HET Seminar -BNL

Dark Matter Interactions in White Dwarfs

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2410.13908

AGENDA

01

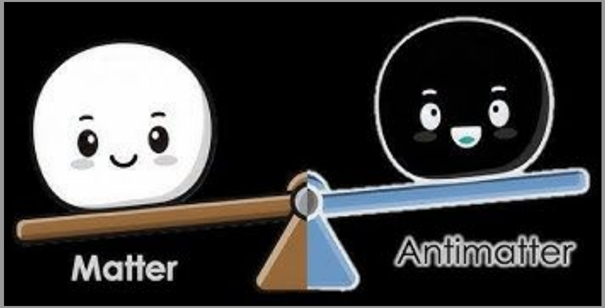
Motivation

Standard Model of particle physics

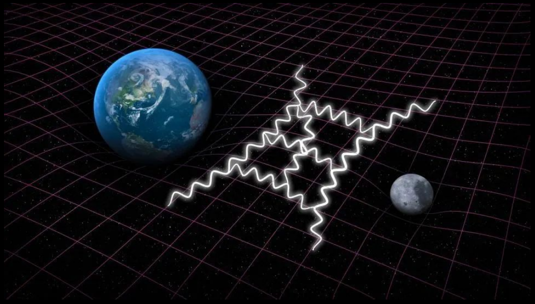
Successful theory, but it cannot explain...

Dominant matter component in the universe

Baryon asymmetry of the universe



Gravitational interactions at quantum level

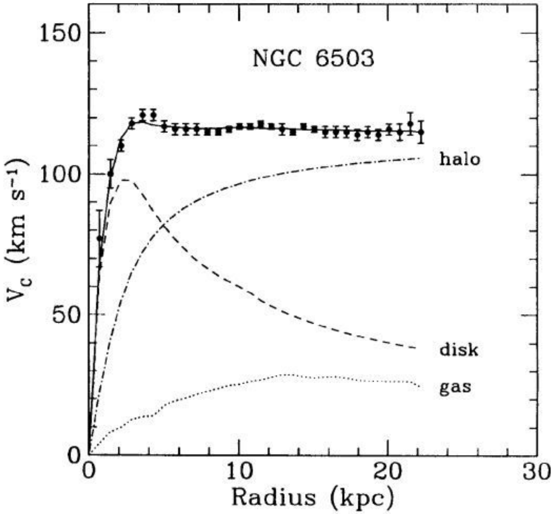


Neutrino oscillations

Why dark matter?

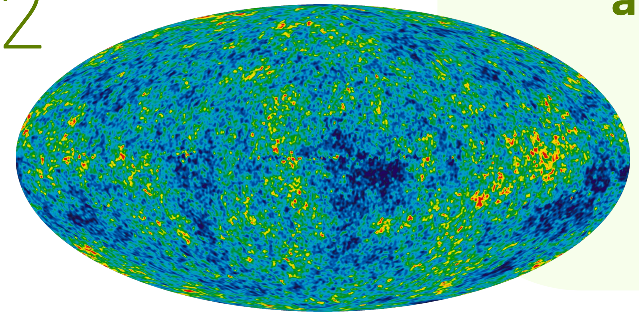
(or something that alters what we know)

01



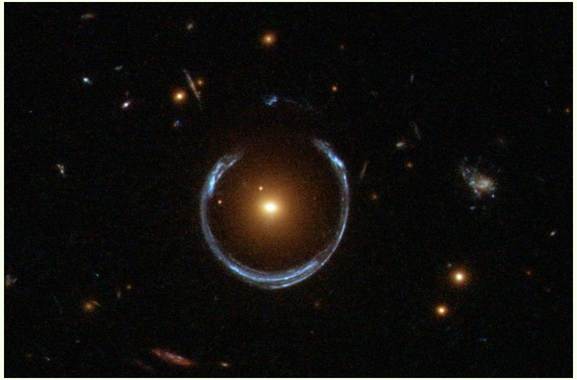
Galaxy rotation curves

02



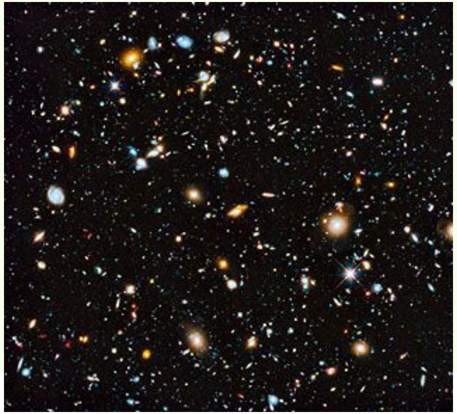
CMB

03



Gravitational lensing

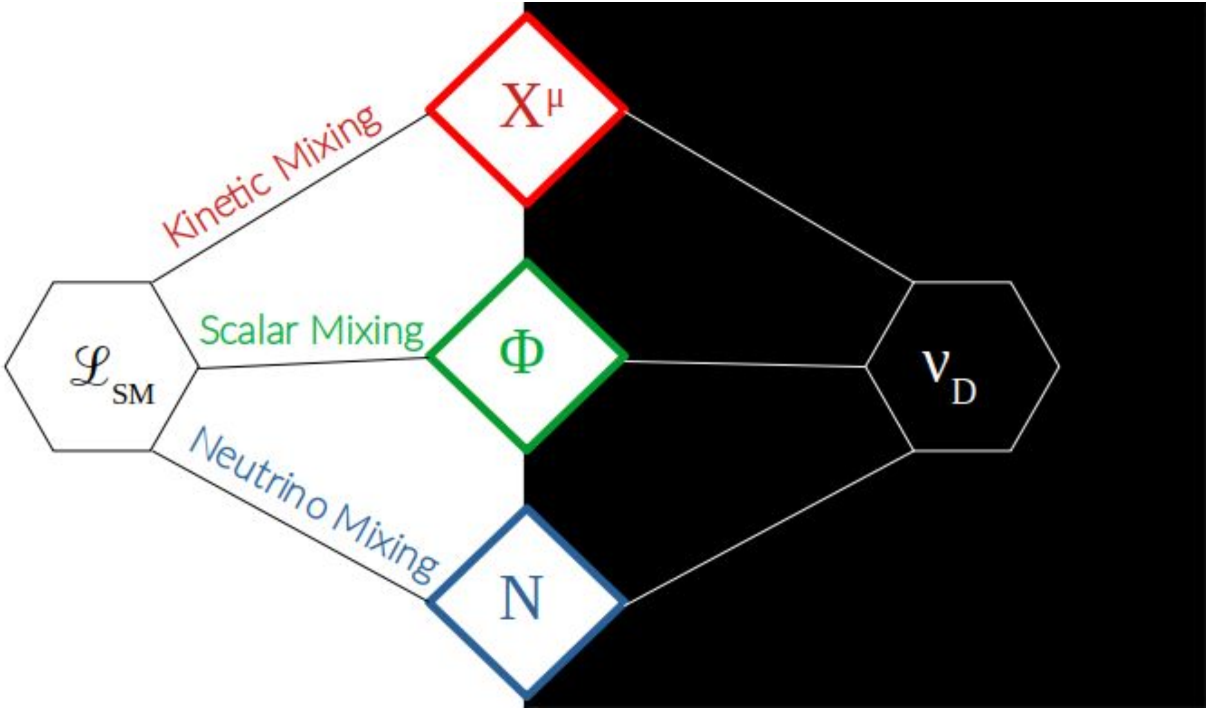
04



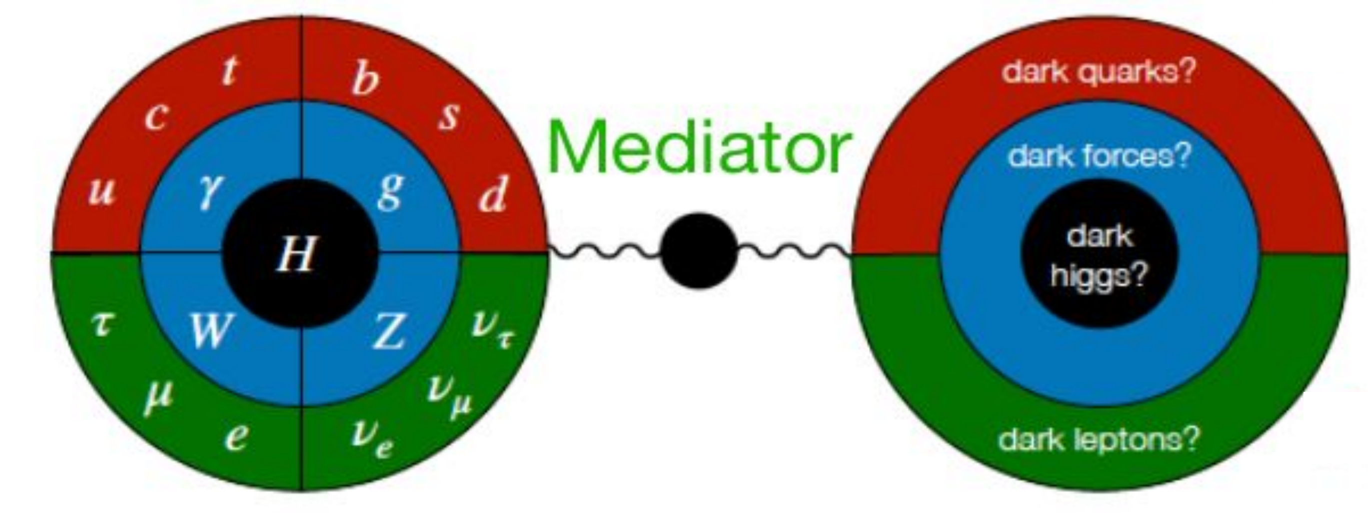
Large scale structure formation

Going even beyond...

Experimental anomalies also call for "dark" extensions of the SM.



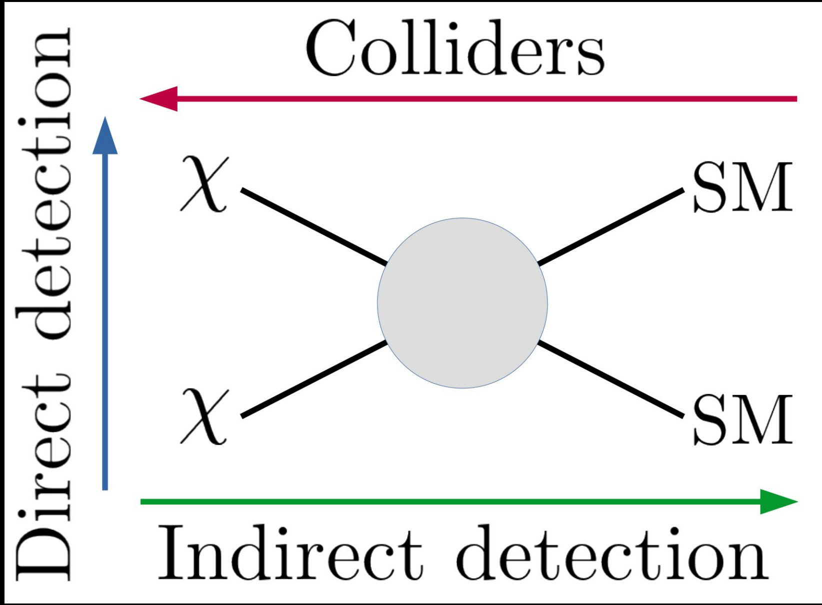
We are searching for new particles that account for those problems

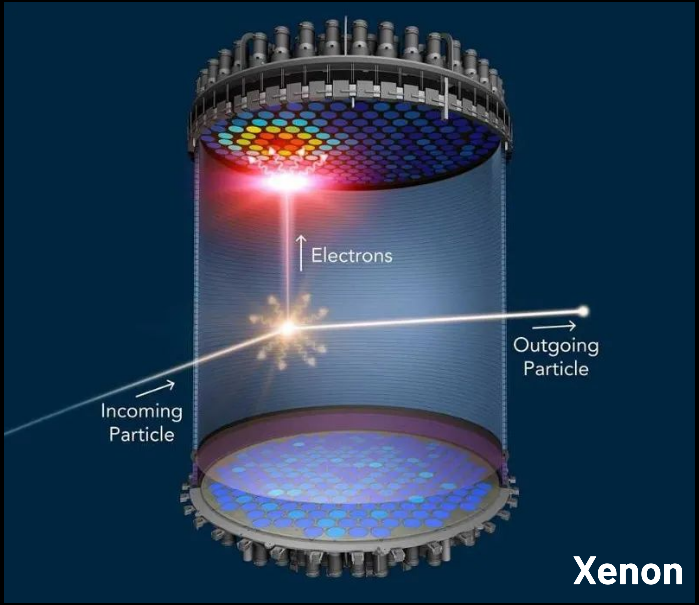


This is not the only kind of solution!

DM experiments

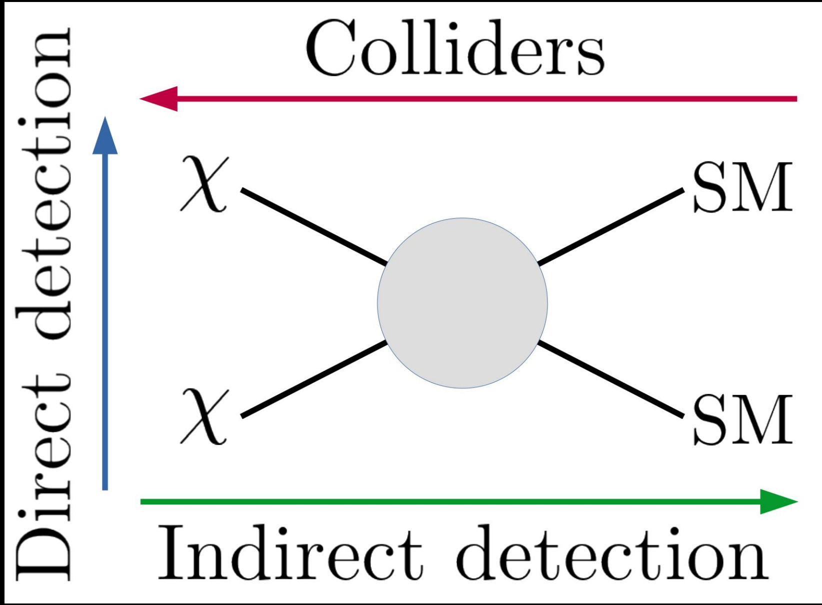
How to detect DM?

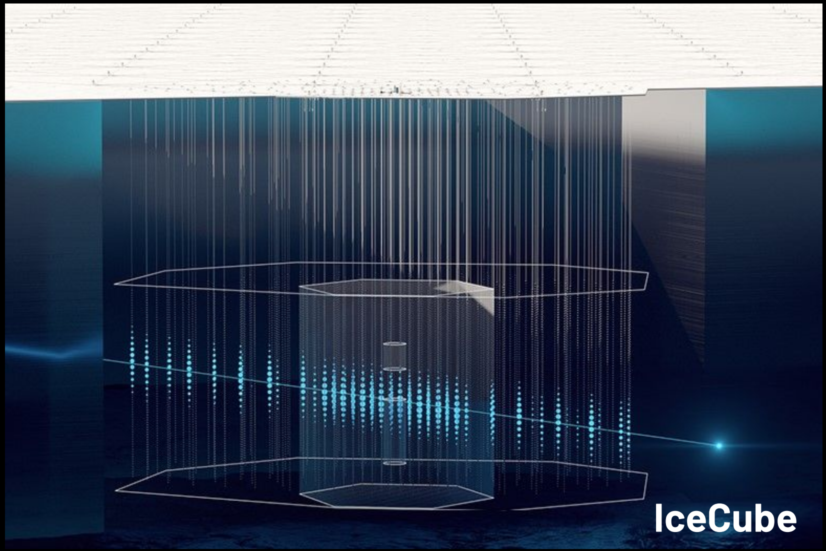




DM experiments

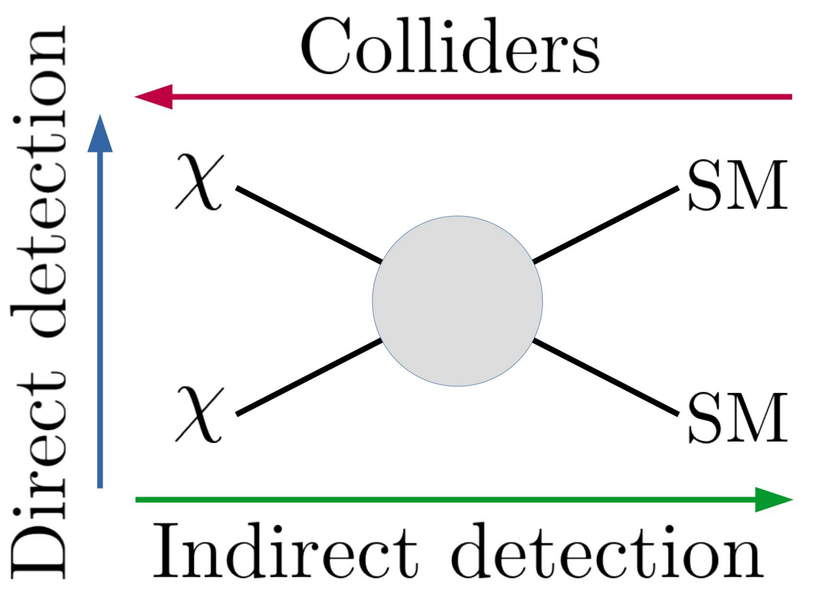
How to detect DM?

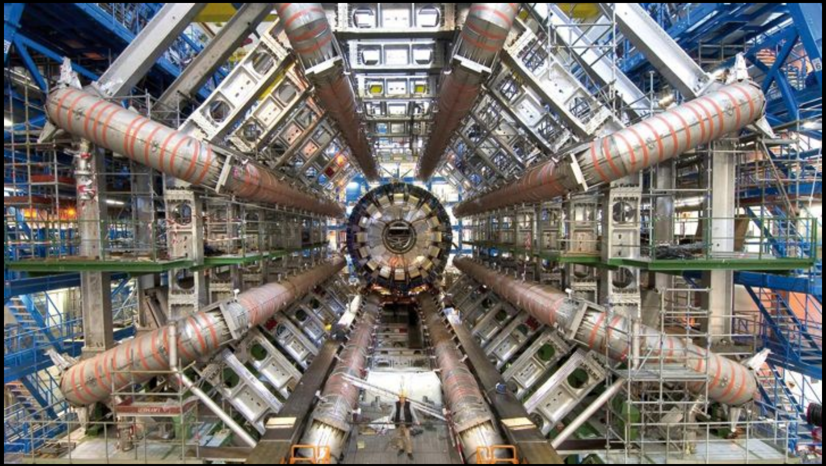




DM experiments

How to detect DM?





ATLAS (LHC)

STARS can be used as huge detectors for DM direct detection!

Especially very compact stars

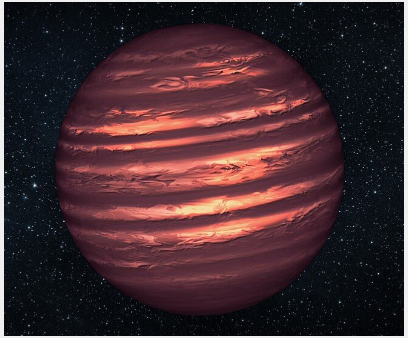
02

White dwarfs

End of life of stars

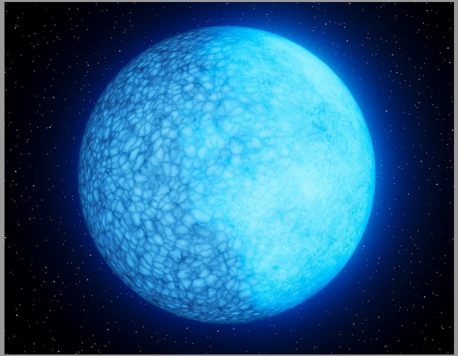
Brown dwarf

$13 - 80\ M_J$



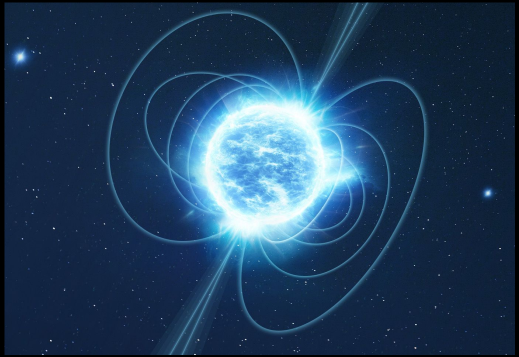
White dwarf

$0.17 - 1.33\ M_\odot$

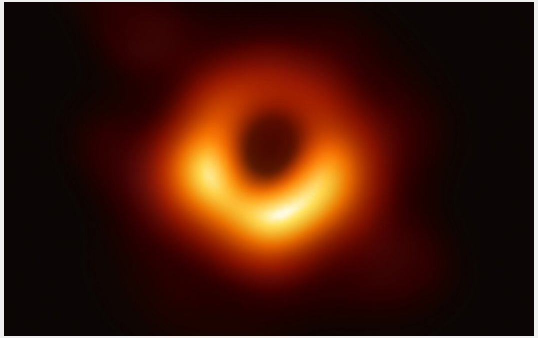


Neutron star

$1.1 - 2.3\ M_\odot$



Black hole

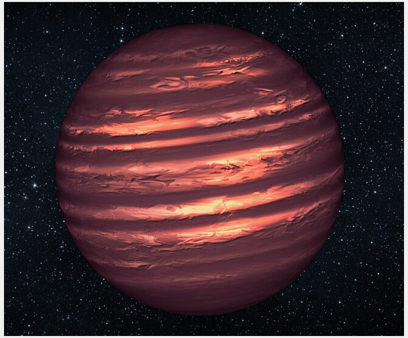


End of life of stars

Compact objects

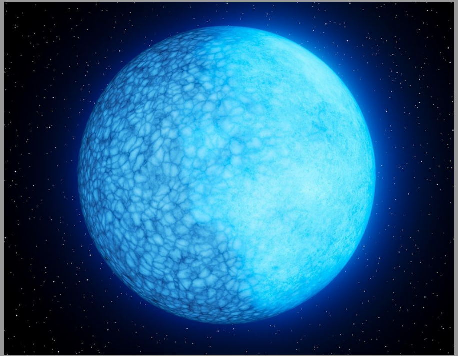
Brown dwarf

13 - 80 M_J



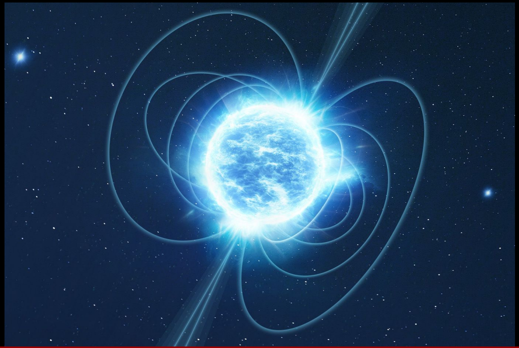
White dwarf

0.17 - 1.33 $M_\odot$

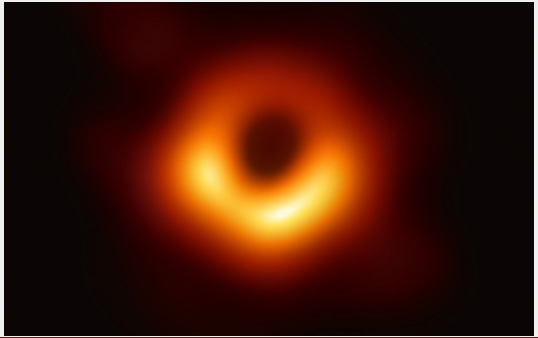


Neutron star

1.1 - 2.3 $M_\odot$



Black hole



Main characteristics of White Dwarfs

Density	• Between 10^6 - 10^9 kg/m^3 • Mainly composed by C or O.
Forces	• Gravitational force • Degenerate pressure of e^- • Coulomb forces
Mass	• Less than ~1.4 M (Chandrasekhar limit)
Eq. of state	• Salpeter + TOV equations (Tolman-Oppenheimer-Volkoff)

03

Capture of DM by stars

EFFECT OF HYPOTHETICAL, WEAKLY INTERACTING, MASSIVE PARTICLES ON ENERGY TRANSPORT IN THE SOLAR INTERIOR

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Received 1984 December 28; accepted 1985 January 28

ABSTRACT

A possible solution to the solar neutrino problem is to posit a massive, stable, neutral particle as part of the Sun's primordial composition. If that particle has a mass between 5 and 60 GeV, then it will populate only the inner "solar neutrino unit–producing" core and not the larger luminosity-producing region. If it has a scattering cross section on protons of 4×10^{-36} cm^2, then a fractional abundance of 10^{-12} will have order-unity effect on the Sun's thermal transport, in the direction of decreasing the expected neutrino signature. For smaller cross sections, the required abundance rises in inverse proportion, so that cross sections as small as 10^{-46} cm^2 are effective if the concentration is as large as $\sim 10^{-2}$. The photino is a possible candidate particle; mirror neutrinos may also be candidates.

Subject headings: elementary particles — neutrinos — nucleosynthesis — Sun: interior

I. INTRODUCTION

e discuss in this paper an unlikely, but possible, solution to the solar neutrino problem. (For recent reviews of this problem Bahcall *et al.* 1982; Bahcall 1985.) Our solution is unlikely only in requiring the existence in the Sun of a stable, neutral pa a mass in the range of 5–60 GeV, and with a scattering cross section on protons in the range of 10^{-26} to 10^{-46} cm^2. ne required range of cross sections is intermediate between strong and weak cross sections of ordinary, nonexotic pa ics. There are, however, hints that significant aspects of particle physics in the required mass range are not now comp rstood: On the experimental side, monojet events, and other events in the UA1 collaboration at CERN (Arnison *et al.* pia 1984), lack theoretical explanation (Ellis 1984; Hall, Jaffe, and Rosner 1984). On the theoretical side, stable particles i ed mass range can arise as "mirror neutrinos" in left-right symmetric theories and vector-like theories (Bagger and D

Spergel and Press (1985)

and solar neutrino problem

WEAKLY INTERACTING, MASSIVE PARTICLES AND THE SOLAR NEUTRINO FLUX[1]

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Received 1985 April 18; accepted 1985 June 3

ABSTRACT

If the Sun contained even a small mass fraction of weakly interacting massive particles, there could be significant effects on central solar structure. The long mean free paths associated with such particles make them very efficient energy conductors. Consequently, an essentially isothermal core can be produced, which removes the central temperature peak responsible for the bulk of the predicted solar neutrinos in Davis's ^{37}Cl neutrino capture experiment. We explore the solar evolutionary consequences of both imposed isothermal core models and models including energy transport by a specific class of weakly interacting particles. In particular, a relative mass fraction, 10^{-8}, of 4 GeV particles reduces predicted neutrino count rates by more than a factor of 3.

Subject headings: elementary particles — neutrinos — Sun: interior

I. INTRODUCTION

e classical solar neutrino problem has now been with us lmost two decades (see Bahcall and Davis 1982 for a rical review and Bahcall 1985 for the most recent ssion). In essence, there is a persistent discrepancy, of three, between the predicted and observing counting

(where they constitute much of § V*c*), the full details were n published. This was in part because other, rapid developm in cosmology and particle physics seemed to rule out the vival of particles in the range of real interest to the s situation. Although we realized that a sufficient mass frac of suitable WIMPs (ideally, with $m_w \gtrsim 4$ GeV and σ_w

Faulkner and Gilliland (1985)

and solar neutrino problem

Gaisser et al. (1986)

and DM

Limits on cold-dark-matter candidates from deep underground detectors

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(Received 2 July 1986)

Weakly interacting massive particles which are candidates for the dark mass in the halo of the Galaxy would be captured by the Sun, accumulate in the solar core, and annihilate. We present a systematic evaluation of the neutrino signal produced by such annihilations. Since most annihilations occur in the dense solar interior, only prompt neutrinos escape with sufficiently high energy to be readily observable in deep underground detectors. We find that existing underground experiments are capable of finding—or excluding—several possible cold-dark-matter candidates.

I. INTRODUCTION

e mass in the Universe is dark, the nonlumi-

energy spectrum of the neutrinos at produc this with the correct energy dependence of interaction cross section in the detector to

RESONANT ENHANCEMENTS IN WEAKLY INTERACTING MASSIVE PARTICLE CAPTURE BY THE EARTH

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Stanford Linear Accelerator Center, Stanford University
Received 1987 March 2; accepted 1987 March 17

ABSTRACT

The exact formulae for the capture of weakly interacting massive particles (WIMPs) by a massive body are derived. Capture by the Earth is found to be significantly enhanced whenever the WIMP mass is roughly equal to the nuclear mass of an element present in the Earth in large quantities. For Dirac neutrino WIMPs of mass 10–90 GeV, the capture rate is 10–300 times that previously believed. Capture rates for the Sun are also recalculated and found to be from 1.5 times higher to 3 times lower than previously believed, depending on the mass and type of WIMP. The Earth alone or the Earth in combination with the Sun is found to give a much stronger annihilation signal from Dirac neutrino WIMPs than the Sun alone over a very large mass range. This is particularly important in the neighborhood of the mass of iron where previous analyses could not set any significant limits.

Subject headings: elementary particles — neutrinos

I. INTRODUCTION

part of their argument that weakly interacting massive particles (WIMPs) could explain both the "dark matter problem solar neutrino problem," Press and Spergel (1985) gave an estimate of the capture rate by a massive body of WIMP well-Boltzmann distribution in the Galactic halo or Galactic disk. Their argument made admittedly crude assumptions 1P phase space which they hoped would introduce errors of no more than a factor of 2. They were satisfied with this le racy because of the "order of magnitude" character of their argument. The Press and Spergel calculation was equally the probability of a given WIMP interacting with the body was of order 1, and when it was much less than 1. This w rtant feature for Press and Spergel because, to solve the solar neutrino problem, it is best to have WIMPs with much weak interaction cross sections.
bsequently a number of workers have realized that if WIMPs and anti–WIMPs were both present in the Galactic halo ld tend to collect in the Sun (Silk, Olive, and Srednicki 1985; Gaisser, Steigman, and Tilav 1986; Srednicki, Olive, and Silk

Gould (1987)

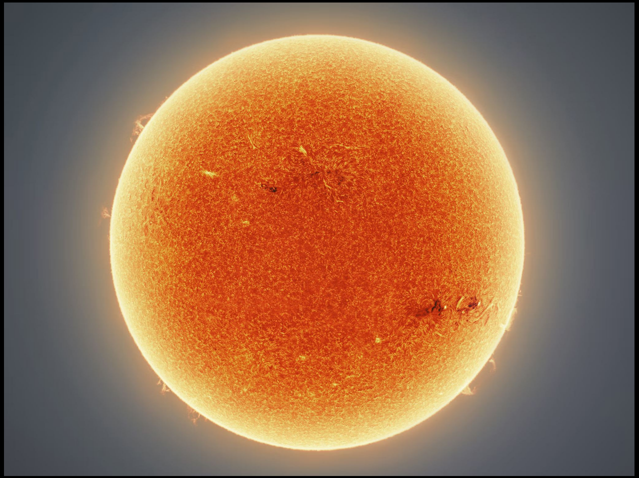
and capture by the Earth (+Sun)

Brief history of the topic

It started with the SOLAR COSMION (WIMP) to solve the solar neutrino problem and the missing mass problem (DM), due to their efficiency in energy transport.

What to measure?

01



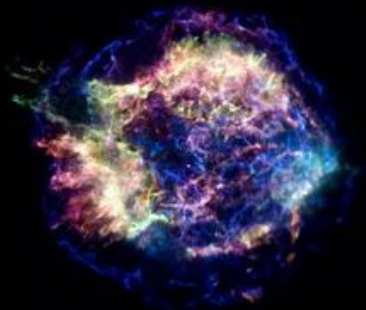
Mass accumulation

02

Heating of star

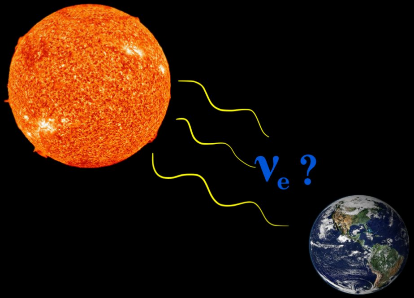
01a

Gravitational effects



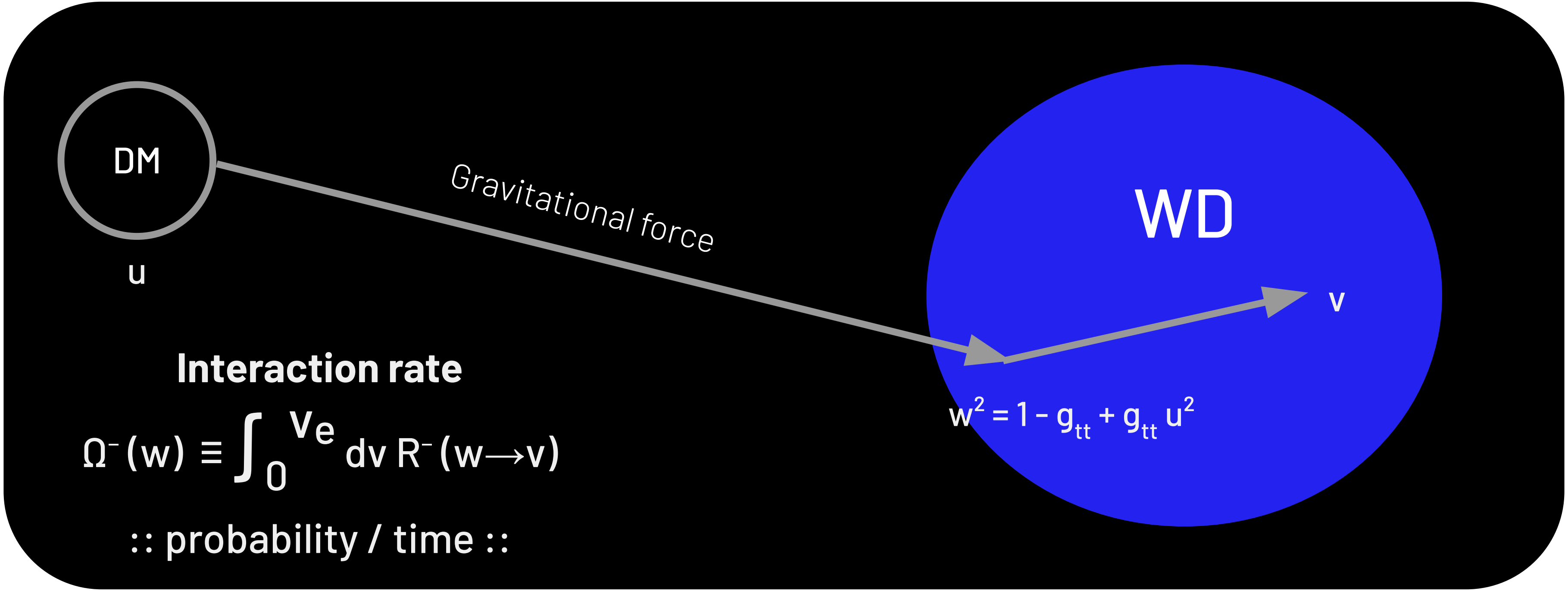
01b

Emission of SM particles



CAPTURE MECHANISM

What happens?



Interaction rate

$$\Omega^-(w) \equiv \int_0^{v_e} dv\, R^-(w \rightarrow v)$$

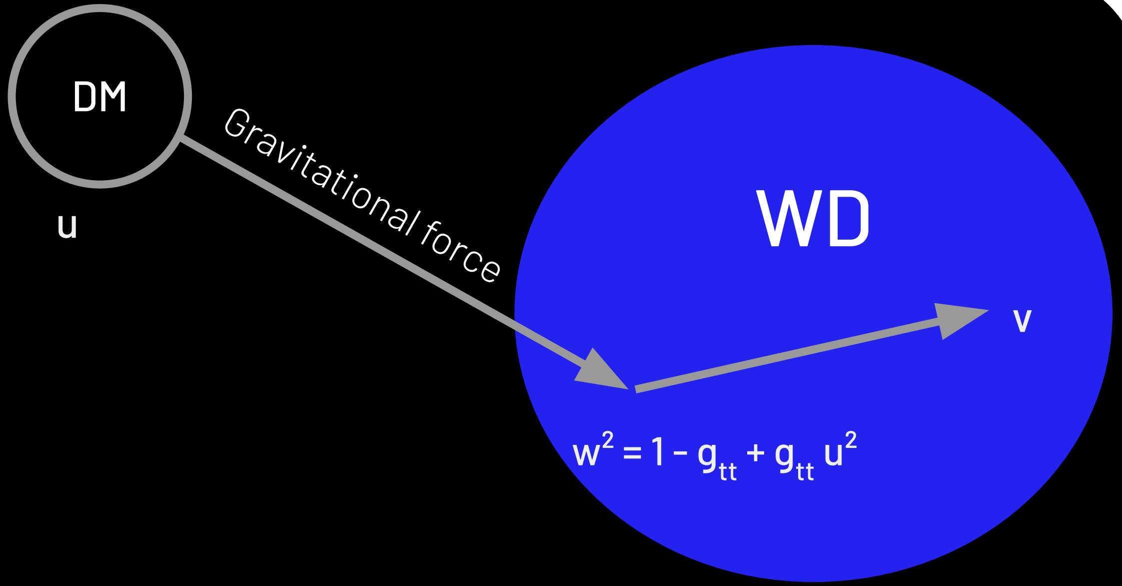
:: probability / time ::

CAPTURE MECHANISM

What happens?

$$R^- \propto d\sigma/dv$$

$$\Omega^- \propto \sigma(v<v_e)$$



CAPTURE MECHANISM

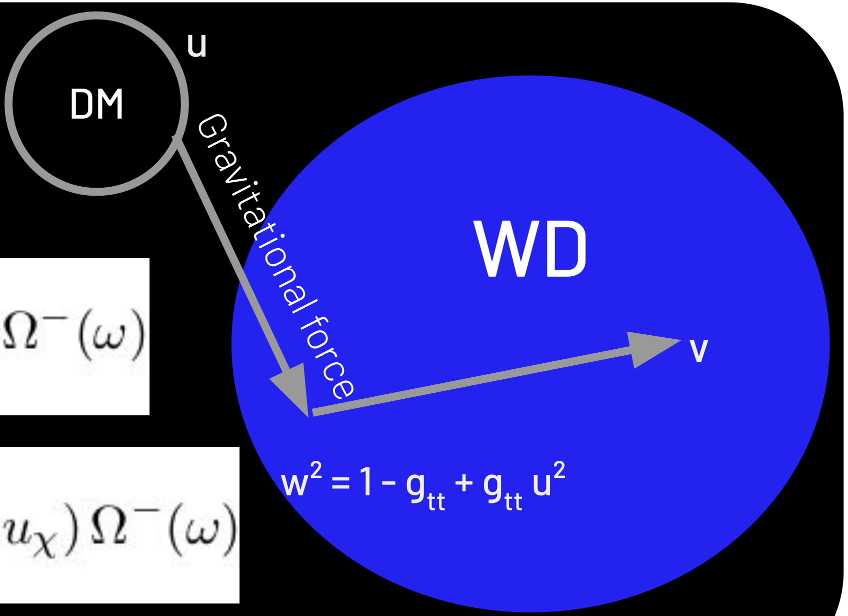
(in the Optically Thin Limit)

What happens?

$$\Omega^-(\omega) = \frac{4}{\sqrt{\pi}} \int_0^{v_e} dv \frac{d\sigma}{dv} \omega^2 n_T(r)$$

$$C = \frac{\rho_\chi}{m_\chi} \int_0^{R\star} dr 4\pi r^2 \int_0^\infty du_\chi \frac{\omega}{u_\chi} f_{\mathrm{MB}}(u_\chi) \Omega^-(\omega)$$

$$\mathscr{C} = \frac{1}{m_\chi} \int_0^{R\star} dr 4\pi r^2 \int_0^\infty du'_\chi \frac{\omega}{u'_\chi} \delta(u'_\chi - u_\chi) \Omega^-(\omega)$$



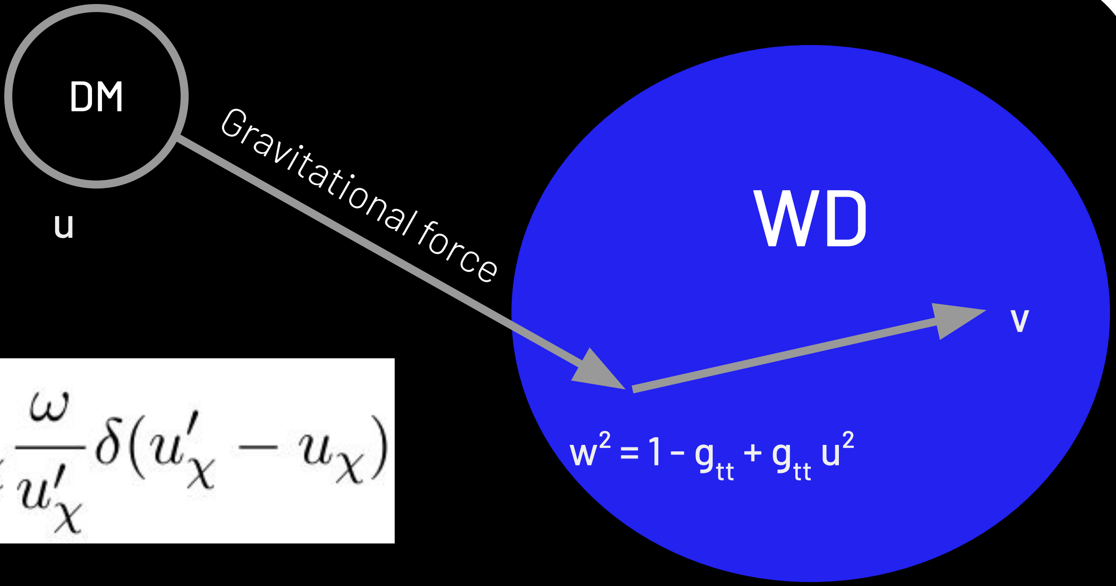
CAPTURE MECHANISM

What happens as you reach a cross section threshold (σ_{th})?
(out of the Optically Thin Limit)

What happens?

$$\eta(r) = \frac{1}{2}\int_{-1}^{1} dz e^{-\tau(r,z)}$$

$$\mathscr{C}_{\mathrm{geom}} = \frac{\pi R_\star^2}{m_\chi}\int_0^\infty du'_\chi \frac{\omega}{u'_\chi}\delta(u'_\chi - u_\chi)$$



04

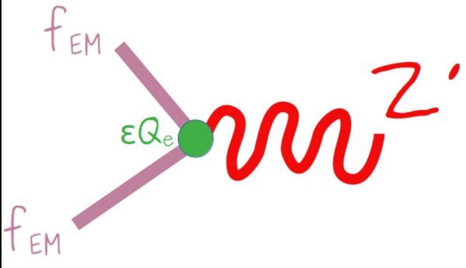
Interactions

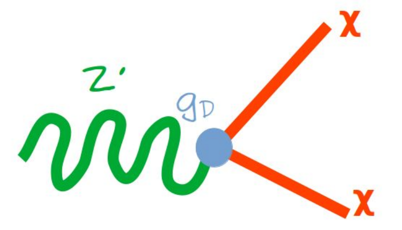
Modelling the cross sections

BSM MODELS

VECTOR, Dark photon

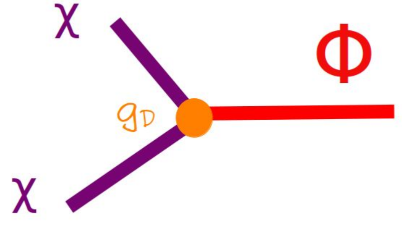
$$\mathcal{L}_{Z'} = -\epsilon e Q_{\rm EM} J^{\mu}_{\rm EM} Z'_{\mu} + g_D \overline{\chi} \gamma^{\mu} (g^{\chi}_V - g^{\chi}_A \gamma^5) \chi Z'_{\mu}$$

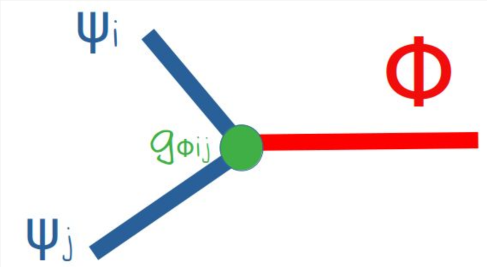




SCALAR

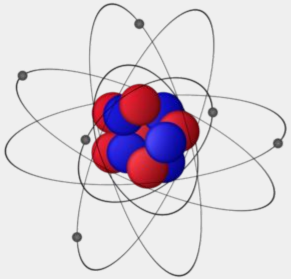
$$\mathcal{L}_{\Phi} = g^{ij}_{\Phi} \overline{\psi}^{i}_{\rm SM} \psi^{j}_{\rm SM} \Phi + g_D \overline{\chi} \chi \Phi$$

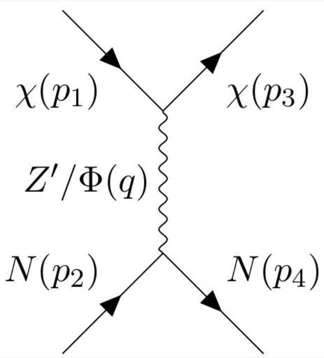




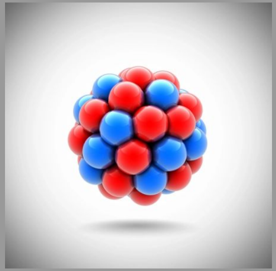
WHAT INTERACTIONS COULD TAKE PLACE?

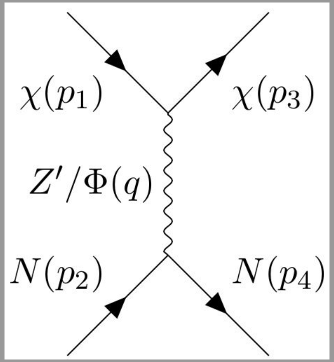
NUCLEAR



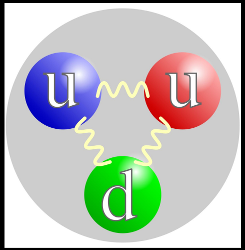


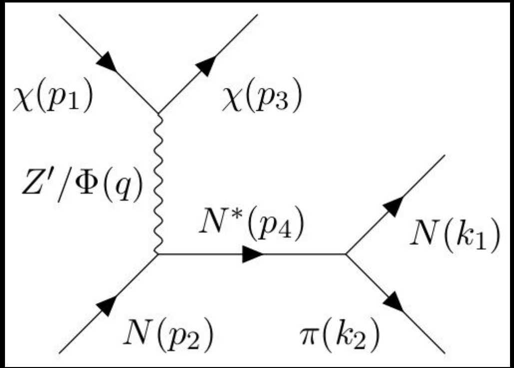
NUCLEON





RESONANT





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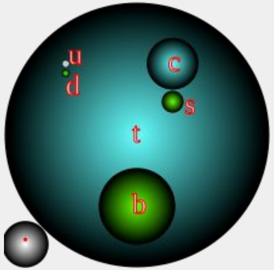
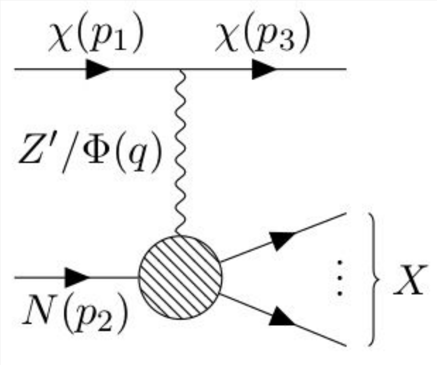
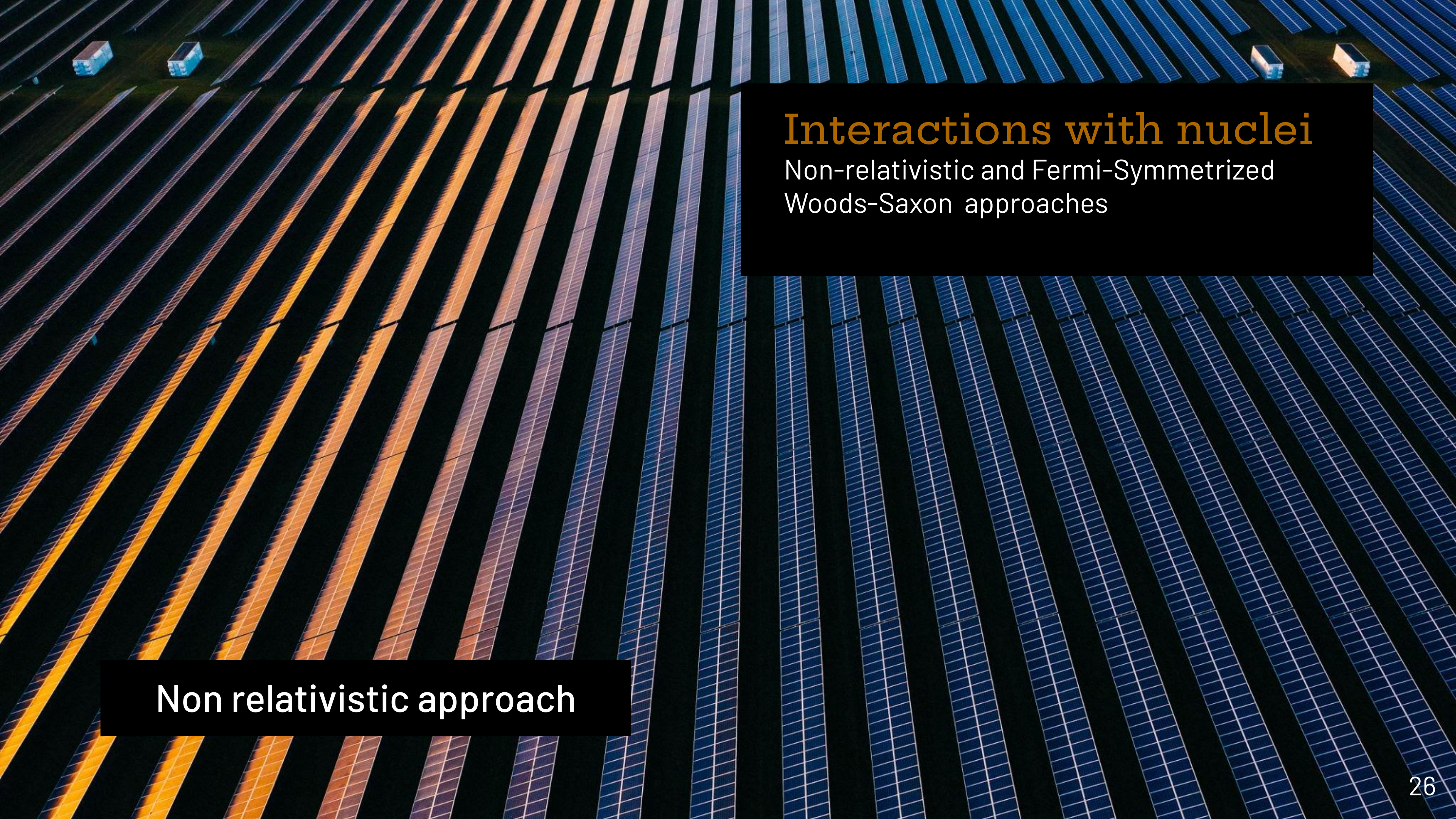






Interactions with nuclei

Non-relativistic and Fermi-Symmetrized Woods-Saxon approaches

Non relativistic approach

Ingredients

Regime

- Non-relativistic, so that we can use Galilean invariance.
- We will use the relative velocity: $\mathbf{v} = \mathbf{p}_\chi/m_\chi - \mathbf{p}_N/m_N$.
- We also need the transverse 3-momentum: $\mathbf{q}$.

Quantum operators

- NR operators: $Id_{\chi N}$, iq/m_N, $v^\perp$, s_χ, s_N.
- They act on the tensor product space of χ and N: $|\mathbf{p}_\chi, j_\chi>$, $|\mathbf{p}_N, j_N>$

Matrix elements

$$\langle \mathbf{p}', j_\chi; \mathbf{k}', j_N | \, i\hat{\mathbf{q}} \, | \mathbf{p}, j_\chi; \mathbf{k}, j_N \rangle = i\mathbf{q} \, e^{-i\mathbf{q}\cdot\mathbf{r}} \, (2\pi)^3 \delta(\mathbf{k}' + \mathbf{p}' - \mathbf{k} - \mathbf{p})$$

$$\langle \mathbf{p}', j_\chi; \mathbf{k}', j_N | \, \hat{\mathbf{v}}^\perp \, | \mathbf{p}, j_\chi; \mathbf{k}, j_N \rangle = \left(\mathbf{v} + \frac{\mathbf{q}}{2\mu_N} \right) e^{-i\mathbf{q}\cdot\mathbf{r}} \, (2\pi)^3 \delta(\mathbf{k}' + \mathbf{p}' - \mathbf{k} - \mathbf{p})$$

Operators

$$\hat{\mathcal{O}}_1 = \mathbb{1}_{\chi N}$$
$$\hat{\mathcal{O}}_3 = i\hat{\mathbf{S}}_N \cdot \left(\frac{\hat{\mathbf{q}}}{m_N} \times \hat{\mathbf{v}}^\perp \right)$$
$$\hat{\mathcal{O}}_4 = \hat{\mathbf{S}}_\chi \cdot \hat{\mathbf{S}}_N$$
$$\hat{\mathcal{O}}_5 = i\hat{\mathbf{S}}_\chi \cdot \left(\frac{\hat{\mathbf{q}}}{m_N} \times \hat{\mathbf{v}}^\perp \right)$$
$$\hat{\mathcal{O}}_6 = \left(\hat{\mathbf{S}}_\chi \cdot \frac{\hat{\mathbf{q}}}{m_N} \right) \left(\hat{\mathbf{S}}_N \cdot \frac{\hat{\mathbf{q}}}{m_N} \right)$$
$$\hat{\mathcal{O}}_7 = \hat{\mathbf{S}}_N \cdot \hat{\mathbf{v}}^\perp$$
$$\hat{\mathcal{O}}_8 = \hat{\mathbf{S}}_\chi \cdot \hat{\mathbf{v}}^\perp$$
$$\hat{\mathcal{O}}_9 = i\hat{\mathbf{S}}_\chi \cdot \left(\hat{\mathbf{S}}_N \times \frac{\hat{\mathbf{q}}}{m_N} \right)$$
$$\hat{\mathcal{O}}_{10} = i\hat{\mathbf{S}}_N \cdot \frac{\hat{\mathbf{q}}}{m_N}$$
$$\hat{\mathcal{O}}_{11} = i\hat{\mathbf{S}}_\chi \cdot \frac{\hat{\mathbf{q}}}{m_N}$$
$$\hat{\mathcal{O}}_{12} = \hat{\mathbf{S}}_\chi \cdot \left(\hat{\mathbf{S}}_N \times \hat{\mathbf{v}}^\perp \right)$$
$$\hat{\mathcal{O}}_{13} = i \left(\hat{\mathbf{S}}_\chi \cdot \hat{\mathbf{v}}^\perp \right) \left(\hat{\mathbf{S}}_N \cdot \frac{\hat{\mathbf{q}}}{m_N} \right)$$
$$\hat{\mathcal{O}}_{14} = i \left(\hat{\mathbf{S}}_\chi \cdot \frac{\hat{\mathbf{q}}}{m_N} \right) \left(\hat{\mathbf{S}}_N \cdot \hat{\mathbf{v}}^\perp \right)$$
$$\hat{\mathcal{O}}_{15} = - \left(\hat{\mathbf{S}}_\chi \cdot \frac{\hat{\mathbf{q}}}{m_N} \right) \left[\left(\hat{\mathbf{S}}_N \times \hat{\mathbf{v}}^\perp \right) \cdot \frac{\hat{\mathbf{q}}}{m_N} \right]$$

General Hamiltonian

$$\hat{\mathcal{H}}(\mathbf{r}) = 2\sum_{k=1}^{15}\left[c_k^p\left(\frac{\mathbb{1}+\tau_3}{2}\right)+c_k^n\left(\frac{\mathbb{1}-\tau_3}{2}\right)\right]\hat{\mathcal{O}}_k(\mathbf{r})$$

$$\hat{\mathcal{H}}(\mathbf{r}) = \sum_{\tau=0,1}\sum_{k=1}^{15} c_k^\tau \hat{\mathcal{O}}_k(\mathbf{r})\, t^\tau$$

We need to build this in position space

$$\hat{\mathcal{H}}_{\mathrm{T}}(\mathbf{r}) = \sum_{i=1}^{A}\sum_{\tau=0,1}\sum_{k=1}^{15} c_k^\tau \hat{\mathcal{O}}_k^{(i)}(\mathbf{r})\, t^\tau_{(i)}$$

$$H_{\mathrm{T}} = \int d^3\mathbf{r}\, \hat{\mathcal{H}}_{\mathrm{T}}(\mathbf{r})$$

$$\langle f|H_{\mathrm{T}}|i\rangle = (2\pi)^3\delta(\mathbf{k}'_T+\mathbf{p}'-\mathbf{k}_T-\mathbf{p})\, i\mathcal{M}_{NR}$$

Results using the nuclear shell model

$$\frac{1}{N_i}\sum_{i,j}|\mathcal{M}_T^{NR}|^2 = \frac{m_T^2}{m_N^2}\sum_{i,j}^{15}\sum_{\alpha,\beta=0,1} c_i^\alpha c_j^\beta F_{ij}^{\alpha\beta}(v^2, q^2, y)$$

y = (bq/2)^2

b = (41.467/(45A$^{-1/3}$ – 25A$^{-2/3}$))$^{1/2}$

$$\frac{d\sigma_T^{NR}}{d\cos\theta} = \frac{1}{32\pi(m_\chi + m_T)^2}\frac{1}{N_i}\sum_{i,j}|\mathcal{M}_T^{NR}|^2$$

From HE theory to a NR one: expansion of bispinors

$$u^s(p) = \begin{pmatrix} \sqrt{p^\mu \sigma_\mu}\,\xi^s \\ \sqrt{p^\mu \bar{\sigma}_\mu}\,\xi^s \end{pmatrix} = \frac{1}{\sqrt{2(p^0+m)}} \begin{pmatrix} (p^\mu \sigma_\mu + m)\,\xi^s \\ (p^\mu \bar{\sigma}_\mu + m)\,\xi^s \end{pmatrix}$$

$$= \frac{1}{\sqrt{4m}} \begin{pmatrix} (2m - \vec{p}\cdot\vec{\sigma})\,\xi^s \\ (2m + \vec{p}\cdot\vec{\sigma})\,\xi^s \end{pmatrix} + \mathcal{O}(\vec{p}^2)$$

$$\bar{u}(p')u(p) \simeq 2m\ ,$$

$$\bar{u}(p')i\,\gamma^5 u(p) \simeq 2i\,\vec{q}\cdot\vec{s}\ ,$$

$$\bar{u}(p')\gamma^\mu u(p) \simeq \begin{pmatrix} 2m \\ \vec{P} + 2i\,\vec{q}\times\vec{s} \end{pmatrix}$$

$$\bar{u}(p')\gamma^\mu\gamma^5 u(p) \simeq \begin{pmatrix} 2\vec{P}\cdot\vec{s} \\ 4m\,\vec{s} \end{pmatrix}\ ,$$

$P \equiv p + p'$

From HE theory to a NR one: dark photon

$$\mathcal{H}_I^{Z'N} = \frac{g_D g_{Z'N}}{m_{Z'}^2}\left[g_V^\chi \overline{u}(p_3)\gamma_\mu u(p_1)\overline{u}(p_4)\gamma^\mu u(p_2) - g_A^\chi \overline{u}(p_3)\gamma_\mu\gamma^5 u(p_1)\overline{u}(p_4)\gamma^\mu u(p_2)\right]$$

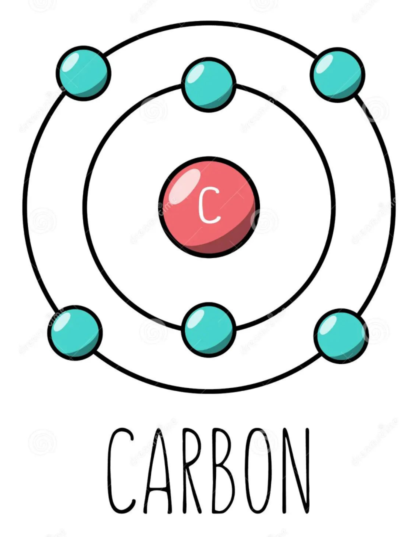
$$\overline{u}(p_3)\gamma_\mu u(p_1)\overline{u}(p_4)\gamma^\mu u(p_2) \simeq 4m_\chi m_N + 2m_\chi m_N\left(i\hat{s}_N\cdot\left[\frac{\hat{q}}{m_N}\times\hat{v}^\perp\right]\right) + 2m_N^2\left(i\hat{s}_\chi\cdot\left[\frac{\hat{q}}{m_N}\times\hat{v}^\perp\right]\right) - \frac{m_\chi}{m_N}\hat{q}^2 - 4\hat{q}^2\hat{s}_\chi\cdot\hat{s}_N + 4m_N^2\left(\hat{s}_\chi\cdot\frac{\hat{q}}{m_N}\right)\left(\hat{s}_N\cdot\frac{\hat{q}}{m_N}\right)$$

$$\overline{u}(p_3)\gamma_\mu\gamma^5 u(p_1)\overline{u}(p_4)\gamma^\mu u(p_2) \simeq 8m_\chi m_N\left(\hat{s}_\chi\cdot\hat{v}^\perp + i\hat{s}_\chi\cdot\left[\hat{s}_N\times\frac{\hat{q}}{m_N}\right]\right)$$

From HE theory to a NR one: scalar

$$\mathcal{H}_I^{\Phi N} = \frac{g_D g_{\Phi N}}{m_\Phi^2}\left[\overline{u}(p_3)u(p_1)\overline{u}(p_4)u(p_2)\right] = 4 g_D g_{\Phi N}\frac{m_\chi m_N}{m_\Phi^2}\mathcal{O}_1^{NR}$$

$$g_{\Phi N} = \sum_{q=u,d,s}\frac{g_{\Phi q}}{m_q}m_N f_{Tq}^{(N)} + \frac{2}{27}f_{TG}^{(N)}\sum_{q=c,b,t}\frac{g_{\Phi q}}{m_q}m_N$$



Interactions with nuclei

Non-relativistic and Fermi-Symmetrized Woods-Saxon approaches

Fermi-Symmetrized Woods-Saxon approach

$$F^{\mathrm{Dip}}(Q) = \frac{1}{1 + \frac{Q^2}{M_D^2}}$$

$$M_D = 1.18 + 0.83 * A^{1/3}$$

From Born approximation

$$d\sigma/d\Omega = (d\sigma/d\Omega)_{\mathrm{point}} |F|^2$$

$$F(\mathbf{q}) = \int \rho(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}} d^3\mathbf{r}.$$

Inelastic and Elastic Scattering of 187-Mev Electrons from Selected Even-Even Nuclei*

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High-Energy Physics Laboratory, Stanford University, Stanford, California

(Received August 27, 1956)

A survey has been made of the differential scattering cross sections for 187-Mev electrons on the even-even nuclei ${}_{12}Mg^{24}$, ${}_{14}Si^{28}$, ${}_{16}S^{32}$, ${}_{18}A^{40}$, and ${}_{38}Sr^{88}$. It has been possible to separate the elastic scattering from the inelastic in all cases and to resolve the inelastic groups from specific nuclear levels for at least one level in all cases. A simple Born-approximation analysis of the elastic data yields values of the effective radii and surface thicknesses of the nuclear charge densities which (if suitably corrected for failure of the Born approximation) are in substantial agreement with the results of Hahn, Ravenhall, and Hofstadter; i.e., a radius parameter of $c \cong 1.08\ A^{\frac{1}{3}} \times 10^{-13}$ cm (radius to half-maximum of the charge distribution) and a surface thickness of $t \cong 2.5 \times 10^{-13}$ cm (thickness from 10% to 90% of the maximum of the charge distribution). Phenomenological analysis of the inelastic scattering along the lines laid down by Schiff yields some tentative multipolarity assignments, and application of some results of Ravenhall yields estimates of (radiative) partial level widths; for the $E2$ transitions these correspond to lifetimes of $\sim 19 \times 10^{-13}$ sec (Mg 1.37 Mev) to $\sim 1.4 \times 10^{-13}$ sec (Sr 1.85 Mev). The observed strengths of the transitions are compared to those predicted by Weisskopf theory.

I. INTRODUCTION

THE elastic scattering of high-energy electrons by atomic nuclei has been the subject of considerable experimental study.[1–8] Recently it has been possible in this laboratory to observe certain examples of

The present experiments were initiated as a survey of the inelastic and elastic scattering from even-even nuclei in the region of intermediate atomic numbers. These target materials were chosen for a number of reasons: First, most of them are known from gamma-ray

Richard Helm, 1956

Inelastic and Elastic Scattering of 18'7-Mev Electrons from Selected Even-Even Nuclei

$$F^{\mathrm{Helm}}(Q)\frac{3|j_1(QR)|}{QR}e^{-Q^2 s^2/2},$$

with $a = 0.523$ fm, $s = 0.9$ fm, and $R = 3.9$ fm.

Nucleus: hard sphere with a diffuse surface layer (thickness: s)

$$F^{\mathrm{FS-WS}}(Q) = \frac{3\pi a}{r_0^2 + \pi^2 a^2} \frac{a\pi \operatorname{cotanh}(\pi Q a) \sin(Q r_0) - r_0 \cos(Q r_0)}{Q r_0 \sinh(\pi Q a)}$$

Fermi Symmetrized Woods-Saxon

Woods-Saxon nuclear potential (1954). V_0: potential depth, R: nuclear radius, a: surface diffuseness parameter, r: radial distance

$$V(r) = \frac{V_0}{1 + e^{(r-R)/a}}$$

Fermi symmetrized function, to account for nuclear surface. c: parameter related to the half-density radius.

$$f_{\mathrm{SF}}(r) = \frac{\sinh(c/a)}{\cosh(r/a) + \cosh(c/a)}$$

Nucleus: smooth density distribution...

$$\rho(r) = \frac{\rho_0}{1 + e^{(r-R)/a}}$$

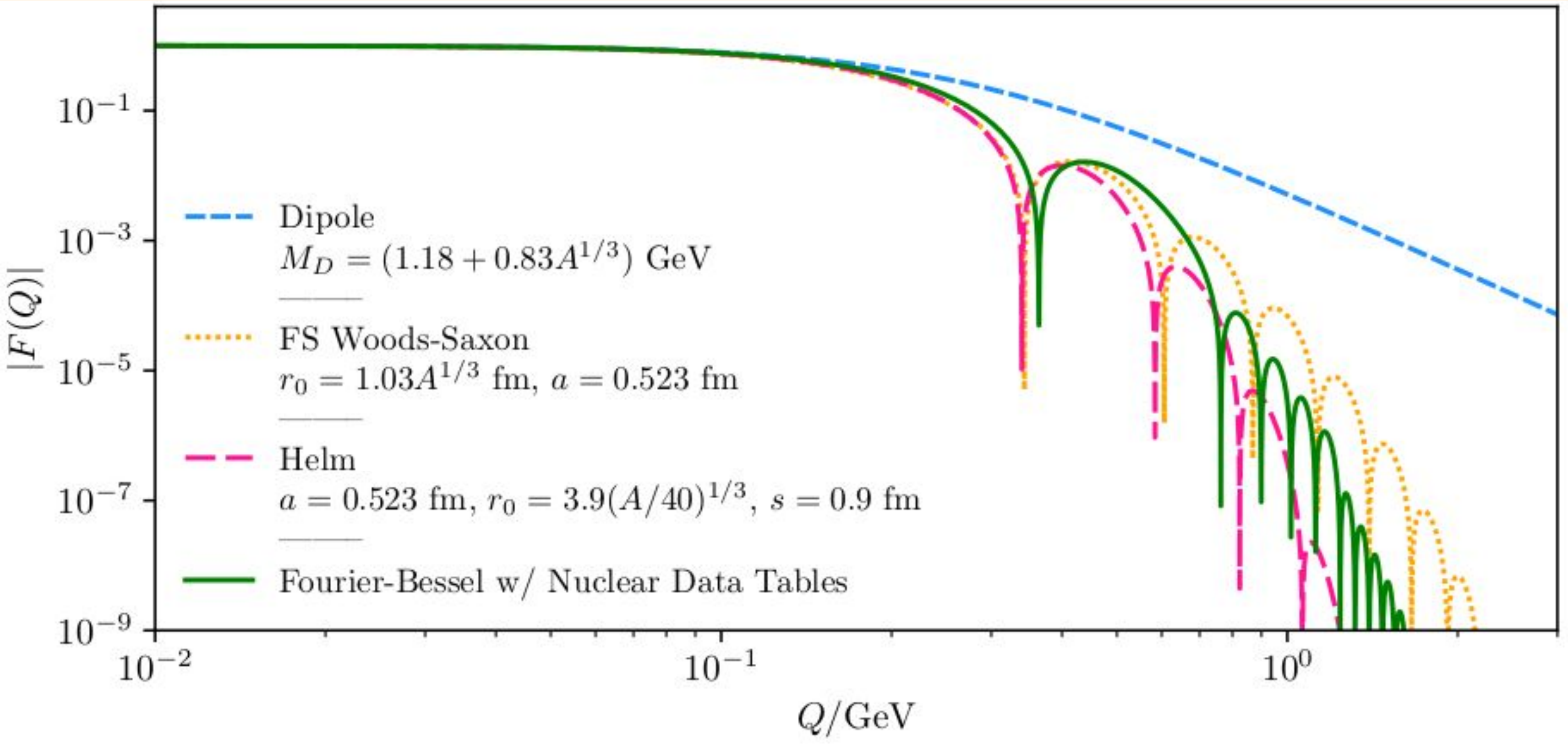
$$F^{\mathrm{FB}}(Q) = N \times \frac{\sin(QR)}{QR} \sum_n \frac{(-1)^n a_n}{n^2\pi^2 - Q^2}$$

Fourier-Bessel parametrization

Nucleus: modeled as a series expansion in spherical Bessel functions

$$\rho(r) = \sum_n a_n j_n(kr)$$

Comparison of form factors for ^{12}C



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For the scalar, the expression is simpler:

$$\frac{d\sigma^N}{dQ^2} = \frac{\sigma_0 E_\chi^2}{4\mu_N (E_\chi^2 - m_\chi^2)} F_H^2(Q^2)$$

$$\sigma_0 = \frac{g_D^2 g_{N\Phi}^2}{8\pi p_\chi^2} \int_0^{x_m} dx \frac{(x + 2m_N^2)(x + 2m_\chi^2)}{(x + m_N^2)(2x + m_\Phi^2)^2}$$

$$x \equiv m_N (E_\chi - E_3)$$

$$x_m = m_N E_\chi - m_N (m_N^2 E_\chi + m_\chi^2 (E_\chi + 2m_N))/(2m_N E_\chi + m_N^2 + m_\chi^2)$$

For the dark photon, the differential cross section has a similar expression, with a different σ_0

Interactions with nucleons

Elastic scattering, Z′

We can follow a similar approach as with the SM:

$$\mathcal{M}_N = i\frac{g_D g_{\text{Had}}}{q^2 - m_{Z'}^2}[\overline{u}(p_3)\gamma^\mu(g_V^\chi - g_A^\chi\gamma^5)u(p_1)]\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_{Z'}^2}\right)\langle N(p_4)|\, j_{Z'Q}^\nu(0)\,|N(p_2)\rangle$$

$$j_{Z'Q}^\nu = \sum_q g_V^q \bar{q}\gamma^\nu q - \sum_q g_A^q \bar{q}\gamma^\nu\gamma^5 q$$

SM currents

$$v_3^\nu = \frac{1}{2}[\overline{u}\gamma^\nu u - \overline{d}\gamma^\nu d]$$

$$j_{AQ}^\nu = \frac{2}{3}\sum_\alpha[\overline{q}_\alpha^U\gamma^\nu q_\alpha^U] - \frac{1}{3}\sum_\alpha[\overline{q}_\alpha^D\gamma^\nu q_\alpha^D]$$

$$v_s^\nu = \sum_{q=s,c,b,t} \overline{q}\gamma^\nu q$$

$$a_3^\nu = \frac{1}{2}[\overline{u}\gamma^\nu\gamma^5 u - \overline{d}\gamma^\nu\gamma^5 d]$$

$$a_0^\nu = \frac{1}{2}[\overline{u}\gamma^\nu\gamma^5 u + \overline{d}\gamma^\nu\gamma^5 d]$$

$$a_s^\nu = \sum_{q=s,c,b,t} \overline{q}\gamma^\nu\gamma^5 q.$$

We need an expression for the Z'Q current in terms of the SM currents we have already measured…

In terms of SM form factors...

$$v^{\nu}_{Z'Q} = -2(g^u_V + 2g^d_V)v^{\nu}_3 + 3(g^u_V + g^d_V)j^{\nu}_{AQ} + (g^u_V + g^d_V + g^s_V)v^{\nu}_s$$
$$- [g^s_V \bar{b}\gamma^{\nu} b + (3g^u_V + 3g^d_V + g^s_V)(\bar{c}\gamma^{\nu} c + \bar{t}\gamma^{\nu} t)]$$
$$a^{\nu}_{Z'Q} = (g^u_A - g^d_A)a^{\nu}_3 + (g^u_A + g^d_A)a^{\nu}_0 + g^s_A a^{\nu}_s - \sum_{q=c,b,t}(g^s_A - g^q_A)\bar{q}\gamma^{\nu}\gamma^5 q$$

$$\langle N(p_4)|\, v^{\mu}_{Z'Q}(0)\, |N(p_2)\rangle = \bar{u}_N(p_4)\left[\gamma^{\mu} F^{Z'N}_1(Q^2) + i\frac{q_{\nu}}{2m_N}\sigma^{\mu\nu} F^{Z'N}_2(Q^2)\right] u_N(p_2),$$
$$\langle N(p_4)|\, a^{\mu}_{Z'Q}(0)\, |N(p_2)\rangle = \bar{u}_N(p_4)\left[\gamma^{\mu}\gamma^5 G^{Z'N}_A(Q^2) + \frac{q_{\mu}}{m_N}\gamma^5 G^{Z'N}_P(Q^2)\right] u_N(p_2).$$

$$F^{Z'N}_i \simeq \mp(g^u_V + 2g^d_V)(F^p_i - F^n_i) + 3(g^u_V + g^d_V)F^N_i + (g^u_V + g^d_V + g^s_V)F^{sN}_i$$
$$G^{Z'N}_k \simeq \pm\frac{1}{2}(g^u_A - g^d_A)G_k + (g^u_A + g^d_A)G^{0N}_k + g^s_A G^{sN}_k,$$

Form factors

Sachs electric and magnetic form factors

$$G_E^N(Q^2) = F_1^N(Q^2) - \frac{Q^2}{4m_N^2} F_2^N(Q^2) = \delta_{Np} G_D(Q^2),$$

$$G_M^N(Q^2) = F_1^N(Q^2) + F_2^N(Q^2) = \frac{\mu_N}{\mu_\mathcal{N}} G_D(Q^2),$$

Dipole function

$$G_D(Q^2) = (1 + Q^2/m_V^2)^{-2}.$$

$$\mu_\mathcal{N} \equiv \frac{e\hbar}{2m_p}$$

nuclear magneton

$$M_V \simeq 0.84\,\text{GeV}.$$

F_1: Dirac ff
F_2: Pauli ff
G_A: axial ff
G_P: pseudoscalar ff

$\mu_p = 2.79\ \mu_\mathcal{N}$, $\mu_n = -1.91\ \mu_\mathcal{N}$

The cross section is straightforwardly found (if you're a particle physicist)

$$\mathcal{M}_N = i\frac{g_D \epsilon e}{q^2 - m_{Z'}^2}[\overline{u}(p_3)\gamma^\mu(g_V^\chi - g_A^\chi\gamma^5)u(p_1)]\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_{Z'}^2}\right)$$
$$\times\, \overline{u}_N(p_4)\left[\gamma^\nu F_1^N(Q^2) + i\frac{q_\lambda}{2m_N}\sigma^{\nu\lambda}F_2^N(Q^2)\right]u_N(p_2).$$

Interactions with nucleons

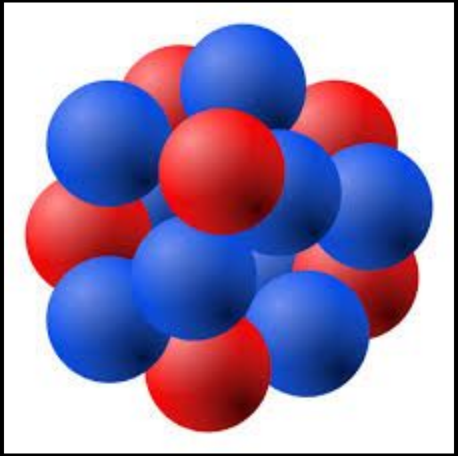
Elastic scattering, Φ

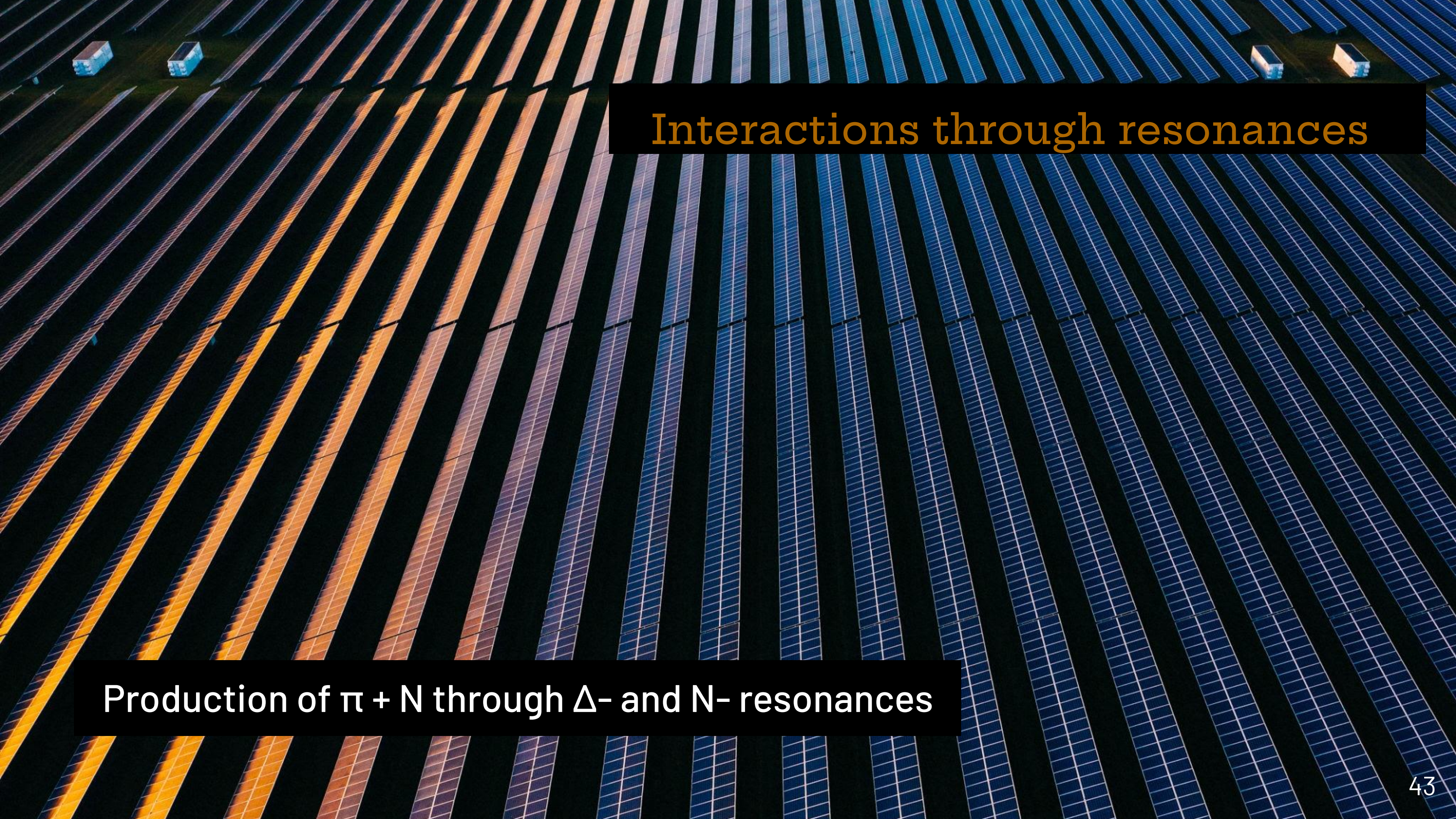
Here the process is more complicated...

$$\frac{d\sigma^N}{dz} = \frac{g_D^2 g_{\Phi N}^2}{8\pi m_N^2 \left(E_1 + E_2\right)^2} \frac{\left(E_1^2 + m_\chi^2 - p_1^2 z\right)\left(p_1^2(1+z) + 2m_N^2\right)\left(2F_1^{\rm SN} m_N^2 - p_1^2 F_2^{\rm SN}(1-z)\right)^2}{\left(2p_1^2(1-z) + m_\Phi^2\right)^2}$$

$z = \cos\theta$

$$F_i^{SN} \simeq \frac{3}{2}\left(\frac{g_{\Phi u}}{m_u} + \frac{g_{\Phi d}}{m_d}\right) F_i^N \mp \frac{1}{2}\left(\frac{g_{\Phi u}}{m_u} + 2\frac{g_{\Phi d}}{m_d}\right)\left(F_i^p - F_i^n\right)$$





Interactions through resonances

Production of $\pi + N$ through Δ- and N- resonances

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN
California Institute of Technology, Pasadena, California

Received 4 January 1964

e that the strong interactions of bary-
s are correctly described in terms of
htfold way" 1-3), we are tempted to
'undamental explanation of the situa-
promised approach is the purely dy-
trap" model for all the strongly in-
cles within which one may try to de-
oin and strangeness conservation and
d symmetry from self-consistency
urse, with only strong interactions,

ber $n_t - n_{\bar{t}}$ would be zero for all kn
mesons. The most interesting exa
model is one in which the triplet ha
$z = -1$, so that the four particles d
exhibit a parallel with the leptons.
A simpler and more elegant sch
constructed if we allow non-integra
charges. We can dispense entirely
baryon b if we assign to the triplet
properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and bar

SPIN AND UNITARY-SPIN INDEPENDENCE IN A PARAQUARK MODEL OF BARYONS AND MESONS

O. W. Greenberg*
Institute for Advanced Study, Princeton, New Jersey
(Received 27 October 1964)

Wigner's supermultiplet theory,[1] transplanted independently by Gürsey, Pais, and Radicati,[2] and by Sakita,[2] from nuclear-structure physics to particle-structure physics, has aroused a good deal of interest recently. In the nuclear supermultiplet theory, the approximate independence of both spin and isospin of those forces

ticle supermultiplet theory, the possible independence of both spin and unitary spin of those forces relevant to the masses of certain low-lying bound states (particles) makes it interesting to classify the states according to irreducible representations of SU(6). Three results associated with this SU(6) classification indi-

Current Matrix Elements from a Relativistic Quark Model*

R. P. Feynman, M. Kislinger, and F. Ravndal
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(Received 17 December 1970)

A relativistic equation to represent the symmetric quark model of hadrons with harmonic interaction is used to define and calculate matrix elements of vector and axial-vector currents. Elements between states with large mass differences are too big compared to experiment, so a factor whose functional form involves one arbitrary constant is introduced to compensate this. The vector elements are compared with experiments on photoelectric meson production, K_{l3} decay, and $\omega \to \pi\gamma$. Pseudoscalar-meson decay widths of hadrons are calculated supposing the amplitude is proportional (with one new scale constant) to the divergence of the axial-vector current matrix elements. Starting only from these two constants, the slope of the Regge trajectories, and the masses of the particles, 75 matrix elements are calculated, of which more than $\frac{3}{4}$ agree with the experimental values within 40%. The problems of extending this calculational scheme to a viable physical theory are discussed.

INTRODUCTION

ficing theoretical adequacy for simplicity. We shall choose a relativistic theory which is naive

Greenberg (1964)

Paraquark model of baryons and mesons

Gell-Mann (1964)

Non-relativistic model for baryons and mesons

FKR (1971)

Relativistic quark model

Neutrino-Excitation of Baryon Resonances and Single Pion Production

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III Physikalisches Institut, Technische Hochschule, Aachen, West Germany

Received October 31, 1980

This is an attempt to describe all existing data on neutrino production of single p
in the resonance region up to $W = 2$ GeV in terms of the relativistic quark mod
Feynman, Kislinger and Ravndal (FKR). We considered single pion production t
mediated by all interfering resonances below 2 GeV. A simple noninterfering, no
sonant background of isospin $\frac{1}{2}$ was added. It improved agreement with experin
particularly in the ratio of isospin amplitudes in charged current reactions, at the exp
of one additional constant. All total cross sections, cross section ratios and W-distr
tions are well reproduced at low and high energies, with charged and neutral cur
(supposing the Salam–Weinberg theory with $\sin^2\theta_w \approx \frac{1}{4}$ to be correct), and for neutr
and antineutrinos, giving predictions where data are lacking. New predictions have

Lepton mass effects in single pion production by neutrinos

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L. M. Sehgal†
Institut für Theoretische Physik (E) der RWTH, Aachen, Germany
(Received 28 September 2007; published 27 December 2007)

We reconsider the Feynman-Kislinger-Ravndal model applied to neutrino-excitation of baryon reso
nces. The effects of lepton mass are included, using the formalism of Kuzmin, Lyubushkin, and
umov. In addition we take account of the pion-pole contribution to the hadronic axial vector current
plication of this new formalism to the reaction $\nu_\mu + p \to \mu^- + \Delta^{++}$ at $E_\nu \sim 1$ GeV gives a
ppressed cross section at small angles, in agreement with the screening correction in Adler's
rward-scattering theorem. Application to the process $\nu_\tau + p \to \tau^- + \Delta^{++}$ at $E_\nu \sim 7$ GeV leads to
prediction of right-handed τ^- polarization for forward-going leptons, in line with a calculation based
an isobar model. Our formalism represents an improved version of the Rein-Sehgal model, incorpo
ing lepton mass effects in a manner consistent with partially conserved axial-vector current.

I: 10.1103/PhysRevD.76.113004 PACS numbers: 13.15.+g, 11.40.Ha, 12.39.K

I. INTRODUCTION

ation of neutrino experiments is under way
g low energy neutrino reactions such as
d $\nu_\mu p \to \mu^- p \pi^+$ with unprecedented sta-

duction in the resonance region up to $W \approx$
attractive feature of the model is its econo
input the vector and axial vector form facto
sielastic channel $\nu_\mu n \to \mu^- p$, it provides
scription of resonance production, embraci

Dark Matter Interactions in White Dwarfs: A Multi-Energy Approach to Capture Mechanisms

Jaime Hoefken Zink,[a,b] Shihwen Hor,[c] and Maura E. Ramirez-Quezada[d,e]

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[d] *Johannes Gutenberg-Universität Mainz, 55099 Mainz, Germany, and*
[e] *Dual CP Institute of High Energy Physics, C.P. 28045, Colima, México.*
E-mail: jaime.hoefkenzink2@unibo.it, shihwen@hep-th.phys.s.u-tokyo.ac.jp, mramirez@uni-mainz.de

ABSTRACT: White dwarfs offer a compelling avenue for probing interactions of dark m
particles, particularly in the challenging sub-GeV mass regime. The constraints de
from these celestial objects strongly depend on the existence of high dark matter den

Rein, Sehgal (1981)

Massless lepton excitation of baryon resonances

Berger, Sehgal (2007)

Massive lepton excitation of baryon resonances

HHR (2024)

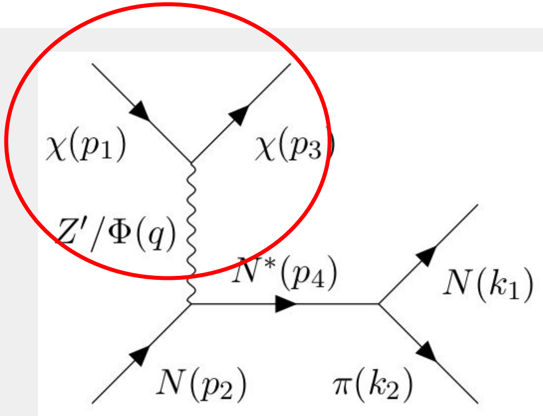
Double massive DM excitation of baryon resonances

HOW TO COMPUTE THE CROSS SECTIONS

Short-version recipe

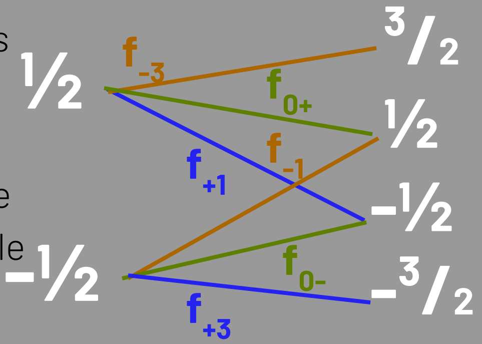
1. VECTOR

We compute the vector **V** as the one involved in the transition N→ N* for each combination of DM-spins in RES frame.



2. FORM FACTORS

We compute the amplitudes using the FKR model (4D harmonic oscillator Hamiltonian)) with a suitable interaction for every possible hadronic spin transition.



3. SUM RESONANCES

Sum the contributions for the 18 resonances ($L_{2I,2J}$). Equal L and J interfere.

4. COMPUTE CROSS SECTION

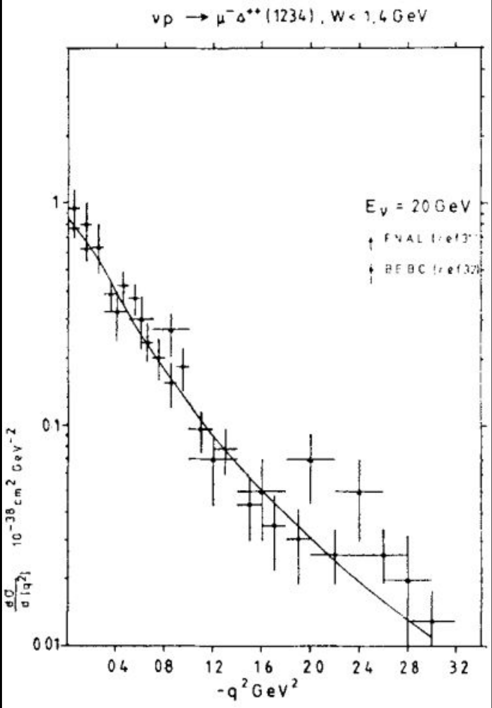
Compute the cross section: χ N → χ N*. Turn the δ(W - M) into a Breit-Wigner factor to account for the decay of the N*:

$$\delta(W - M) \rightarrow \frac{1}{2\pi} \cdot \frac{\Gamma}{(W-M)^2 + \Gamma^2/4}$$

Then, we are also considering the attached process: N* → N π, so that N* is nearly on-shell.

The process has to be done in the channels:

1. $\chi + p \to \chi + p + \pi^0,$
2. $\chi + p \to \chi + n + \pi^+,$
3. $\chi + n \to \chi + n + \pi^0,$
4. $\chi + n \to \chi + p + \pi^-.$



Prediction of Rein-Sehgal model for neutrinos: $\nu\ p \to \mu^- \Delta^{++}$

Just to see how it looks like… For the dark photon:

$$
\begin{aligned}
&\mathcal{M}(\chi(p_1,\lambda_1)N(p_2)\to\chi(p_3,\lambda_2)N^*(p_4)) \\
&= \frac{g_D\epsilon e}{q^2-m_{Z'}^2}\left[\overline{u}_{p_3\lambda_2}\gamma_\mu\left(g_V-g_A\gamma^5\right)u_{p_1\lambda_1}\right]\left(g^{\mu\nu}-q^\mu q^\nu/m_{Z'}^2\right)\langle N^*|\,J_\nu^+(0)\,|N\rangle \\
&= 2M\frac{g_D\epsilon e}{q^2-m_{Z'}^2}V^\mu_{\lambda_1\lambda_2}\langle N^*|\,F_\mu^V\,|N\rangle\,,
\end{aligned}
$$

$$
\frac{d\sigma}{dW\,dq^2}=\frac{\alpha g_D^2\epsilon^2}{\pi\left(q^2-m_{Z'}^2\right)^2}\frac{W}{m_N}\sum_{\lambda_1\lambda_2}\left[\left|C_L^{\lambda_1\lambda_2}\right|^2\sigma_L+\left|C_R^{\lambda_1\lambda_2}\right|^2\sigma_R+\left|C_s^{\lambda_1\lambda_2}\right|^2\sigma_s^{\lambda_1\lambda_2}\right]
$$

Just to see how it looks like... For the scalar:

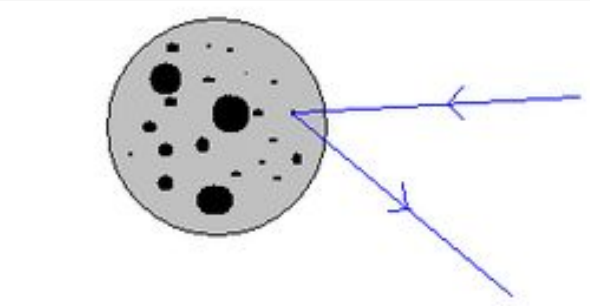
$$\mathcal{M}(\chi(p_1,\lambda_1)N(p_2)\to\chi(p_3,\lambda_2)N^*(p_4)) = \frac{g_D g_{\Phi N}}{q^2-m_\Phi^2}\left[\overline{u}_{p_3\lambda_2}u_{p_1\lambda_1}\right]\langle N^*|\,J_S^+(0)\,|N\rangle$$
$$= 2M\frac{g_D g_{\Phi N}}{q^2-m_\Phi^2}V^S_{\lambda_1\lambda_2}\langle N^*|\,F_S\,|N\rangle\,,$$

$$\frac{d\sigma}{dW\,dq^2} = \frac{g_D^2 g_{\Phi N}^2}{64\pi\left(q^2-m_\Phi^2\right)^2}\frac{W}{m_N^2|\vec{p}_1|^2}\sum_{\lambda_1\lambda_2}|V^S_{\lambda_1\lambda_2}|^2\sum_{i=\pm}\langle N^*|\,F^S_{0\pm}\,|N\rangle$$

Deep inelastic scattering (DIS)

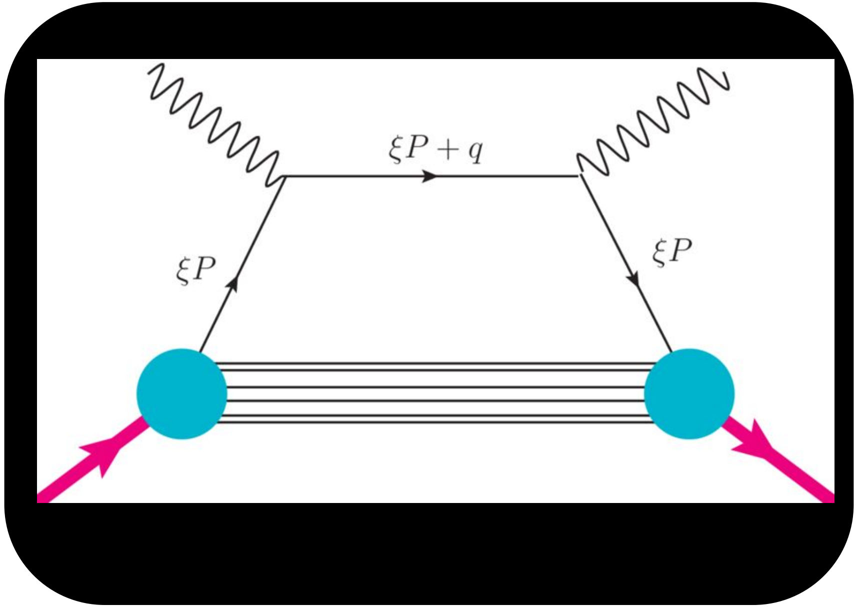
Partonic approach

DIS and partons



Partonic approach

Due to asymptotic freedom, interaction among quarks in a nucleon can be neglected and they can be considered to carry a “part” of the momentum of the nucleon: ξp_N.

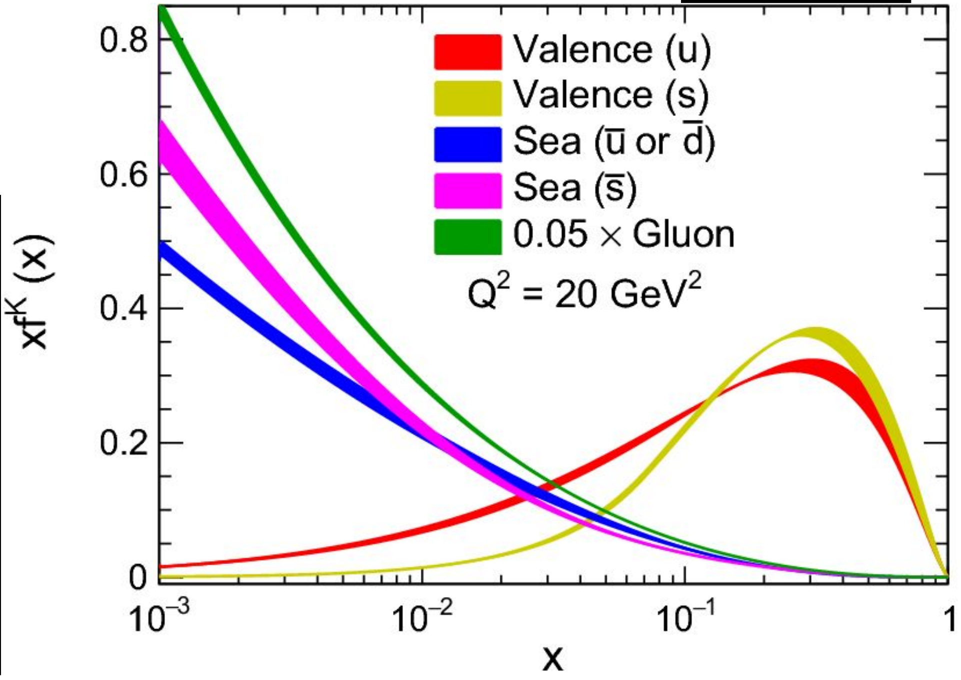


Feynman, "Very High-Energy Collisions of Hadrons”, 1969.

Main variables and ingredients:

- *Momentum transfer:* $Q^2 \equiv -q^2$
- *Energy transfer:* $\nu \equiv \frac{p_2 \cdot q}{m_N}$, such that m_N is the mass of the nucleon.
- *Inelasticity:* $y \equiv \frac{p_2 \cdot q}{p_2 \cdot p_1}$
- *Bjorken scaling variable:* $x \equiv \frac{Q^2}{2 p_2 \cdot q}$.

$\xi \rightarrow x$ (high energies)



Cross sections

Vector

$$\frac{d^2\sigma}{dxdy} = \frac{g_D^2}{4\pi m_{Z'}^4} \frac{E_\chi^2 m_N x}{(1+Q^2/m_{Z'}^2)^2} \frac{\sqrt{E_\chi^2(1-y)^2 - m_\chi^2}}{(1-y)(E_\chi^2 - m_\chi^2)}$$
$$\times \sum_q g_{Z'q}^2 \left((A_q A_\chi + C_q C_\chi) y^2 - 2(A_q A_\chi - C_q C_\chi) y - \frac{A_q m_\chi^2}{E_\chi m_N x} B_\chi y + 2 A_q A_\chi \right) f_q(x).$$

$$A_k \equiv (g_V^k)^2 + (g_A^k)^2,\ B_k \equiv (g_V^k)^2 - (g_A^k)^2,\ \text{and}\ C_k \equiv 2 g_k^V g_k^A$$

Scalar

$$\frac{d^2\sigma}{dxdy} = \frac{g_D^2}{16\pi m_\Phi^4} \frac{y E_\chi^2 (Q^2 + 4m_\chi^2)}{(1+Q^2/m_\Phi^2)^2 (E_\chi^2 - m_\chi^2)} \times \sum_q g_{\Phi q}^2 f_q(x)$$

Benchmark points to study

01

$m_{Z'}$
100 MeV

m_{χ}
100 MeV

$\varepsilon = 10^{-5}$
$g_D = 0.1$

Vector
Light mediator

02

$m_{Z'}$
10 GeV

m_{χ}
100 MeV

$\varepsilon = 10^{-5}$
$g_D = 0.1$

Vector
Heavy mediator

03

m_{Φ}
1 GeV

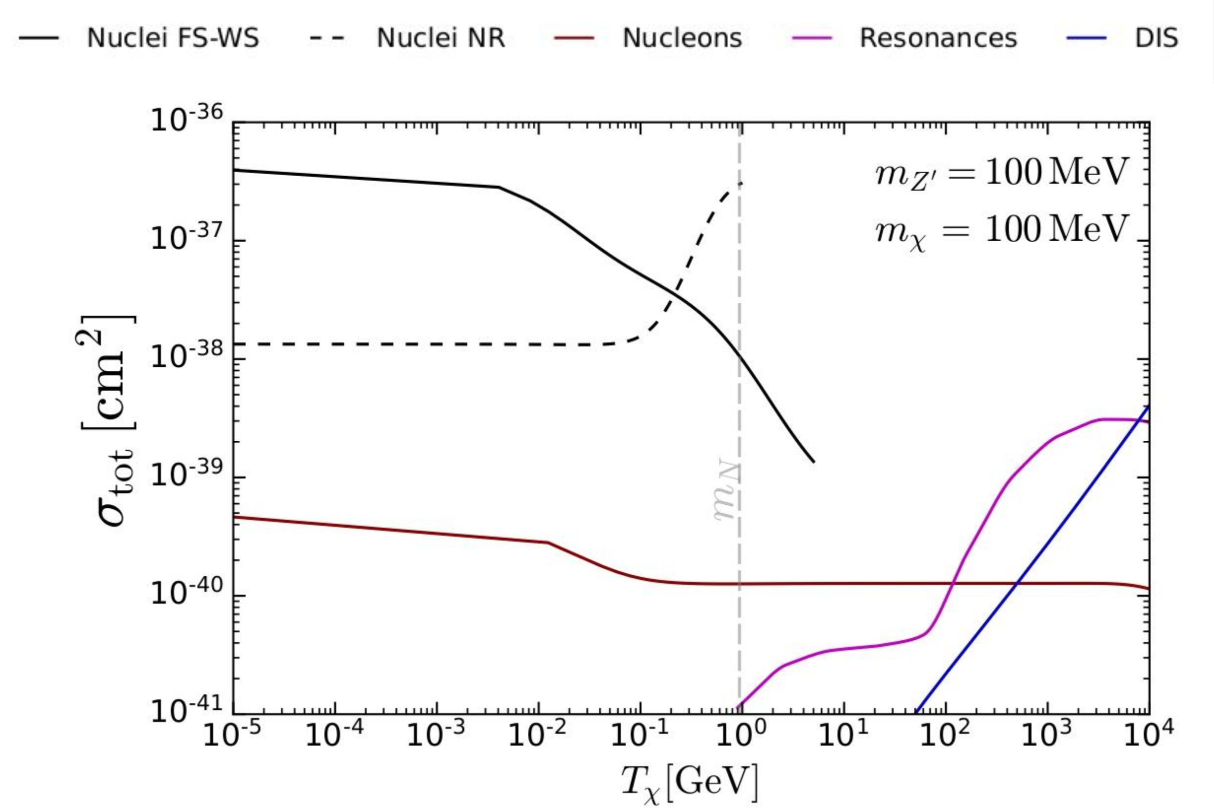
m_{χ}
100 MeV

$g_{N\Phi} = 10^{-5}$
$g_D = 0.1$

Scalar
Heavy mediator

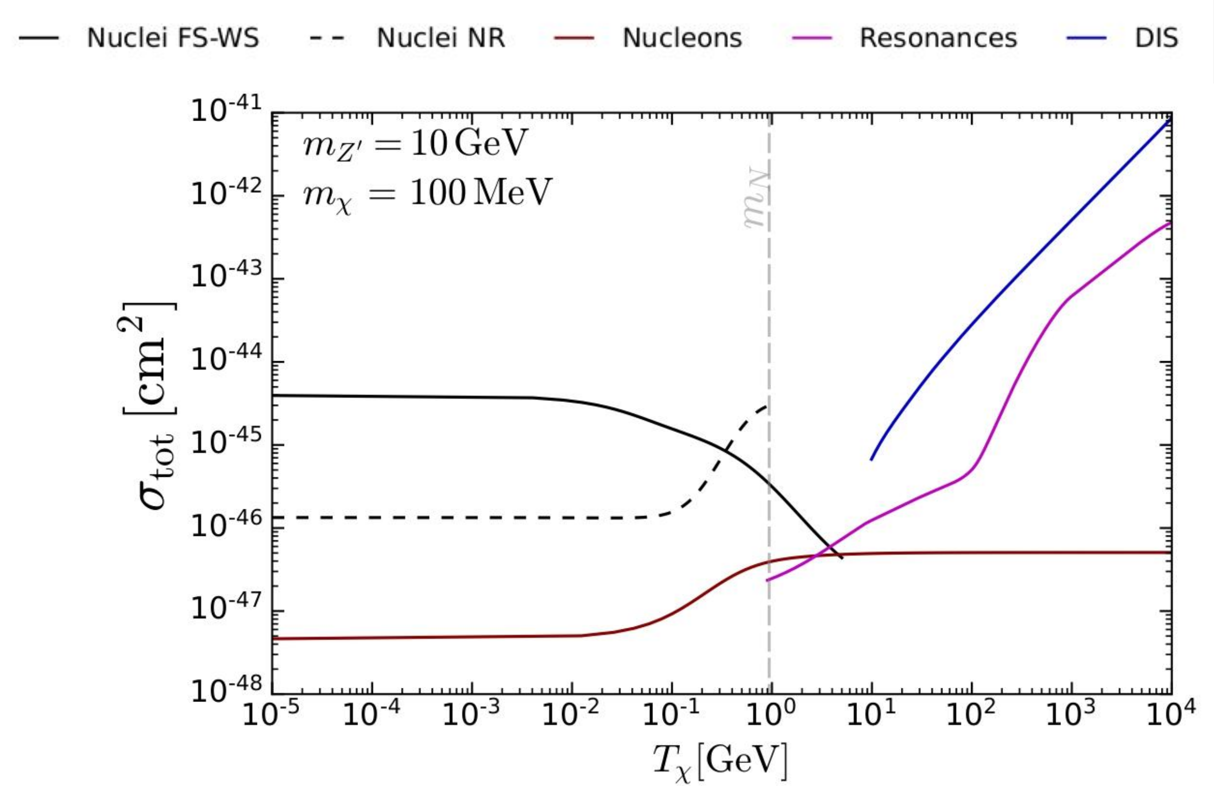
BP 1

Light vector



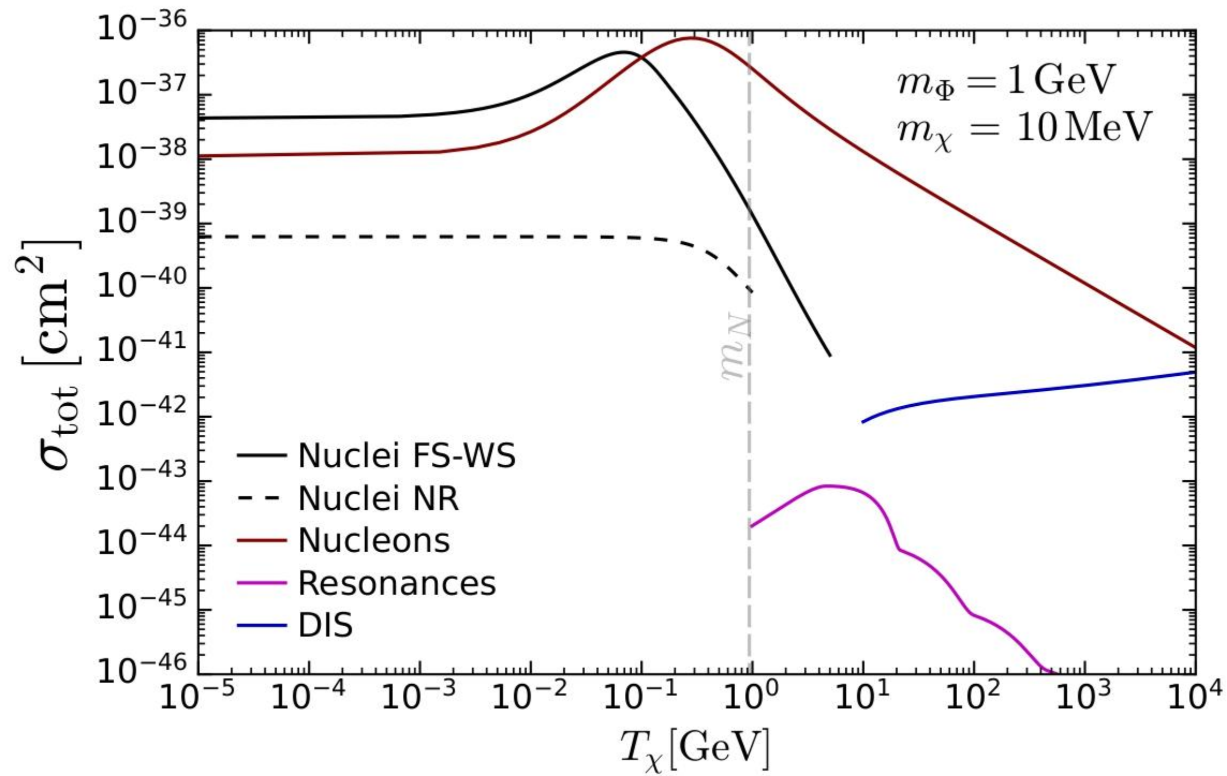
BP 2

Heavy vector



BP 3

Heavy scalar

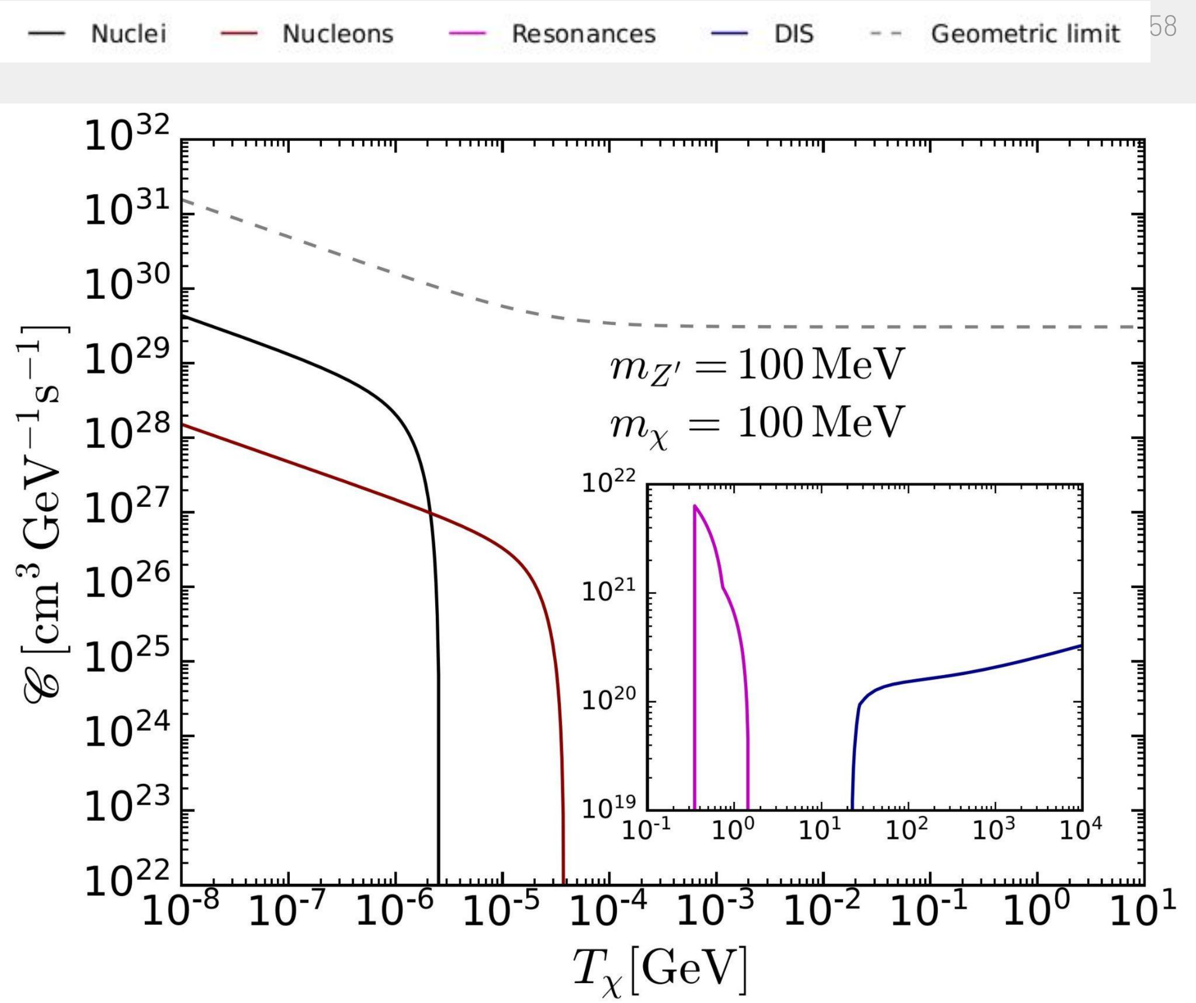


05

Results

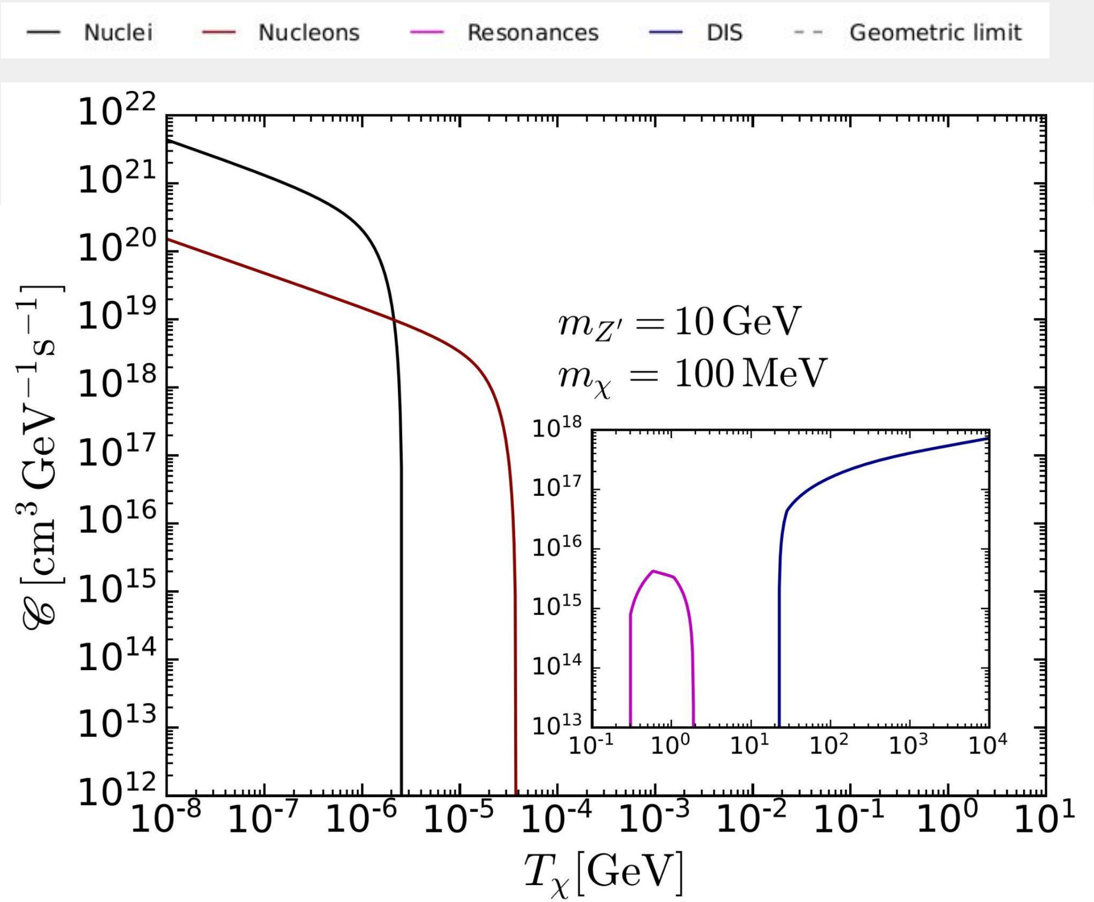
BP 1

Light vector



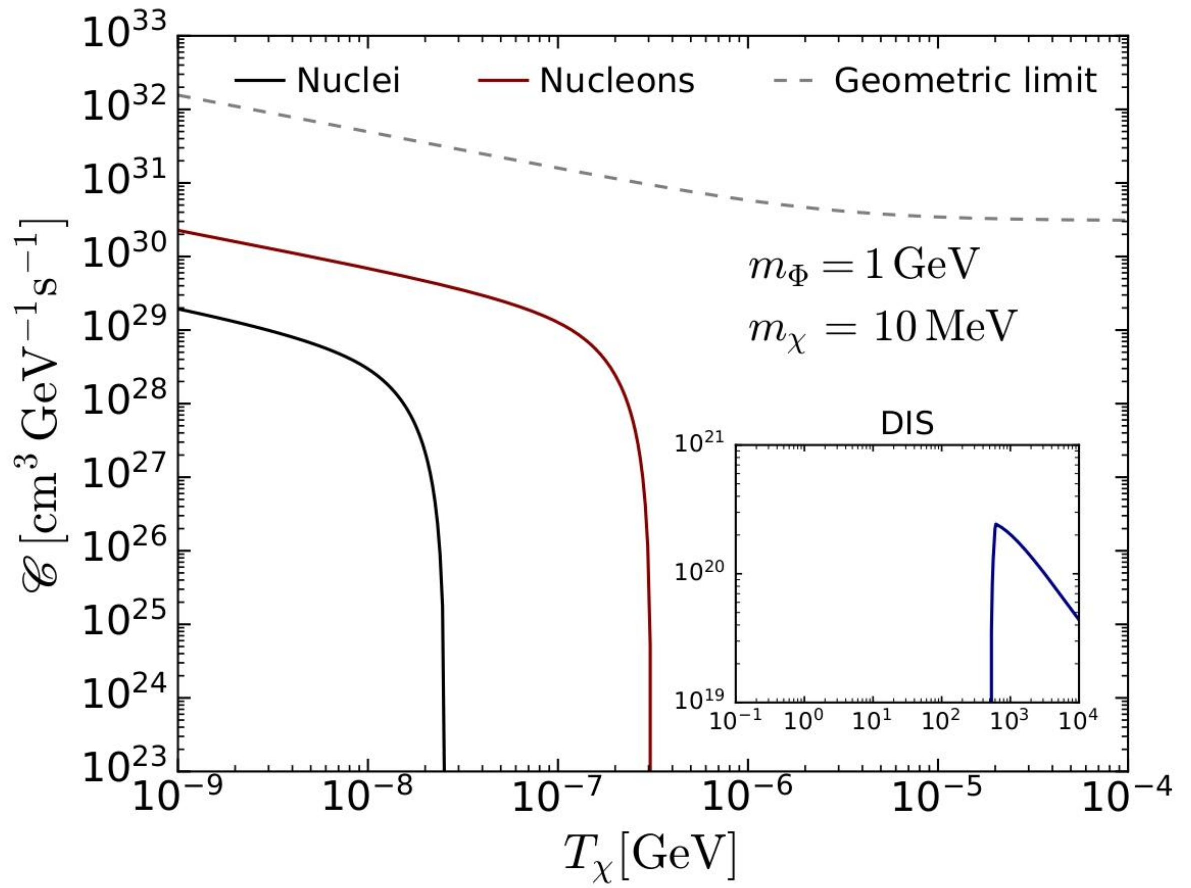
BP 2

Heavy vector



BP 3

Heavy scalar



06

Conclusions

CONCLUSIONS

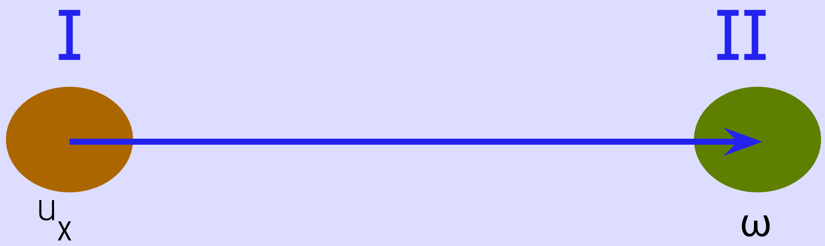
What was shown	• Mechanism of capture rate of dark particles in WDs. • DM - N (SM) cross sections (vector/scalar). • Capture rate density (sensitivity) for different energies.
What we found	• The least energetic DM is easier to capture (under same fluxes) • For a vector mediator, resonant and DIS scatterings could also be visible for high fluxes.
Pheno tasks	• Test sensitivity for different fluxes. • FInd limits on WD lifetimes to set bounds on the different DM models.
Future prospects	• Study these effects in neutron stars, red giants... • Compute the capture of kinetic energy and the heating of old stars.

THANK YOU

A

Extra slides

WD accelerating DM



$$d_s^2 = \begin{cases} -g_{tt}\,dt^2 + g_{rr}\,dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2), & \text{at } r = R_\star \\ -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2), & \text{at } r = \infty \end{cases}$$

$$A = g_{tt} \text{ and } B = g_{rr}.$$

1

$$E = -g_{\mu\nu}p^\mu \xi^\nu, \text{ where } \xi = (1,0,0,0).$$

$$E_{\rm I} = \frac{m_\chi}{\sqrt{1-u_\chi^2}} \qquad \text{where } u_\chi^t = 1/(A\sqrt{1-u_\chi^2}).$$

$$E_{\rm II} = g_{tt} m_\chi u_{\rm II}^t,$$

2

$$g_{\mu\nu}u_{\rm II}^\mu u_{\rm II}^\nu = -1 \qquad g_{tt}u_{\rm II}^t u_{\rm II}^t + g_{rr}u_{\rm II}^r u_{\rm II}^r = -1$$

$$A(u_{\rm II}^t)^2 + B(u_{\rm II}^r)^2 = -1$$

$$u_{\rm II}^r = \sqrt{\frac{1}{A\,B\,(1-u_\chi^2)} - \frac{1}{B}}$$

$$\frac{u_{\rm II}^r}{u_{\rm II}^t} \equiv \frac{dr}{dt} = A\sqrt{\frac{1}{A\,B} - \frac{(1-u_\chi^2)}{B}}$$

3

4

$$g_{\hat t\hat t} = -1 \text{ and } g_{\hat r\hat r} = 1 \qquad e_t^{\hat t} = \sqrt{A} \text{ and } e_r^{\hat r} = \sqrt{B}.$$

$$\omega = \frac{d\hat r}{d\hat t} = \frac{e_r^{\hat r}}{e_t^{\hat t}}\frac{u_{\rm II}^r}{u_{\rm II}^t} \qquad \boxed{\omega^2 = v_e^2 + (1-v_e^2)u_\chi^2.}$$

B

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