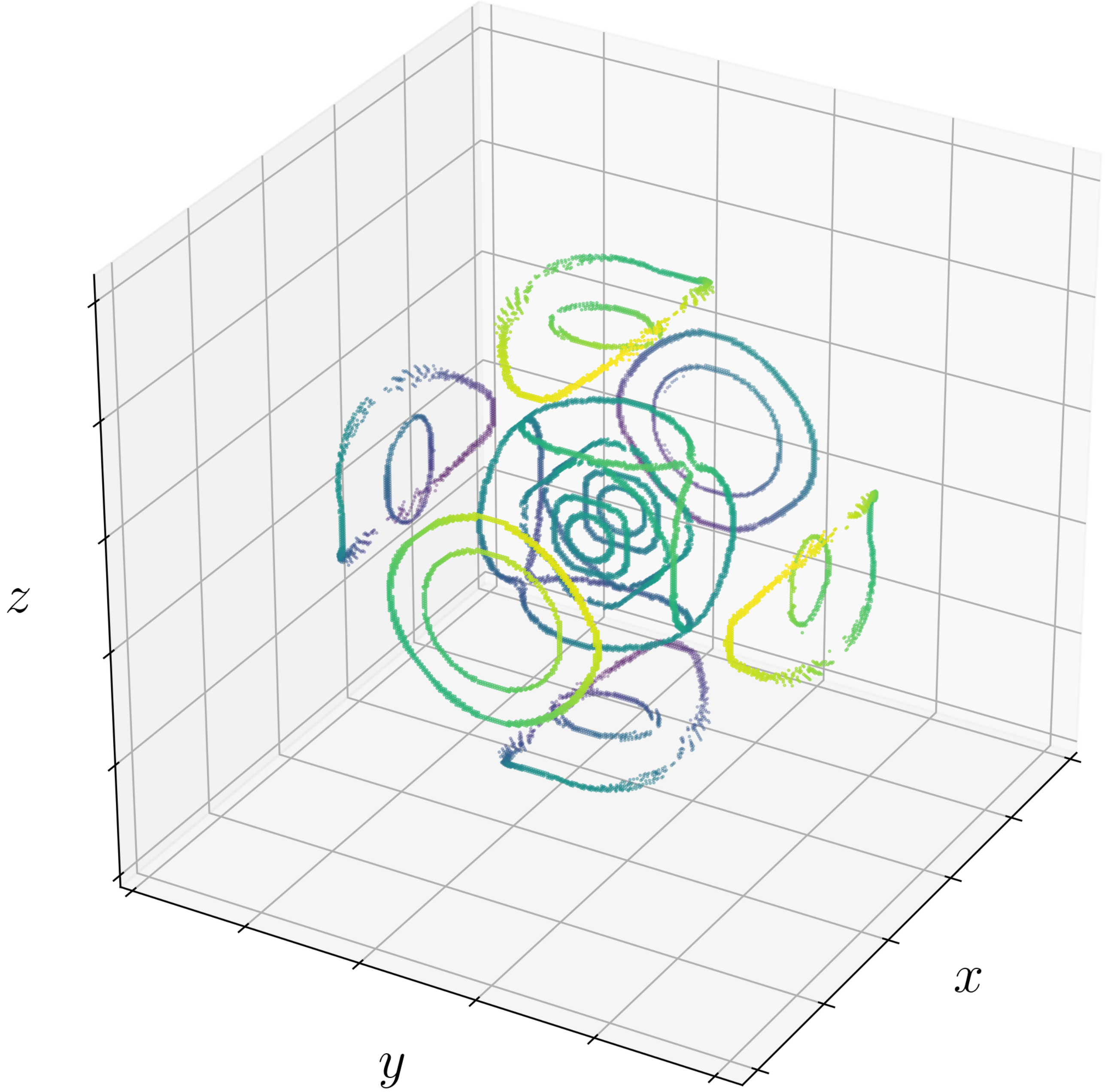


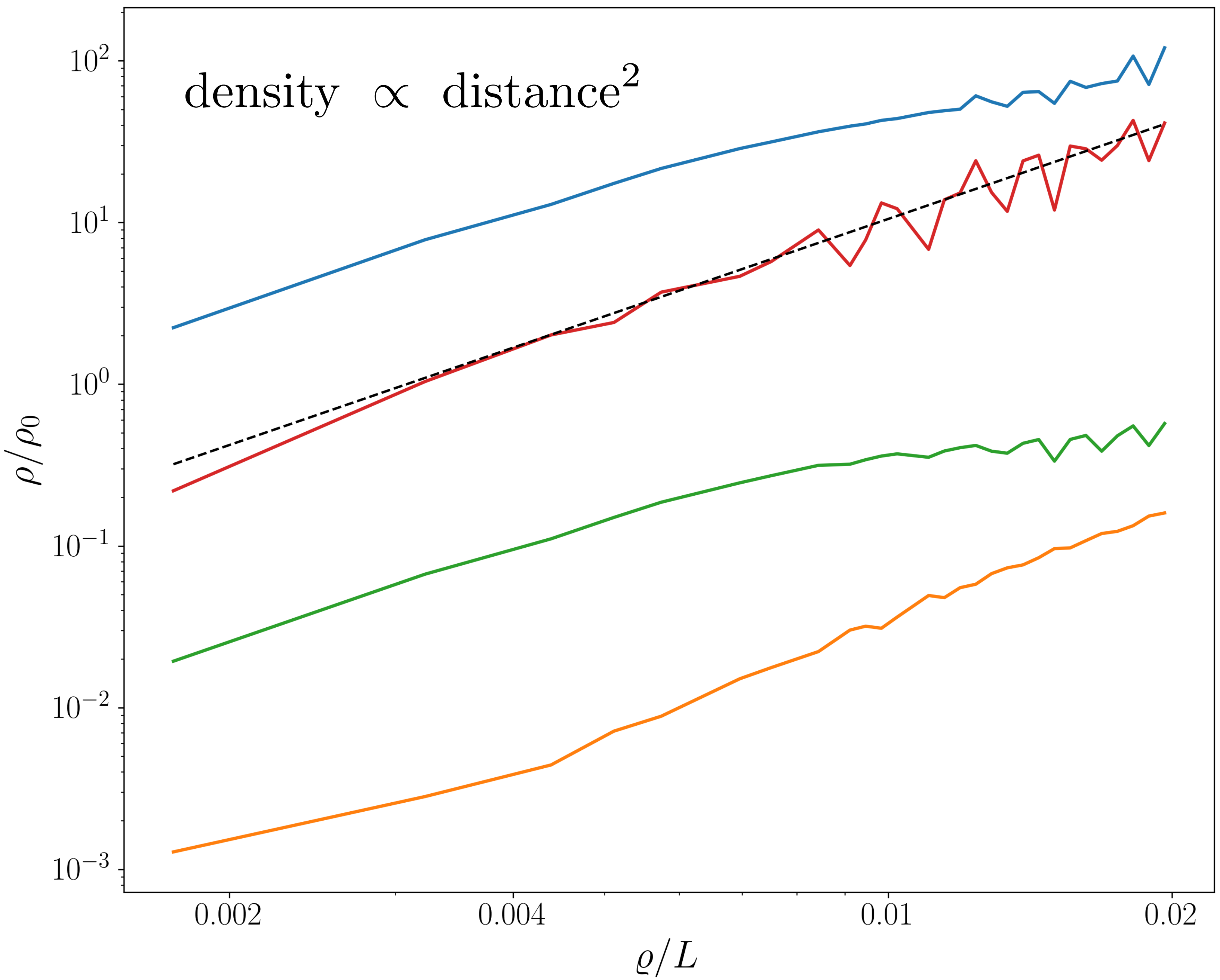
- Taylor expansion reveals further details (case of $n = 1$):

$$\psi(\vec{x}) \sim \cancel{\psi(0)} + \vec{x} \cdot \vec{\partial}\psi\big|_0 + \ldots \qquad \rho \sim r^2 \qquad (\text{also } v \sim 1/r)$$

- Vortex generally takes the form of a loop i.e. vortex ring.
  Statistically, expect 1 vortex ring per de Broglie volume.

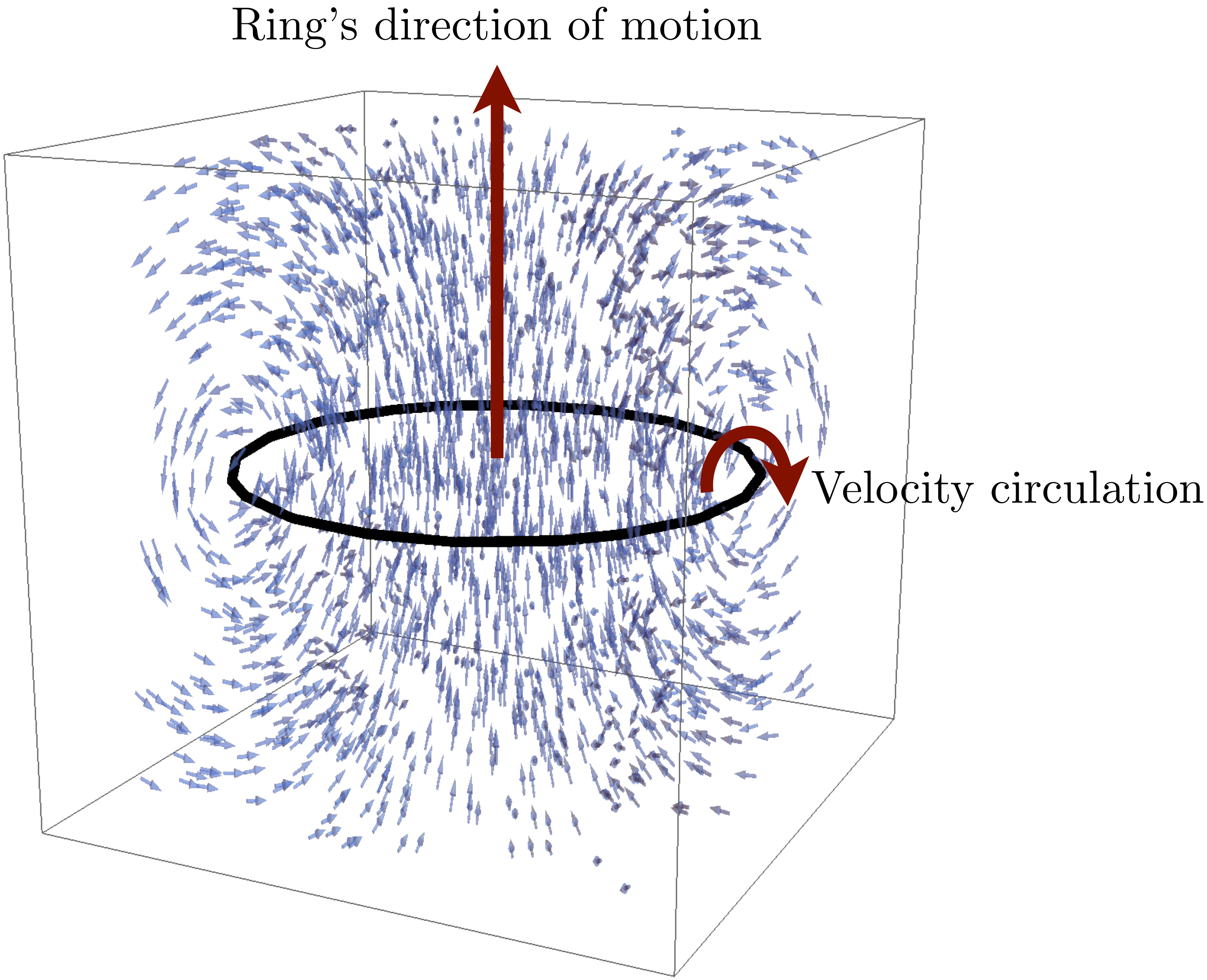
Note: this is not the usual axion string.

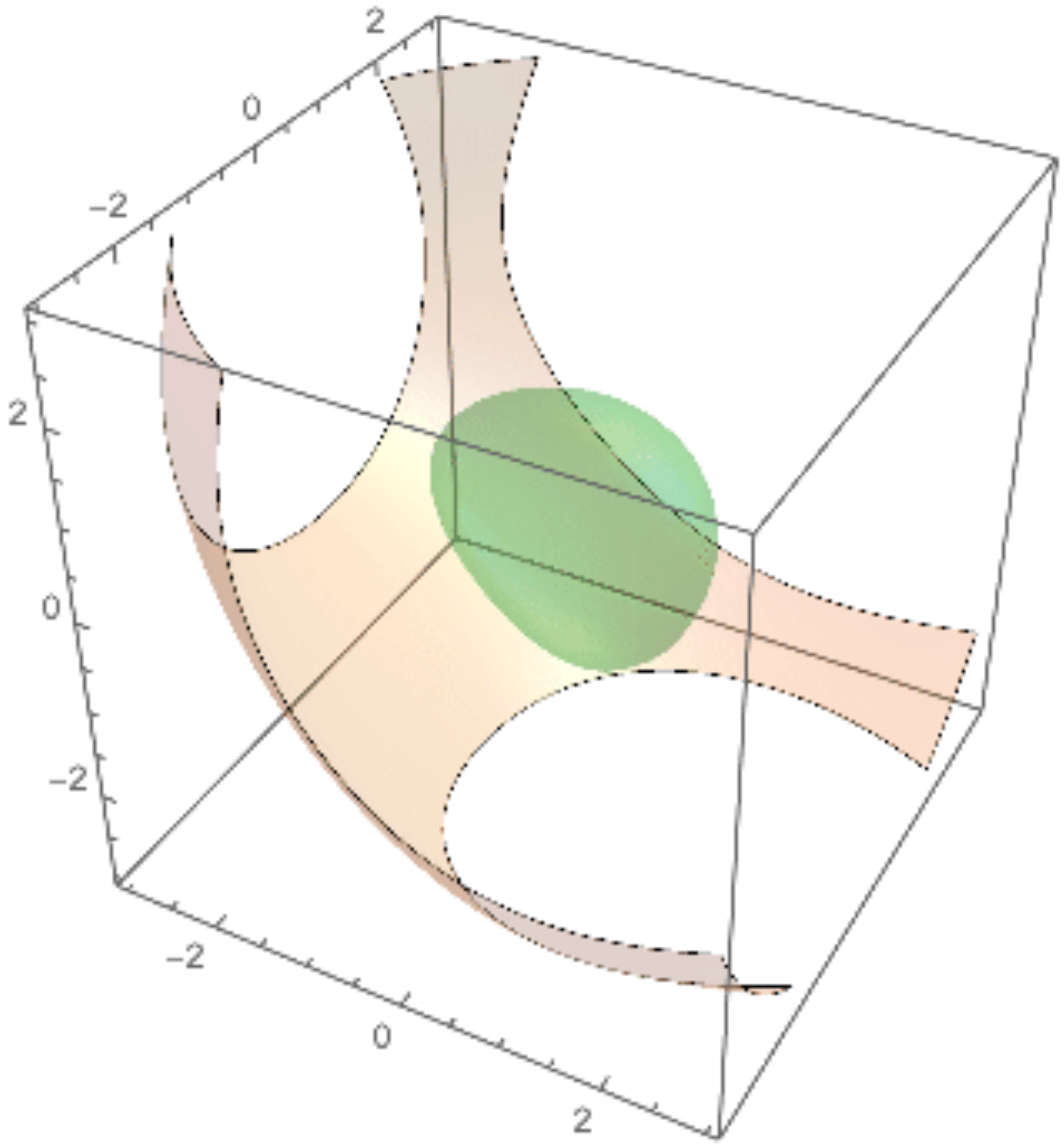

z
y
x



density

distance from vortex


Ring's direction of motion
Velocity circulation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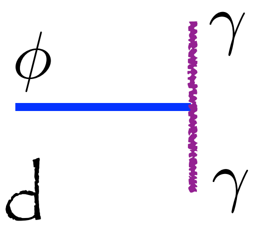
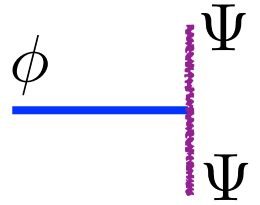
0
−2
2
0
−2
−2
0
2

Experimental implications (light DM e.g. QCD axion):

- Experiments mostly make use of
  - coupling of the axion ϕ to photon γ
    e.g. axion produces photon in magnetic field



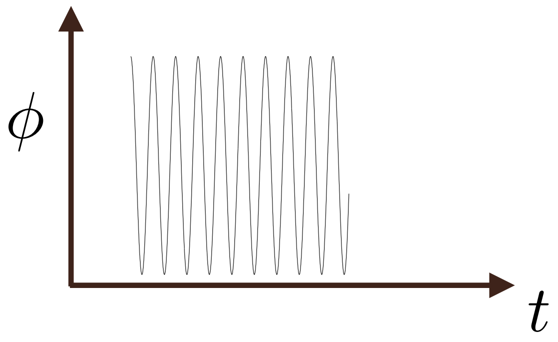
  - coupling of the axion ϕ to fermion Ψ
    e.g. axion causes fermion spin to precess



$$\mathcal{L} \sim \frac{\phi}{f} F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{\partial_\mu \phi}{f} \bar{\Psi}\gamma^5\gamma^\mu\Psi$$

Reviews: Sikivie 2003
Graham et al. 2015, Marsh 2016

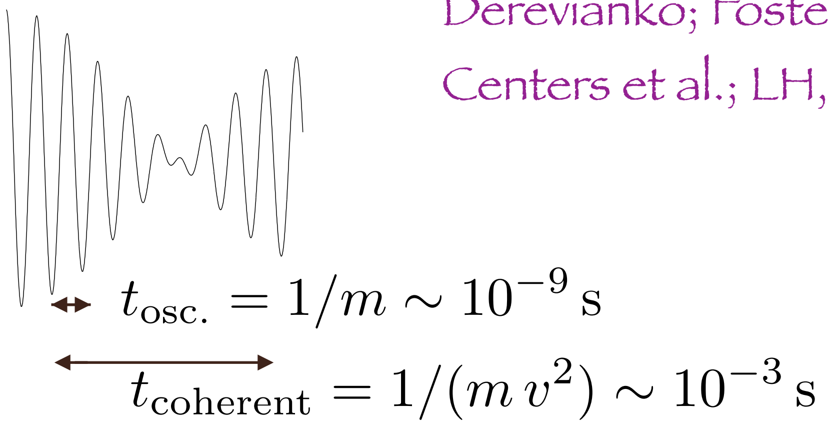
- Instead of



wave interference means:

$$\phi \sim \psi e^{-imt} + \psi^* e^{imt}$$

slow fast



$t_{\mathrm{osc.}} = 1/m \sim 10^{-9}\,\mathrm{s}$

$t_{\mathrm{coherent}} = 1/(m\,v^2) \sim 10^{-3}\,\mathrm{s}$

Derevianko; Foster, Rodd, Safdi;
Centers et al.; LH, Joyce, Landry, Li

- Useful to think of axion detection experiment as measuring correlation functions
- At vortices $\phi = 0$ but $\vec{\nabla}\phi \neq 0$.
- Existence of vortices suggests oscillation phase is interesting: $\phi \sim |\psi| \cos(mt - \theta)$

Summary:

Particle physics motivations

Wave dynamics and phenomenology

Astrophysical implications (ultra-light DM)

Experimental implications (light DM)

Caustic fringes applied to tidal shells

Heating constraints

Vortices

Wave interference for axion detection