FFAG Optics and Design

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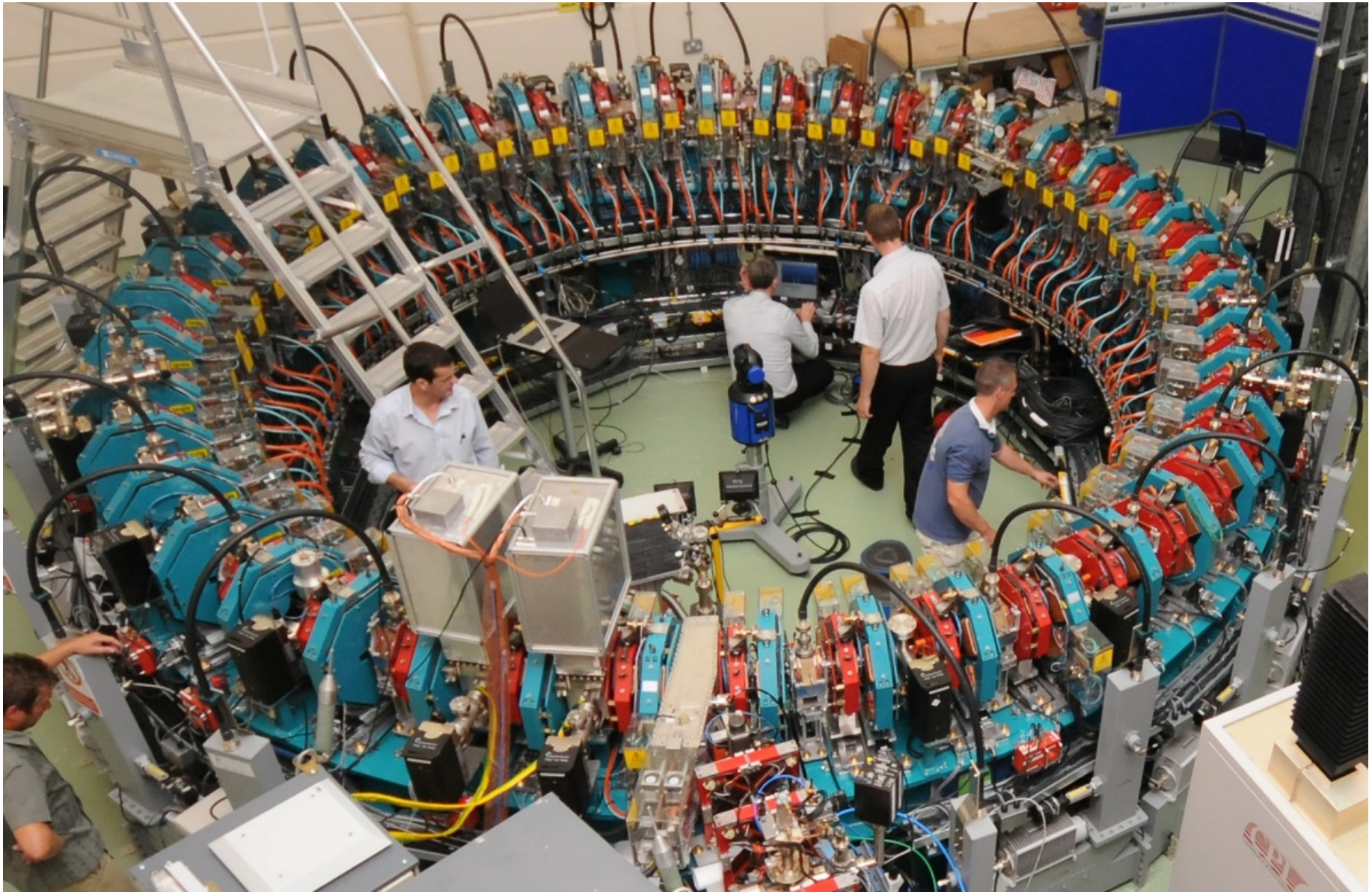
Key Concepts

- Superperiods
- Energy-dependent periodic orbits

Superperiods

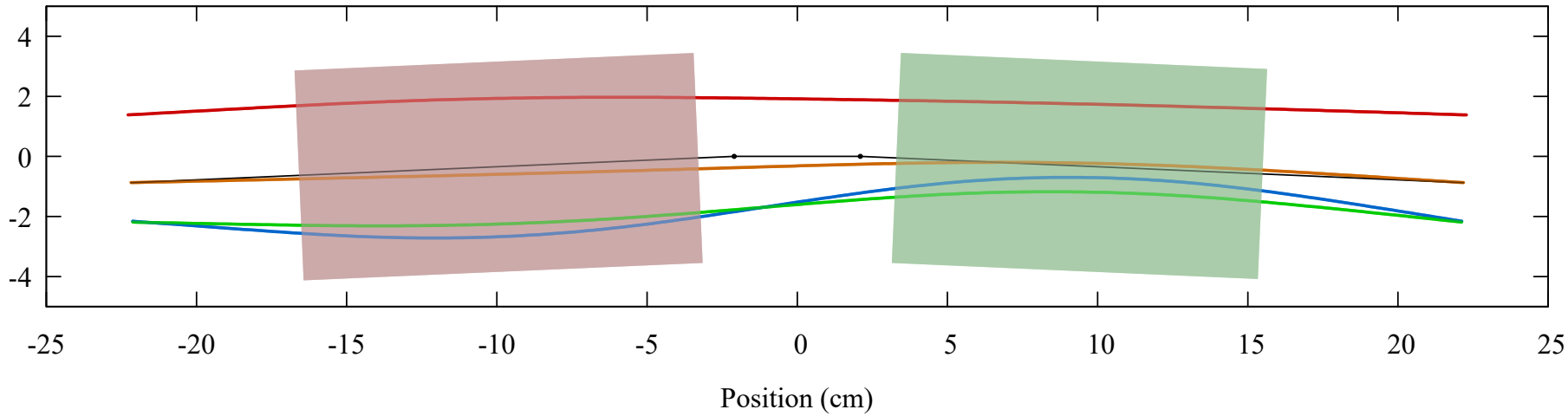
- FFAGs generally consist entirely of a single simple, identical cells, repeated around the ring
  - The cell generally only has two magnets, though occasionally more
- This means we can focus only on the superperiod: a single cell
  - We only need worry about the linear resonances of the single cell
  - This is what allows a wide range of energy stability

Superperiods



Periodic Orbits

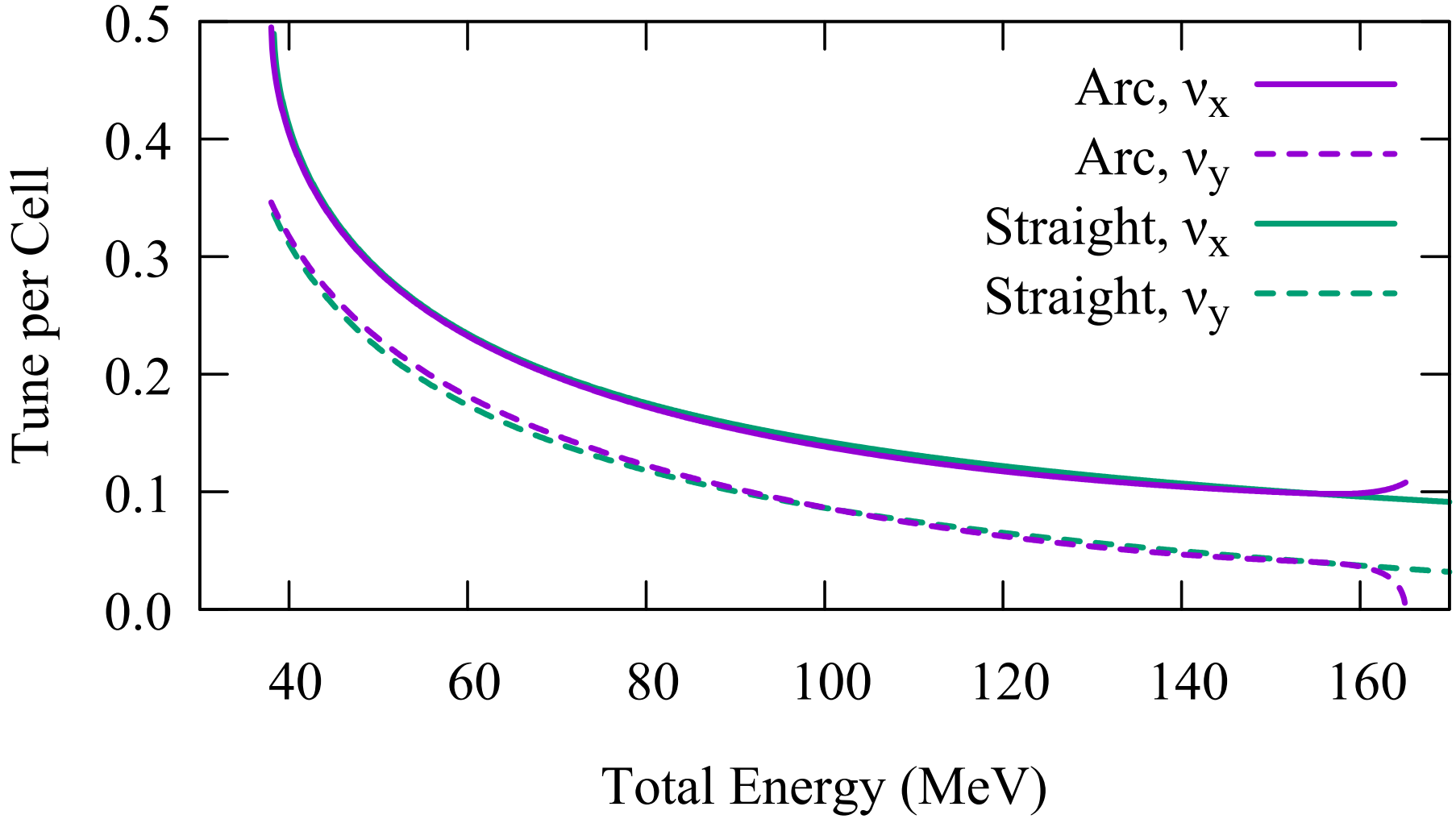
- Ignore RF, energy doesn't change
- For some range of energies, you can find an orbit whose phase space coordiates at the beginning of the cell are identical to its phase space coordinates at the end of a cell: periodic orbit



Periodic Orbits

- Phase space coordinates are relative to a local coordinate system
  - A cell has a design bend; the coordiante system rotates by this amount as well
  - The periodic orbit bends by a the design angle in floor coordinates
  - That angle is the same for every energy
- Each orbit has a tune; this tune is for the single cell
- The tune depends on energy
- For a linear FFAG, this tune variation bounds the energy range
  - Half integer at low energy, zero tune at high energy

Periodic Orbits



Bmad

- Construct a single cell
- Compute periodic orbits
  - `parameter[geometry]=closed`
- Use multiple universes for different energies
  - `n_universes` in `&tao_design_lattice`
  - `change n@beam_start pz @(dp_on_p)` to set energy for universe `n`
- Can plot versus energy
  - `curve(n)%data_type_x='beam_start[pz]'`

Getting Numbers from Tao

- `show lat -at attribute` will show a given element attribute for every element
- `show val expression` will evaluate an expression
  - See chapter 6 of the Tao manual
- Expression syntax; primitives are:
  - `u@ele::ename[attrib]`: element attribute
  - `u@lat::dtype[ename]`: tao "data type" (`dtype`) for given element (`ename`): see Tao manual section 8.8
  - `u@var::vname`: Tao variable
  - `u@data::dname`: Tao data item

Exercises: 1

- Execute tao on the example given
- Identify the horizontally focusing and defocusing magnets
- Where longitudinally is the beta function the lowest?
- The orbits are evenly spaced in energy; are their horizontal positions evenly spaced?
- Have Bmad display the tune at the lowest energy
- Create a plot with horizontal beta functions or dispersions vs. position for different energies
- Plot the beta function vs. energy at the defocusing quad center

Design Principles

- Want to keep apertures reasonably small
- Reduce variation of periodic orbit position with energy
- This is related to dispersion (which is the local derivative of orbit position with respect to energy)

Normalized Dispersion

- For coordinates x and momenta p_x, we can write

$$x = \sqrt{\beta} w_1 \qquad p_x = -(\alpha w_1 + w_2)/\sqrt{\beta}$$

- $w_1^2 + w_2^2$ is invariant for a linear lattice
- Consider similar quantities for dispersion

$$D_x = \sqrt{\beta_x}\xi_1 \qquad D_p = -(\alpha_x \xi_1 + \xi_2)/\sqrt{\beta_x}$$

- $\xi_1^2 + \xi_2^2$ is *not* invariant; but it only changes in magnetic fields

$$\frac{d(\xi_1^2 + \xi_2^2)}{ds} = 2\sqrt{\beta_x}\frac{qB_y}{p}\cos\phi_\xi$$

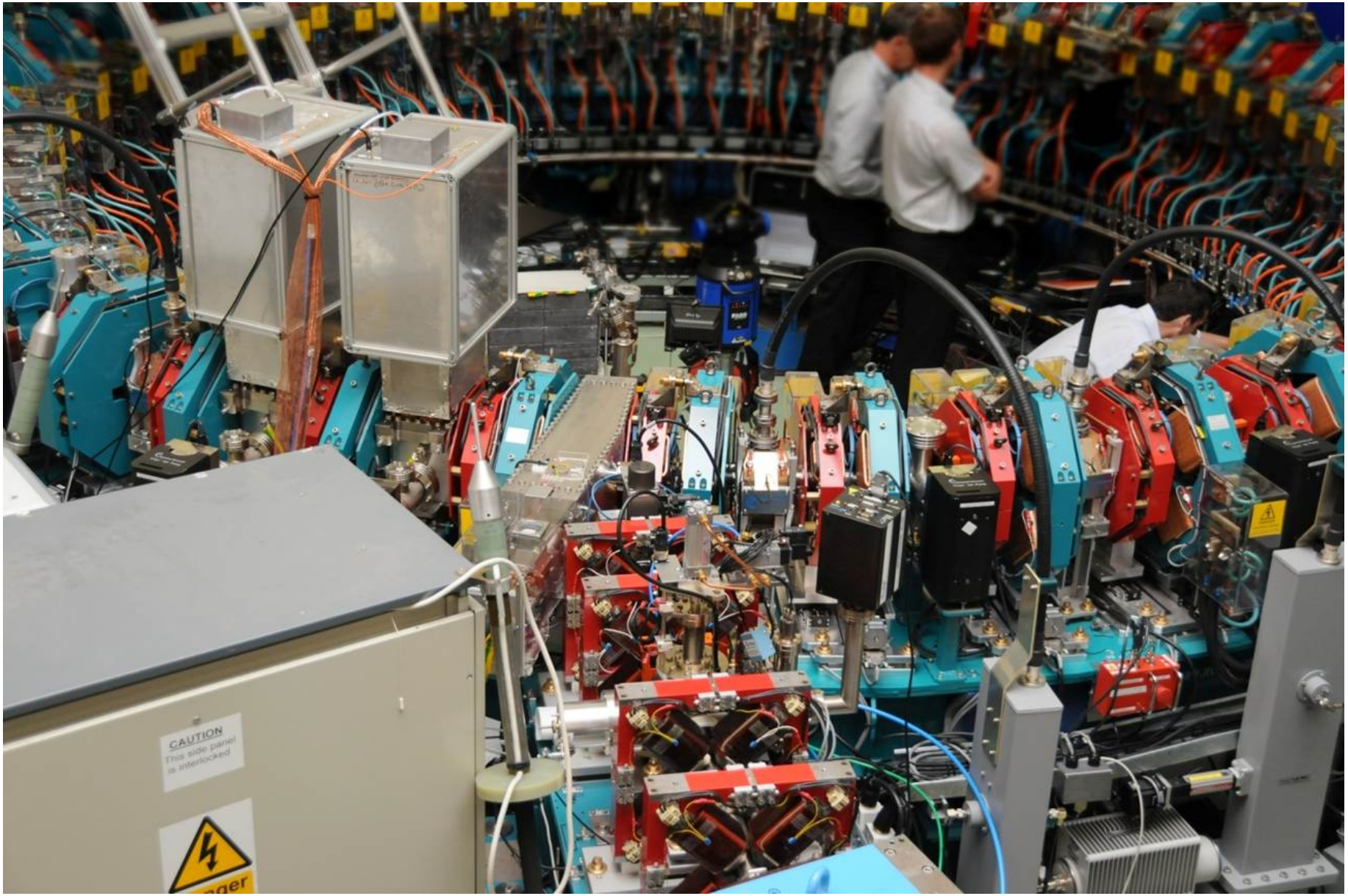
Normalized Dispersion

- Generally, horizontal beta functions are smallest at a defocusing quadrupole
- Thus, want to bend near the defocusing quadrupole
- Thus, at least in non-scaling FFAGs, we used combined-function defocusing quadrupoles
  - On average, more bend happens in the defocusing quadrupole
  - But that's not true for every orbit

Length Scaling

- Generally dispersion is proportaionl to $L\theta$: the cell length times the bending angle
- Generally try to make the cell length as short as possible
- Hardware will limit your cell length
- Generally have at least one "long drift"; needed for RF, injection/extraction, diagnostics, etc.
  - Even if you don't need these things in every cell, every cell must have the long drift
- Often make other drifts "short" to reduce cell length
  - Still have some realistic lower limits

Length Scaling



Magnets

- Consider magnet strengths
  - Shorter magnets require higher fields: inversely with magnet length
  - Shortening drifts increase magnetic fields also, roughly inversely with cell length
- Ratio of magnet aperture to magnet length should not be too large

Magnets

- Magnets will still generally be short
  - Magnets don't simply have "body fields"
  - Maxwell's equations force certain additional field patterns on the ends
  - Fields are not localized within the magnet "body"; length scale proportional to aperture
  - The relative effect of these is larger with shorter magnets
- Magnet "end fields" can be modelled in Bmad
  - `fringe_type`
  - Soft-edge parameter `fq1` and `fq2`

Tunes

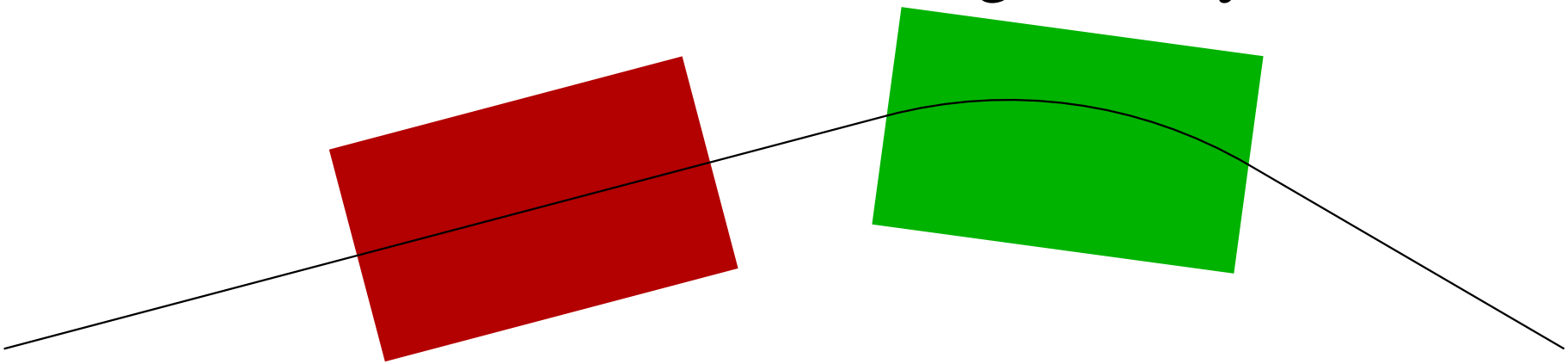
- Higher horizontal tunes will reduce dispersion
- Tunes rise very rapidly as approach the half-integer per cell
- Best use of magnets is generally to have low vertical tunes (which have little impact on orbits), high horizontal tunes
- Linear magents: tunes naturally reduce at higher energy
  - Roughly as $1/p$ for very high energies
  - Dispersion rises, orbits spread out
  - This gives a practical limitation on energy range
- Beware of vertical instability at high energies

Geometry

- Magnet layout is generally described with some coordinate system
  - Representation in design code is not always straightforward
- There is no “reference orbit” that follows the coordinate sytem
  - There can be, but this is generally not the best representation
- There are many ways to do this
- The coordinate system should be related to your hardware
  - How your beam pipe is laid out
  - What your magnets look like
    - Field representation should correspond to the actual magnet

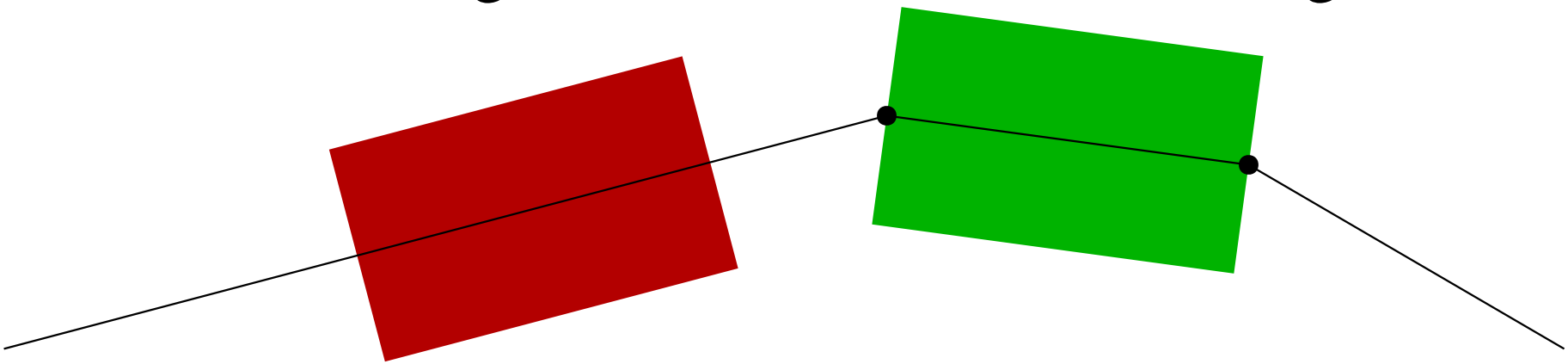
Geometry

Traditional MAD-like geometry



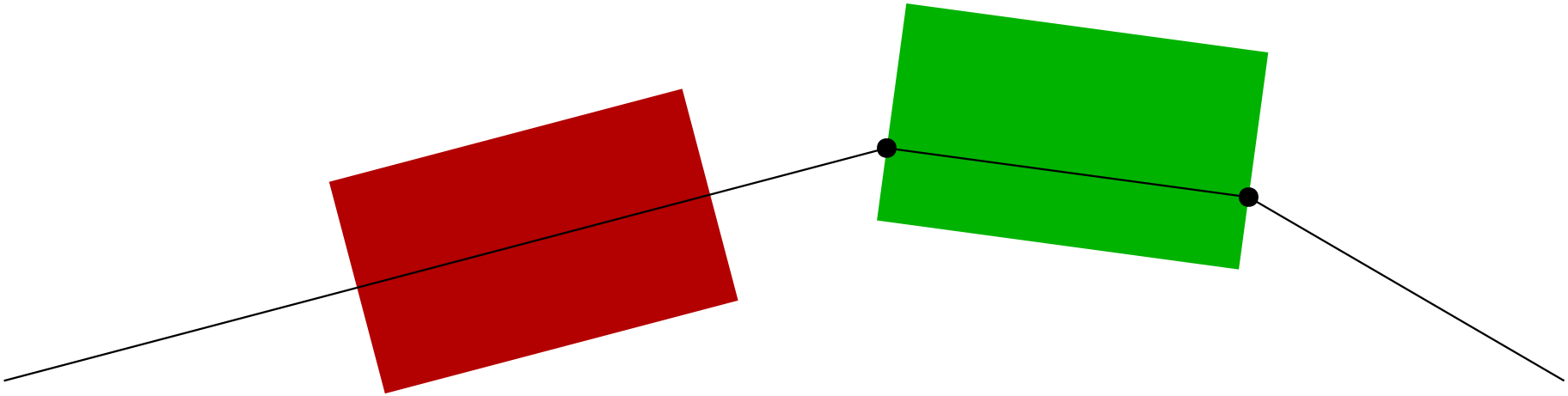
- Combined-function defocusing quadrupole
- All coordinate bending in that magnet
- Lines of constant field are arcs
  - Your magnet probably doesn't do this
- No coordinate patches
- There can be an orbit that follows the geometry

True rectangular combined-function magnet



- Lines of constant field are straight lines
  - `sbend, b_field=0, b_field_err=...`
  - `b_field` determines the geometry
  - `b_field+b_field_err` determines the field
- Patches at magnet ends to rotate coordinate system
  - `patch, x_pitch=...`

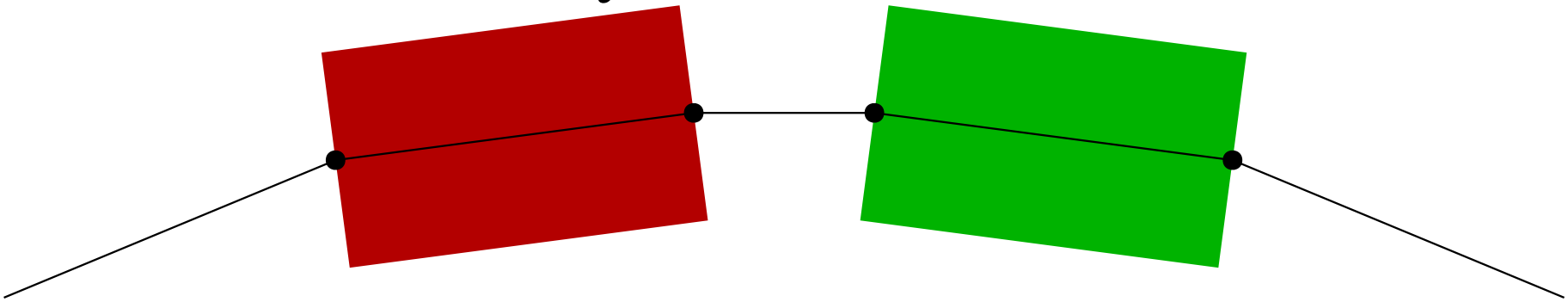
Shifted quadrupoles



- Use shifted quadrupole instead of combined-function magnet
  - `quadrupole, x_offset=...`

Geometry

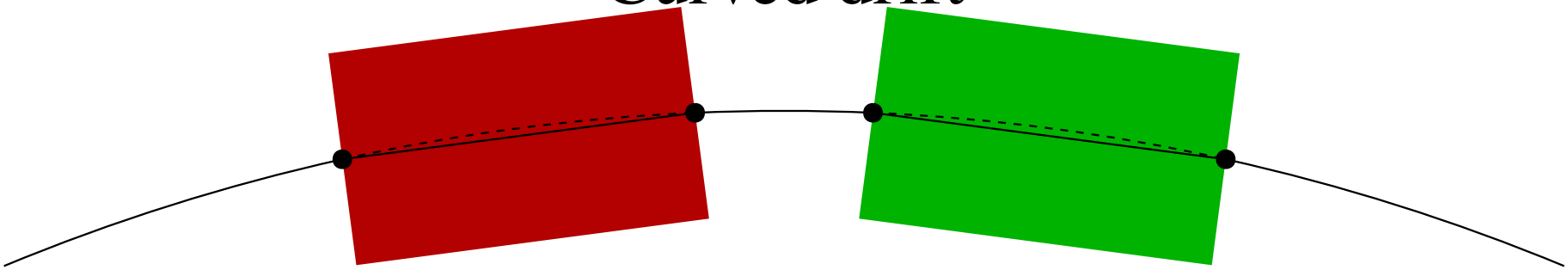
Layout on a circle



- Segments are chords of a circle of a given radius
- Coordinate patchces between segments

Geometry

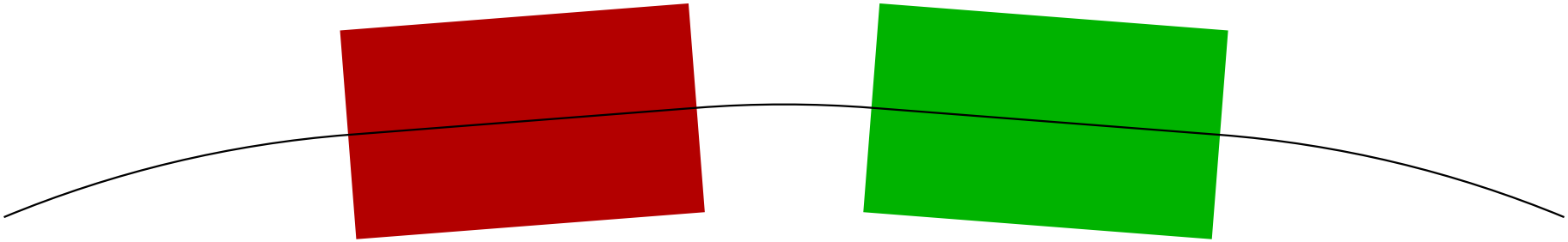
Curved drift



- As before, now drifts are in a curved coordinate system
  - `sbend, g=g0, g_err=-g0`
- Still have coordinate patches

Geometry

Curved drift, no patches



- All bending in drifts
- No more patches

Exercises: 2

- There are 6 lattices matching the above geometries
- Copy the tao files into two directories (for comparison)
- Renaming the desired example to `cell.bmad`
- Which examples should be the same? Are they? Check tunes, orbits.
- Set `fringe_type` to `full`; what changes?
- Change a dipole field and describe the effects
- Change the quadrupole strengths and describe the effects

Lattice Design

- The cell has four parameters: two dipole fields, two quadrupole fields
- There are three "obvious" criteria
  - Centering the beam in the pipe
  - Low energy horizontal tune high, but not too high
  - High energy vertical tune low, but not too low
- Tune choices are are affected by other criteria at times (resonances, magnet fields, etc.)
- The remaining free variable can optimize various things
  - Aperture
  - Magnet design parameters
  - "Cost"

Exercises: 3

- Optimization criteria have been set up; what do they do?
- I modified one of my lattices from the previous exercises; why?
- How would the variables be modified if you were using shifted quadrupoles?
- Run the optimization; describe the results.
- Change the target tunes, one at a time; what happens to the orbits?
- Optimize fitting only the tunes and center the orbits. One variable should not be varied (which?). Examine results as you change that variable.

Exercise: Design CBETA

- Criteria
  - 5 m radius of curvature
  - Minimum drifts: 11 cm long, 6 cm short
  - Energy range: 42 MeV to 150 MeV
- Use one of the existing examples as a starting point
- Make plots showing four energies: 42, 78, 114, 150 MeV