Spin dependent scattering of sub-GeV Dark Matter

in crystal targets (arxiv: 2506.11191)

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Bethany Suter[2]

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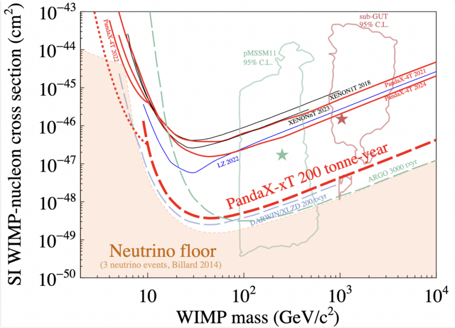
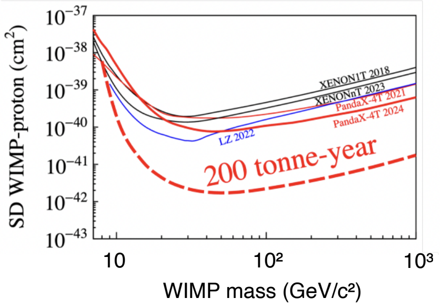
Motivation and Outline

Present bounds on Spin Dependent (SD) interactions are roughly 5-6 orders of magnitude weaker than on Spin Independent (SI) interactions.





Panda-X collaboration, Sci.China Phys.Mech.Astron. 68 (2025)

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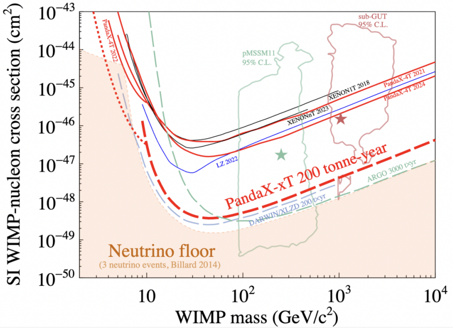

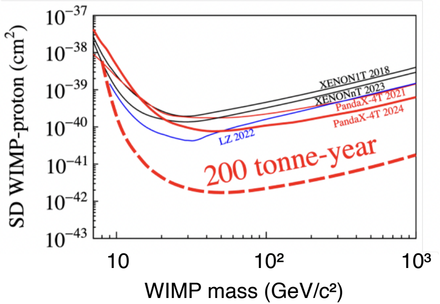

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Lose coherent enhancement ⇒ SD operators are generically harder to probe.

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Sub-GeV DM has sufficiently small momentum transfer $q \sim (2\pi/a) \sim$ keV that modes with wavelengths longer than the interatomic spacing are excited.

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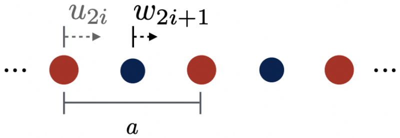

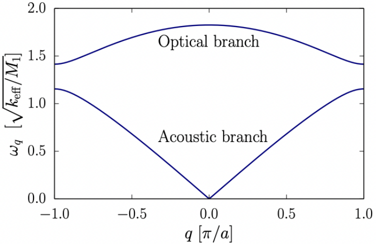

T. Lin, TASI lectures on dark matter models and direct detection, 1904.07915

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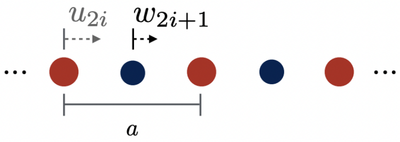

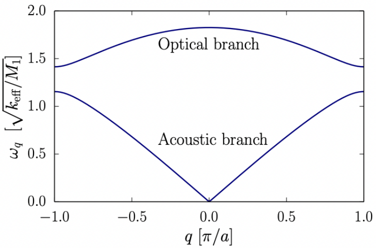

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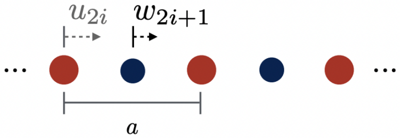

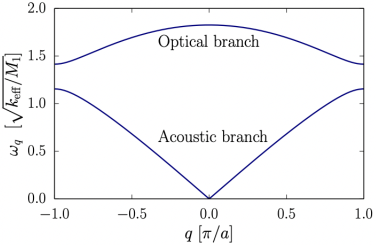

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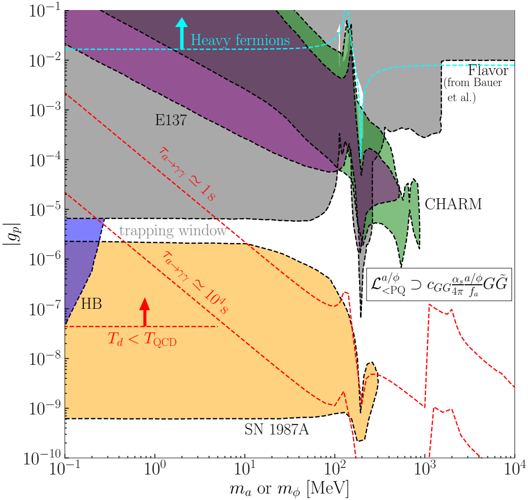
- Motivated models for DM-nucleon interactions and their bounds.
- Computation of scattering rate with crystal and expected sensitivities of GaAs and sapphire.

Effective Lagrangians at the ~GeV scale

$$\mathcal{L}_\phi = \phi\left[g_\chi\bar{\chi}\chi + g_p\bar{p}\gamma^5 p + g_n\bar{n}\gamma^5 n\right]$$
$$\mathcal{L}_a = a\left[g_\chi\bar{\chi}\gamma_5\chi + g_p\bar{p}\gamma^5 p + g_n\bar{n}\gamma^5 n\right]$$
$$\mathcal{L}_{A'} = A'^\mu\left[g_\chi\bar{\chi}\gamma_\mu\gamma_5\chi + g_p\bar{p}\gamma_\mu\gamma_5 p + g_n\bar{n}\gamma_\mu\gamma_5 n\right]$$

Constraints on a or ϕ mediator

Model: $\mathcal{L}^{a/\phi}_{<\mathrm{PQ}} \supset c_{GG}\frac{\alpha_s}{4\pi}\frac{a}{f_a}G\tilde{G}$.
Running c_{GG} from $\Lambda \sim f_a$ to GeV scale generates g_p, g_n.



Model-dependent

- Horizontal Branch Stars.

Gori, Knapen, Lin, PM, Suter 2506.11191

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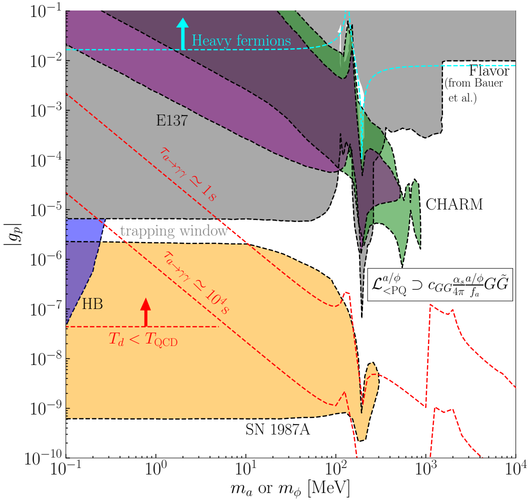

Model-dependent

- Horizontal Branch Stars.
  Changes ratio of horizontal branch stars to red giant branch stars in globular clusters.
  P. Carenza, O. Straniero, B. D¨obrich, M. Giannotti, G. Lucente, and A. Mirizzi, Phys. Lett. B 809, 135709

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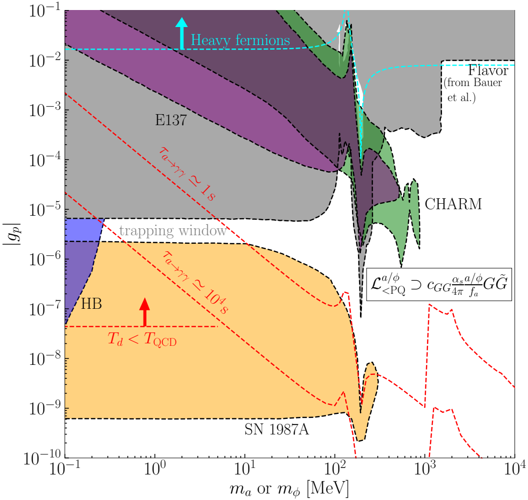

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- Horizontal Branch Stars.
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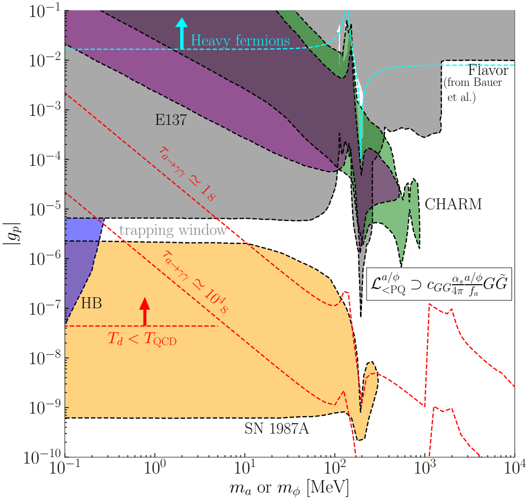

<u>Model-dependent</u>

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  Modifies the observed neutrino spectrum in Kamiokande-II detector.
  A. Lella, P. Carenza, G. Co', G. Lucente, M. Giannotti, A. Mirizzi, and T. Rauscher, Phys. Rev. D 109, 023001

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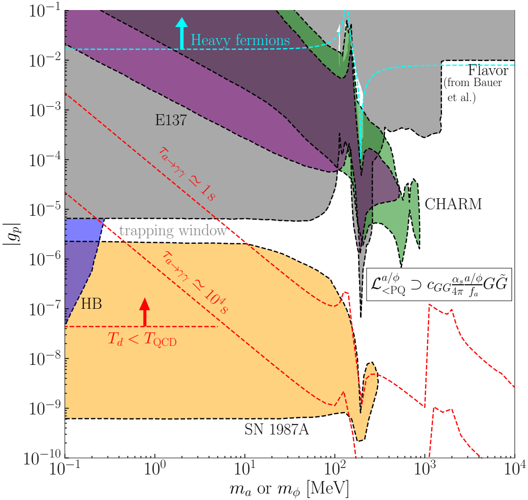

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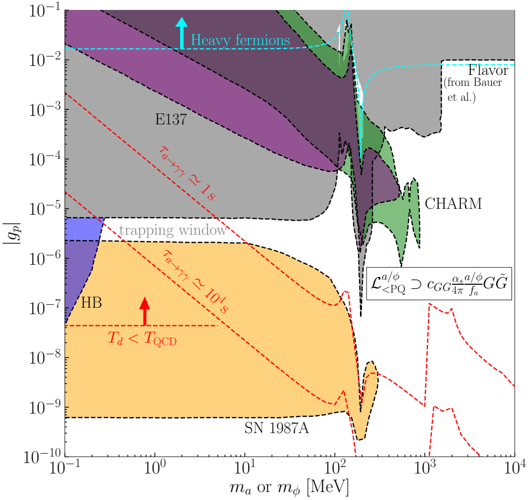

Model-dependent

- Horizontal Branch Stars.
- Supernova SN1987A.
- Rare meson decays.
  $K \to \pi\bar{\nu}\nu$ (NA62)
  $B \to K\mu\mu$ (LHCb)
  top quark dipole moment
  ...
  M. Bauer, M. Neubert, S. Renner, M. Schnubel, and A. Thamm,JHEP04,063(2021)

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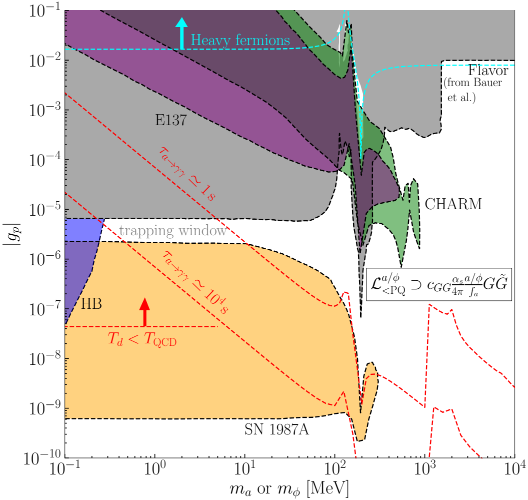

Model-dependent

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- Beam-dump experiments : CHARM, E137.

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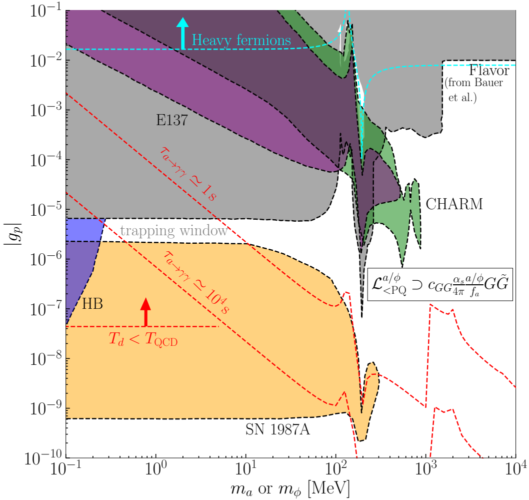

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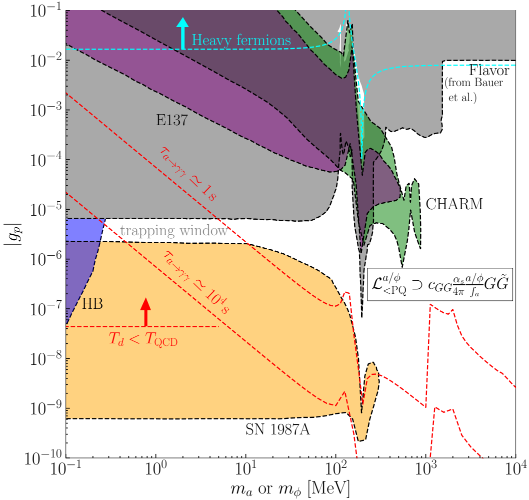

Model-dependent

- Horizontal Branch Stars.
- Supernova SN1987A.
- Rare meson decays.
- Beam-dump experiments : CHARM, E137.
- Big Bang Nucleosynthesis
  - EM injection energy photodisssociates Helium, Deuterium
  - ΔN_{eff}.

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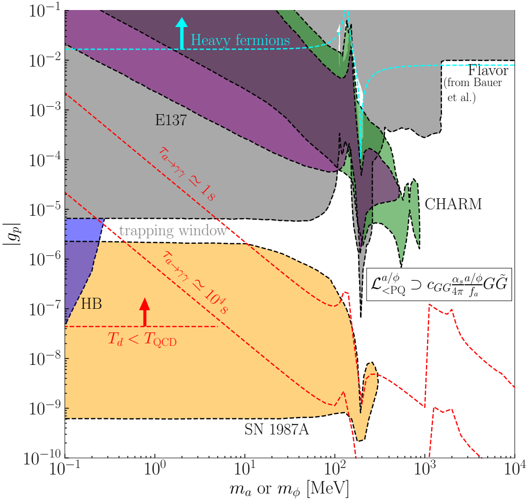

Model-dependent

- Horizontal Branch Stars.
- Supernova SN1987A.
- Rare meson decays.
- Beam-dump experiments : CHARM, E137.
- Big Bang Nucleosynthesis
- Heavy fermions (model-dependent) $\mathcal{L}_{\mathrm{KSVZ}} \supset y_\star \Phi \bar{\psi}_L \psi_R + \mathrm{h.c.}$

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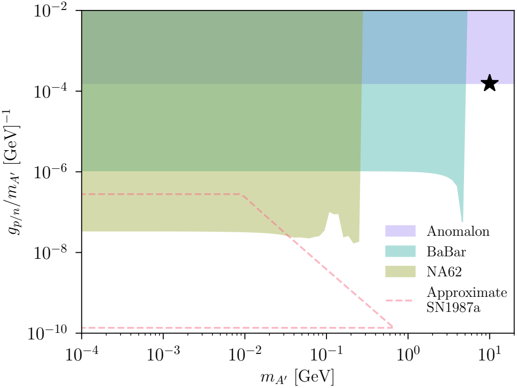
Constraints on A' mediator

<u>Model</u> : $\mathcal{L} \supset g' A'^{\mu} \left[\sum_q \bar{q}\gamma_\mu\gamma_5 q - 2\bar{\nu}\gamma_\mu P_R \nu - \frac{1}{2}\bar{\chi}\gamma_\mu\gamma_5\chi \right],$



<u>Model-dependent</u>

- Supernova SN1987A.
- NA62 : $K \to \pi A' \to \pi\nu\bar{\nu}$
- BaBar : $B \to K A' \to K\nu\bar{\nu}$.
- Avoid gauge anomalies
  $\Rightarrow$ Heavy fermions in UV completion
  $\Rightarrow$ Anomalons (LHC) :
  $$g_p \lesssim \frac{m_{A'}}{5\ \text{TeV}}$$

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★ is our chosen benchmark for direct detection.

Self-interacting DM constraints on g_χ I

$$\sigma_V = \int d\Omega \, \frac{d\sigma}{d\Omega}(1 - \cos^2\theta) \lesssim 1.1\,\mathrm{cm}^2\mathrm{g}^{-1} m_\chi$$

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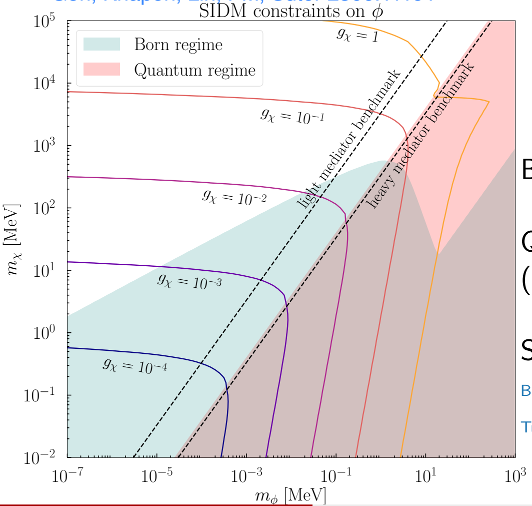


$$R \equiv \frac{m_\chi v}{m_{\rm med}}$$

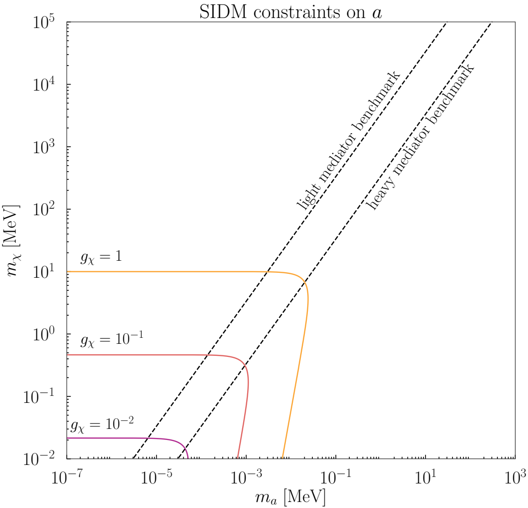
Born regime : $g_\chi^2 m_\chi/(4\pi m_\phi) \ll 1.$

Quantum regime : $R \lesssim 1$
(S-wave scattering dominates).

Semi-classical regime : $R \gtrsim 1.$

B. Colquhoun, S. Heeba, F. Kahlhoefer, L. Sagunski, and S. Tulin, Phys. Rev. D 103, 035006 (2021)

Self-interacting DM constraints on g_χ II



P. Agrawal, A. Parikh, and M. Reece, JHEP 10, 191 (2020)

Born approximation works everywhere.

Terms in the non-relativistic potential that could potentially have given rise to Sommerfeld enhancement are suppressed by $m_{\mathsf{a}}^2/m_\chi^2$ when $m_{\mathsf{a}} \ll m_\chi$.

Weaker bounds than on ϕ due to additional velocity suppression in σ_V.

Coupling to a nucleus

Form factors f_p, f_n are estimated by using the Odd Group Model or computed more precisely using the shell model. The crystals that we consider have $f_p \gg f_n$.

$$\mathcal{H}^N = -\frac{g_\chi g_p f_p}{q_0^2 + m_{\text{med}}^2} F(\mathbf{q}) \mathcal{O}\left(\mathbf{J}_\chi, \mathbf{q}\right) \cdot \mathbf{J}\, e^{i\mathbf{q}\cdot\mathbf{r}}$$

where $\mathcal{O}_\phi\left(\mathbf{J}_\chi, \mathbf{q}\right) = \frac{\mathbf{q}}{m_p}$, $\mathcal{O}_a\left(\mathbf{J}_\chi, \mathbf{q}\right) = \left(\mathbf{J}_\chi \cdot \mathbf{q}\right)\frac{\mathbf{q}}{m_p m_\chi}$, $\mathcal{O}_{A'}^{\text{heavy}}\left(\mathbf{J}_\chi, \mathbf{q}\right) = 4\mathbf{J}_\chi$.
The (momentum) form factor is given by

$$F(\mathbf{q}) = \frac{q_0^2 + m_{\text{med}}^2}{q^2 + m_{\text{med}}^2}$$

Scattering calculation

Dark matter couples to all atoms in the crystal :

$$\mathcal{H}^c = -\frac{g_\chi g_p}{q_0^2 + m_{\text{med}}^2} F(\mathbf{q}) \sum_{\ell,d} (f_p)_{\ell,d} \mathcal{O}\left(\mathbf{J}_\chi, \mathbf{q}\right) \cdot \mathbf{J}_{\ell,d}\, e^{i\mathbf{q}\cdot\mathbf{r}_{\ell,d}}.$$

l indexes the unit cells in the lattice.
d indexes the atoms in a unit cell.

Scattering calculation

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Fermi's golden rule :

$$\Gamma = \frac{2\pi}{V} \sum_{i,f} \sum_{i_\chi, f_\chi} w_i w_{i_\chi} \int \frac{d^3\mathbf{q}}{(2\pi)^3} |\langle f, f_\chi | \mathcal{H}^c(\mathbf{q}) | i, i_\chi \rangle|^2 \delta(E_f - \omega - E_i)$$

$|i\rangle$ is the initial state of crystal, $|i_\chi\rangle$ is the initial DM spin.
w_i, w_{i_χ} are weights for the initial states. ω is the energy transferred.

Scattering calculation II

- Assume that the phonons do not affect the nuclei spins $\Rightarrow$ factorize spin and phonon states such that $|i\rangle = \underbrace{|i_s\rangle}_{\text{spin}} \otimes \underbrace{|0\rangle}_{\text{phonon g.d state}}$.
- Assume that nuclear,DM spins are randomly oriented and unpolarized.
- Neglect higher-order phonon interactions e.g. 3-point (can be important in $m_\chi \simeq 1 - 10$ MeV range).

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Therefore,

$$\boxed{\Gamma = \frac{g_\chi^2 g_p^2}{(q_0^2 + m^2)^2} \frac{N}{V} \int \frac{d^3\mathbf{q}}{(2\pi)^3} |F(\mathbf{q})|^2 G(\mathbf{q}) S(\mathbf{q}, \omega)}$$

Dark matter spin form factor:
$G(\mathbf{q}) \equiv \frac{1}{3} \sum_{i_\chi} w_{i_\chi} \langle i_\chi | \mathcal{O}(\mathbf{J}_\chi, \mathbf{q}) \cdot \mathcal{O}(\mathbf{J}_\chi, \mathbf{q}) | i_\chi \rangle$

N is the number of unit cells, V is the volume of the crystal.

Target-specific : encapsulates the response of the target to DM coupling to nucleons.

$$S(\mathbf{q},\omega) \equiv \frac{1}{N}\sum_{\ell,d}(f_p^2)_{\ell,d} C_{\ell,d}(\mathbf{q},\omega) \underbrace{\sum_{i_s} w_{i_s}\langle i_s|\mathbf{J}_{\ell,d}\cdot\mathbf{J}_{\ell,d}|i_s\rangle}_{\text{spin response}}$$

$C_{\ell,d}(\mathbf{q},\omega) \equiv \int_{-\infty}^{+\infty} dt\, \langle 0|e^{-i\mathbf{q}\cdot\mathbf{r}_{\ell,d}(0)} e^{i\mathbf{q}\cdot\mathbf{r}_{\ell,d}(t)}|0\rangle e^{i\omega t}$ characterizes the response of the phonons.

The auto-correlation function $C_{\ell,d}$ is an expansion over the number of phonons. Each phonon contributes $\frac{q}{\sqrt{2m_d\bar{\omega}_d}}$.

B. Campbell-Deem, S. Knapen, T. Lin, and E. Villarama, Phys. Rev. D 106, 036019 (2022)

Rate per target mass

Folding in the DM velocity, multiplying by the number of DM particles in the target and dividing by the target mass, we get

$$R = \frac{1}{\rho_T}\frac{\rho_\chi}{m_\chi}\int d^3\mathbf{v}\, f(\mathbf{v})\,\Gamma(v).$$

Results are implemented using DarkElf package.

<u>Cross sections</u>

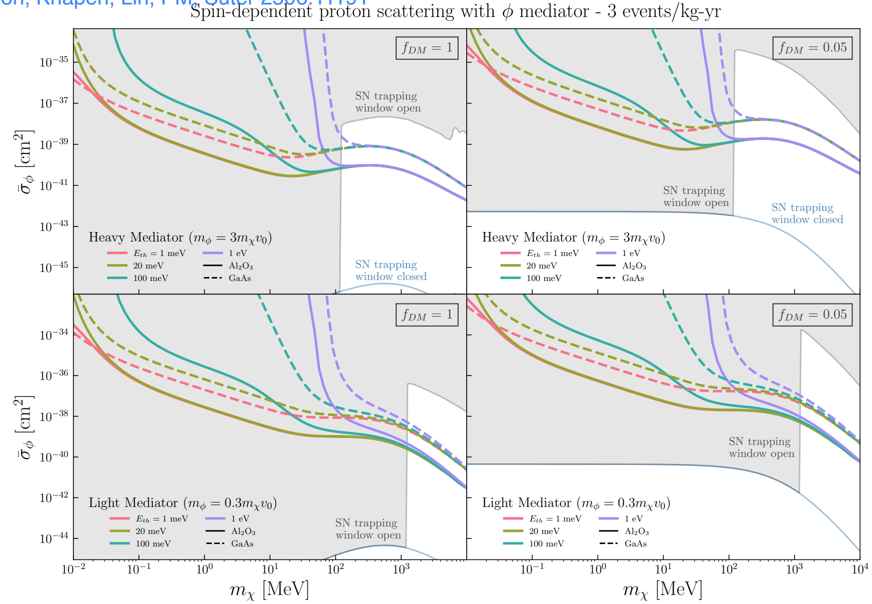
$$\bar{\sigma}_\phi = \frac{g_p^2 g_\chi^2}{2\pi m_p^2}\frac{v_0^2\mu_{p\chi}^4}{(q_0^2+m_\phi^2)^2}$$

$$\bar{\sigma}_a = \frac{g_p^2 g_\chi^2}{3\pi m_p^2 m_\chi^2}\frac{v_0^4\mu_{p\chi}^6}{(q_0^2+m_a^2)^2}$$

$$\bar{\sigma}_{A'}^{\text{heavy}} = 3\frac{g_p^2 g_\chi^2}{\pi}\frac{\mu_{p\chi}^2}{m_{A'}^4}$$

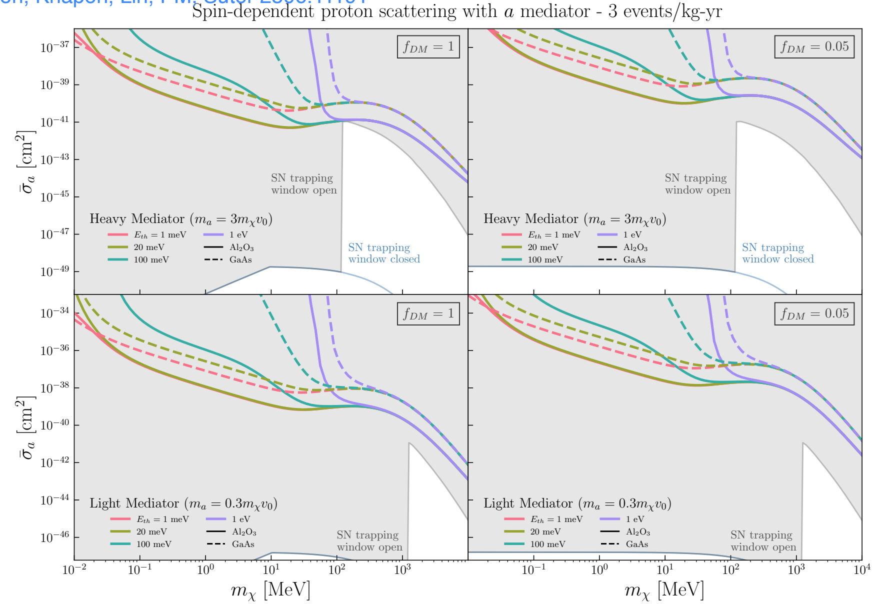
Direct Detection cross sections (ϕ mediator)

Gori, Knapen, Lin, PM, Suter 2506.11191



Direct Detection cross sections (a mediator)

Gori, Knapen, Lin, PM, Suter 2506.11191



Direct Detection cross sections (A' mediator)

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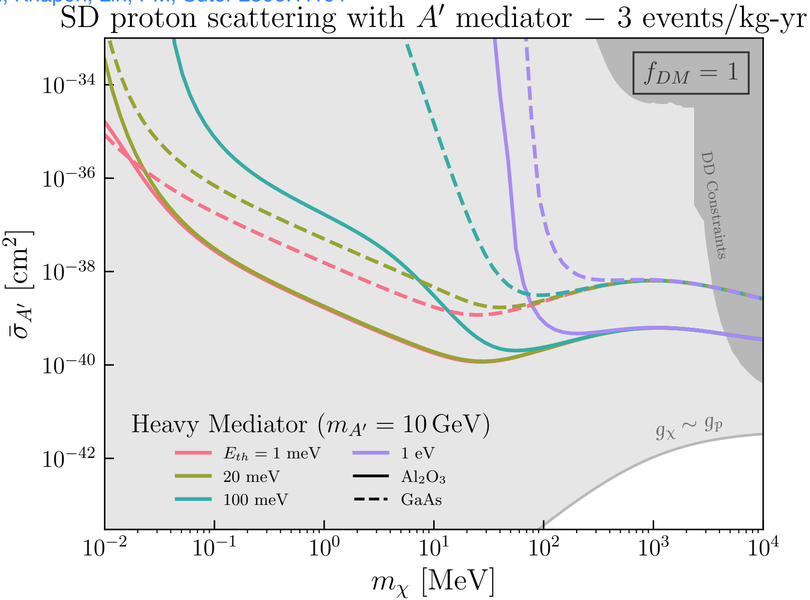

Conclusion

Spin dependent (SD) interactions appear in theoretically well-motivated models such as axions.

The phonon response of crystals offers an additional probe of SD interactions.

Multiple phonons can allow for a larger energy threshold than having only 1-phonon.

Backup slides (Scattering I)

$$\Gamma = \frac{1}{V}\int_{-\infty}^{+\infty} dt \int \frac{d^3\mathbf{q}}{(2\pi)^3} \sum_{i,f}\sum_{i_\chi, f_\chi} w_i w_{i_\chi} \langle i, i_\chi | \mathcal{H}^{c\dagger} | f, f_\chi\rangle \times \langle f, f_\chi | e^{-iE_f t}\mathcal{H}^c e^{iE_i t} | i, i_\chi\rangle e^{i\omega t}.$$

$$\Gamma = \frac{1}{V}\int_{-\infty}^{+\infty} dt\, e^{i\omega t} \int \frac{d^3\mathbf{q}}{(2\pi)^3} \sum_{i,i_\chi} w_i w_{i_\chi} \langle i, i_\chi | \mathcal{H}^c(0)^\dagger \mathcal{H}^c(t) | i, i_\chi\rangle.$$

$$\Gamma = \frac{1}{V}\frac{g_\chi^2 g_p^2}{(q_0^2 + m_{\text{med}}^2)^2} \int_{-\infty}^{+\infty} dt\, e^{i\omega t} \int \frac{d^3\mathbf{q}}{(2\pi)^3} |F_{\text{med}}(\mathbf{q})|^2 \times$$

$$\sum_{\alpha,\beta=x,y,z}\left[\sum_{i_\chi} w_{i_\chi} \langle i_\chi | \mathcal{O}(\mathbf{J}_\chi, \mathbf{q})^\alpha \, \mathcal{O}(\mathbf{J}_\chi, \mathbf{q})^\beta \, | i_\chi\rangle\right] \times$$

$$\left[\sum_i w_i \sum_{\ell,d;\ell',d'} \lambda_{\ell,d}\lambda_{\ell',d'} \langle i | J_{\ell,d}^\alpha e^{-i\mathbf{q}\cdot\mathbf{r}_{\ell,d}(0)} J_{\ell',d'}^\beta e^{i\mathbf{q}\cdot\mathbf{r}_{\ell',d'}(t)} | i\rangle\right].$$

$$J_{\ell,d}^\alpha J_{\ell',d'}^\beta \to \frac{1}{3}\delta^{\alpha\beta}\delta_{\ell\ell'}\delta_{dd'}\mathbf{J}_{\ell,d}^2.$$

Backup slides (Scattering II)

$$S(\mathbf{q},\omega) = \sum_d \overline{\lambda_d^2 J_d(J_d+1)} C_{\ell,d}(\mathbf{q},\omega),$$

$$C_{\ell,d}(\mathbf{q},\omega) = 2\pi e^{-2W_d(\mathbf{q})} \sum_{n=1}^{\infty} \frac{1}{n!} \left(\frac{q^2}{2m_d}\right)^n \times \prod_{i=1}^{n} \int d\omega_i \frac{D_d(\omega_i)}{\omega_i} \delta\left(\sum_{i=1}^{n} \omega_i - \omega\right).$$

Backup slides (Aprime UV)

We must include a set of heavy fermions to cancel the gauge anomalies, a scalar field ϕ which breaks the $U(1)'$ spontaneously, and two new Higgs inert doublets H'_u and H'_d, which are responsible for generating the SM Yukawa interactions.

$$\mathcal{L} \supset \lambda_Q \phi Q' \bar{Q}' + \lambda_u \phi^\dagger u' \bar{u}' + \lambda_d \phi^\dagger d' \bar{d}' + \frac{\phi^2}{\Lambda_N} N' \bar{N}'. \tag{1}$$

The LHC bounds on the colored fermions $Q', \bar{Q}'$, etc., depend on their assumed decay and production channels. We therefore require that their masses $\gtrsim 1$ TeV for all twelve vector-like quarks. Thus we need $\langle \phi \rangle \gtrsim 1$ TeV to satisfy bounds on the anomalons. The field ϕ also generates the mass of the A', such that we can obtain the bound

$$g' \lesssim \frac{1}{\sqrt{2}} \frac{m_{A'}}{\mathrm{TeV}}. \tag{2}$$

In particular, to cancel the gauge anomalies we need a pair of Weyl fermions $\chi_{1,2}$ with charge $-1/2$ and along with a pair $\bar{\chi}_{1,2}$ with charge $1/2$. The dark sector Yukawa interactions are

$$y_\chi \phi \chi_1 \chi_2 + \bar{y}_\chi \phi^\dagger \bar{\chi}_1 \bar{\chi}_2 \ + \mathrm{h.c.} \tag{3}$$

We identify the Dirac fermion composed out of the Weyl fermions $\chi \equiv (\chi_1, \chi_2^\dagger)$ with the dark matter.

Backup slides (Aprime UV)

	SU(3)	SU(2)	$U(1)_Y$	$U(1)'$
Q	$\square$	$\square$	1/6	$+1$
u	$\overline{\square}$	1	-2/3	$+1$
d	$\overline{\square}$	1	1/3	$+1$
L	1	$\square$	-1/2	0
e	1	1	$+1$	0
N	1	1	0	$+2$
H	1	$\square$	-1/2	0
Q'	$\square$	$\square$	1/6	-1
u'	$\overline{\square}$	1	-2/3	-1
d'	$\overline{\square}$	1	1/3	-1
$\bar{Q}'$	$\overline{\square}$	$\square$	-1/6	0
$\bar{u}'$	$\square$	1	2/3	0
$\bar{d}'$	$\square$	1	-1/3	0
N'	1	1	0	-2
$\bar{N}'$	1	1	0	0
H'_u	1	$\square$	1/2	-2
H'_d	1	$\square$	-1/2	-2
ϕ	1	1	0	$+1$
χ_1	1	1	0	$-1/2$
χ_2	1	1	0	$-1/2$
$\bar{\chi}_1$	1	1	0	$1/2$
$\bar{\chi}_2$	1	1	0	$1/2$

Table: All fermion fields are chosen to be left-handed Weyl fermions by convention. The second block contains a set of heavy fermions. ϕ is a scalar field which generates the masses of the anomalons, the A' and the dark matter. The third block contains the dark matter and a set of heavier dark fermions which cancel the dark matter's contribution the $U(1)'$ gauge anomalies.