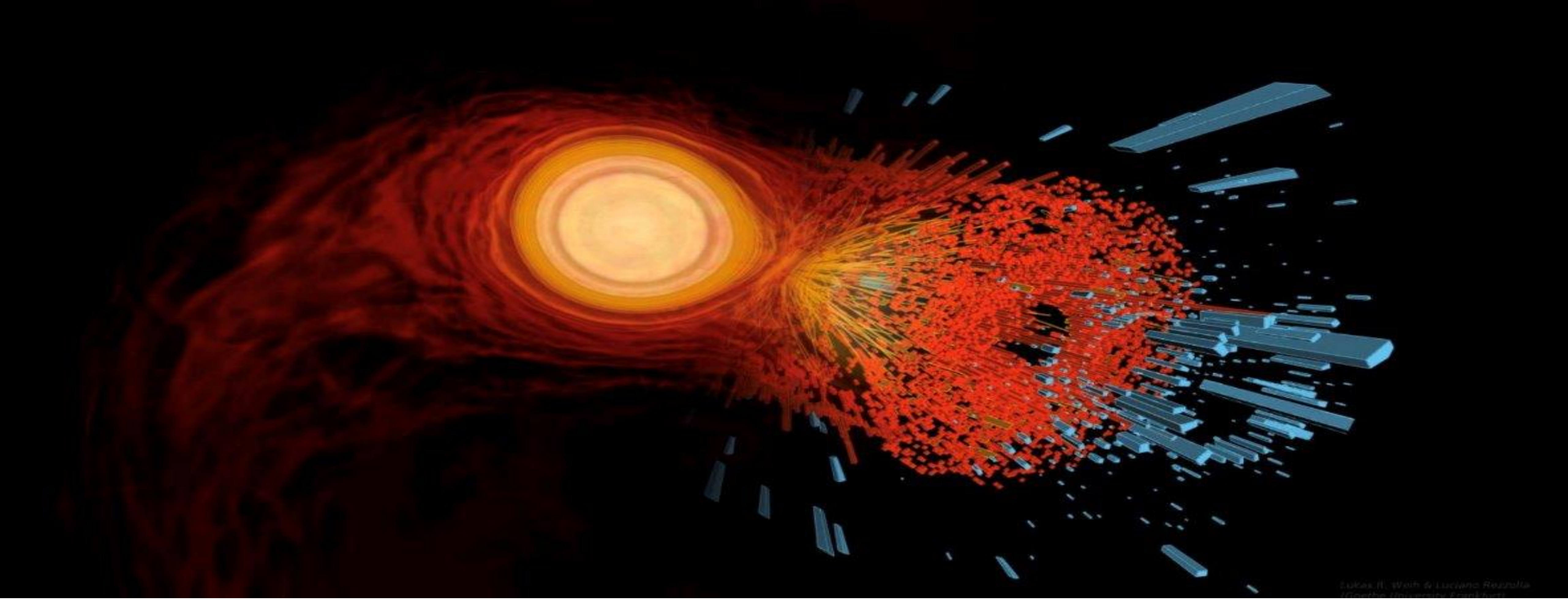

THE QCD PHASE DIAGRAM: REVIEW OF THE RECENT PROGRESS

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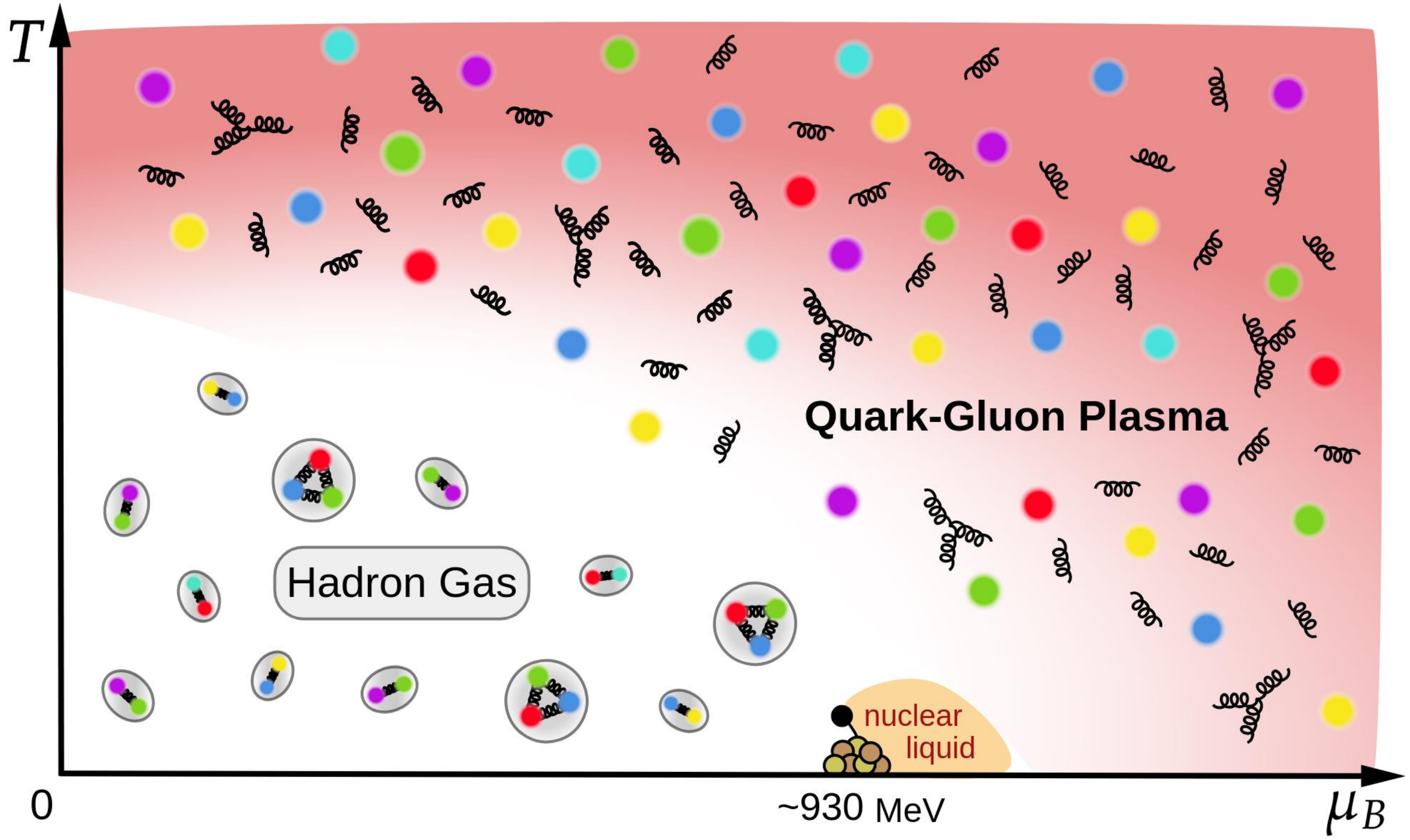
muses

PHASE DIAGRAM OF NUCLEAR MATTER

- Representing different phases and transitions as a function of thermodynamic variables
  (temperature, pressure, entropy density...)

- What do we know about the nuclear phase diagram?
  (for sure, from observation)
  - Atomic nuclei
  - Hadron gas / nuclear liquid
  - Quark-gluon plasma (QGP)

...and that's all.

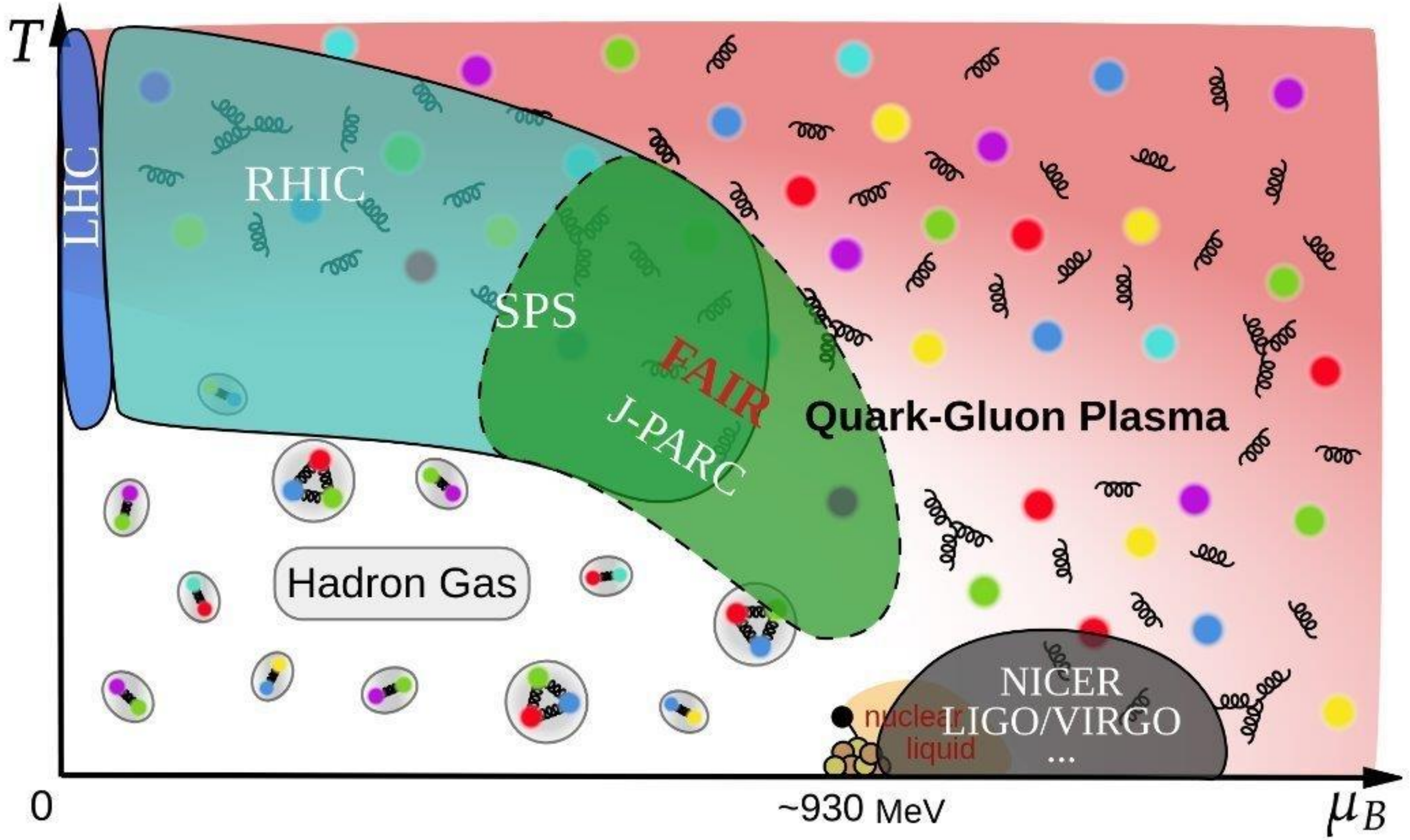


HOW TO EXPLORE THE PHASE DIAGRAM?

Essentially 2 ways to learn about the structure of the phase diagram:

- Down on Earth
  - LHC @ CERN
  - RHIC @ BNL / SPS @ CERN
  - J-PARC @ Tokai / FAIR @ GSI

- Looking up at the sky
  - Study neutron star structure + mergers from gravitational waves
  - NICER / LIGO-VIRGO



OUTLINE

1. Equation of state at finite μ_B
2. Structures of the phase diagram
3. Exploring other dimensions
4. MUSES: one framework to capture them all

LATTICE QCD – JUST A REMINDER

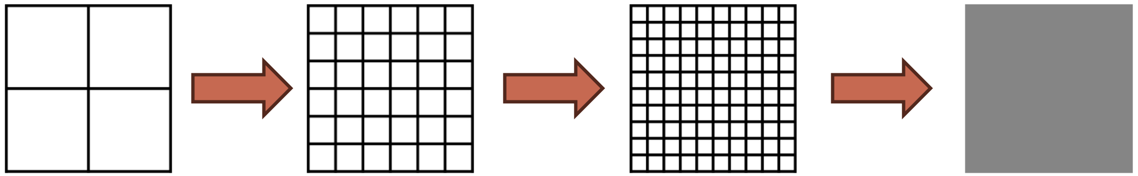
In the non-perturbative regime, no analytical solutions of QCD can be calculated.

<u>Solution:</u> discretizing space-time on a lattice and compute path integral using Monte-Carlo sampling for many configurations U

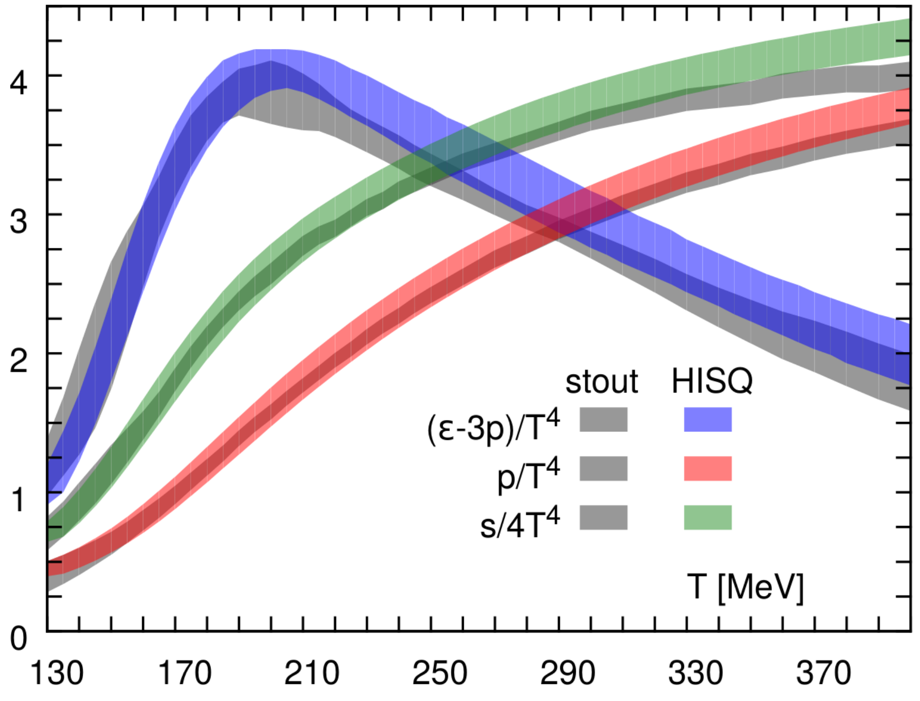
$$\langle \hat{O} \rangle = \frac{1}{Z} \int \mathcal{D}U \, \hat{O} \, \det M[U] \, e^{-S_G[U]}$$

for any observable $\hat{O}$.

Simulations achieved at different volumes V and different lattice spacings a to obtain in the end continuum limit results in an infinite volume.



Results at $\mu_B = 0$ are consistent across different collaborations.



Bazavov et al., PRD 90 (2014),094503

Borsányi et al.,B 730 (2014) 99-104

LATTICE QCD – GOING TO FINITE DENSITY

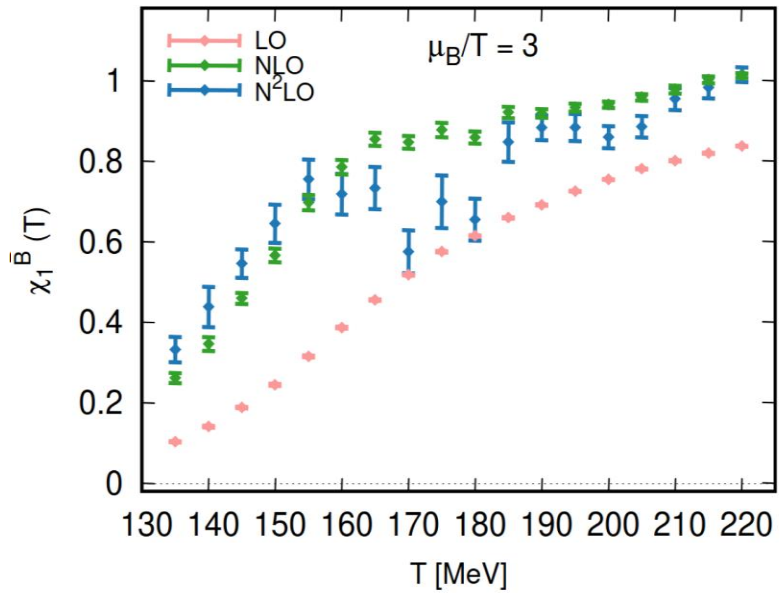
The **sign problem** in lattice QCD prevent from direct computation of thermodynamics at real finite μ_B ⟶ need to employ **expansion methods**.

Taylor series expansion

$$\frac{P(T,\mu_B)}{T^4} = \sum_i \frac{1}{i!} \chi_i^B(T,\mu_B=0) \left(\frac{\mu_B}{T}\right)^i$$

Allows to compute an EoS at finite density from expansion coefficients, called susceptibilities, computed at $\mu_{B/Q/S} = 0$ from lattice QCD.

$$\chi_i^B = \left.\frac{\partial^i (P/T^4)}{\partial \hat{\mu}_B{}^i}\right|_{\hat{\mu}_B=0}$$



Borsányi et al., *JHEP* 10 (2018) 205

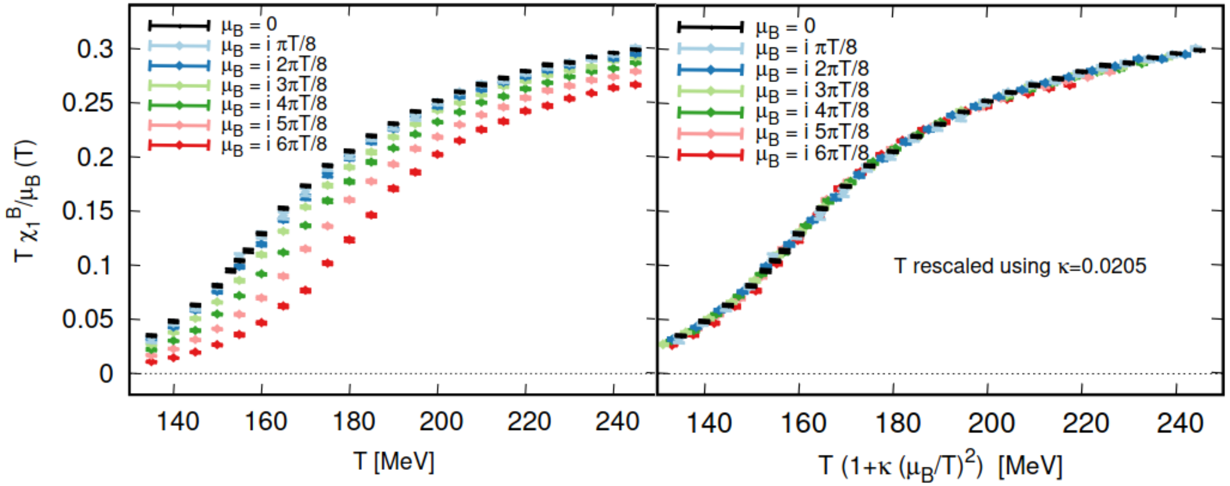
Limitations:

- Expansion achieved at constant T, missing out curvature of transition line
- Large errors due to high-order terms leading at large μ_i/T

⟶ **Expansion limited to $\mu_i/T \lesssim$ 2.5~3**

LATTICE QCD – GOING TO FINITE DENSITY

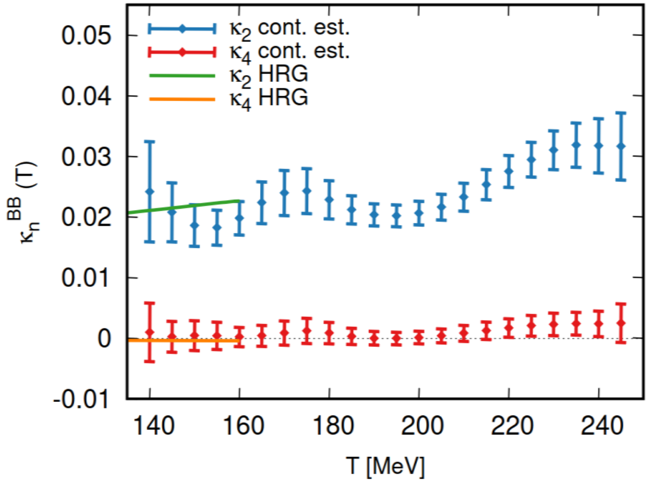
T-Expansion Scheme (TExS)



Empirical observation:

- all 1st order susceptibilities scale when defining a μ_B-dependent temperature $T'(T, \mu_B)$
- scales like: $\frac{\chi_1^B(T,\hat{\mu}_B)}{\hat{\mu}_B} = \chi_2^B(T',0)$

[Borsányi et al., *PRL* 126 (2021) 23, 232001]



one can thus redefine temperature and use a T-expansion scheme **(TExS)**:

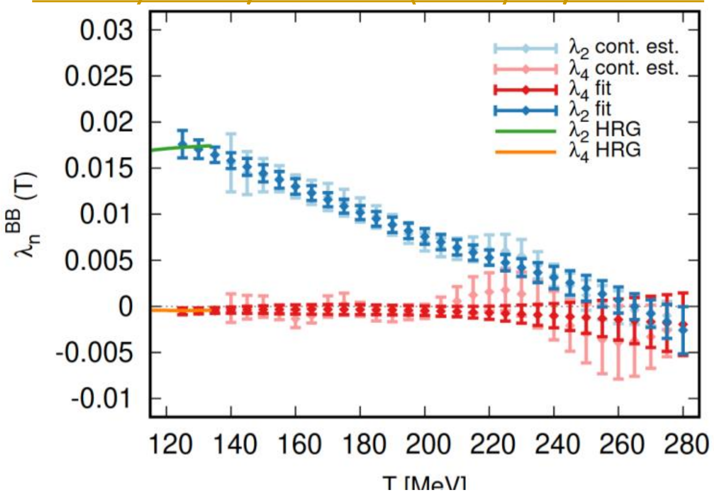
$$T'(T,\hat{\mu}_B) = T\left(1 + \kappa_2^{BB}(T)\hat{\mu}_B^2 + \kappa_4^{BB}(T)\hat{\mu}_B^4 + \mathcal{O}(\hat{\mu}_B^6)\right)$$

with expansion coefficients κ, related to susceptibilities:

$$\kappa_2^{BB}(T) = \frac{1}{6T}\frac{\chi_4^B(T)}{\chi_2^{B\prime}(T)} \qquad \kappa_4^{BB}(T) = \frac{1}{360\chi_2^{B\prime}(T)^3}\left(3\chi_2^{B\prime}(T)^2\chi_6^B(T) - 5\chi_2^{B\prime\prime}(T)\chi_4^B(T)^2\right)$$

LATTICE QCD – GOING TO FINITE DENSITY

Borsányi et al., *PRD* 105 (2022) 11, 114504



T-Expansion Scheme (TExS)

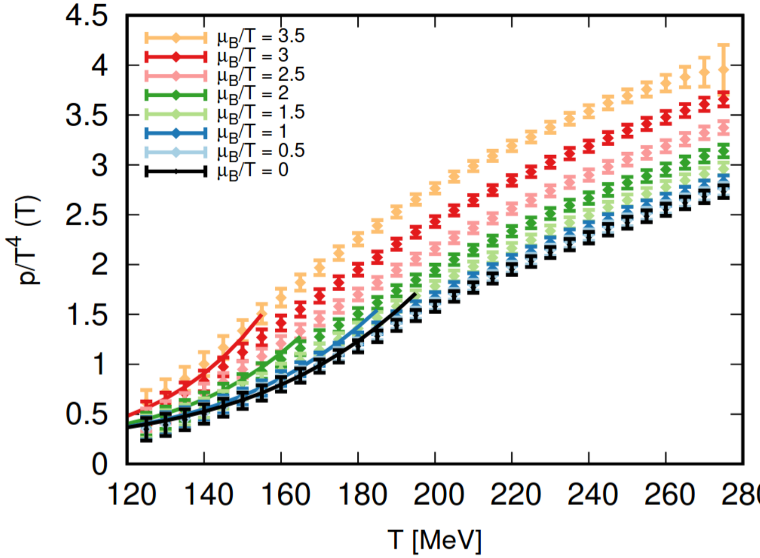
New method is essentially a resummation of Taylor expansion, defined with correct Stefan-Boltzmann limits ($T\to\infty$) as:

$$\frac{\chi_1^B(T,\hat{\mu}_B)}{\overline{\chi_1^B(\hat{\mu}_B)}} = \frac{\chi_2^B(T'_B,0)}{\overline{\chi_2^B(0)}}$$

Main identity

$$T'_B(T,\hat{\mu}_B) = T\left(1+\lambda_2^B(T)\hat{\mu}_B^2+\ldots\right)$$

$$\lambda_2^B(T) = \frac{1}{6T\chi_2^{B\prime}(T)}\left(\chi_4^B(T) - \frac{\overline{\chi_4^B}}{\overline{\chi_2^B}}\chi_2^B(T)\right)$$



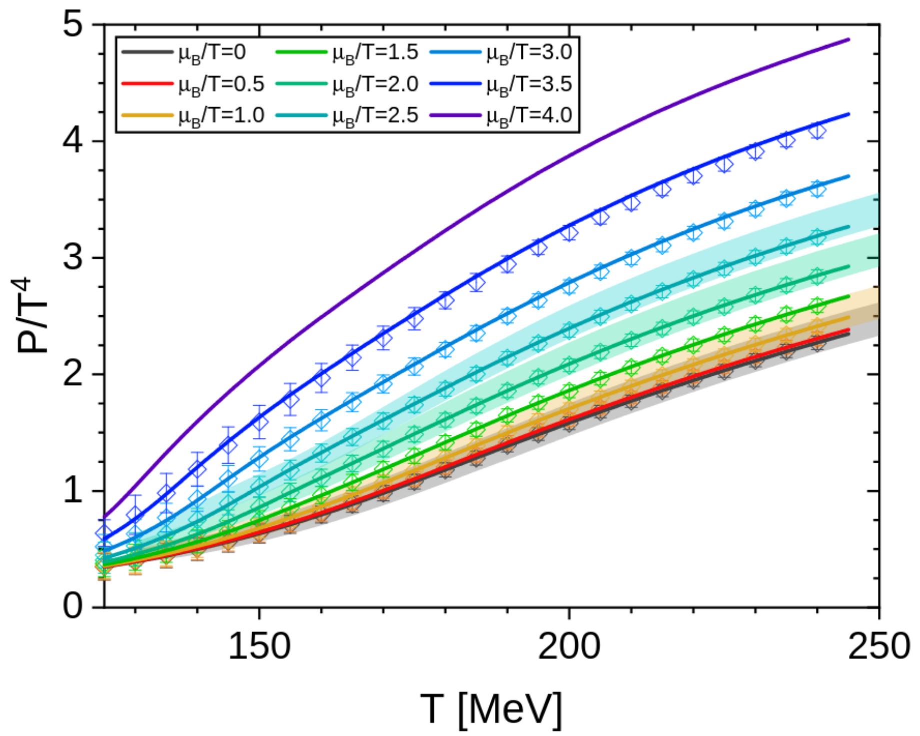
- To compute the complete EoS, one has to integrate χ_1^B to get pressure

- Separation in magnitude between expansion coefficients (related to χs) hints at better convergence than Taylor expansion

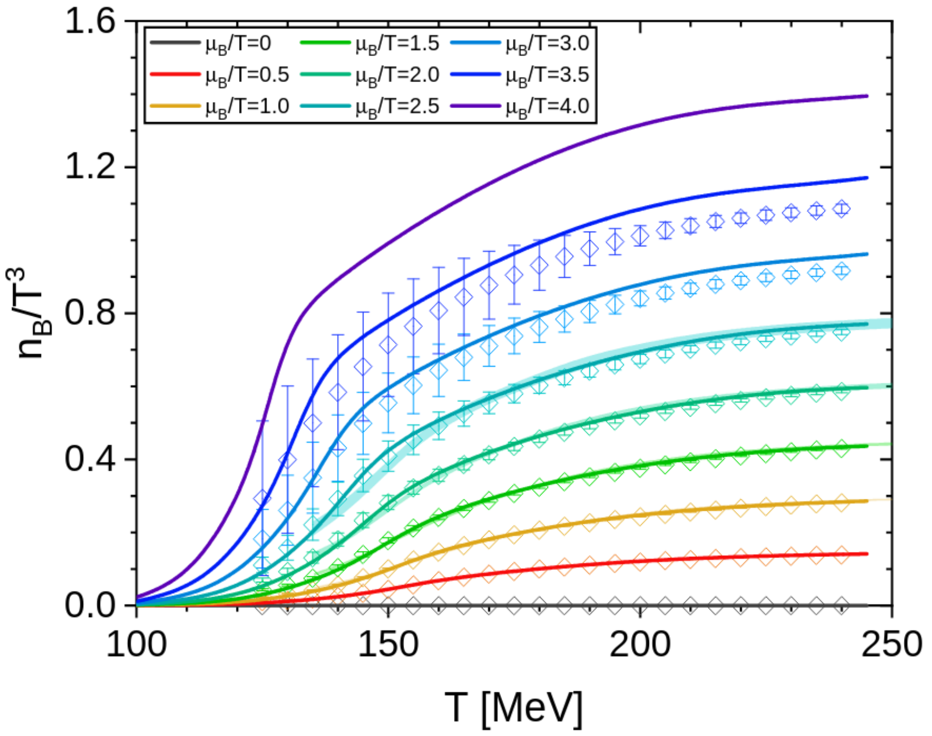
→ **Trusted up to $\mu_B/T = 3.5$**

PREDICTIONS FROM FUNCTIONAL METHODS

- Based on a truncated expansion of QCD action functional
  ⟶ Allow for direct calculations at finite density



- Recent progress towards better control of the error allows computation of thermodynamics

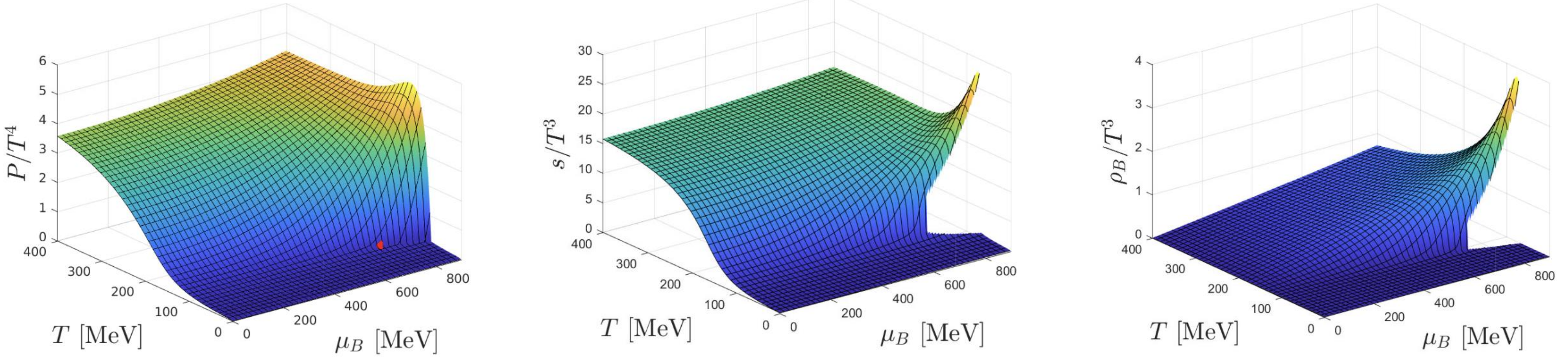


Lu et al., arXiv:2504.05099

EQUATION OF STATE FROM HOLOGRAPHY

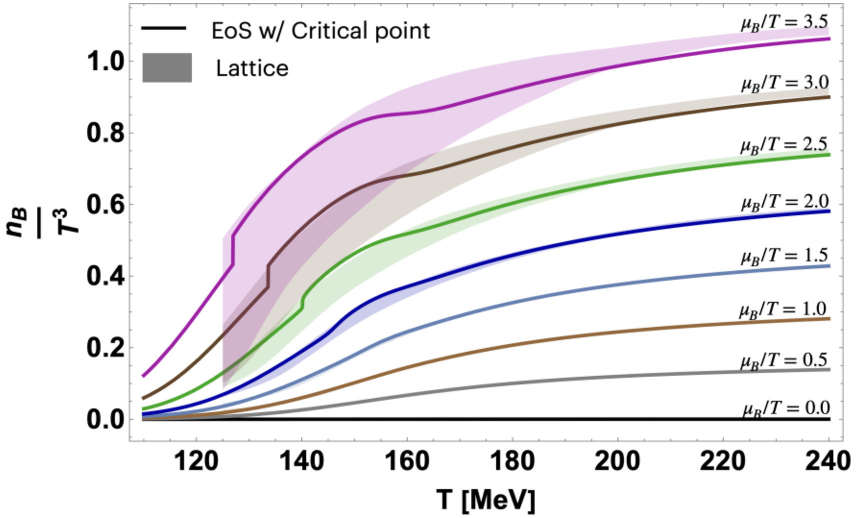
- Use gauge/gravity (AdS/QCD) correspondence to obtain QCD thermodynamics from an EMD Holographic model
- Fix the parameters to reproduce lattice QCD results at $\mu = 0$
- Calculate equation of state at finite density, but only for finite μ_B

Grefa et al., *PRD* 104 (2021) 3, 034002



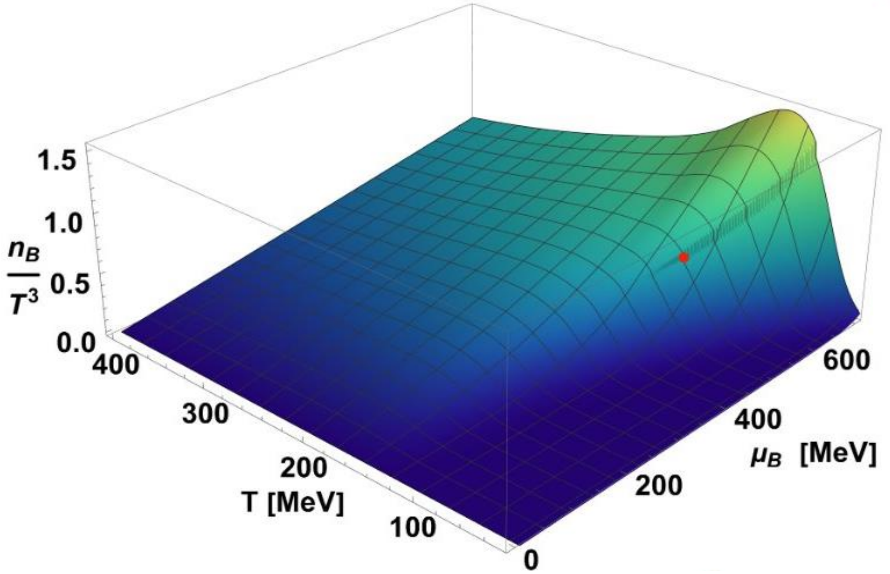
ISING-2D T.EX.S EQUATION OF STATE FROM L-QCD

Using the new lattice TExS EoS, one can implement a CP mapped from the 3D-Ising model, offering a tool to study critical effect in the QCD EoS



Improvement w.r.t. the BEST EoS:

- Expansion carried along constant baryon density line
- Critical point can now be parametrized up to μ_B=700 MeV
- Study of the stability dependence of the CP strength parametrization



Kahangirwe, J.J. et al., D 109 (2024) 9, 094046

OUTLINE

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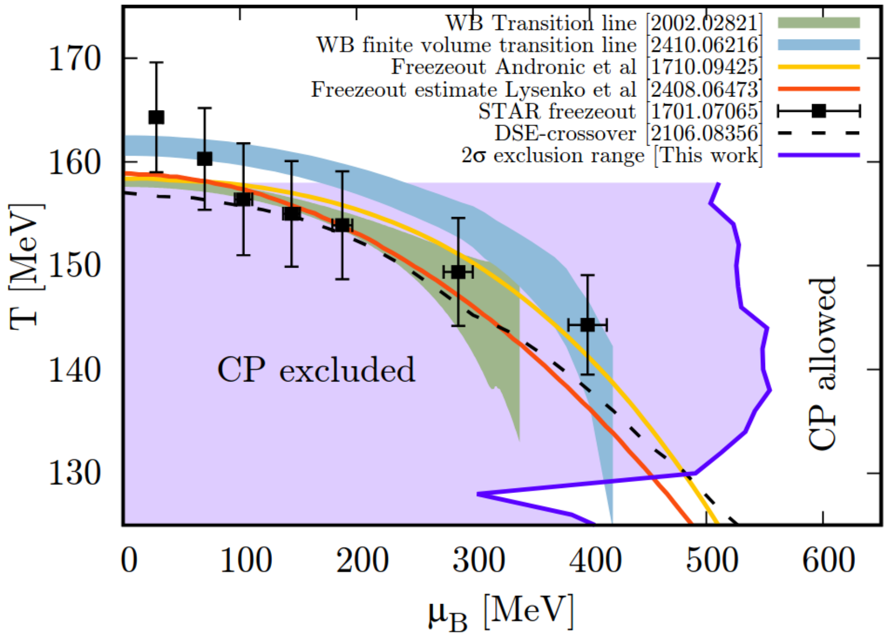
LATTICE QCD – THE NATURE OF THE TRANSITION

By extrapolating from complex μ_B lattice simulations to real μ_B, one can compute the shape of the transition line:

$$\frac{T_c(\mu_B)}{\mathbf{T_c(\mu_B=0)}} = 1 + \kappa_2 \left(\frac{\mu_B}{T_c(\mu_B)}\right)^2 + \kappa_4 \left(\frac{\mu_B}{T_c(\mu_B)}\right)^4 + \mathcal{O}(\mu_B^6)$$

(**location**, **curvature**, **"hyper-curvature"**, ...)

Borsányi et al., arXiv:2502.10267



Most recent results:

$$T_c(\mu_B = 0) = 158.0 \pm 0.6 \text{ MeV}$$

$$\kappa_2 = 0.0153 \pm 0.0018 \qquad \kappa_4 = 0.00032 \pm 0.00067$$

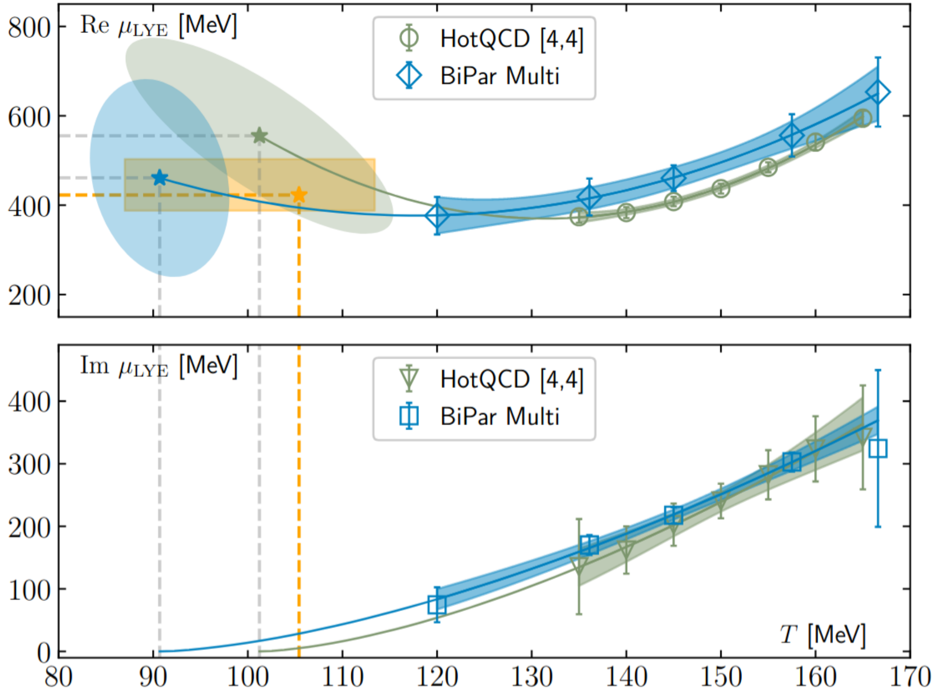
Borsányi et al., *PRL* 125 (2020), 052001

→ Existence of **critical point excluded** for μ_B <450 MeV by most recent lattice QCD analysis

LOCATION OF A CRITICAL POINT FROM LATTICE QCD

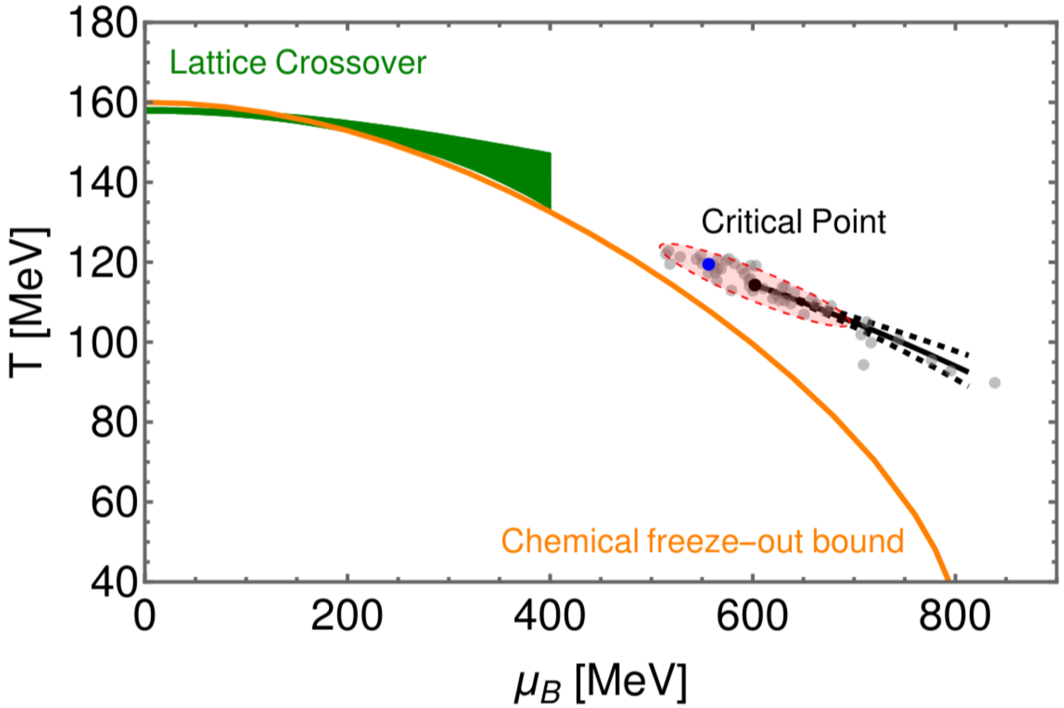
- **Padé approximants**
  Clarke et al., arXiv:2405.10196

Prediction of a **critical point** from **lattice QCD** from simulations at imaginary baryon chemical potential, extrapolated to the real plane.



- **Constant entropy contour**
  Shah et al., arXiv:2410.16206

Prediction of a **critical point** from **lattice QCD** extrapolated **contours of constant entropy density**

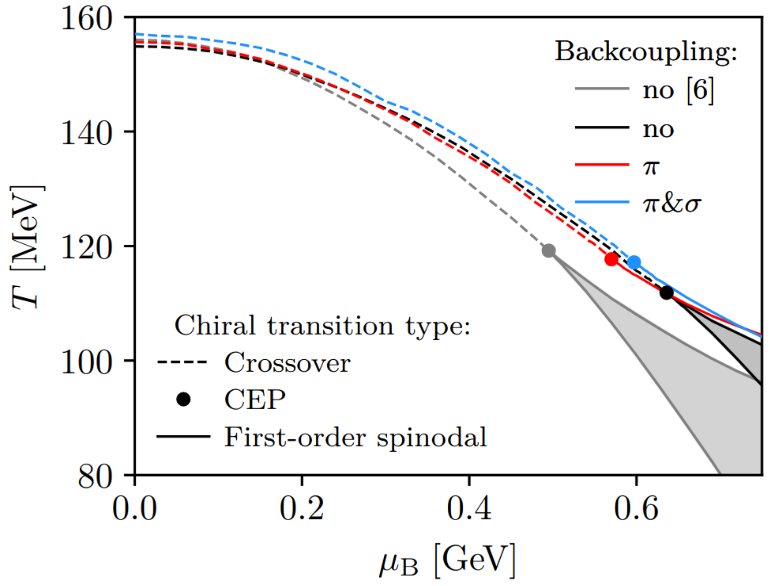


QCD EFFECTIVE THEORIES

Several (radically) different **effective non-perturbative approaches to QCD** predict the **existence** of a **critical point** at finite baryon chemical potential, among which the most recent ones:

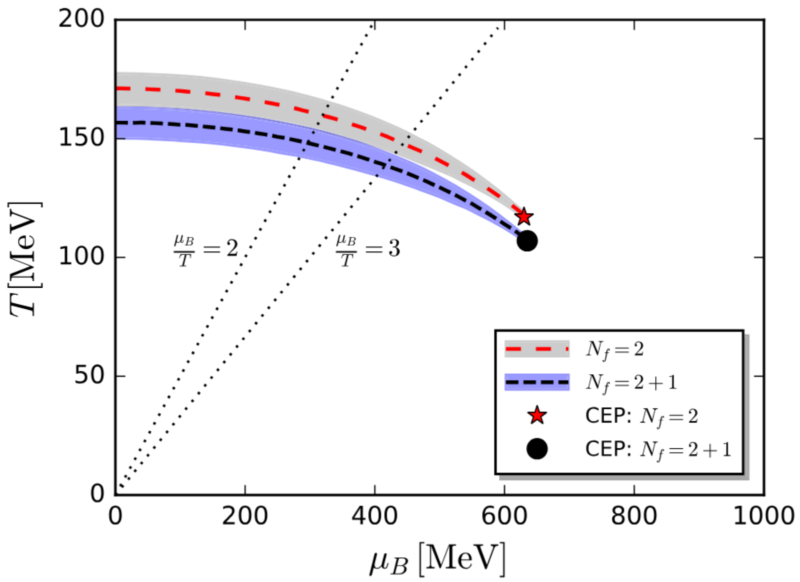
- **Dyson-Schwinger equations**

Phys.Rev.D 104 (2021) 5, 054022



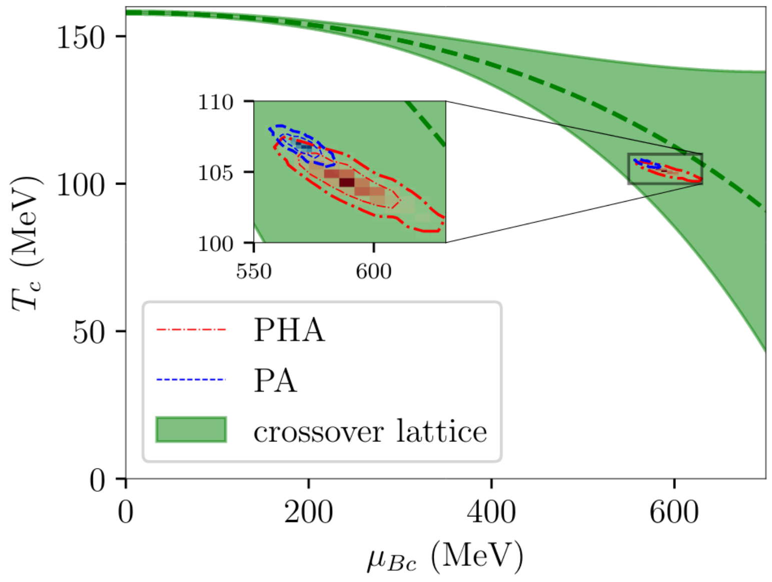
- **Functional Renormalization Group**

Phys.Rev.D 101 (2020) 5, 054032



- **Holographic model**

Phys.Rev.D 110 (2024) 9, 094006



SUMMARY OF QCD CRITICAL POINT PREDICTIONS

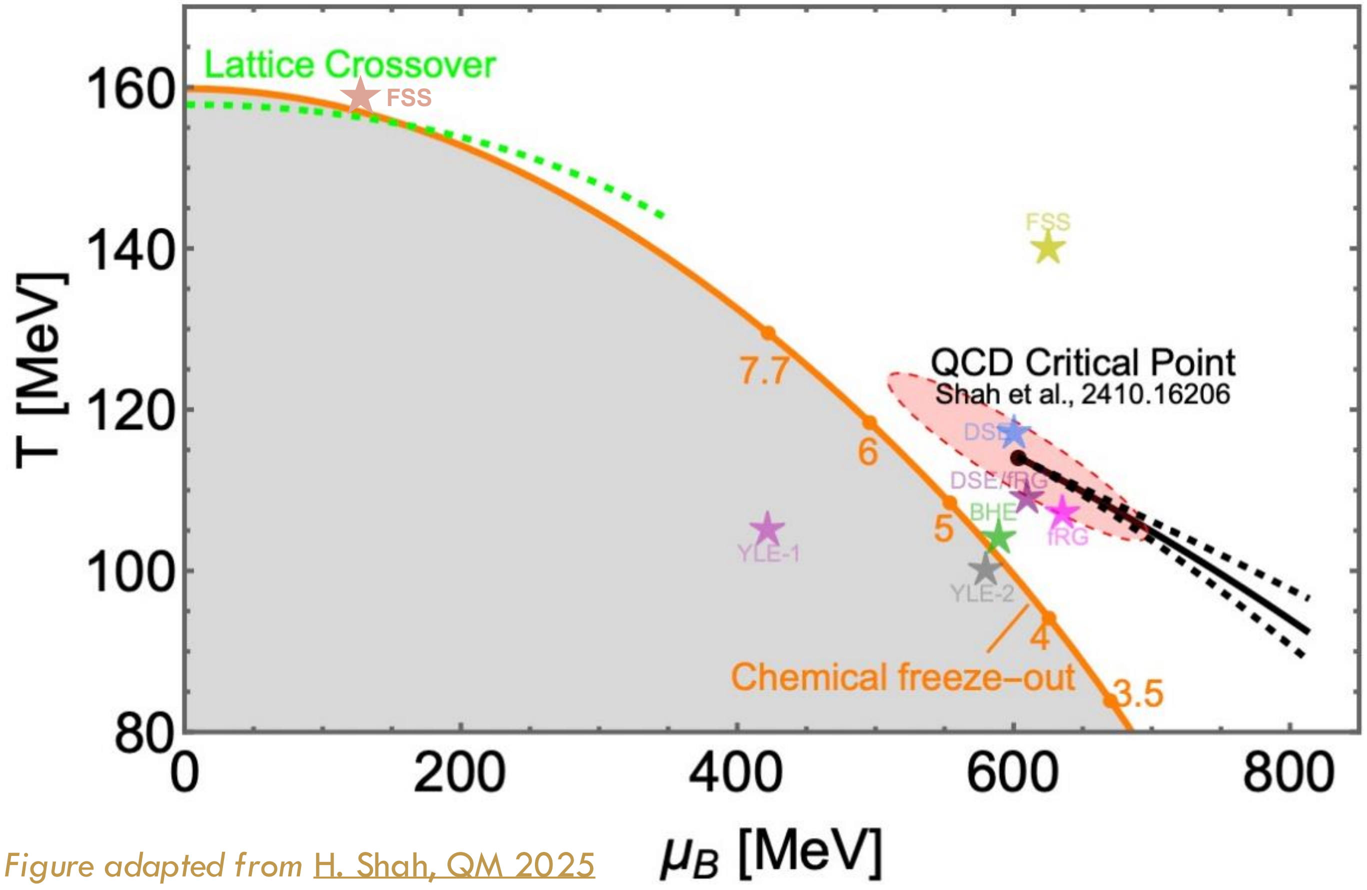


Figure adapted from H. Shah, QM 2025

YLE-1: D.A. Clarke et al, arXiv:2405.10196

YLE-2: G. Basar, PRC 110, 015203 (2024)

BHE: M. Hippert et al., PRD 110, 094006 (2024)

FRG: W-J. Fu et al., PRD 101, 054032 (2020)

DSE: P.J. Gunkel et al., PRD 104, 052202 (2021)

DSE/fRG: Gao, Pawlowski., PLB 820, 136584 (2021)

FSS: A. Sorensen et al., arXiv:2405.10278

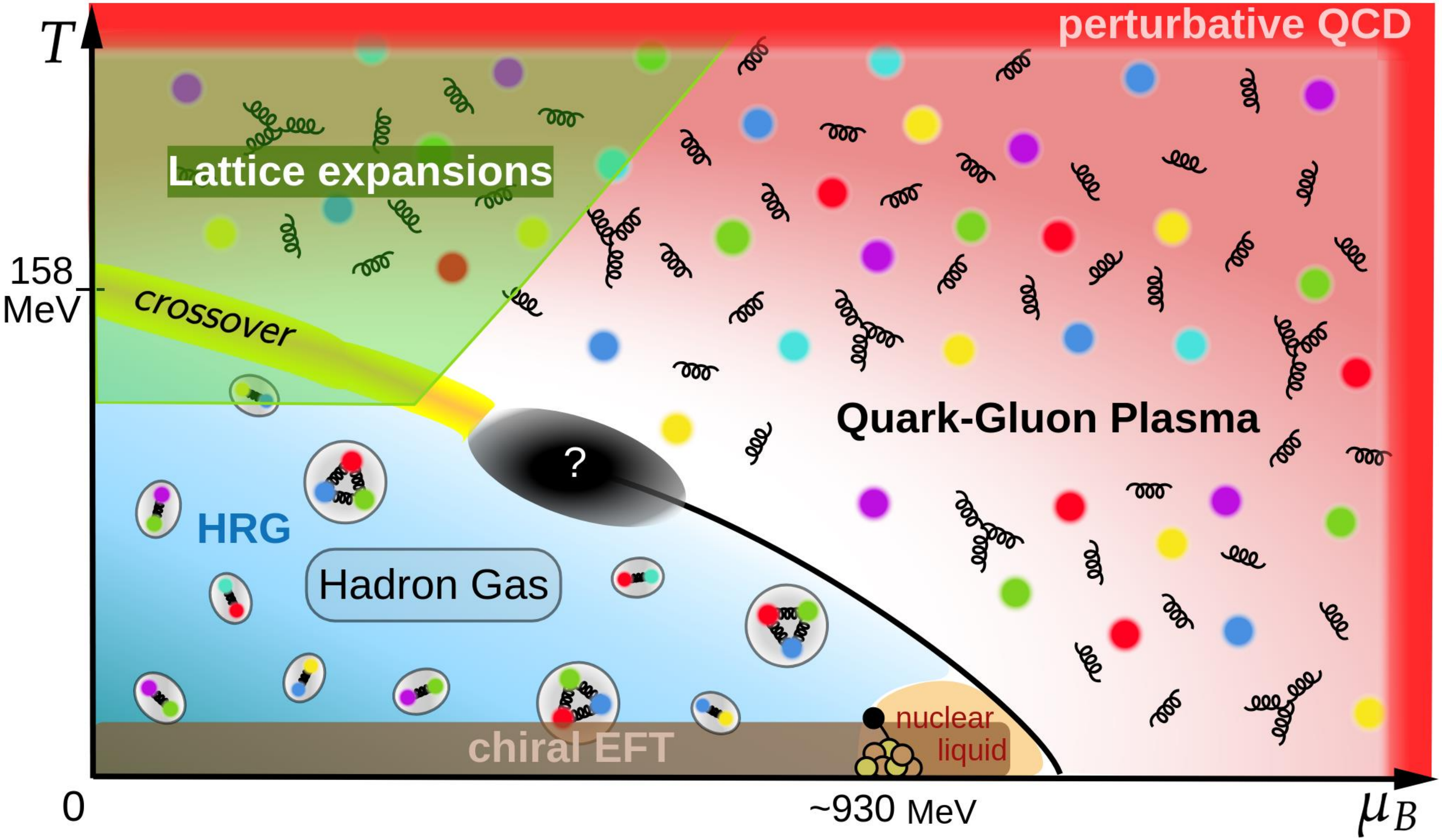
FSS: Roy Lacey, arXiv:2411.09139

Compared with **new chemical freeze-out line** obtained from **constant ε/n** line obtained with **HRG** at $\mu_Q = \mu_S = 0$

→ **Lower bound for CEP existence**

Lysenko et al., arXiv:2408.06473

NUCLEAR PHASE DIAGRAM (IN 2D)



OUTLINE

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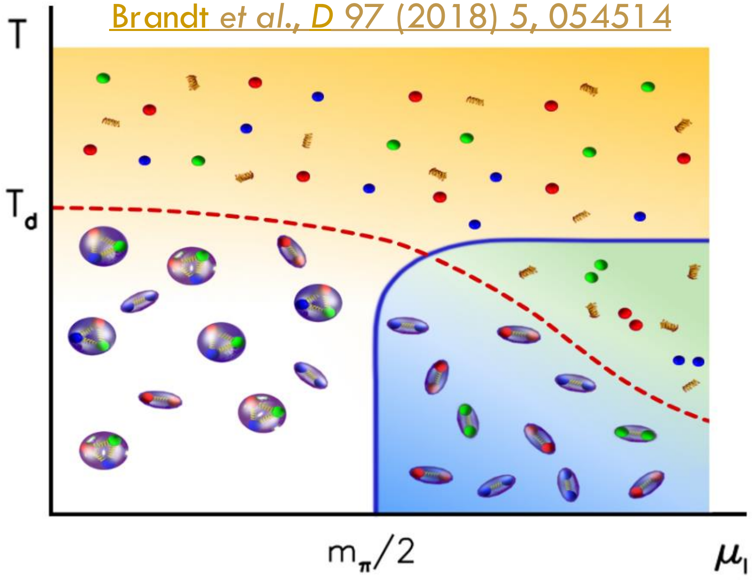
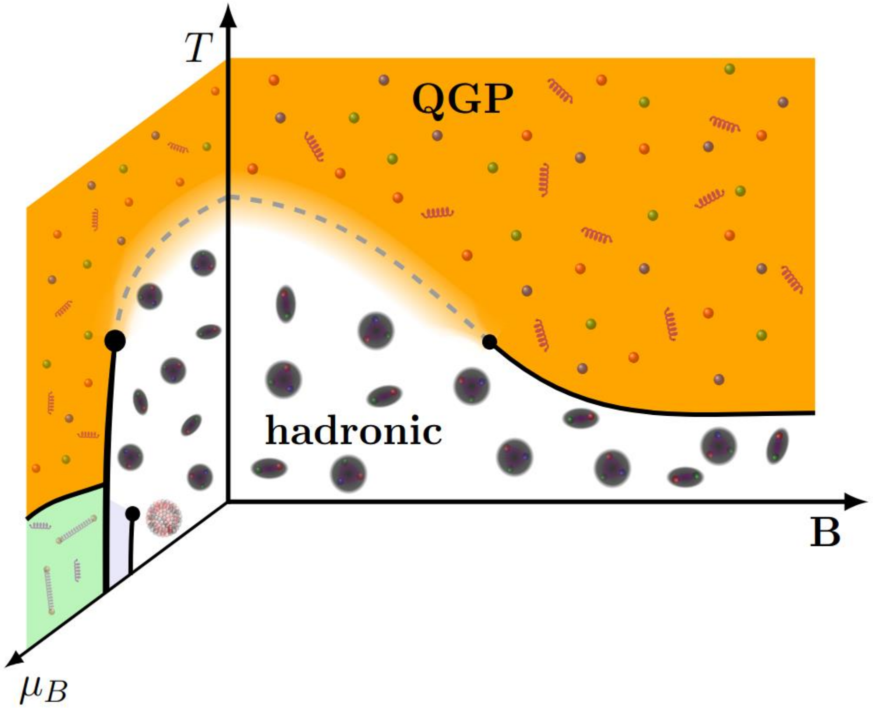
GETTING AROUND THE SIGN PROBLEM

- Simulation at finite isospin do not suffer from the sign problem
  - Indication of the formation of a BEC at $\mu_Q > m_\pi$ and $T < 155$ MeV
  - Finite-T thermodynamics obtained in continuum limit
    Brandt *et al.*, *JHEP 07 (2023) 055*

Brandt *et al.*, *D 97 (2018) 5, 054514*





- Finite magnetic fields simulations can also be done
  - Display transition to 1st order at very high-B
    D'Elia *et al.*, 105 (2022) 3, 034511
  - Strong sensitivity of χ^{BQ} indicates potential use as thermometer for HICs
    Ding *et al.*, arXiv:2503.18467

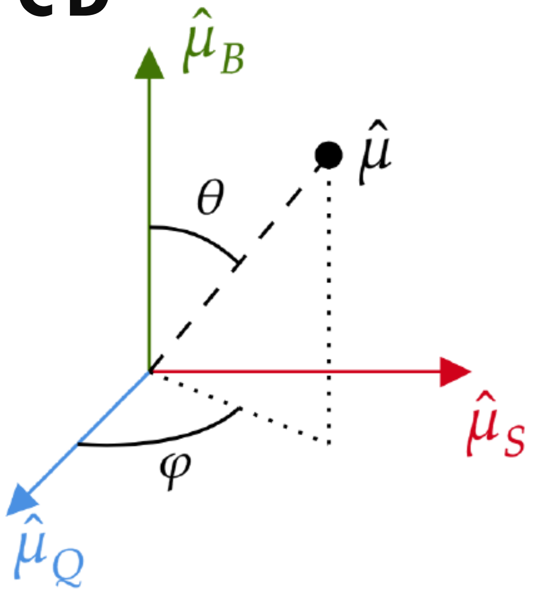
4D-T.EX.S EQUATION OF STATE FROM L-QCD

- Generalization of the previous 2D T'-Expansion Scheme to 3 conserved charges by projecting the "cartesian" (μ_B, μ_Q, μ_S) coordinates to spherical ones

$$\hat{\mu} = \sqrt{\hat{\mu}_B^2 + \hat{\mu}_Q^2 + \hat{\mu}_S^2}$$

$$\hat{\mu}_B = \hat{\mu} \,.\, \cos(\theta)$$
$$\hat{\mu}_Q = \hat{\mu} \,.\, \sin(\theta)\cos(\varphi)$$
$$\hat{\mu}_S = \hat{\mu} \,.\, \sin(\theta)\sin(\varphi)$$

$\longrightarrow$ still a 2D-TExS expansion, along a constant μ/T line



Abuali, **J.J.** *et al.*, arXiv:2504.01881

- Calculate expansion coefficient λ_2 based on so-called "generalized susceptibilities" $X_{2/4}$ *(linear combinations of lattice QCD susceptibilities)* + their Stefan-Boltzmann limits

$$\lambda_2^{\theta,\varphi}(T) = \frac{1}{6T}\frac{1}{X_2'^{\,\theta,\varphi}(T)} \times \left(X_4^{\theta,\varphi}(T) - \frac{\overline{X}_4^{\theta,\varphi}(0)}{\overline{X}_2^{\theta,\varphi}(0)} X_2^{\theta,\varphi}(T) \right)$$

$$X_2^{\theta,\varphi}(T) = c_\theta^2 \cdot \chi_2^B(T) + s_\theta^2 c_\varphi^2 \cdot \chi_2^Q(T) + s_\theta^2 s_\varphi^2 \cdot \chi_2^S(T) + \dots$$

$$X_4^{\theta,\varphi}(T) = c_\theta^4 \cdot \chi_4^B(T) + s_\theta^4 c_\varphi^4 \cdot \chi_4^Q(T) + s_\theta^4 s_\varphi^4 \cdot \chi_4^S(T) + \dots$$

4D-T.EX.S EQUATION OF STATE FROM L-QCD

- Compute the "generalized charge density" X_1 along the projected line using the expanded temperature T' and the T.Ex.S main identity (modified to match with Stefan-Boltzmann limit at $T \longrightarrow \infty$)

$$X_1^{\theta,\varphi}(T,\hat{\mu}) = \frac{\overline{X}_1^{\theta,\varphi}(\hat{\mu})}{\overline{X}_2^{\theta,\varphi}(0)} \times X_2^{\theta,\varphi}\left(T'^{\theta,\varphi}(T,\hat{\mu}),0\right)$$

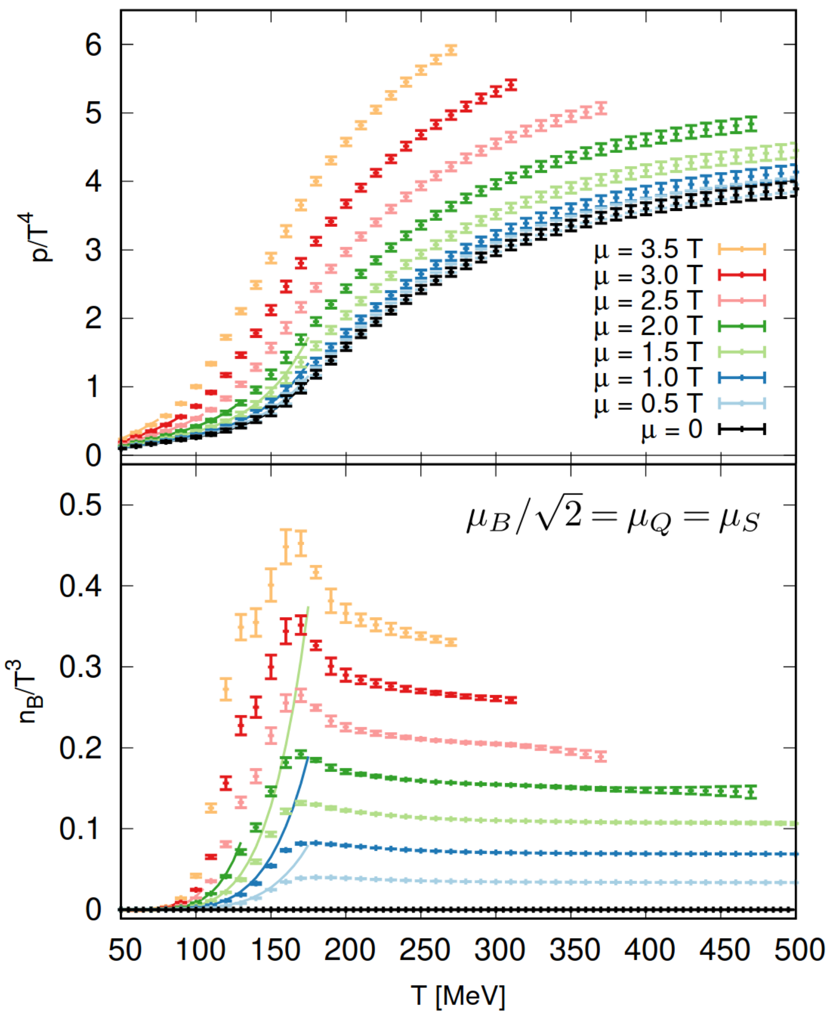
with $$T'^{\theta,\varphi}(T,\hat{\mu}) = T\left(1+\lambda_2^{\theta,\varphi}(T)\hat{\mu}_B^2\right)$$

- Obtain pressure by integrating X_1, allowing then to compute all thermodynamics

- Comes with a new complete set of continuum-estimated B,Q,S lattice susceptibilities of order 2 & 4
(based on N_τ = 10, 12, 16, 20, 24 results from Wuppertal-Budapest)

J.J., P. Parotto *et al.*, DOI:10.5281/zenodo.15123622

Abuali, J.J. *et al.*, arXiv:2504.01881



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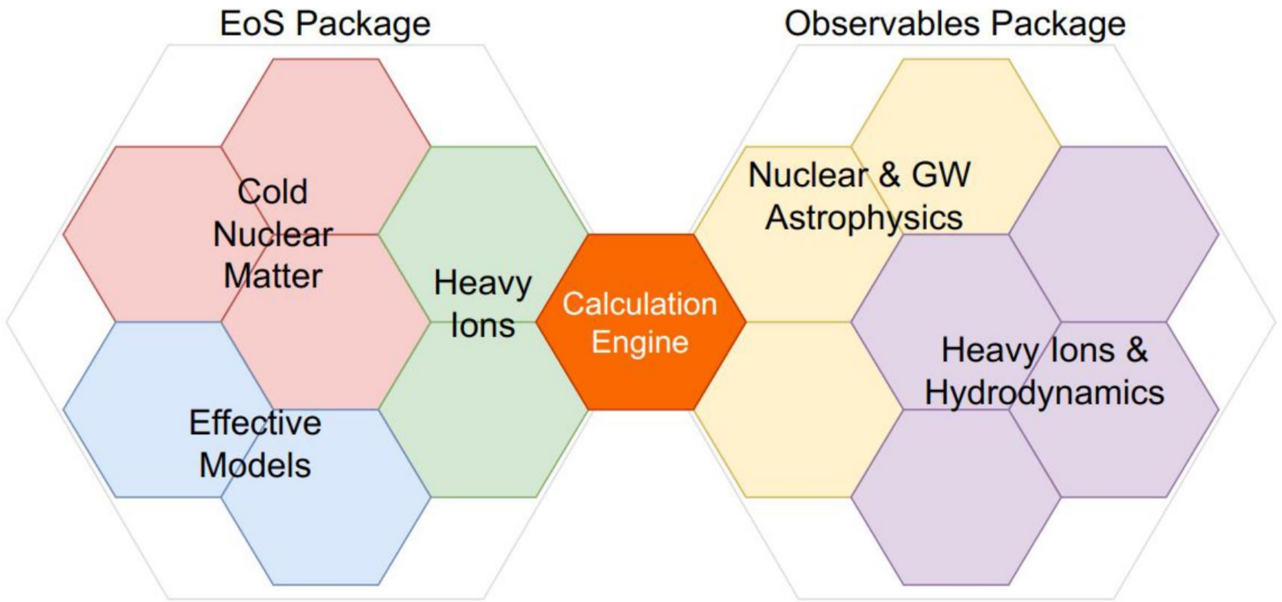
MODULAR UNIFIED SOLVER OF THE EQUATION OF STATE

Gathers physicists from heavy-ion, neutron star and low-energy nuclear physics and computer scientists

- **Modular:** different modules computing EoS for different regions of the nuclear phase diagram
  + associated observables

- **Unified:** modules are integrated in a single framework, to ensure
  i. Maximum coverage of phase space
  ii. Respect of their constraints



A.T. Manning, MUSES Collaboration Meeting 2023 muses

Click here for more

MODULAR UNIFIED SOLVER OF THE EQUATION OF STATE

Public release of the calculation engine early 2025!

Pelicer, **J.J.** *et al.*, *PRD* 111 (2025) 10, 103037

Manning *et al.*, DOI:10.5281/zenodo.14721911

Available:

- 4D lattice Taylor-expanded EoS
- Ising – 2D TExS EoS
- Chiral EFT
- Crust DFT
- CMF++ EoS
- Holographic EoS
- (Pseudo) universal QLIMR relations
- Beta equilibrium
- 1D EoS merger

Next to come:

- HRG vdW EoS
- 4D TExS EoS
- Partial pressures
- Thermal fits
- …

CONCLUSIONS

Recent progress in finite density calculations of the QCD equation of state

- Higher reach at finite μ_B + Extension to other finite densities
  important for simulations at BES energies
- Is the coverage of these new EoS sufficient enough? Can we go further?
- Observation of pionic BEC: what about finite strangeness?

Many new predictions of the CEP location point toward the same region

- Waiting for more RHIC results to help deciphering
- How sensitive to errors on lattice data?
- Can we use the Ising-2D TExS family of EoS to infer the existence of the CEP? *(i.e. do we have the adequate framework?)*
- What about more realistic scenario for HICs? *(S-neutrality, 4D EoS)*

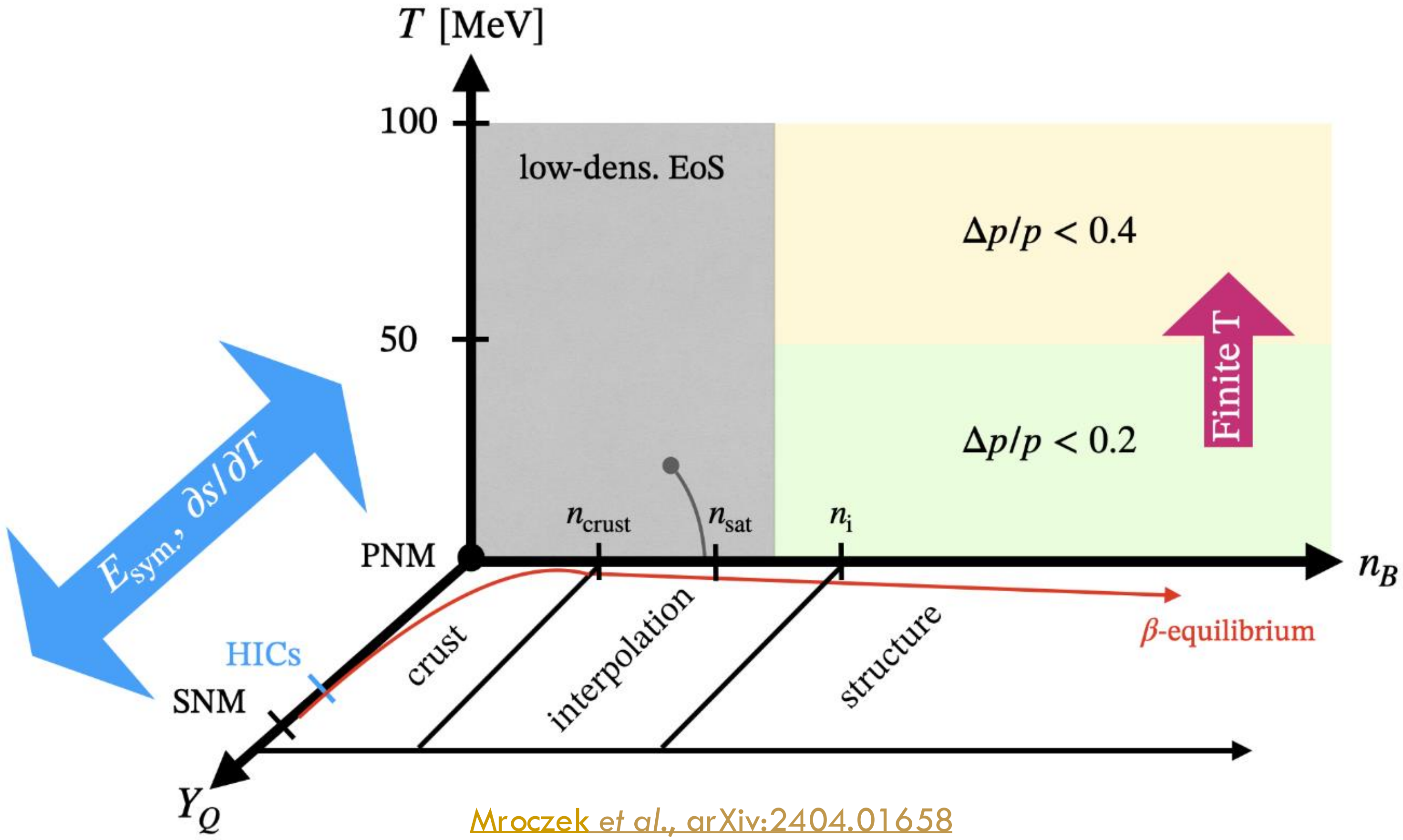
Theoretical collaborative efforts to create standardized, open-source and long-lasting codes for EoS computation & tools

- New release of MUSES in early 2025
- What should we add?

BACKUP

EXPANSION AT FINITE T FOR T=0

$$p(T,\vec{\mu}) = p_{T=0} + \frac{1}{2}\frac{\partial s}{\partial T}\bigg|_{T=0,\vec{\mu}} T^2 + \frac{1}{6}\frac{\partial^2 s}{\partial T^2}\bigg|_{T=0,\vec{\mu}} T^3$$



Mroczek *et al.*, arXiv:2404.01658

THERMODYNAMICS RELATIONS

To compute an **equation of state (EoS)**, one usually start by calculating one quantity as a function ***$\mathcal{A}$(T, μ//n)*** and derive all other quantities from there.

Basic thermodynamic relations, in the **grand-canonical limit** (from the partition function $\mathscr{Z}$):

Pressure: $$P = -T \frac{\partial \ln(\mathscr{Z})}{\partial V}$$

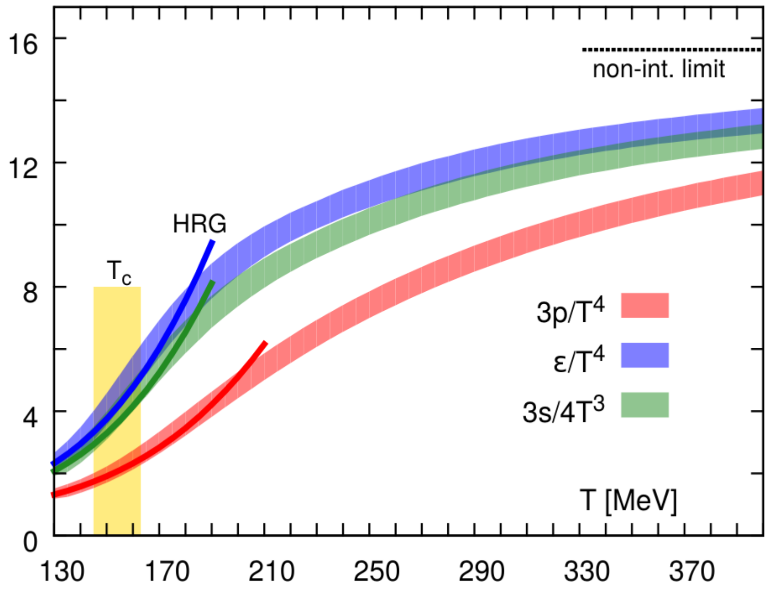
Entropy density: $$s = \left(\frac{\partial P}{\partial T}\right)_{\mu_i}$$

Charge densities: $$n_i = \left(\frac{\partial P}{\partial \mu_i}\right)_{T,\mu_{j\neq i}}$$

Energy density: $$\varepsilon = sT - P + \sum_i \mu_i . n_i$$

HADRON RESONANCE GAS MODEL

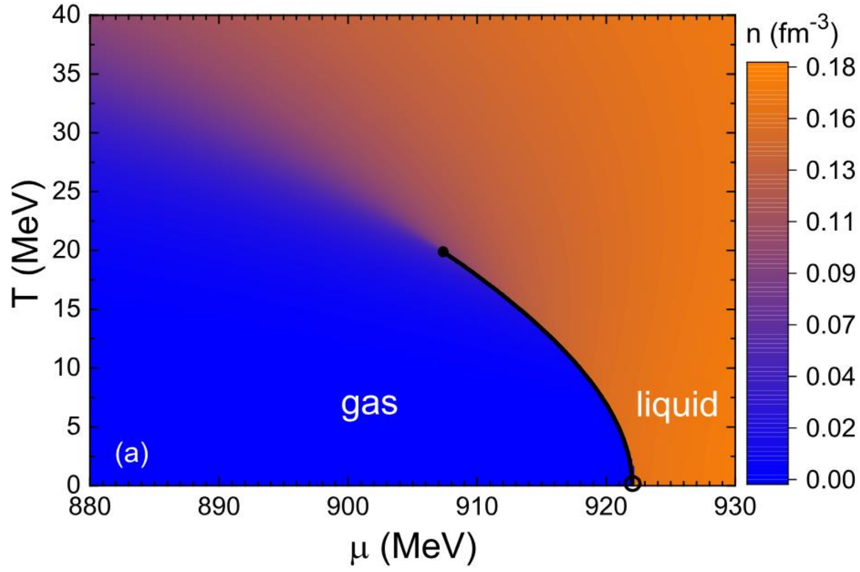
- Thermal model based on Fermi-Dirac & Bose-Einstein statistics, assuming a gas of interacting hadrons in their ground states can be modeled by a gas of non-interacting hadrons and resonances.



Bazavov et al., PRD 90 (2014),094503

- Describes the hadronic phase only *(blows up at the transition)*
  → used as a reference for low-T QCD, as it matches with lattice QCD *(which is too costly to compute below T~120 MeV)*

- Can be improved by adding excluded volume correction and van der Waals attractive interaction
  → describes the nuclear liquid-gas phase transition

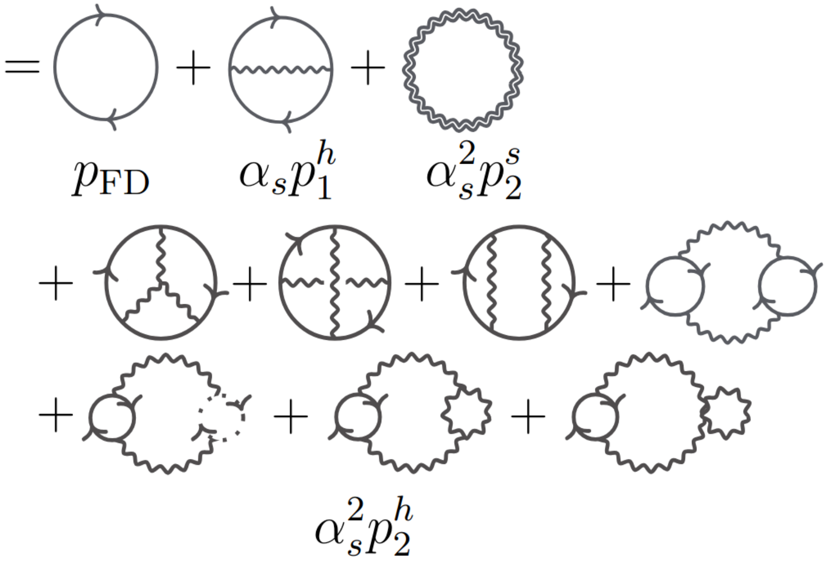


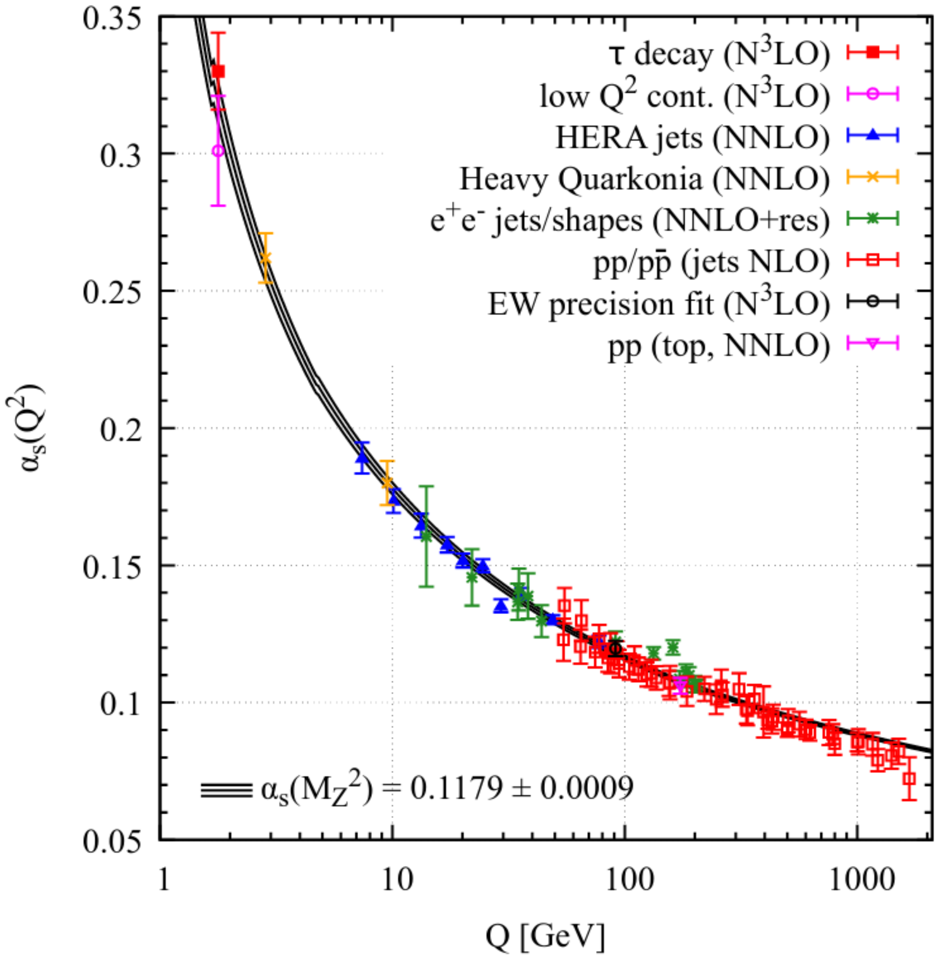
Vovchenko et al., *Phys.Rev.C* 92 (2015) 5, 054901

THE RUNNING COUPLING CONSTANT OF QCD

- One **fundamental property of QCD** is the fact that its **coupling** α_s is **changing magnitude** depending on the energy involved in the process considered.
- Direct calculation of the pressure from QCD is only possible through perturbation methods for small α_s :

$$p = p_{\mathrm{FD}} + \alpha_s p_1^h + \alpha_s^2 p_2^h + \alpha_s^2 p_2^s + \ \ldots$$



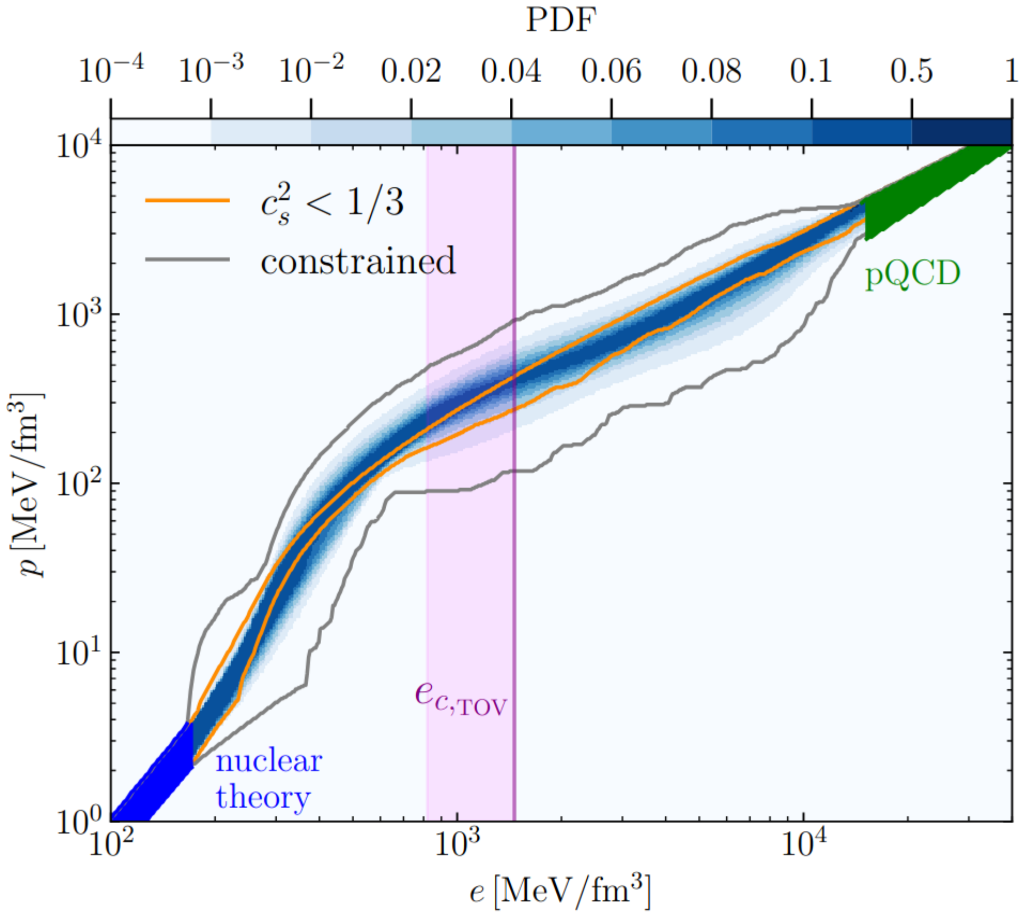


PERTURBATIVE QCD

- Perturbative calculation from Feynman diagrams are only possible at very high T/n_B
- In recent years, the approach have been generalized at NNLO to cover all T and n_B

  Gorda et al., Phys.Rev.D 104 (2021) 7, 074015

- Only available for $n_B \geq 40\ n_{sat}$ at low-T, pQCD helps to constrain the nuclear EoS through Bayesian analysis when considering:
  - Constraint from nuclear theory at low density
  - Imposing causal and stable EoS
  - Using known constraints from astronomical observations



Altiparmak, Eckert & Rezzolla, *Astrophys.J.Lett.* 939 (2022) 2, L34

LATTICE QCD - THE SIGN PROBLEM

When using Monte-Carlo sampling: $\langle \hat{O} \rangle = \frac{1}{Z} \int \mathcal{D}U \, \hat{O} \, \boxed{\det M[U] \, e^{-S_G[U]}}$

$\det M[U] \, e^{-S[U]}$ is used as a statistical weight

(least probable configurations U are ignored)

- **When $\mu_B^2 = 0$:** $\det M[U] \, e^{-S[U]}$ is real
- **When $\mu_B^2 > 0$:** $\det M[U] \, e^{-S[U]}$ becomes complex and has highly oscillating phase

→ **Can't be interpreted as a statistical weight!**

BUT! For purely imaginary μ_B (when $\mu_B^2 < 0$), $\det M[U] \, e^{-S[U]}$ is real again: **simulations possible...**

LATTICE QCD – THE NATURE OF THE TRANSITION

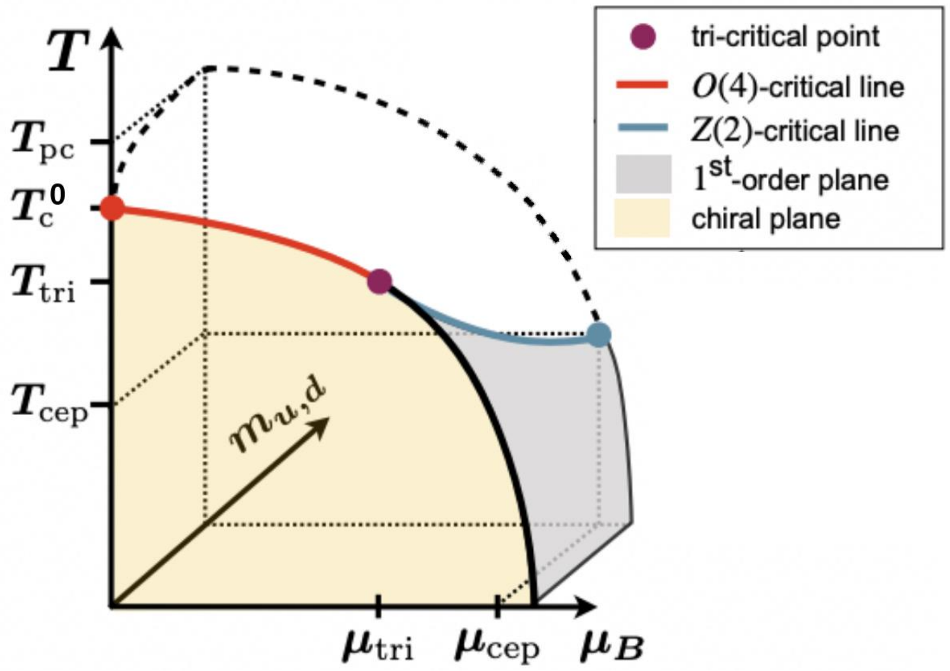
If one considers the case where $m_u = m_d = 0$ (chiral limit), the **phase transition** is expected to be of **2nd order**.

However, in real-life QCD, $m_u \simeq m_d \neq 0$, and the chiral symmetry is broken.

→ smooth crossover from hadron gas to QGP

(no discontinuity in 1st and 2nd order derivatives)

🫠 Hadrons basically melt like butter at room temperature 🫠



Ding et al., *Phys.Rev.D* 109 (2024) 11, 114516

ISING-2D T.EX.S EQUATION OF STATE FROM L-QCD

Implement scaling behavior of 3D-Ising model EoS:

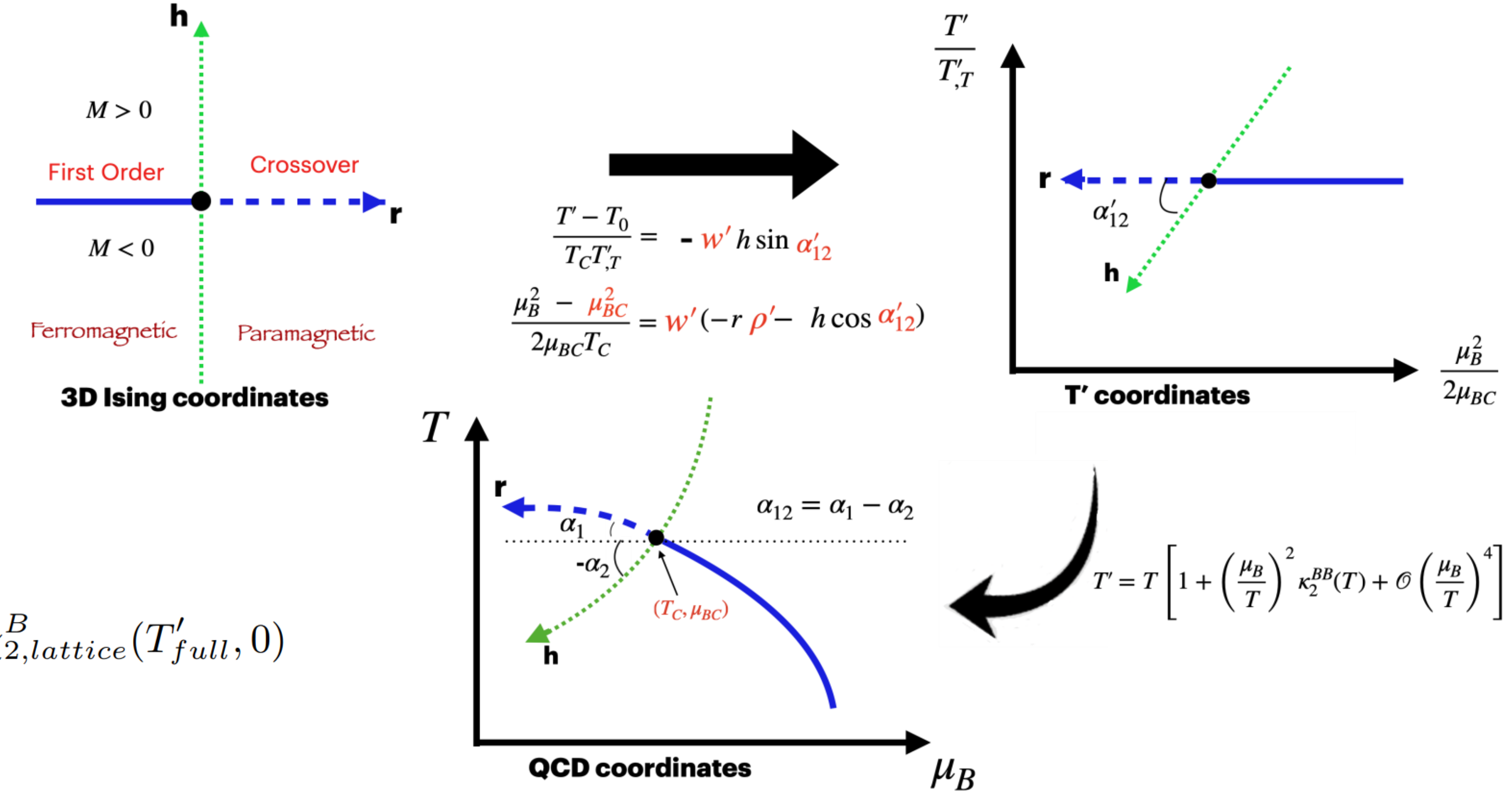
- Define map from 3D-Ising model to QCD

- Estimate contribution to Taylor coefficients from 3D-Ising model critical point

- Reconstruct full baryon density $\frac{n_B^{full}(T,\mu_B)}{(\mu_B/T)T^3} = \chi^B_{2,lattice}(T'_{full},0)$

  with

$$T'_{full}(T,\mu_B) = \underbrace{T'_{lattice}(T,\mu_B)}_{\text{lowest orders in }(\mu_B/T)} + \underbrace{T'_{crit}(T,\mu_B) - Taylor[T'_{crit}(T,\mu_B)]}_{\text{higher order in }(\mu_B/T)}$$



$$T'_{crit}(T,\mu_B) \approx \left(\frac{\partial \chi^B_{2,lat}(T,0)}{\partial T}\bigg|_{T_0}\right)^{-1} \frac{n_B^{crit}(T,\mu_B)}{(\mu_B/T)}$$

M. Kahangirwe, J.J., C. Ratti et al., PRD (2024); arXiv:2402.08636

LATTICE QCD – THE NATURE OF THE TRANSITION

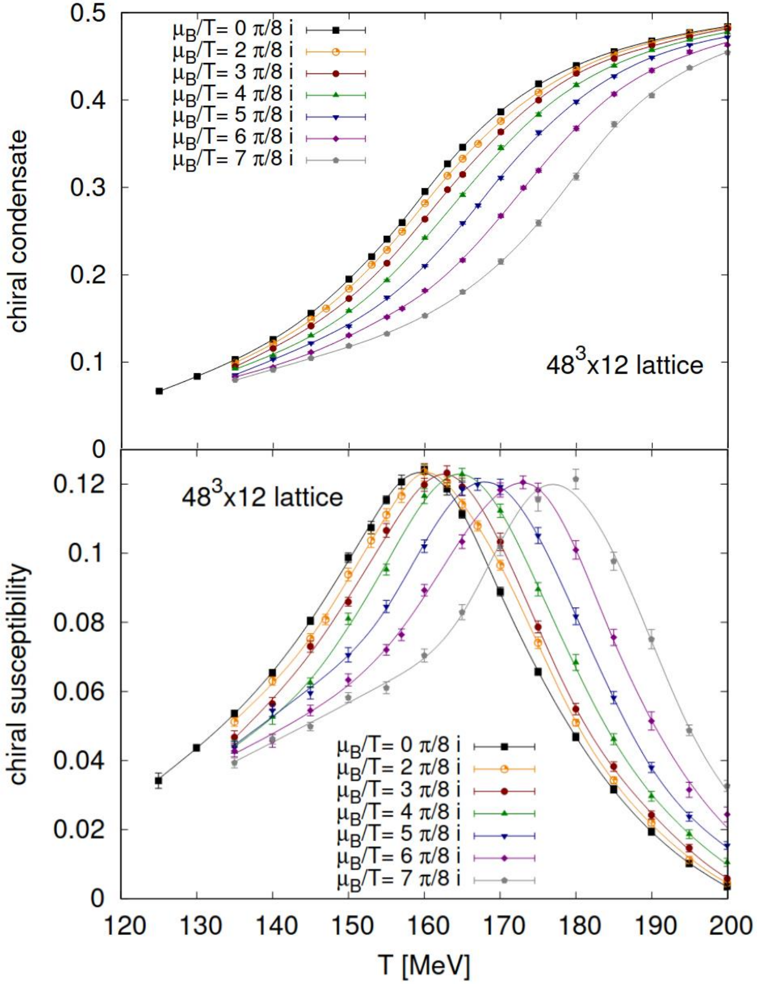
One can simulate lattice QCD at purely imaginary chemical potential, and determine the transition line by looking at:

- inflection point of the chiral condensate $\langle\bar{\psi}\psi\rangle = \frac{T}{V}\frac{\partial \ln Z}{\partial m_{ud}}$
- peak of the chiral susceptibility $\chi = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial m_{ud}^2}$

→ smooth crossover from hadron gas to QGP

(no discontinuity in 1st and 2nd order derivatives)

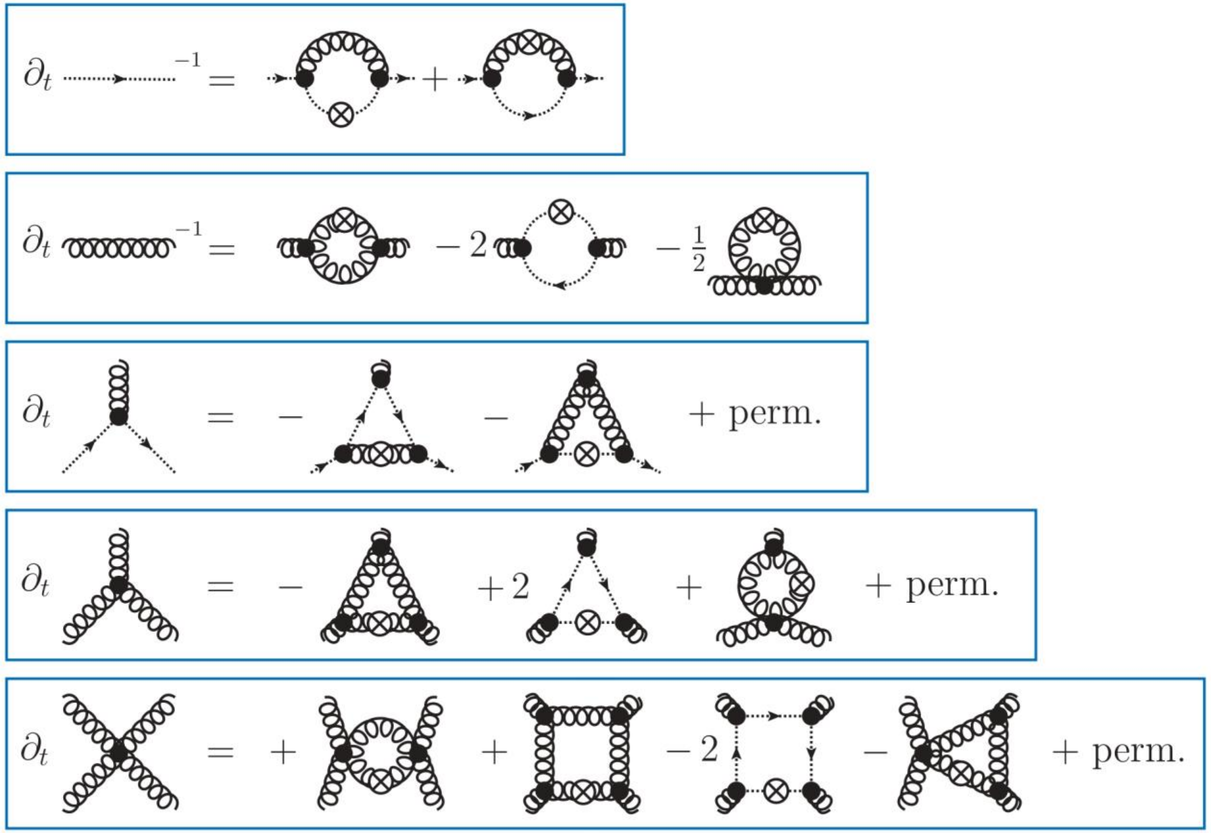
Hadrons basically melt like butter at room temperature



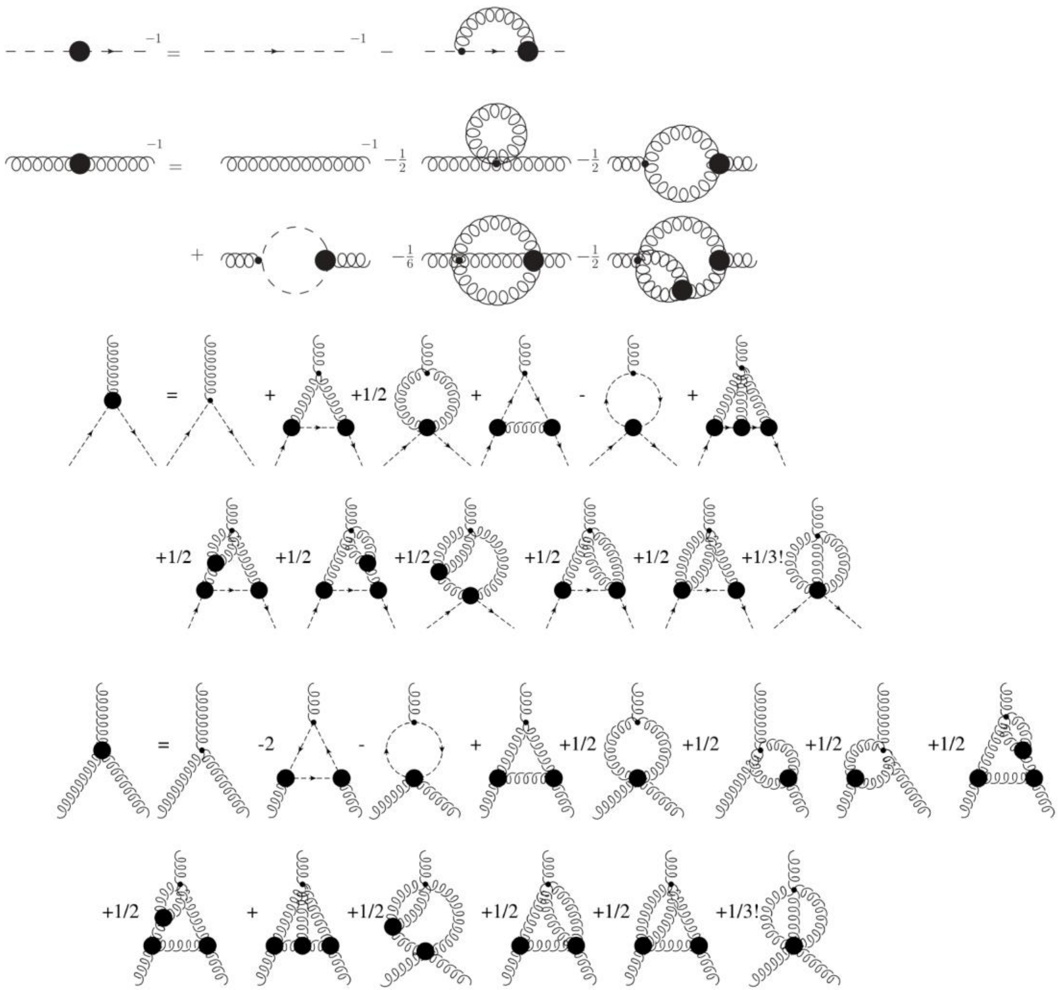
FUNCTIONAL QCD - A GLIMPSE

What we solve - in the gauge sector

FRG [Cyrol et al, 1605.01856]



DSE [Huber, Maas, von Smekal, 1207.0222]



+…+ quark contributions

Slide from *F. Rennecke, QM 2025*

EQUATION OF STATE FROM HOLOGRAPHY

String theory/Classical gravity in 5D	⟷	Quantum Field Theory in 4D

By solving the equations of motion (EoM) for a 5D Einstein-Maxwell-Dilaton (EMD) model defined by the following action:

$$S = \frac{1}{2\kappa_5^2}\int_{\mathcal{M}_5} d^5x\sqrt{-g}\left[R - \frac{(\partial_\mu\phi)^2}{2} - V(\phi) - \frac{f(\phi)F_{\mu\nu}^2}{4}\right]$$

(simplest action reproducing a realistic 4D QCD EFT)

...one can obtain the following thermodynamic quantities by

- using the UV behavior of the EMD fields
- fixing free parameters Λ, κ_5 and the functional form of $V(\phi)$ and $f(\phi)$ by matching with lQCD results at $\mu_B = 0$

$$T = \frac{1}{4\pi\phi_A^{1/\nu}\sqrt{h_0^{far}}}\Lambda \qquad s = \frac{2\pi}{\kappa_5^2\phi_A^{3/\nu}}\Lambda^3$$

$$\mu_B = \frac{\Phi_0^{far}}{\phi_A^{1/\nu}\sqrt{h_0^{far}}}\Lambda \qquad \rho_B = -\frac{\Phi_2^{far}}{\kappa_5^2\phi_A^{3/\nu}\sqrt{h_0^{far}}}\Lambda^3$$

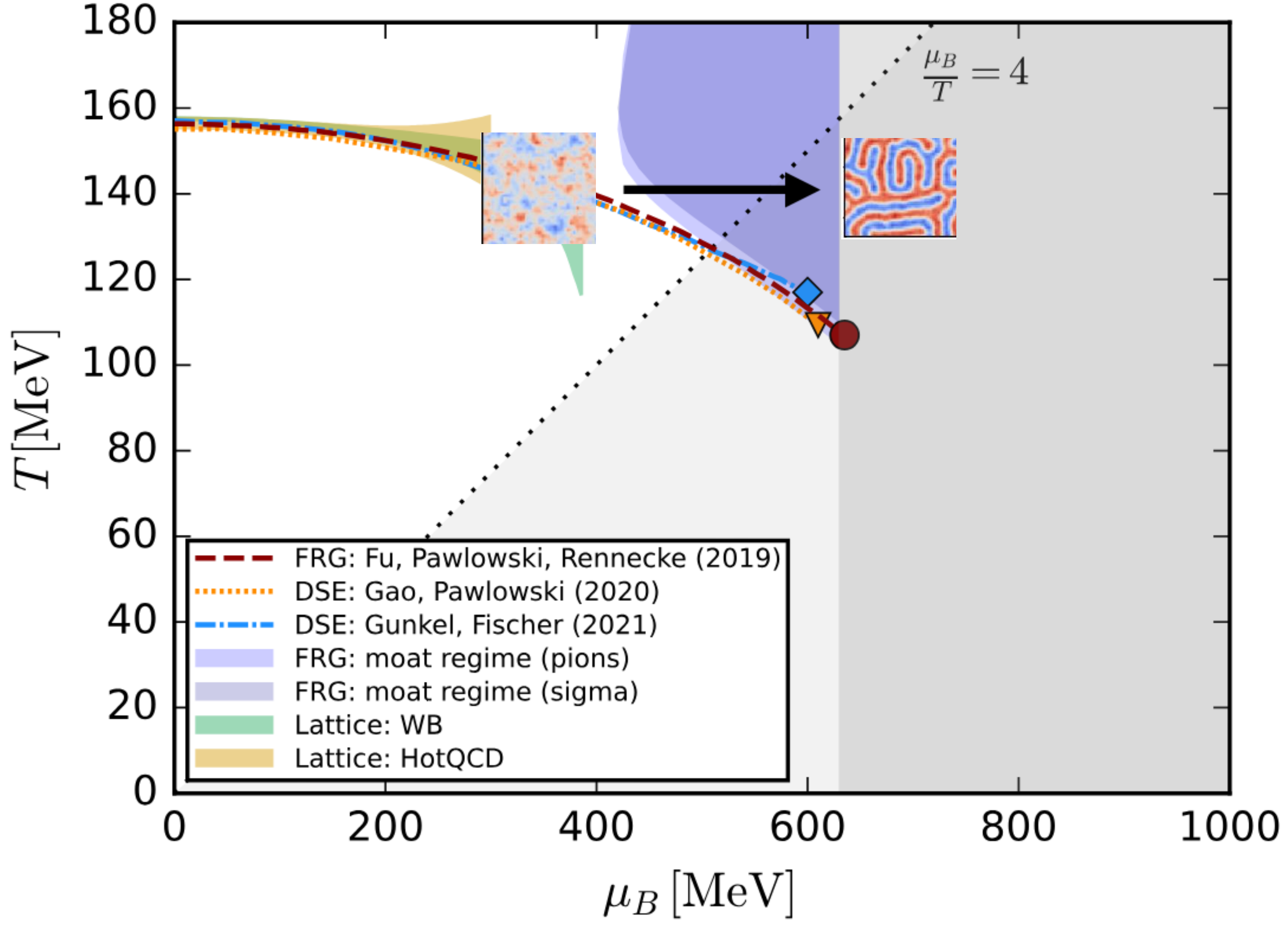
➞ **Only** describes **strongly-coupled QGP**

Grefa et al., *Prog.Part.Nucl.Phys.C* 135 (2024) 104093

PREDICTIONS FROM FUNCTIONAL METHODS

- Prediction of the "moat regime", an inhomogeneous phase of QCD matter

Braun *et al.*, *PRD* 111 (2025) 9, 094010



FREEZE-OUT COORDINATES

Using the HRG model, one can extract information for **temperature** and **chemical potentials** at:

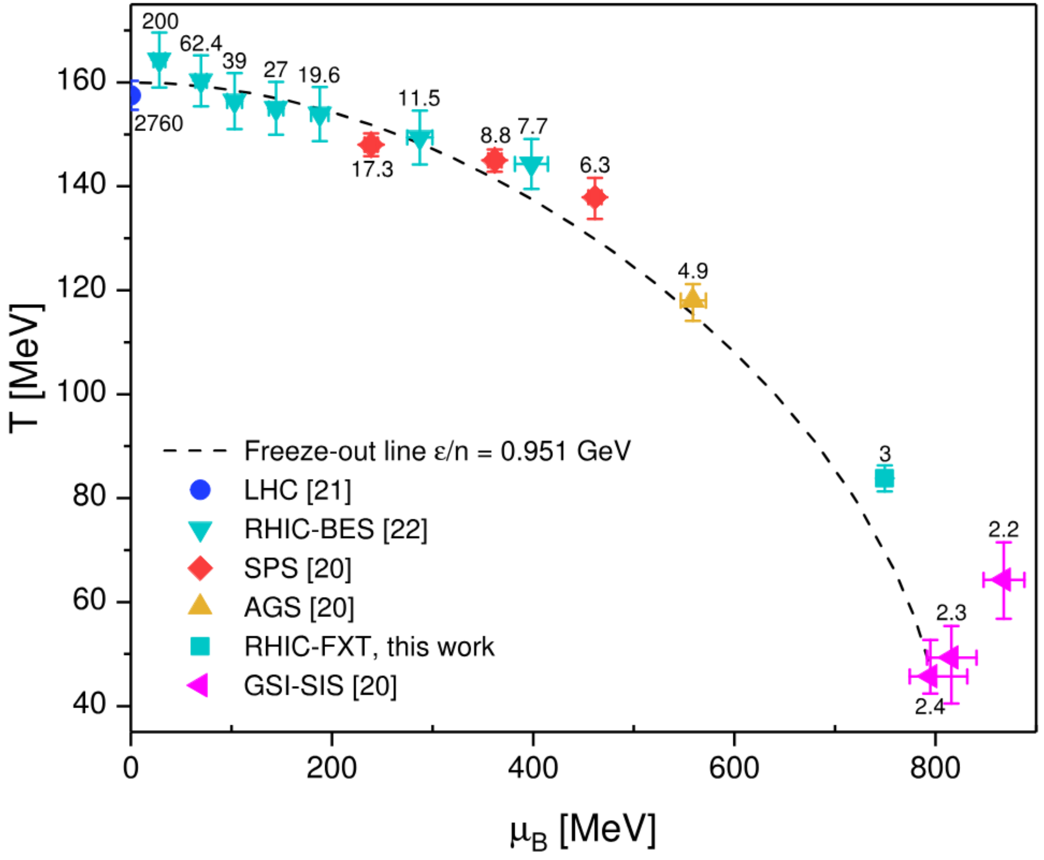
- **Chemical freeze-out:** when inelastic interactions cease, by fitting particle abundances through density

$$n_i^{\text{id}}(T,\mu_i) = \frac{d_i}{2\pi^2}\int dm f_i(m)\int_0^\infty k^2 dk\left[\exp\left(\frac{\sqrt{k^2+m^2}-\mu_i}{T}\right) + \eta_i\right]^{-1}$$

- **Kinetic freeze-out:** when all interaction cease, by fitting the transvers momentum spectra

$$\frac{dN}{p_T\, dp_T} \propto \int_0^R r\, dr\, m_T I_0\left(\frac{p_T \sinh\rho(r)}{T_{\text{kin}}}\right)\times K_1\left(\frac{m_T\cosh\rho(r)}{T_{\text{kin}}}\right)$$

assuming a simple radial flow velocity profile $\beta = \beta_S (r/R)^n$



Lysenko et al., arXiv:2408.06473

HOW TO LINK THEORY TO THE EXPERIMENTS?

The best way to confront theoretically calculated EoS with experimental results is to run **simulations**.

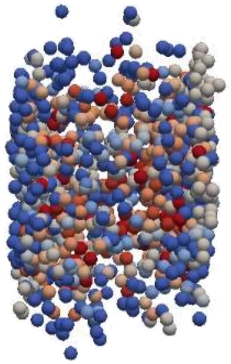
- Making data prediction/interpretation using pre-computed EoS
- Helps to constrain properties of the EoS by comparing to measurements

This is where FAIR enters the game!

Heavy-Ion collisions

Microscopic transport

- Hadrons/quarks as d.o.fs



SMASH

Event generators:

- EPOS4
- HIJING
- Angantyr
- AMPT
- UrQMD
- SMASH

…

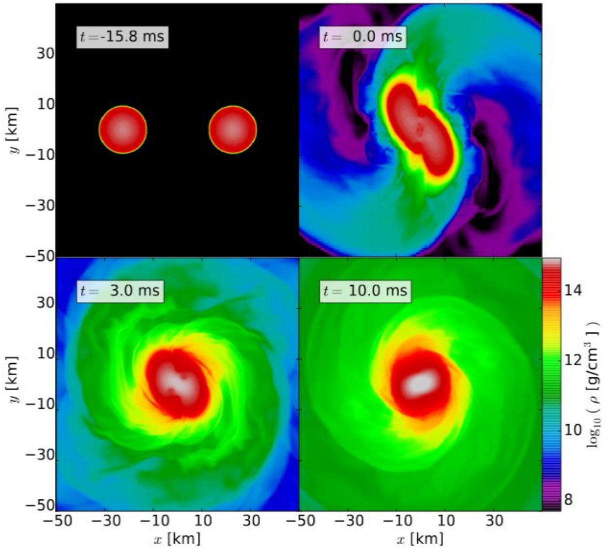
Relativistic hydrodynamics

- Mesoscopic scale (using densities)

MADAI

Neutron star mergers & Kilonovae

Relativistic Magnetohydrodynamics



Takami et al., *Phys.Rev.D* 91 (2015) 6, 064001

EXPERIMENTAL SEARCH OF THE CRITICAL POINT

Fluctuations of conserved charges

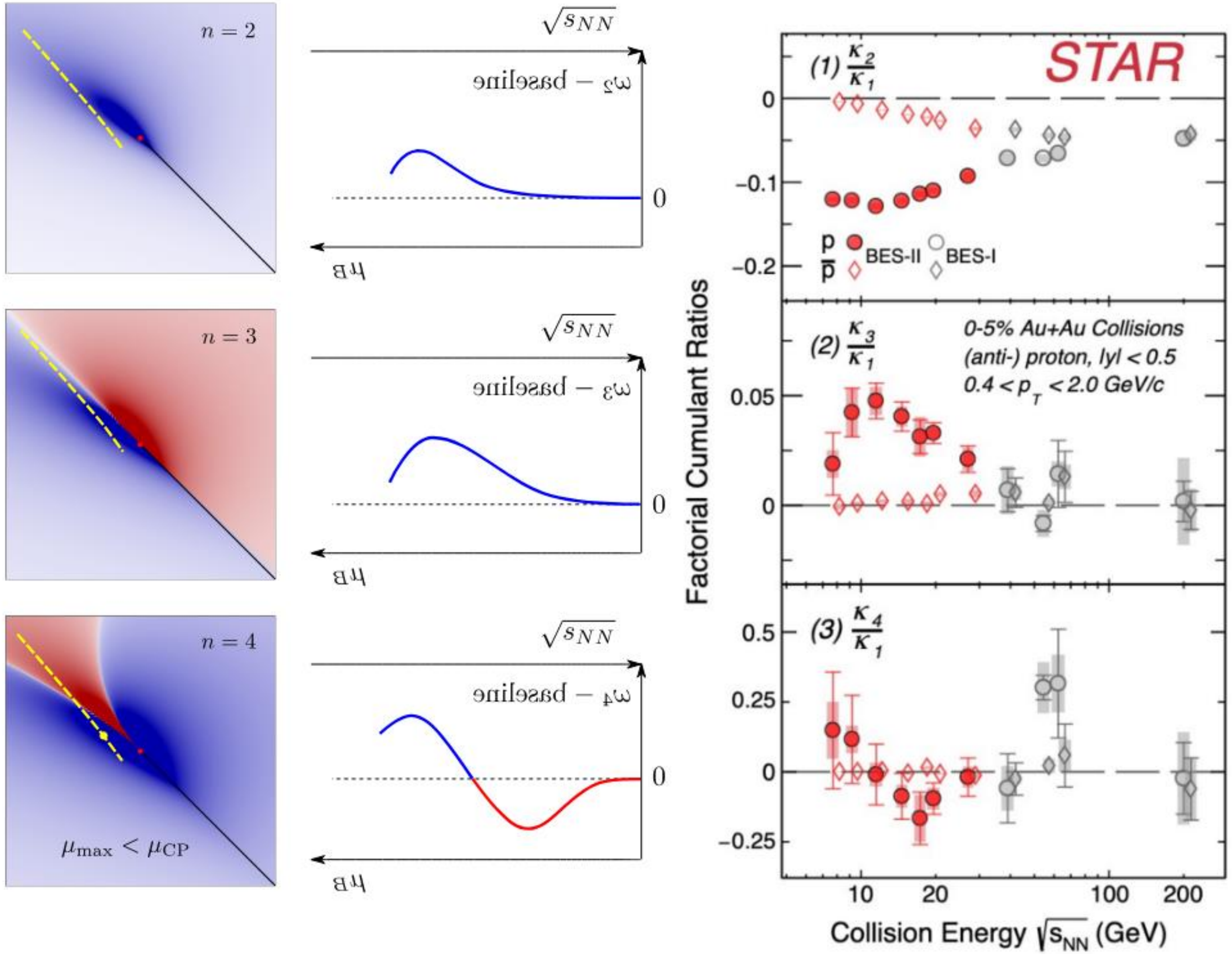
Cumulants of conserved charges are sensitive to the critical point
(diverge in its vicinity, in theory)

$$\kappa_n \propto \frac{\partial^n (\ln Z^{\rm gce})}{\partial \mu^n}$$

→ using **hadron species** as **proxies** for **conserved charges**

- Differences between highly dynamic and short-lived HICs make comparison far from straight-forward:
  - choice of proxies
  - finite-volume effects
  - volume fluctuations
  - acceptance

  …



Stephanov, *Strangeness in Quark Matter* 2024

INFERRING THE EOS FROM SPACE

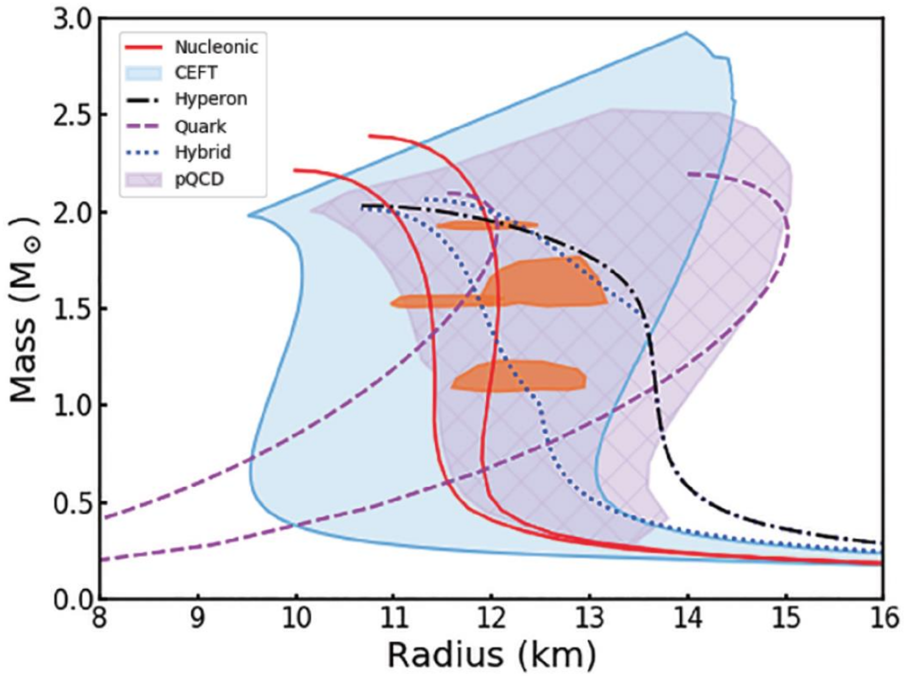
- **Properties of neutron stars**

  Using the Tolman-Oppenheimer-Volkoff equation, one can compute *(M,R)* relations for a given nuclear EoS:

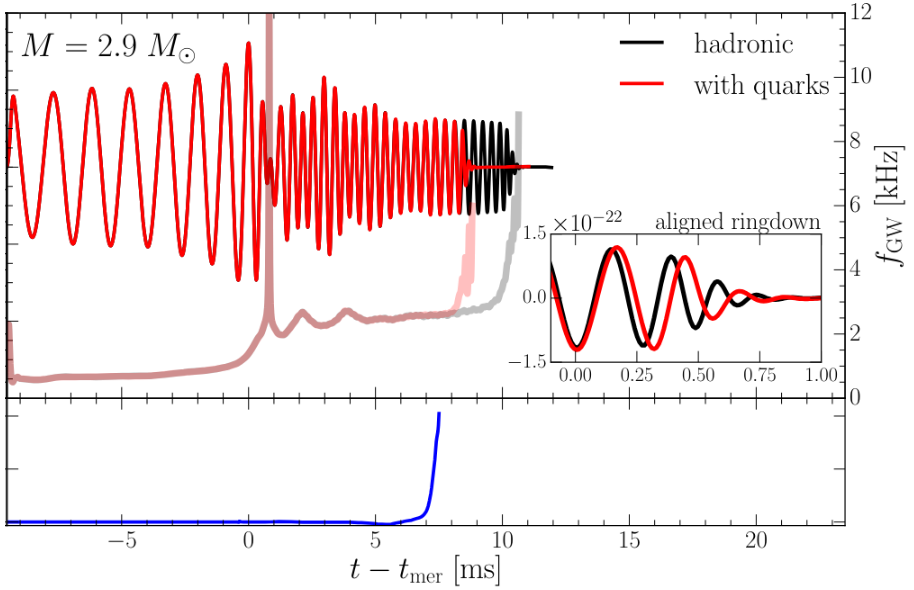
$$\frac{dP}{dr} = -\frac{G\epsilon(r)m(r)}{c^2r^2}\left[1+\frac{P(r)}{\epsilon(r)}\right]\left[1+\frac{4\pi r^3P(r)}{m(r)c^2}\right]\left[1-\frac{2Gm(r)}{c^2r}\right]^{-1}$$

  and confront the results to current measurements
  + other observables *(tidal deformability, quad. moment…)*

Tolos & Fabbietti, *Prog.Part.Nucl.Phys.* 112 (2020) 103770



Most et al., *Phys.Rev.Lett.* 122 (2019) 6, 061101

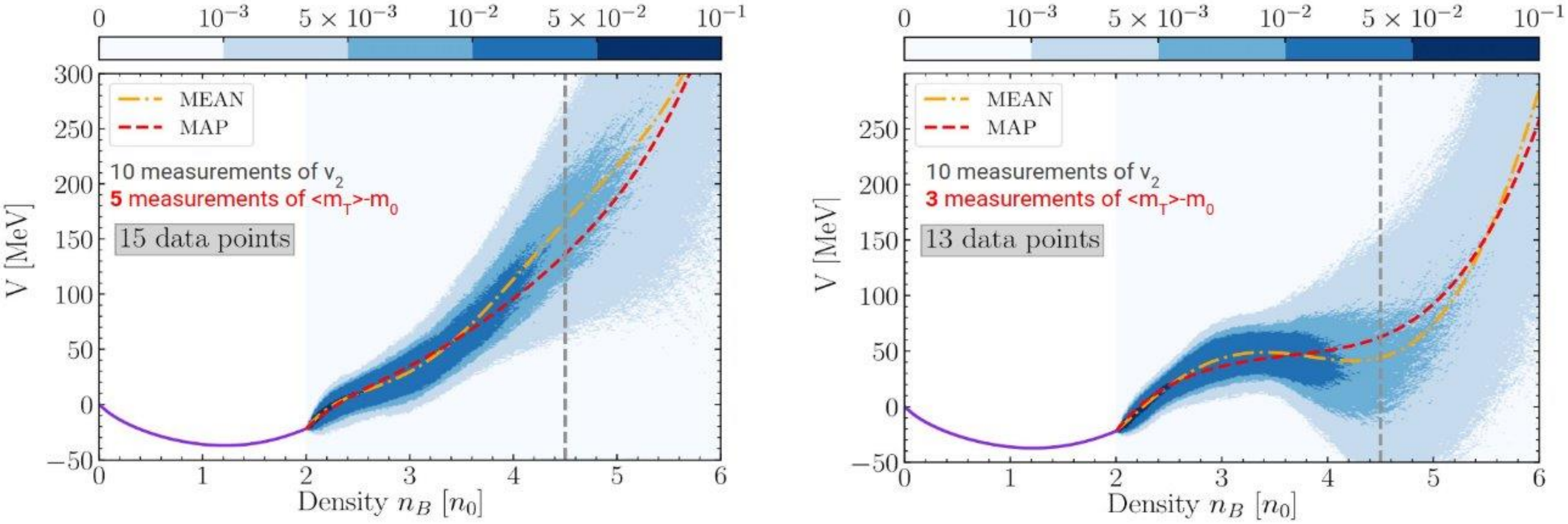


- **Gravitational waves detection**

  GW signals detected from neutron star mergers can help to infer NS properties, as well as the nature of the composition, and hence of the transition type

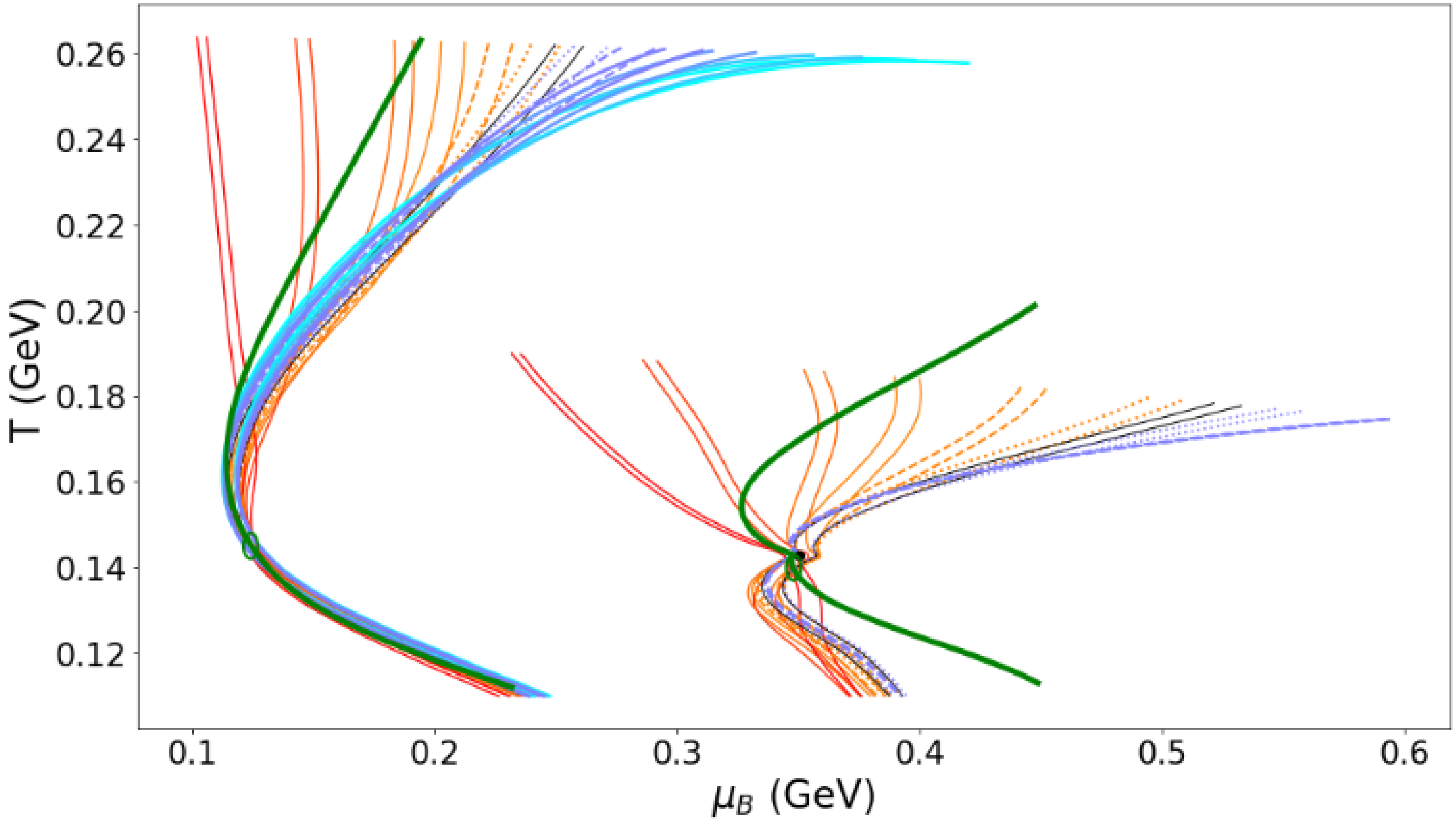
BAYESIAN INFERENCE OF THE EOS THROUGH HIC

M.O. Kuttan, *Strangeness in Quark Matter* 2024



Simulation achieved with UrQMD at different collision energies, using CMF equations of state.

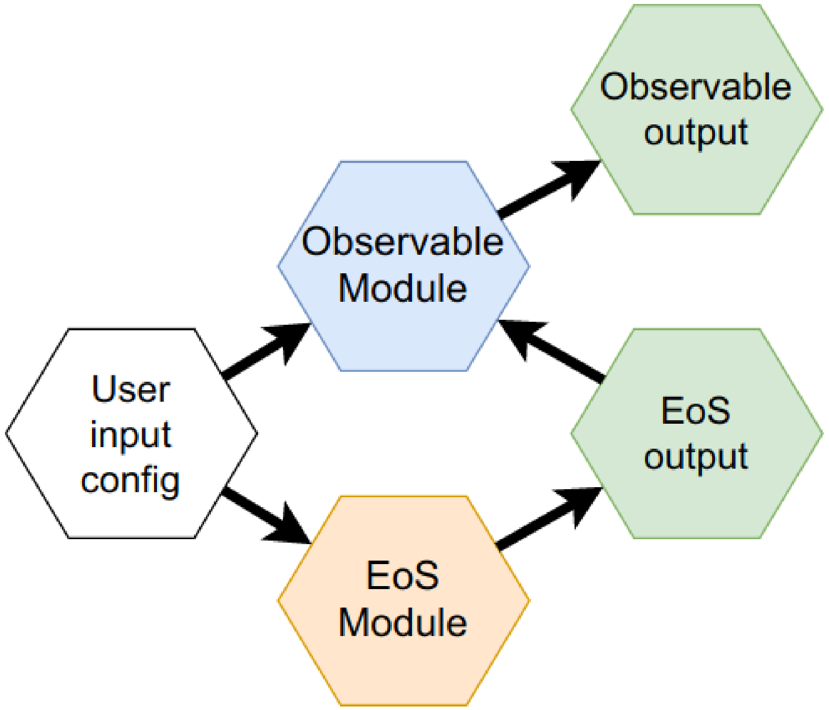
TRAJECTORIES ACROSS THE PHASE DIAGRAM



Dore et al., *J.Phys.Conf.Ser.* 1602 (2020) 1, 012017

WORKFLOWS IN MUSES

- Example of a typical workflow within MUSES, implying EOS generation + observable calculation



- More complex workflows can also be defined