New Angles on Energy Correlators

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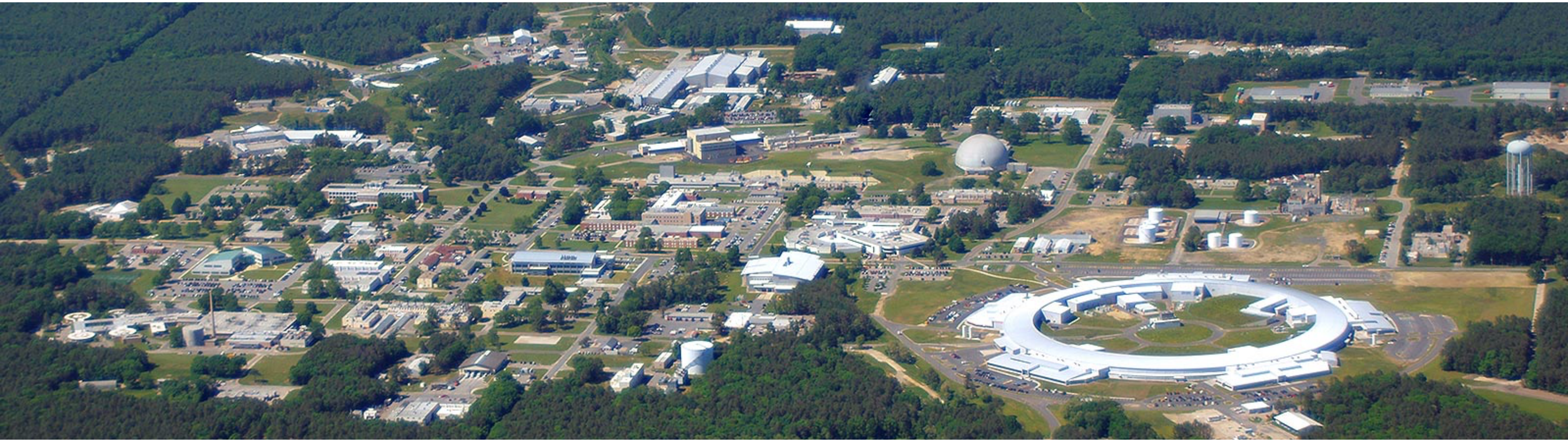
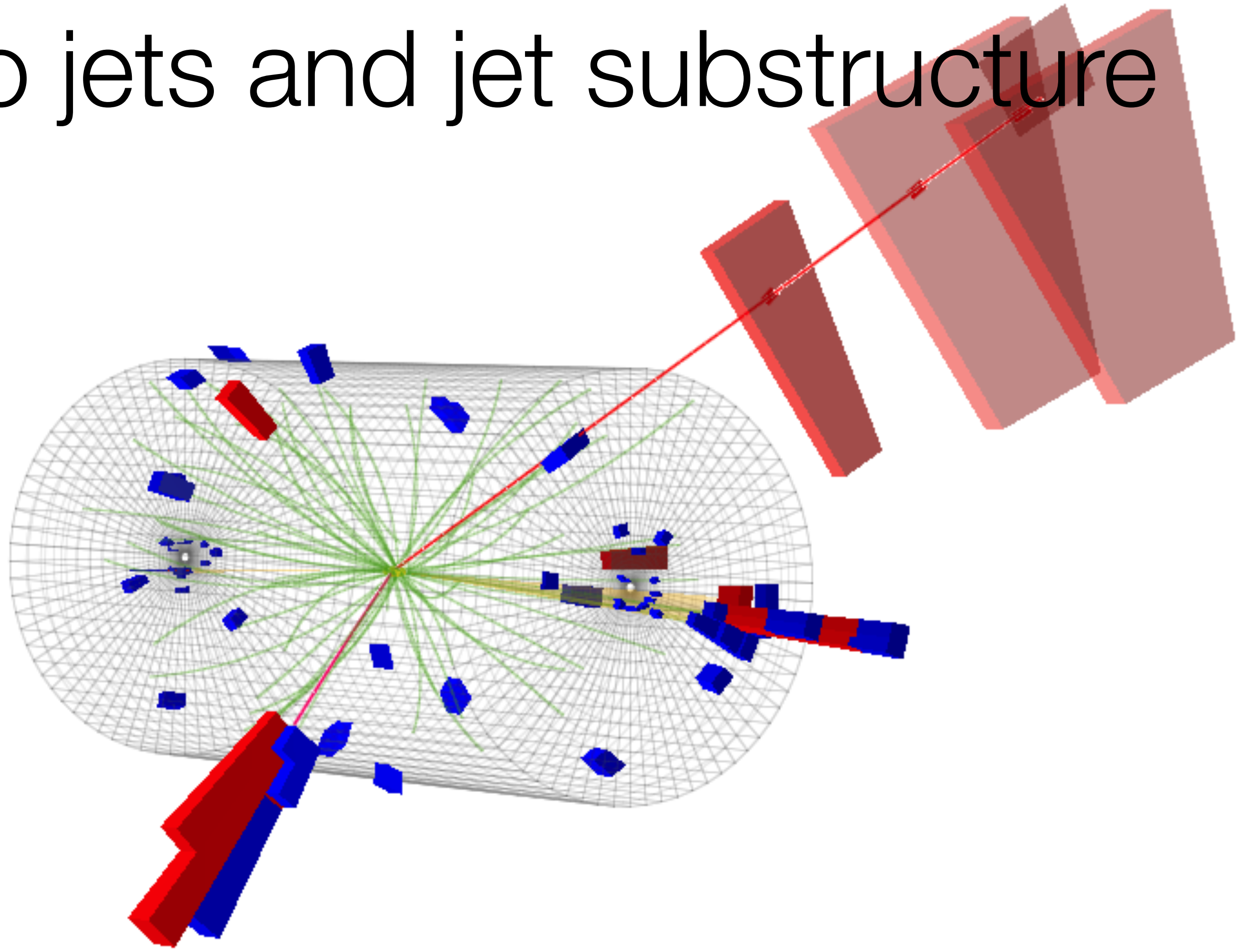


BNL High Energy Theory Seminar - June 12

Outline

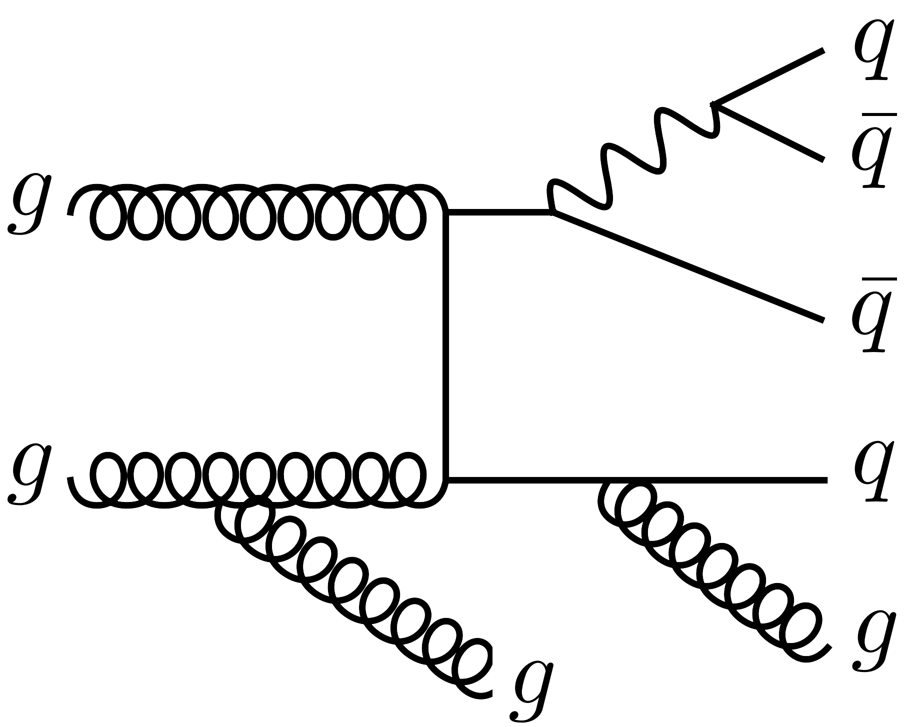
1. Introduction to jets and jet substructure
2. Introduction to energy correlators
3. Energy-energy correlator: on track to high precision
4. Analytic continuation and small-x physics
5. New angles on energy correlators
6. Bonus
7. Conclusions

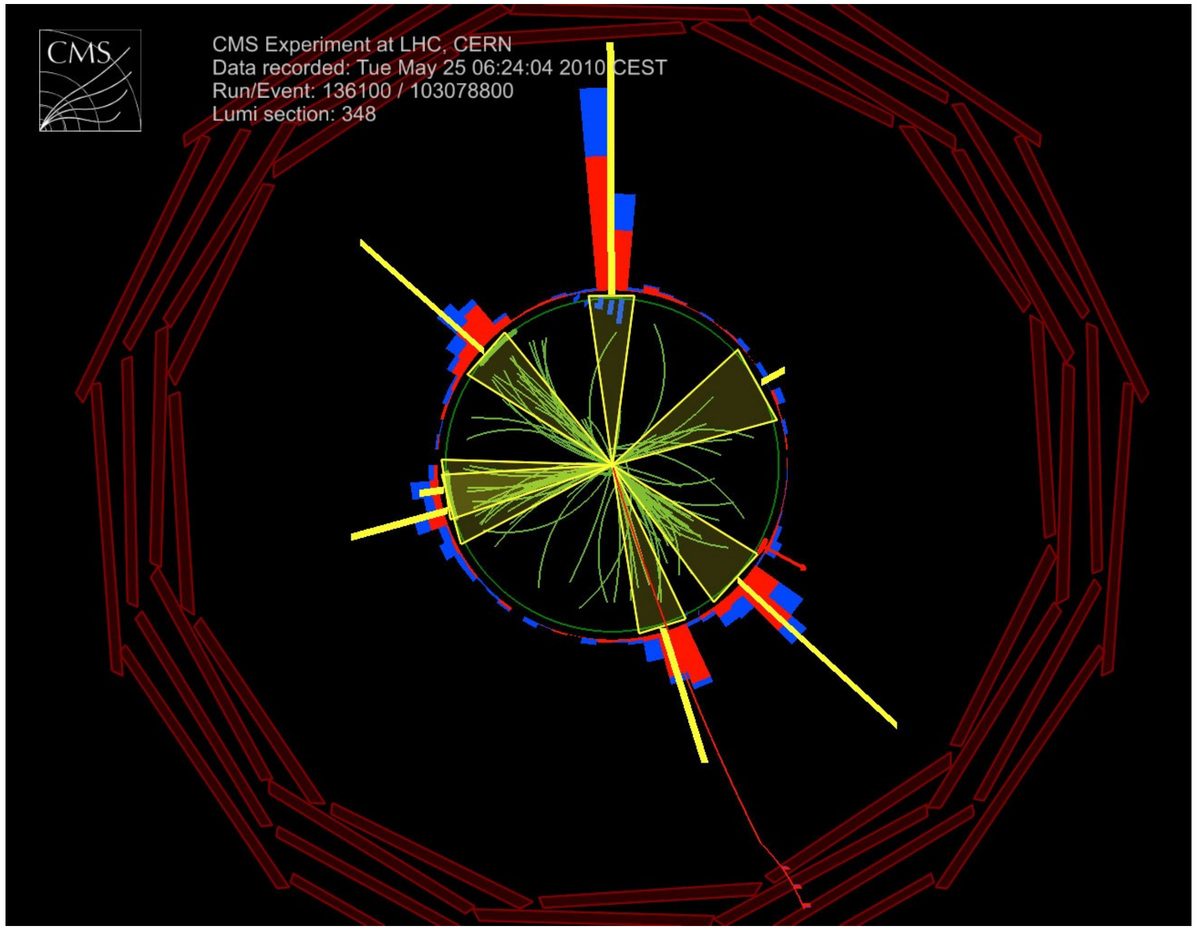
1. Introduction to jets and jet substructure



What is a jet?

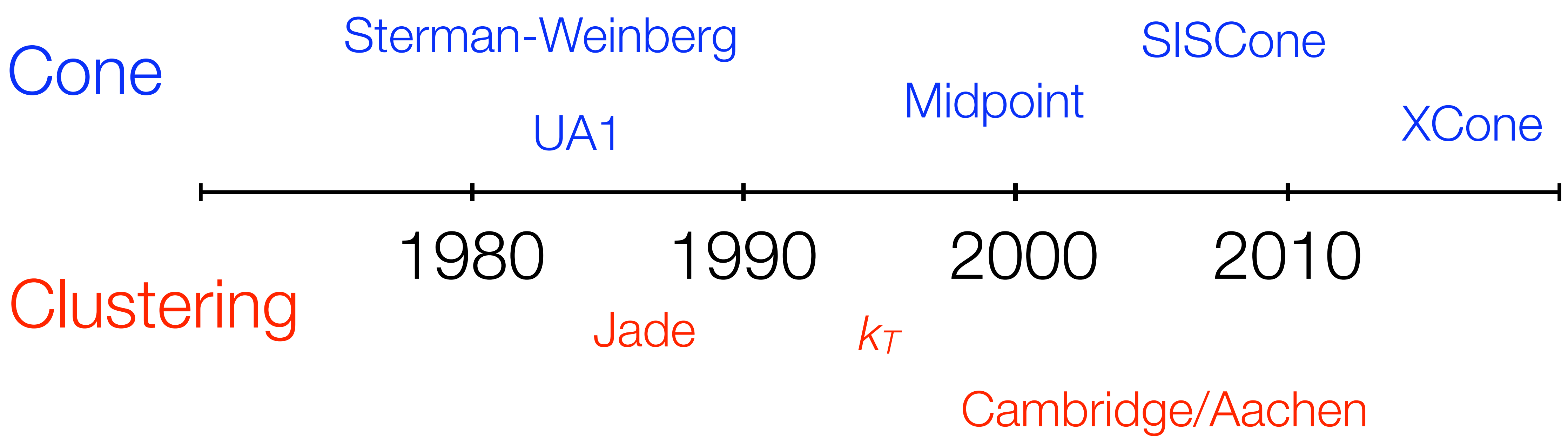
- Quarks and gluons produced in colliders radiate and hadronize
  → result in collimated streams of hadrons.

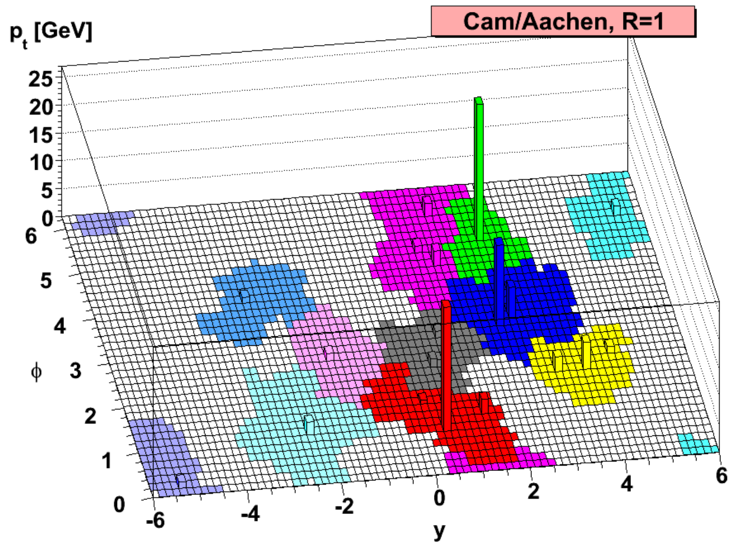


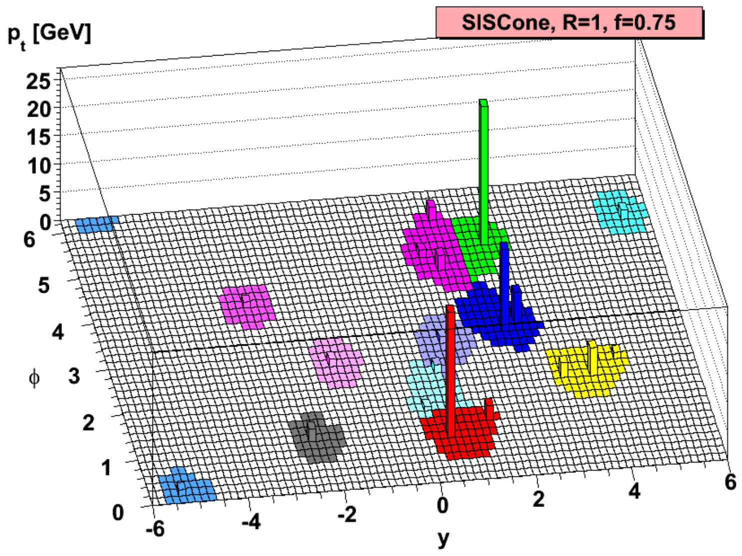


A brief history of jet definitions

Should be:
- infrared and collinear safe
- easy to implement in theory & experiment







[Cacciari, Salam, Soyez]

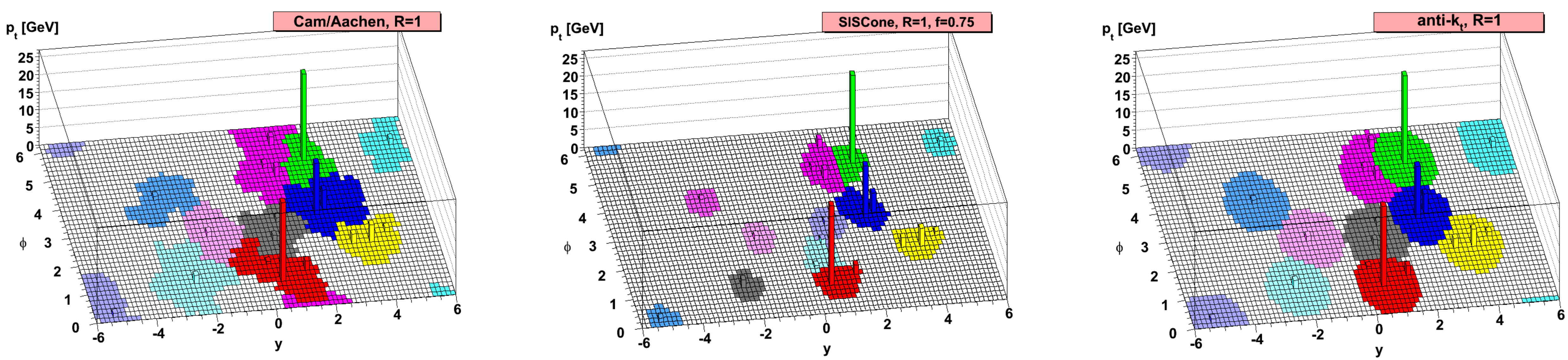
A brief history of jet definitions

Should be:
- infrared and collinear safe
- easy to implement in theory & experiment

Cone: Sterman-Weinberg, UA1, Midpoint, SISCone, XCone

1980 — 1990 — 2000 — 2010

Clustering: Jade, k_T, Cambridge/Aachen, anti-k_T



[Cacciari, Salam, Soyez]

Jets matter

- Jets enter in most LHC analyses as signal or background.
- Study parton evolution with jets → improve parton showers

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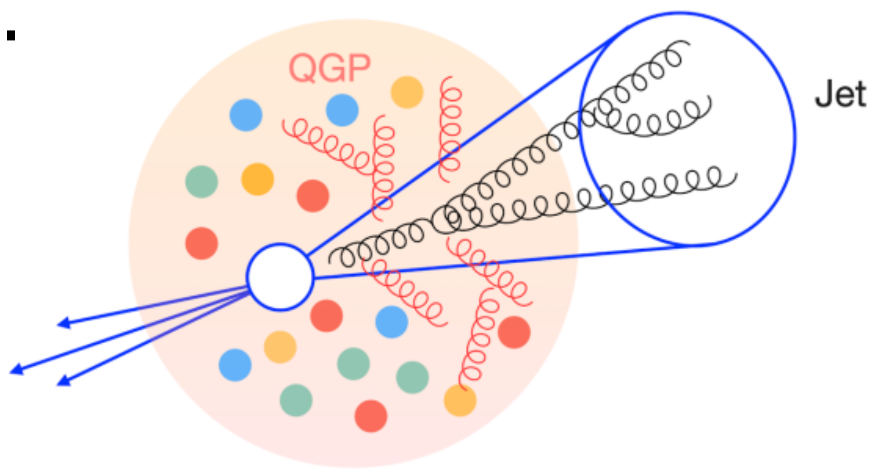
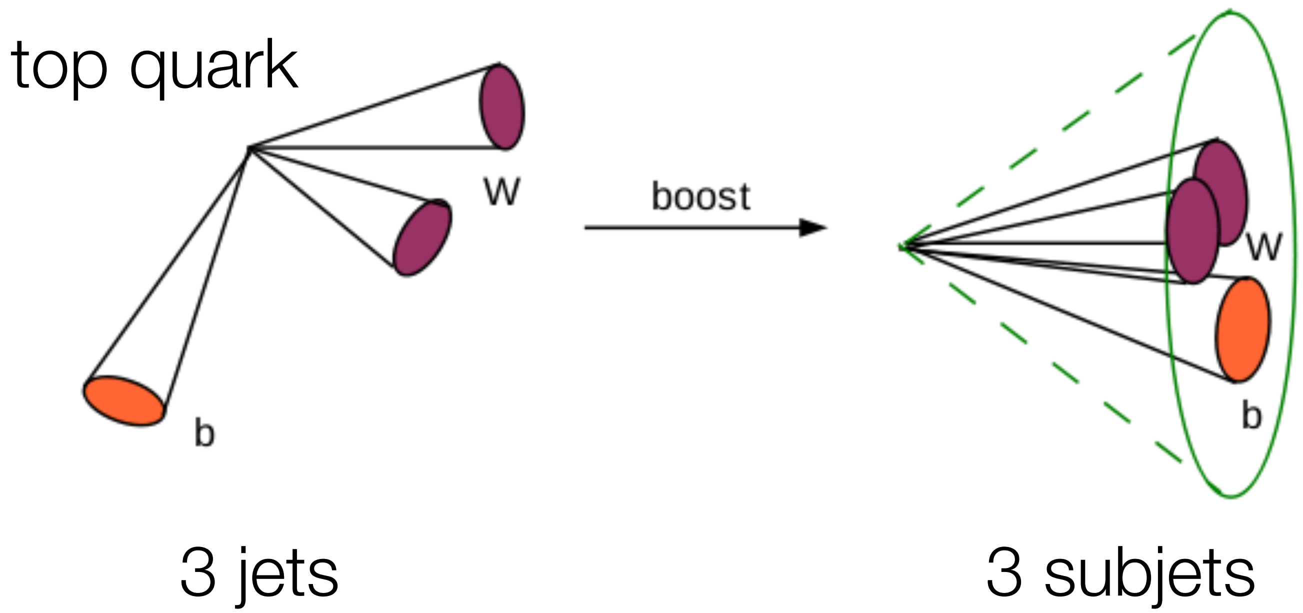
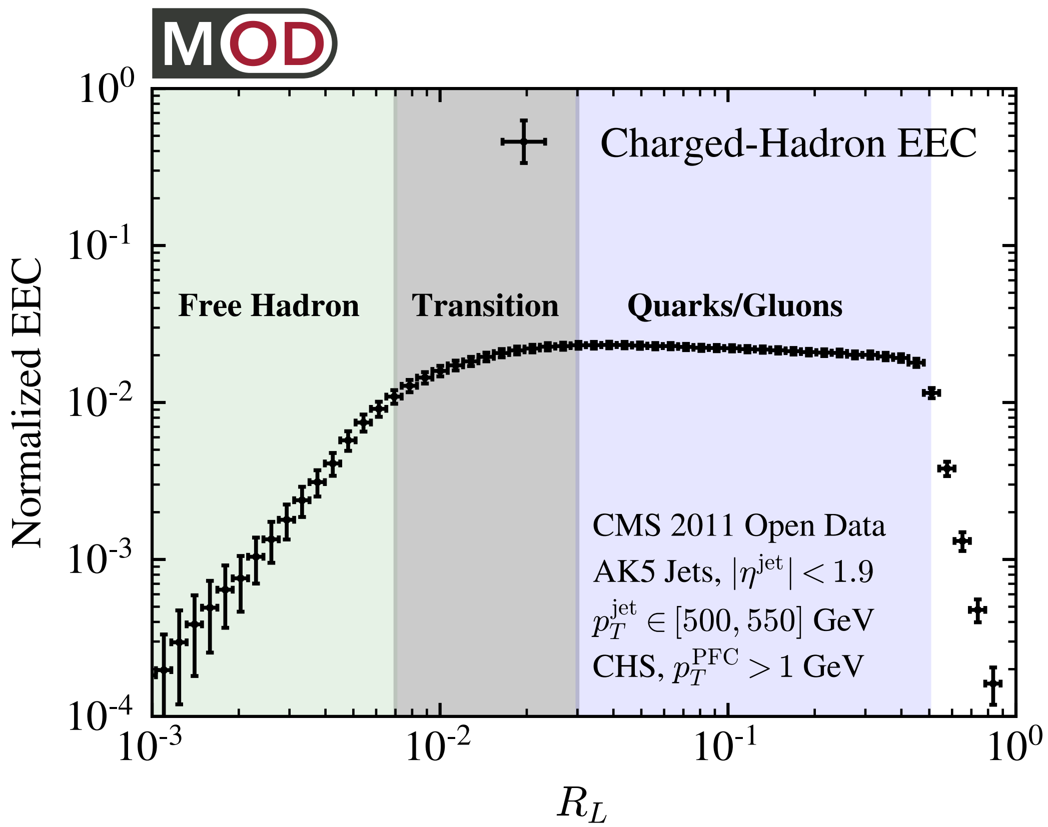
- Jets enter in most LHC analyses as signal or background.
- Study parton evolution with jets → improve parton showers, probe quark-gluon plasma.



- Jet substructure can e.g. identify boosted heavy particles.



2. Introduction to energy correlators

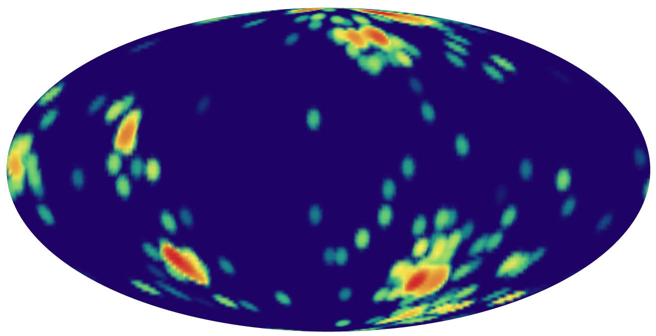


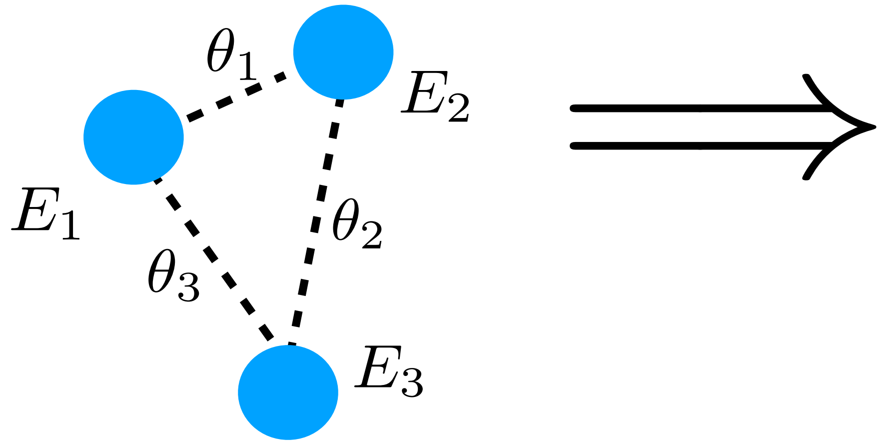
Introduction to energy correlators

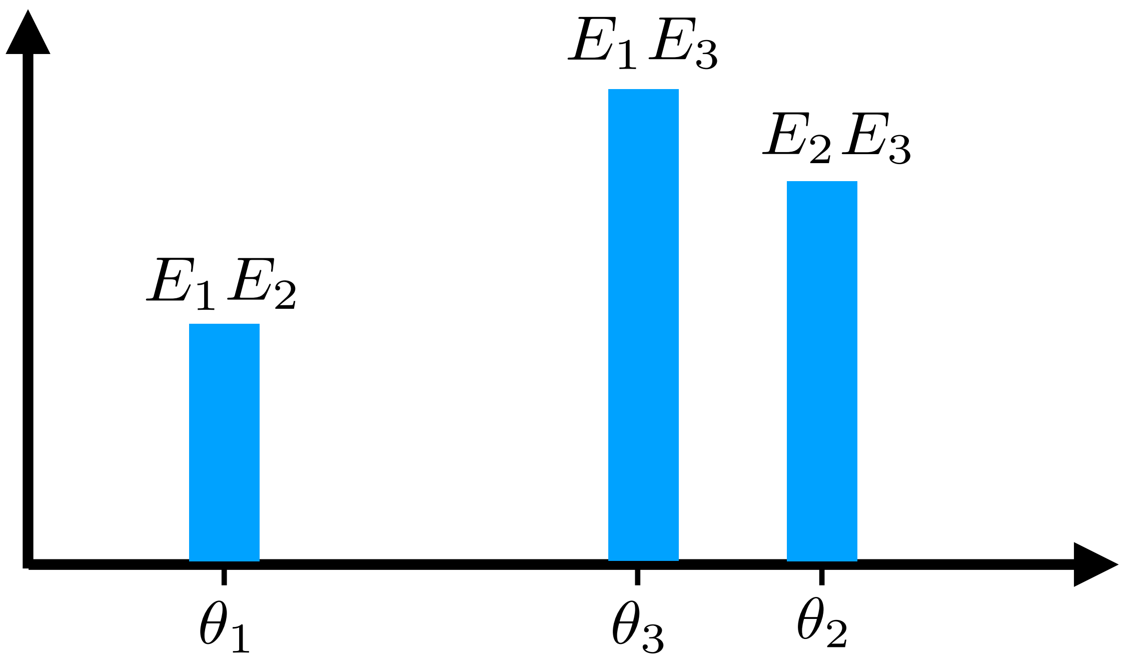
- Event (or jet) shapes describe it through one number.
- Energy-Energy Correlator probes **correlations** in energy flow:

$$\frac{d\sigma}{d\theta} = \int d\sigma \sum_{i,j} \frac{E_i E_j}{(\sum_k E_k)^2} \delta(\theta - \theta_{ij})$$

[Basham, Brown, Ellis, Love]







Introduction to energy correlators

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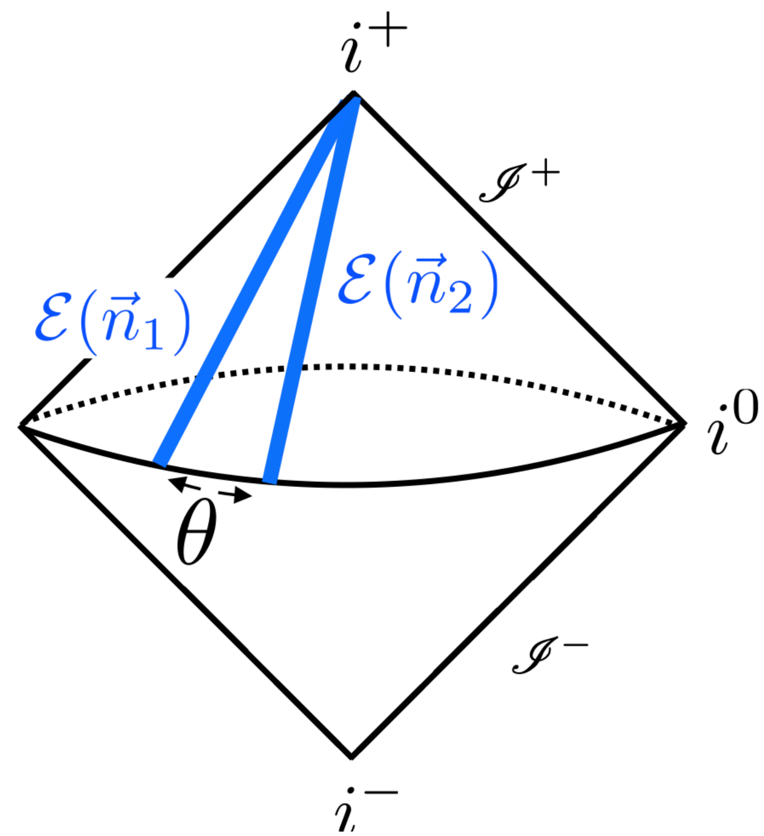
$$\frac{d\sigma}{d\theta} = \int d\sigma \sum_{i,j} \frac{E_i E_j}{(\sum_k E_k)^2} \delta(\theta - \theta_{ij}) \sim \langle \mathcal{E}(\hat{n}_1) \mathcal{E}(\hat{n}_2) \rangle$$

[Basham, Brown, Ellis, Love]

$$\mathcal{E}(\hat{n}) = \lim_{r \to \infty} \int_0^\infty dt\, r^2 n^i T_{0i}(t, r\hat{n})$$



Why the hype?

Recent interest in energy correlators has been driven by:

✓Natural **separation** of physics at different scales.

✓**Simpler** theoretical description → better interpretation.

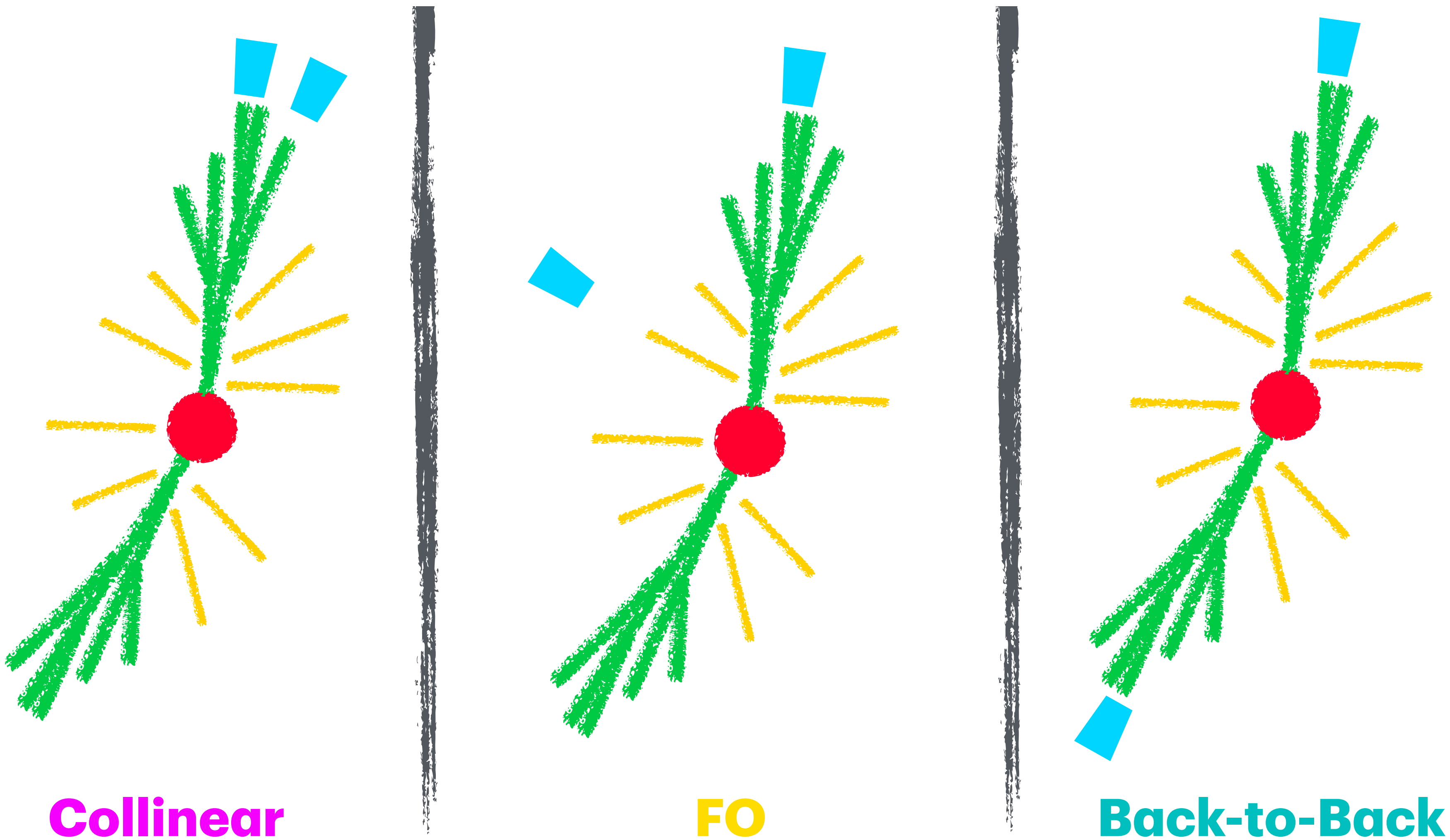
✓Suppression of soft contamination (no grooming).

Wide range of applications:

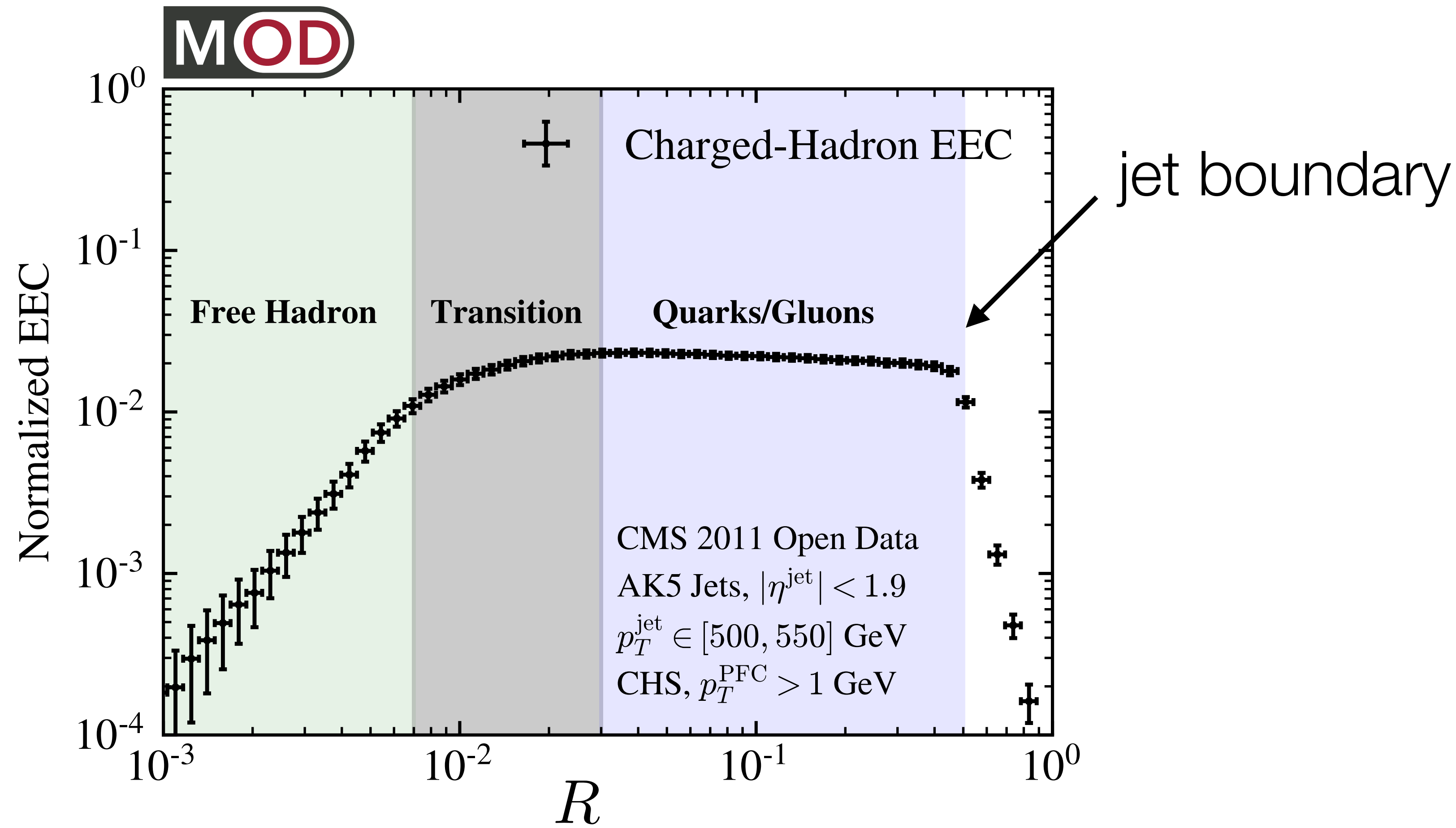
- Strong coupling determination,
- Top quark mass determination,
- Probing quark-gluon plasma,
- Dead cone for heavy quarks, …

Different physics at different angles

- Collinear: power-law scaling, determined by DGLAP evolution.
- Back-to-back: Sudakov, described by TMD factorization.



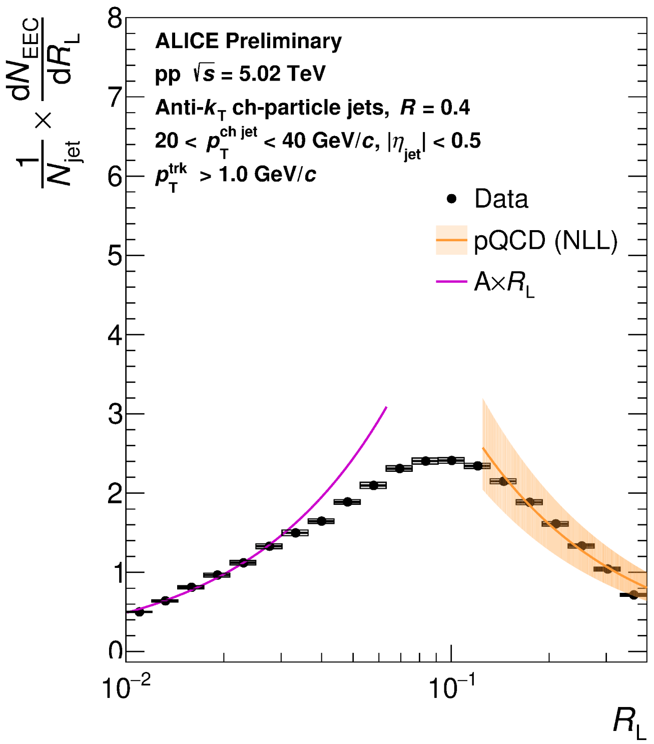
Collinear region

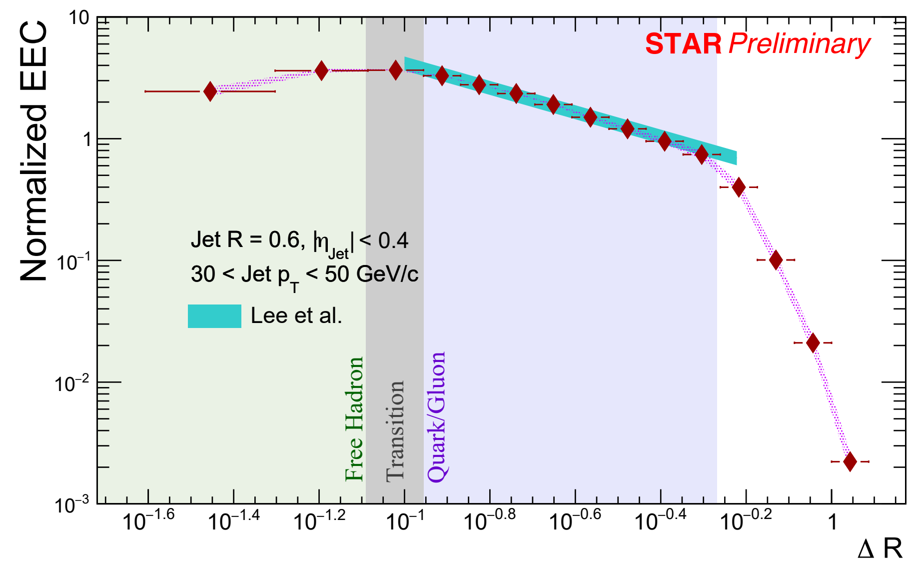


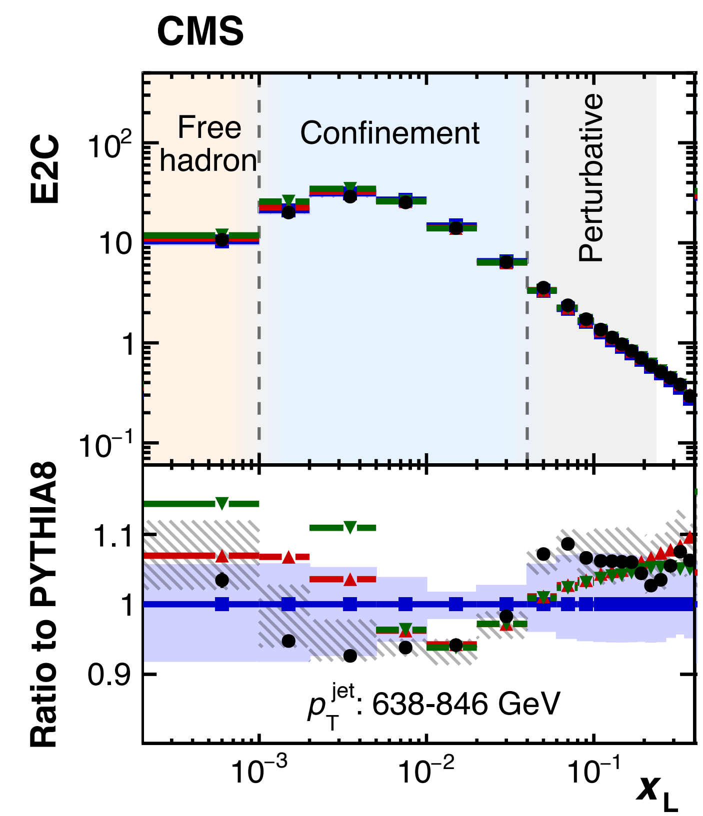
[Komiske, Moult, Thaler, Zhu]

- At the LHC, $(E, \theta) \to (p_T, R)$.
- Perturbative region: $\sim R^\gamma$ with γ set by DGLAP.
- Nonperturbative region: $\sim R^2$, free hadron gas.

Recent measurements







✓Scaling of EEC in perturbative and nonperturbative regimes observed by ALICE, STAR and CMS over **wide** energy range

(Note factor R difference compared to the previous slide.)

N-point energy correlator

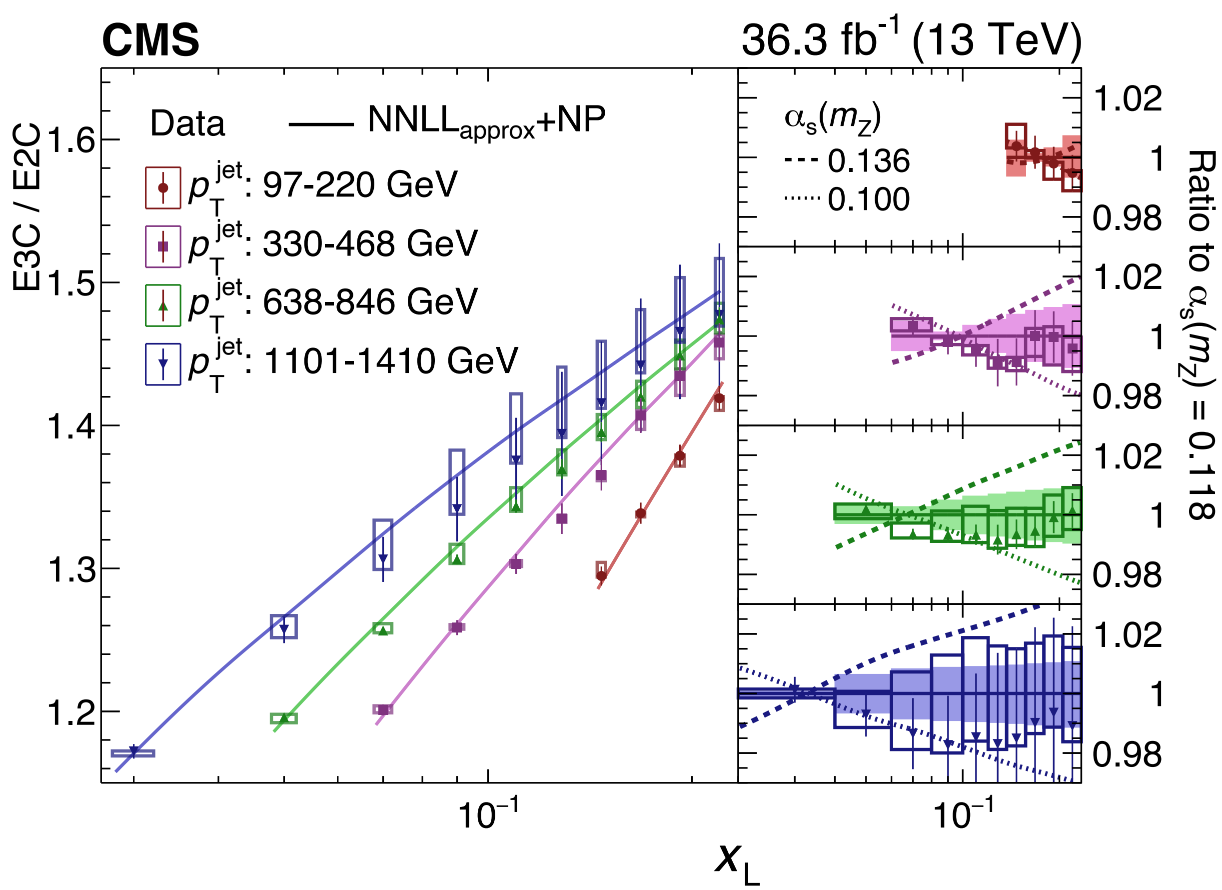
- N-point correlators parametrized by all pairs of angles θ_{ij}
- One can project onto largest angle θ_L

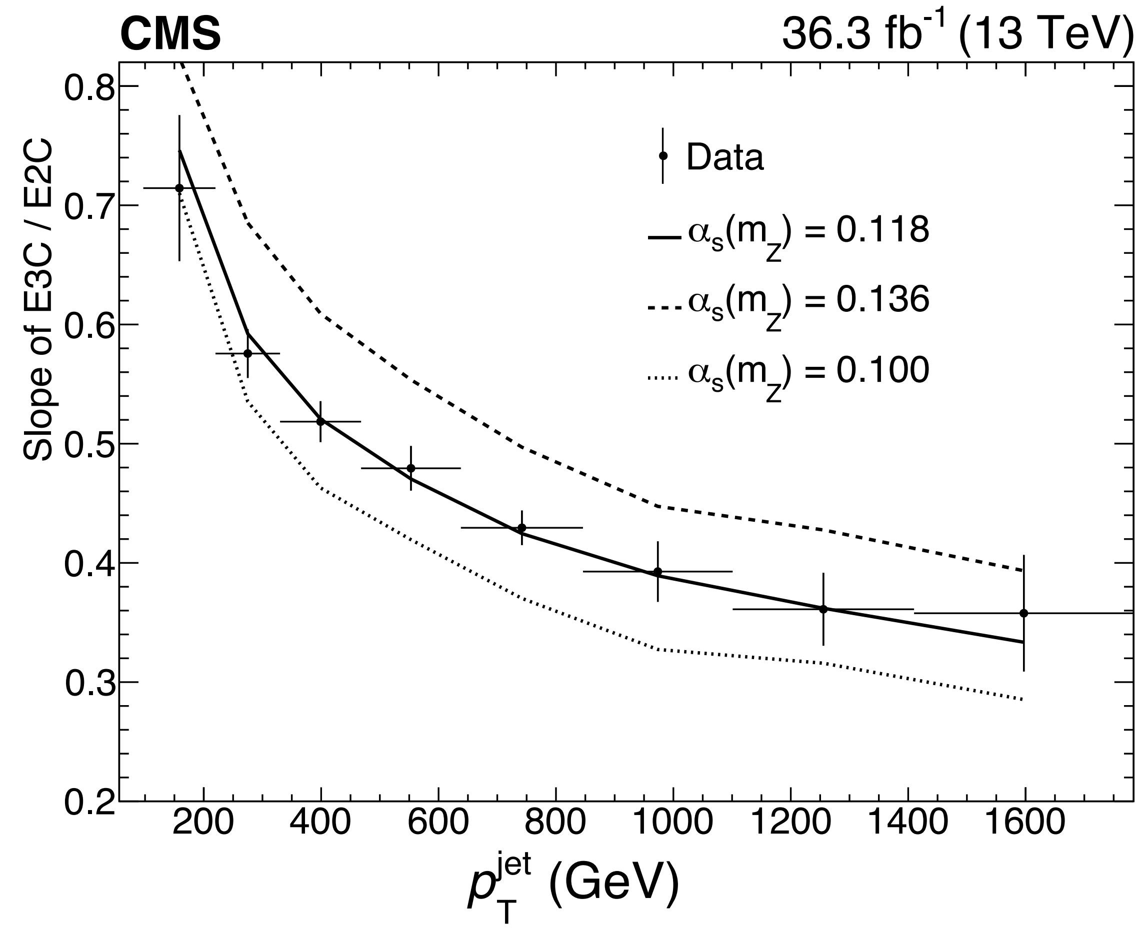
$$\frac{\mathrm{d}\sigma^{[N]}}{\mathrm{d}\theta_L} = \int \mathrm{d}\sigma \sum_{i,j,k,\ldots} \frac{E_i E_j E_k \cdots}{(\sum_m E_m)^N} \, \delta(\theta_L - \max\{\theta_{ij}, \theta_{ik}, \cdots\})$$

[Chen, Moult, Zhang, Zhu]

- Projected N-point correlator (ENC) again has power-law in collinear region.
- Uncertainties reduced in ratio of N-point and 2-point.

Application: α_s

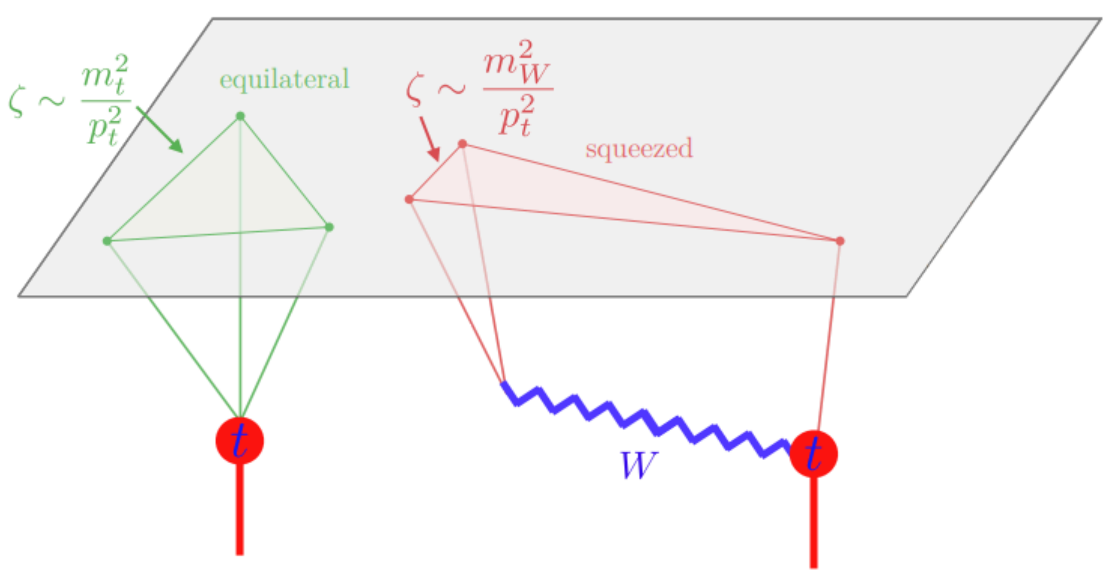


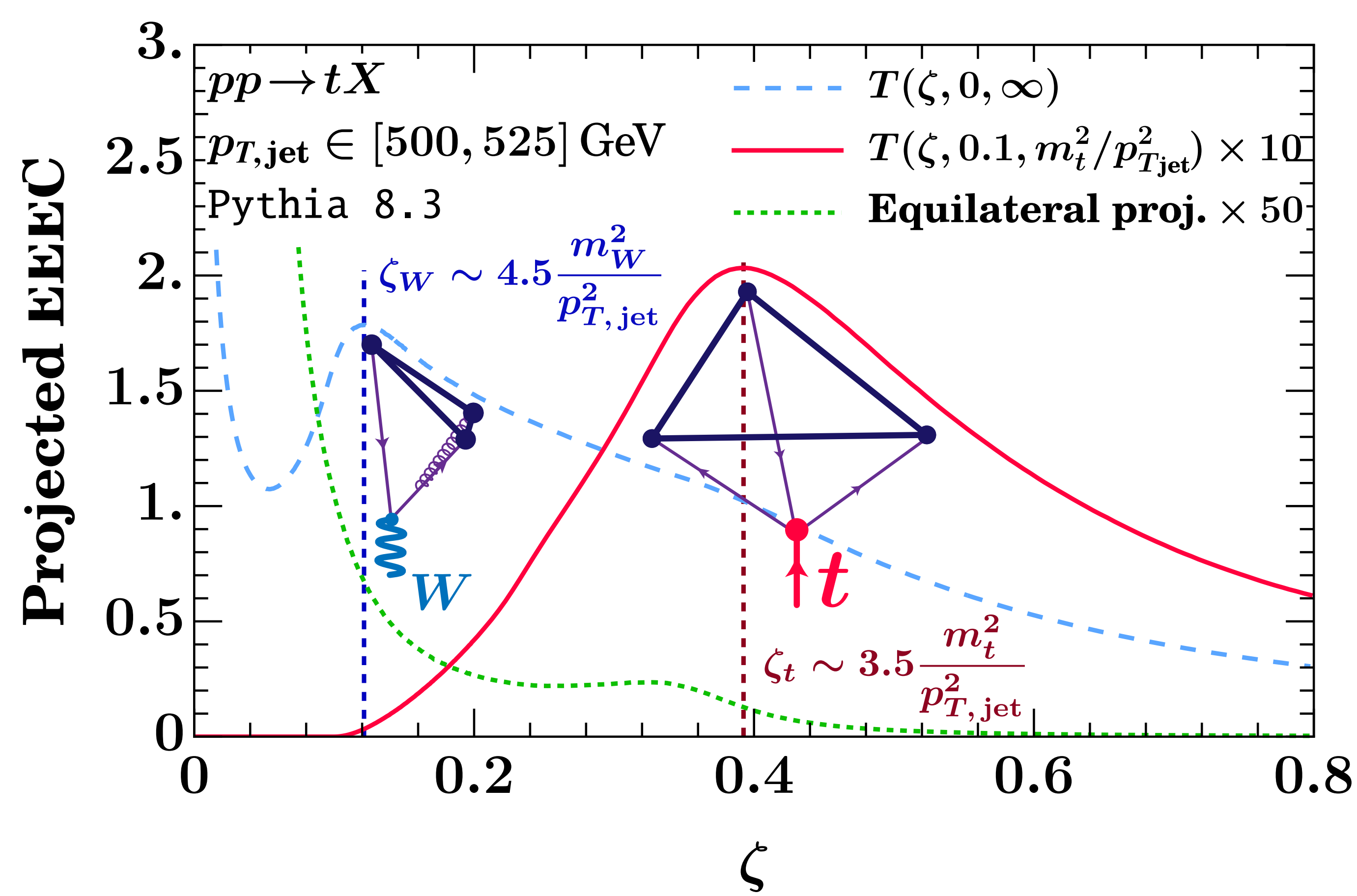


- Extract $\alpha_s(m_Z)$ from slope of E3C/EEC, compare to NLO+NNLL.
- Best fit $\alpha_s(m_Z) = 0.1229^{+0.0014}_{-0.0012}$ (stat.) $^{+0.0023}_{-0.0036}$ (syst.) $^{+0.0030}_{-0.0033}$ (theory) is **most precise** measurement from jet substructure.

Application: top quark mass

- Existing approaches offer **either** good theoretical control **or** good sensitivity to top quark mass → try energy correlators.
- Convert the top quark peak position into a mass using *W*.



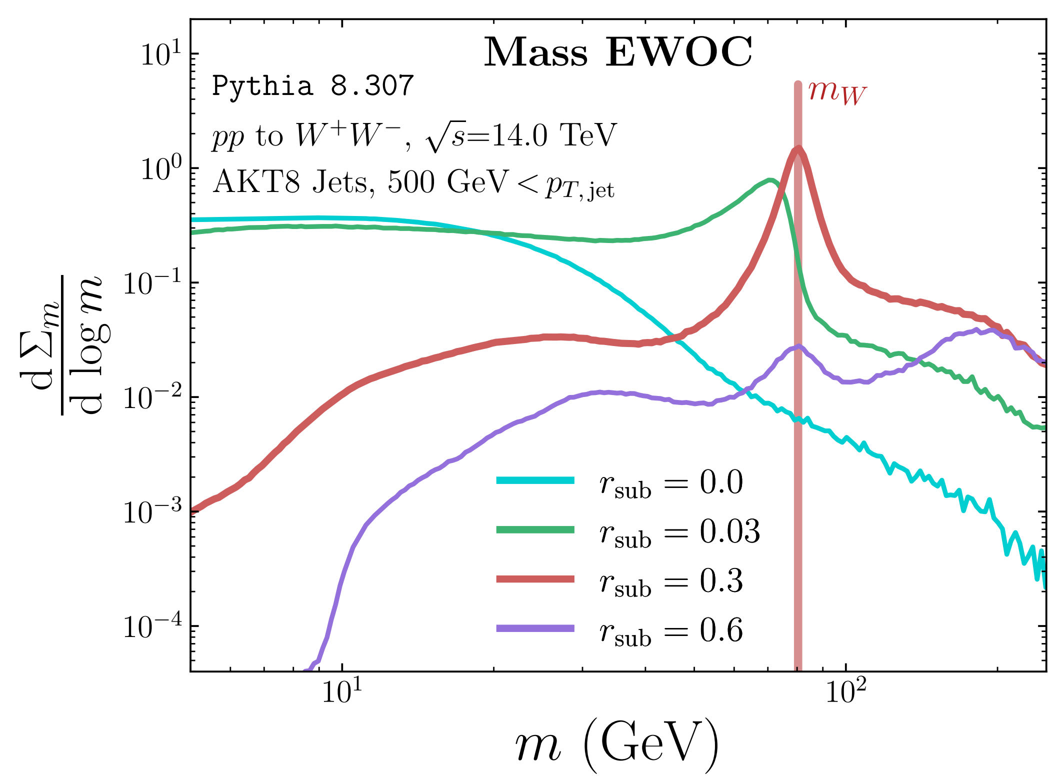


[Holguin, Moult, Pathak, Procura, Schofbeck, Schwarz]

Energy Weighted Observable Correlations

- Motivation: directly study correlations in e.g. mass.
- Collinear unsafe → regularize using subjet radius $r_{\rm sub}$
- Example: mass EWOC for hadronically decaying *W* boson

$$\frac{d\sigma}{dm} = \sum_{\text{subjets } i,j} \int d\sigma \, z_i z_j \, \delta(m - m_{ij})$$

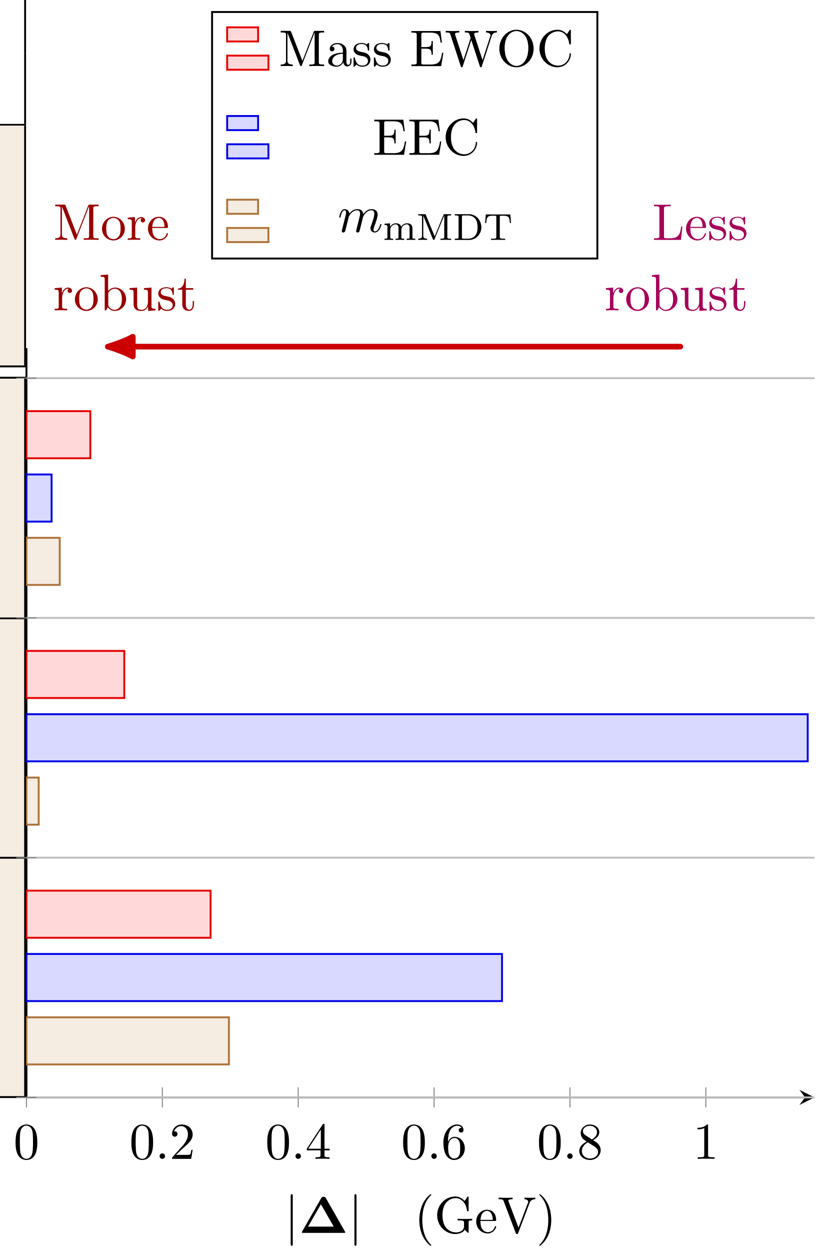


[Alipour-Fard, WW]

Mass EWOC for hadronic W

Shift in m_W Determination
from the peak of each distribution

Δ	**Mass EWOC** k_t subjets, $r_{\mathrm{sub}} = 0.3$	**EEC**	m_{mMDT} $z_{\mathrm{cut}} = 0.1$
Smearing (cf ref. [57])	94 MeV	-37 MeV	49 MeV
Parton vs. Hadron	-144 MeV	-1150 MeV	18 MeV
UE (MPI) On/Off	271 MeV	700 MeV	298 MeV

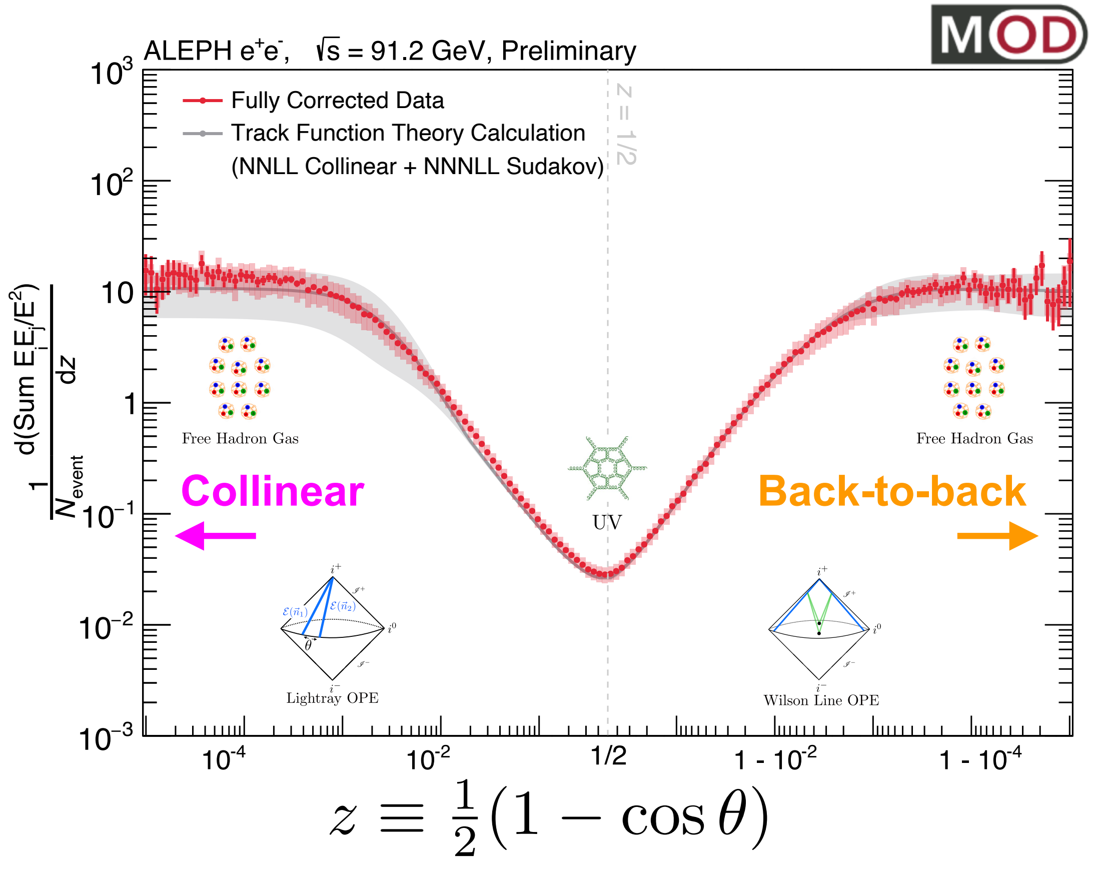


✓ EWOC competitive with soft drop mass.

- For EEC, it is essential to use m_W to extract m_t

[Alipour-Fard, WW]

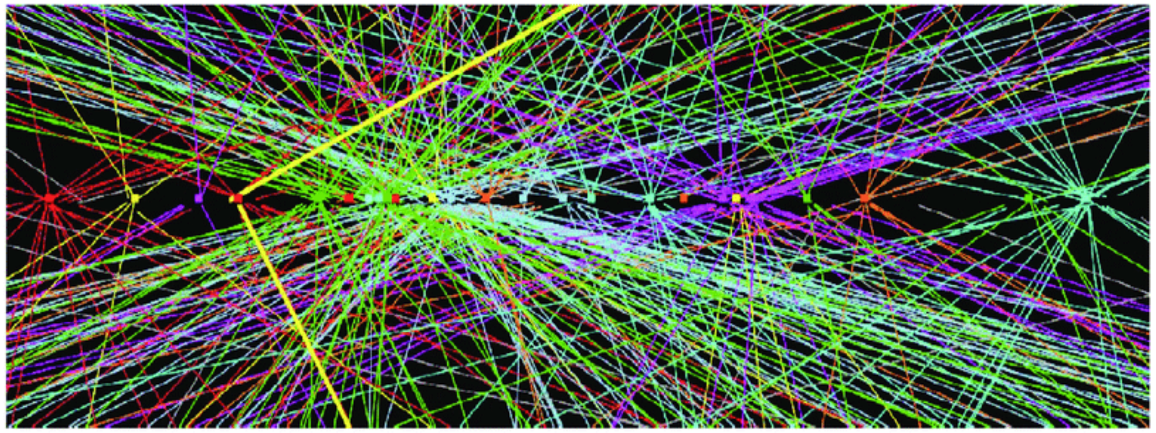
3. Energy correlator: on track to high precision



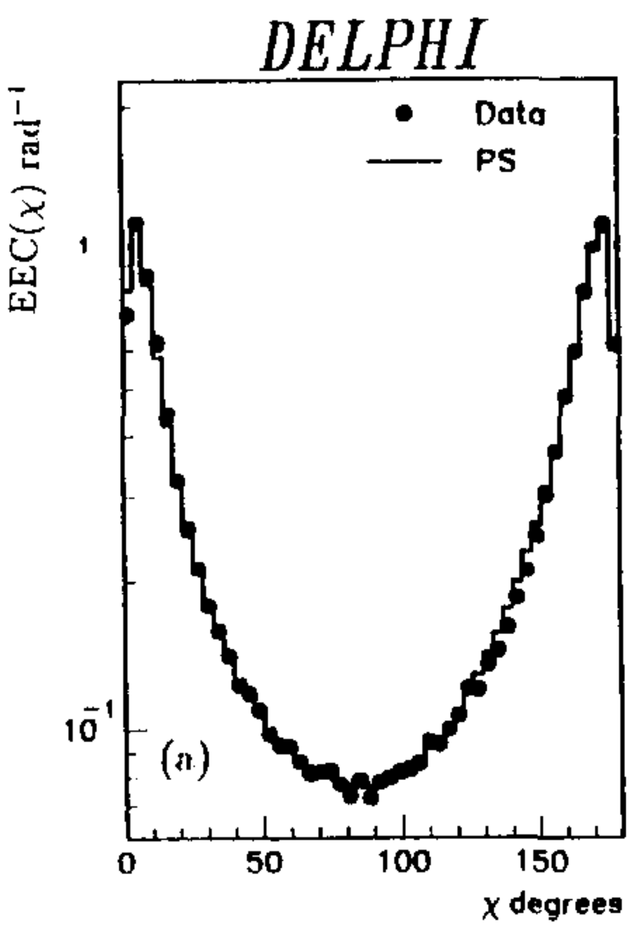
Motivation for track-based measurements

✓Pile-up removal.

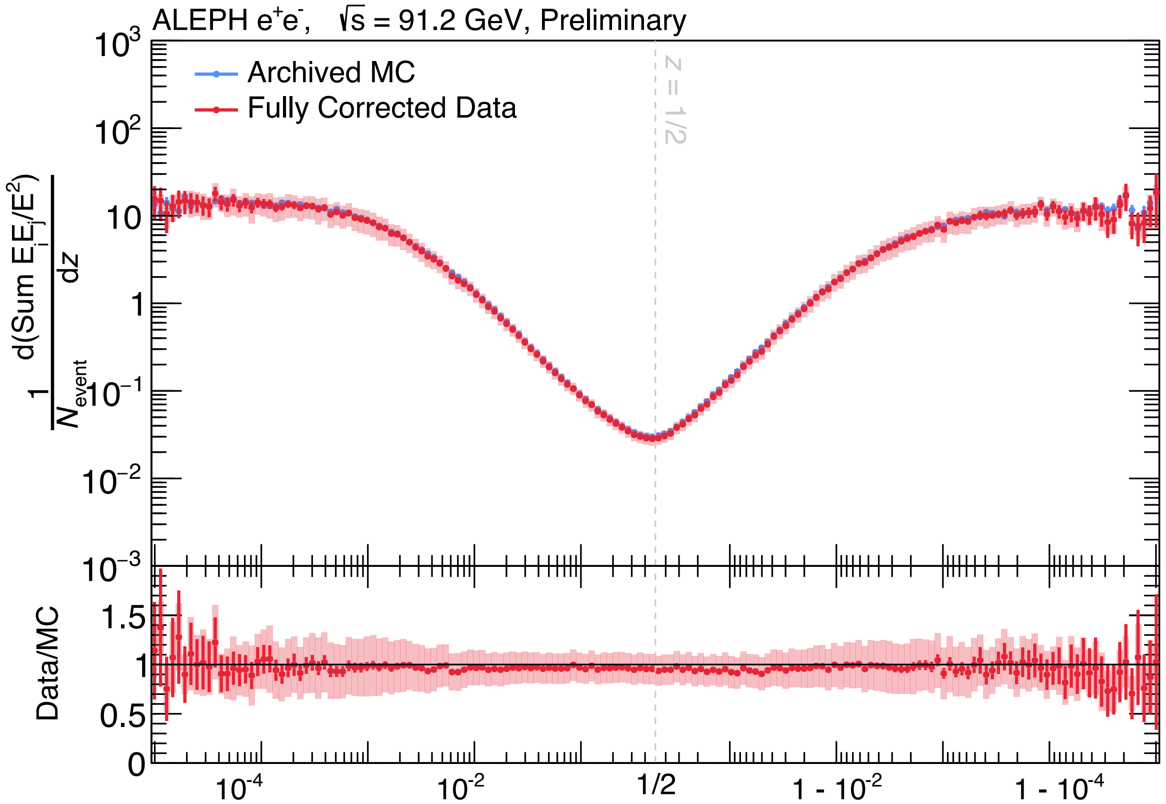
✓Superior angular resolution
→ good for jet substructure.



All particles:



Charged particles:



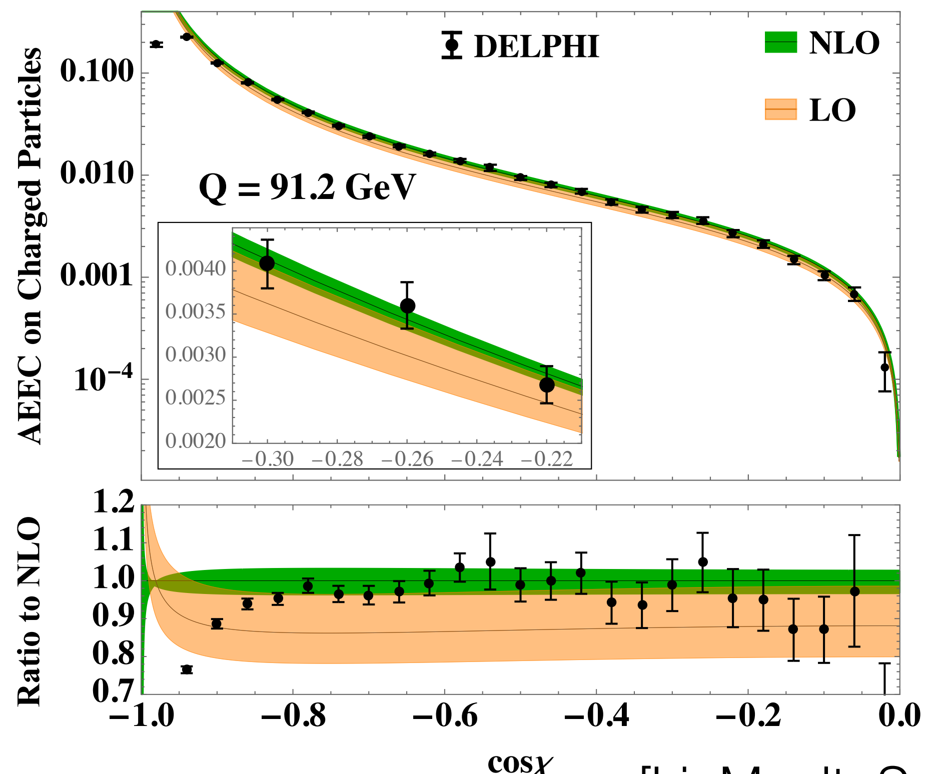
$$z \equiv \frac{1}{2}(1 - \cos\theta)$$

[Y.-C. Chen's talk at Hard Probes 2024]

Main message on track-based predictions

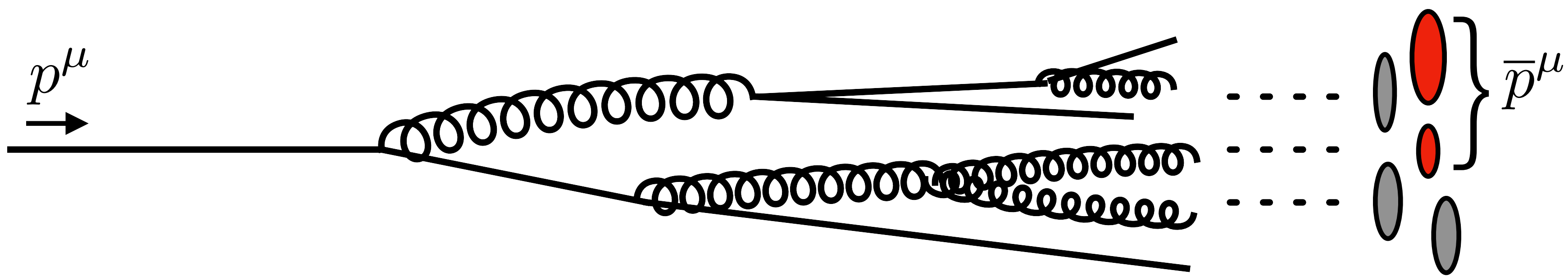
- Track-based measurements are sensitive to hadronization.
- Instead of hadronization models in parton showers, track functions offer **systematically** improvable framework.
- Recently extended to $\mathcal{O}(\alpha_s^2)$ → high precision possible!

✓ For energy correlators, track functions are easy to implement (only moments).



[Li, Moult, Schrijnder van Velzen, WW, Zhu]

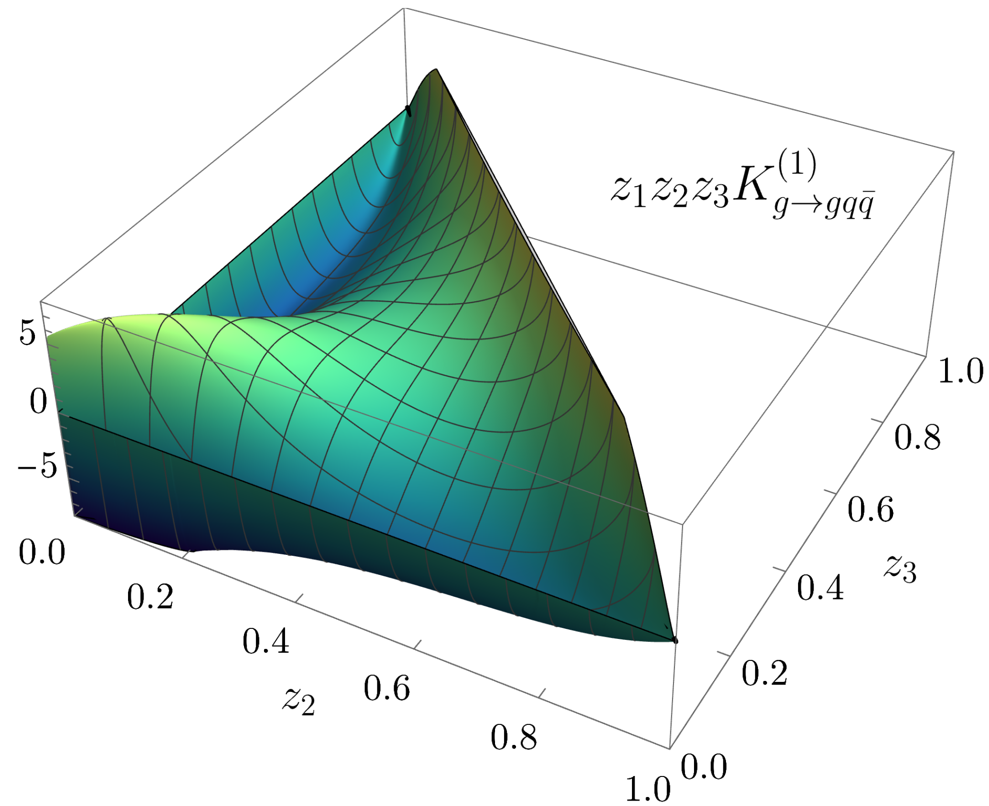
Track functions 101

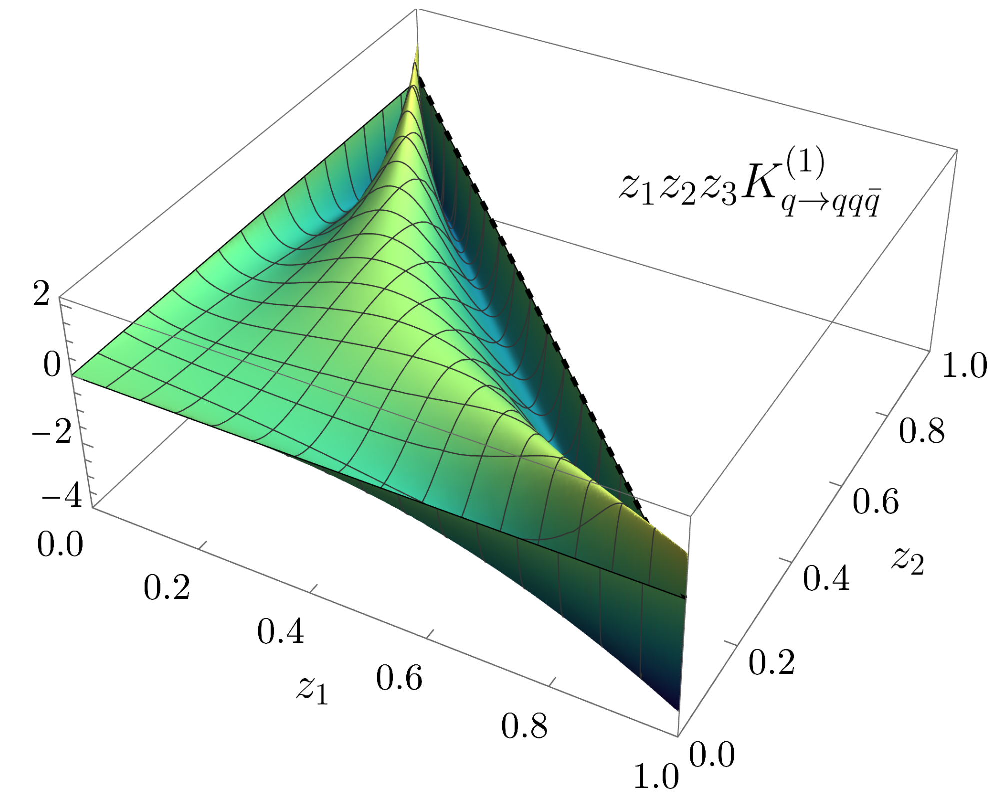


[Chang, Procura, Thaler, WW]

- $T_i(x,\mu)$ describes **total** momentum fraction x of initial parton i converted to tracks, i.e. $\bar{p}^\mu = x p^\mu + \mathcal{O}(\Lambda_{\mathrm{QCD}})$
- Nonperturbative, process-independent function.
- Conservation of probability: $\int_0^1 \mathrm{d}x\, T_i(x) = 1$
- Similar matching and evolution as for PDFs and fragmentation functions, but **nonlinear**.

Track function evolution at NLO

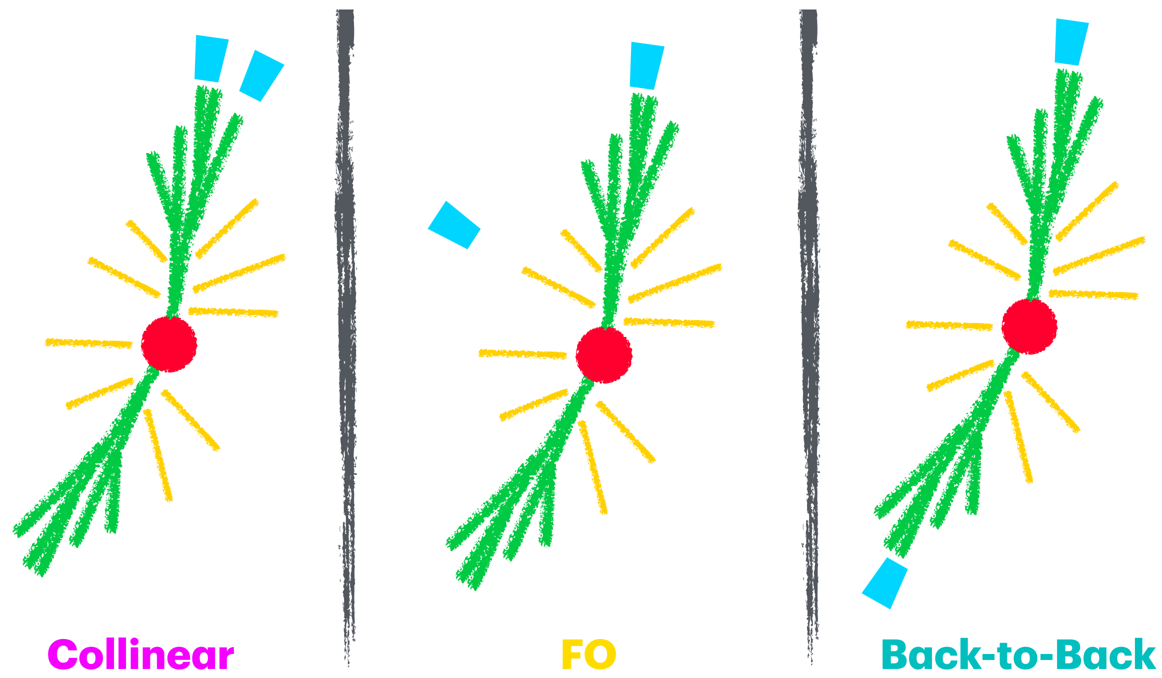




$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2}T(x,\mu) = a_s\Big[K^{(0)}_{1\to 1}\otimes T(x,\mu) + K^{(0)}_{1\to 2}\otimes TT(x,\mu)\Big]$$
$$+\, a_s^2\Big[K^{(1)}_{1\to 1}\otimes T(x,\mu) + K^{(1)}_{1\to 2}\otimes TT(x,\mu) + K^{(1)}_{1\to 3}\otimes TTT(x,\mu)\Big]$$

- Projects onto DGLAP, but also yields evolution of **multi-**hadron fragmentation functions
- Related IR poles needed for matching, **simplifies** for integer moments. [Chen, Jaarsma, Li, Moult, WW, Zhu]

Ingredients for track-based EEC



Collinear region ($z \to 0$)

- NNLL resummation of single logarithms of z.
- Nonperturbative plateau (modelled).
- Jet function matched onto track functions:

$$J_i = \mathcal{J}_{i\to j} T_j(2) + \mathcal{J}_{i\to jk} T_j(1) T_k(1)$$

Back-to-back region ($z \to 1$)

- (N)NNNLL resummation of double logarithms of 1-z.
- TMD factorization, nonperturbative Collins-Soper kernel.
- Jet function matched onto $T(1)$, soft function only contributes through recoil.

Fixed-order region

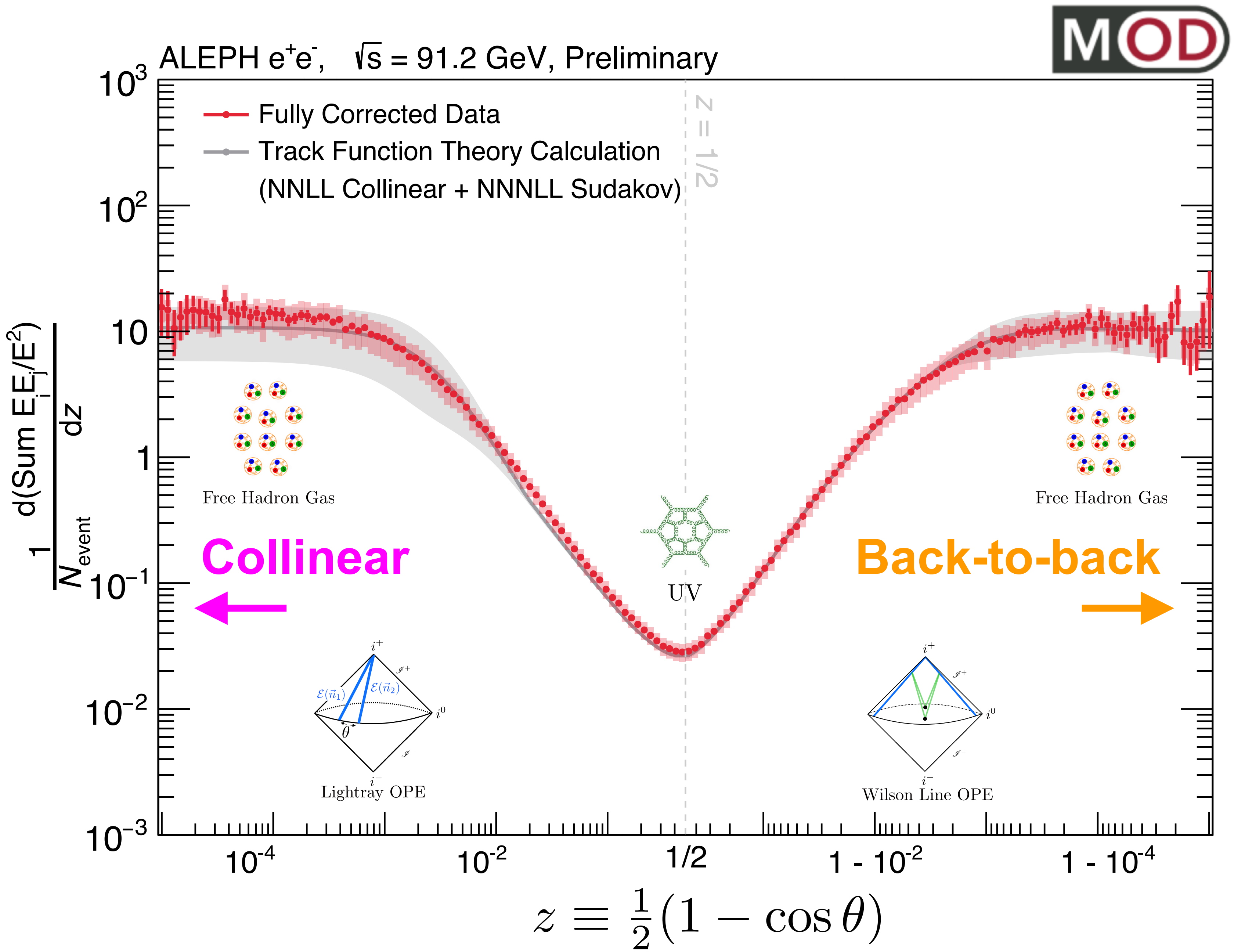
- Order α_s^3 from CoLoRFulNNLO.

All regions:

- Leading nonperturbative correction described by Ω_1, rescaled by $T_g(1)$
- Transition between regions using profile functions.

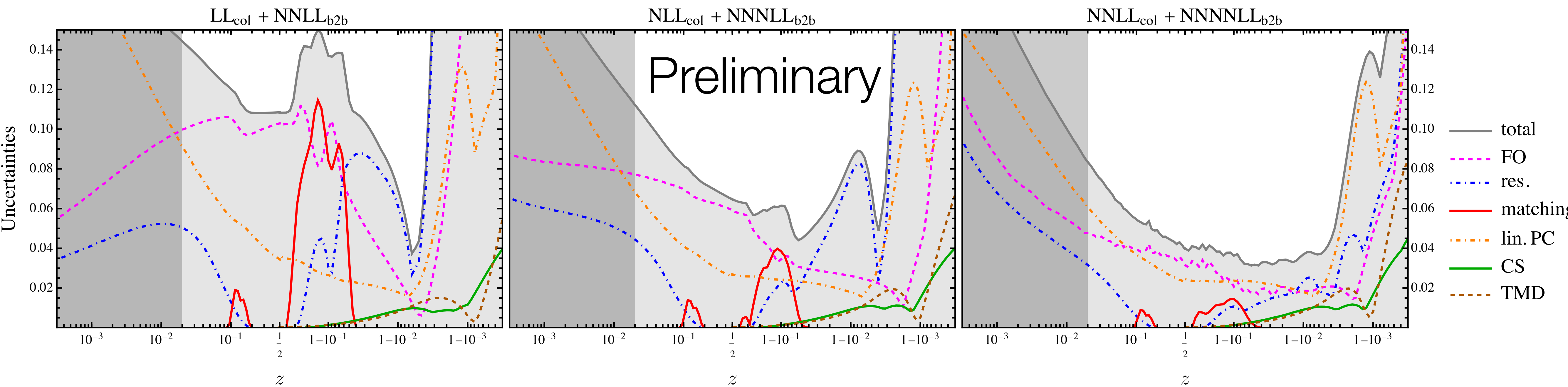
Results for track-based EEC

- A first comparison to archived ALEPH data:



[Y.-C. Chen's talk at Hard Probes 2024 - theory input: Jaarsma, Li, Moult, WW, Zhu]

Theory uncertainties

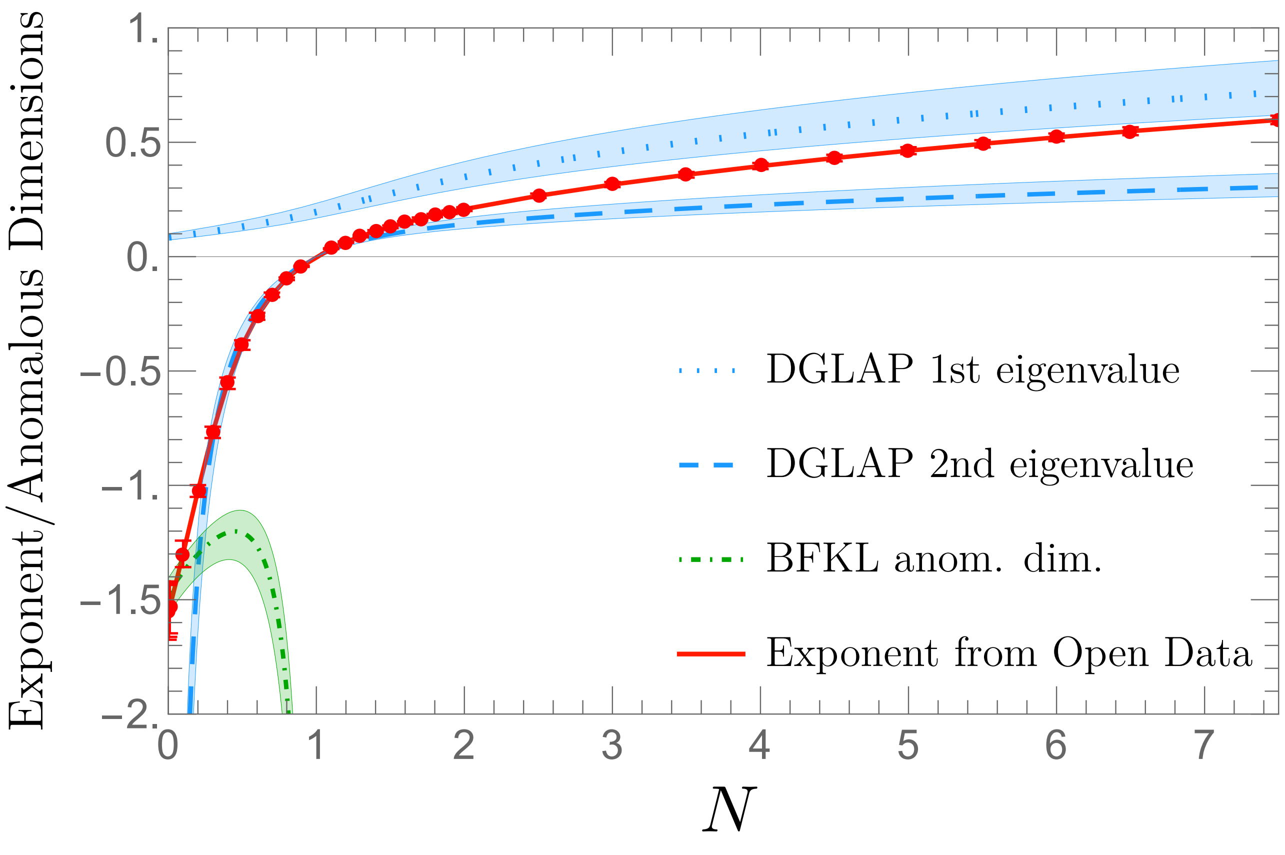


[Jaarsma, Li, Moult, WW, Zhu]

✓ Uncertainties reduce at higher orders.

- Important remaining uncertainty from leading nonperturbative correction, for which we don't have complete resummation.

4. Analytic continuation and small-x physics



Motivation for analytic continuation

- *N*-point correlator has power-law scaling $\sim R_L^{\gamma(N)}$ with

$$\gamma(N) \sim \int_0^1 \mathrm{d}x\, x^N P(x)$$

  the *N*-th moment of the DGLAP splitting functions $P(x)$.

- For $N \to 0$ we can study small-*x* physics using jets.

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- This scaling follows from:

$$\int^{R_L} \mathrm{d}R_L' \frac{\mathrm{d}\sigma^{[N]}}{\mathrm{d}R_L'} = \int_0^1 \mathrm{d}x\, x^N \vec{H}\left(x, \frac{Q}{\mu}\right) \cdot \vec{J}^{[N]}\left(\ln \frac{R_L x Q}{\mu}\right)$$

hard scattering — jet formation

[Dixon, Moult, Zhu; Chen, Moult, Zhang, Zhu]

where *H* satisfies the usual DGLAP evolution.

Analytic continuation in N

- The projected correlator can be rewritten as:

$$\frac{d\sigma^{[N]}}{dR_L} = \sum_X \int d\sigma_X \sum_{S\subset X} \mathcal{W}^{[N]}(S)\, \delta(R_L - \max\{R_{ij}\}_{i,j\in S})\,,$$

$$\mathcal{W}^{[N]}(\emptyset) = 0\,, \qquad \mathcal{W}^{[N]}(S) = \left(\sum_{i\in S} z_i\right)^N - \sum_{S'\subsetneq S} \mathcal{W}^{[N]}(S')\,.$$

[Chen, Moult, Zhang, Zhu]

- E.g. for two particles:

$$\mathcal{W}^{[2]} = (z_1+z_2)^2 - z_1^2 - z_2^2 = 2z_1z_2$$

$$\mathcal{W}^{[3]} = (z_1+z_2)^3 - z_1^3 - z_2^3 = 3z_1^2z_2 + 3z_1z_2^2$$

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[Chen, Moult, Zhang, Zhu]

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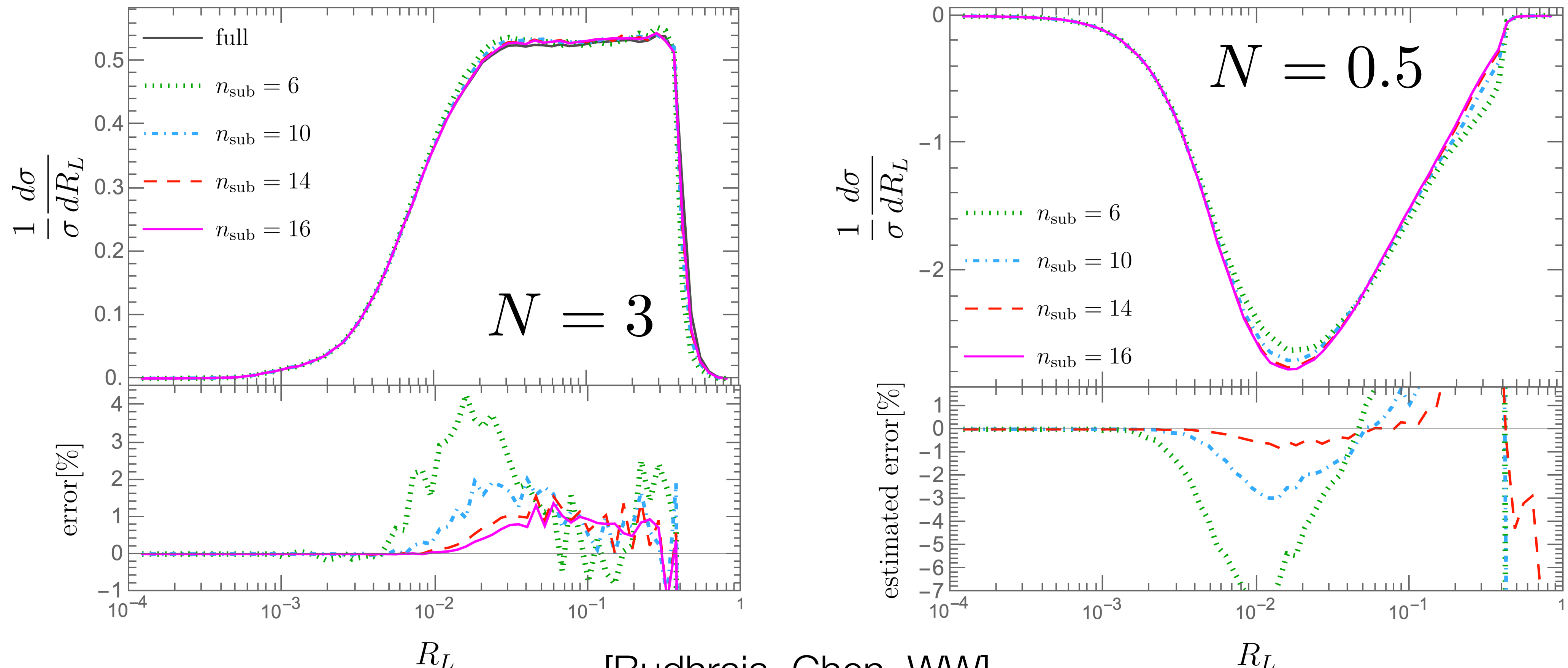
- This form can be analytically continued in N.
- **Prohibitive** computation time: $\mathcal{O}(2^{2M})$ for M particles.

Speeding up

- Avoid nested sums over subsets by storing intermediates:
Time: $\mathcal{O}(2^{2M}) \to \mathcal{O}(2^M)$, Memory: $\mathcal{O}(M) \to \mathcal{O}(M2^M)$

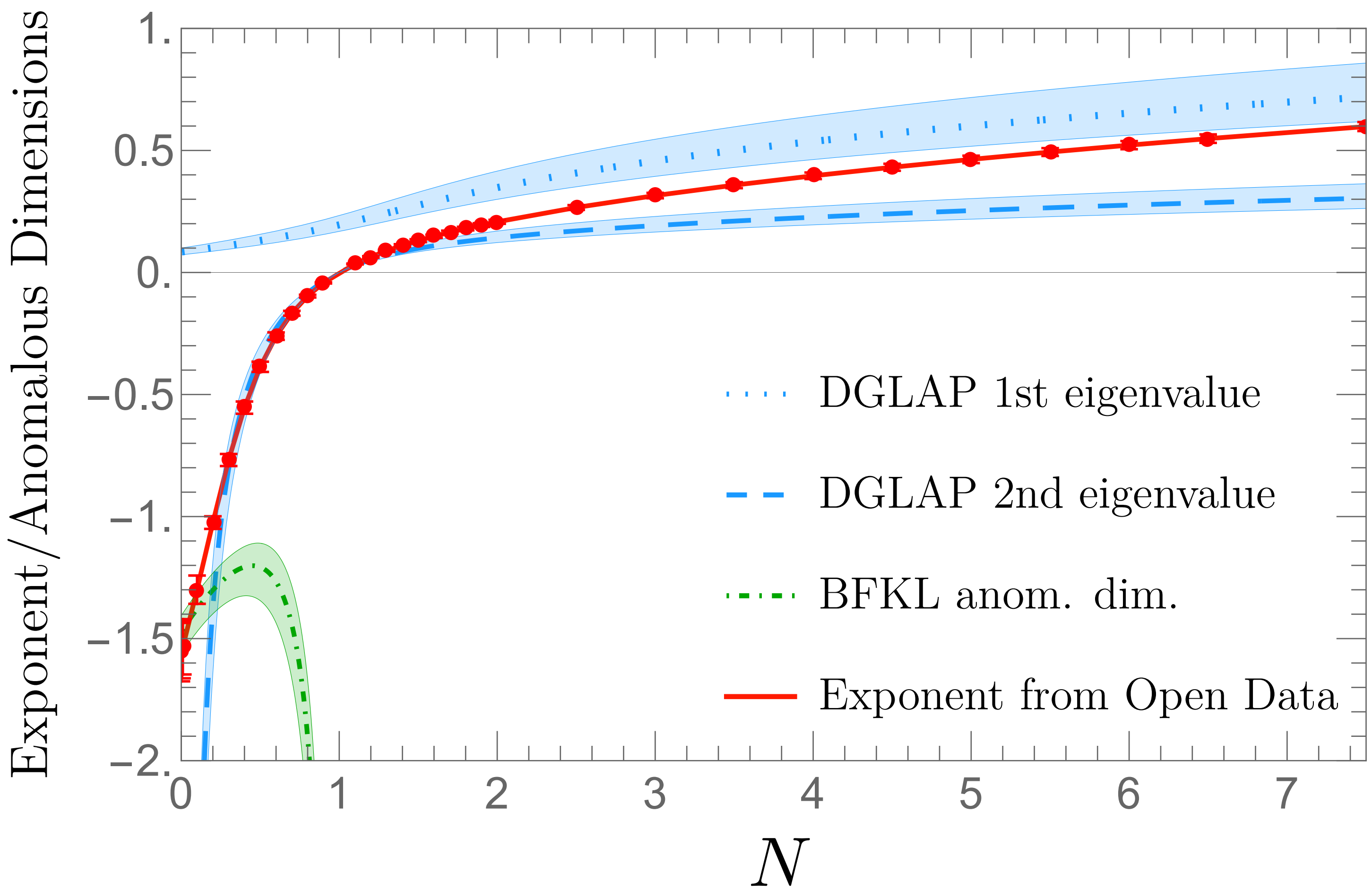
- Approximation: replace *M* by subjets instead of particles, with a maximum number of subjets n_{sub}

✓Validation:



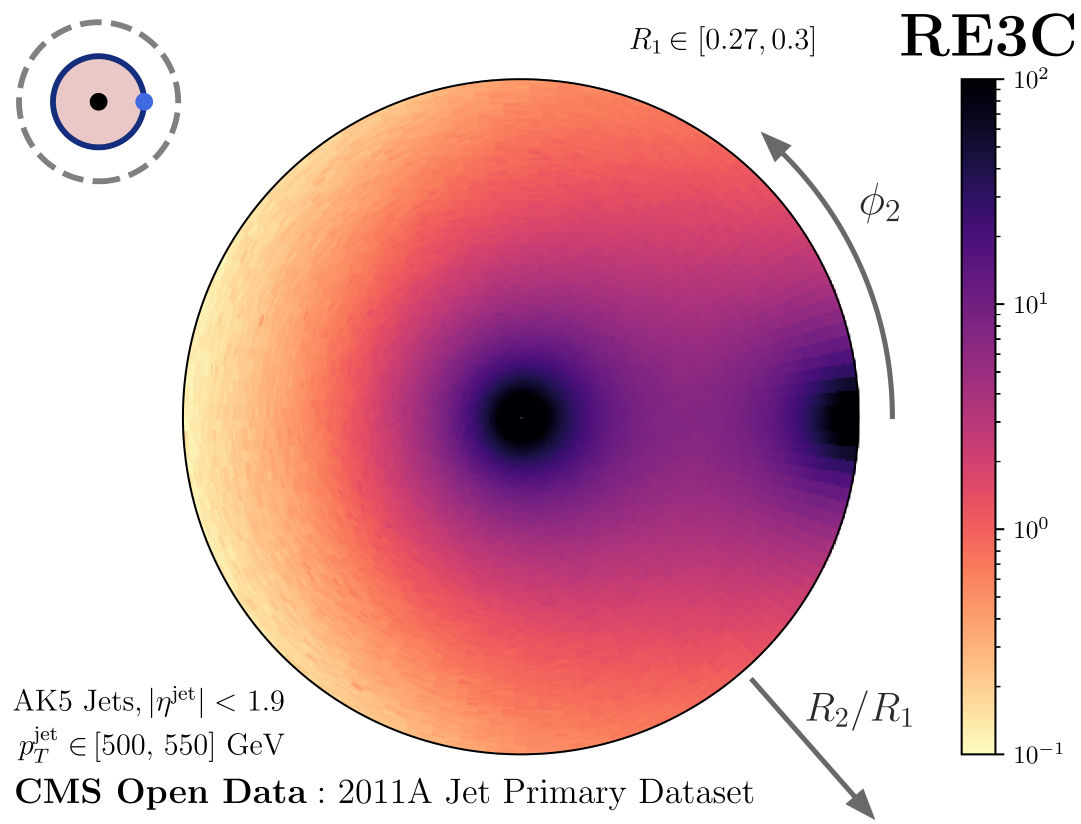
[Budhraja, Chen, WW]

Power-law as function of N



- Fit CMS open data to power law.
- Due to quark/gluon mixing not just one power-law exponent → plot both DGLAP eigenvalues.
- Interestingly, approaches BFKL for $N \to 0$.

5. New angles on energy correlators

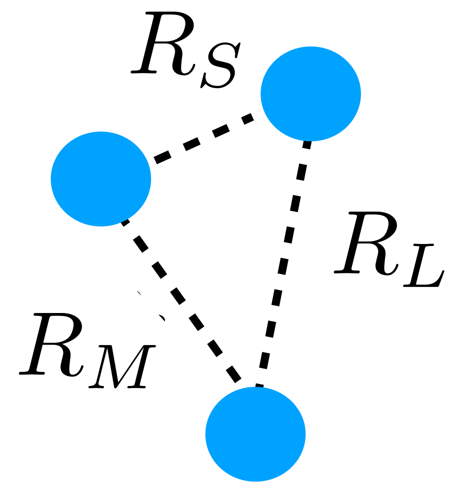


Issues

- Computation time: $\mathcal{O}(M^N)$ or $\mathcal{O}(2^M)$.
- Parametrization in terms of all distances is redundant:

$$\binom{N}{2} > 2N - 3 \quad \text{for} \quad N > 3.$$

- Orientation is not preserved. E.g. for 3-point, all 6 permutations are mapped to same R_L, R_M, R_S.



New parametrization

Isolate a special point s and **only** consider the distance to it:

$$\frac{\mathrm{d}\sigma^{[N]}}{\mathrm{d}R_1} = \int \mathrm{d}\sigma \sum_{s} z_s \sum_{i,j,k,\ldots} z_j z_k \cdots \delta(R_1 - \max\{R_{si}, R_{sj}, \cdots\})$$

- Time is $\mathcal{O}(M^2 \ln M)$ for projected correlator for all *N*!
- Clear from cumulative:

$$\Sigma^{[N]}(R_1) = \int^{R_1} \mathrm{d}R_1' \frac{\mathrm{d}\sigma^{[N]}}{\mathrm{d}R_1'} = \int \mathrm{d}\sigma \sum_{s} z_s [z_{\mathrm{disk}}(s, R)]^{N-1}$$

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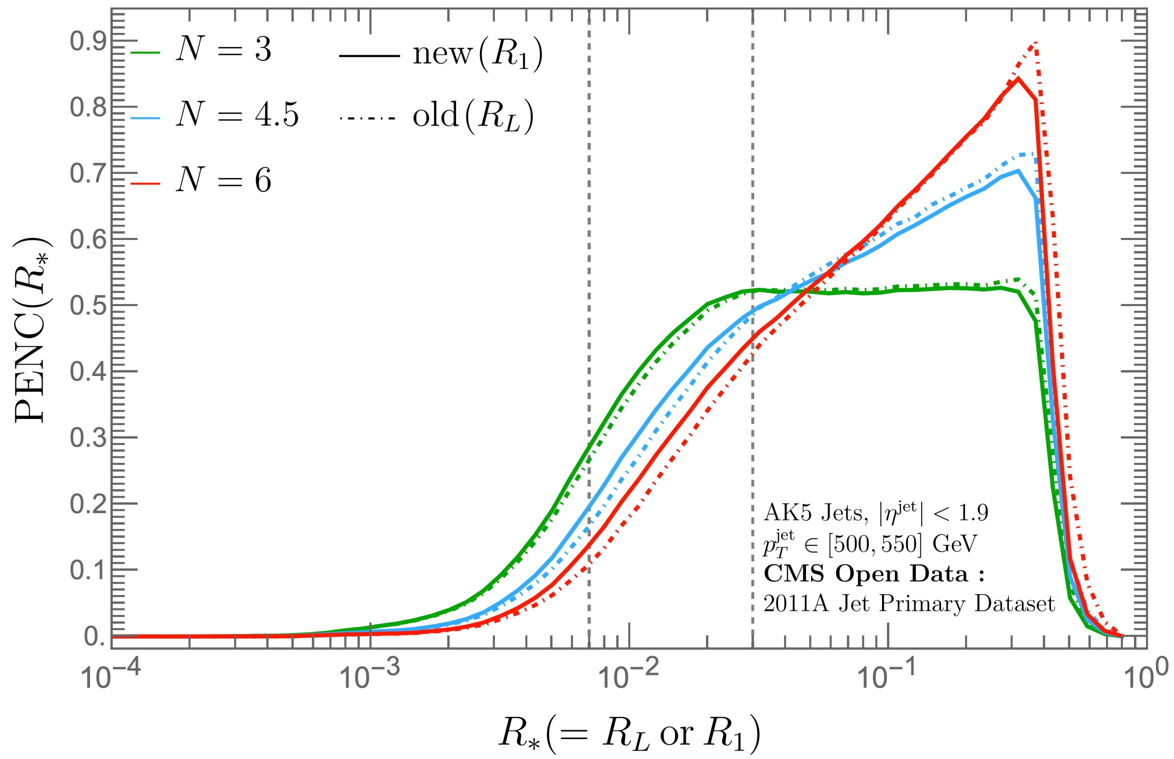
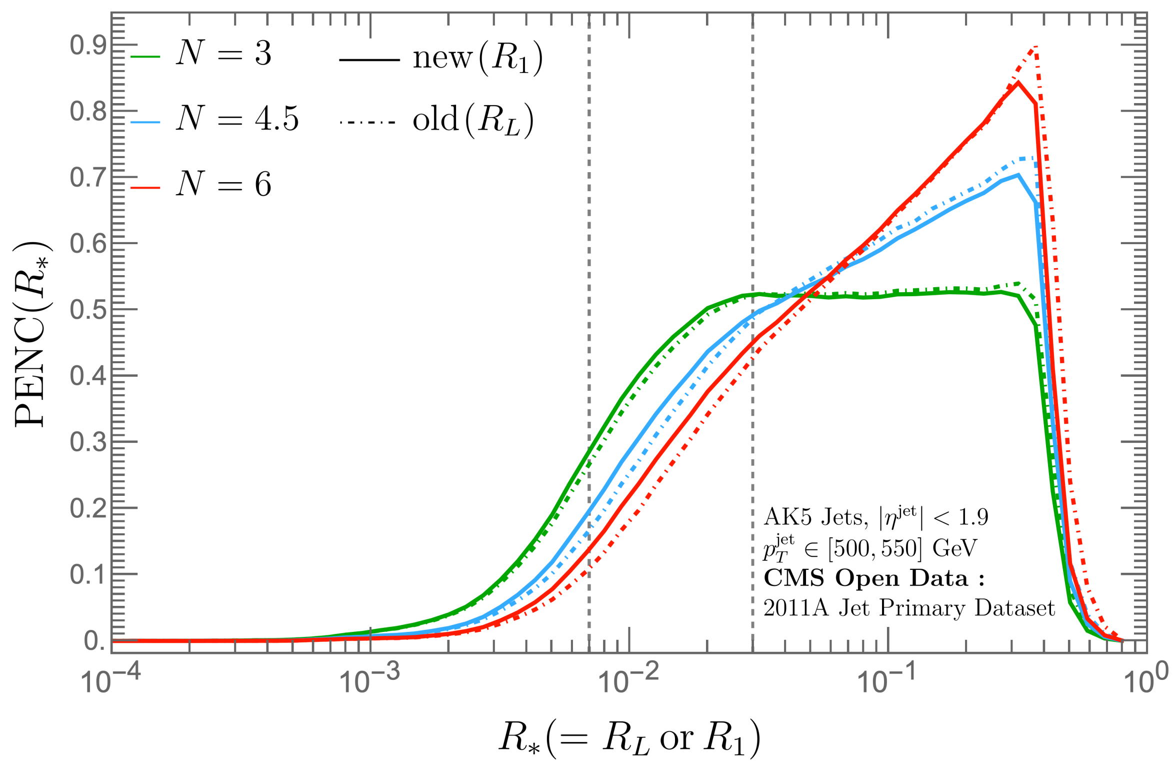
- $R_1 \leq R_L \leq 2R_1$, so R_1 is good measure of overall scale.
- Same theory framework. First difference is in $\mathcal{O}(\alpha_s^2)$ constant → NNLL effect → $R_L = R_1[1 + \mathcal{O}(\alpha_s)]$.

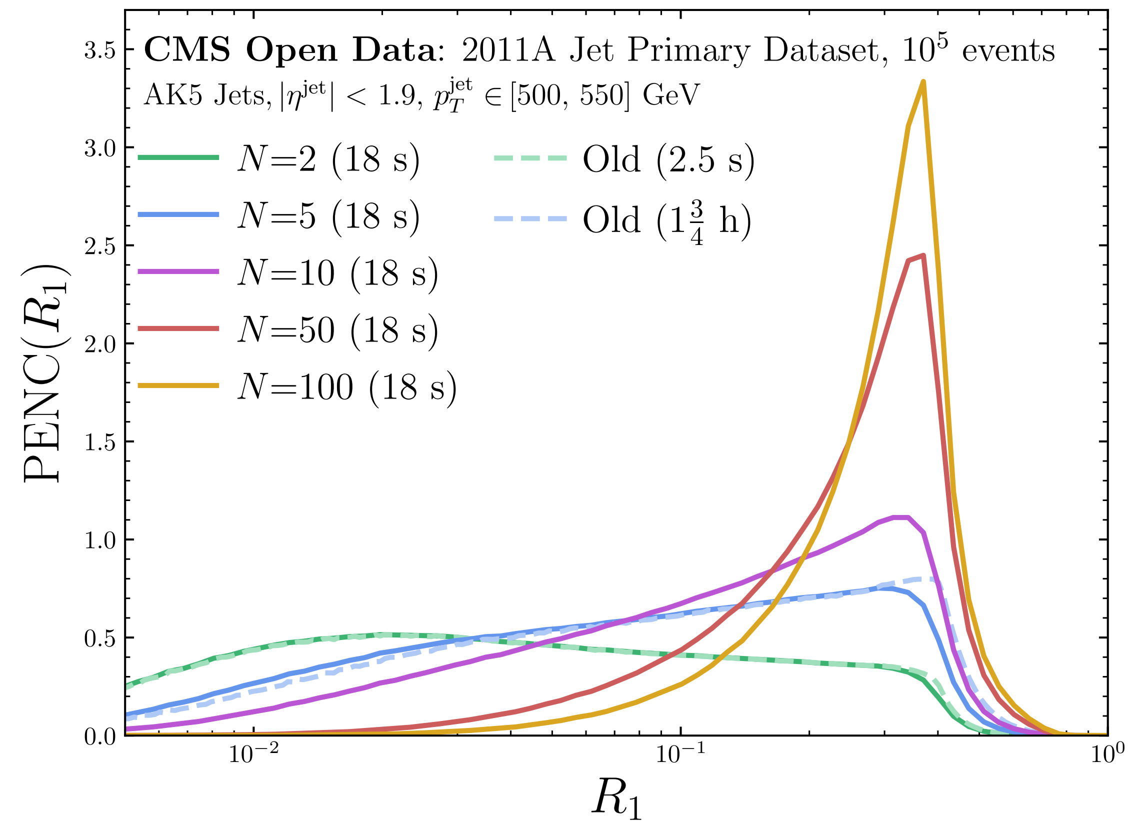
Comparing old and new projected correlator



- Difference small. Most visible in transition region.

Comparing old and new projected correlator

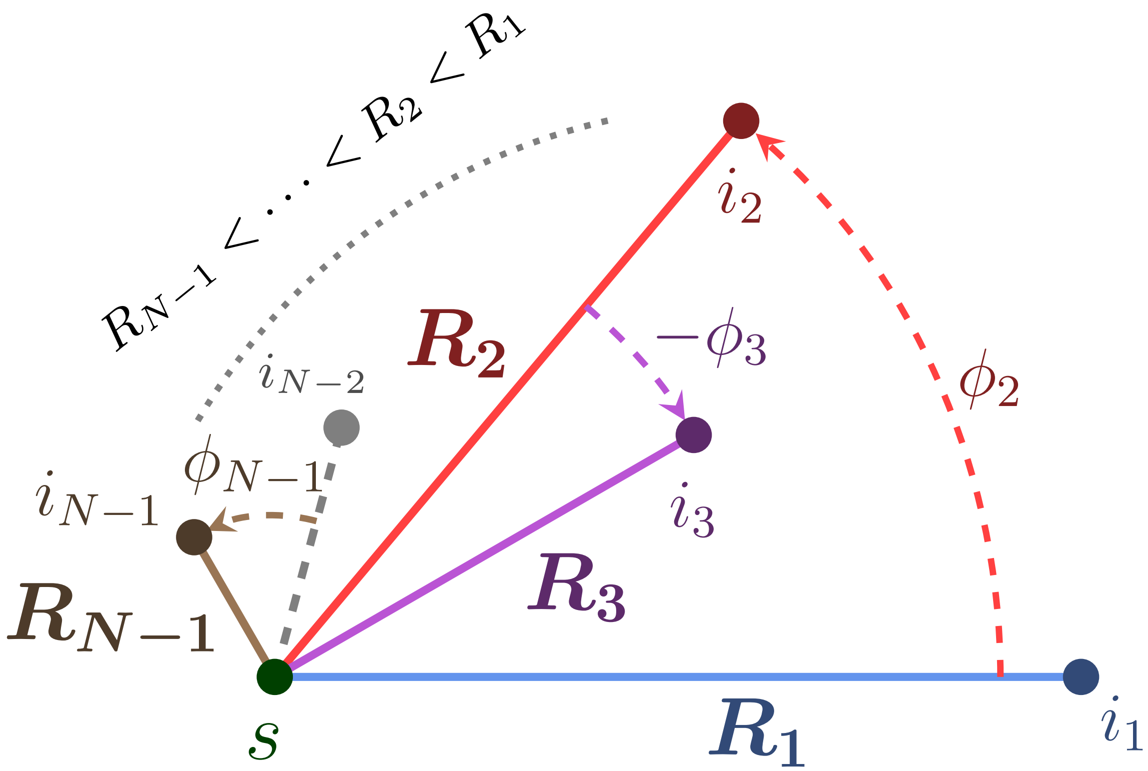




- Difference small. Most visible in transition region.

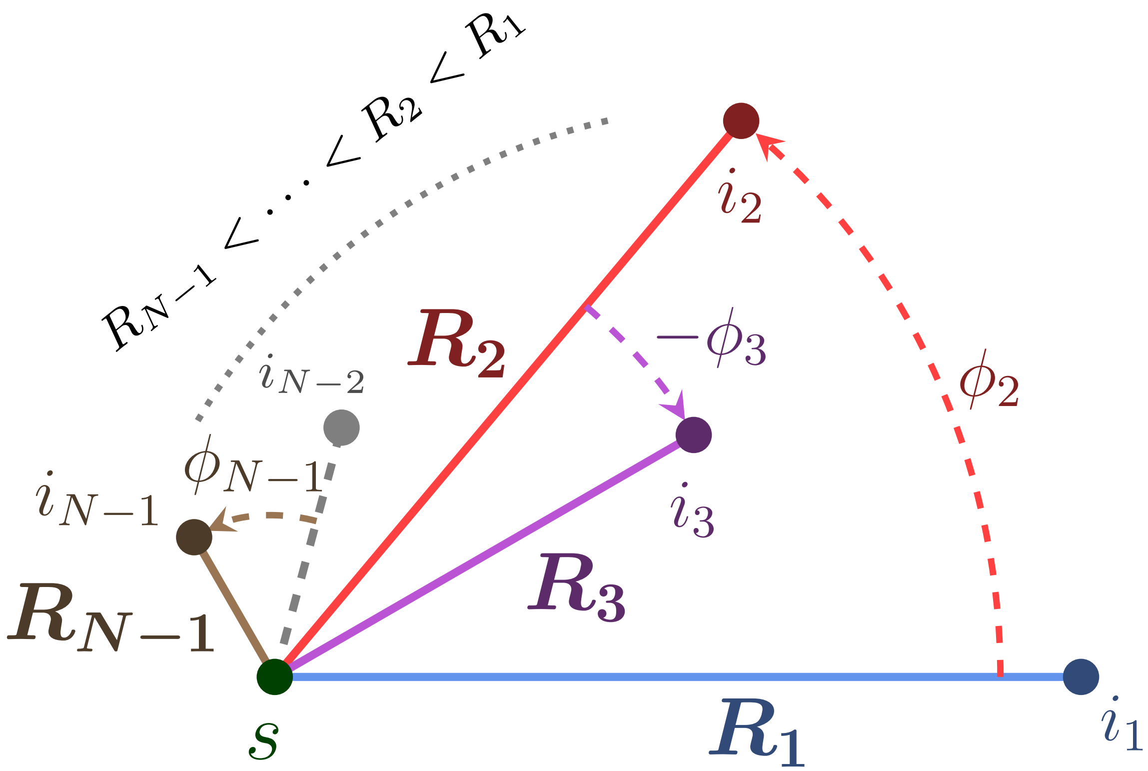
✓ New parametrization is much faster.

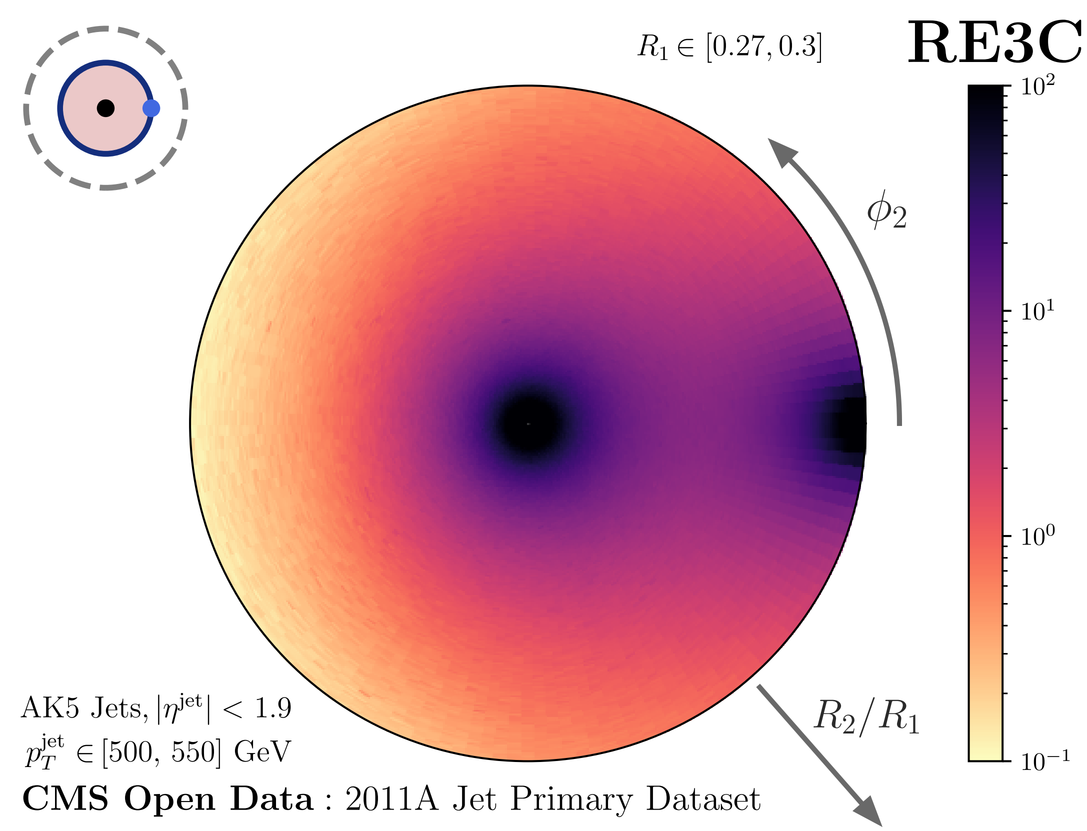
Resolved energy correlator



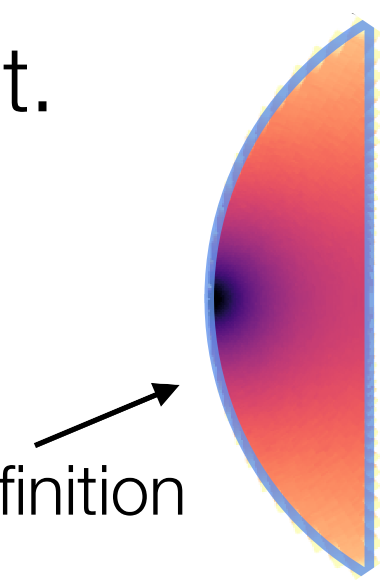
- Use polar coordinates around the special point.
- Nonredundant.

Resolved energy correlator

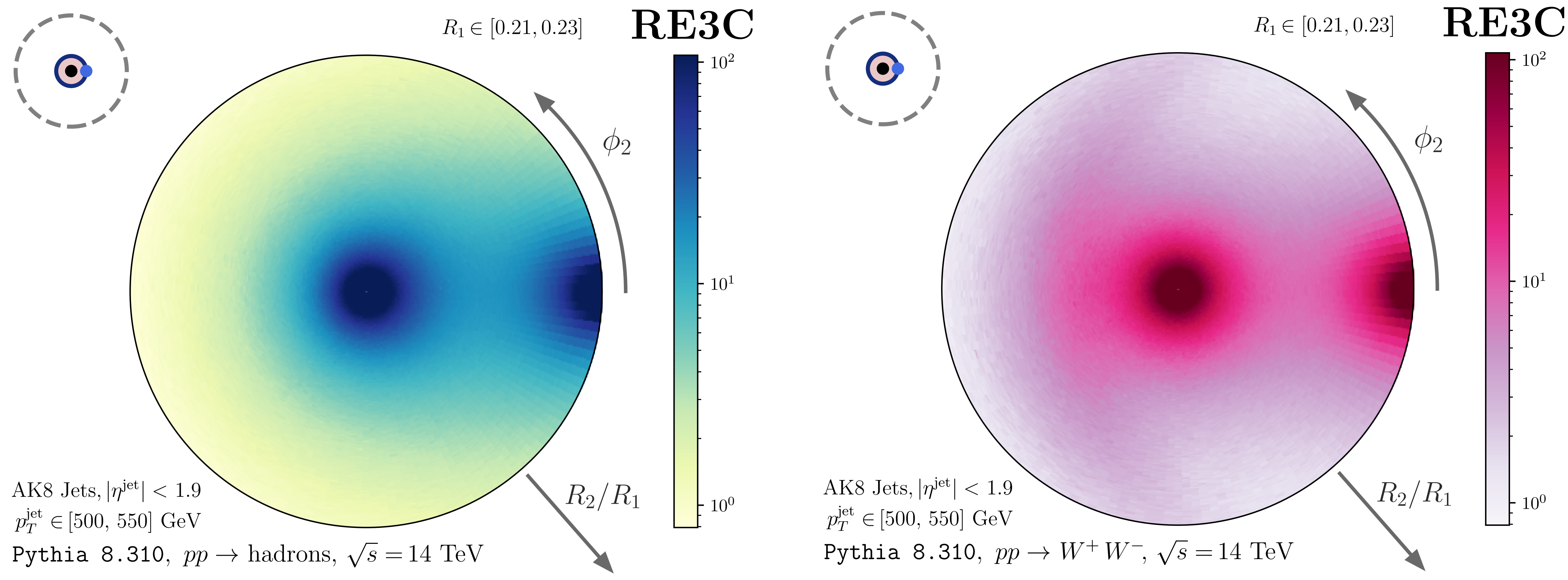




- Use polar coordinates around the special point.
- Nonredundant.
- Maintains orientation.



Bulls-eye for different jets



- Comparing QCD and W jets.
- Qualitative differences

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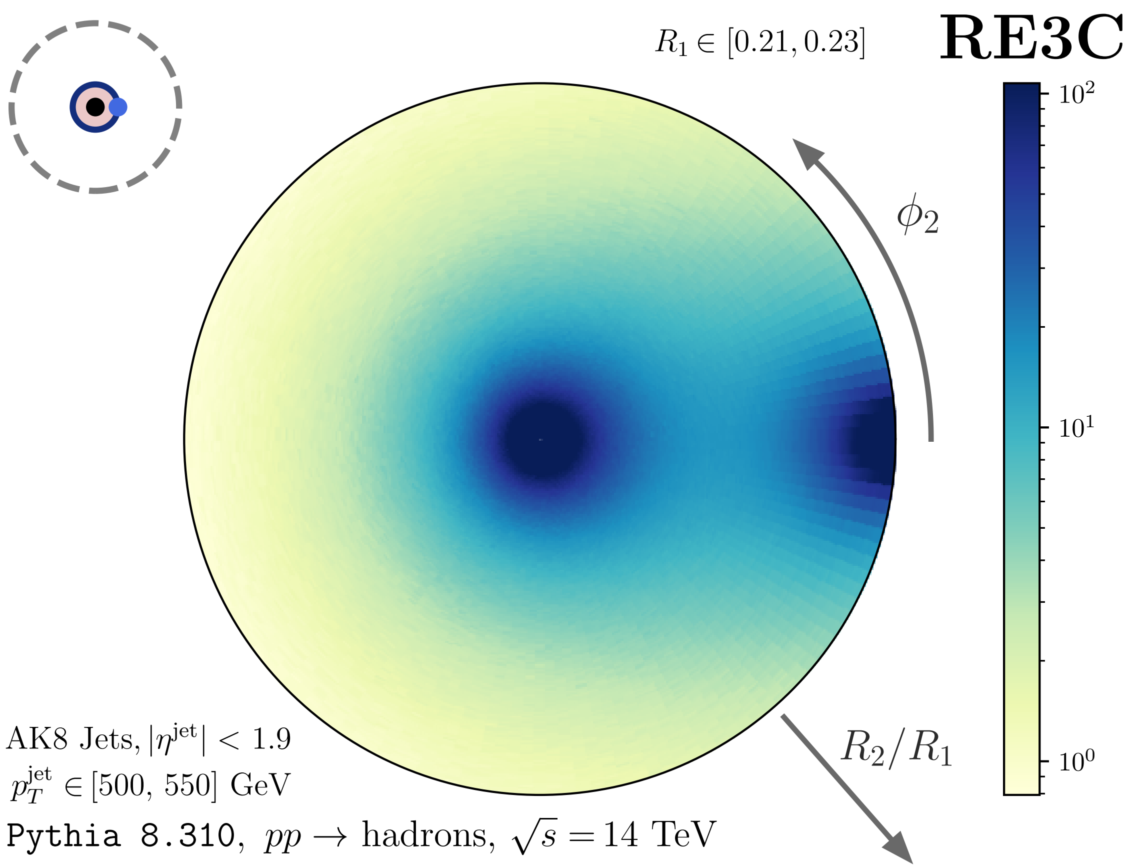
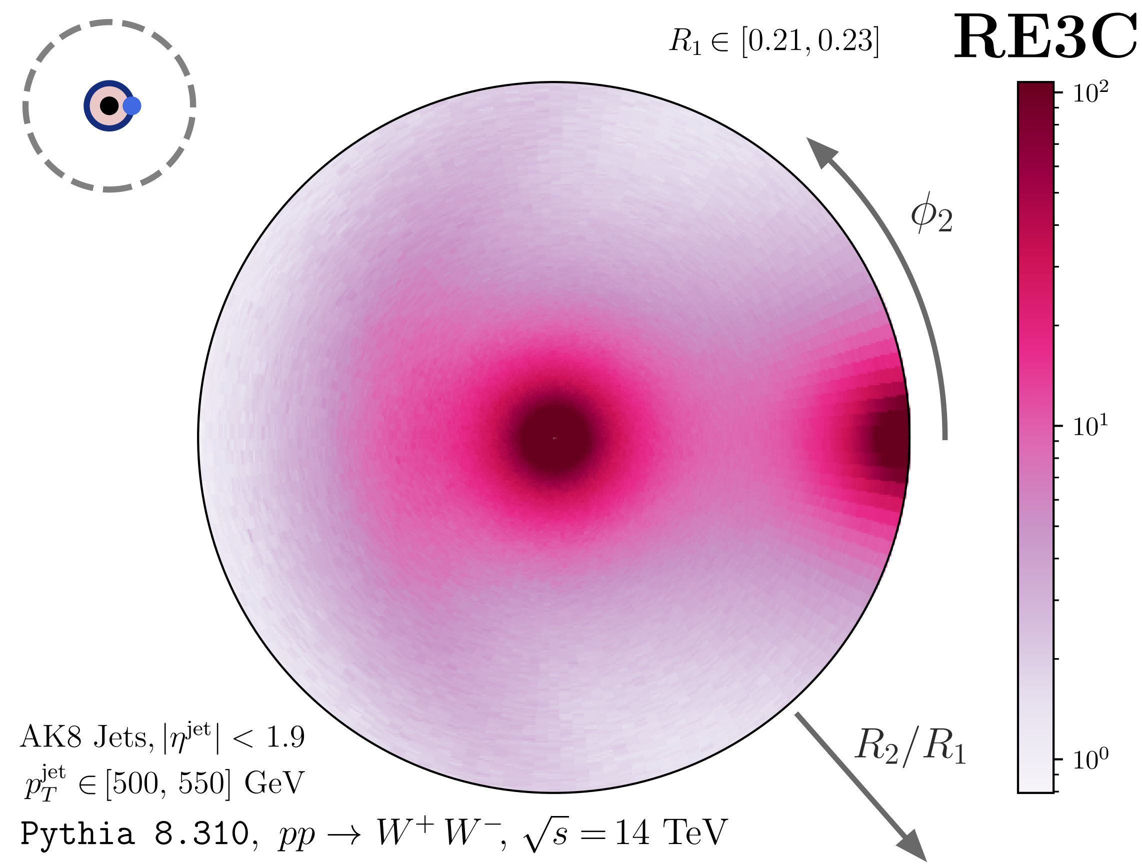
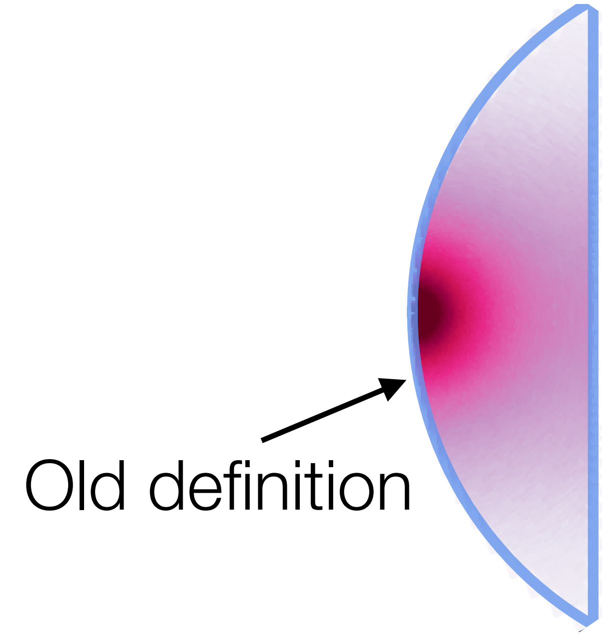
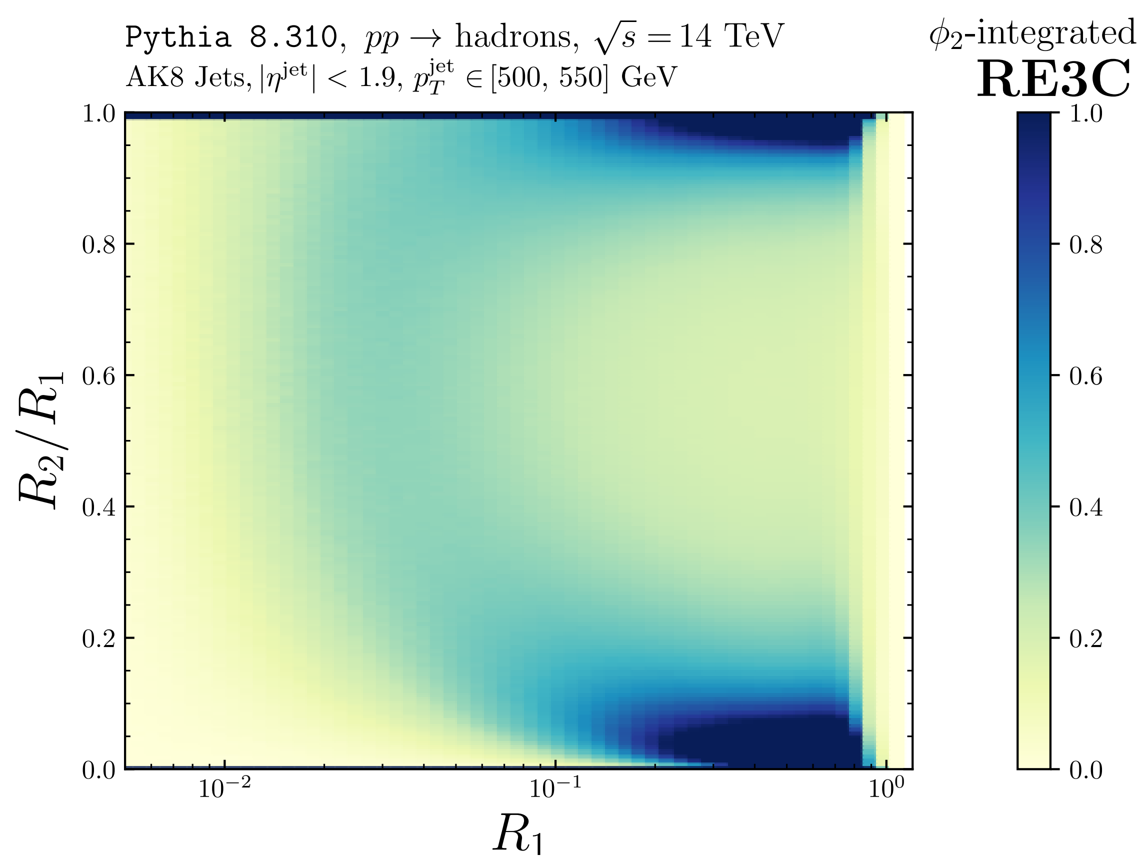


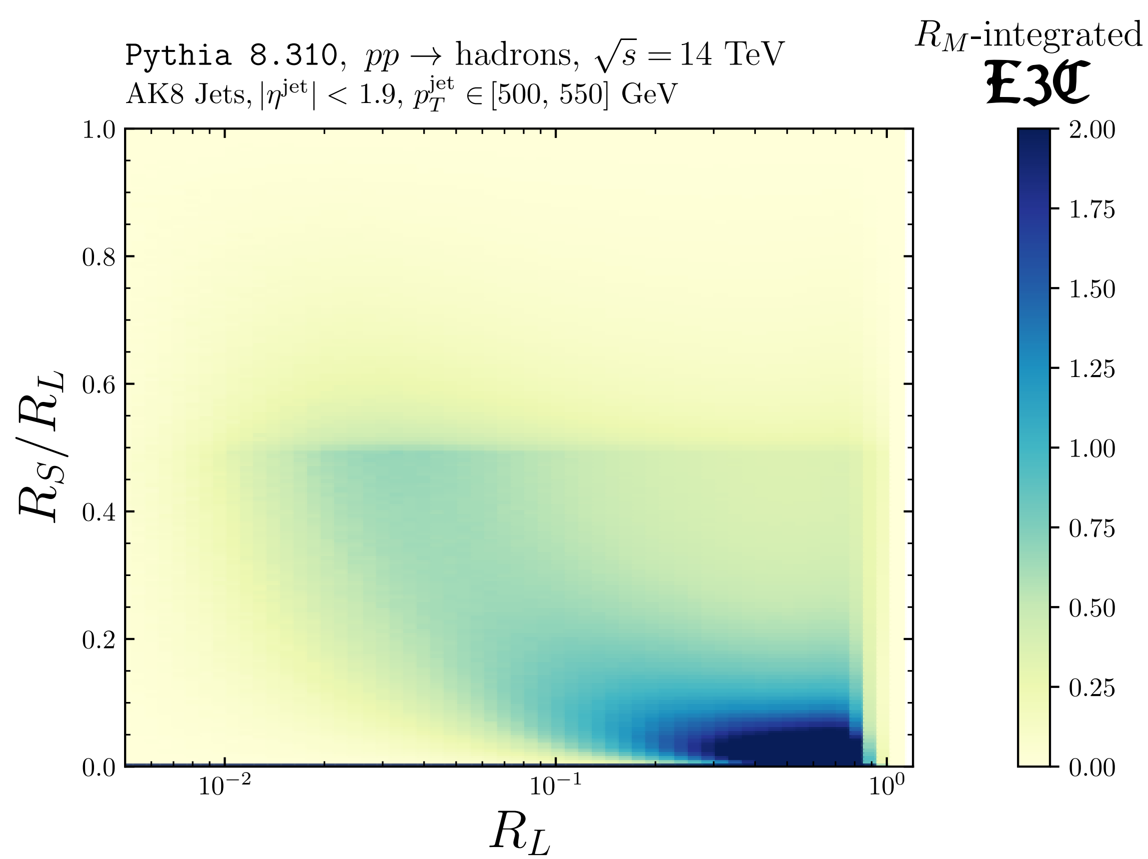


- Comparing QCD and W jets.
- Qualitative differences, not visible in old parametrization.



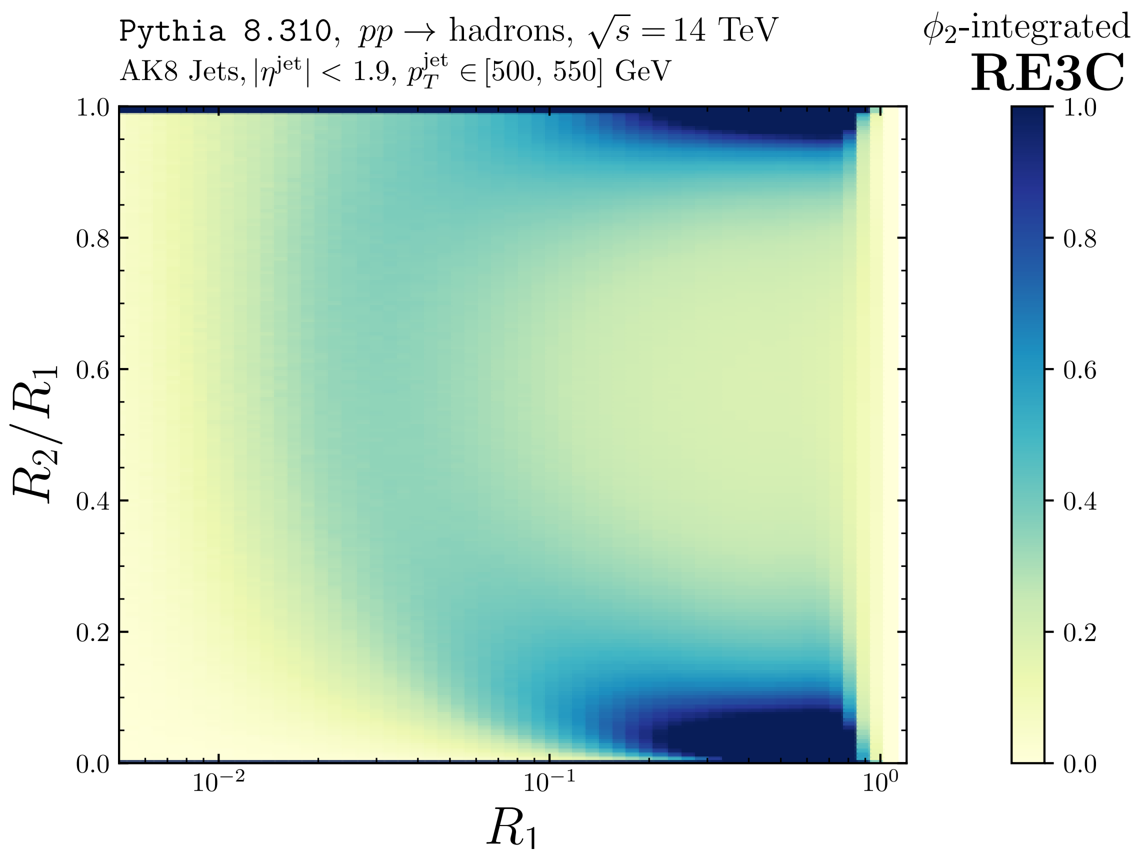
Radial distribution for different jets

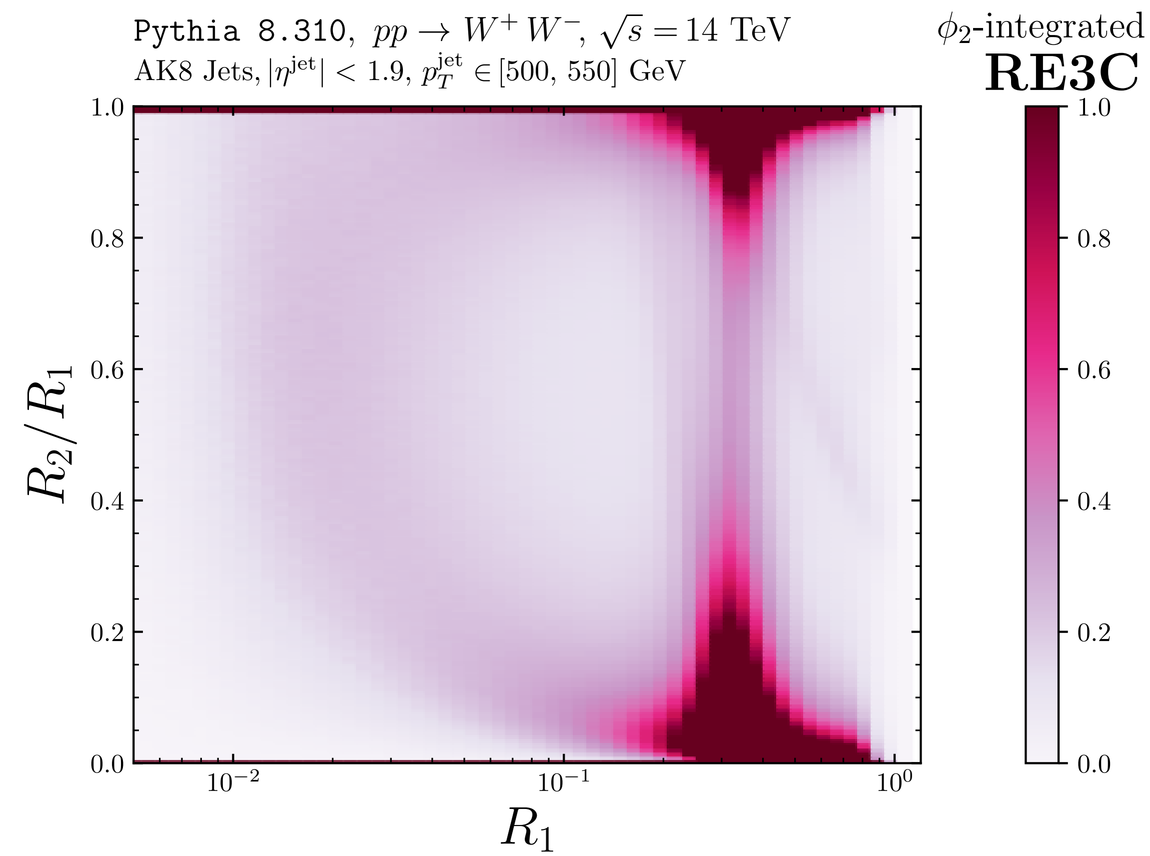


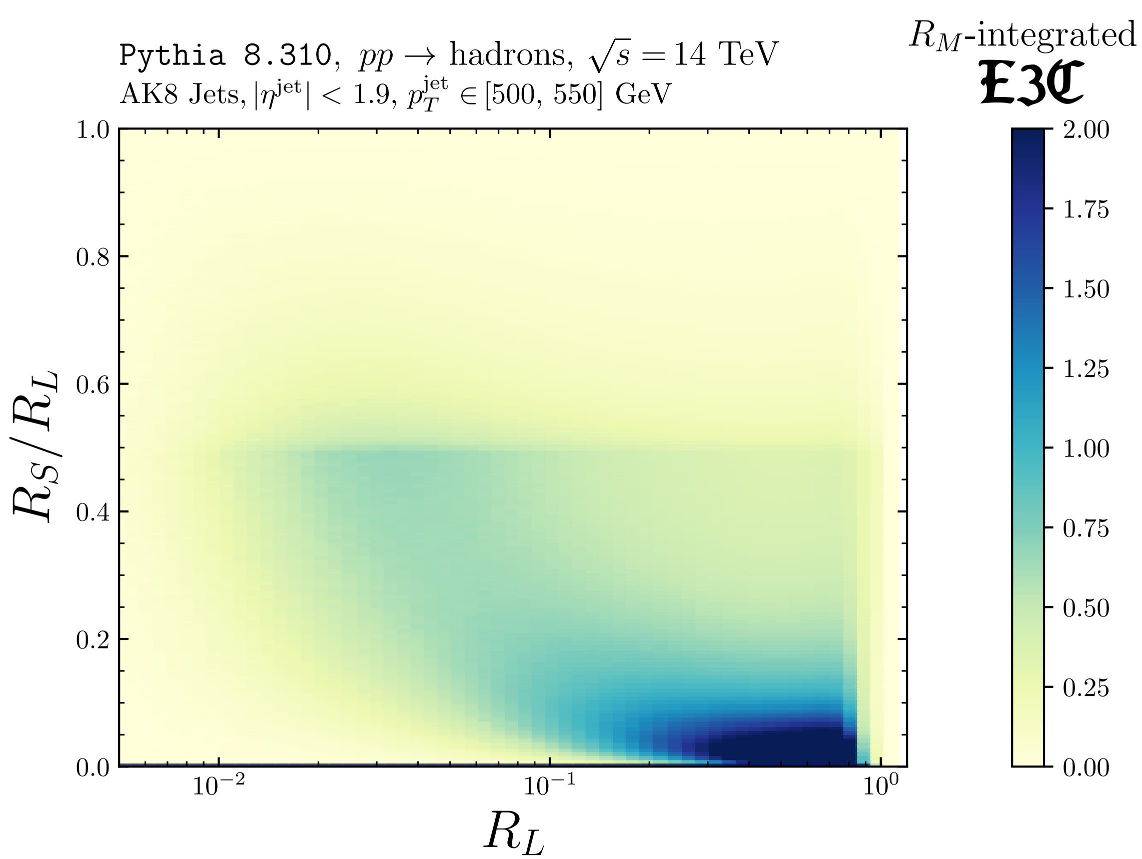


- Old and new agree on "lower half".

Radial distribution for different jets



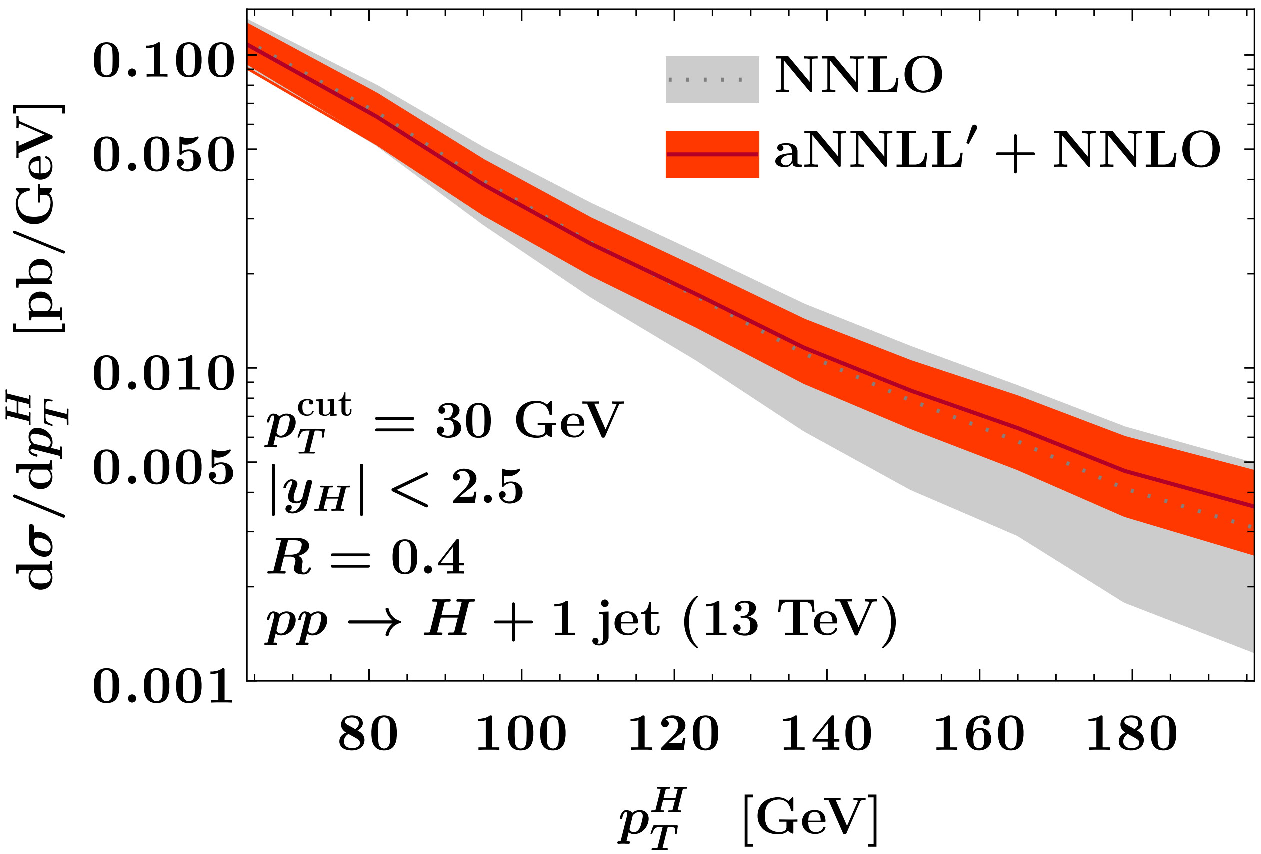




- Old and new agree on "lower half".
- *W* boson mass imprinted.

6. Bonus

Higgs + 1 jet at aNNLL'+NNLO



[Cal, Lim, Scott, Tackmann, WW]

Higgs + 1 jet production with a veto on additional jets:

- Extra "N" compared to previous study [Liu, Petriello].
- Resum leading nonglobal logarithms, logarithms of jet radius.

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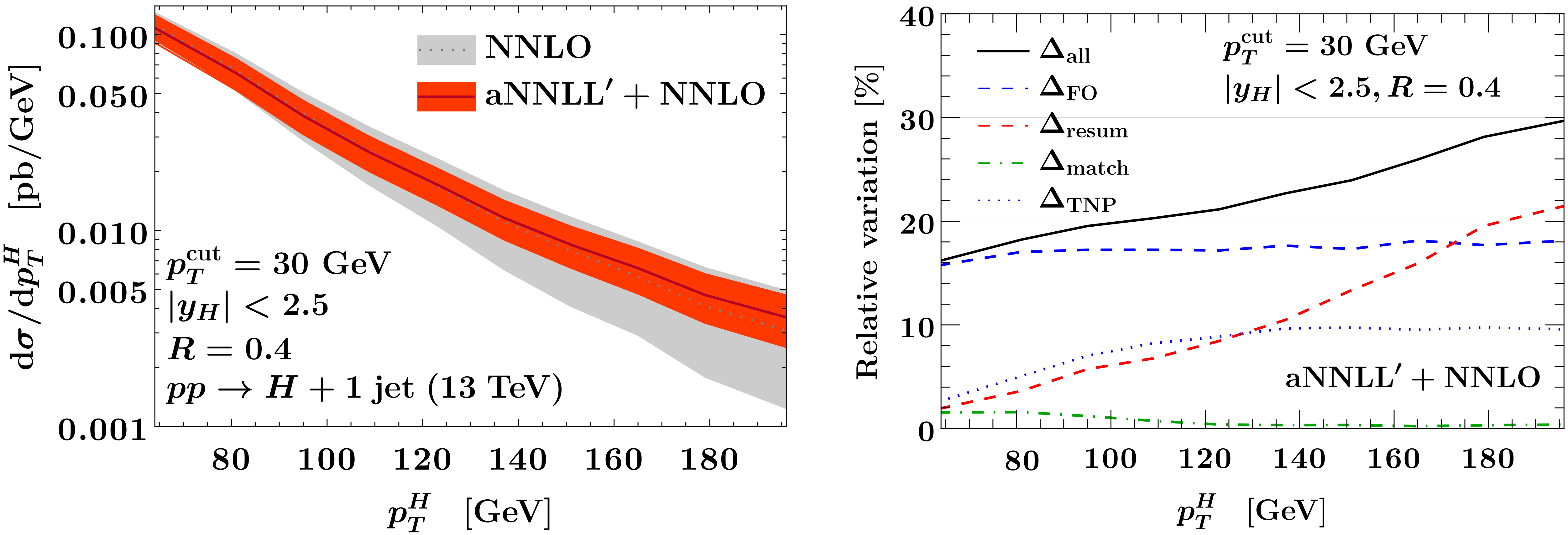

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Higgs + 1 jet production with a veto on additional jets:

- Extra “N” compared to previous study [Liu, Petriello].
- Resum leading nonglobal logarithms, logarithms of jet radius.
- Missing pieces parametrized by theory nuisance parameters.

q_T slicing with multiple jets

- For color-singlet production, cancel IR divergences by q_T slicing

$$\frac{\mathrm{d}\sigma}{\mathrm{d}X} = \int_0^\delta \mathrm{d}q_T \, \frac{\mathrm{d}\sigma_{\mathrm{SCET}}}{\mathrm{d}X\,\mathrm{d}q_T}[1+\mathcal{O}(\delta^p)] + \int_\delta^\infty \mathrm{d}q_T \, \frac{\mathrm{d}\sigma_{\mathrm{QCD}}}{\mathrm{d}X\,\mathrm{d}q_T}$$

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- q_T fails for jets, because emissions inside jets leave $q_T = 0$.

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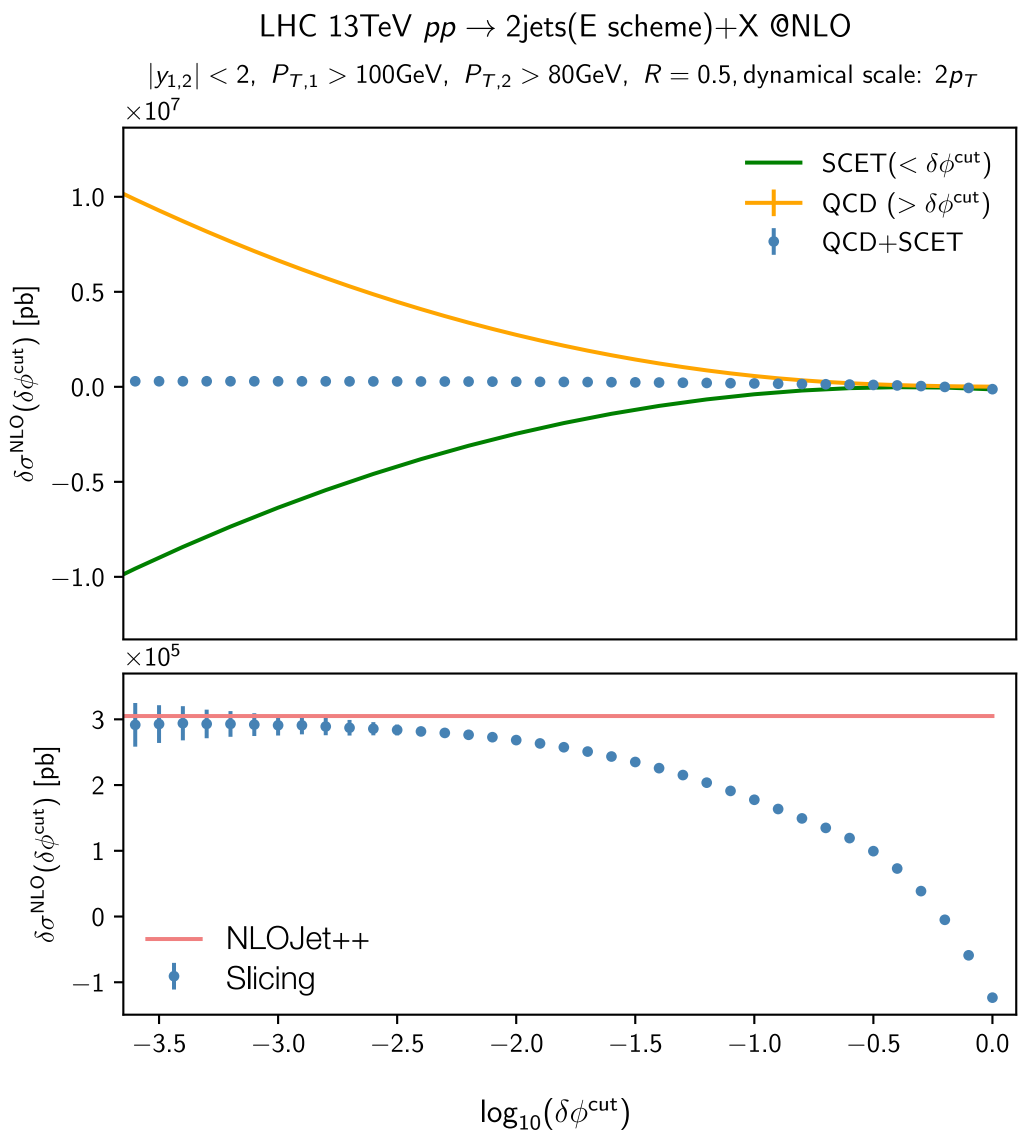
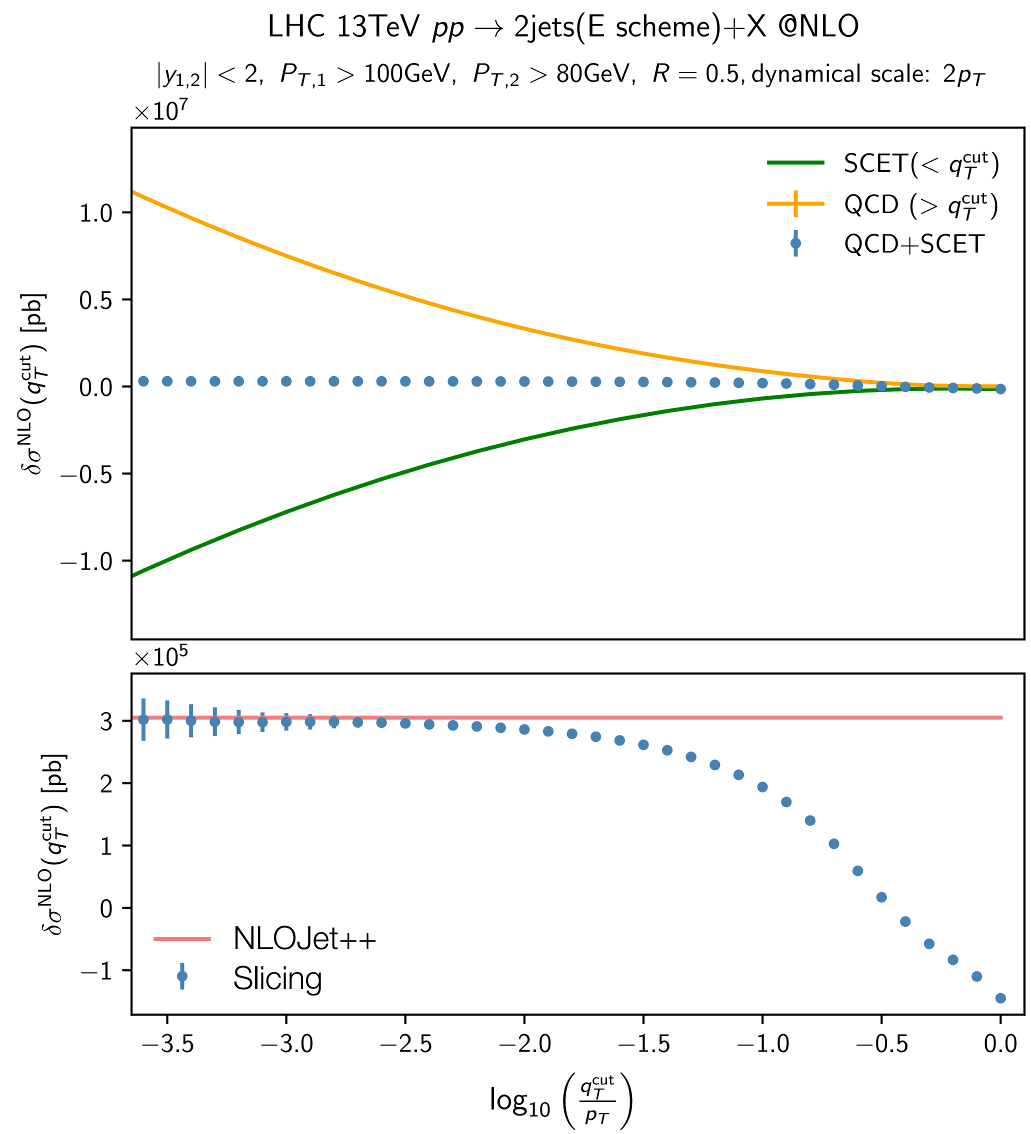
✓ q_T works when using **winner-take-all axis** [Salam; Bertolini, Chan, Thaler].

- Planar processes: component transverse to plane is simple.

q_T slicing with multiple jets

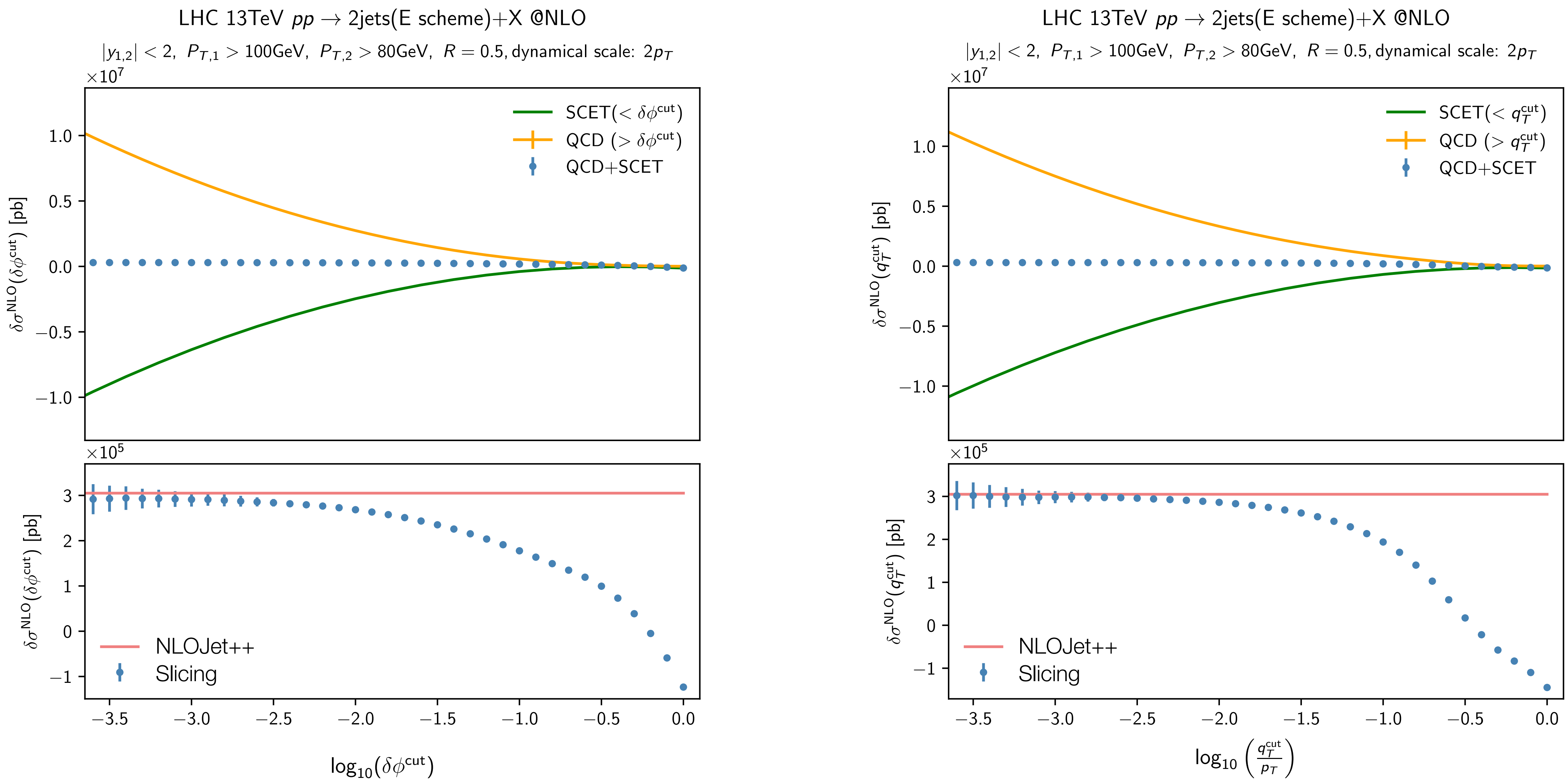




[Fu, Rahn, Shao, WW, Wu]

Proof of concept at NLO.

q_T slicing with multiple jets



[Fu, Rahn, Shao, WW, Wu]

Proof of concept at NLO. At NNLO:

- For planar case ($\delta\phi$) only need constant of two-loop gluon jet.
- For q_T also need two-loop soft function (expand in R).

Conclusions

- Energy correlators separate scales, suppress soft radiation, simple(r) theory → applications: α_s, m_{top}, …
- Track-based energy correlators can be calculated at high precision, and only involve a few moments of track functions.
- Analytic continuation in *N* gives access to small *x* in jets.
- New parametrization enables fast evaluation of higher-point correlators and qualitative differences between jet samples.
- Now studying nonperturbative effects, back-to-back region, as well as new applications (heavy ions) with new definition.

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Thank you!