Theory of Quarkonia

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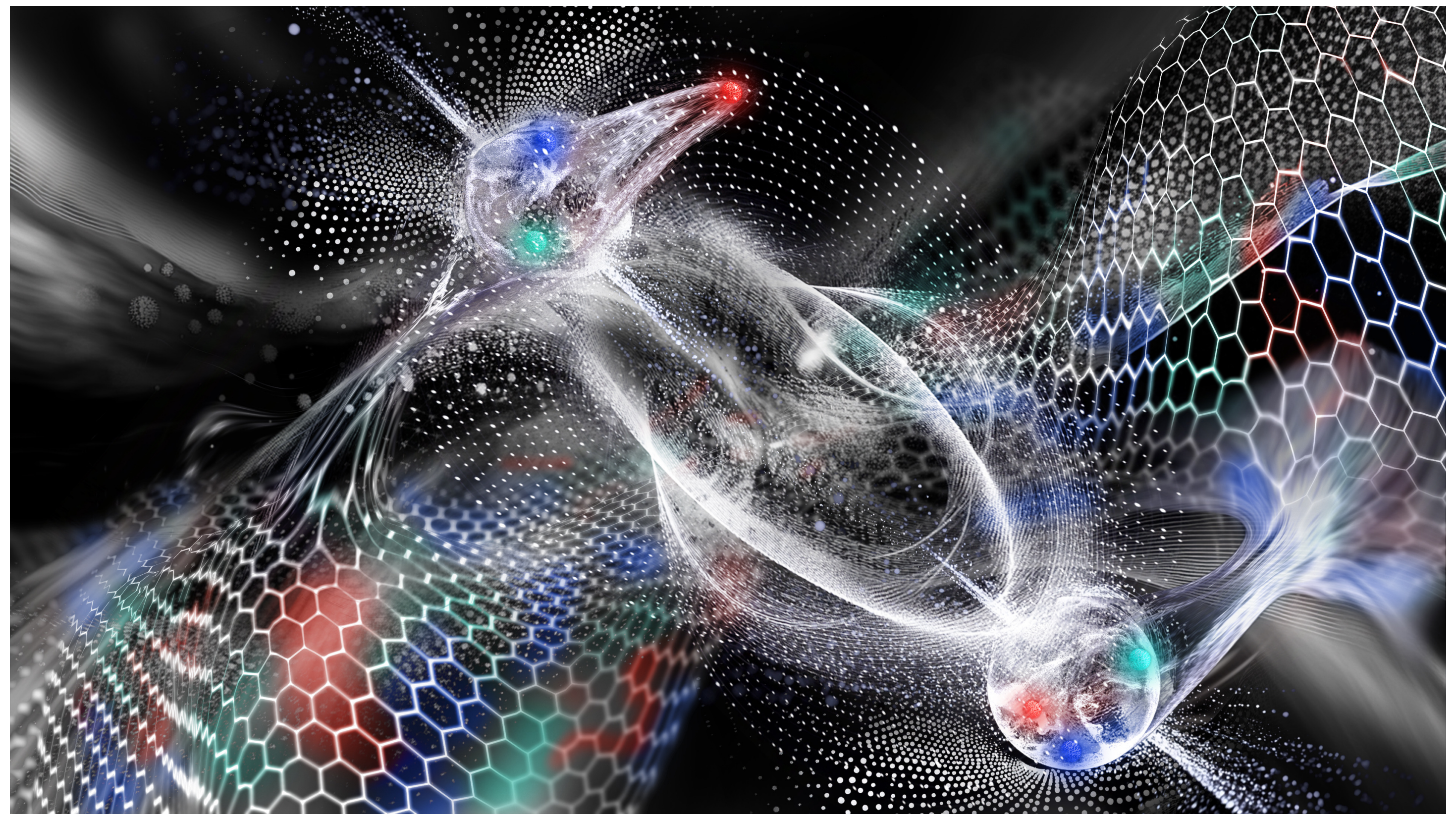
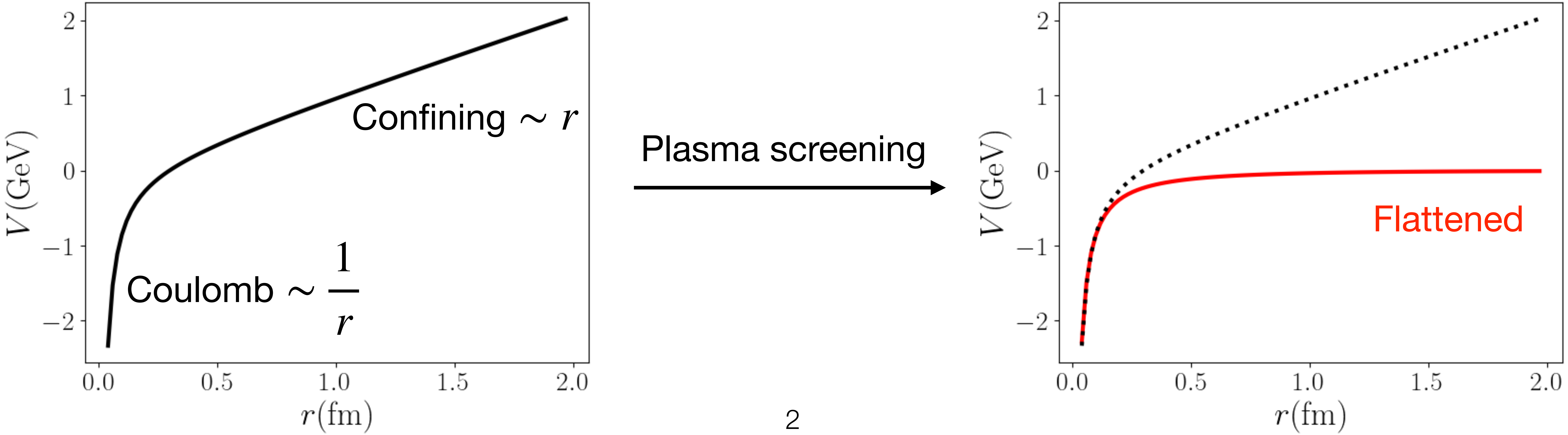
InQubator for Quantum Simulation



2025 RHIC/AGS Annual Users' Meeting
Brookhaven National Laboratory
May 20, 2025

Quarkonium Probes Deconfinement: Screening

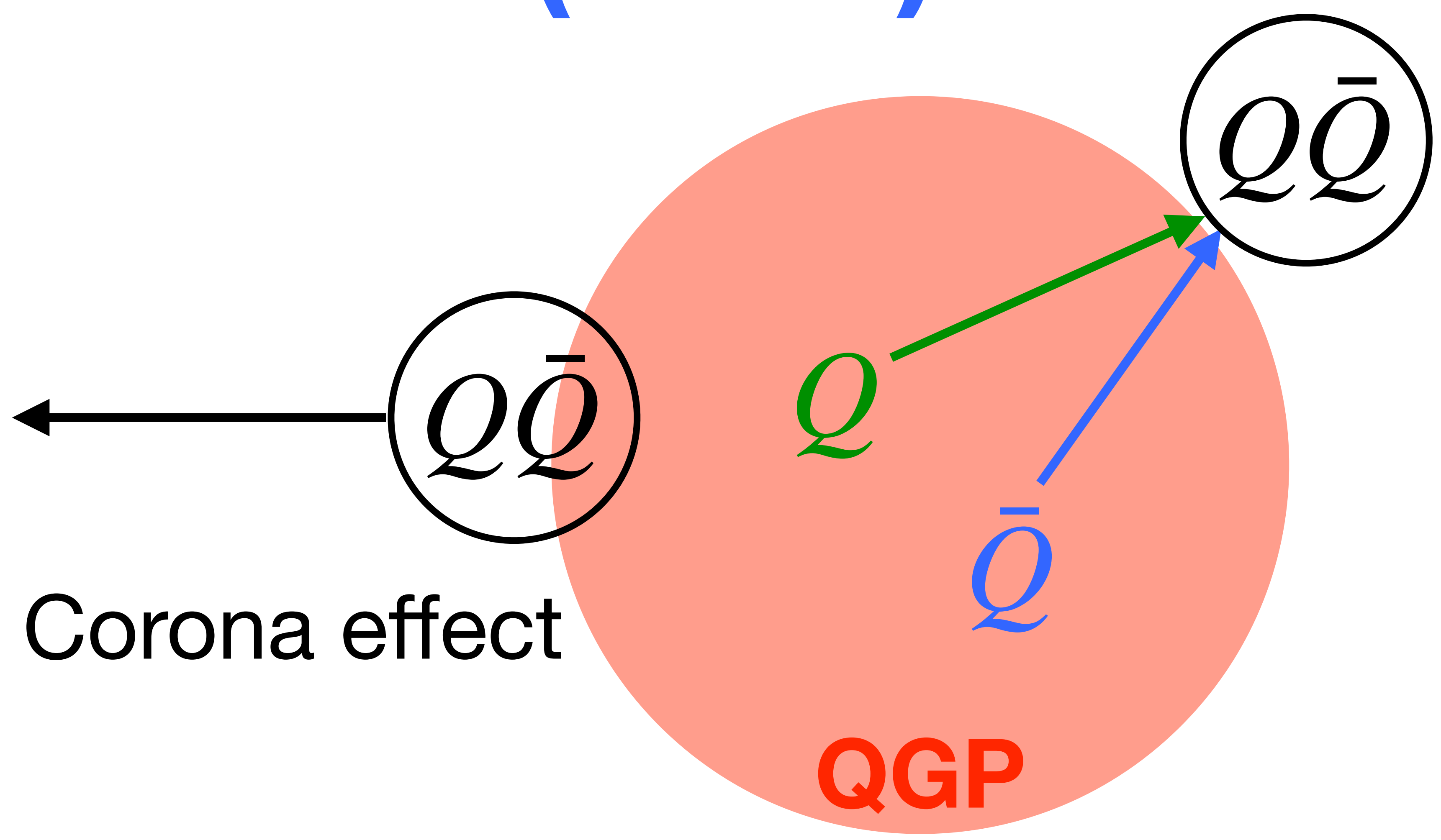
- **Quarkonium: bound state of heavy $Q\bar{Q}$ pair**
- **Screening of potential at finite T**

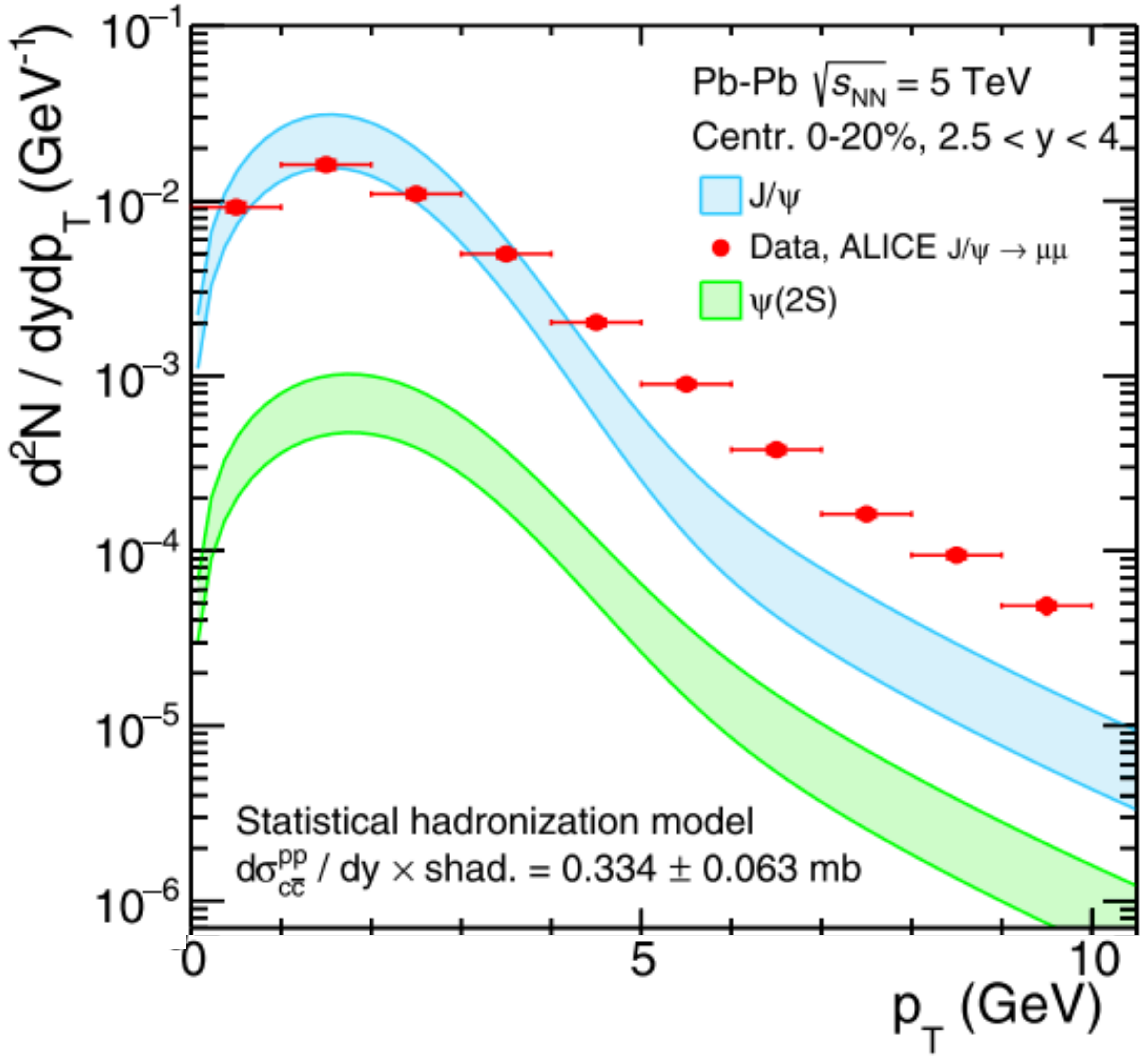


Assume quarkonium not exist inside QGP, only (re)generated at phase transition

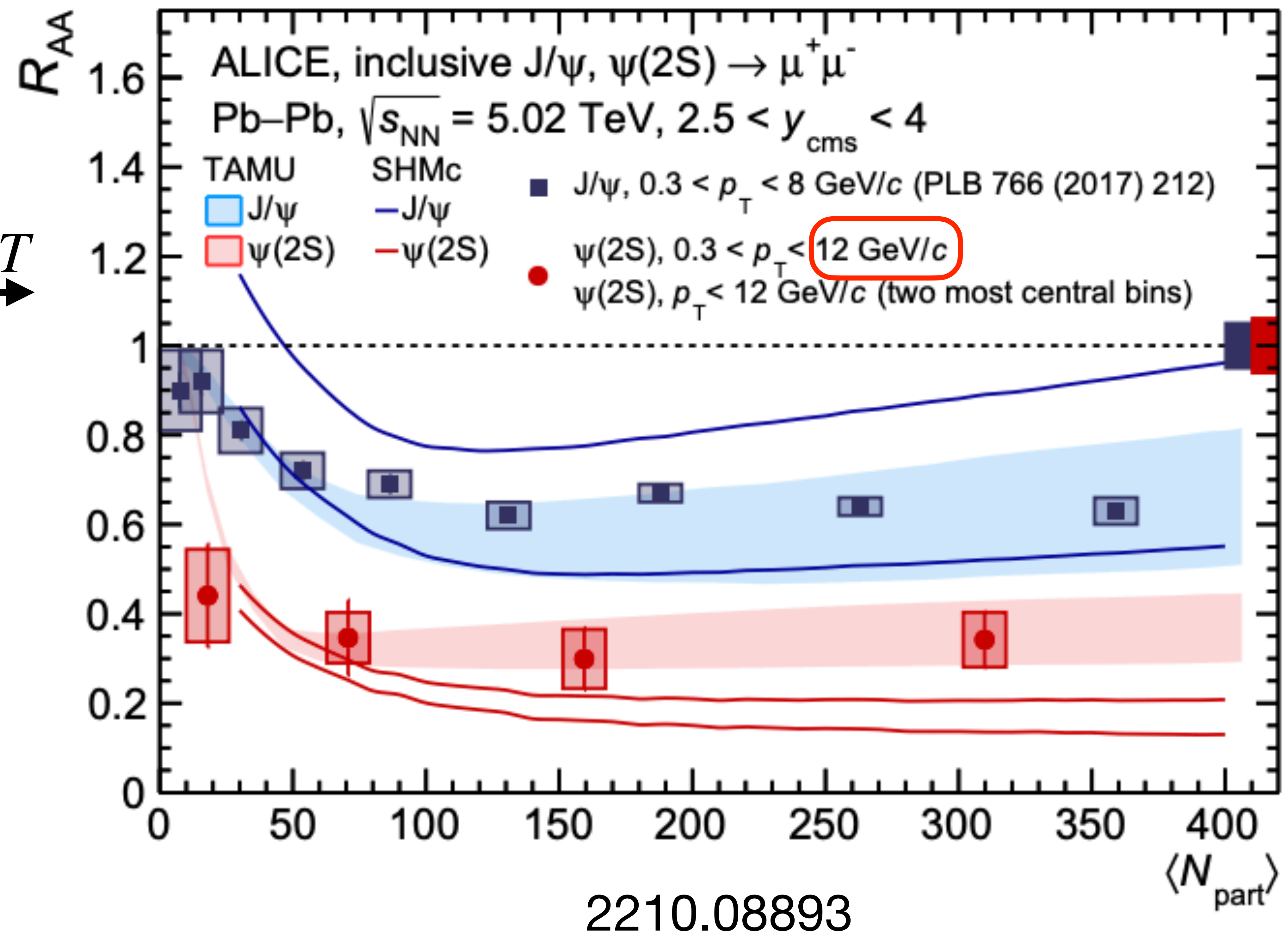
Statistical Hadronization Model (SHM)

- In QGP: charms unbound, reach **kinetic equilibrium** (large mass)
- Instantaneous hadronization at freezeout





Valid at low p_T

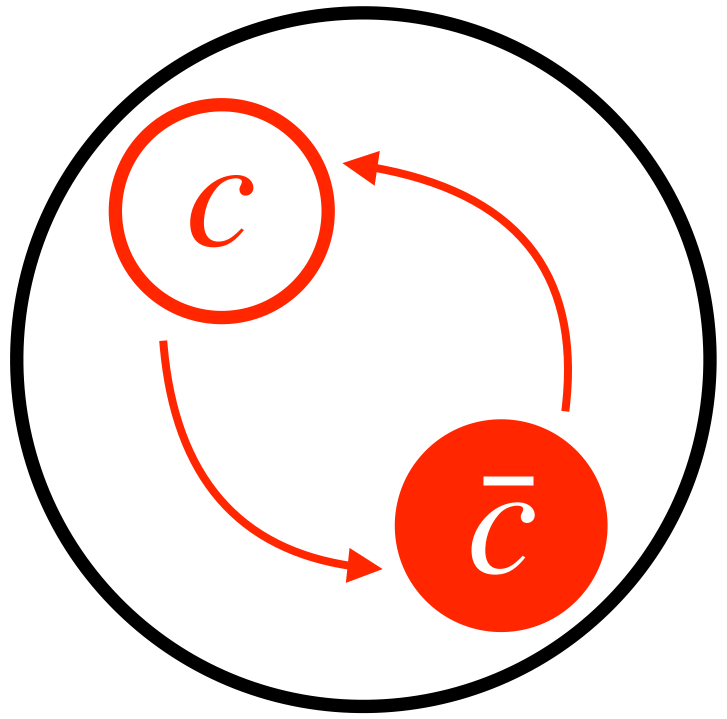


2210.08893

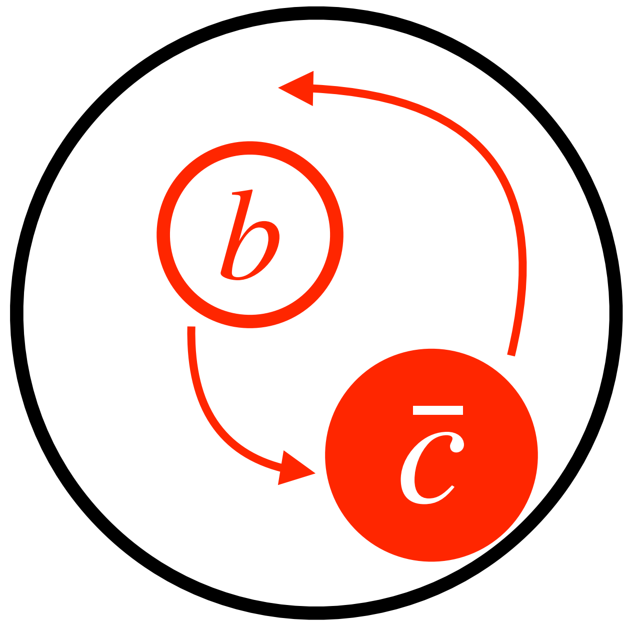
Andronic, Braun-Munzinger, Köhler, Redlich, Stachel, 1901.09200
Andronic, Braun-Munzinger, Redlich, Stachel, 1710.09425

B_c Meson Production

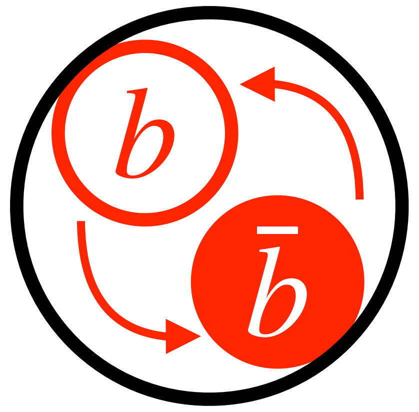
- B_c more "similar" to J/ψ than Υ(1S)



$$r[J/\psi] \sim 0.4 \text{ fm}$$

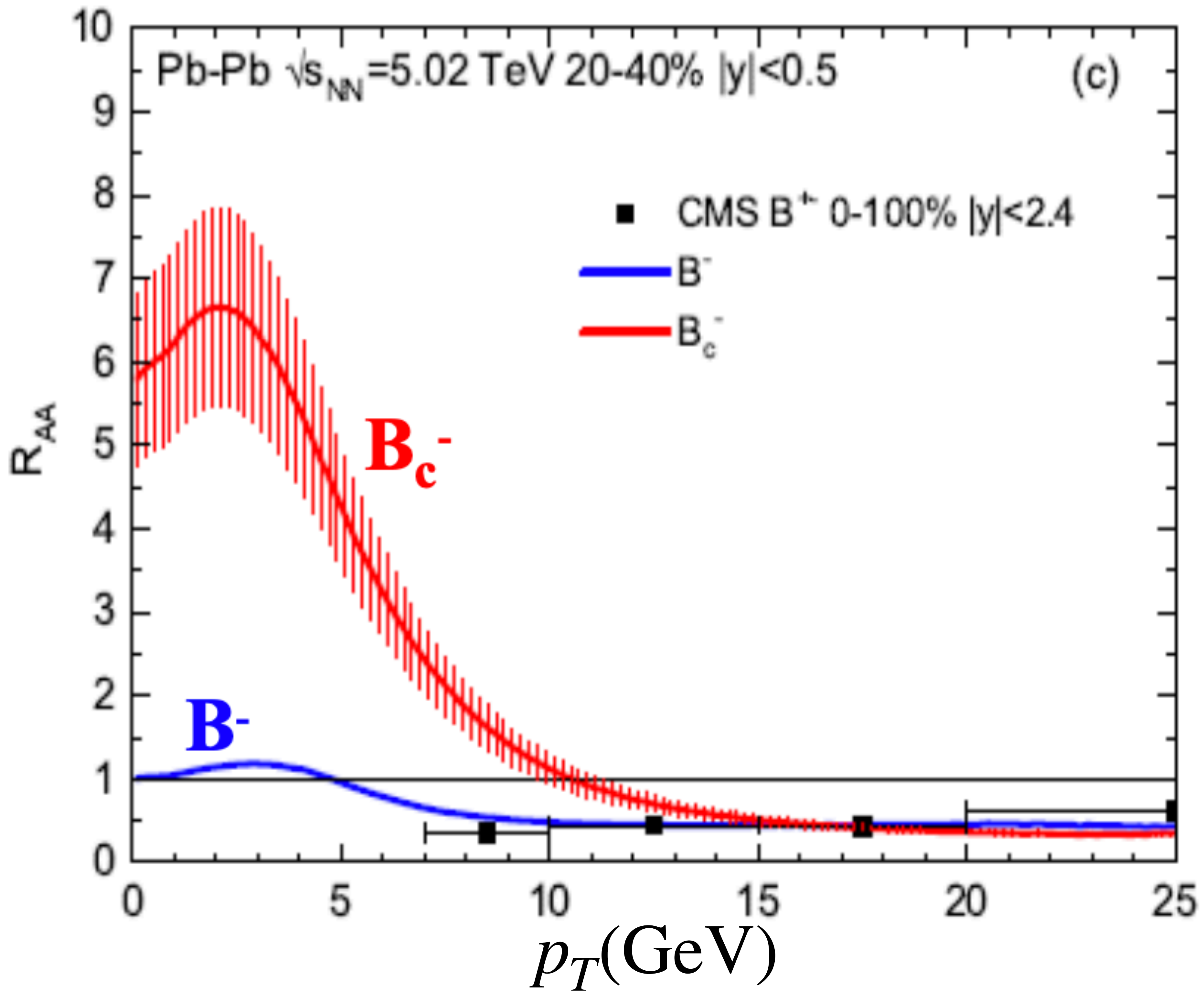


$$r[B_c] \sim 0.34 \text{ fm}$$



$$r[\Upsilon(1S)] \sim 0.2 \text{ fm}$$

- Langevin for unbound heavy quarks
- Instantaneous coalescence at freezeout: one bottom surrounded by multiple charms



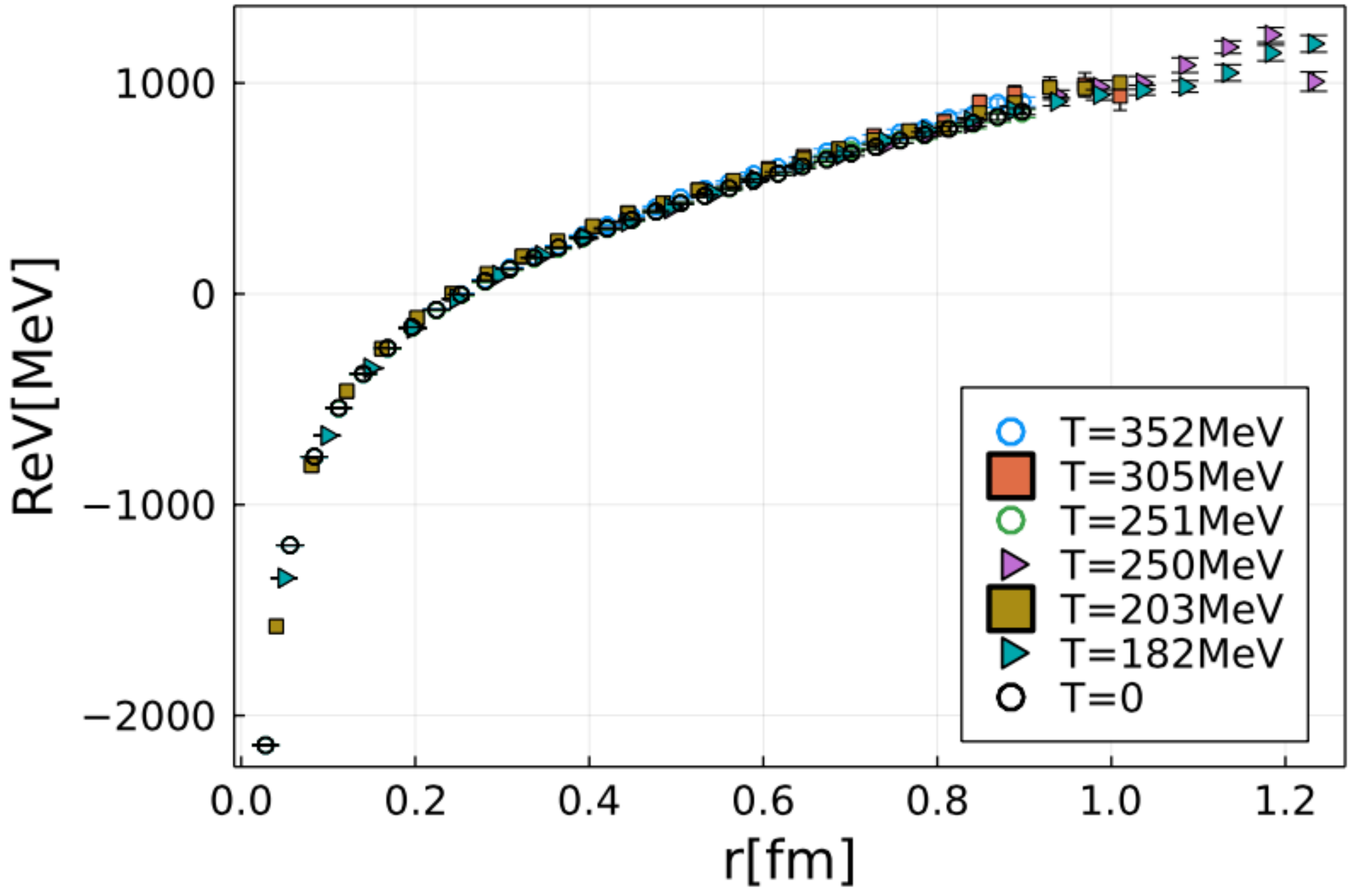
Zhao, He, 2407.05234

Can Quarkonium Exist inside QGP?

- **Deeply bound state at low T ≈ small color dipole, e.g. $rT \sim 0.2$ for Υ(1S) at $T = 200$ MeV**

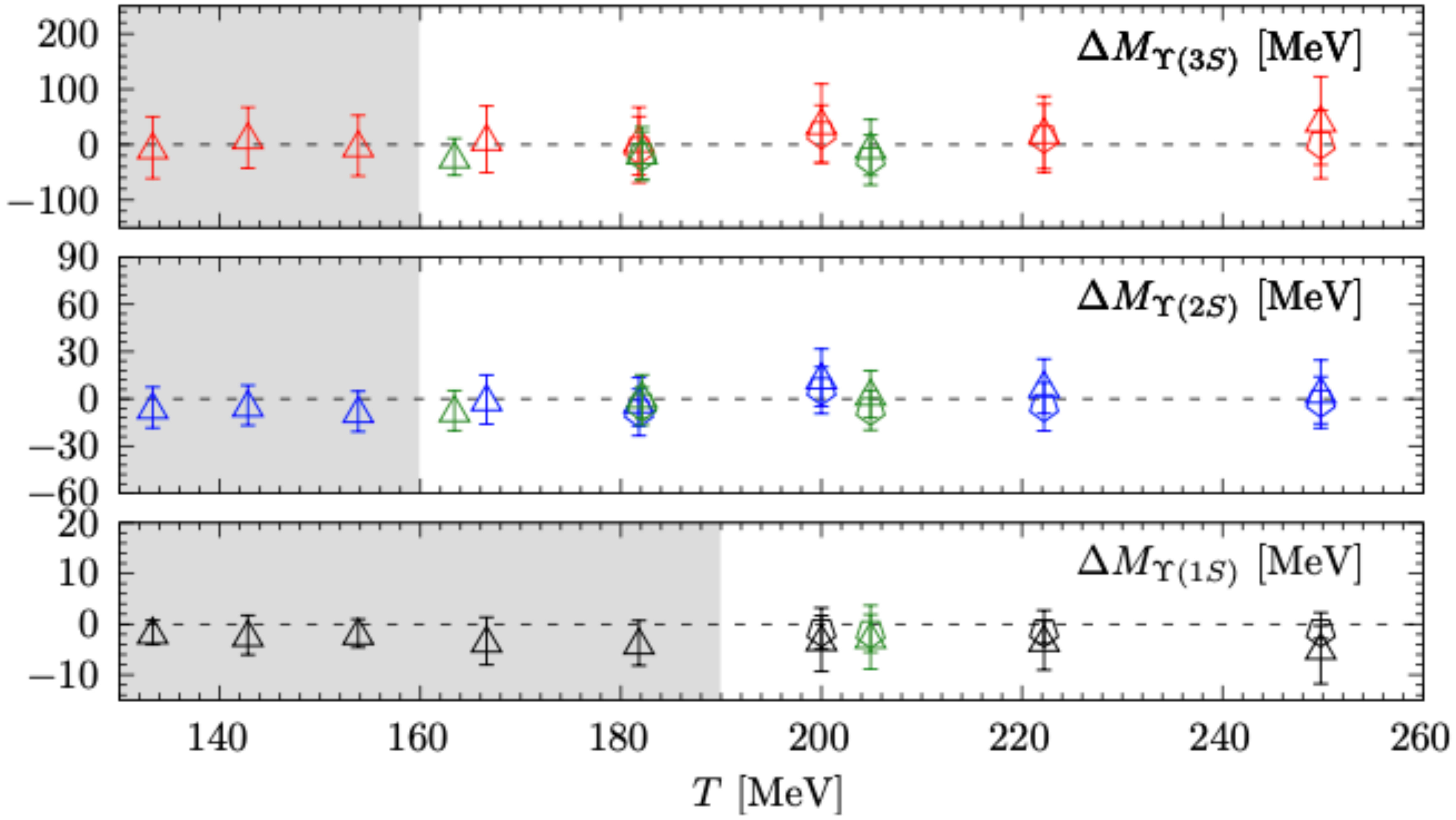
- **Lattice hint at static limit $M_Q \to \infty$:**
  no screening of potential

$$\langle \mathrm{Tr}[W(\tau,r)W^\dagger(\tau,0)]\rangle_T = \int_\omega \rho(\omega,r)e^{-\beta\omega}$$



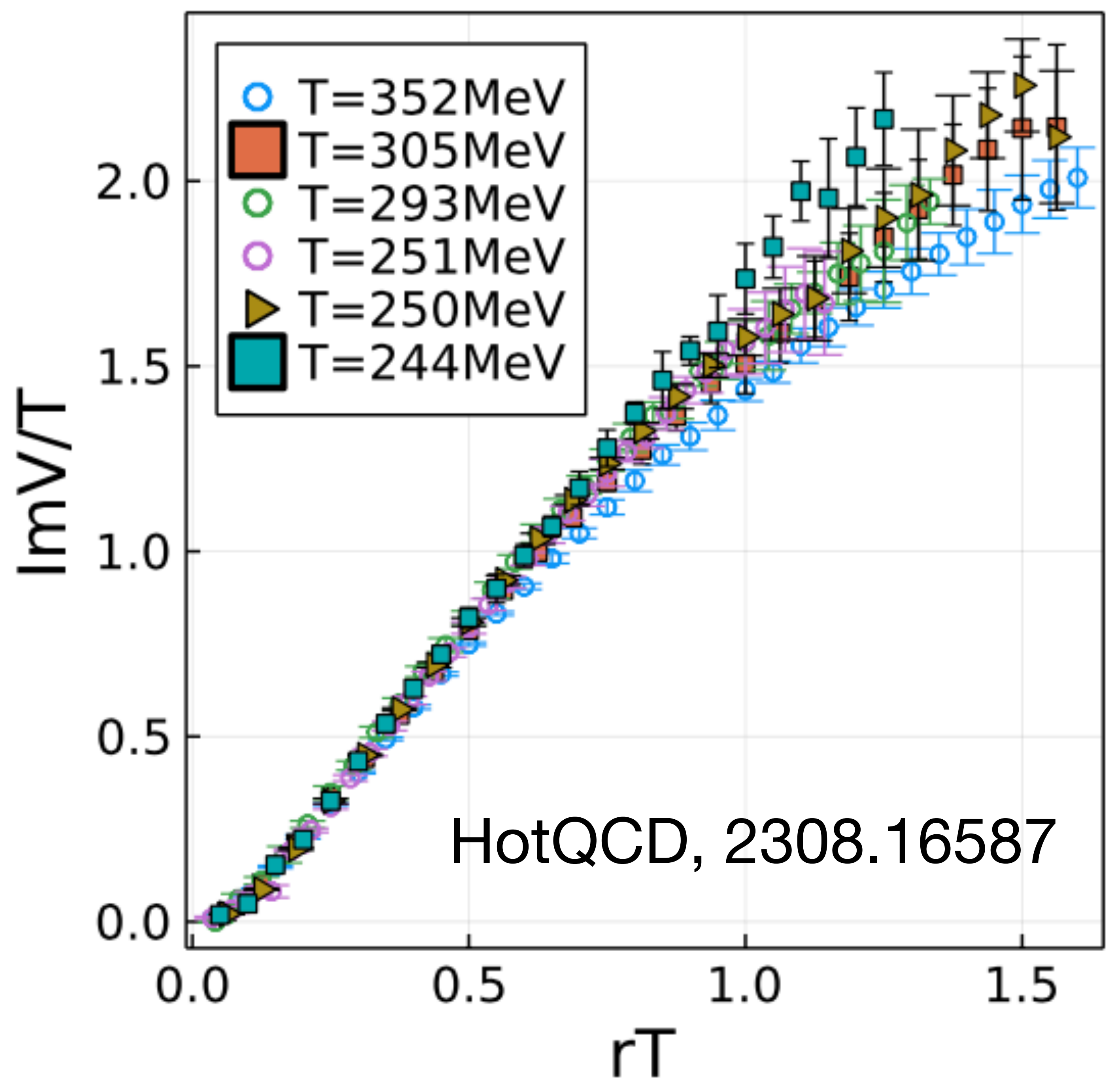
HotQCD, 2308.16587

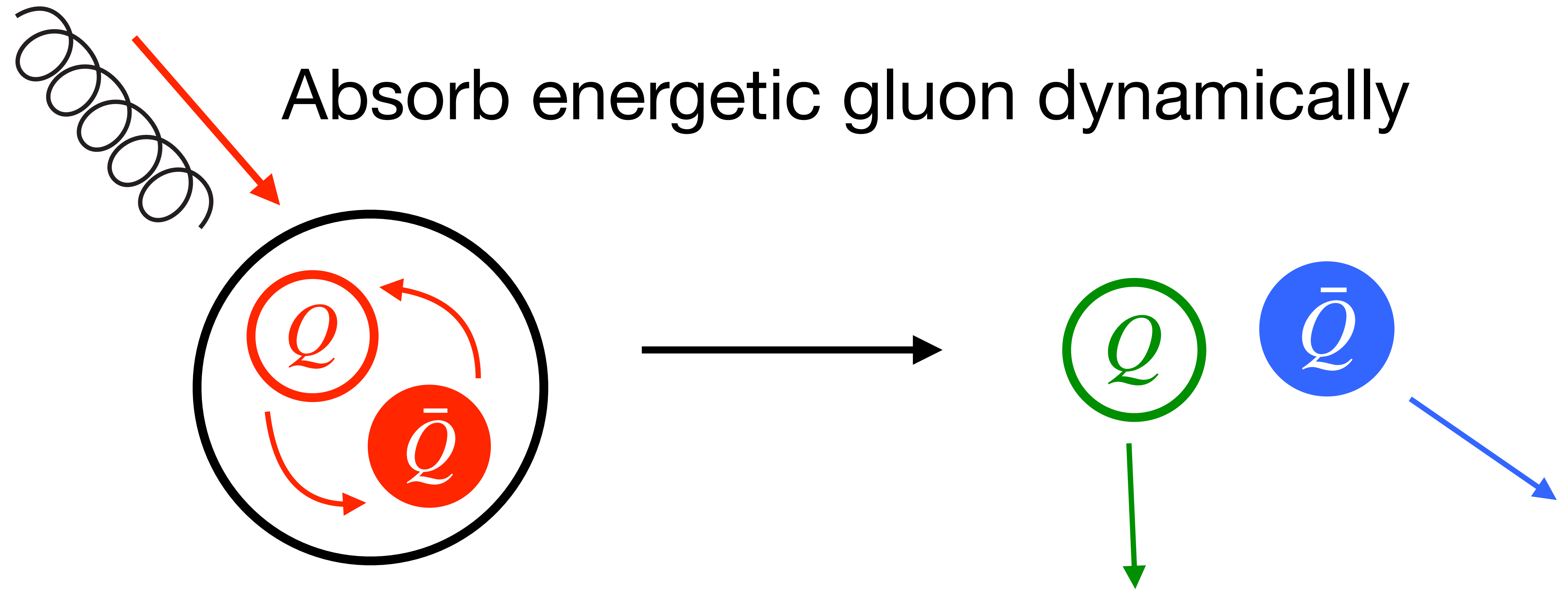
Lattice NRQCD at finite bottom quark mass



Dissociation and Regeneration

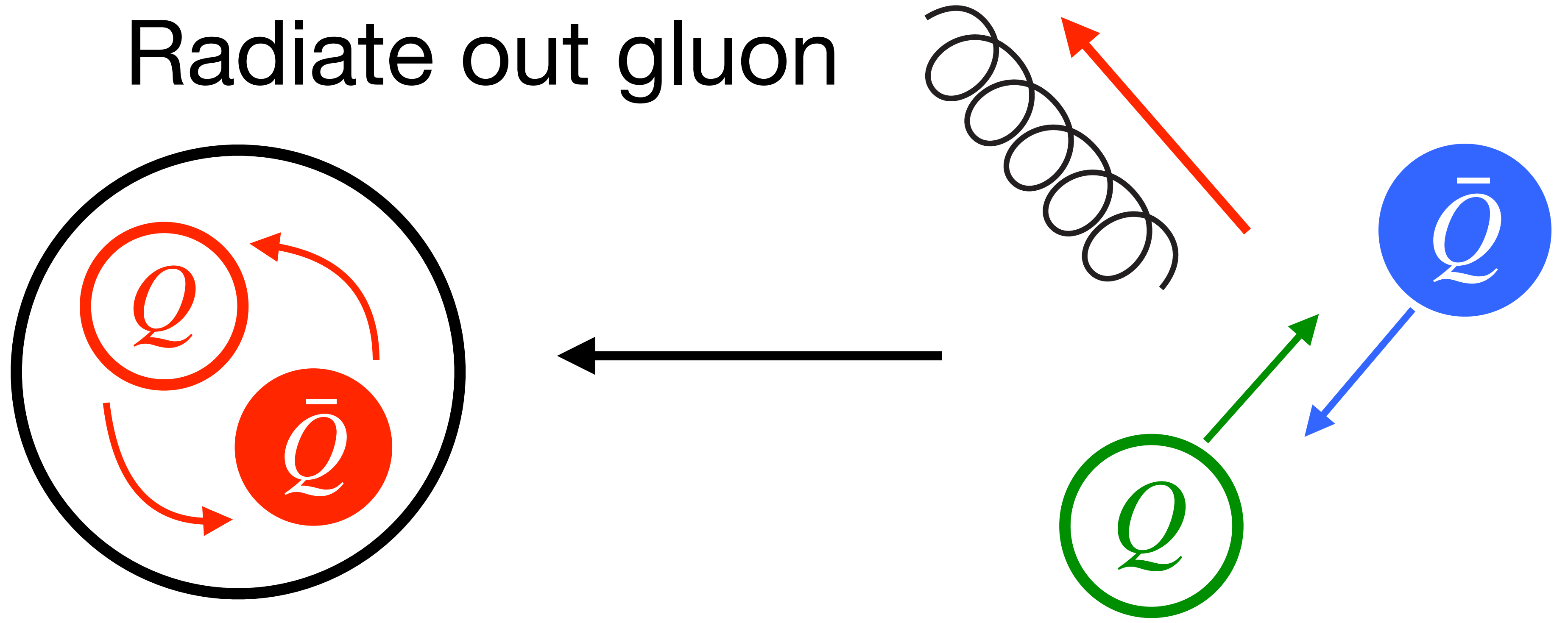
- **Same lattice calculation shows thermal width** → **Quarkonium can dissociate**





Laine, Philipsen, Romatschke, Tassler, hep-ph/0611300
Beraudo, Blaizot, Ratti, 0712.4394
Brambilla, Ghiglieri, Petreczky, Vairo, 0804.0993

- **If quarkonium exists inside QGP, *recombination* can also happen, as inverse process of dissociation**



Thews, Schroedter, Rafelski, hep-ph/0007323

Phenomenology with Dissociation and Regeneration

- **Texas A&M University (TAMU) model**

$$\frac{\mathrm{d}N_{\mathcal{B}}(\tau)}{\mathrm{d}\tau} = -\Gamma_D[T(\tau)]\Big(N_{\mathcal{B}}(\tau) - N_{\mathcal{B}}^{\mathrm{eq}}[T(\tau)]\Big)$$

#(bound state) change rate

Equilibrium number, time-dependent

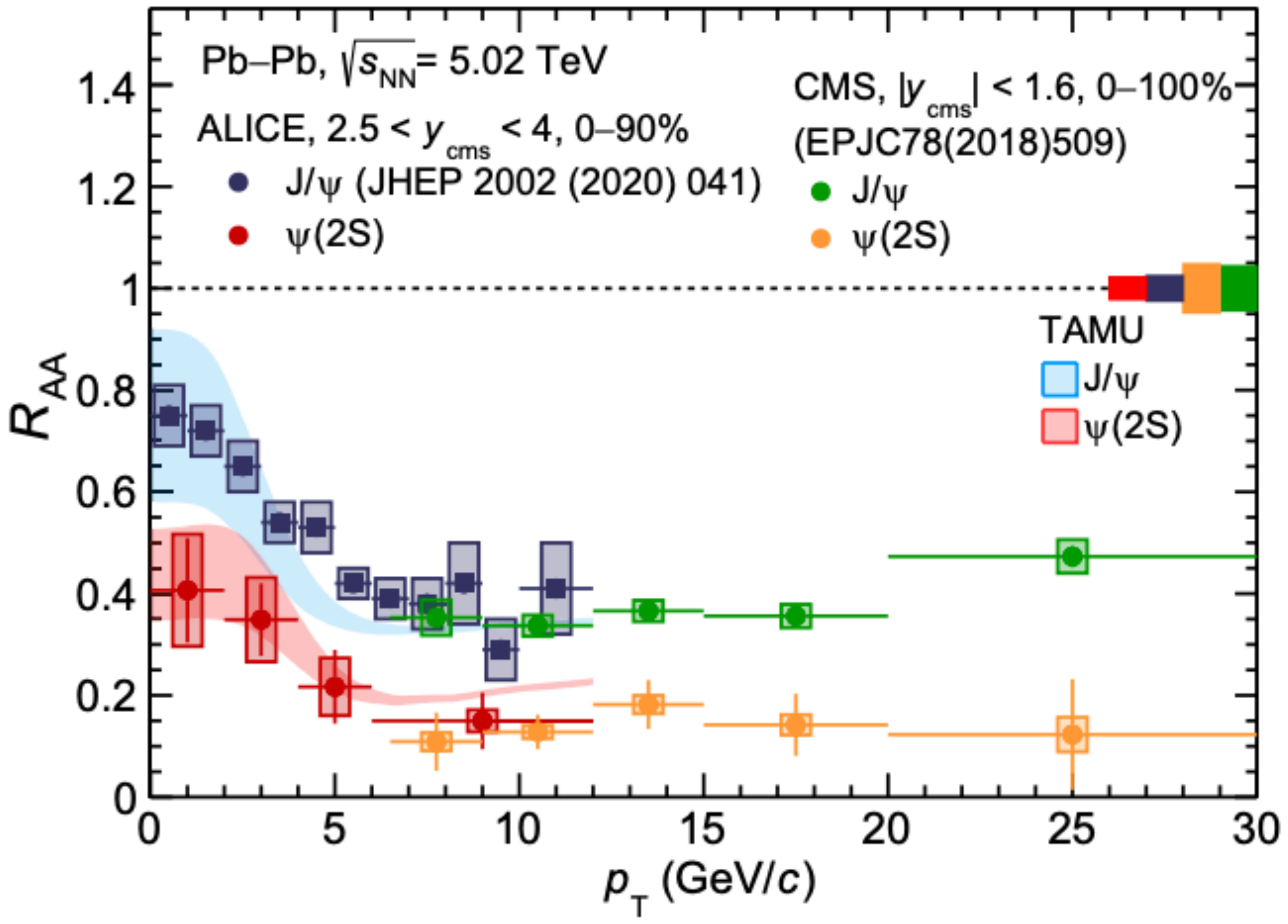
- Dissociation rate from $g + \mathcal{B} \to Q + \bar{Q}$

$$2 \times (q/g + Q \to q/g + Q)$$

- Phenomenological factors, interference

- New development:
  T-matrix summation + lattice input

Wu, Tang, Rapp, 2503.10089



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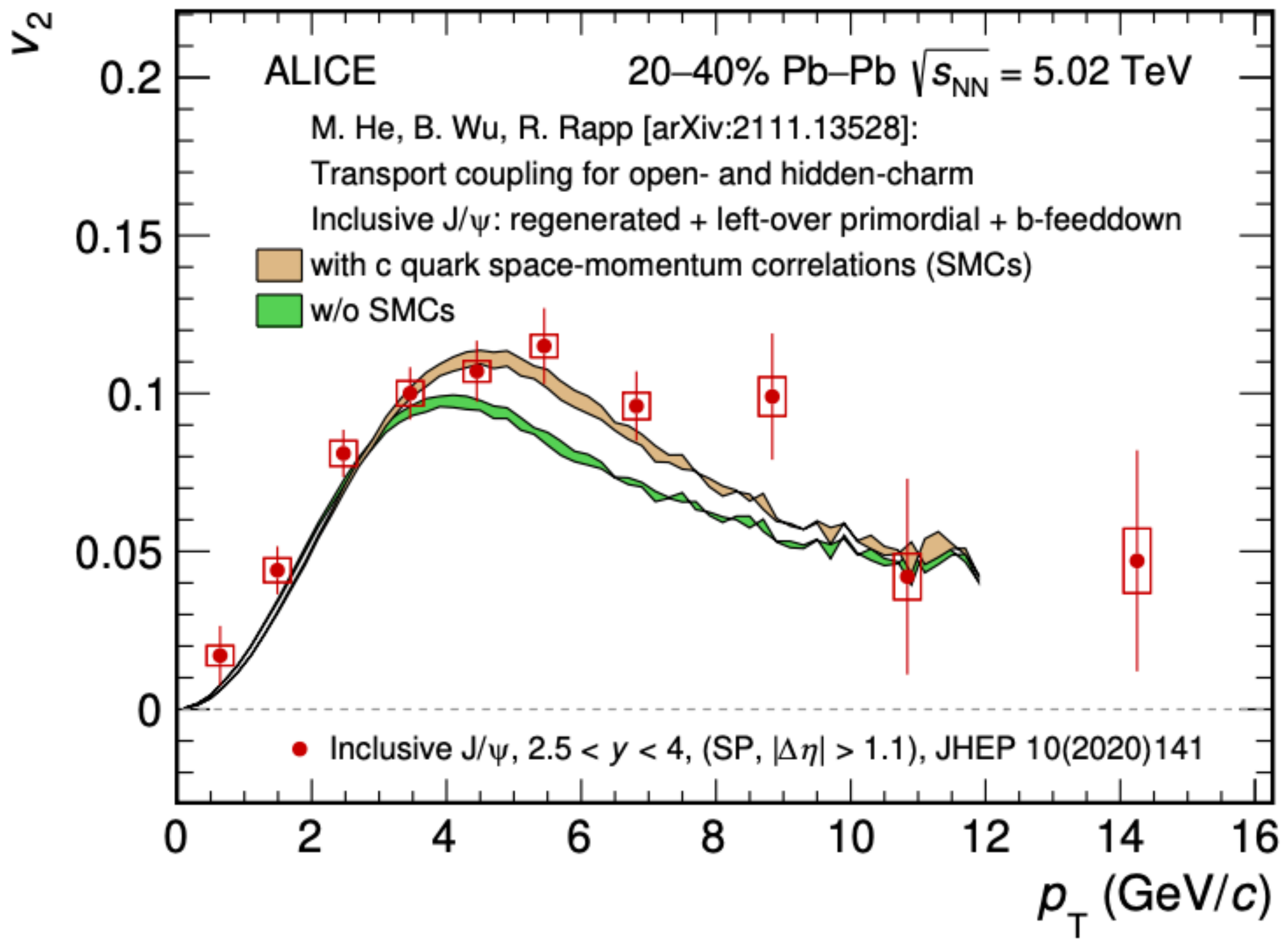


2005.14518

Phenomenology with Dissociation and Regeneration

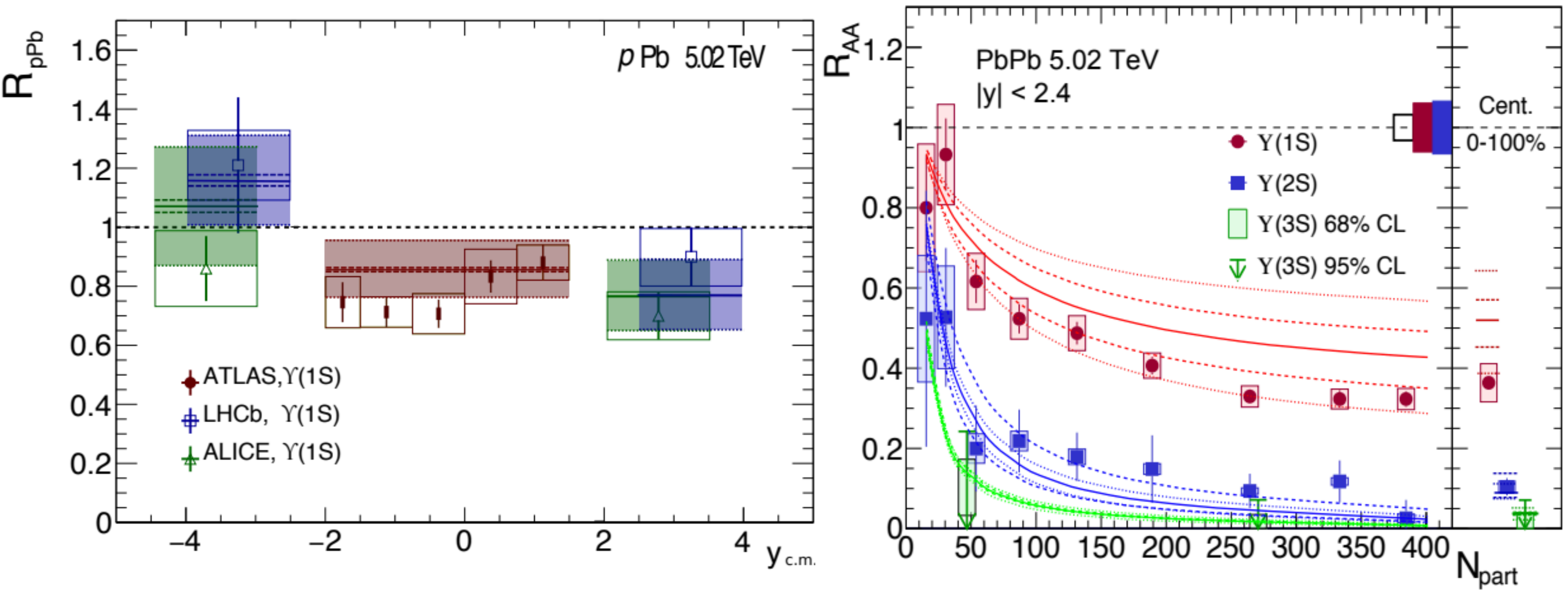
- **Santiago comover interaction model**

$$\tau \frac{\mathrm{d}\rho_{\mathcal{B}}(\tau)}{\mathrm{d}\tau} = -\sigma_{\mathrm{co}-\mathcal{B}} \left(\rho_{\mathrm{co}} \rho_{\mathcal{B}} - \rho_{Q} \rho_{\bar{Q}} \right)$$

↓

Input: comover-quarkonium cross section

No regeneration for bottomonium



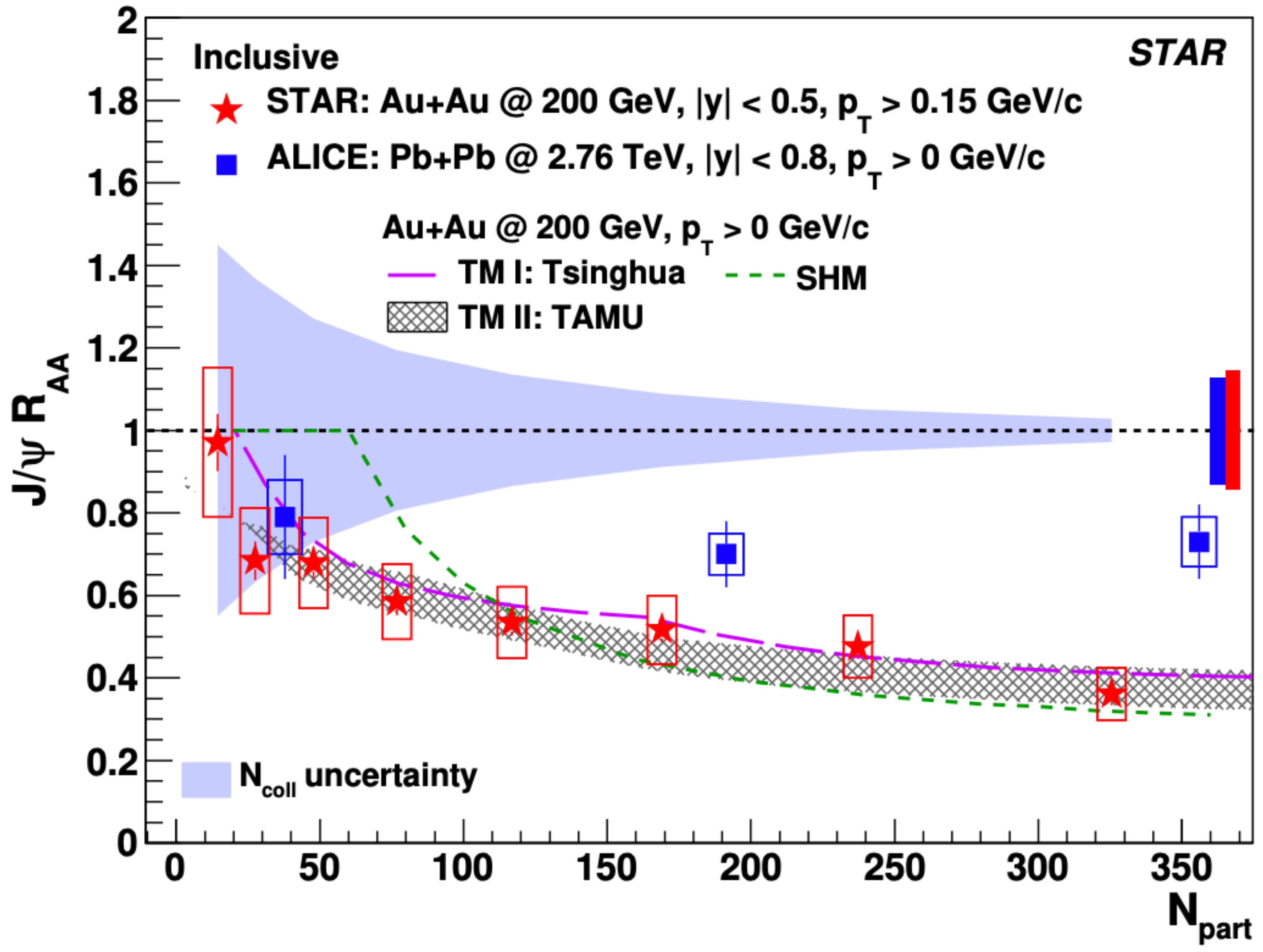
Ferreiro, Lansberg, 1804.04474

- **Tsinghua model: relativistic Boltzmann**

$$\left[\cosh(y-\eta) \frac{\partial}{\partial \tau} + \frac{\sinh(y-\eta)}{\tau} \frac{\partial}{\partial \eta} + \boldsymbol{v}_T \cdot \nabla_T \right] f_{\mathcal{B}} = -\alpha f_{\mathcal{B}} + \beta$$

Only leading-order processes included

$$g + \mathcal{B} \leftrightarrow Q + \bar{Q}$$



1905.13669

What Properties of QGP are Probed?

- **Temperature?** ⟶ Input from well-studied hydro
- **Dissociation/recombination rate?** ⟶ Property of quarkonium

Separate intrinsic properties of quarkonium from intrinsic properties of QGP

↓

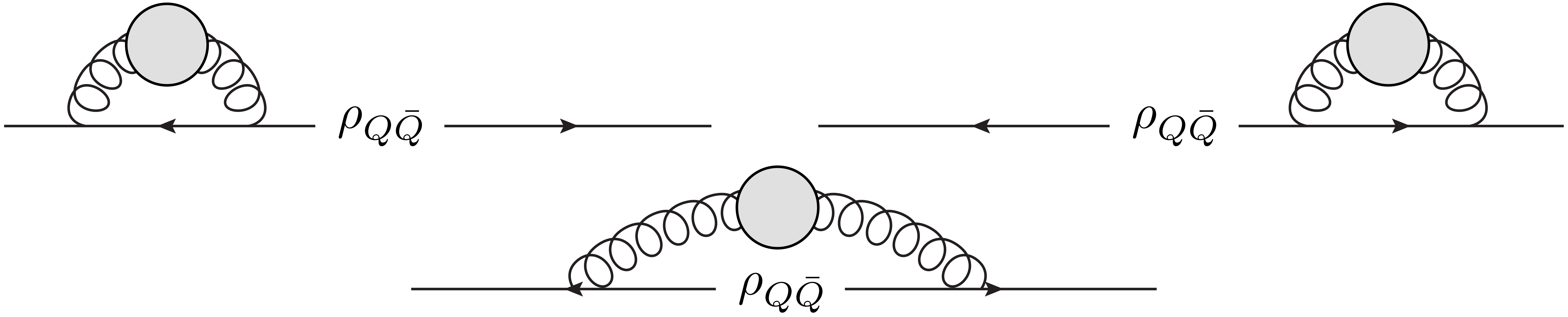
Open quantum system (OQS) + effective field theory (EFT)

Open Quantum System for Quarkonium

- **QGP + $Q\bar{Q}$ = closed quantum system; $Q\bar{Q}$ = open quantum system**

$$\rho_{Q\bar{Q}}(t) = \mathrm{Tr}_{\mathrm{QGP}}[U(t)\rho_{\mathrm{tot}}(0)U^{\dagger}(t)]$$

$$\frac{\mathrm{d}\rho_{Q\bar{Q}}(t)}{\mathrm{d}t} =$$



Akamatsu, Rothkopf, 1110.1203; Akamatsu, 1403.5783

Brambilla, Escobedo, Soto, Vairo, 1612.07248, 1711.04515; Blaizot, Escobedo, 1711.10812;
Miura, Akamatsu, Asakawa, Rothkopf, 1908.06293, Yao, Mehen, 1811.07027, 2009.02408

Reviews of OQS for quarkonia: Rothkopf, 1912.02253; Akamatsu, 2009.10559, Sharma, 2101.04268; Yao, 2102.01736

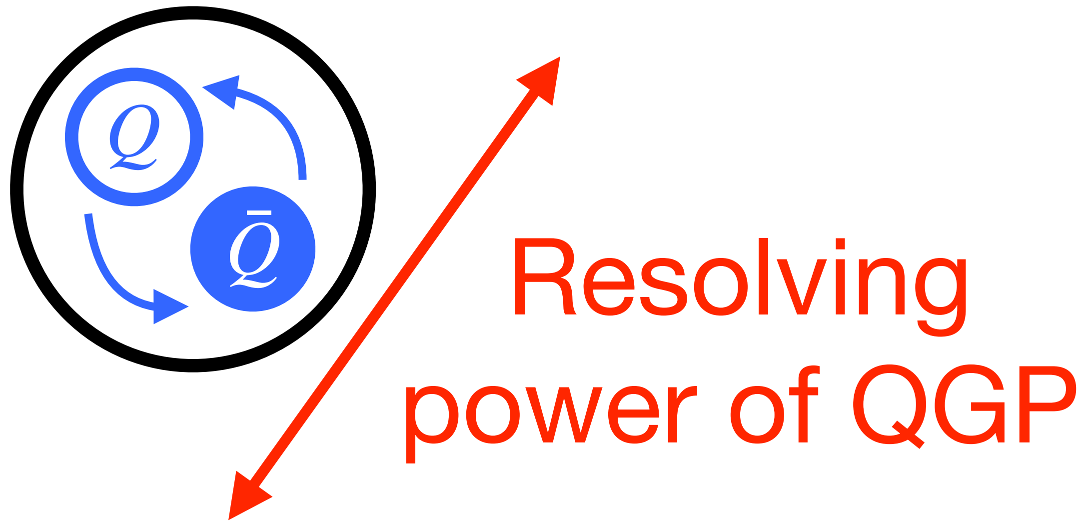
Separation of Energy Scales and Nonrelativistic EFT

In vacuum $M \gg Mv \gg Mv^2$

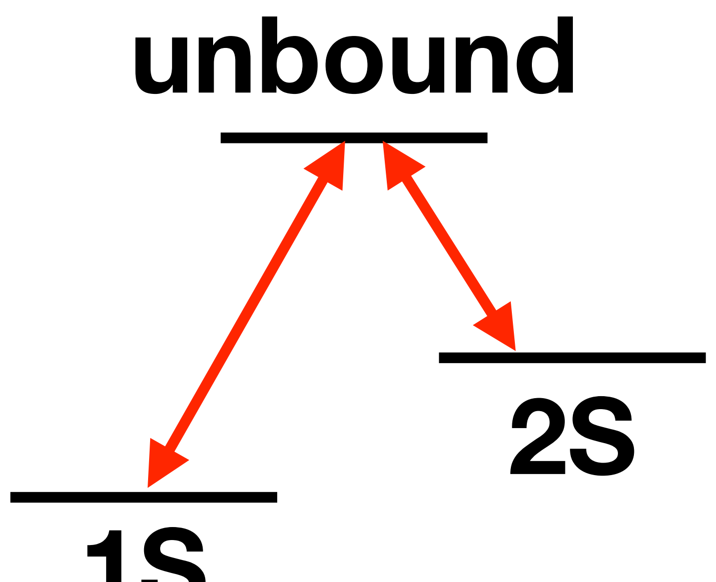
EFT	Scale		$c\bar{c}$	$b\bar{b}$
QCD				
	M	heavy quark mass	1.3	4.5 GeV
NRQCD				
	Mv	inverse of quarkonium size r^{-1}	0.7	1.1 GeV
pNRQCD				
	Mv^2	quarkonium binding energy	0.5	0.5 GeV

Medium case depends on where T fits

- **Quantum optical limit: low T**

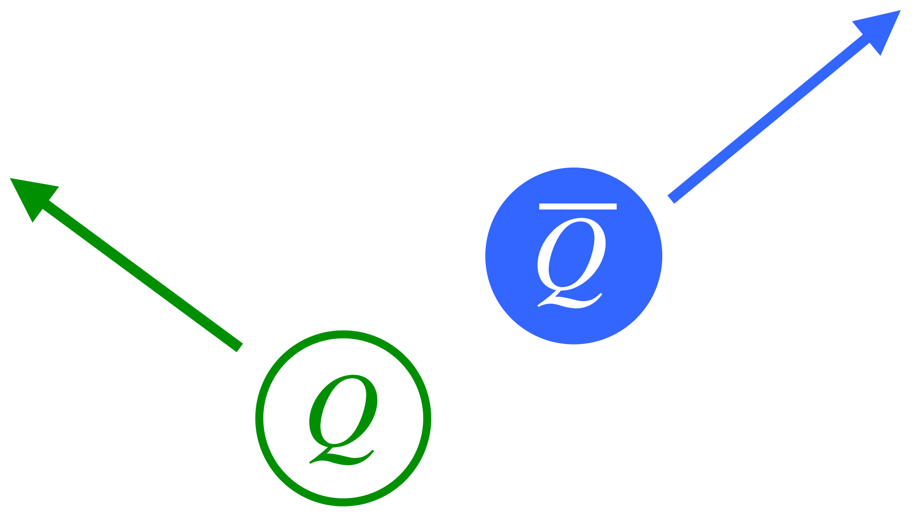


Transitions between levels



- **Quantum Brownian motion: high T**

Decoherence of $Q\bar{Q}$ pair, “diffusion”



Separation of Energy Scales and Physical Scenarios

Hierarchy of energy scales	EFT	Quantum Description	Classical Limit
$M \gg T \gg Mv^2, \Lambda_{QCD}$	NRQCD $\alpha_s(T)$ small	Lindblad (quantum Brownian motion)	Diffusion equation (semiclassical limit)
$M \gg Mv \gg T, \Lambda_{QCD}$ $T \gg Mv^2$, expand Mv^2/T	pNRQCD rT small	Lindblad (quantum Brownian motion)	
$M \gg Mv \gg T, \Lambda_{QCD}$ No expansion of Mv^2/T	pNRQCD rT small		Boltzmann equation (quantum optical and semiclassical limits)

Case 1: Lindblad Equation in Quantum Brownian Motion

- $M \gg T \gg Mv^2, \Lambda_{\mathrm{QCD}}$, 1D numerics

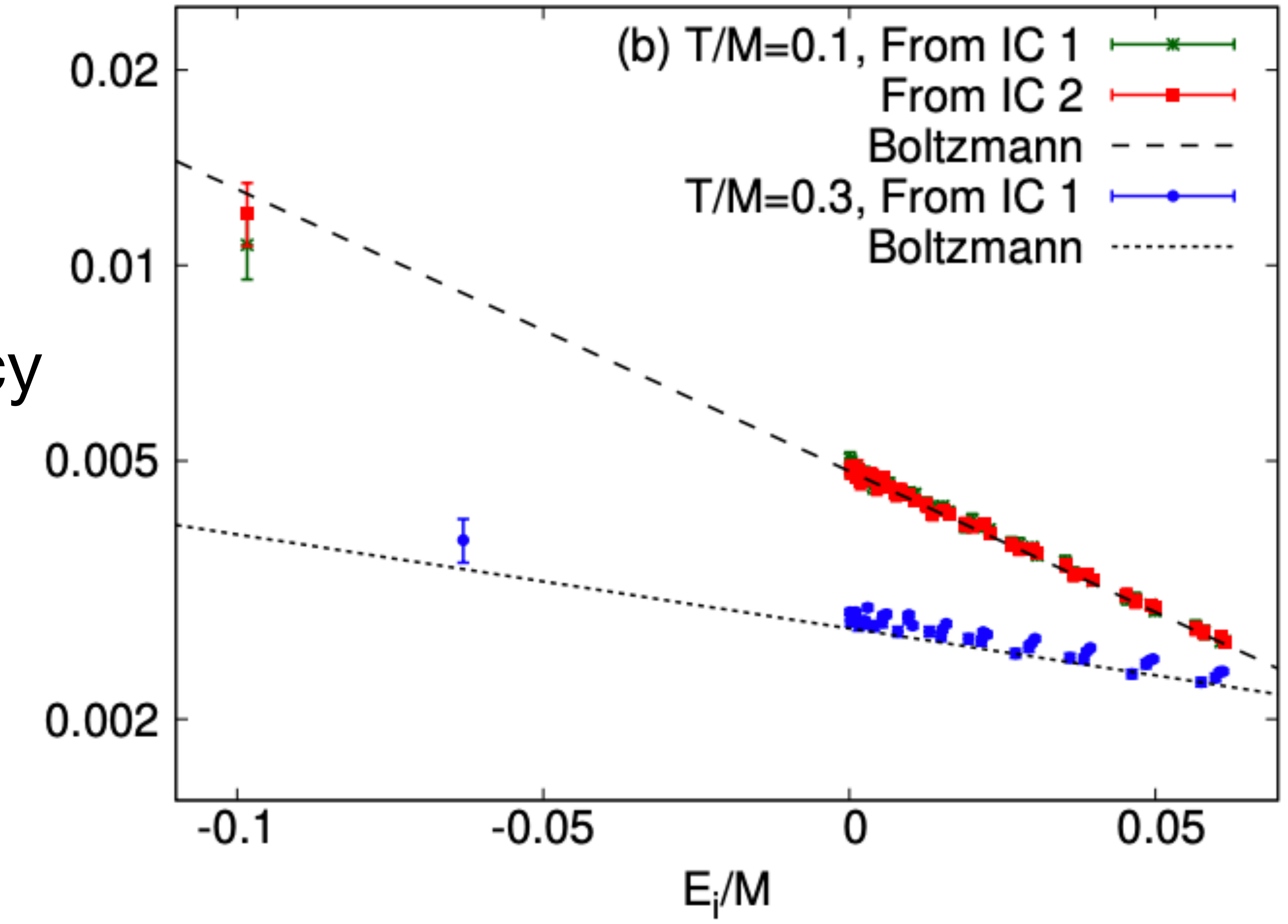
$$\frac{\mathrm{d}\rho_{Q\bar{Q}}(t)}{\mathrm{d}t} = -i[H, \rho_{Q\bar{Q}}(t)] + \sum_{ij} D_{ij}\left(L_i \rho_{Q\bar{Q}}(t) L_j^\dagger - \frac{1}{2}\{L_j^\dagger L_i, \rho_{Q\bar{Q}}(t)\}\right)$$

H = kinetic + potential

L_i : singlet-octet, octet-octet transitions

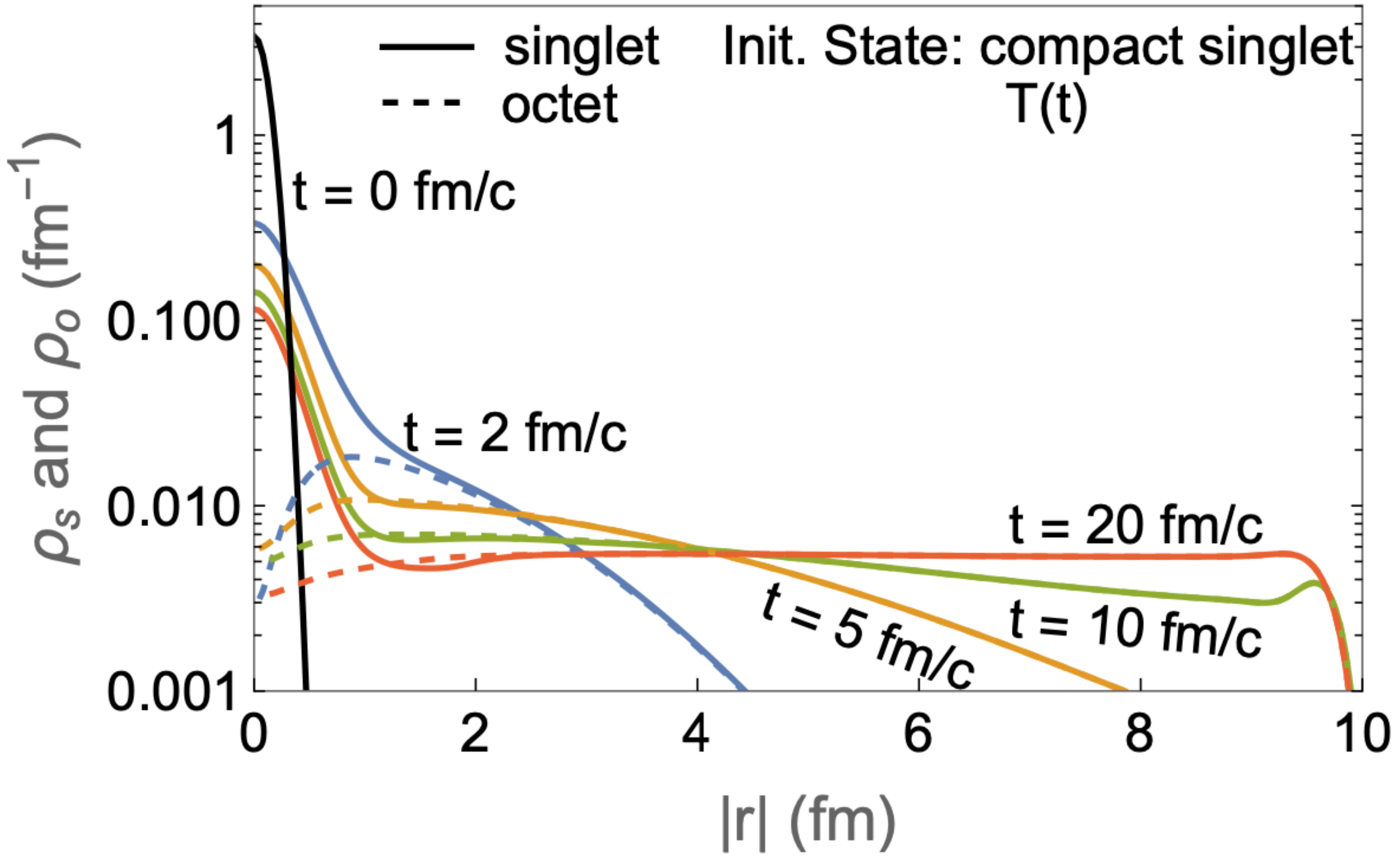
- **Osaka group: momentum basis $D_{ij} \propto \delta_{ij}$**

Occupancy in steady state



Miura, Akamatsu, Asakawa, Kaida, 2205.15551

- **Nantes: position basis D_{ij} has off-diagonal**



Delorme, Katz, Gousset, Gossiaux, Blaizot, 2402.04488

Case 2: Lindblad Equation in Quantum Brownian Motion

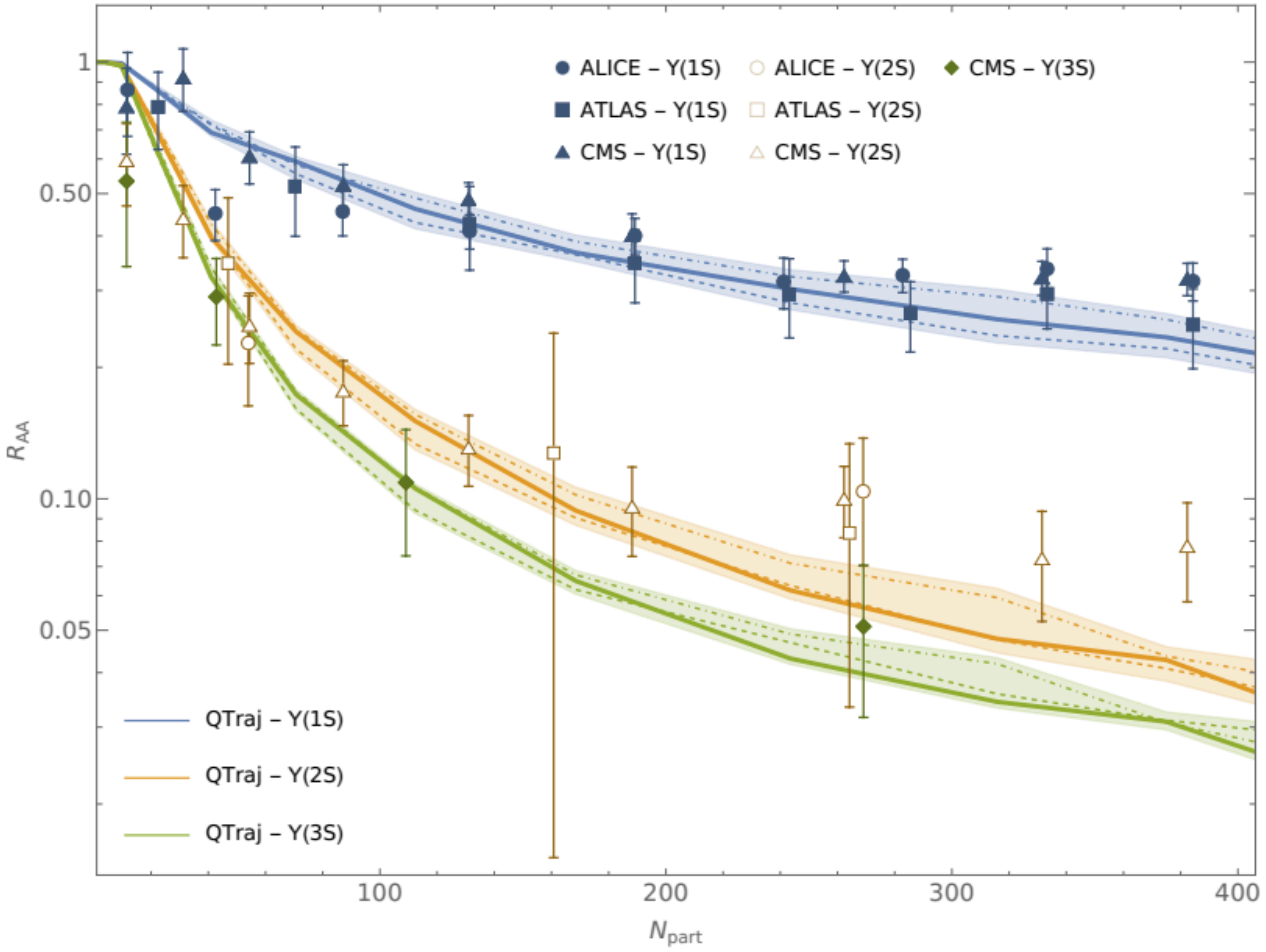
- **Munich-Kent approach:** $M \gg Mv \gg T, \Lambda_{\text{QCD}}$ and $T \gg Mv^2$

$$\frac{\mathrm{d}\rho_{Q\bar{Q}}(t)}{\mathrm{d}t} = -i\left[H + \gamma_{\text{adj}}\Delta h,\, \rho_{Q\bar{Q}}(t)\right] + \kappa_{\text{adj}}\left(L_{\alpha i}\rho_{Q\bar{Q}}(t)L_{\alpha i}^{\dagger} - \frac{1}{2}\{L_{\alpha i}^{\dagger}L_{\alpha i},\, \rho_{Q\bar{Q}}(t)\}\right)$$

- Perturbative at Mv: **3-loop QCD potential**, beyond Coulomb
  Nonperturbative transport parameters: κ_{adj} and γ_{adj} from **chromoelectric correlator**

$$\kappa_{\text{adj}} + i\gamma_{\text{adj}} \equiv \frac{g^2 T_F}{3N_c}\int \mathrm{d}t \langle \mathcal{T} E_i^a(t)\mathcal{W}^{ab}(t,0)E_i^b(0)\rangle_T$$

- New development: perturbative beyond dipole limit



Brambilla, Magorsch, Strickland, Vairo, Griend, 2403.15545

Case 3: Boltzmann Equation in Quantum Optical & Classical

- **Rigorous derivation** $M \gg Mv \gg T, \Lambda_{\rm QCD}$

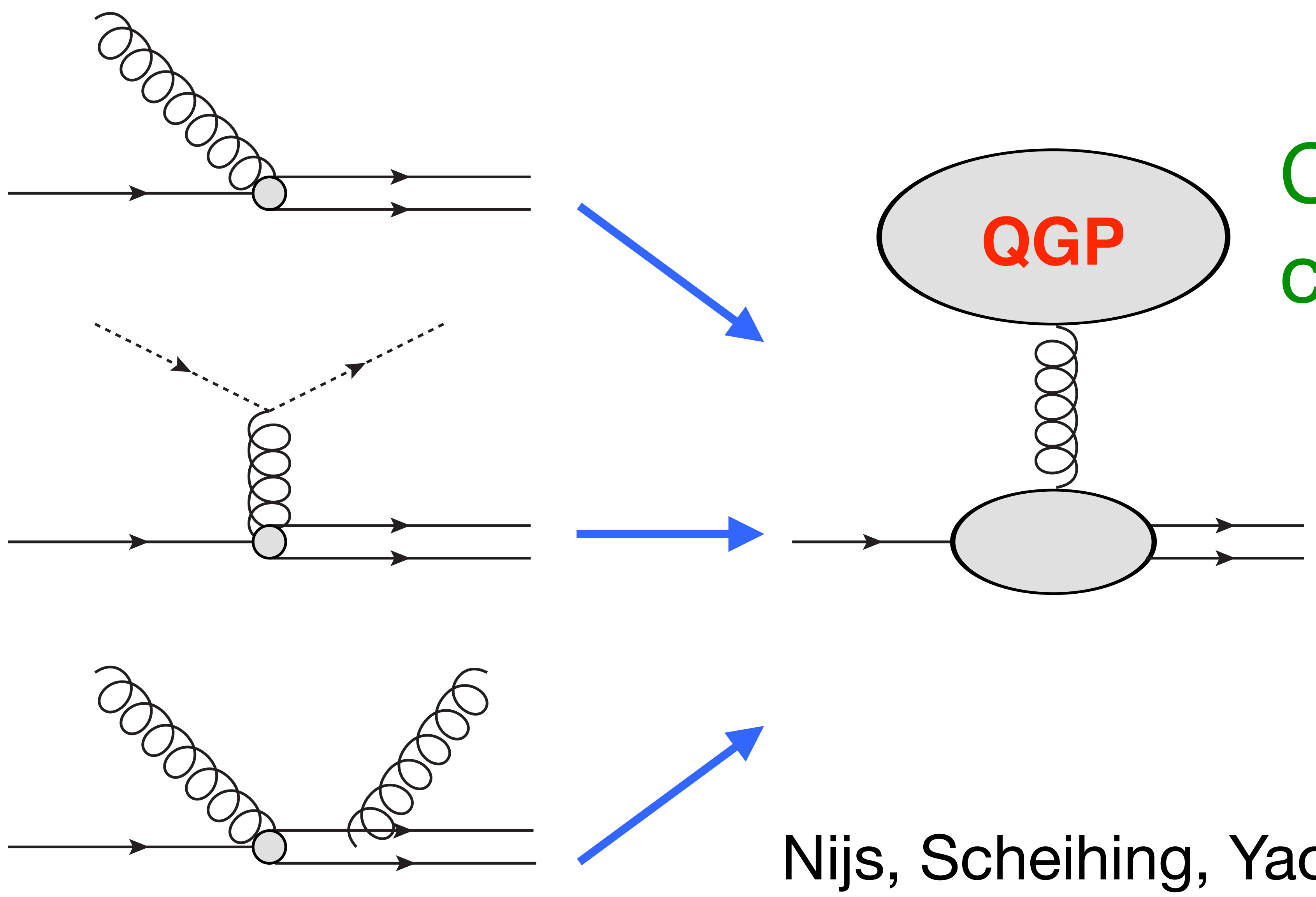
$$\frac{\mathrm{d}n_{\mathcal{B}}(t,\mathbf{x})}{\mathrm{d}t} = -\Gamma\, n_{\mathcal{B}}(t,\mathbf{x}) + F(t,\mathbf{x})$$

Yao, Mehen, 1811.07027, 2009.02408

$$\Gamma = \int \frac{\mathrm{d}^3 p_{\rm rel}}{(2\pi)^3} |\langle \psi_b | \boldsymbol{r} | \Psi_{\boldsymbol{p}_{\rm rel}} \rangle|^2 [g_{\rm adj}^{++}]^{>}(-\Delta E)$$

$$F = \int \frac{\mathrm{d}^3 p_{\rm cm}}{(2\pi)^3} \frac{\mathrm{d}^3 p_{\rm rel}}{(2\pi)^3} f_{Q\bar{Q}} |\langle \psi_b | \boldsymbol{r} | \Psi_{\boldsymbol{p}_{\rm rel}} \rangle|^2 [g_{\rm adj}^{--}]^{>}(\Delta E)$$

$$f_{Q\bar{Q}} \neq f_Q f_{\bar{Q}}$$



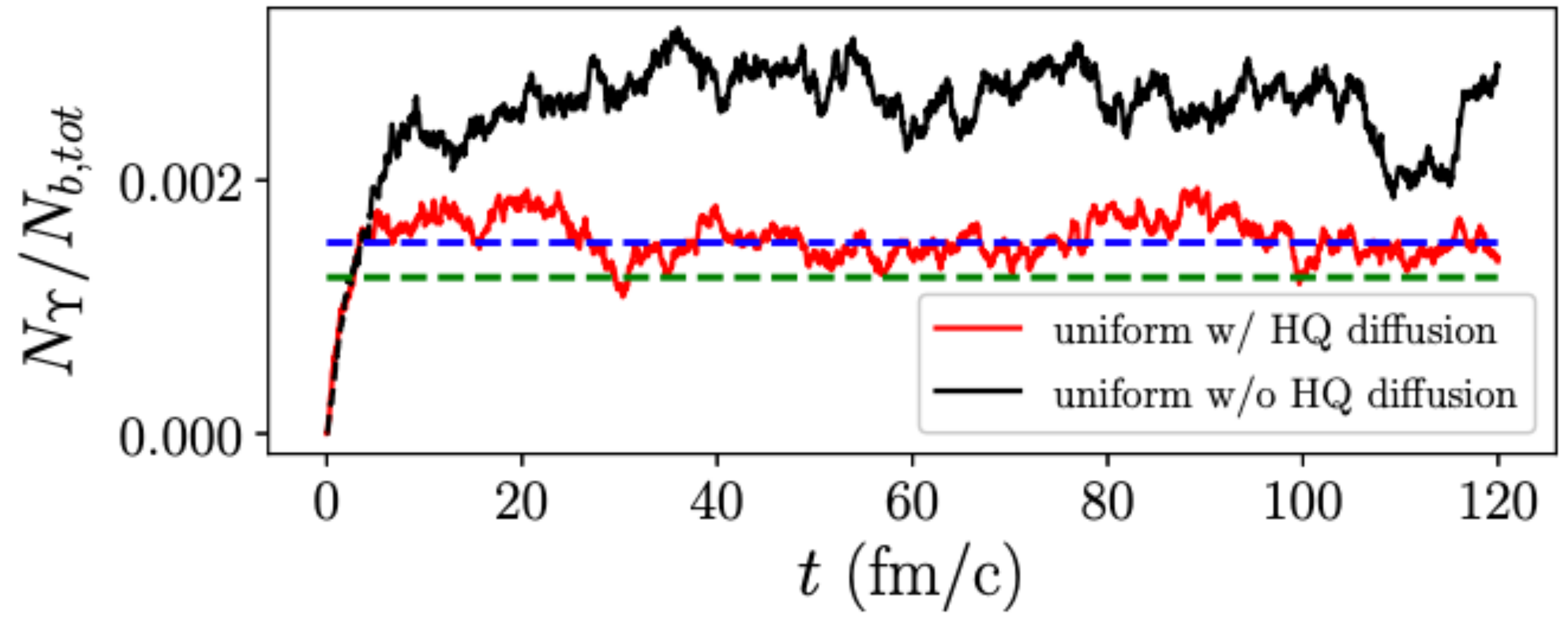
Chromoelectric correlator

Quarkonium transition

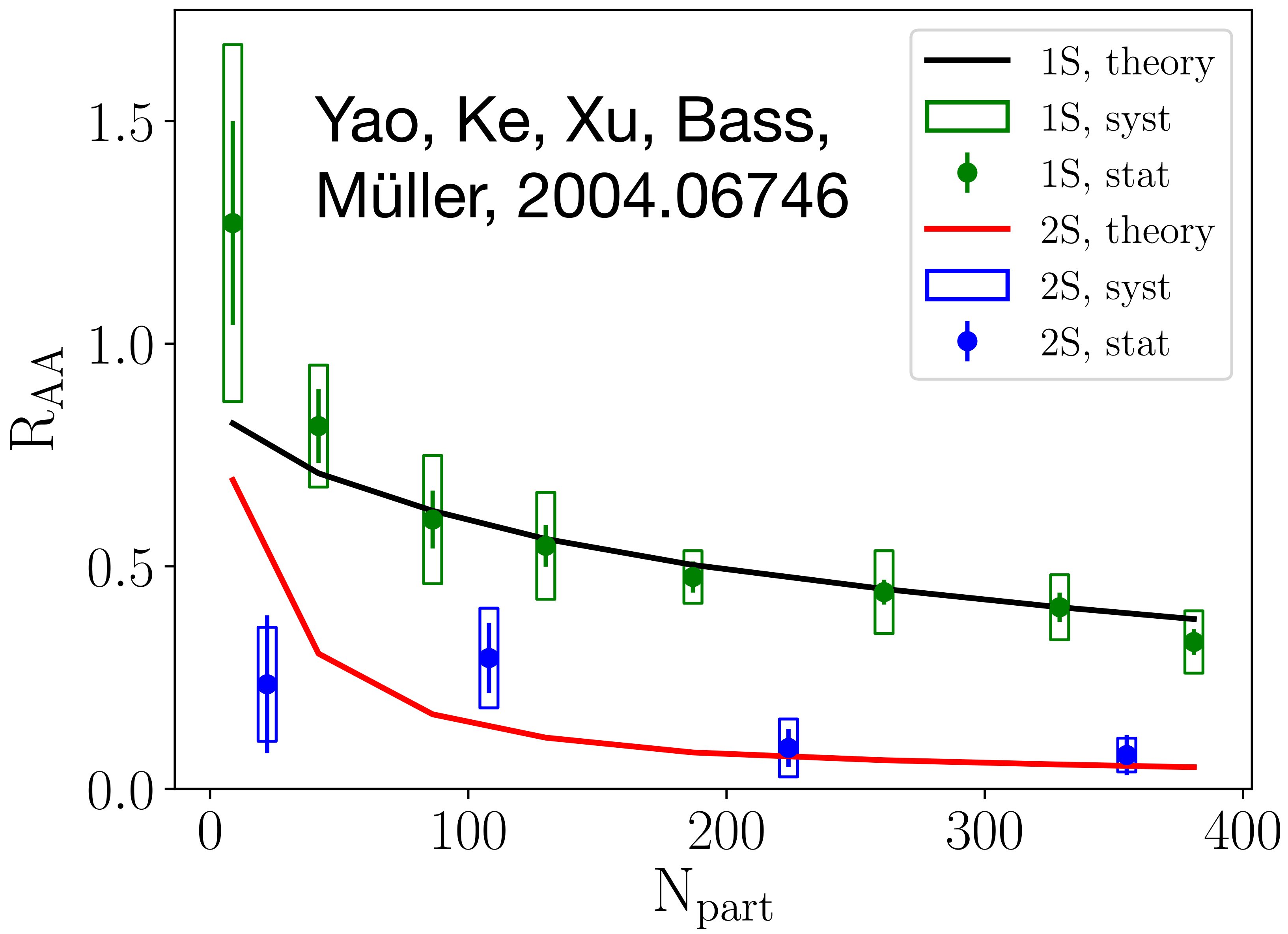
Nijs, Scheihing, Yao, 2310.09325

- **Duke-MIT: coupled Boltzmann equations for open heavy quarks and quarkonia**

Open heavy quark dynamics crucial for quarkonium equilibrium



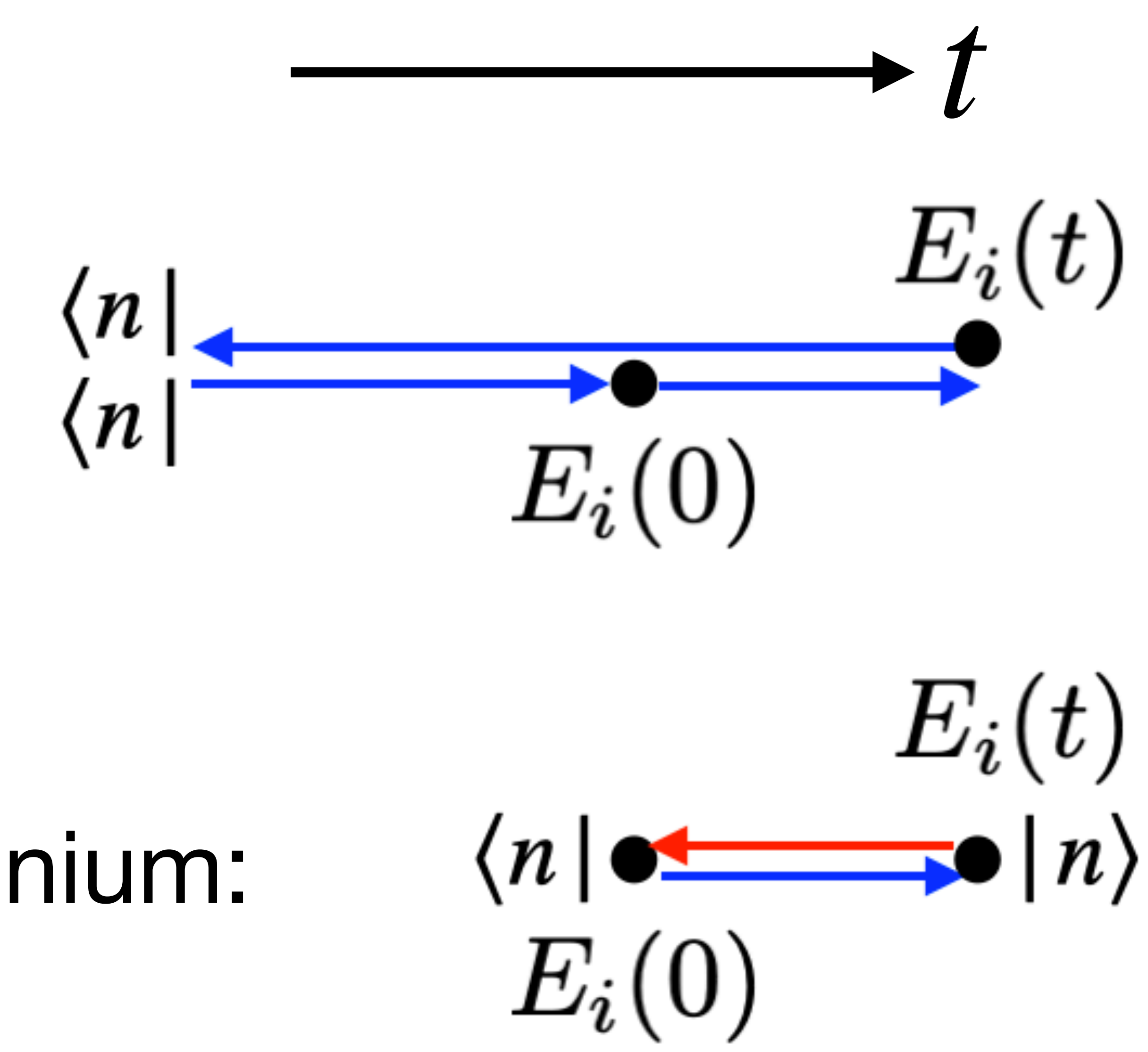
Yao, Müller, 1709.03529

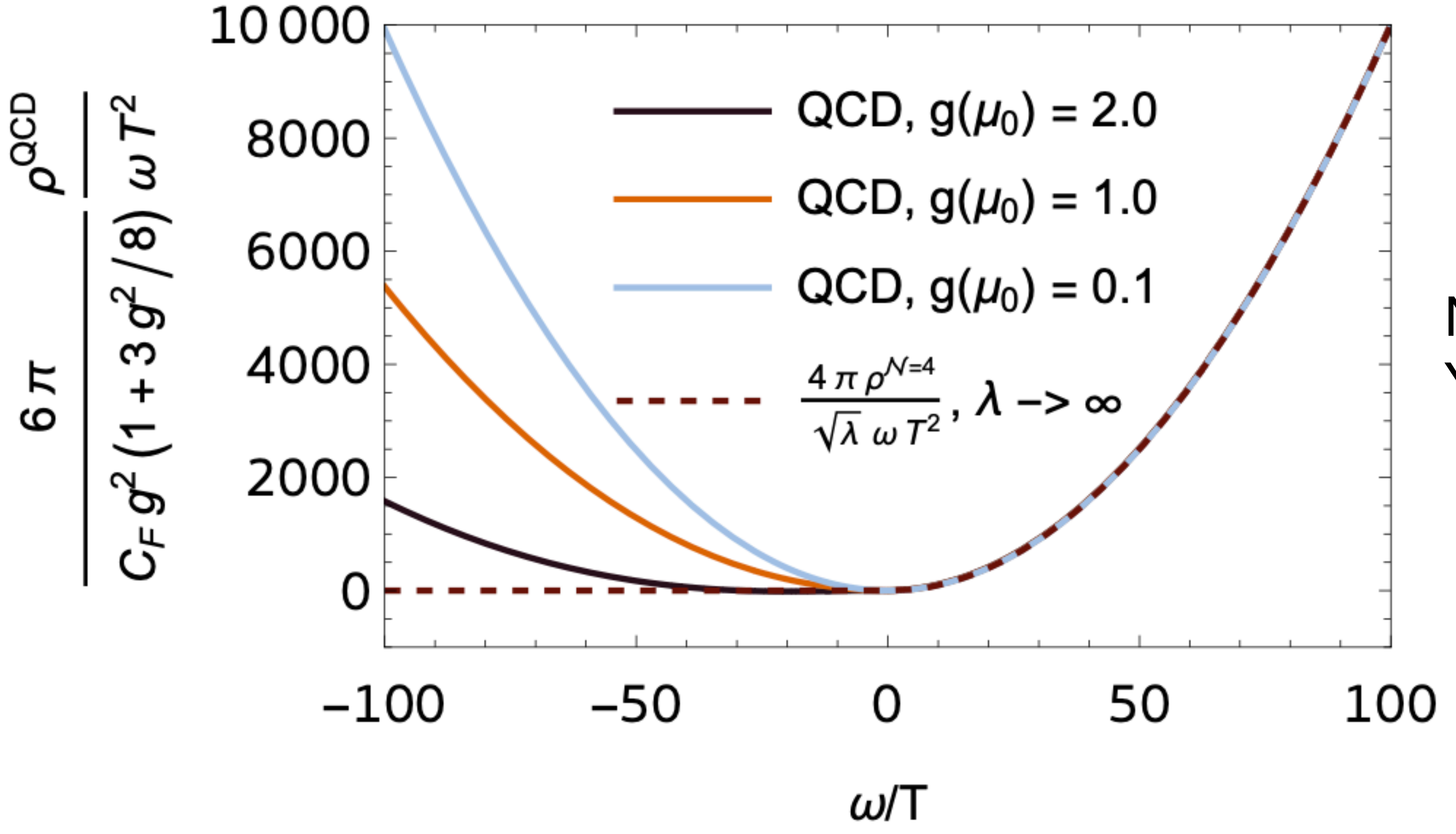


Chromoelectric Correlators for Heavy Quark and Quarkonium

- **HQ diffusion** $$\kappa_{\rm fund} = \frac{g^2}{3N_c} {\rm Re} \int {\rm d}t \langle {\rm Tr_c}[U(-\infty,t)E_i(t)U(t,0)E_i(0)U(0,-\infty)] \rangle_{T,Q}$$
- **Quarkonium** $$\kappa_{\rm adj} = [g^{++}_{\rm adj}]^{>}(\omega=0) = \frac{g^2 T_F}{3N_c} \int {\rm d}t \langle E_i^a(t) \mathcal{W}^{ab}(t,0) E_i^b(0) \rangle_T$$
- **Operator orderings are different;** evident difference seen in lattice calculation

Mayer-Steudte, PoS LATTICE2024 (2025) 205





Nijs, Scheihing, Yao, 2310.09325

Eller, Ghiglieri, Moore, 1903.08064; Binder, Mukaida, Scheihing, Yao, 2107.03945; Scheihing, Yao, 2205.04477, 2306.13127

Need new dynamical description to access strong coupling effect

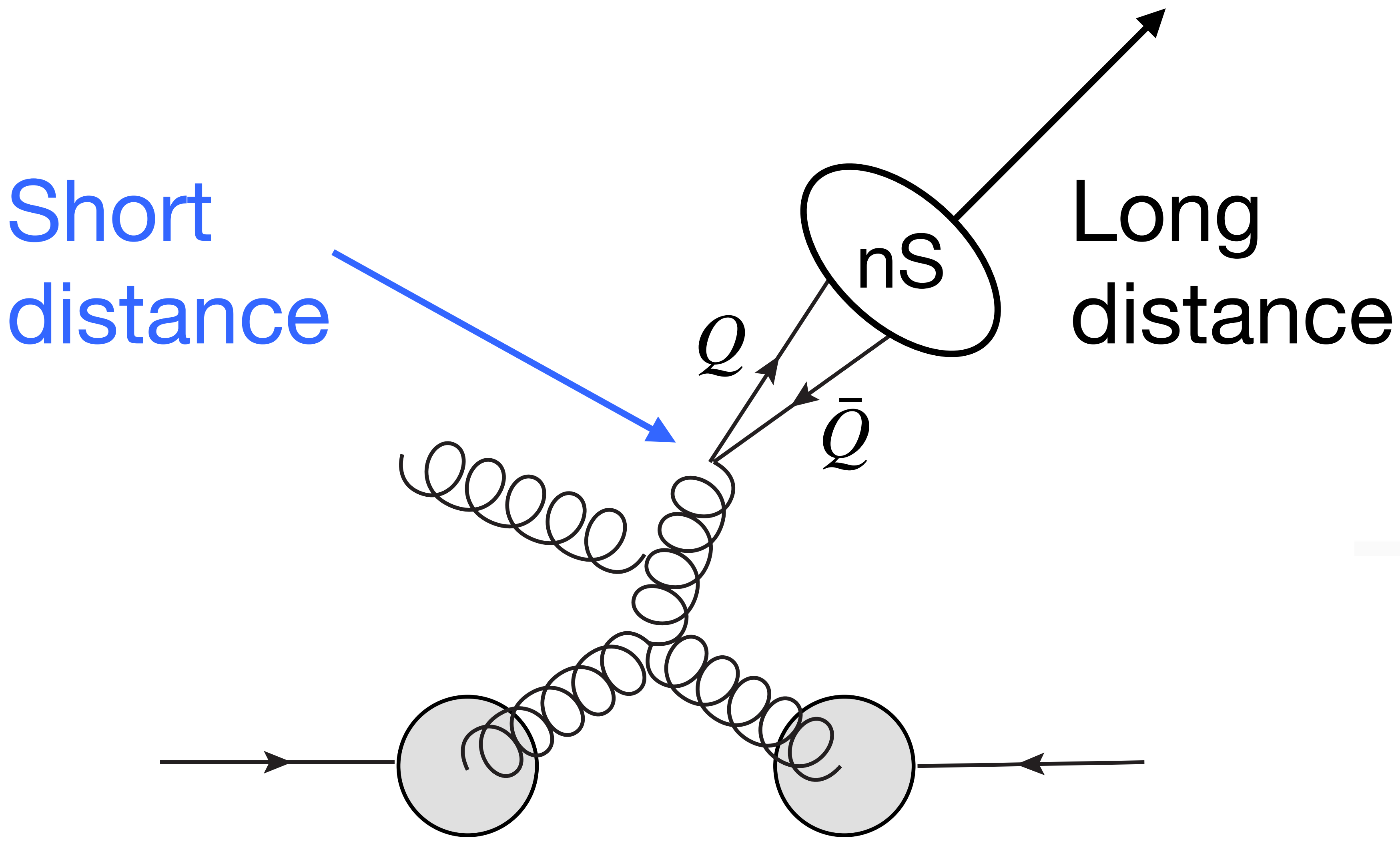
What Properties of QGP are Probed?

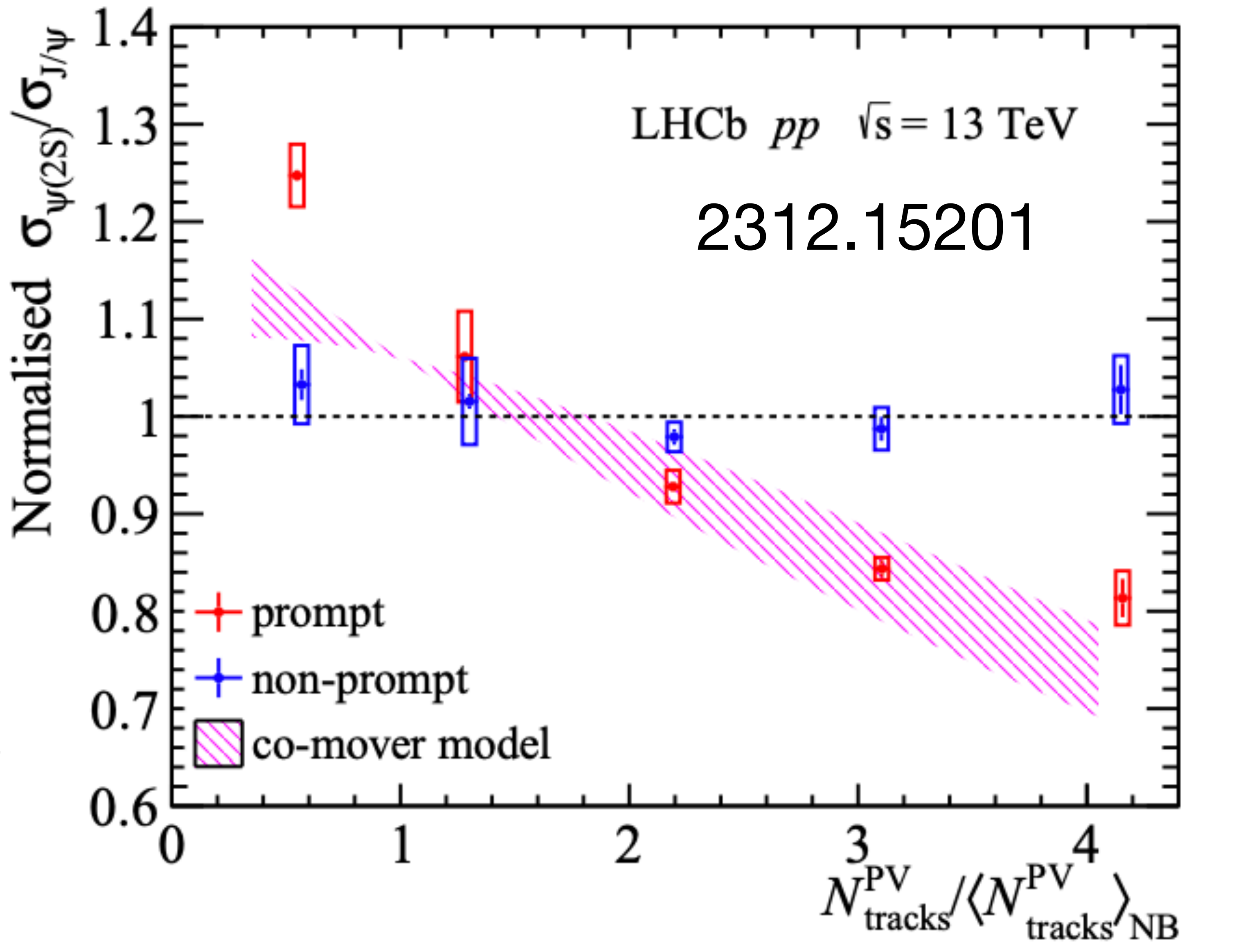
- Light hadrons v_n and $\langle p_T \rangle$ → Shear and bulk viscosities η, ζ
- High-p_T particles suppression → Jet quenching parameter $\hat{q}$
- Open heavy flavors R_{AA} and v_n → Heavy quark drag and diffusion coefficients η_D, κ_{fund}
- **Low-p_T quarkonium R_{AA} and v_n** → **Quarkonium transport parameters κ_{adj} and γ_{adj}, and more generally, chromoelectric correlator**

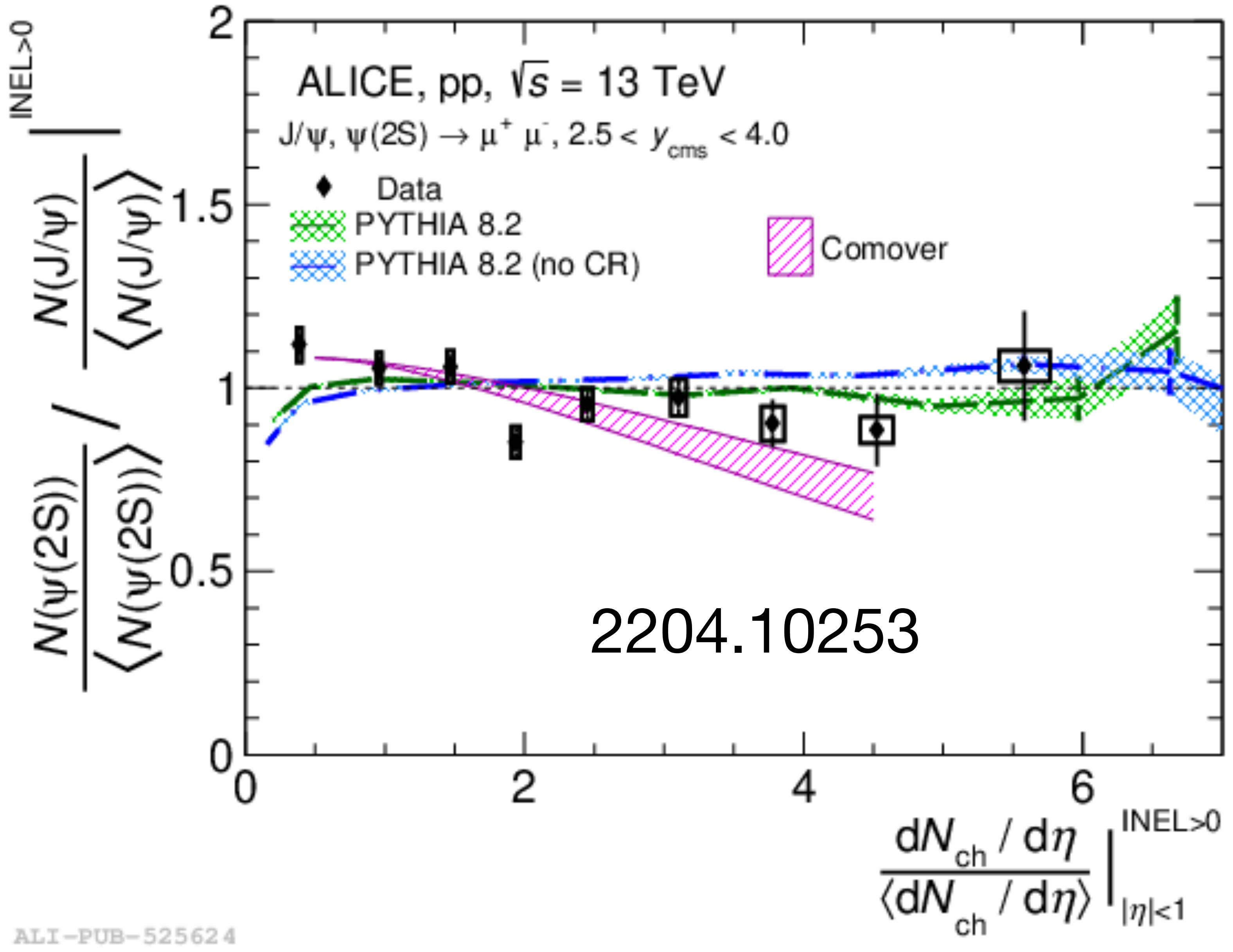
$$\langle E_i^a \mathcal{W}^{ab} E_i^b \rangle_T(\omega)$$

Quarkonium Production from AA to pp, pA

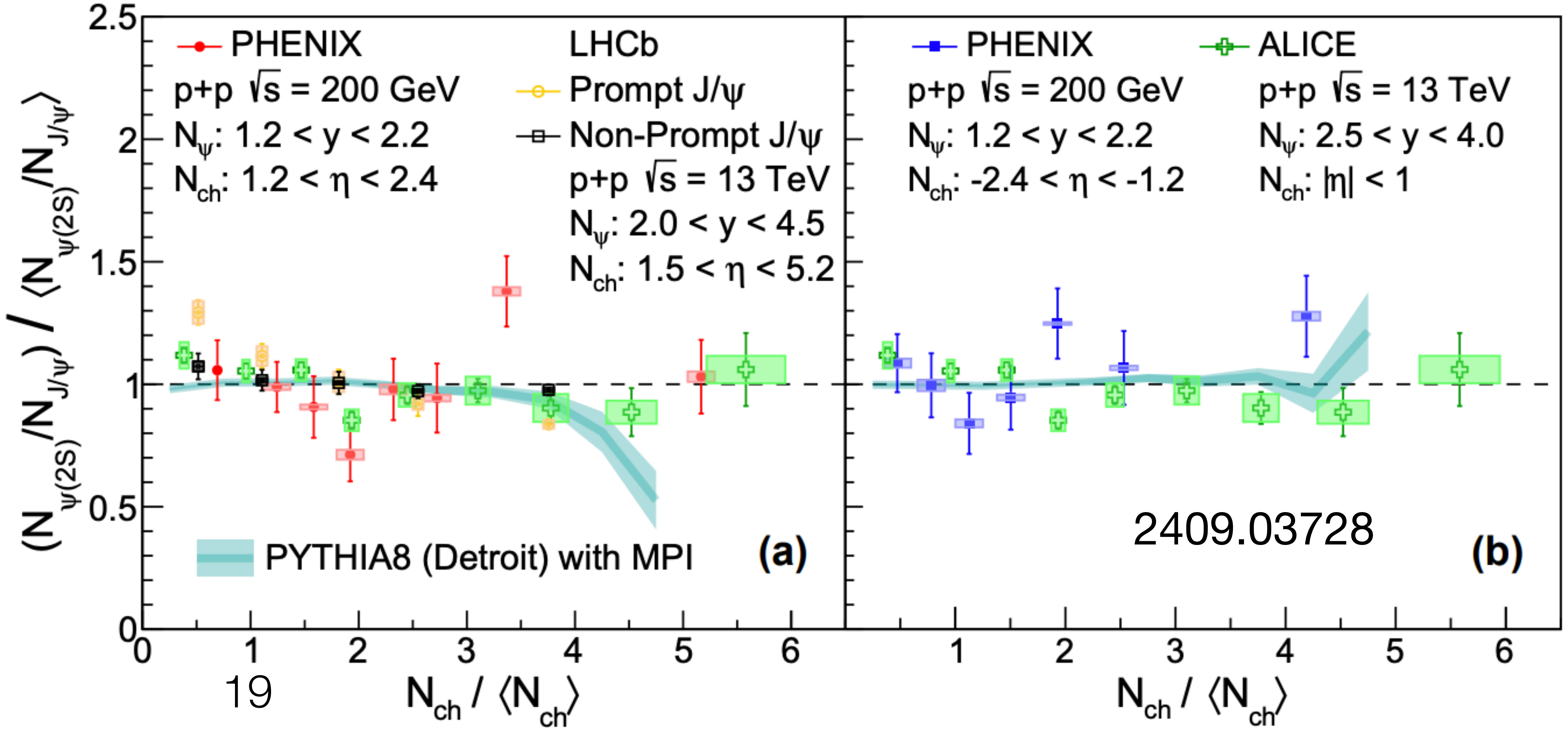
- $\frac{\psi(2S)}{J/\psi}$ v.s. event activity in pp





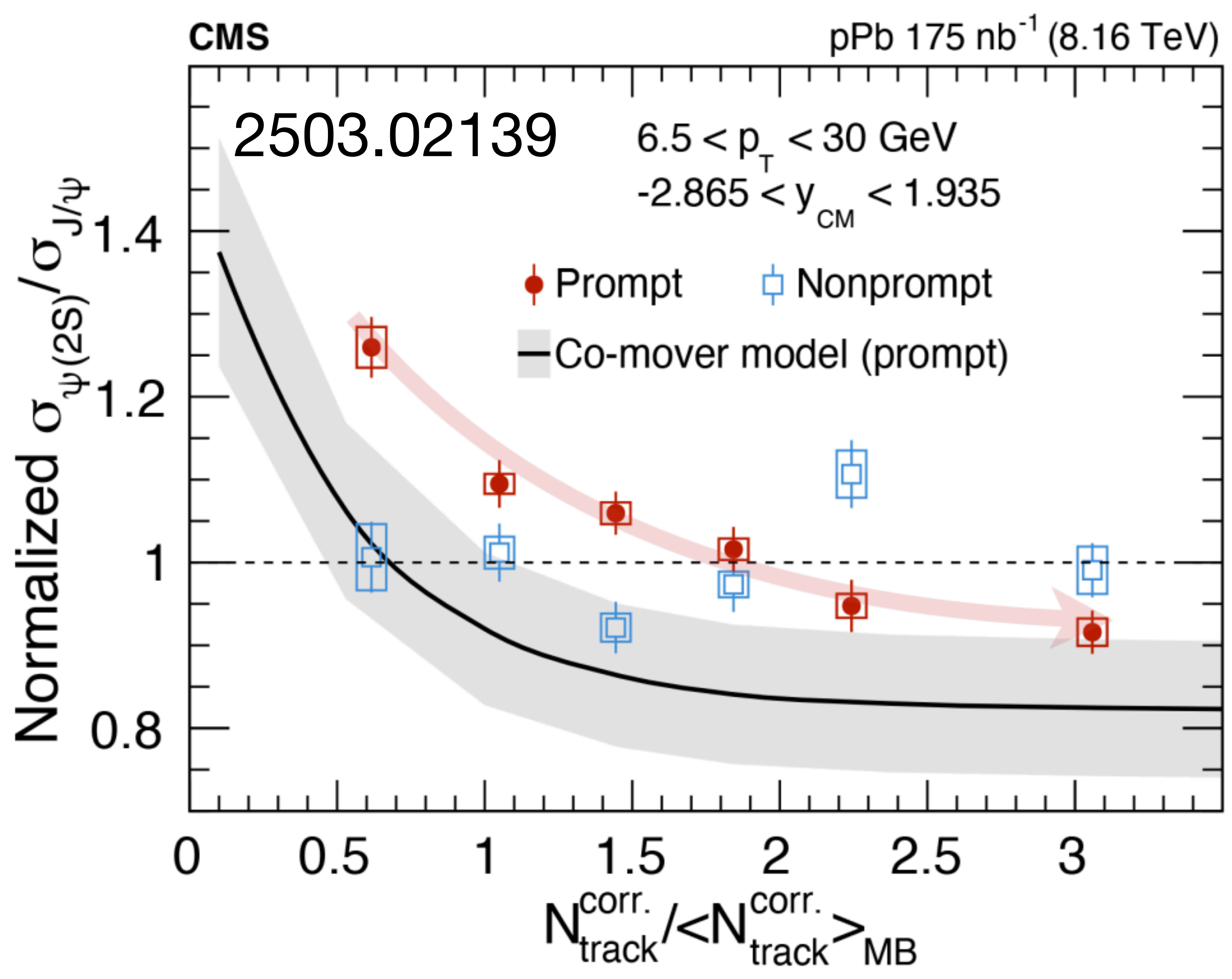


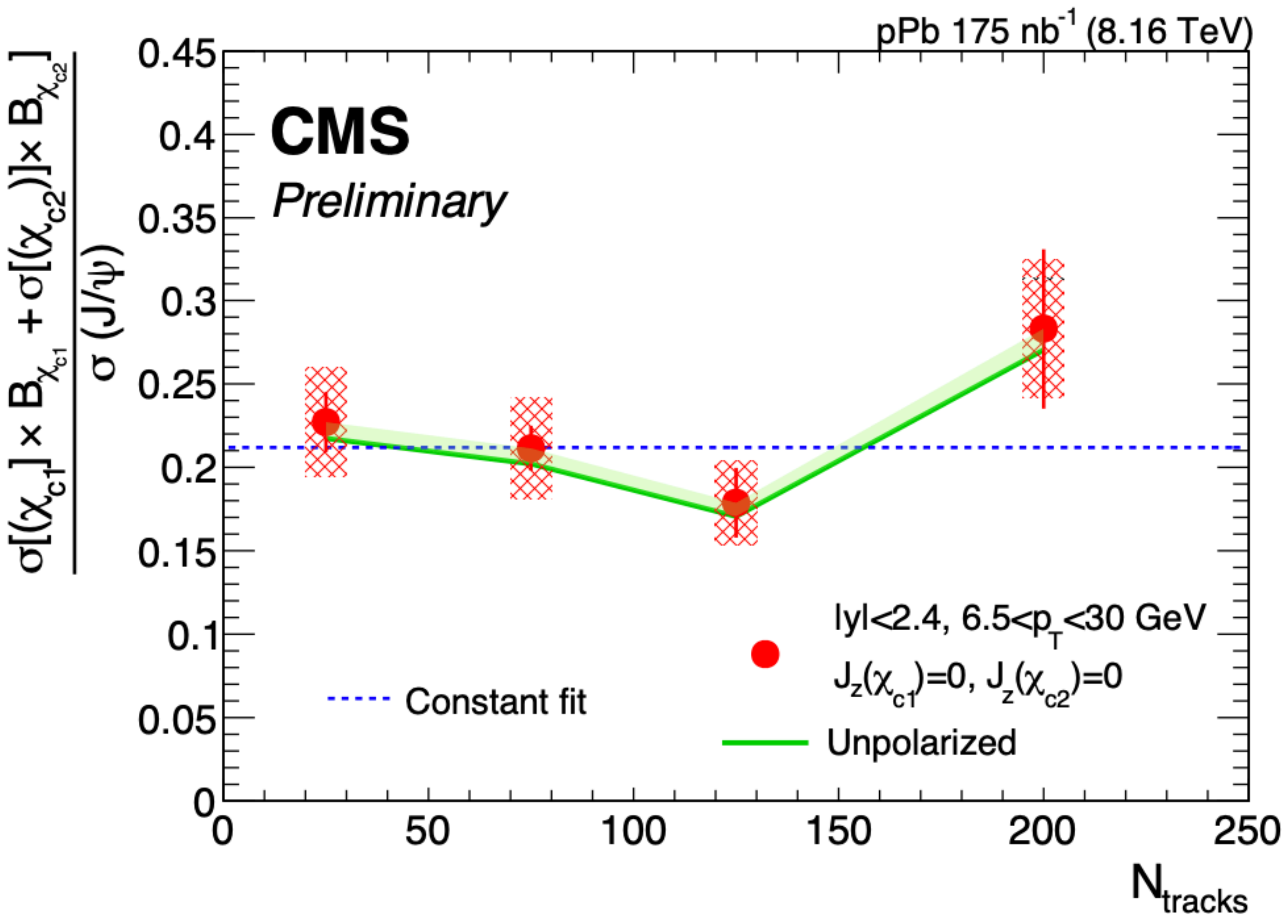
Universality of long-distance may break down in pp?

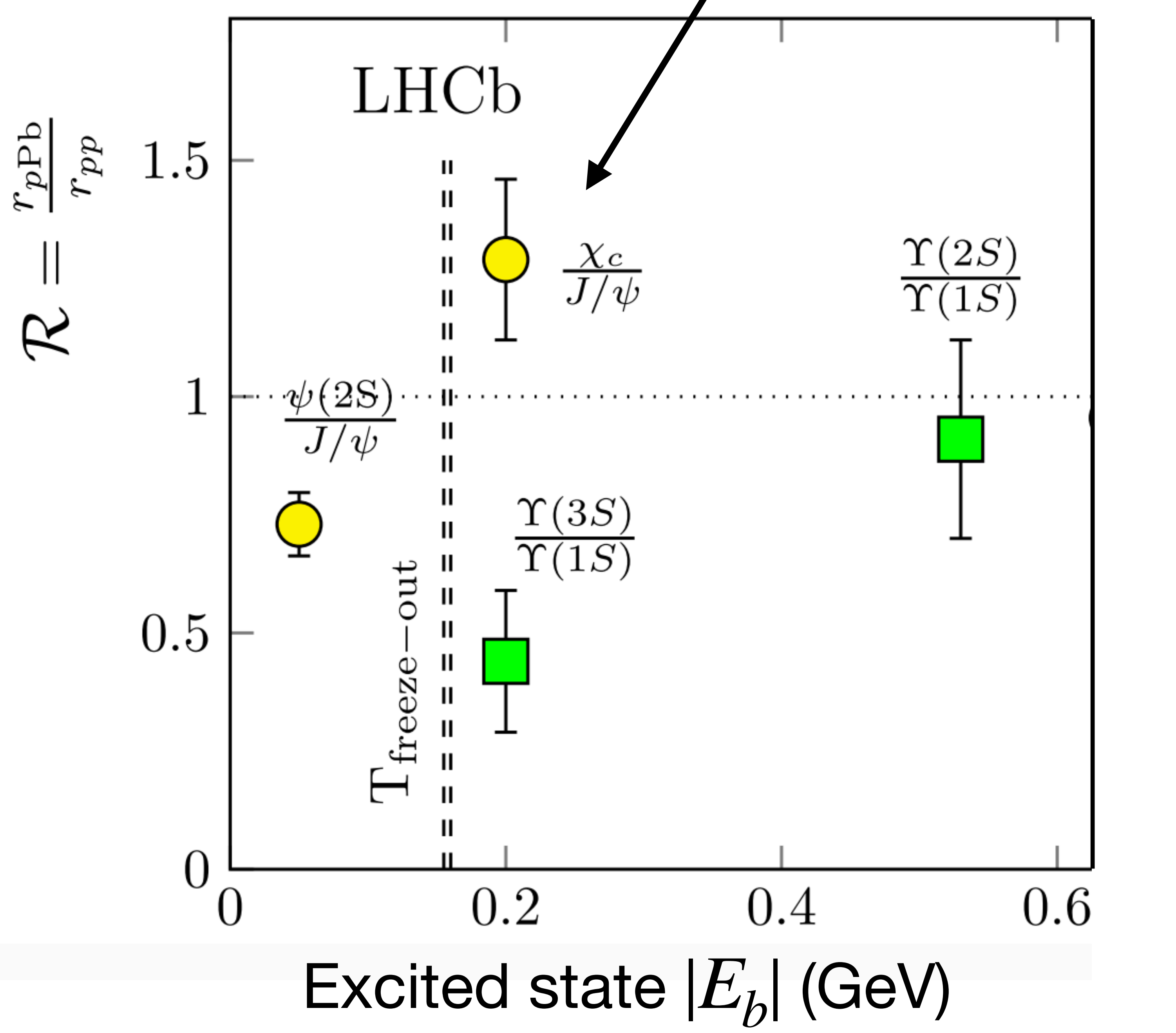


Quarkonium Production from AA to pp, pA

- **Final-state interaction effect in quarkonium production in pA collisions**







- **Consistent theoretical description needed**

Nature of “environment”, how its property depends on multiplicity

How formation of quarkonium is affected by “environment”, dependence on size & binding energy

Interaction time with “environment”, p_T/m dependence

Summary and Future Prospects

- Quarkonium probes deconfinement in AA
- **Uniquely** probes **chromoelectric correlator** of QGP in the dipole limit
- From AA to pp, pA
- Details of model descriptions: **Andronic, et al, 2402.04366**, summarizing efforts of EMMI Rapid Reaction Task Force started at GSI 2019
- Other interesting directions: (1) quarkonium spin alignment/polarization

  (2) novel techniques to solve Lindblad equations

  Neutral network: Lin, Luo, Yao, Shanahan, 2402.06607

  Hydro formulation: Rais, van Hees, Greiner, 2502.21047

  (3) quarkonium production at intermediate/high p_T, within jet