François Méot
Collider-Accelerator Department
Brookhaven National Laboratory

Large amplitude characterization of CBETA arc cell using 3-D OPERA field maps

Contents

1 Introduction

◇ Two things are addressed in these slides :

- the paraxial amplitude properties of CBETA arc cell.

This is in order essentially to make sure with tackle the next point with the correct hypotheses, namely

- large amplitude properties of CBETA arc cell.

◇ The investigation aims

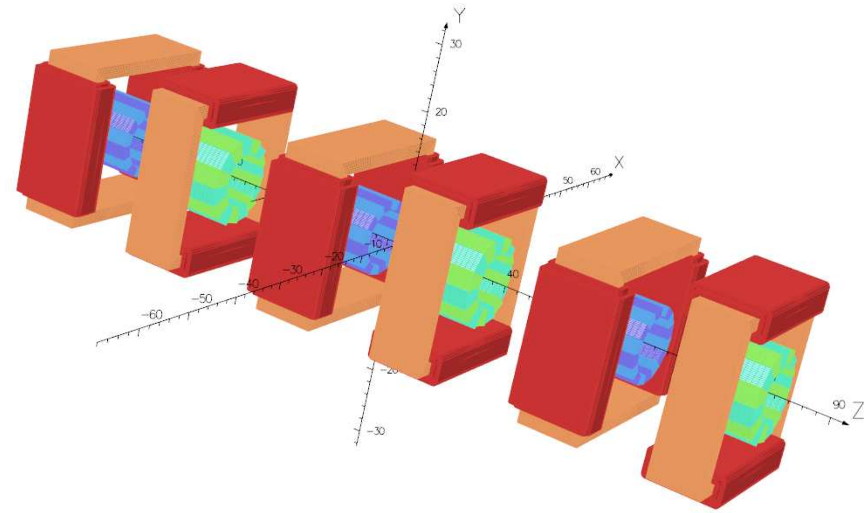
- to validate, or to feedback on, the cell parameters and/or OPERA outcomes,
- with the criterion that the expectations are met (tune footprint, dynamical admittance, etc.)

◇ The exercise, back and forth between OPERA computation and ray-tracing, including the numerous changes in the lattice parameters, has kept us busy the past 2∼3 years.

2 Field maps

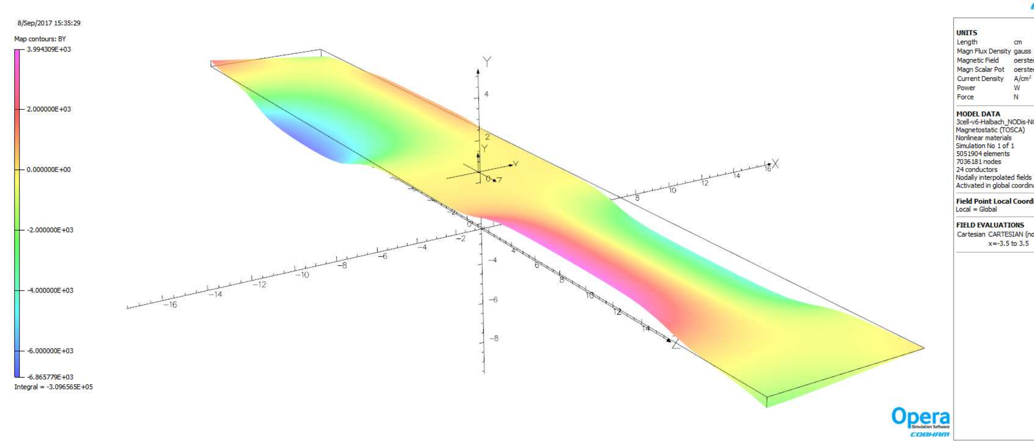
◇ The 3D field maps of concern have been produced by Nick Tsoupas

◇ As field maps are used, the study is *fully based on stepwise ray-tracing* techniques : particles are pushed across the field map step-by-step, typically millimeter steps.



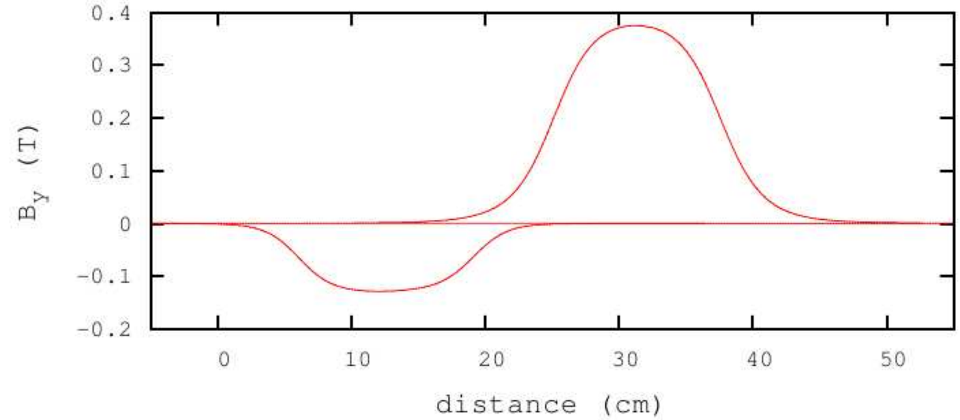
◇ Two models of the cell can be used for ray-tracing :

- either a single field map comprising QF and BD
- or two separate field maps.



The latter is the case in these slides, two OPERA file that we called
"QF_v6 x=+-4p1y=+-1p4z=+-25 stp=1mm integral N35EH.table"
"BD_v6 x=+-4p1y=+-1p4z=+-25 stp=1mm integral N35EH.table"

◇ The cell magnets being short, the field along the orbits does not feature any plateau.

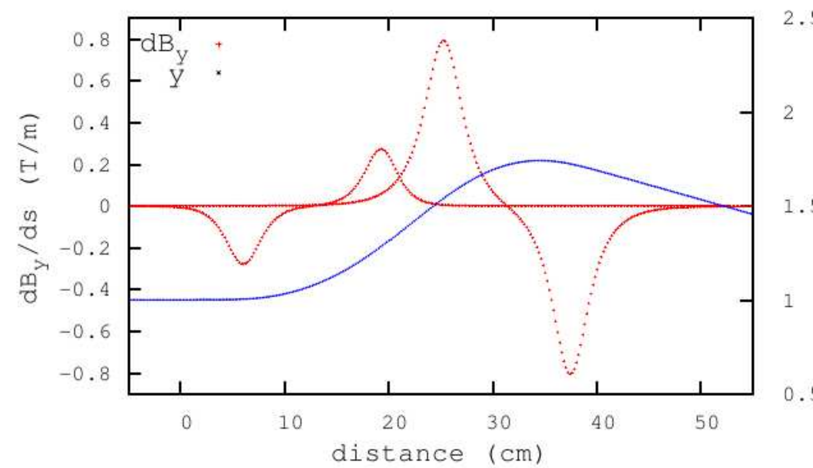


◇ **As a consequence it has a rich content in derivatives, as results from**

$$V_1(s,x,y) = G_1(s)y - \frac{G_1^{(2)}(s)}{8}(x^2+y^2)y + \frac{G_1^{(4)}(s)}{192}(x^2+y^2)^2 y + \ldots \quad \text{(dipole)}$$

$$V_2(s,x,y) = G_2(s)xy - \frac{G_2^{(2)}(s)}{12}(x^2+y^2)xy + \frac{G_2^{(4)}(s)}{384}(x^2+y^2)^2 xy + \ldots \quad \text{(quadrupole)}$$

◇ **This is the origin of the huge reduction of the dynamical admittance, compared to that of linear magnets (slide #10) (the latter is limited as well though, a sufficient cause for that are kinematic non-linearities)**



First order field derivative dB_y/ds (red) along a trajectory off-mid-plane (blue).

◇ **An ideal cell without any random errors (neither field not alignment) is considered.**

◇ **It does include systematic non-linearities, namely,**

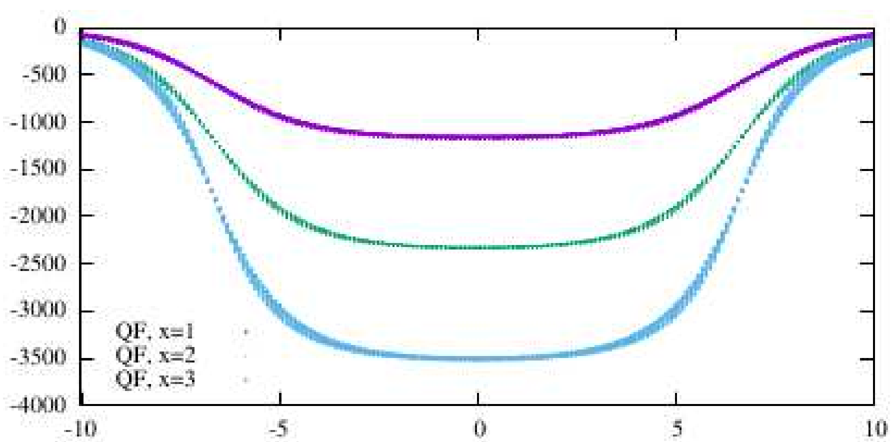
(i) the multipole content intrinsic to the magnet geometry in the OPERA simulation, and

(ii) the non-linearity content resulting from the longitudinal form factor as addressed above. Note that the latter includes all orders in x, y.

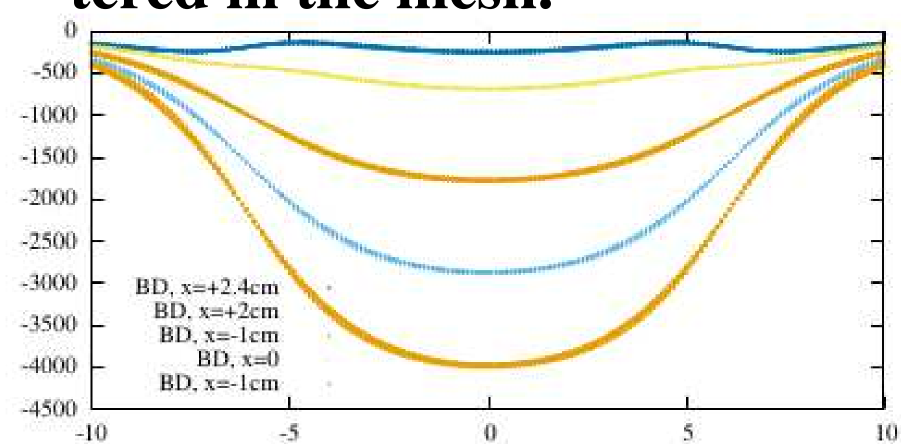
◇ **In addition, as stepwise ray-tracing is used the kinematic non-linearities are present in the motion, always.**

• Check the positioning of the magnet with respect to the map frame (which is also the mesh frame). They are centered on the center of the field map.

◇ Field from QF field map file, along x=1, 2 and 3 cm. The field is symmetric wrt. s=0 plane → magnet centered in the mesh.



◇ Field from BD field map file, along lines at various constant-x values. The field is symmetric wrt. s=0 plane → magnet centered in the mesh.



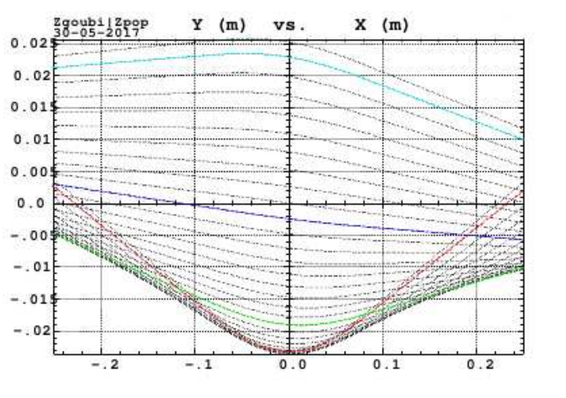
• Check the centering of extreme orbits (42 and 150 MeV) in QF and BD maps.

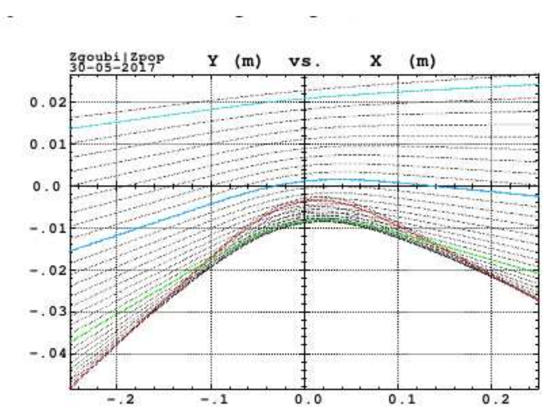
(Orbits over a 40-153 MeV range are plotted here, the design energies are colored.)

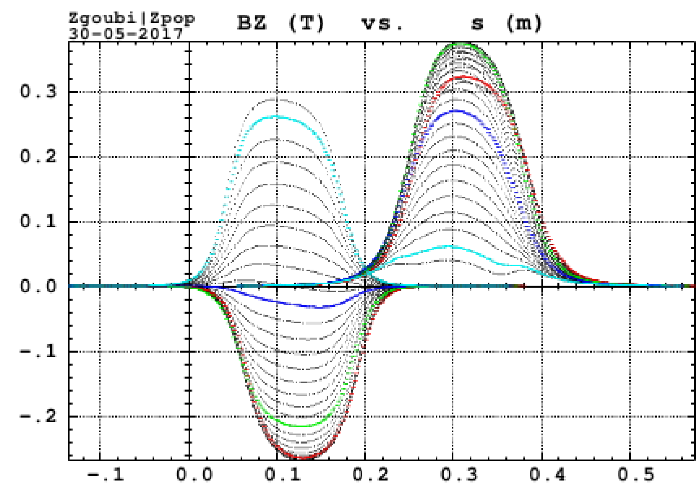
◇ **QF : The 42 and 150 MeV orbit excursions reach ±23.33 mm : the beam is correctly centered.**

◇ **Orbit excursion in BD magnet (centered on 0, length 12 cm).**

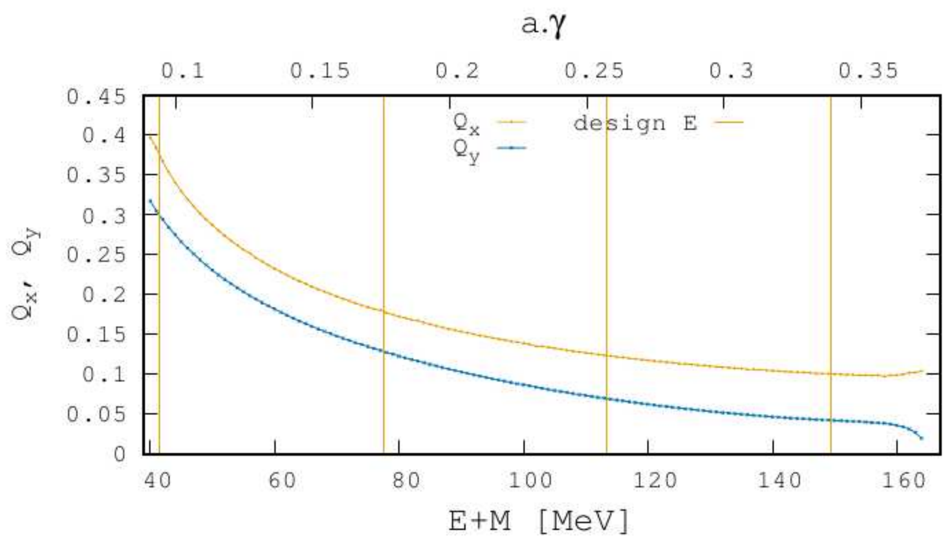
◇ **Field along 30 different orbits in QF and BD (40 to 153 MeV).**



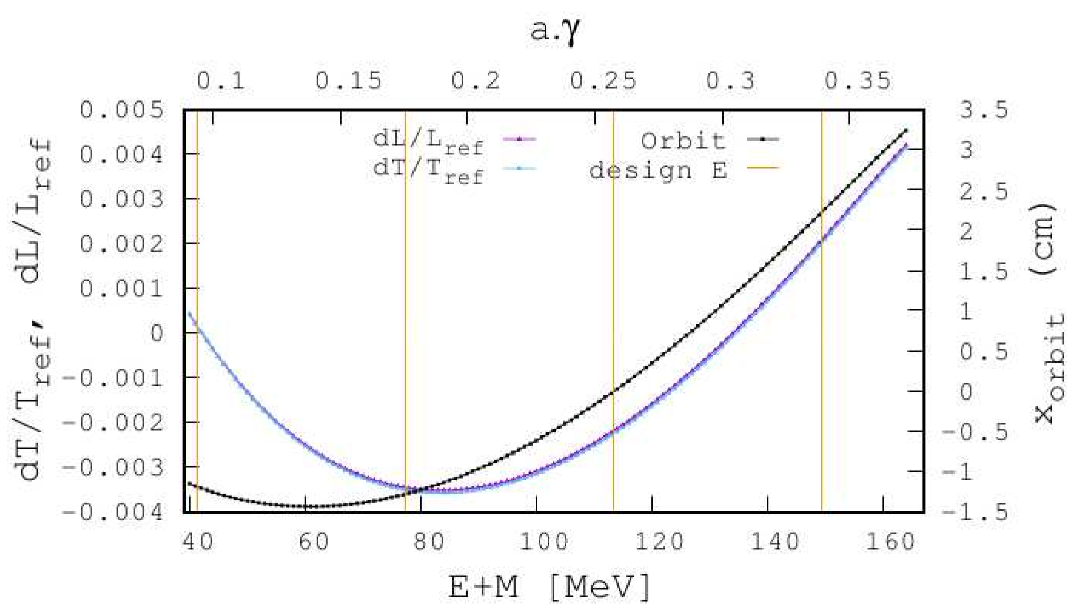




• **Energy dependence of cell tunes (from first order mapping), from 40 MeV up to maximum stable energy (≈166 MeV).**



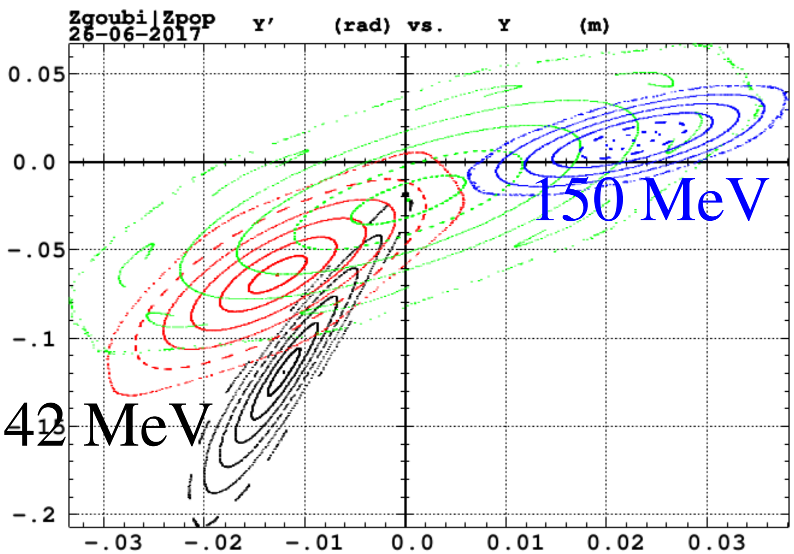
• **Energy dependence of orbit position (at center of long drift), path length and time of flight.**



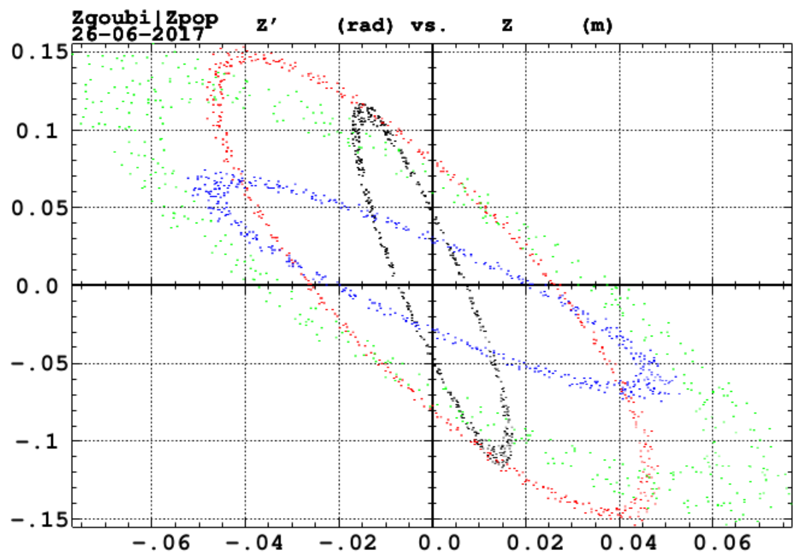
3 Large amplitude

- **Maximum stable invariants, for each one of the 4 design energies.**

HORIZONTAL



VERTICAL

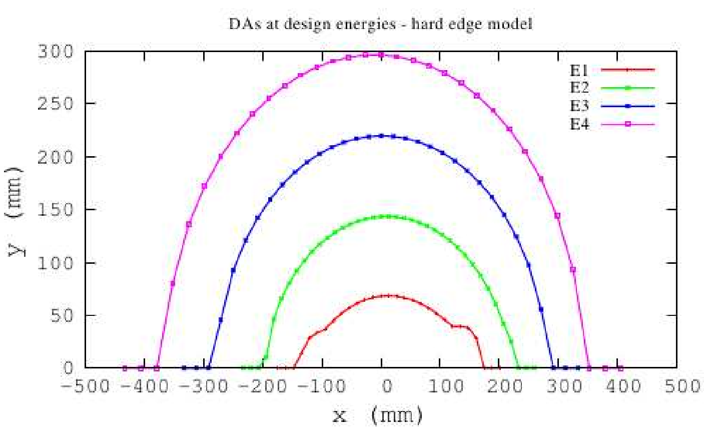


- **A parenthesis concerning the "dynamical admitttance".**

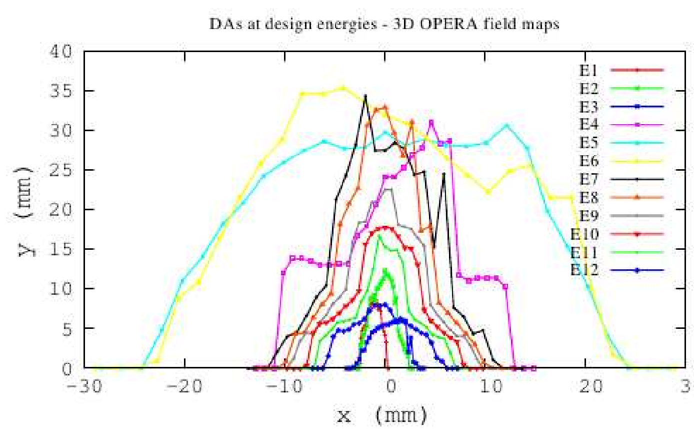
Typical behavior of hard-edge model versus realistic fields :

E1 to E4 : 42, 78, 114 and 150 MeV

Hard edge model



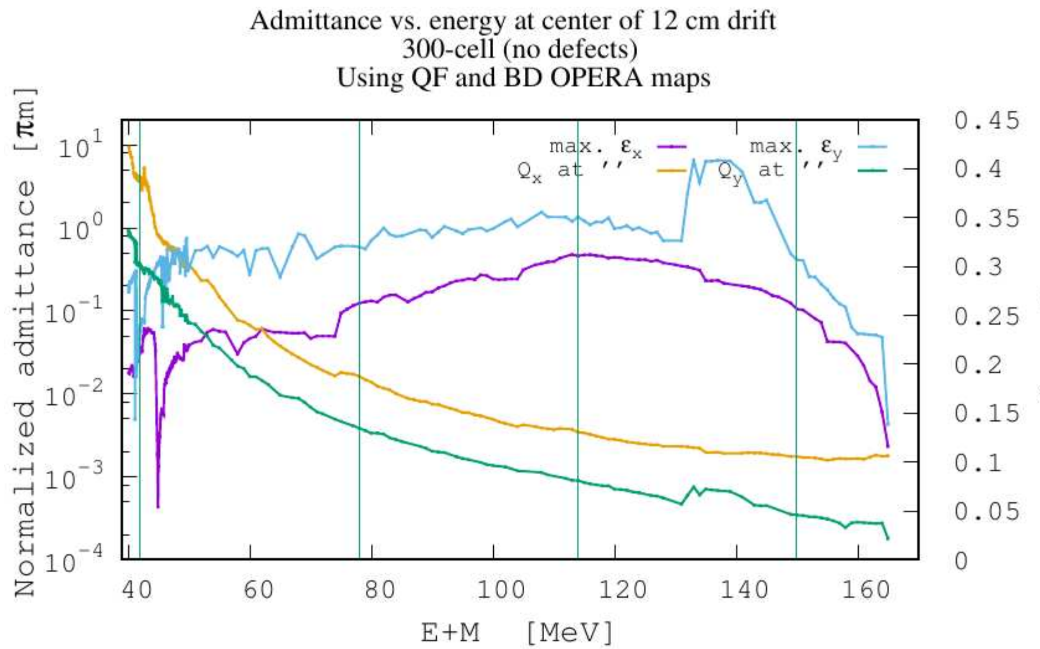
3D OPERA field maps



E1 to E12 : 40, 41, 42, 78 114, 119, 150, 152, 154, 156, 158, 160 MeV.

- **A scan of the H and V dynamical admittances of a 300-cell beam line, over 40-166 MeV (this is the surface of the phase space invariant at maximum sable amplitude, respectively H and V, previous slide).**

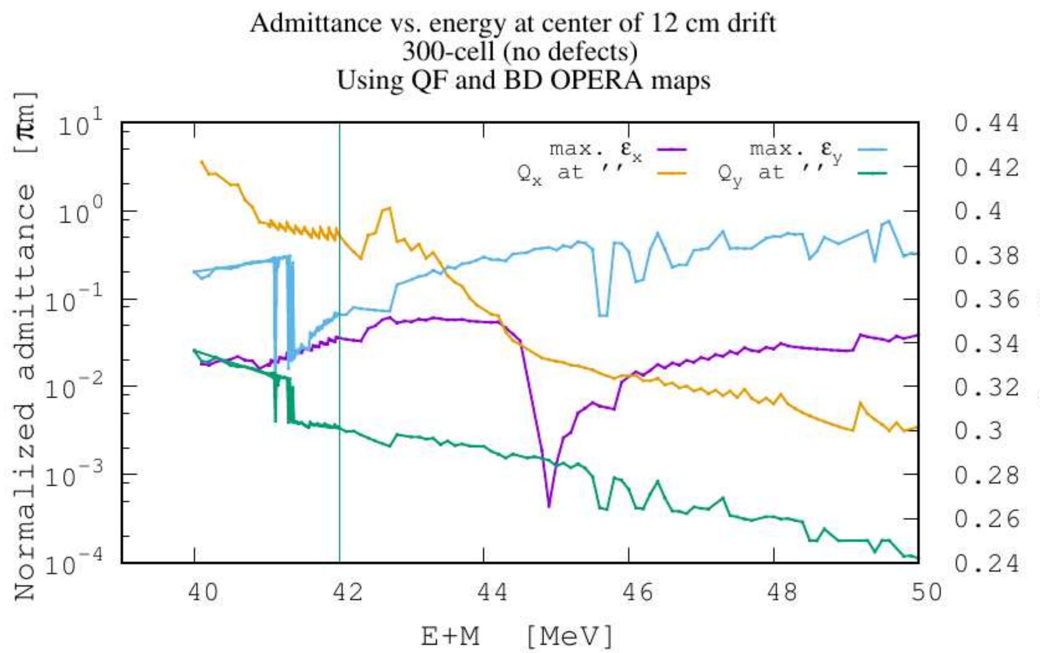
◇ **The vertical bar is at 42 MeV.**



- **A zoom in the 40-50 MeV region.**

◇ **The A_y dip at 41+ MeV is attributed to $\mathrm{Q_x} + 2\mathrm{Q_y} = \mathrm{int.}$ sum resonance line, TBC ! source : skew sextupole excitation (see middle and bot. Q-diagrams in slide #4).**

◇ **The A_x dip at 45- MeV is due to $3\mathrm{Q_x} = \mathrm{integer}$ (upright sextupole).**



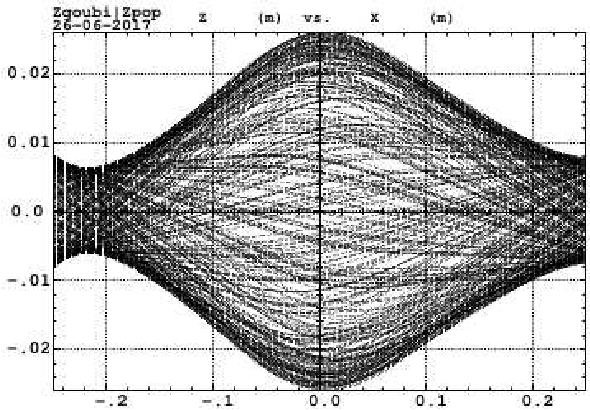
- **$\mathrm{Q_x}$ and $\mathrm{Q_y}$ tunes in these plots are for the motion at maximum stable amplitude (*i.e.*, the tunes for the maximum invariants, previous slide).**

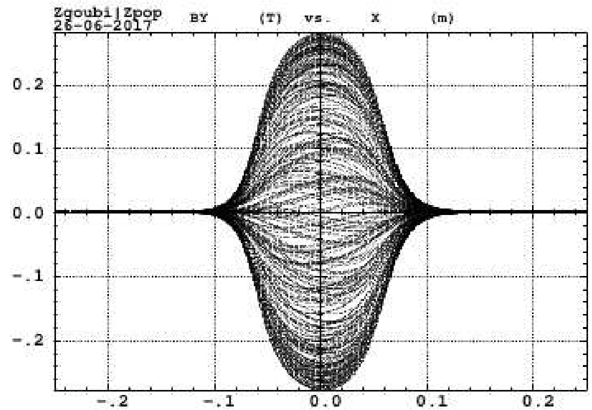
• Because the maximum stable vertical invariant *really is* at large amplitude (± 2.6 cm, see left plot below), need to check that the field at such large amplitude is still correctly computed (at least, "looks realistic"), otherwise DA computations are questionable...

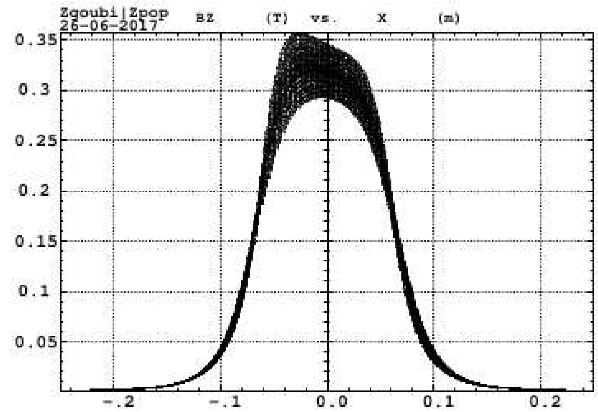
A 42 MeV electron is launched on maximum vertical stable invariant, and observed across BD over a few 10s of passes (that's sufficient for the electron to reach large mplitudes/fields) :

◇ y-projection of the vertical motion.

◇ Radial (left) and vertical (right) components of the field experienced. Does not look to badly computed...







4 Tune footprint

- **Spreading of Qx, Qy under the effect of horizontal (dQx/dax , dQx/day) and vertical (dQy/dax, dQy/day) anharmonicities.**

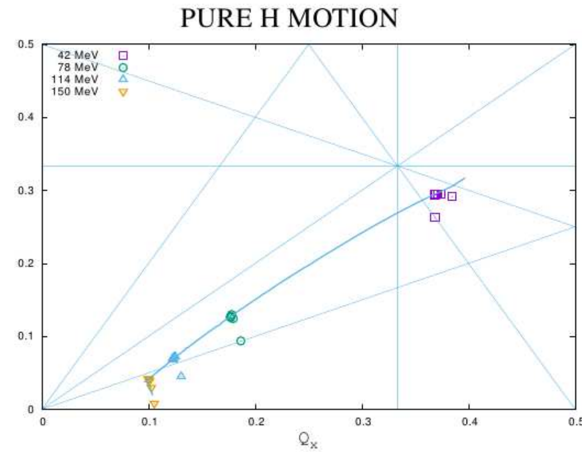
◇ For each of the 4 design energies, 7 invariants from paraxial to maximum stable are tracked and the tunes are computed.

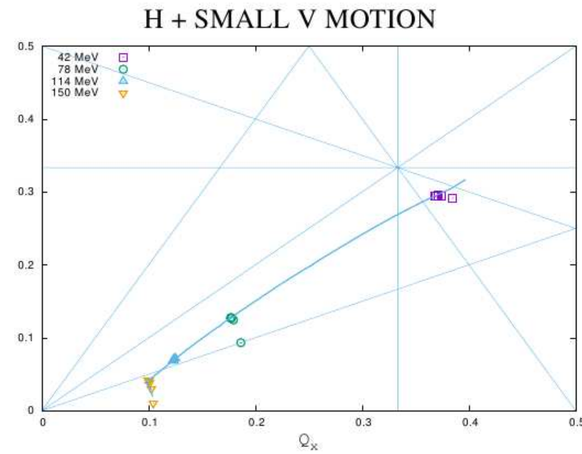
- Top : case of pure H motion

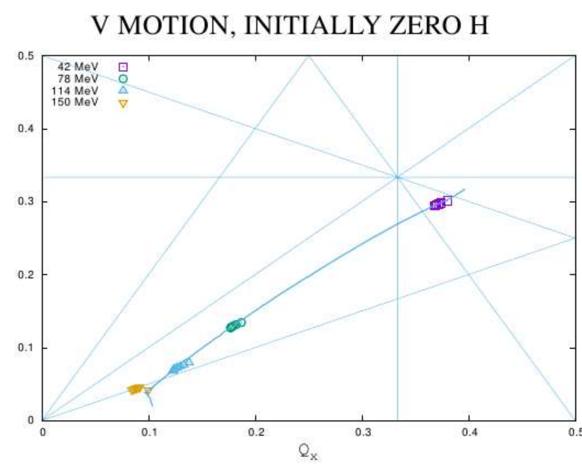
- Middle : initial H + subliminal V

- Bottom : initial V + subliminal H

The solid line spans the 42-150 MeV range, case of paraxial tunes.







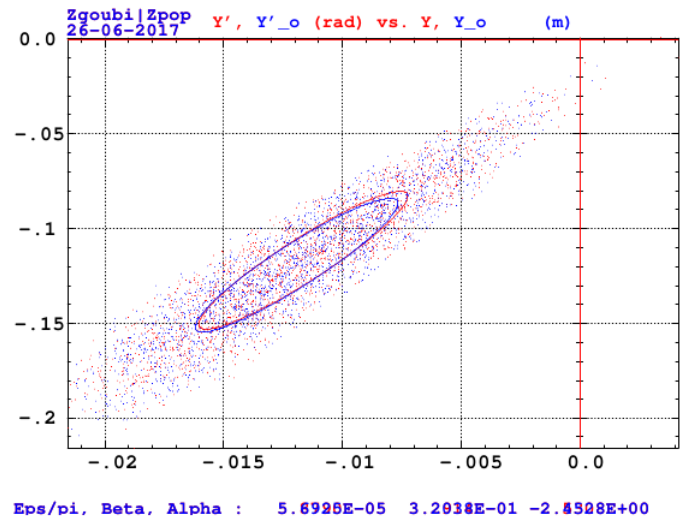
5 Transmission

• The focus here is on 42 MeV, the region of smaller admittance (previous slides).

- 10000 particles are launched in a wide $\epsilon_x/\pi = 10^{-4}$ m, $\epsilon_y/\pi = 10^{-3}$ m, geometric.

- Of these, 21% make it to the end of the 300-cell channel. This determines the H, V admittance of the 42 MeV channel, at its entrance.

• x, x' coordinates (initial (blue) and final (red)) of the particles that make it thru, and matching ellipses. The geometrical admittance is $\epsilon_x/\pi = 5.7\,10^{-5}$ m, i.e., $\epsilon_x/\pi = 4.8$ mm, normalized.



• y, y' coordinates (initial (blue) and final (red)) of the particles that make it thru, and matching ellipses. The geometrical admittance is $\epsilon_y/\pi = 1.7\,10^{-4}$ m, i.e., $\epsilon_y/\pi = 14.3$ mm normalized.