Theory of Open Heavy-Flavor in Heavy-ion Collisions

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Outline

- Introduction
- Modeling of heavy flavor evolution
  - Transport models
  - Hadronization
- Transport Coefficients
  - Quantum many-body theory (T-matrix)
- Results on HF Observables from HEFTY
  - Nuclear modification factor: R_{AA}
  - Elliptic flow: v_2
  - Hadro-chemistry (Λ_c^+/D^0)
- Summary and Outlook

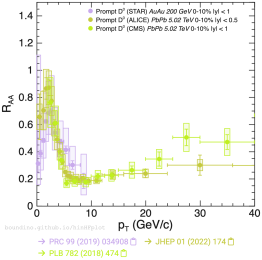
Introduction

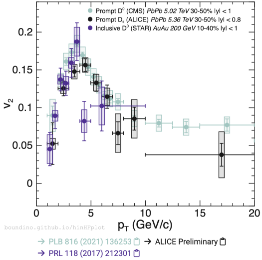
Open heavy flavor in a nutshell

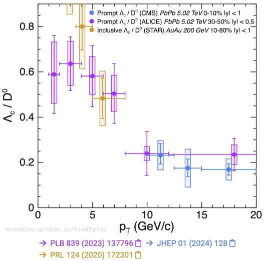
- Properties:
  - large mass
    - $m_Q \gg \Lambda_{QCD}$:produced in hard processes; pQCD calculable.
    - $m_Q \gg T_{QGP}$: production in QGP negligible
    - $\tau_{prod.} \sim \frac{1}{2m_Q}$: witness the full QGP evolution
  - strong interactions with the QGP conserve flavor

- Heavy quarks are effective probes of:
  - presence of deconfined color charge and hadronization schemes
  - mechanisms of quark-medium interaction
  - strength of the collective expansion of the system

HF observables







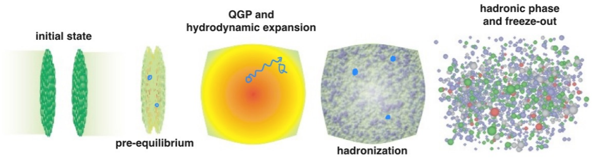
- Study the energy loss process of heavy quarks
- Characterizes medium modifications at high-pT

- Study the thermalization process of heavy quarks
- Constrain the color potential of HQ-medium interaction

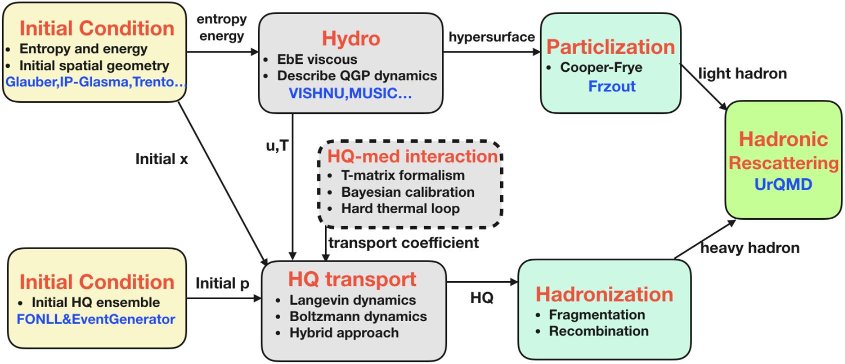
- Study the hadronization process of heavy quarks
- Constrain the in-medium hadron wave-function

Modeling of heavy flavor evolution

Modeling of heavy-quark dynamics in heavy-ion collision



Modular description of HQ evolution in heavy-ion collisions :

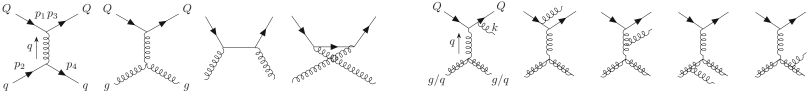


Heavy quark transport

Real-time tool for non-equilibrium evolution:

Boltzmann equation: $\frac{df_Q(t,x,p)}{dt} = \mathcal{C}_{2\leftrightarrow 2}[f_Q, f_{q,g}] + \mathcal{C}_{2\leftrightarrow 3}[f_Q, f_{q,g}]$

- $\mathcal{C}_{2\leftrightarrow 2} + \mathcal{C}_{2\leftrightarrow 3}$:



  - **perturbative** scattering; **high** momentum transfer
  - treat medium as **thermal partons**
  - $2 \rightarrow 2$ **elastic** scattering + $2 \leftrightarrow 3$ **inelastic** scattering

Heavy quark transport

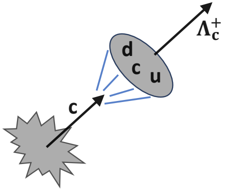
Soft-momentum transfer:

Fokker-Planck equation: $\frac{df_Q}{dt} = \frac{\partial}{\partial p_i}(A_i(\vec{p}) - \frac{1}{2}\frac{\partial}{\partial p_i}B_{ij})f_Q$

$$A_i(\vec{p}) = \int d^3\vec{k}w(\vec{p},\vec{k})k_i, \quad B_{ij}(\vec{p}) = \frac{1}{2}\int d^3\vec{k}w(\vec{p},\vec{k})k_ik_j$$

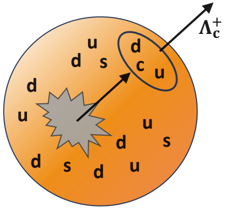
- Stochastic realization:
  - Fokker-Planck $\Leftrightarrow$ **Langevin** dynamics:
    $\Delta\vec{x} = \frac{\vec{p}}{E}\Delta t, \quad \Delta\vec{p} = -\eta_D\vec{p}\Delta t + \vec{\xi}(t)\Delta t$
  - Correlation of thermal **random force**
    $\left\langle \xi_i(t)\xi_j'(t')\right\rangle = \left(\kappa_L\frac{p_ip_j}{p^2} + \kappa_T(\delta_{ij} - \frac{p_ip_j}{p^2})\right)\delta(t-t')$
  - Einstein relation: $\eta_D = A = \frac{\kappa_L}{2ET} - \frac{\partial\kappa_L}{\partial p^2} - \frac{\kappa_L-\kappa_T}{p^2}$
    where $A_i = Ap_i \quad B_{ij} = \kappa_L\frac{p_ip_j}{p^2} + \kappa_T(\delta_{ij} - \frac{p_ip_j}{p^2})$
  - Does not require quasi-particle medium

Heavy-quark hadronization



Fragmentation:

- At **high** p_T, parton spectrum is pQCD power law;
- HQs fragment into HF hadrons.
  - HQ Effective Theory FF
  - Peterson FF



Recombination:

- At **low** p_T, parton spectrum is thermal;
- HQs recombine with medium quarks into hadrons locally.
  - Resonance Recombination Model (RRM)
  - improved Sudden Recombination Model (iSRM)

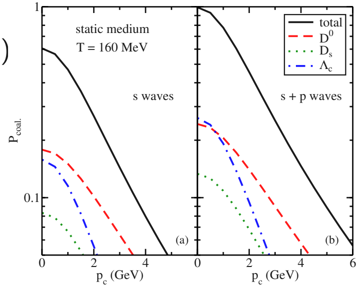
HQET FF:*Braaten et al PRD*51(1995)
Peterson FF:*Peterson et al PRD*27(1983)
iSRM:*Cao et al PLB*807(2020)
RRM:*Ravagli+Rapp PLB*655(2007)

improved Sudden Recombination Model

iSRM:

- $\frac{dN_M}{d^3\vec{P}_M} = g_M \int d^3\vec{p}_q d^3\vec{p}_Q f_q(\vec{p}_q) f_Q(\vec{p}_Q) W_M(\vec{k}) \delta^3(\vec{p}_q + \vec{p}_Q - \vec{P}_M)$
  - $f_q(\vec{p}_q) = g_q \frac{V}{(2\pi)^3(e^{E_q/T}+1)}$
  - Meson Wigner function:
    s-wave: $W_M^s = g_h \frac{(2\sqrt{\pi}\sigma)^3}{V} e^{-\sigma^2 k^2}$ p-wave: $W_M^p = g_h \frac{(2\sqrt{\pi}\sigma)^3}{V} \frac{2}{3}\sigma^2 k^2 e^{-\sigma^2 k^2}$
    σ: radius parameter for each hadron; $\vec{k} \equiv \vec{p}_q - \vec{p}_Q$

- Improvement:
  - Energy conservation:
    E(combined quarks) = E(off-shell hadron)
    $\xrightarrow{\text{decay}}$ on-shell hadron + pion
  - Coalescence probability:
    Include all s,p-wave hadron states
    allowed by spin-orbit coupling.



S. Cao, et al; arXiv:1911.00456 [nucl-th]

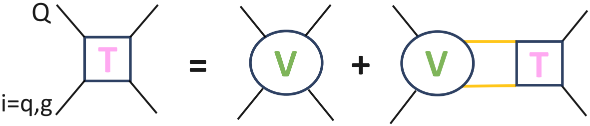
Resonance Recombination Model

RRM:

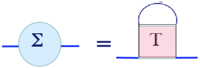
- Transport approach for hadronization via a resonant interaction:
  $\frac{d}{dt} f_M(R, P_M) = -\frac{\Gamma}{\gamma_p} f_M(t, R, P_M) + g(R, P_M)$
  - Gain term: $g(P_M) \sim \int d^3p_1 d^3p_2 f_q(p_1) f_Q(p_2) \sigma(s) v_{rel} \delta^3(P_M - p_1 - p_2)$
  - Resonant interaction:
    - Breit-Wigner: $\sigma(s) = g_\sigma \frac{4\pi}{k_{cm}^2} \frac{(\Gamma m)^2}{(s-m^2)^2+(\Gamma m)^2}$
    - Connect to T-matrix interactions in QGP near T_c: $\sigma \sim |T_{qQ}|^2$
- In equilibrium: $f_M(R, P_M) = \frac{\gamma_M}{\Gamma} g(R, P_M)$
- $\frac{dN_M}{d^3\vec{P}_M} = \int d^3R \; f_M(R, P_M)$
- Explicit energy conservation

Transport coefficients

HQ-medium interaction: T-matrix formalism



- Strong coupling: $T_{Qi} = V_{Qi} + \int V_{Qi} D_Q D_i T_{Qi}$
- In-medium propagator $D_i = \frac{1}{\omega - \omega_k - \Sigma_i(\omega, k)}$

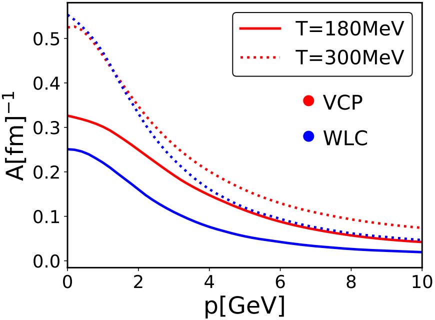


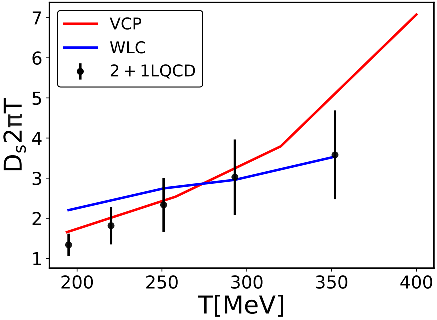
  - Self-energy $\Sigma_i(\omega, k)$ self-consistently computed from T_{Qi}
- Key input: in-medium potential $V_{Qi} = V_{Qi}^{Coulomb} + V_{Qi}^{Conf}$
- Two scenarios (embedded in lQCD EoS)
  - 40% Vector Confinement Potential (vacuum spin splittings) - VCP
  - Wilson Line Correlators from lattice QCD - WLC

VCP: *Tang+Rapp PRC*108(2023)
WLC: *Tang et al EPJA*60(2024)
T-matrix: *Riek+Rapp PRC*82(2010); *Liu+Rapp PRC*97(2018)

Diffusion coefficient from T-matrix

- Drag coefficient: $A \sim \int d^3\vec{k}\{\frac{\vec{p}\cdot(\vec{p}-\vec{k})}{p^2}\}|T_{Qi}(\vec{p},\vec{k})|^2$
- Spatial diffusion coefficient: $D_s = \frac{T}{MA(p\to 0)}$
- VCP vs. WLC



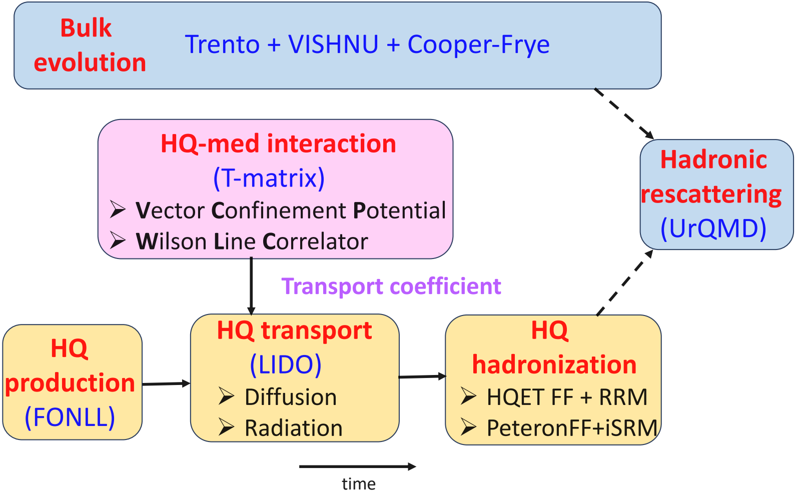


VCP: *Tang+Rapp PRC*108(2023)
WLC: *Tang et al EPJA*60(2024)
2+1 LQCD: *Altenkort et al PRL*130(2023)

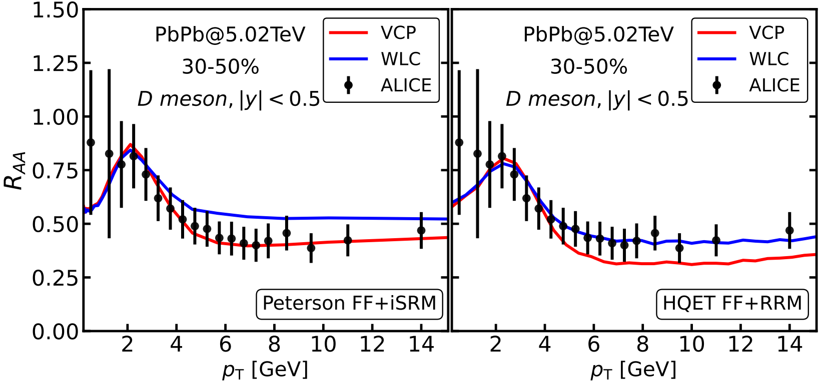
Results From HEFTY collaboration

Framework of open HF dynamics for HEFTY

Goal: Comprehensive framework for the transport and hadronization of heavy quarks with lQCD-derived transport coefficients



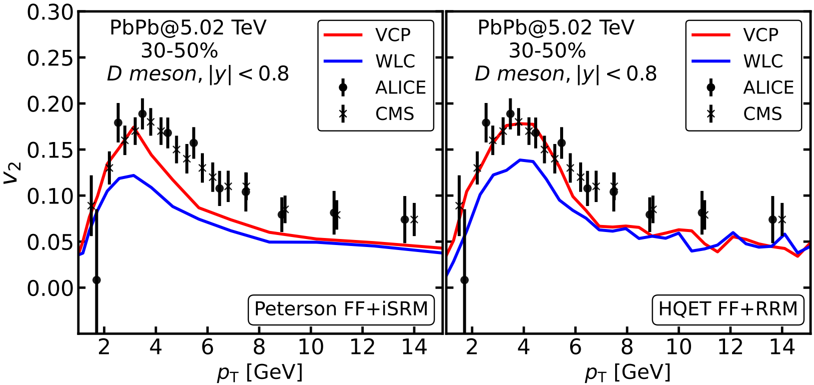
Nuclear modification factor: R_{AA}



- Sensitive to lQCD constraints via microscopic interactions
- Fair agreement between two hadronization models

ALICE: *JHEP*01(2022)

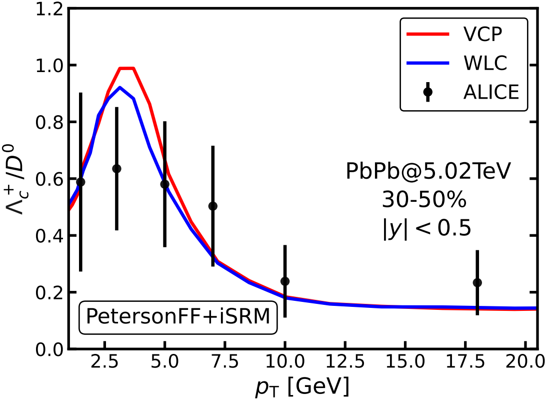
Elliptic flow: v_2



- Larger sensitivity to in-medium QCD force

ALICE: *PLB*813(2021)
CMS: *PLB*816(2021)

Baryon-to-meson ratio: Λ_c^+/D^0



- Systematic investigation of hadro-chemistry underway

ALICE: *PLB*839(2023)

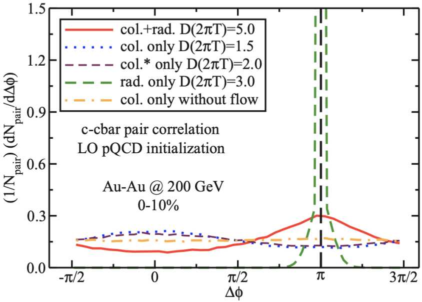
Summary and outlook

- Summary
  - comprehensive state-of-the-art framework for HQ evolution in QGP:
    - HQ transport (Lido): Diffusion + Radiation
    - HQ-medium interaction (T-matrix) based on lQCD: VCP & WLC
    - HQ hadronization: iSRM vs. RRM
  - demonstrated sensitivity to different microscopic descriptions
  - preliminary data-theory comparison promising
- Outlook
  - combine first-principle results for transport coefficients with rigorous statistical methods for data analysis
  - additional observables: HF chemistry, pair angular correlation, bottom

Thank you!

Backup

HF pair angular correlation



- Medium-induced gluon radiation is mostly dominated by the k_T kicks exerted on the radiated gluons; its effect on the propagation directions of the heavy quarks is very mild.
- Diffusion is very effective in changing the heavy quark propagation directions, and the final angular distribution is widely spread out

S. Cao, et al; arXiv:1505.01869[nucl-*th*]

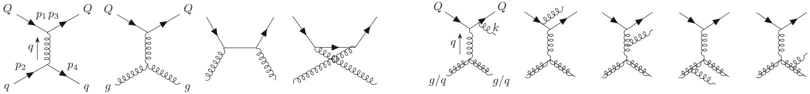
Heavy-quark transport

LIDO: Combine the linearized Boltzmann and Langevin approach

$$\frac{df_Q}{dt} = \mathcal{D}[f_Q] + \mathcal{C}^{1\leftrightarrow 2}[f_Q] + \mathcal{C}^{2\leftrightarrow 2}[f_Q] + \mathcal{C}^{2\leftrightarrow 3}[f_Q]$$

(separated by a momentum cutoff $q_{cut} \sim 4$GeV)

- $\mathcal{C}^{2\leftrightarrow 2}[f_Q] + \mathcal{C}^{2\leftrightarrow 3}[f_Q]$: Boltzmann for high-pt scattering + radiation



  - perturbative scattering; high momentum transfer
  - treat medium as thermal partons
  - $2 \rightarrow 2$ elastic scattering + $2 \leftrightarrow 3$ inelastic scattering

Heavy-quark transport

LIDO: Combine the linearized Boltzmann and Langevin approach

$$\frac{df_Q}{dt} = \mathcal{D}[f_Q] + \mathcal{C}^{1\leftrightarrow 2}[f_Q] + \mathcal{C}^{2\leftrightarrow 2}[f_Q] + \mathcal{C}^{2\leftrightarrow 3}[f_Q]$$

(separated by a momentum cutoff $q_{cut} \sim 4$GeV)

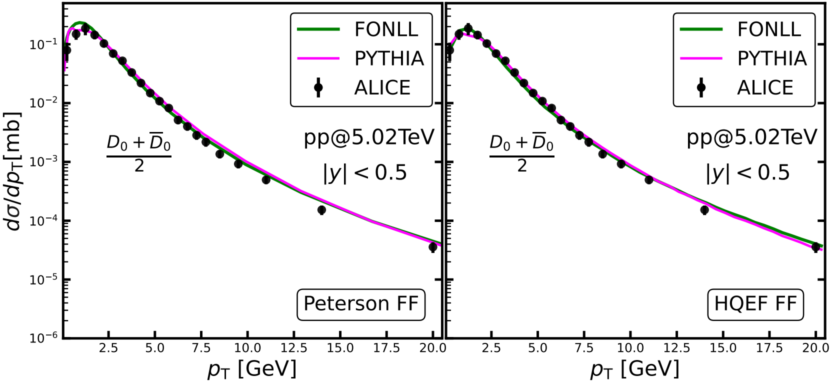
- $\mathcal{D}[f_Q]$: Fokker-Planck for diffusion with low momentum transfer
  - Fokker-Planck $\Leftrightarrow$ Langevin dynamics:
    $\Delta\vec{p} = -\eta_D \vec{p}\Delta t + \vec{\xi}(t)\Delta t \quad \left\langle \xi_i(t)\xi_j'(t')\right\rangle = \kappa\delta_{ij}\delta(t-t') \quad (\kappa = 2TE\eta_D)$
- $\mathcal{C}^{1\leftrightarrow 2}[f_Q]$: Diffusion-induced radiation
  - Monte Carlo implementation of medium induced splitting $P_{1\to 2}$
  - Modeling of LPM effect compatible with opacity expansion

improved Sudden Recombination Model(iSRM)

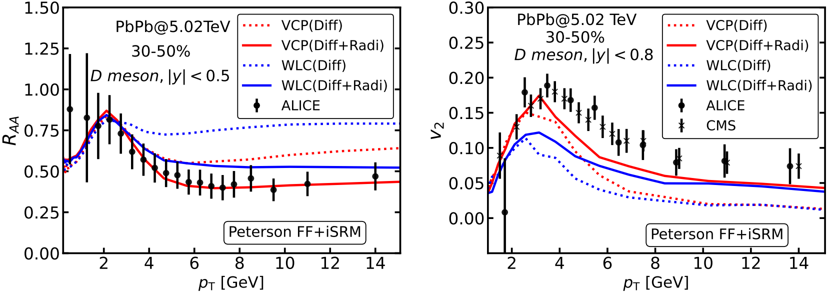
Sudden Recombination Model

- Coalescence probability of $q + Q \to meson$ is expressed as the overlap between the wavefunctions of free quarks and bound state.
  $P_{coal} \propto |\int d^3\vec{r}_q \int d^3\vec{r}_Q \langle \vec{P}_M; \vec{R}, \Delta\vec{R} | \vec{p}_q, \vec{p}_Q; \vec{r}_q, \vec{r}_Q \rangle|^2$
- Meson Wigner function: $W_M(\vec{r}, \vec{k}) = \int d^3 s \phi_M^*(\vec{r} + \frac{\vec{s}}{2}) \phi_M(\vec{r} - \frac{\vec{s}}{2}) e^{-i\vec{k}\cdot\vec{r}}$
  - s-wave: $W_M^s = g_h \frac{(2\sqrt{\pi}\sigma)^3}{V} e^{-\sigma^2 k^2}$
  - p-wave: $W_M^p = g_h \frac{(2\sqrt{\pi}\sigma)^3}{V} \frac{2}{3}\sigma^2 k^2 e^{-\sigma^2 k^2}$
  - $\sigma = 1/\sqrt{\mu\omega}$ with μ being the reduced mass of the quark and antiquark in the meson, and ω is the oscillator frequency.
- $\frac{dP_{coal}}{d^3\vec{p}_Q} = g_M \int d^3\vec{p}_q \int d^3\vec{r} W_M(\vec{r}, \vec{k}) f_q(\vec{p}_q) \delta^3(\vec{p}_q + \vec{p}_Q - \vec{P}_M)$
  - $f_q(\vec{p}_q) = g_q \frac{V}{(2\pi)^3 (e^{E_q/T} + 1)}$

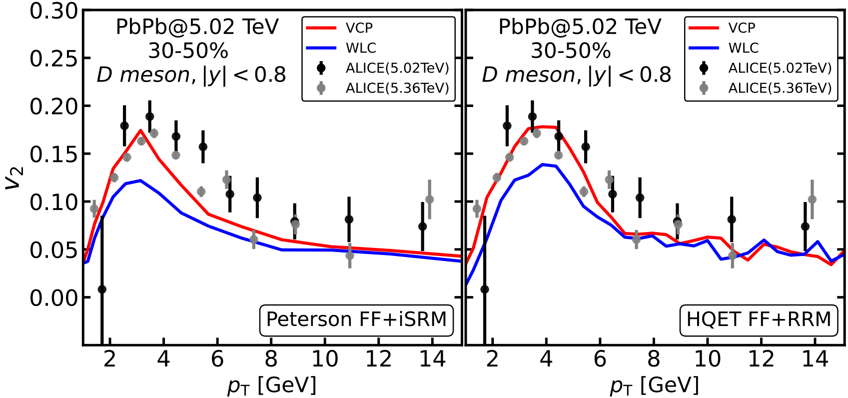
D meson spectrum in pp



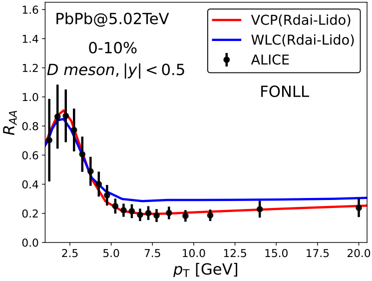
Diffusion vs. Diffusion+Radiation

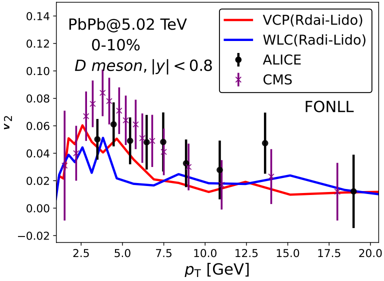


Elliptic flow: v_2



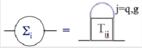
0-10 % centrality





T-matrix formalism with different in-medium potential

from Zhanduo Tang

		(1) [F. Riek+R. Rapp'10] [K. Huggins+R. Rapp '12] U-potential	(2) [S. Liu+R. Rapp'18, '19] SCS	(3) [Z. Tang+R. Rapp'23] VcConf
Potential (V)	Lorentz structure	$V^{vec} = V_{Coul}$ $V^{sca} = V_{conf}$	Same as (1)	$V^{vec} = V_{Coul} + (1-\chi)V_{conf}$ $V^{sca} = \chi V_{conf}$ χ: mixing coefficient
	Value	Taking internal potential (U-potential) from lQCD as input static (R=1) potential	Constrained by QQbar free energy from lQCD	Same as (2)
Parton width (Γ) /Selfenergy (Σ) (Γ=-2ImΣ)		• Quasiparticle (on-shell) • Small constant ImΣ~0	• Parton described by spectral function (off-shell) • Σ solved self-consistently  	Same as (2)
Light parton mass ($m_{q,g}$)		• $m_{q,g}$~gT • Quasiparticle EoS in line with lQCD	• $m_{q,g}$ tuned • EoS (with broad spectral function) agrees with lQCD	Same as (2)
Charm mass (m_Q)		$m_Q = m^0_Q + V(r\to\infty)/2$, m^0_Q constrained by vacuum spectroscopy	Same as (1)	Same as (1)