ePIC DVCS event variables

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1 Deeply Virtual Compton Scattering

Deeply Virtual Compton Scattering (DVCS) is the procedure by which an electron can scatter off a proton, producing an additional photon in the final state. The Feynman diagram for the DVCS process is in figure 1.

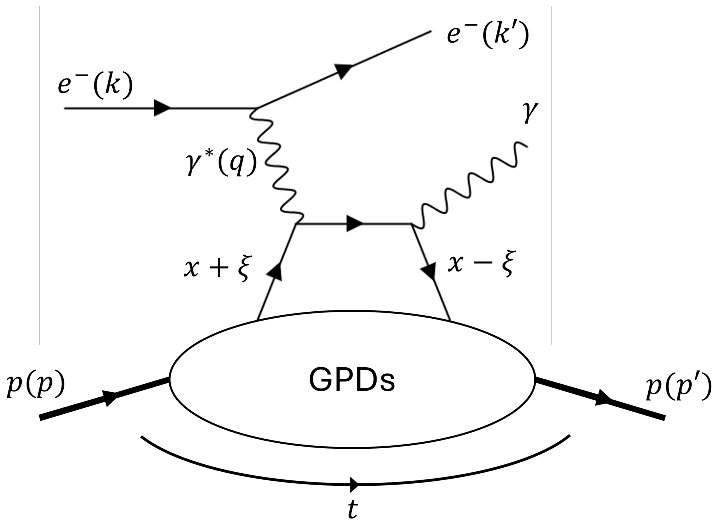

Figure 1: Handbag diagram of DVCS on the proton.

The DVCS interaction is usually written as

$$e(k) + p(p) \rightarrow e'(k') + p'(p') + \gamma \,,$$

where the 'unprimed' variables represent the incoming beams (or electron beam and fixed proton target), the 'primed' variables are the scattered electron and proton, and $q = k - k'$ is the 4-momentum carried by the virtual photon.

2 Electron vertex kinematics

These kinematic variables describe the interaction behaviour at the scattering electron vertex. The variables discussed here are:

- the 4-momentum transfer squared, carried by the virtual photon - Q^2,
- the event inelasticity - y, and
- the Bjorken scaling variable - x or x_B.

Whilst there are more ways to calculate these variables than listed below, only those methods which are implemented into the EICrecon reconstruction framework are actually discussed. It should also be noted that academic references for some of the methods listed below differ from the stated formulae; this note instead lists the equations used by EICrecon for such calculations.

2.1 Electron method

This could be considered the 'default' method to calculate the electron vertex kinematics. The formulae listed below depend on the initial state electron and proton (k and p respectively), and the scattered electron k'. EICrecon uses the electron method on the raw Monte Carlo events in order to create a comparative 'truth' within the InclusiveKinematicsTruth branch of its output trees.

$$Q^2 = -(k - k')^2$$

$$x = \frac{Q^2}{2q.p}$$

$$y = \frac{q.p}{k.p}$$

2.2 JB method

The Jaquet-Blondel (JB) method for reconstructing kinematics relies on the detection of all final state particles that are *not* the scattered beam electron. This collection is referred to as the hadronic final state (HFS), and in the case of DVCS is the sum of the scattered proton and real photon 4-vectors (p' and γ). For the calculations of Q^2, x and y, the following quantities need to be calculated:

- the transverse momentum of the HFS, $p_{T,h}$

$$p_{T,h}^2 = (\Sigma\, p_x)^2 + (\Sigma\, p_y)^2$$

- the energy imbalance of the HFS, Σ_h

$$\Sigma_h = \Sigma\, E - \Sigma\, p_z$$

This leads to the following equations for Q^2, x and y, where E_e and E_p are the energies of the electron and proton beams respectively.

$$y = \frac{\Sigma_h}{2E_e}$$

$$Q^2 = \frac{p_{T,h}^2}{1 - y}$$

$$x = \frac{Q^2}{4E_e E_p y}$$

2.3 DA method

The Double Angle (DA) method reconstructs kinematic variables without using any energy measurements, relying only on the angles made by tracks with respect to the z-axis. This makes the DA method a powerful tool if the detectors being used have poor energy or track momentum resolution. The equations used in the DA method introduce the angle of the electron track with respect to the z-axis, θ_e, and use the definitions of $p_{T,h}$ and Σ_h as listed previously to calculate an angle γ for the HFS,

$$\gamma = \cos^{-1}\left(\frac{p_{T,h}^2 - \Sigma_h^2}{p_{T,h}^2 + \Sigma_h^2}\right)$$

Using these angles, the electron vertex kinematics can be expressed as

$$y = \frac{\tan(\theta_e/2)}{\tan(\theta_e/2) + \tan(\gamma/2)}$$

$$Q^2 = \frac{4E_e^2}{\tan(\theta_e/2)} \frac{1}{\tan(\theta_e/2) + \tan(\gamma/2)}$$

$$x = \frac{Q^2}{4E_e E_p y}$$

2.4 Σ method

Another method which combines information from the scattered electron and HFS is the Sigma (Σ) method. Introduced in the Σ method are the transverse momentum of the scattered electron track, $p_{T,e}$, and the electron energy imbalance of the scattered electron, $\Sigma_e = E - p_z$. This is then used to calculate a total energy imbalance $\Sigma_{tot} = \Sigma_e + \Sigma_h$, giving formulae for Q^2, x and y.

$$y = \frac{\Sigma_h}{\Sigma_{tot}}$$

$$Q^2 = \frac{p_{T,e}^2}{1-y}$$

$$x = \frac{Q^2}{4E_e E_p y}$$

2.5 e-Σ method

The e-Sigma (e-Σ) method is another way to calculate kinematics using a combination of the information from the scattered electron and HFS. It uses formulae from the electron method to calculate Q^2 and from the Σ method to calculate x, then combines those to calculate y.

- Using the electron method,

$$y_e = 1 - \frac{\Sigma_e}{2E_e}$$

$$Q_e^2 = \frac{p_{T,e}^2}{1-y_e}$$

- Using the Σ method,

$$y_\Sigma = \frac{\Sigma_h}{\Sigma_{tot}}$$

$$Q_\Sigma^2 = \frac{p_{T,e}^2}{1-y_\Sigma}$$

$$x_\Sigma = \frac{Q_\Sigma^2}{4E_e E_p y_\Sigma}$$

- Combining the two,

$$y_{e\Sigma} = \frac{Q_e^2}{4E_e E_p x_\Sigma}$$

2.6 Comparing methods

Figure 2 compares the InclusiveKinematicsTruth branch of the EICrecon output trees with manual calculations of Q^2, x and y performed with all of the methods listed above. For this comparison, 100 files were used from the ePIC simulation campaign, using EICrecon version 25.01.1. In total, these files contain 147,847 DVCS events, created using the 10x100 GeV beam setting.

As can be seen in figure 2, all of the kinematic reconstruction methods work well on the raw Monte Carlo candidates. The only notable discrepancy is that the JB method introduces a non-negligible low-Q^2 shoulder to the distribution; as the generated events have a minimum Q^2 of 1 GeV^2, this shoulder is unphysical.

Figures 3 to 7 show the values for Q^2, x and y, calculated both manually and by EICrecon, using all of the methods discussed above. In addition to these, all figures show the same distributions from the 'Truth' branch as consistent comparison plots.

As would be expected, the values of Q^2, x and y from the electron method agree very closely between the manual calculations and the values automatically calculated in EICrecon. Regarding the methods which involve the HFS, the histograms suggest that EICrecon reconstructs around twice the number of events that manual selection yields. This is likely due to poor proton reconstruction efficiency; a combination of geometric effects and current RP reconstruction algorithms which require all layers of the Roman Pots to be hit in order for a proton track to be recognised.

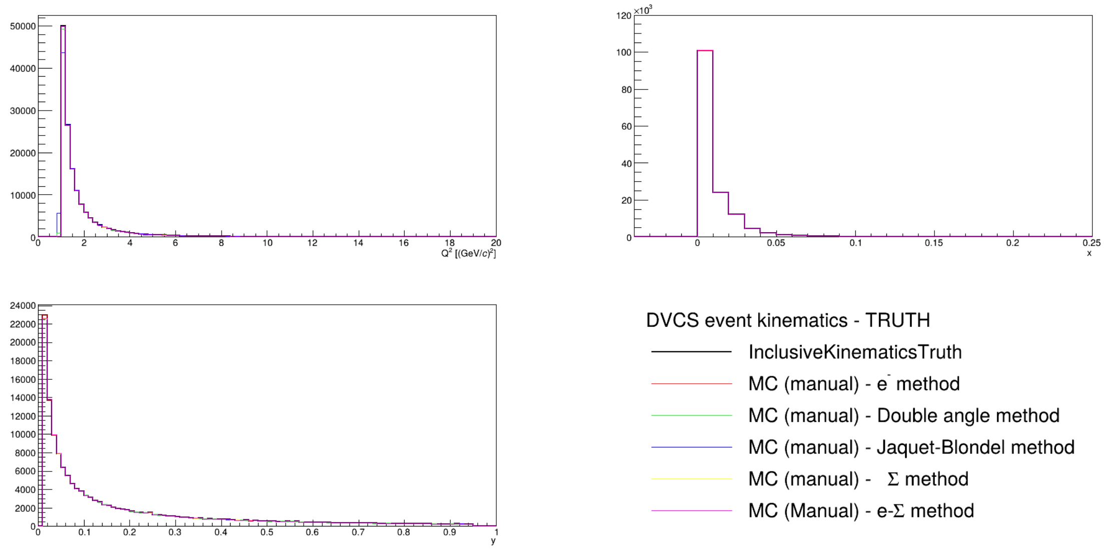

Figure 2: Comparing different methods of calculating Q^2 (top left), x (top right) and y (bottom left) with the “truth” information from eicRecon.

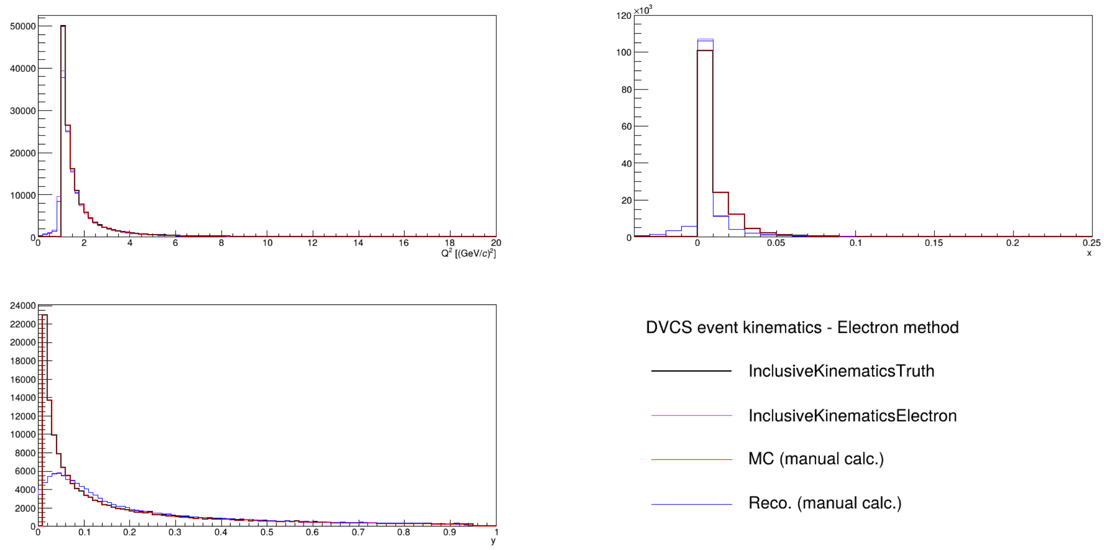

Figure 3: Plots of Q^2 (top left), x (top right) and y, comparing the InclusiveKinematics ‘truth’ data (black) with that reconstructed by EICrecon (purple), or manually calculated (using MC particles - red - or reconstructed particles - blue) using the electron method.

3 Proton 4-momentum transfer, t

At the proton vertex, the Mandelstam variable t denotes the 4-momentum transfer between the scattered proton and the beam proton. It is one of the parameters which Generalised Parton Distributions (GPDs) depend on, alongside x, and the fractional change in parton momentum, ξ.

The default way to calculate t is to use the detected final state proton:

$$t = (p' - p)^2 \ , \tag{1}$$

however, this is not the sole method for such a calculation. Using conservation of momentum, one can re-write equation 1 without the need to measure the scattered proton:

$$t = (k - k' - \gamma)^2 \, , \tag{2}$$

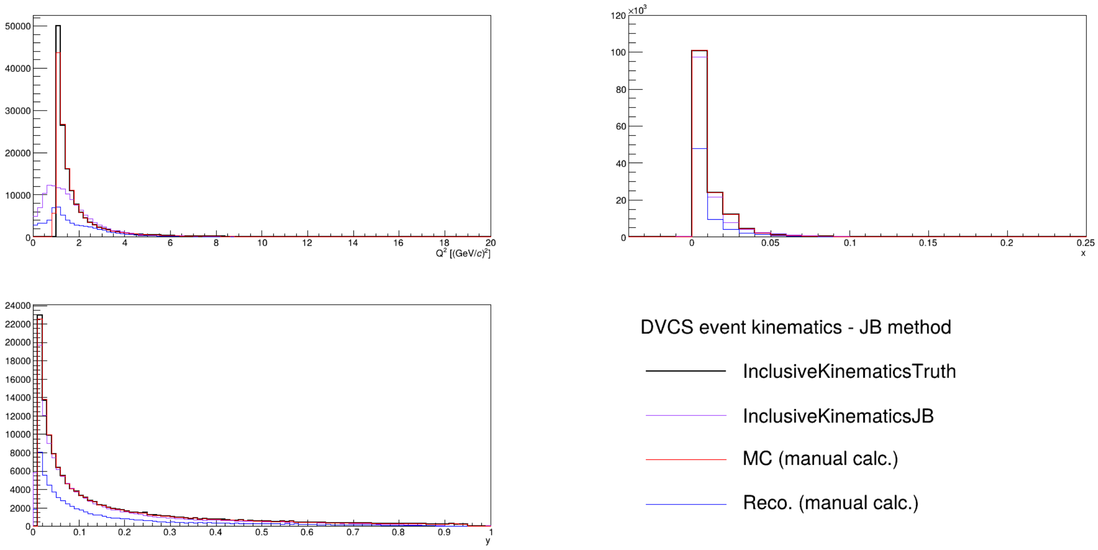

Figure 4: Plots of Q^2 (top left), x (top right) and y, comparing the InclusiveKinematics 'truth' data (black) with that reconstructed by EICrecon (purple), or manually calculated (using MC particles - red - or reconstructed particles - blue) using the JB method.

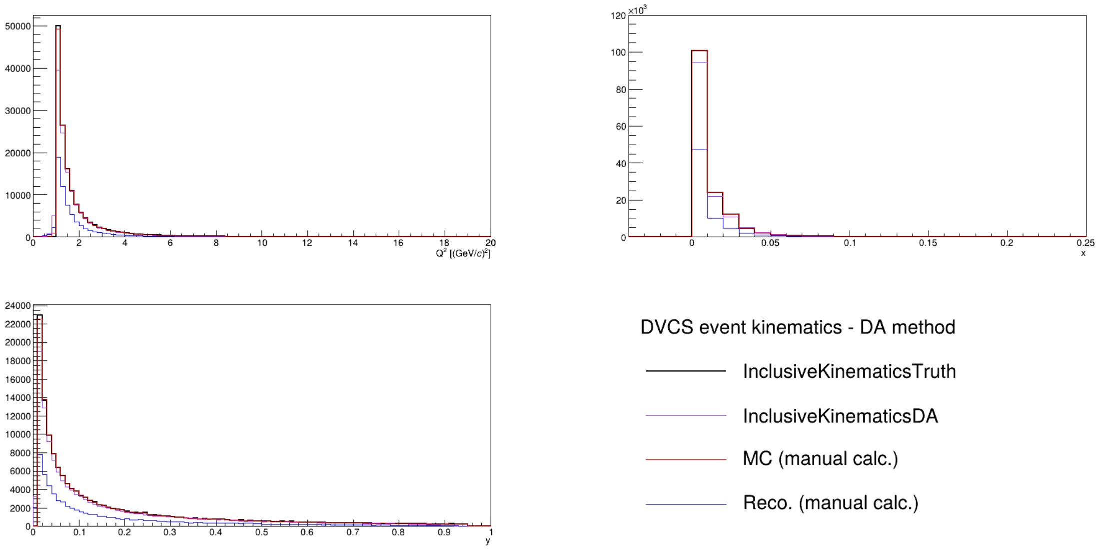

Figure 5: Plots of Q^2 (top left), x (top right) and y, comparing the InclusiveKinematics 'truth' data (black) with that reconstructed by EICrecon (purple), or manually calculated (using MC particles - red - or reconstructed particles - blue) using the DA method.

where γ is the real photon 4-vector.

Another method by which t can be calculated is given in the ECCE exclusive, diffractive and tagging physics paper (Bylinkin, A. *et al.*, *Detector requirements and simulation results for the EIC exclusive, diffractive and tagging physics program using the ECCE detector concept*, NIM. A., 2023), within the context of DVCS on e-He. On the assumption that the ion remnant is not detected, t can be calculated using the scattered electron and photon information:

$$t = -\frac{MQ + 2M\nu\left(\nu - \sqrt{\nu^2 + Q^2}\cos\left(\theta_{\gamma^*\gamma}\right)\right)}{M + \nu - \sqrt{\nu^2 + Q^2}\cos\left(\theta_{\gamma^*\gamma}\right)}, \tag{3}$$

where $\nu = E - E'$ is the difference between the energy of the beam and scattered electron; $M = m_p$ is the mass of the proton, and $\theta_{\gamma^*\gamma}$ is the angle between the virtual and real photon vectors.

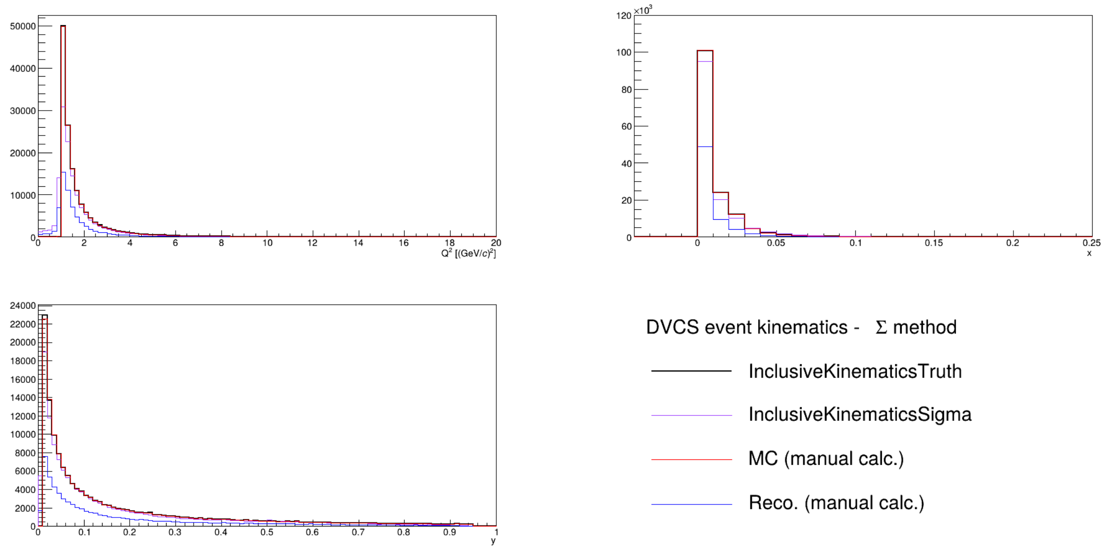

Figure 6: Plots of Q^2 (top left), x (top right) and y, comparing the InclusiveKinematics 'truth' data (black) with that reconstructed by EICrecon (purple), or manually calculated (using MC particles - red - or reconstructed particles - blue) using the Σ method.

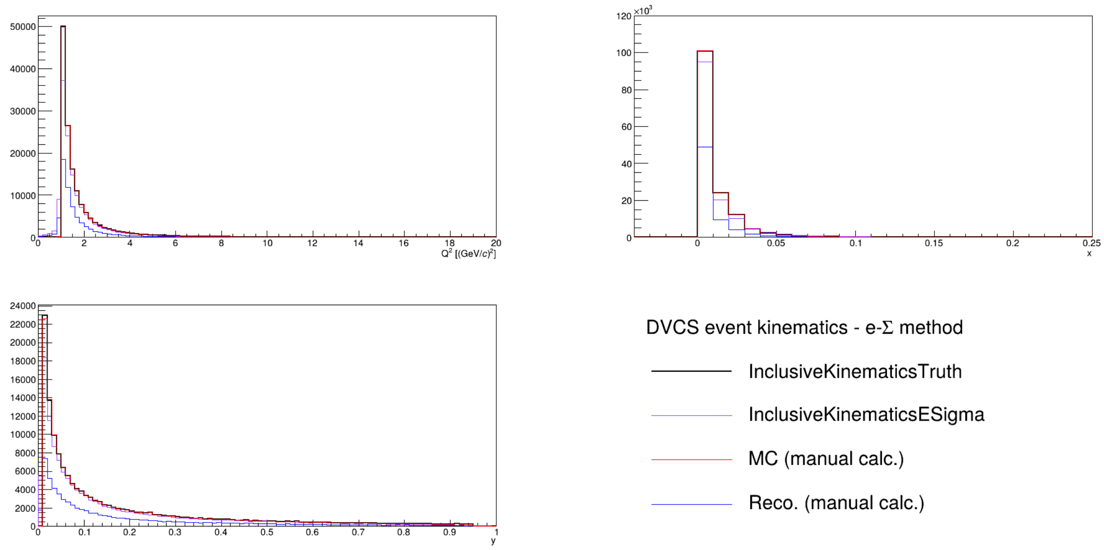

Figure 7: Plots of Q^2 (top left), x (top right) and y, comparing the InclusiveKinematics 'truth' data (black) with that reconstructed by EICrecon (purple), or manually calculated (using MC particles - red - or reconstructed particles - blue) using the e-Σ method.

Figure 8 shows the distribution of t calculated with equations 1, 2 and 3, using the same event sample as used for the inclusive kinematic method tests. Upon intial viewing, figure 8 suggests that the e-He method is much more reliable than the 'default' calculation for t, however, this requires further investigation.

4 Other variables: ϕ_h

Cross-sections and asymmetries measured with the DVCS process are normally expressed as a function of the angle between the leptonic and hadronic planes, ϕ_h. Expressions for this quantity are given in Bachetta, A. *et al.*, *Single-spin asymmetries: the Trento conventions*, Phys. Rev. D 70, 2004. The

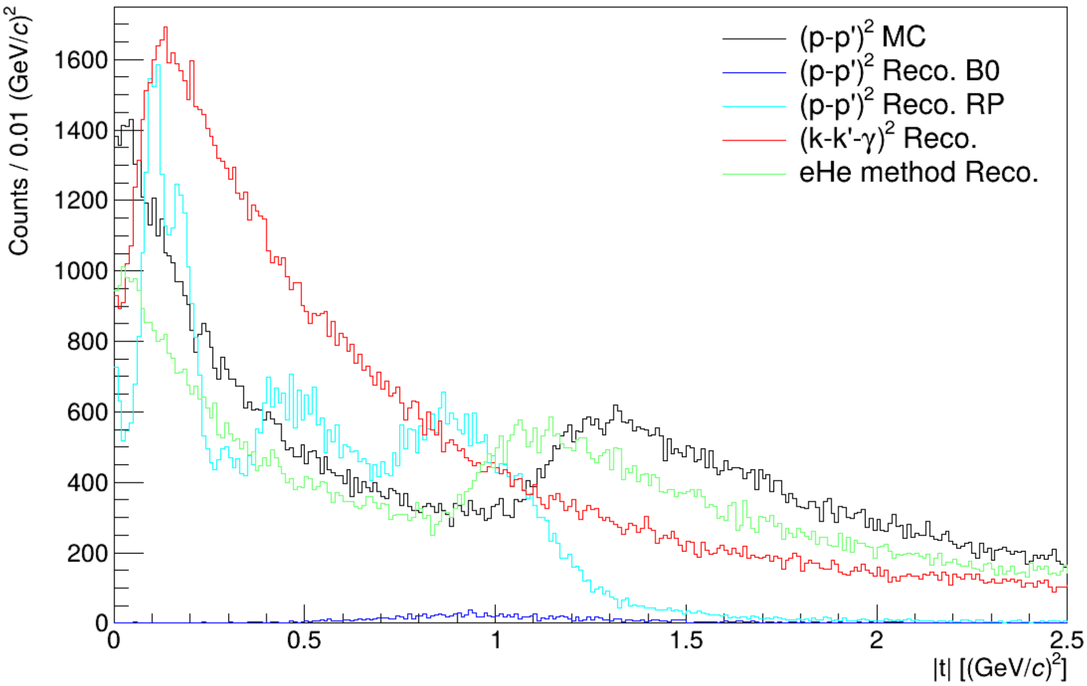

Figure 8: Comparing the value of t calculated with equations 1 (using MC particles, black; reconstructed protons within the B0, dark blue, and from tracks in the Roman Pots, light blue), 2 (red) and 3 (green).

simplest formalism for calculating ϕ_h is equation 16 in the above source:

$$\cos\phi_h = \frac{(\hat{\mathbf{q}}\times\mathbf{k})}{|\hat{\mathbf{q}}\times\mathbf{k}|}\cdot\frac{(\hat{\mathbf{q}}\times\mathbf{p'})}{|\hat{\mathbf{q}}\times\mathbf{p'}|}\,, \tag{4}$$

where k is equivalent to l and p' is the equivalent to P_h from the paper.

The complication involved with using equation 4 is that it is stated within the 'target rest frame'. For fixed target experiments (such as those at Jefferson Lab, where DVCS is a regularly studies channel), this is simply the lab frame and as such is simple to use. For ePIC, which uses colliding beams, this frame *should* be found by boosting by the beam proton 4-vector, however, this is made more complicated when considering the beam crossing angle and geometry of the different detector elements.

A Lorentz invariant form of equation 4 is also given as equation 17 in the paper:

$$\cos\phi_h = -\frac{g_\perp^{\mu\nu}k_\mu p'_\nu}{|k_\perp|\,|p'_\perp|}\,,$$

where $k_\perp = \sqrt{-g_\perp^{\mu\nu}k_\mu k_\nu}$ and $p'_\perp = \sqrt{-g_\perp^{\mu\nu}p'_\mu p'_\nu}$. Here, a perpendicular projection tensor $g_\perp^{\mu\nu}$ is introduced:

$$g_\perp^{\mu\nu} = g^{\mu\nu} - \frac{q^\mu p^\nu + p^\mu q^\nu}{p.q\,(1+\gamma^2)} + \frac{\gamma^2}{1+\gamma^2}\left(\frac{q^\mu q^\nu}{Q^2} - \frac{p^\mu p^\nu}{m_p^2}\,,\right)$$

with $\gamma = 2xm_p/Q$, where x is the normal Bjorken scaling variable and m_p is the mass of the proton. Studies are currently ongoing to test the validity of these different methods to calculate ϕ_h, and to decide which inertial frame to boost the detected particle 4-vectors into.

5 Other variables: event exclusivity

A common set of cut variables for exclusive analyses are 'missing' kinematics. Ostensibly, any exclusive analysis should involve the detection of all final state particles; from conservation of energy and momentum, the sum of all energies and momenta should be the same between the initial and final states. Any 'missing' energy, momentum or mass in the final state can insinuate that a particle that was produced in the interaction was not detected; by requiring these missing kinematic quantities to be small, one can reduce the amount of combinatorial background created from incomplete event reconstruction or misidentified final state particles.

The missing momentum of a DVCS event can be calculated by the vector components of the the initial and final state 4-vectors,

$$\underline{p}_{miss} = (\underline{k} + \underline{p}) - (\underline{k'} + \underline{p'} + \underline{\gamma})$$

and similarly the missing energy can be calculated with the energies of the beam and detected particles,

$$E_{miss} = (E_e + E_p) - (E_{e'} + E_{p'} + E_\gamma) \, .$$

From these 2 quantities, the missing mass of a DVCS event can be calculated:

$$M^2_{miss} = E^2_{miss} - \underline{p}^2_{miss} \, ,$$

which should be zero for a perfectly reconstructed event.