# Lossless crossing of ½ resonance stopband via synchrotron oscillation at NSLS-II

Guimei Wang, Timur Shaftan, Victor Smalyuk, Yongjun Li, Richard Rand*
NSLS-II, Brookhaven National Lab
*Dept. of Mathematics and Dept. of Mechanical & Aerospace Engineering, Cornell University

Sept. 6-11, 2017, Cornell University
International Workshop on FFAG Accelerators

# Outline

- Introduction
- Dynamics of the particle crossing a static resonance stopband
- Controlling the resonance stopband width
- Parametric oscillator excited at half-integer resonance
- Concept of the experiment
- Experimental results
- Conclusions

# Introduction

- Lattice design common practice: place machine tunes clear of major resonances on tune diagram → tight limits on the magnitude of tune shifts with amplitude and momentum
- Difficulty to keep off-momentum tune footprint inside the resonance lines in modern ring lattices design (such as APS-U, HEPS) due to high nonlinearities.
- Half-integer (½) resonance, one of the major resonances, is always treated as an unstable working point that may cause beam loss and poses concerns in many circular accelerators, such as modern synchrotron light sources, heavy ion medical accelerators and non-scaling fixed-field alternating-gradient (FFAG) accelerators [1].
- APS-U latitce chromatic tune spread across the half-integer stopband, but tracking result did not show particle losses, in contrast to the experiments result [2, 3] on resonance crossing where the beam losses were observed. The authors explained this due to rapid transition through stopband together with substantial tune shift with amplitude.
- We investigated the beam dynamics during crossing of a major resonance in NSLS-II, one of the lowest-emittance storage rings worldwide and demonstrated that it is possible, both by design and by experiment, to achieve the storage ring conditions where the beam crosses the ½ resonance without particle loss.

[1] S. Machida, PR-ST AB, 11, 094003 (2008),
[2] T. Uesugi, S. Machida and Y. Mori, PR-ST AB, Volume 5, 044201 (2002), Heavy Ion Medical Accelerator in Chiba
[3] S. L. Sheehy et. al, Prog. Theor. Exp. Phys., 073G01 (2016), KURRI proton FFAG accelerator at Kyoto University

# Modern synchrotrons lattice: chromatic footprint

[1] Hybrid Seven-Bend-Achromat Lattice for the Advanced Photon Source Upgrade,

M. Borland, V. Sajaev, Y. Sun, A. Xiao, in Proc. of IPAC-2015

[2] APS-U Conceptual Design Report (2016)

[3] Statistical analysis of the limitation of half integer resonances on the available momentum acceptance of the High Energy Photon Source,

Y. Jiao and Z. Duan, NIMA 841, 97-103 (2017)

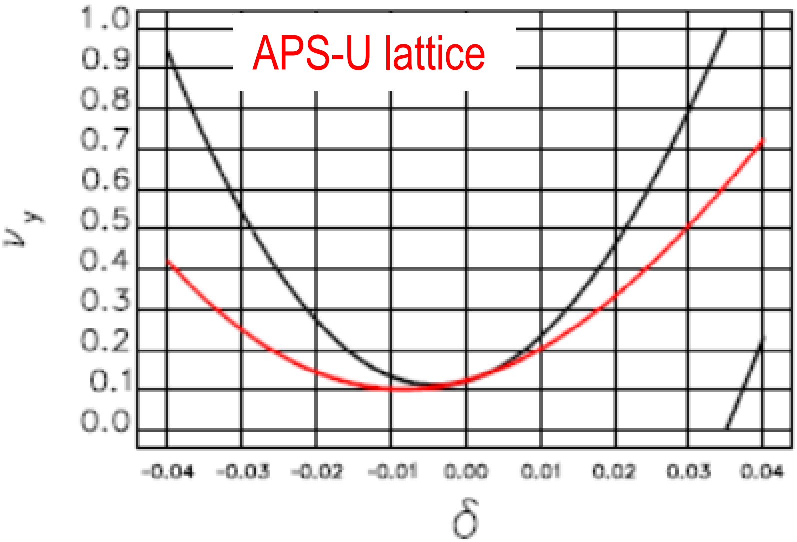



*Figure 3.9. Chromatic tune dependence for the nominal lattice with no errors.*

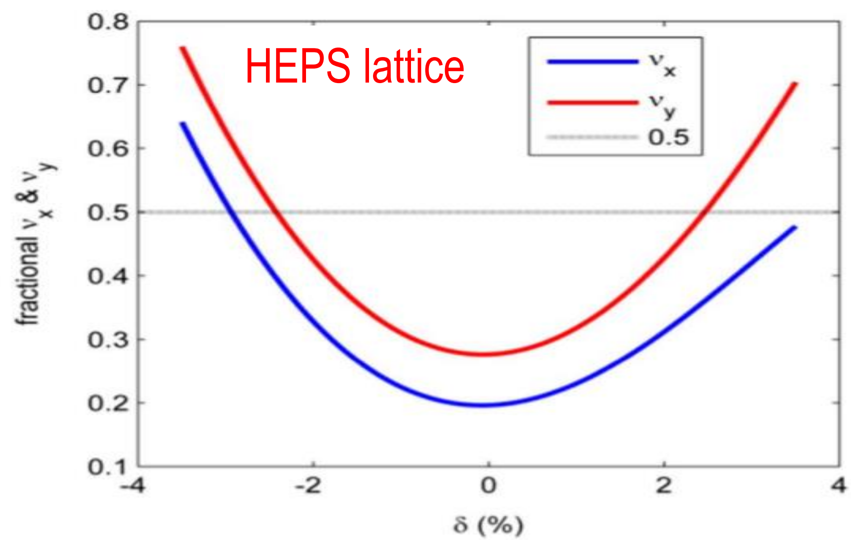



**Fig. 3.** Tune shifts with $\delta$ at $x$=1 μm of the presented HEPS lattice design

# NSLS II overview

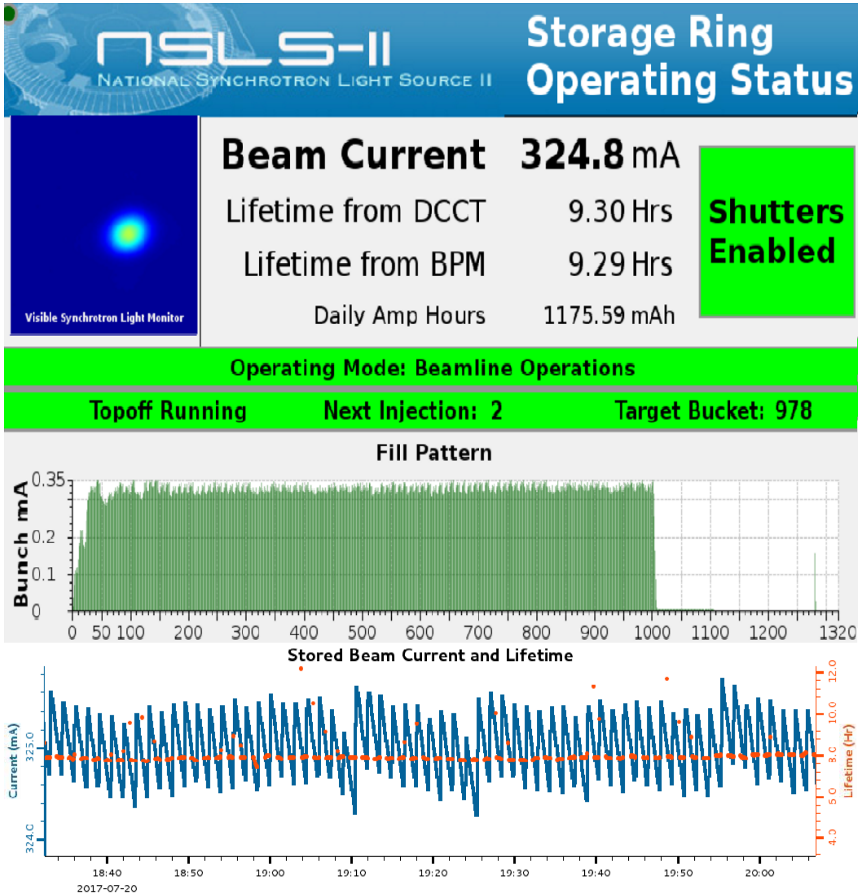
- National Synchrotron Light Source (NSLS-II) is a new 3 GeV, 500 mA, high-brightness synchrotron light source facility at the Brookhaven National Laboratory, funded U.S. Department of Energy (DOE).

- SR circumference is 792 m with 1 nm-rad horizontal and 8 pm-rad vertical emittance.

- NSLS II provides with:
  - wide spectral range
  - high average spectral brightness
  - high flux density
  - 60~80 beamlines

- 20 beamlines in routine top off operation at 325 mA.
  Stored beam current up to 400 mA.

- Beam emittance and beam orbit stability met the goals.



# DYNAMICS OF THE PARTICLE CROSSING A STATIC RESONANCE STOPBAND

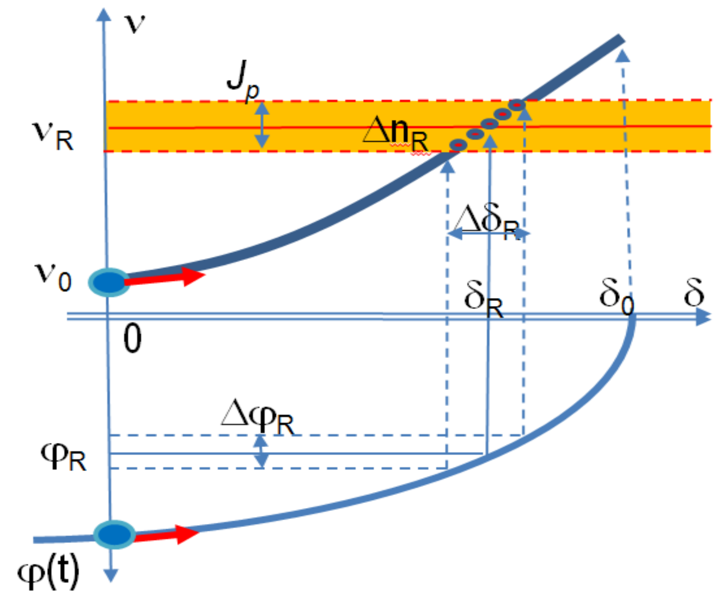
# Dynamics of crossing stopband

- Consider a storage ring model with large chromatic tune shift

  $$\nu(\delta) = \nu_0 + \xi_1\delta + \xi_2\delta^2 + O(\delta^3)$$

- 2-D case (y, $\delta$)
- Design lattice with small $\mathrm{d}\nu/\mathrm{d}J$
- Excite particle synchrotron oscillation

  $$\delta(n) = \delta_0 \sin(2\pi\nu_s n)$$




Cartoon of particle to crossing resonance via synchrotron oscillations

- Vary the tune crossing stopband speed by controlling chromaticity and synchrotron oscillation amplitude
- How many turns will it take to cross the stopband?

# Dynamics of crossing stopband

- Define $\delta_R = \delta(\nu_R)$ as the value of the energy deviation where the particle's tune crosses the resonance $\nu_R$
- Energy deviation boundaries corresponding to the resonance stopband $J_p$ are

$$\delta_R \pm \frac{\Delta\delta_R}{2} = \sqrt{\frac{(p/2 - \nu_0) \pm J_p/2}{\xi_2}}$$

- **Number of turns** to cross the stopband

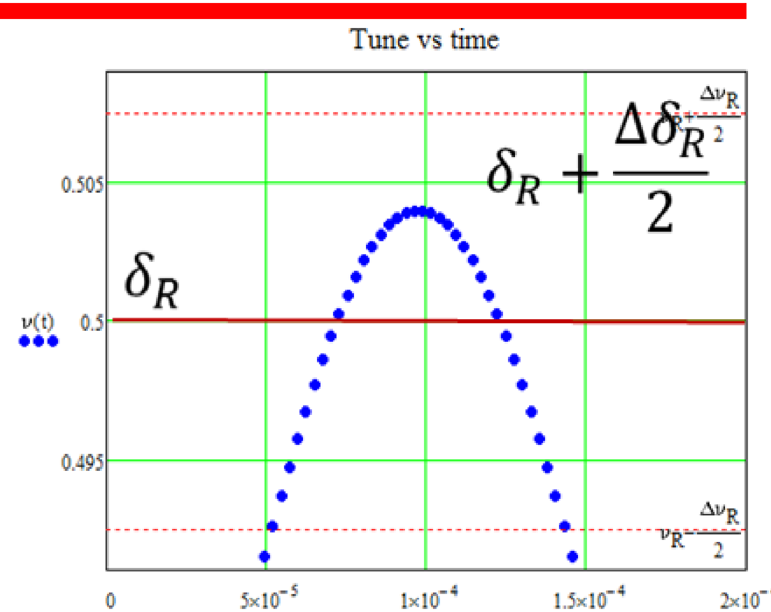
$$\Delta n_R \approx \begin{cases} \dfrac{(\mathrm{asin}(B_+) - \mathrm{asin}(B_-))}{2\pi\nu_s}, & \delta_0 > \delta_R + \dfrac{\Delta\delta_R}{2} \\ \dfrac{\mathrm{acos}(B_-)}{\pi\nu_s}, & \delta_R - \dfrac{\Delta\delta_R}{2} < \delta_0 \le \delta_R + \dfrac{\Delta\delta_R}{2} \end{cases}$$

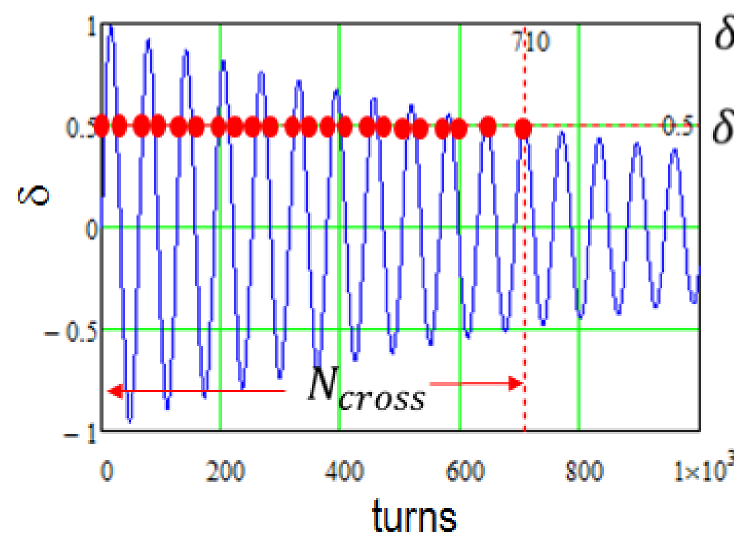
- Here $B_\pm = (\delta_R \pm \Delta\delta_R/2)\delta_0^{-1}$

- **Number of stopband crossings** during radiation damping of synchrotron oscillation

$$N_{cross} = -2\frac{\tau_s}{T_s} ln\left(\frac{\delta_R}{\delta_0}\right)$$

- $\tau_s$ damping time, $T_s$ synchrotron period



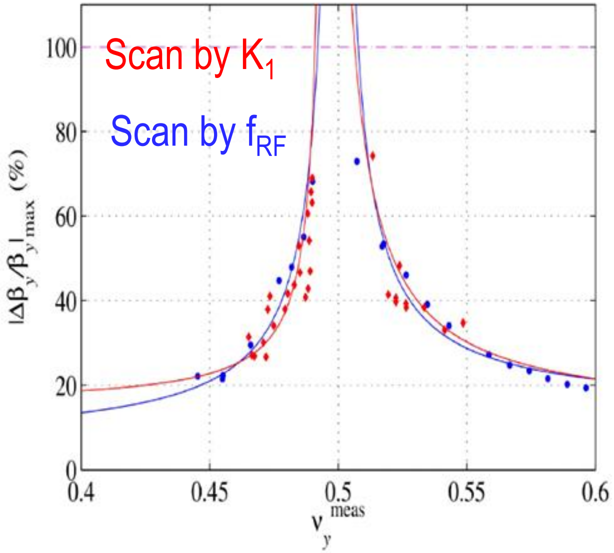
# CONTROLLING RESONANCE STOPBAND WIDTH

# Controlling resonance stopband width

- Stopband width is determined by quadrupole imperfections $\Delta k_1 L$
- Quadrupole perturbations around the machine cause
  - Tune shift (0th harmonics): $\Delta\nu_t = \frac{1}{4\pi}\sum_q \beta_q (\Delta k_1 L)_q$
  - Finite stopband width ($p$th harmonics): $J_p = \frac{1}{2\pi} \left| \sum_q \beta_q (\Delta k_1 L)_q e^{-ip\phi_q} \right|$
- The **resonance stopband width** is defined as the boundary of the tune range where the peak beta-beat $\Delta\beta/\beta = \frac{\beta-\beta_0}{\beta_0}$ reaches 100%

***Characterize and control NSLS-II resonance stopband***

- Park vertical tune in the vicinity of resonance $\nu$y=16.5
- Measured resonance stopband by measuring beta-beat
  - Move tune across ½ resonance with change of quads $K_1$
  - Move tune via the momentum deviation by shifting RF frequency
- The measured stopband width is 0.015
- Increase stopband width:
  - Select $N_q$ quadrupoles separated by n·π + π/2
  - Changed their strength by $\Delta k_1 L$
  - Yielding the maximum stopband width change of $\frac{1}{2\pi}\sum_{q=1}^{N_q} \beta_q (\Delta k_1 L)$




Measurement of the ½ resonance stopband width at NSLS-II

# PARAMETRIC OSCILLATOR EXCITED AT ½ RESONANCE

# Beam motion equation

- Hill's equation with small perturbation $\Delta k_1 \ll K_1$:
$$y'' + (K_1 + \Delta k_1)y = 0$$
- Transforming Hill equation into Floquet equation with normalized variables
$$\frac{d^2\eta}{d\phi^2} + \nu^2\eta = -\nu^2\beta^2\Delta k_1(\phi)\eta$$
  - $\eta = \frac{y}{\sqrt{\beta}},\ \phi = \frac{1}{\nu}\int_0^s \frac{ds}{\beta}$
  - $\nu = \frac{p}{2} + \Delta\nu$ is betatron tune, $p$ is an integer number, $\Delta\nu$ << $\frac{p}{2}$ is detuning from ½ resonance

- Floquet equation near parametric resonance → Mathieu equation:
$$\frac{d^2\eta}{d\phi^2} + \left(\nu^2 + 2\nu J_p\, cos(p\phi)\right)\eta = 0$$
- General solutions of static detuning with slow varying amplitude method
$$\eta(\phi) = \begin{cases} \sqrt{2(C_1^2 + C_2^2)}\left[\mu_1 \sin(\mu p\phi + \varphi_0)\cos\frac{p\phi}{2} + \mu\cos(\mu p\phi + \varphi_0)\sin\frac{p\phi}{2}\right], \frac{J_p}{2} < |\Delta\nu| & \text{Stable motion} \\ C_1\sqrt{\mu_1^2 + \mu^2}e^{\mu p\phi}\sin\left(\frac{p\phi}{2} + \varphi_1\right) - C_2\sqrt{\mu_1^2 + \mu^2}e^{-\mu p\phi}\sin\left(\frac{p\phi}{2} - \varphi_1\right), \frac{J_p}{2} > |\Delta\nu| & \text{Exp. increases} \end{cases}$$
$$C_{1,2} = f(\eta(0), \eta'(0)),\ \mu_{1,2} = \frac{\Delta\nu}{p} \mp \frac{J_p}{2p},\ \mu = \sqrt{\mu_1\mu_2}$$
- Fit machine parameters $\Delta\nu, J_p$… with TBT motion

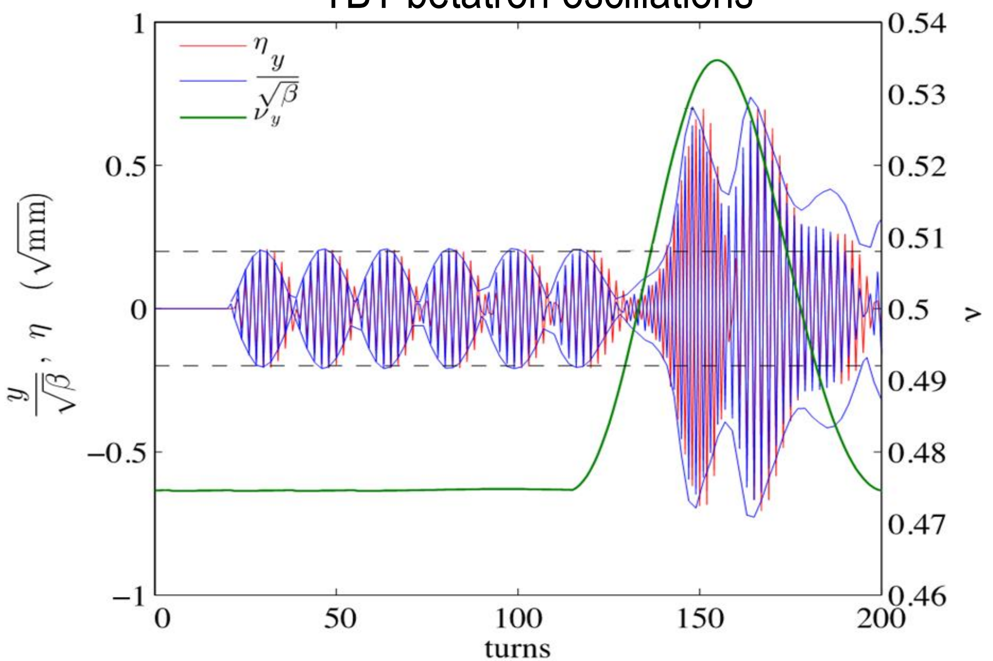
# Detuning vary during resonance crossing

- In the experiment, detuning is not constant by change every turn with synchrotron oscillation

$$\Delta\nu = \Delta\nu(n) = \Delta\nu_0 + \xi_1\delta(n) + \xi_2\delta(n)^2$$

- Slow change in $\nu$: $\frac{1}{\nu}\frac{d\nu}{dn} \ll 1 \rightarrow \mu$ varies slowly and a large number of betatron oscillations to cross stopband
- Numerical solution and Particle tracking agree well

TBT betatron oscillations




Equation of numerical solution

Particle tracking using code Elegant

Tune evolution with stopband borders in dashed lines

- Estimation of increase of betatron oscillation amplitude before and after a single passage of the stopband:

$$\left(\frac{\eta_f}{\eta_i}\right)_{max} \approx exp(\pi J_p \Delta n_R)$$

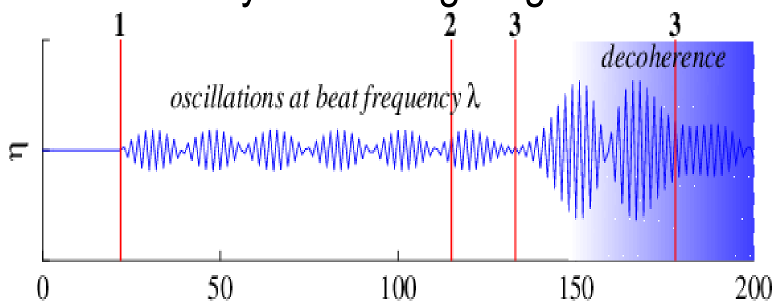
- Assumed instantaneous growth rate $\mu$ at the maximum growth rate

# CONCEPT OF THE EXPERIMENT

# Setup Steps

- Preparations
  - Design a lattice with large 2nd order chromaticity, $\xi_{1y}$=+1, $\xi_{2y}$=+300 (similar to the APS-U design lattice) by changing ring sextupoles
  - Optimize lattice
  - Characterize machine parameters, including chromaticity, 1st and 2nd order dispersion, stopband width
  - Moved the y plane tune $\nu_0$ to the proximity of half integer resonance at 16.5
- Experiment setup
  - Set BPM trigger to get TBT data acquisition
  - Pulse transverse pinger → excite betatron oscillations
  - Align RF trigger timing along transverse pinger, BPMs
  - Pulse RF jump/pinger → excite synchrotron oscillation
  - Save RF data of A, φ and BPM TBT data of x, y, sum

System timing diagram



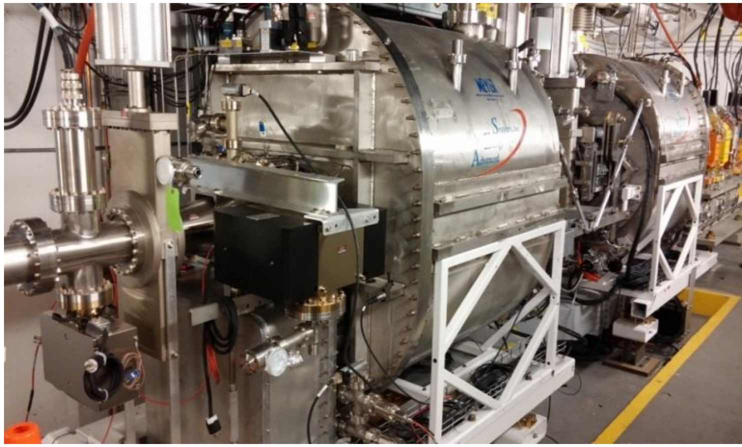

1: H/V kicker, 2: RF pinger, 3: resonance crossing

- Data process
  - TBT energy oscillation: retrieved from TBT x of the BPMs located in the dispersion region
  - TBT y tune modulation: TBT y and TBT energy with measured chromaticity
  - Particle loss: TBT sum

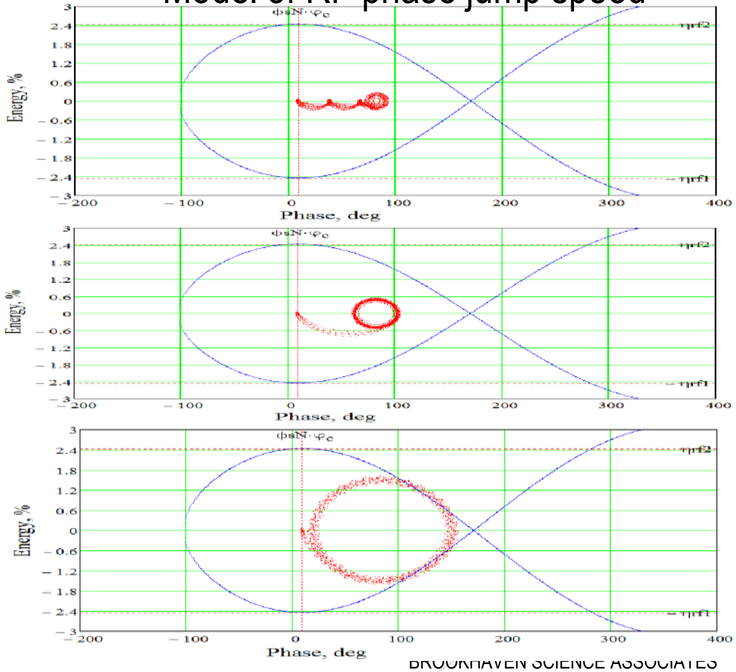
# RF pinger

- **RF pinger** = Sudden change of RF cavity phase or voltage → induce longitudinal beam oscillations
- NSLS-II: two SRF cavities with 300 kW transmitters, operated up to 1.8 MV
- LLRF controller (developed at NSLS-II): capable of generating flexible set-point tables and control parameters of RF feedback
- Manipulate $\varphi(t)$ and $A(t)$ within short timescale ($\frac{1}{4}T_s$) with flexible and fast digital LLRF controller
- Speed of RF jump ($d\varphi/dt$) is limited by transmitter trip due to high $P_{reflect}$ or waveguide arc
- Developed a simple 1D model with various ramp shapes → Model study RF phase jump with short, medium or long transition period
- Optimized RF gain $K_p$, $K_i$ to excite large synchrotron motion without RF system trip

RF cavities in NSLS-II tunnel



Model of RF phase jump speed



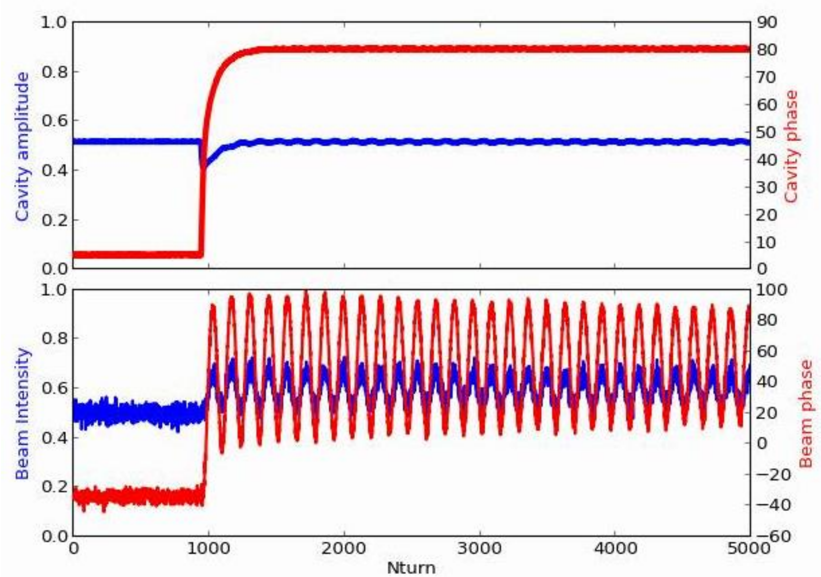
# EXPERIMENTAL RESULTS

# Experiment condition

*2 stopband scenarios:*

- "Small" stopband: $J_p$~0.016
  - Beam crosses stopband in up to 25 turns
- "Large" stopband: $J_p$~0.038. Changed 10 quadrupole knobs separated by $n \cdot \pi + \pi/2$ in phase advance to increase the width.
  - Beam takes up to 50 turns for crossing stopband
- RF jump in ~50…80 turns

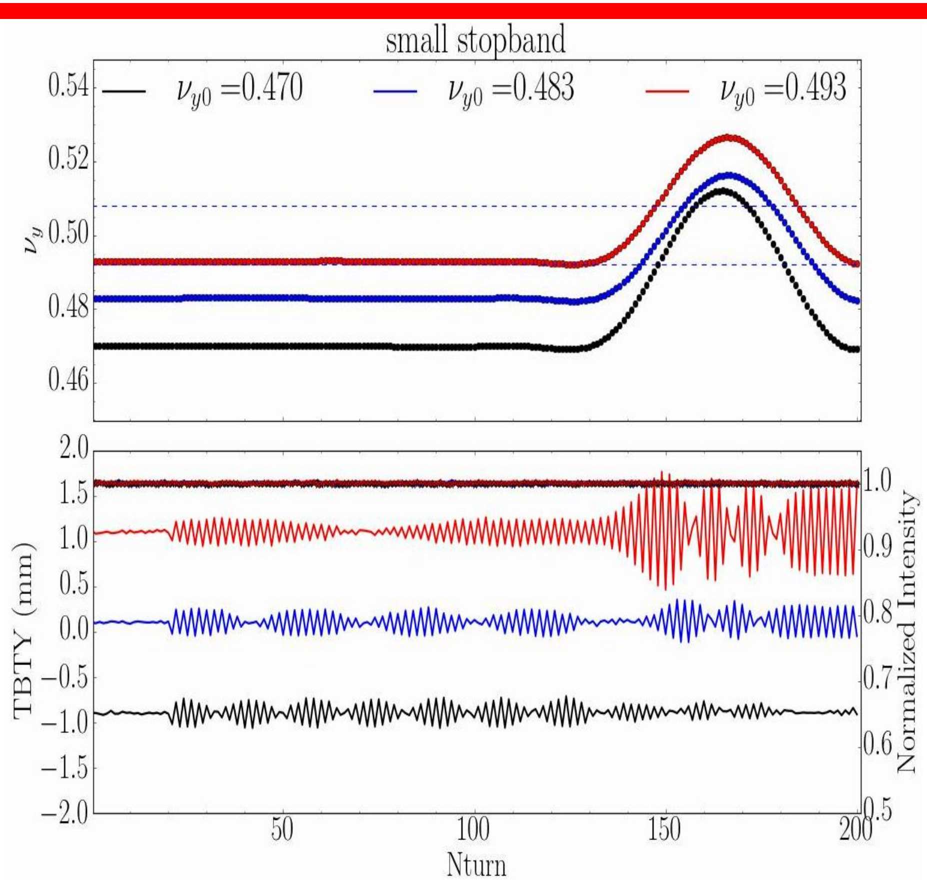
- Beam current: a few mA to minimize beam loading/collective effect
- NSLS-II small beam size and energy spread allowed coherent oscillations during the crossing of resonance stopband as if the beam was a single particle
- Transverse and longitudinal pinger amplitudes are ~ 100 times larger than natural beam divergence and energy spread

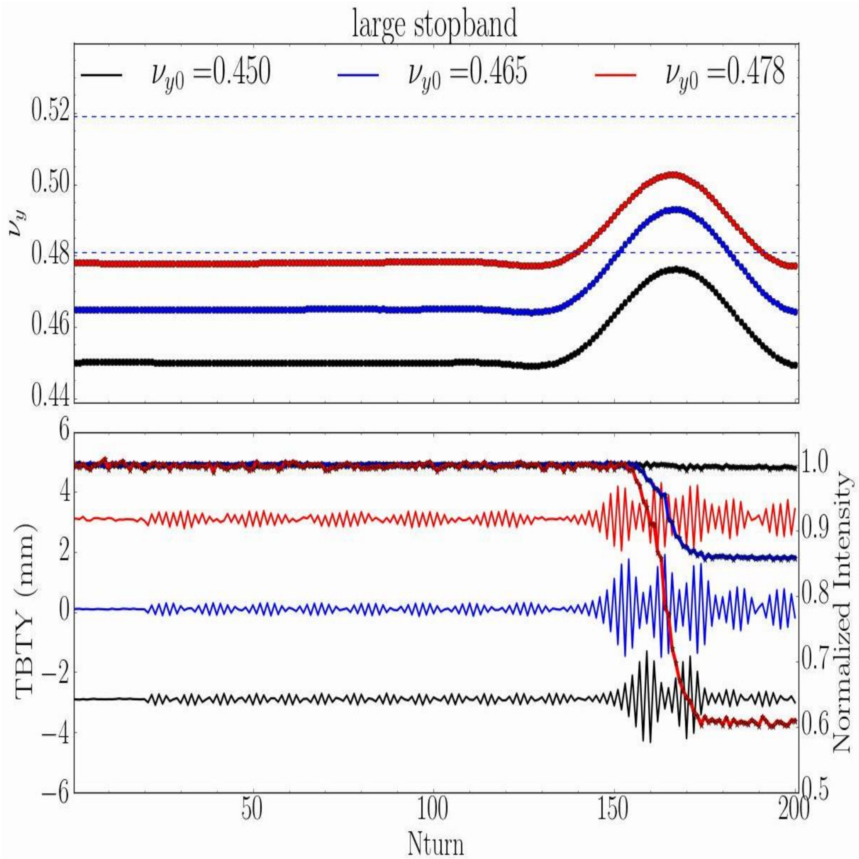



NSLS-II storage ring beam parameters

| Parameters | Value |
|---|---|
| Vertical betatron tune | 16.26…16.55 |
| Revolution period, μsec | 2.64 |
| Synchrotron tune | 0.00625 |
| Damping time (x/y/z), msec | 55.3/55.3/27.7 |
| Vertical emittance, pm·rad | 30 |
| Energy spread, % | 0.05 |
| Vertical betatron tune | 16.26…16.55 |

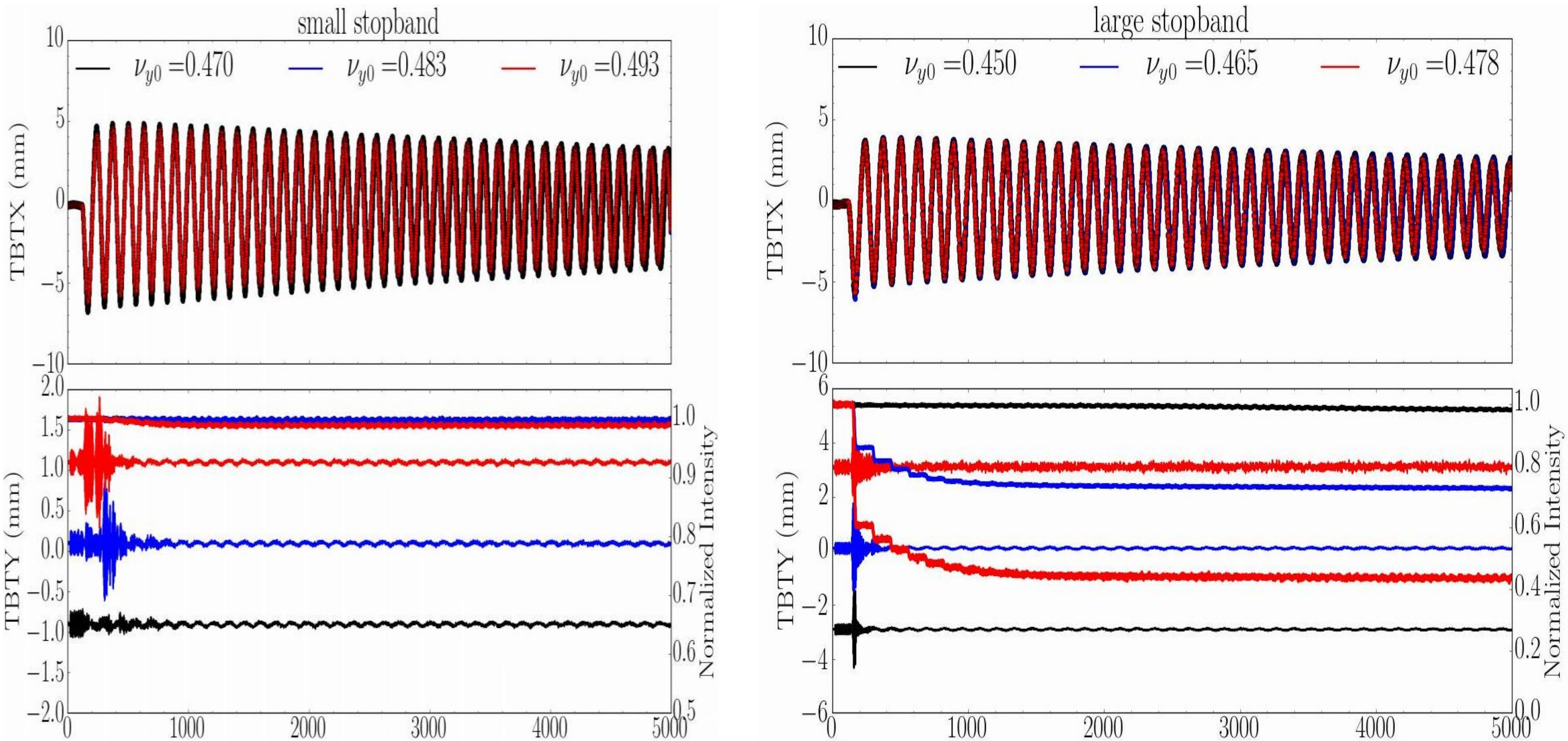
# Small vs Large stopband




- Beam energy oscillation amplitude is about +/-1.4%
- Position initial tune at different value to vary number of turns to cross ½ stopband
- Vertical plane motion exhibits the behavior of typical parametric resonance
- With small stopband, beam can cross ½ without loss
- With large stopband, beam loss occurs only when the beam is moving through the resonance for about 40 turns and then repeat during subsequent synchrotron oscillations.

# Long period experimental data




- Beam position and beam intensity evolution over half of the radiation damping time
- TBT x waveform asymmetric due to contributions of the second order dispersion
- The oscillation envelope decays due to the radiation damping
- Limitations: clear exponential-like growth of betatron motion is visible only during the first resonance crossing. During the next few synchrotron oscillations the BPM TBT signal blurs due to the decoherence and filamentation as the particles are repetitively passing through the stopband

# Summary and outlook

- A study focused on beam dynamics in a circular machines with tune footprint that can span across major resonances
- It is possible, both by design and by experiment, to achieve the storage ring conditions where the beam crosses the ½ resonance without particle loss:
  - This can be accomplished if the stopband is narrow due to small residual magnets field errors and is further controlled by harmonic field errors cancellation around the ring
  - The combination of the small stopband width with a large magnitude of nonlinear chromaticity leads to the rapid crossing of the resonance, which does not cause loss of the particles as demonstrated by our experiments.
- This is the first time in storage ring experiments that the beam crossed the half-integer resonance without losses due to the fast passage and narrow resonance stopband width
- We studied the motion of particles in the vicinity of a single resonance and could not cross the whole range of tunes covered by the footprint similar to APS-U/HEPS lattice with limited momentum oscillations from RF-pinger technique

# Acknowledgement


- We would like to acknowledge RF group members, control group members and diagnostics group members, Feng Gao, Brian Holub, Carlos Marques, John Cupolo, Anton Derbenev, Juri Tagger and Weixing Cheng for their support.

- We are grateful to F. Willeke and S.Y. Lee for helpful discussions and advice related to this paper. T. Shaftan is thankful to N. A. Vinokurov and E. B. Levichev for insightful discussions on particle dynamics.

# References

[1] E. D. Courant and H. S. Snyder (1957), in Annals of Physics 281, p 360 (2000)
[2] APS-U Conceptual Design Report (2016)
[3] Y. Jiao and Z. Duan, NIMA 841, 97-103 (2017)
[4] Y. Cai and Y. Nosochkov, SLAC-PUB-9812 (2013), http://www.slac.stanford.edu/cgi-wrap/getdoc/slac-pub-9812.pdf
[5] T. Uesugi, S. Machida and Y. Mori, PR-ST AB, Volume 5, 044201 (2002)
[6] S. Machida, PR-ST AB, 11, 094003 (2008)
[7] S. L. Sheehy et. al, Prog. Theor. Exp. Phys., 073G01 (2016)
[8] R. Chasman, et al., Proc. IXth International Con. On High Energy Accelerators, SLAC, p 604 (1974)
[9] L. R. Evens and J. Gareyte, IEEE Trans. Nucl. Sci. NS-24, 2397 (1983)
[10] H. Bruck, Particle Accelerators, 11, p 37 (1980)
[11] J. Shi, R.L. Gluckstern and S. Ohnuma, 3rd European Particle Accelerator Conference, Tech. Univ. of Berlin, March 24-28, BNL preprint 47477 (1992)
[12] A.W. Chao and M. Month, Nuclear Instruments and Methods, Volume 121, Issue 1, p 129 (1974)
[13] C. Pellegrini and A. M. Sessler, Nuclear Instruments and Methods, Volume 84, p 109 (1970)
[14] L.J. Laslett and W. A. Perkins, Nuclear Instruments and Methods, Volume 97, p 523 (1971)
[15] W. P. Lysenko, Particle Accelerators, 5, p 1 (1973)
[16] S.Y. Lee, Phys. Rev. Lett. 97 104801 (2006)
[17] X. Pang, S.-Y. Lee, et al., Hadron Beam 2008, p 118 (2008)
[18] S.Y. Lee et al., Phys. Rev. Lett. 110, 094801 (2013)
[19] S.R. Mane and W.T. Weng, Nuclear Instruments and Methods, 306, p 9 (1991)
[20] O. Anchugov et al., Journal of Experimental and Theoretical Physics, Vol. 109, No. 4, p 590 (2009)
[21] R. Baartman and G.H. Mackenzie, East Lansing, Michigan, USA, p 40 (1984)
[22] J. Garland, https://www.escholar.manchester.ac.uk/uk-ac-man-scw:216851 (2013)
[23] S.Y. Lee, Accelerator Physics, World Scientific, Singapore, 3rd edition, (2012)
[24] V. Smaluk et al., in Proceedings of IPAC 2016 conference, Busan, Korea, p. 3400 (2016)
[25] J. Milutinovich, et al., RHIC Tech. Note No. 56 (1989)
[26] G.M. Wang and T. Shaftan, BNL Tech. No. 234 (2016)
[27] M. Borland, "elegant: A Flexible SDDS-Compliant Code for Accelerator Simulation," Advanced Photon Source Note LS-287, September 2000
[28] S.Y. Lee et al., New Journal of Physics. 8, 291 (2006)
[29] L. Ng, R. Rand and M. O'Neil, Journal of Vibration and Control June, Volume 9, No. 6, p 685 (2003)
[30] G.M. Wang et al., Rev. Sci. Instrum. 87, 033301 (2016)
[31] G. M. Wang et al., in Proceedings of IPAC 2016 conference, Busan, Korea, p 3161 (2016)