François Méot
Collider-Accelerator Department
Brookhaven National Laboratory

An introduction to

Fixed Field Alternating Gradient Accelerators
Bibliography

References


[17] SIX-SECTOR FFAG RING TO DEMONSTRATE BUNCH ROTATION FOR PRISM, A. Sato et al., THPP007, proceedings of EPAC08, Genoa, Italy.


1 Introduction

○ Today’s status

◊ Eleven FFAG rings have been operated up to now:
  - 3 electron rings by MURA, 1953 - 1967 [1, 8, 9, 10]
  - 2 proton rings at KEK, 500 keV and 150 MeV, 1999 - 2003 (the latter, moved to Kyushu, 2005) [11, 13]
  - a 150 MeV 3-ring cascade at KURRI, for ADS-R R&D, 2005 on [14]
  - an internal-target storage ring for n production, KURRI, 2007 [16]
  - PRISM, a muon source, Osaka University [17]
  - EMMA electron NS-FFAG, 10-20 MeV, Daresbury, 2007 - 2012 [28]

◊ in addition, have been operated in the recent past
  - a prototype spiral FFAG dipole at SIGMAPHI, 2009, as part of RACCAM protontherapy FFAG study, 2005-2010 [19]
  - a scaling FFAG straight section at KURRI, 2012 [18]
  - a prototype NS-FFAG arc at BNL, 2017 [29]

◊ There has been a number of design and prototyping studies in the recent past:
  - muon acceleration in the Neutrino Factory [22, 23, 24]
  - superconducting FFAG magnets [3]
  - proton drivers for ADS [20, 27]
  - medical application [15, 16, 19]
  - eRHIC recirculator arcs,
  - CBETA now
  - and more...
The landscape at the birth of the FFAG concept, 1953

Background: nuclear physics research
- High-voltage generator: Cockcroft-Walton, 1930-32 (0.7 MV), Van de Graaf, 1929-30 (1.5 MV in 1931, ultimate 25 MV)
  - Cyclotron (classical): E.O. Lawrence, 1928-32
  - Isochronous cyclotron: Thomas, 1938 (590 MeV, 1.4 MW at PSI today; medical and isotope cyclotrons all over the planet)
  - Betatron: Kerst, 1940 (2.2 MeV; ultimate 300 MeV, 1950)
  - Alvarez RF-linac, 1946
  - Pulsed synchrotron (concept Oliphant, 1943; electron POP UK 1945); led to the weak focusing Cosmotron (3.3 GeV, BNL, 1953), Bevatron (6.2 GeV, Berkeley, 1954), Synchro-Phasotron (10 GeV, Dubna, 1957)
  - Electron linacs, 1947 (following from high power, high frequency RF system developments)

In terms of beam optics and acceleration:
- Transverse focusing (cyclotron)
- Isochronous acceleration (cyclotron, betatron)
- Phase focusing, McMillan and Veksler (synchro-cyclotron, synchrotron)
  - Separated focusing (applied to synchrotron, linac)

And then came the FFAG...
- A quasi-simultaneous T. Ohkawa in Japan, K. Symon and D. Kerst in the United States, A. Kolomensky in USSR, ~1953
2 MURA electron FFAGs

• Motivations for MURA, in the early 1950s
  - Stimulate accelerator R/D for high energy physics, and build accelerators! in the Midwest
  - Explore alternate routes to AG synchrotrons, high intensities

• A major contribution to accelerator science [1]

  (i) beam stacking,
  (ii) Hamiltonian theory of longitudinal motion,
  (iii) colliding beams (in itself a quite old idea),
  (iv) storage rings (independently invented by O’Neill),
  (v) spiral-sector geometry used in isochronous cyclotrons,
  (vi) lattices with zero-dispersion and low-β sections for colliding beams,
  (vii) multturn injection into a strong-focusing lattice,
  (viii) first calculations of the effects of nonlinear forces in accelerators,
  (ix) first space-charge calculations including effects of the beam surroundings,
  (x) first experimental measurement of space-charge effects,
  (xi) theory of negative-mass and other collective instabilities and correction systems,
  (xii) the use of digital computation in design of orbits, magnets, and rf structures,
  (xiii) proof of the existence of chaos in digital computation, and
  (xiv) synchrotron-radiation rings
The first model, radial sector FFAG, “MARK I”

- Main features: fixed field ring, \( B = B_0 \left( \frac{r}{r_0} \right)^K \mathcal{F}(\theta) \), strong focusing, scaling gap
- Objectives: demonstrate the FFAG principle. Studies included optics, injection, RF manipulations, effects of misalignments, exploring resonances.

**FFAG ring parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_{\text{inj}} - E_{\text{max}} )  keV</td>
<td>25 - 400</td>
</tr>
<tr>
<td>Orbit radius ((C/2\pi))  m</td>
<td>0.34 - 0.50</td>
</tr>
<tr>
<td><strong>Optics</strong></td>
<td></td>
</tr>
<tr>
<td>Lattice</td>
<td>FD</td>
</tr>
<tr>
<td>Number of cells</td>
<td>8</td>
</tr>
<tr>
<td>Field index (K)</td>
<td>3.36</td>
</tr>
<tr>
<td>( \nu_r / \nu_z )</td>
<td>2.2-3 / 1-3</td>
</tr>
<tr>
<td><strong>Magnet</strong></td>
<td></td>
</tr>
<tr>
<td>Radial sector</td>
<td>( B = B_0 \left( \frac{r}{r_0} \right)^K \mathcal{F}(\theta) )</td>
</tr>
<tr>
<td>( \theta_F, \theta_D ) deg</td>
<td>25.74, 10.44 sector angles</td>
</tr>
<tr>
<td><strong>Acceleration</strong></td>
<td></td>
</tr>
<tr>
<td>Swing</td>
<td>Started with betatron yoke...</td>
</tr>
<tr>
<td>... added RF system, later</td>
<td></td>
</tr>
<tr>
<td>Freq. swing (MHz)</td>
<td>10 in [35, 75] MHz</td>
</tr>
<tr>
<td>Gap voltage (V)</td>
<td>50</td>
</tr>
</tbody>
</table>

F magnet, \( B > 0 \), H-focusing scaling gap \( g \propto r \)
- Basic theory

◊ Combined function optics

- A rule that yields the orientation of $\vec{F}$:
- $I\vec{dl}$, $\vec{B}$ and $\vec{F}$, in that order, form a direct triedra

(in this sketch I assume field obtained from pole shaping, just for simplicity)
Solution of the motion across a combined function magnet

A reference trajectory can be defined, characterized by \( B_0 \rho_0 = \frac{p_0}{q} \).

The equations of small amplitude motion \((x = \rho - \rho_0, y)\) of a particle, in the Serret-Frenet frame attached to that reference curve, are derived from the Lorentz force equation:

\[
\frac{d\vec{p}}{dt} = q\vec{v} \times \vec{B}
\]

using a transverse expansion of the magnetic field \( \vec{B}(s) \) along the trajectory:

\[
B_x = -n\frac{B_0}{\rho_0}y \quad [\text{+ non-linear terms}], \quad B_y = B_0(1 - n\frac{x}{\rho_0}) \quad [\text{+ non-linear terms}]
\]

In expanding \( \vec{B} \) the field index has been introduced:

\[
n = -\frac{\rho_0}{B_0} \frac{\partial B}{\partial x}
\]

The FFAG index \( K = \frac{r}{B} \frac{\partial B}{\partial r} \) (of \( B = B_0(\frac{r}{r_0})^K \)) relates to \( n \) by \( K \approx -nr/\rho \).

Calculations (found in text books) lead to the linear approximation:

\[
\frac{d^2x}{ds^2} + \frac{1 - n}{\rho_0^2} x = \frac{1}{\rho_0} \frac{\Delta p}{p} \quad \frac{d^2y}{ds^2} + \frac{n}{\rho_0^2} y = 0
\]
What approximation leads to $K \approx -\frac{nR}{\rho}$?
Solving these two differential equations yields the coordinates across a magnet (with \( \mathcal{L} = (s - s_0) \) being the path length along the trajectory arc):

**Radial motion**

if \((1 - n) > 0\):
\[
\begin{align*}
x &= x_0 \cos \frac{\sqrt{1 - n}}{\rho_0} \mathcal{L} + \frac{x_0'}{\rho_0} \sin \frac{\sqrt{1 - n}}{\rho_0} \mathcal{L} + \frac{\rho_0}{\sqrt{1 - n}} (1 - \cos \frac{\sqrt{1 - n}}{\rho_0} \mathcal{L}) \frac{\Delta p}{p} \\
x' &= -x_0 \frac{\sqrt{1 - n}}{\rho_0} \sin \frac{\sqrt{1 - n}}{\rho_0} \mathcal{L} + x_0' \cos \frac{\sqrt{1 - n}}{\rho_0} \mathcal{L} + \frac{\rho_0}{\sqrt{1 - n}} \sin \frac{\sqrt{1 - n}}{\rho_0} \mathcal{L} \frac{\Delta p}{p}
\end{align*}
\]

if \((1 - n) < 0\):
\[
\begin{align*}
x &= x_0 \cosh \frac{\sqrt{n - 1}}{\rho_0} \mathcal{L} + \frac{x_0'}{\rho_0} \sinh \frac{\sqrt{n - 1}}{\rho_0} \mathcal{L} + \frac{\rho_0}{n - 1} (1 - \cosh \frac{\sqrt{n - 1}}{\rho_0} \mathcal{L}) \frac{\Delta p}{p} \\
x' &= x_0 \frac{\sqrt{n - 1}}{\rho_0} \sinh \frac{\sqrt{n - 1}}{\rho_0} \mathcal{L} + x_0' \cosh \frac{\sqrt{n - 1}}{\rho_0} \mathcal{L} + \frac{\rho_0}{n - 1} \sinh \frac{\sqrt{n - 1}}{\rho_0} \mathcal{L} \frac{\Delta p}{p}
\end{align*}
\]

**Axial motion**:

if \(n > 0\):
\[
\begin{align*}
y &= y_0 \cos \frac{\sqrt{n}}{\rho_0} \mathcal{L} + \frac{y_0'}{\sqrt{n}} \sin \frac{\sqrt{n}}{\rho_0} \mathcal{L} \\
y' &= -y_0 \frac{\sqrt{n}}{\rho_0} \sin \frac{\sqrt{n}}{\rho_0} \mathcal{L} + y_0' \cos \frac{\sqrt{n}}{\rho_0} \mathcal{L}
\end{align*}
\]

if \(n < 0\):
\[
\begin{align*}
y &= y_0 \cosh \frac{\sqrt{-n}}{\rho_0} \mathcal{L} + \frac{y_0'}{\sqrt{-n}} \sinh \frac{\sqrt{-n}}{\rho_0} \mathcal{L} \\
y' &= y_0 \frac{\sqrt{-n}}{\rho_0} \sinh \frac{\sqrt{-n}}{\rho_0} \mathcal{L} + y_0' \cosh \frac{\sqrt{-n}}{\rho_0} \mathcal{L}
\end{align*}
\]
In transport matrix notations (with, for short, $k_x = |1 - n|/\rho_0^2$, $k_y = |n|/\rho_0^2$)

\[
\begin{pmatrix}
  x \\
  x' \\
  y \\
  y' \\
  \delta p \\
  p
\end{pmatrix}
= \begin{pmatrix}
  \cos \sqrt{k_x} \mathcal{L} & \frac{1}{\sqrt{k_x}} \sin \sqrt{k_x} \mathcal{L} & 0 & 0 & \frac{1}{\rho k_x} (1 - \cos \sqrt{k_x} \mathcal{L}) \\
  -\sqrt{k_x} \sin \sqrt{k_x} \mathcal{L} & \cos \sqrt{k_x} \mathcal{L} & 0 & 0 & \frac{1}{\rho k_x} \sin \sqrt{k_x} \mathcal{L} \\
  0 & 0 & \cosh \sqrt{k_y} \mathcal{L} & \frac{1}{\sqrt{k_y}} \sinh \sqrt{k_y} \mathcal{L} & 0 \\
  0 & 0 & \sqrt{k_y} \sinh \sqrt{k_y} \mathcal{L} & \cosh \sqrt{k_y} \mathcal{L} & 0 \\
  0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
  x_0 \\
  x'_0 \\
  y_0 \\
  y'_0 \\
  \delta p \\
  p
\end{pmatrix}
\]

If $n < 0$ : The dipole is horizontally focusing and vertically defocusing

If $0 < n < 1$ : The dipole is focusing in both planes. aka “weak focusing” (cyclotrons)

If $n > 1$ : The dipole is horizontally defocusing and vertically focusing.
“MARK I” uses strong focusing, BF is focusing, BD is defocusing

- The (BF,BD) doublet is globally focusing in both $x$ and $y$ planes:
Wedge focusing is required to complete the (BF,BD) cell matrix
- It stems from the orbit geometry:

The following geometrical relationships are evident from the figure, where \( N \) is the total number of sectors, and the approximate expressions are valid when the angles are small:

\[
N = \frac{2\pi}{\beta_1 \cdot \beta_2} = \frac{2\pi}{\sigma_1 \cdot \sigma_2}
\]

(either sector)

The length of either sector is

\[
s = \beta \cdot \theta
\]

The total length of orbit is

\[
L = N(s_1 + s_2) = 2\pi \frac{\sigma_1 \cdot \sigma_2}{\beta_1 \cdot \beta_2}
\]

MARK I is a ring, one more ingredient is needed to make it operational:
periodic stability

- Let's oversimplify: we forget drifts, wedge focusing,
- let's also consider magnets with same field \((B_{0,F} = B_{0,D})\) and same index \((k_F = k_D = k)\),
- thus the transport matrix for the \((BF,BD)\) cell writes:

\[
\begin{pmatrix}
\cosh \sqrt{k_D} L_D & \frac{1}{\sqrt{k_D}} \sinh \sqrt{k_D} L_D \\
\sqrt{k_D} \sinh \sqrt{k_D} L_D & \cosh \sqrt{k_D} L_D
\end{pmatrix}
\times
\begin{pmatrix}
\cos \sqrt{k_F} L_F & \frac{1}{\sqrt{k_F}} \sin \sqrt{k_F} L_F \\
-\sqrt{k_F} \sin \sqrt{k_F} L_F & \cos \sqrt{k_F} L_F
\end{pmatrix}
=
\begin{pmatrix}
\cosh \sqrt{k} L_D \times \cos \sqrt{k} L_F - \sinh \sqrt{k} L_D \times \sin \sqrt{k} L_F \\
\sinh \sqrt{k} L_D \times \sin \sqrt{k} L_F + \cosh \sqrt{k} L_D \times \cos \sqrt{k} L_F
\end{pmatrix}
\]

- Periodic stability requires

\[
\frac{1}{2} \text{Trace[CellMatrix]} < 1 \quad \text{for both planes!}
\]

i.e.,

\[
\cosh \sqrt{k} L_D \times \cos \sqrt{k} L_F < 1 \quad \text{and} \quad \cos \sqrt{k} L_D \times \cosh \sqrt{k} L_F < 1 \quad \text{(noting} \sqrt{k} L = \sqrt{k} \mathcal{L})
\]
HOME WORK:

Plot the stability diagram, i.e., the region of periodic stability in the \((\sqrt{kL_F}, \sqrt{kL_D})\) space (take \(\sqrt{kL_F} \in [0, 2\pi]\), \(\sqrt{kL_D} \in [0, 2\pi]\)).
• Constant tunes

○ Re-write the linearized equation of motion (slide #2) with the transformation \( s = r \theta \) yields

\[
\begin{align*}
\frac{d^2 x}{d\theta^2} + \left( \frac{r(\theta)^2}{\rho^2(r, \theta)} \right) \left[ 1 - n \right] x &= 0 \\
\frac{d^2 y}{d\theta^2} + \left( \frac{r(\theta)^2}{\rho^2(r, \theta)} n \right) y &= 0
\end{align*}
\]

(\( r \) is the local radius of the trajectory wrt center of the ring, \( \rho \) is the local curvature radius).

○ Thus, two sufficient conditions to have both the vertical and horizontal betatron oscillations constant with respect to the momentum (i.e., the forcing terms in the above equations constant), are:

\[
\begin{align*}
\left. \frac{\partial n}{\partial p} \right|_{\theta = \text{const}} &= 0 \\
\left. \frac{\partial}{\partial p} \left( \frac{r}{\rho} \right) \right|_{\theta = \text{const}} &= 0
\end{align*}
\]

- the first one expresses the constancy of the field index with respect to the momentum
- the second one expresses the similarity of the closed orbits.

This defines a “zero-chromaticity” optics:

\[
\frac{\delta \nu}{\delta p/p} = 0
\]
Show that, with the FFAG scaling law $B = B_0 (r/r_0)^K \mathcal{F}(\theta)$, this happens.
(take a step function for $\mathcal{F}(\theta)$: 1 inside the magnets, 0 outside)
• Longitudinal motion, longitudinal stability

○ If synchrotron style of RF operation is used, then the longitudinal motion satisfies the regular phase-stability principles,

\[ \Phi'' + \frac{\Omega^2}{\cos \phi_s} (\sin \phi - \sin \phi_s) = 0 \]

synchrotron frequency \( f_s = \frac{\Omega_s}{2\pi} = \frac{c}{\mathcal{E}} \left( \frac{\hbar \eta \cos \phi_s q \hat{v}}{2\pi E_s} \right)^{1/2} \), bucket height \( \pm \frac{\Delta p}{p} = \pm \frac{1}{\beta_s} \left( \frac{2q \hat{v}}{\pi \hbar \eta E_s} \right)^{1/2} \), etc.

• Betatron damping

Introducing the velocity term and its variation, the previous differential equations change to:

\[
\begin{align*}
| & x'' + \left( \frac{\beta \gamma'}{\beta \gamma} \right) x' + \frac{1 - n}{\rho_0^2} x = \frac{1}{\rho_0} \frac{\Delta p}{p} \\
& y'' + \left( \frac{\beta \gamma'}{\beta \gamma} \right) y' + \frac{n}{\rho_0^2} y = 0
\end{align*}
\]

Solving (text books ...) yields (same for \( y, y' \))

\[ x \propto \sqrt{\frac{r}{\beta \gamma}}, \quad x' \propto \sqrt{\frac{1}{r \times \beta \gamma}} \]
• Second model, spiral sector FFAG, “MARK V”

◊ The idea in the spiral FFAG was to avoid the “wrong sign” curvature and bring the circumference factor $C = R/\rho$ close to 1. The wedge angles provide the vertical focusing.
◊ R&D objectives: spiral FFAG POP - first extensive use of computers to determine magnetic field and machine parameters; long-term orbit stability; RF acceleration methods.

**First operation Aug. 1957 at the MURA Lab., Madison.**

**MARK V PARAMETERS**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{inj} - E_{max}$, keV</td>
<td>35 - 180</td>
</tr>
<tr>
<td>orbit radius, m</td>
<td>0.34 - 0.52</td>
</tr>
<tr>
<td>$E_{tr} / r_{tr}$, keV/m</td>
<td>155 / 0.49</td>
</tr>
<tr>
<td>Optics, strong focusing, scaling</td>
<td></td>
</tr>
<tr>
<td>number of sectors</td>
<td>6</td>
</tr>
<tr>
<td>number of sectors</td>
<td></td>
</tr>
<tr>
<td>field index $K$</td>
<td>0.7</td>
</tr>
<tr>
<td>flutter $F_{eff}$</td>
<td>1.1</td>
</tr>
<tr>
<td>$\nu_r / \nu_z$</td>
<td>1.4 / 1.2</td>
</tr>
<tr>
<td>$\beta_r / \beta_z$</td>
<td>0.45-1.3 / 0.6-1.4</td>
</tr>
<tr>
<td>Magnet, spiral sector, $B = B_0(r/r_0)^K F(N(tan(\zeta \ln(r/r_0) - \theta))$</td>
<td></td>
</tr>
<tr>
<td>gap, deg, cm</td>
<td>46 / 16.5 - 7</td>
</tr>
<tr>
<td>Injection, cont. or pulsed</td>
<td></td>
</tr>
<tr>
<td>Acceleration, betatron and RF</td>
<td></td>
</tr>
<tr>
<td><strong>$\gamma_{tr} = (1 + K)^{1/2}$</strong></td>
<td></td>
</tr>
<tr>
<td><strong>RF exprmnts</strong></td>
<td></td>
</tr>
<tr>
<td><strong>coil windings</strong></td>
<td>tunable 0.2-1.16</td>
</tr>
<tr>
<td>tuning coils / 0.57 - 1.60</td>
<td>tunable</td>
</tr>
<tr>
<td>min-max</td>
<td></td>
</tr>
<tr>
<td>$g/r = \text{Cte}$</td>
<td></td>
</tr>
<tr>
<td>$e$-gun + $e$-inflector</td>
<td></td>
</tr>
<tr>
<td>extensive RF tests</td>
<td></td>
</tr>
</tbody>
</table>
On the optics in the spiral FFAG

The following form for the field preserves the scaling property in an N-periodic spiral FFAG:

\[ B(r, \theta)|_{z=0} = B_0 \left( \frac{r}{r_0} \right)^K \mathcal{F} \left( N(\tan \zeta \times \ln \frac{r}{r_0} - \theta) \right) \]

\( \mathcal{F} \) is the axial modulation of the field (“flutter”). One can for instance think of

\[ \mathcal{F} = 1 + f \sin \left( N(\tan \zeta \ln \frac{r}{r_0} - \theta) \right), \quad f \approx 0.25. \]

- The logarithmic spiral edge \( r = r_0 \exp((\theta - \theta_0)/\tan \zeta) \) ensures constant angle between spiral sector edges and radius.
- The in and out wedge angles are different, \( \text{V-defocusing, and V-focusing (larger), overall effect is vertical focusing.} \)

Effect field fall-off extent on vertical focusing

\[
\begin{pmatrix}
 x \\
 x'
\end{pmatrix} = \begin{pmatrix}
 1 & 0 \\
 \frac{\tan \epsilon}{\rho} & 1
\end{pmatrix} \begin{pmatrix}
 x_0 \\
 x'_0
\end{pmatrix},
\begin{pmatrix}
 y \\
 y'
\end{pmatrix} = \begin{pmatrix}
 1 & 0 \\
 -\frac{\tan(\epsilon - \psi)}{\rho} & 1
\end{pmatrix} \begin{pmatrix}
 y_0 \\
 y'_0
\end{pmatrix},
\]

where \( \psi = \frac{I_1 \cdot \lambda \cdot (1 + \sin^2(\epsilon))}{\rho \cdot \cos(\epsilon)}, \) with \( I_1 = \int B_z(s) \cdot (B_0 - B_z(s)) \cdot \frac{\lambda \cdot B_0^2}{\rho}. ds, \) \( \lambda \) is the fringe field extent.

Expansion of the equations of motion around the scalloped orbit in the linear approximation yields the approximate tunes

\[ \nu_r \approx \sqrt{1 + K}, \quad \nu_z \approx \sqrt{-K + F^2(1 + 2 \tan^2 \zeta)} \quad (F = \frac{B^2}{B_0^2} - 1 \xrightarrow{hard-edge} \frac{R}{\rho} - 1) \]
• Second radial sector, 50 MeV, 2-way

- Preliminary studies early 1957. The spiral sector e-model was not yet completed - this determined the choice of radial sector: easier to design, better understood. Pole is not scaling: \( g \propto 1/r \), constant tunes require tweaking the flutter, and pole face windings.
- Study objectives: RF stacking, high circulating \( I \), 2-way storage.
- First start Dec. 1959, 2-beam mode, 27 MeV; disassembled in 60, magnets corrected; second start Aug. 61, single beam, 50 MeV.

---

**FFAG parameters**

\[
\begin{array}{l|l|l|l}
\text{parameter} & \text{value} \\
\hline
E_{\text{inj}} - E_{\text{max}} & \text{MeV} & 0.1 - 50 & \text{reasonable size & beam life-time} \\
\text{orbit radius} & m & 1.20 - 2.00 & B_\rho : 0.001 \rightarrow 0.17 \text{ (T.m)} \\
\hline
\text{Optics} \\
\text{lattice} & \text{FODO} \\
\text{number of cells} & 16 \\
K & 9.25 \\
\nu_r / \nu_z & 4.42 / 2.75 \\
\hline
\text{Magnet} \\
\theta, \text{ core} & \text{deg} & 6.3 \\
\text{peak field} & T & 0.52 \\
gap & \text{cm} & 8.6 \\
power & kW & 100 \\
\hline
\text{Injection} \\
e\text{-gun + e-inflector} \\
\hline
\text{Acceleration} \\
\text{swing} & \text{MHz} & 20 - 23 \\
\text{harmonic} & & 1 \\
voltage p-to-p & \text{kV} & 1.3 - 3 \\
cycle rep. rate & Hz & 60 \\
\hline
\end{array}
\]

- Ended up injector to the first dedicated light source storage ring - by MURA, Tantalus.
3 Mid-1960s to late 1990s

- After MURA, reduced activity. On alternative proposals in high power proton beam projects mostly.

  - **ESS accelerator facility** should serve two target stations, a 5MW 50 Hz and a 1MW 10Hz.

    Structure: a 1.33GeV H- Linac followed by 2 accumulator rings that compress the beam pulse to 0.4µs (H- injection, 1000 turns), 2.5MW throughput each, 2.3e14ppp, 25Hz, radius 26m, ia in each ring=63A.

    Alternative FFAG scheme (early 1990’s) : 0.4 GeV H- Linac followed by either 1.6 or 3 GeV FFAG.

    Finally rejected, considered difficult option: injection (drifts too short), large magnets, high cost.

| Beam Power  | MW | 5 |
| Top E       | GeV| 3 |
| Ppp         | GeV| 2e14 |
| Rep. Rate   | Hz | 50 |
| < I >       | mA | 1.7 |
| Max. Radius | m | 45 |

- Injection

  - Multiturn, charge exchange

  - Space charge tune shift constraint

  - 300 µs

- Extraction

  - Single turn, fast kicker

  - Favors lower intensity (hence higher top E and – stronger magnets)

- FFAG Optics

  - DFD triplet, K

  - Feasibility of the magnets was demonstrated / \( \gamma_{tr} > \gamma_{max} \)

  - Considered too short for injection

  - Yields “very massive magnets”

  - SC magnets considered - MAFIA calculations performed.

- RF

  - Freq

  - Voltage (×10 cavities)

  - MHz | 0.8 - 1 |
  - kV  | 20 |
Two options 1: synchrotron, Linac
Possible option 3: FFAG, because it is supposed to feature large acceptance, high repetition rate.
Two optics explored: spiral (issue of RF space) and radial (magnets and $\beta_V$ too big).

An additional drawback was: difficult to install in existing accelerator complex.

<table>
<thead>
<tr>
<th>p-Driver</th>
<th>FFAG radial sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>GeV</td>
</tr>
<tr>
<td>$E - \text{injection}$</td>
<td>MeV</td>
</tr>
<tr>
<td>beam power</td>
<td>MW</td>
</tr>
<tr>
<td>p/bunch</td>
<td></td>
</tr>
<tr>
<td>circumference</td>
<td>m</td>
</tr>
<tr>
<td>optics</td>
<td>DFD</td>
</tr>
<tr>
<td># of sectors</td>
<td></td>
</tr>
<tr>
<td>K value</td>
<td></td>
</tr>
<tr>
<td>radial extent</td>
<td>m</td>
</tr>
<tr>
<td>rep. rate</td>
<td>Hz</td>
</tr>
<tr>
<td>b/pulse (RF harmonic)</td>
<td></td>
</tr>
<tr>
<td>p/pulse</td>
<td></td>
</tr>
<tr>
<td>RF frequency</td>
<td>MHz</td>
</tr>
<tr>
<td>RF peak power</td>
<td>kW</td>
</tr>
<tr>
<td>$&lt;I&gt;$</td>
<td>$\mu$A</td>
</tr>
<tr>
<td>$\beta \gamma \epsilon_{x,z}$</td>
<td>$10^{-6}$ m.rad</td>
</tr>
<tr>
<td>$\beta \gamma \epsilon_{i}$</td>
<td>eV.s</td>
</tr>
<tr>
<td># of injections to MI</td>
<td></td>
</tr>
<tr>
<td>(inj. time 400 ms)</td>
<td></td>
</tr>
<tr>
<td>cost estimates</td>
<td>M$</td>
</tr>
</tbody>
</table>

- FFAG: need more than one ring?
- synchrotron $\frac{1}{7} \rightarrow$ FFAG
- FFAG needs bunch rotation for inj. into RF peak power
- 7 times lower circulating current in FFAG
- an advantage of Option 2 compared to Rough, for a 0.8-2.5 GeV, 5 MW design

\[ \frac{1}{7} \rightarrow \text{FFAG} \]
4 The Neutrino Factory - a source of innovations

It has triggered a strong activity in the domain of FFAG design, and lead to the development of new concepts.

Europe NuFact

The Europe and the two US NuFact studies propose to accelerate muons up to the storage energy (20 or 50 GeV) by means of one or two 4- or 5-pass RLA’s. RLA’s are complicated machines (spreaders, combiners), hence expensive.

The Japan NuFact

J-Parc: 50-GeV, $3.3 \times 10^{14}$ ppp at 0.3 Hz (15 µA) / 0.75 MW
Four muon FFAG’s : 0.2-1 GeV, 1-3, 3-10 (SC), 10-20 (SC).
No cooling, compact (R≈200m)
30ns/300±50% MeV bunch

Acceleration rate is lower than RLA, requires larger distance, but, acceptance is larger both transversally (twice : DA $3\pi$ cm norm. at $\delta p = 0$) and longitudinally ($\approx 5$ eV.s). Hence achieve comparable production rate : $\approx 10^{20}$ muon decays per year (1 MW p power).
5 KEK proton prototypes

- POP - Proof of principle, the first proton FFAG

[Typical] data

\[ E_{\text{inj}} - E_{\text{max}} \] keV 50 - 500
orbit radius m 0.8 - 1.14

Optics
- lattice
- number of cells DFD 8
- K 2.5
- \( \beta_r, \beta_z \) max. m 0.7
- \( \nu_r / \nu_z \) 2.2 / 1.25

Magnet
- high field, non-linear gradient
- \( \theta_D / \theta_F, \) core deg 2.8 / 14
- \( B_D / B_F \) T 0.04-0.13 / 0.14-0.32
- gap cm 30-9

Injection
- multi- or single-turn

Extraction
- massless septum exprmnt

Acceleration
- Amorphous MA cavity
  - swing MHz 0.6 - 1.4
  - harmonic 1
  - voltage p-to-p kV 1.3 - 3
  - cycle time ms 1
  - rep. rate kHz 1
  - \( \dot{B} \) T/s 180
  - broad band, high \( \vec{E} \) RF
  - 2-beam accel.

\[ B = B_0 (r/r_0)^K F(\theta) \]

\( r_{\text{inj}} \rightarrow r_{\text{max}} \)

\( \text{gap} = g_0 (r_0/r)^K \)

\{ electrostatic inflector + 2 bumpers \}

\( B \)
• The 150 MeV machine

First operation 2003.

[Typical] data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{\text{inj}} - E_{\text{max}}$</td>
<td>MeV 12 - 150</td>
</tr>
<tr>
<td>Orbit radius</td>
<td>m 4.47 - 5.20</td>
</tr>
</tbody>
</table>

**Optics**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lattice</td>
<td>DFD</td>
</tr>
<tr>
<td>Number of cells</td>
<td>12</td>
</tr>
<tr>
<td>K</td>
<td>7.6</td>
</tr>
<tr>
<td>$\beta_r / \beta_z$ max.</td>
<td>m 2.5 / 4.5</td>
</tr>
<tr>
<td>$\nu_r / \nu_z$</td>
<td>3.7 / 1.3</td>
</tr>
<tr>
<td>$\alpha$, $\gamma_{tr}$</td>
<td>0.13, 2.95</td>
</tr>
<tr>
<td>$R/\rho</td>
<td>E_{\text{max}}$</td>
</tr>
</tbody>
</table>

**Magnet**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_D / \theta_F$ deg</td>
<td>3.43 / 10.24</td>
</tr>
<tr>
<td>$B_D / B_F$ T</td>
<td>0.2-0.78 / 0.5-1.63</td>
</tr>
<tr>
<td>Gap cm</td>
<td>23.2 - 4.2</td>
</tr>
</tbody>
</table>

**Injection**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multi-turn</td>
<td></td>
</tr>
</tbody>
</table>

**Extraction**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single-turn</td>
<td></td>
</tr>
</tbody>
</table>

**Fast kicker (1kG, 150 ns)**

**Acceleration**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amorphous MA, broad band, high gradient RF</td>
<td></td>
</tr>
<tr>
<td>Swing MHz</td>
<td>1.5 - 4.5</td>
</tr>
<tr>
<td>Harmonic</td>
<td>1</td>
</tr>
<tr>
<td>Voltage p-to-p kV</td>
<td>2</td>
</tr>
<tr>
<td>$\phi_s$ deg</td>
<td>20</td>
</tr>
<tr>
<td>$\nu_s$</td>
<td>0.01 - 0.0026</td>
</tr>
<tr>
<td>$B$ T/s</td>
<td>300</td>
</tr>
<tr>
<td>Rep. rate Hz</td>
<td>250</td>
</tr>
</tbody>
</table>

[Typical] data

$E_{\text{inj}} - E_{\text{max}}$ = MeV 12 - 150

**Magnet**

- Return yoke free magnet

- $\theta_D / \theta_F$ deg 3.43 / 10.24
- $B_D / B_F$ T 0.2-0.78 / 0.5-1.63
- Gap cm 23.2 - 4.2

**Injection**

- Multi-turn

**Extraction**

- Single-turn

**Fast kicker (1kG, 150 ns)**

**Acceleration**

- Amorphous MA, broad band, high gradient RF
- Swing MHz 1.5 - 4.5
- Harmonic 1
- Voltage p-to-p kV 2
- $\phi_s$ deg 20
- $\nu_s$ 0.01 - 0.0026
- $B$ T/s 300
- Rep. rate Hz 250

First operation 2003.
6 KURRI KUCA

An ADS-R experiments, proton driver R/D

- First coupling to an ADS-R core, March 2009, 100 MeV beam
- Thorium-loaded ADS-R experiment, March 2010: 100 MeV, 30 Hz, 5 mW

100-150 MeV proton, repetition rate 20-50 Hz

- Upgrades:
  - On-going: H- charge exchange injection
  - Towards 10s of µAmp

Planned: neutron flux increased by a factor 30. Options: additional 700 MeV spiral lattice FFAG, or 400 MeV quasi-isochronous FFAG
The magnet gap is non-scaling: parallel faces. Pole face windings control the r-dependence of the vertical tune.
“Energy Recovery Internal Target”, at KURRI, Kyoto University.

- A compact proton storage ring for the production of 10 MeV BNCT neutrons
- High neutron flux is needed at patient: \( \approx 2 \times 10^{13} \) neutrons in 30 minutes for typical tumor volume
- Today, a 5-10 MW reactor is used, there is needed for hospital environment compliant equipment: ERIT

**Injector (425 MHz RFQ + IH-DTL)**

H-, kinetic energy 11 MeV  
Peak/average beam current 5 mA / > 100 \( \mu \)A  
Repetition rate 200 Hz, d.c. 2%

**FFAG ring**

FDF lattice, 8 cells  
H- injection on internal Be target (5 - 10 \( \mu \)m thick)  
proton energy 11 MeV  
circulating current 70 mA

**ERIT system**

Beam survival 500-1000 turns  
Target lifetime > 1 month  
\( \Delta E / \text{turn} \) 70 keV  
**RF cavity**

Operated CW, 100 kW input power  
RF voltage / frequency 250 kV / 18.1 MHz  
Harmonic number 5
8 PRISM

- A muon bunch phase rotator
  - An R/D program started in 2003
  - FFAG used as phase rotator, for momentum compression
    p=68MeV/c +/-20% down to +/-2% in 6 turns
  - Advantage of FFAG optics : large geometrical acceptance, zero chromaticity
  - A difficult task : injection and extraction

  - FFAG ring characteristics :
    - DFD lattice 14t triplet yoke, 120 kW/triplet
    - $K, B_F / B_D$ variable → quasi-decoupled $\nu_x, \nu_z$ adjustments
    - H / V apertures : 1 / 0.3 m
    - acceptance : $4\pi$ cm.rad $\times$ 0.65 $\pi$ cm.rad
    - RF : 5-gap cavity, 33 cm gap, 150-200 kV/m, 2MV/turn, saw-tooth waveform

  - 2005: downsized to 6 cells for POP,
    - central orbit radius 3 meter
    - 2.1 MHz (h=5) RF, gap voltage 33 kV peak
    - operated using 100 MeV/c alphas from an $^{241}$Am source.
9 RACCAM

- Working frame: Neutrino factory R/D and medical applications.
- A feasibility study of a rapid-cycling, variable energy, spiral lattice scaling FFAG
- Magnet prototype (built by SIGMAPHI) proved \{ gap shaping spiral sector scaling FFAG field, including flutter

Tracking in measured field maps (3D Hall-probe, by SIGMAPHI) proved \{ constant tunes large dynamical aperture

- Outcome:
  - demonstration of gap shaping scaling spiral dipole feasibility. First of the kind.
  - a cost-effective multiple-beam delivery hadrontherapy installation.
10 Straight S-FFAG line

- Scaling FFAG accelerators can be designed not only in a ring shape, but also with no overall bend.
- This requires a mid-plane guide field of the form

\[ B_y(x, s) = B_0 e^{m(x-x_0)} F(s) \]

- Similarly to the cylindrical \((r, \theta)\) case, \(B_0\) is some reference field value, taken at some arbitrary reference \(x_0\), \(F(s)\) is a flutter function.

\[ m = \frac{\rho}{B_0} \frac{\partial B}{\partial x} \]

is the normalized field gradient.

The experiment used the 11 MeV linac (injector of the 150 MeV FFAG). The FDF cell is moved horizontally (bellows) to match the incident beam momentum to the proper FFAG orbit.

The experiment measured the orbit location, optical functions including phase advances, for various momenta, and showed good agreement with outcomes of tracking in field maps.
Back to the neutrino factory, US-Study-2a: based on linear FFAG

- FFAG based on linear optical elements (quadrupoles)
- orbits no longer scale, tunes are allowed to vary with energy

This has a series of consequences:

- $R/\rho < 2$ - this decreases the machine size compared to classical (scaling) FFAG
- horizontal beam excursion is reasonable (small $D_x$) → magnets apertures are much smaller
- yields large transverse acceptance ← fields are linear ($3\pi \text{cm}$ achieved)
- small $\delta\text{TOF}$ over energy span, allows fast acceleration high gradient RF (200 MHz type SCRF cavities)
- Above 5 GeV, non-scaling linear FFAG method yields lower cost/GeV than RLA.
● The EMMA experiment.

○ An experimental model to investigate the new concept of “linear FFAG”:
  - linear magnets (quadrupoles) \(\rightarrow\) yields huge acceptance
  - fixed field \(\rightarrow\) yields fast acceleration
  - fast acceleration \(\rightarrow\) requires a lot of RF, and fixed frequency, gutter acceleration

A model of Study IIa FFAG
10 to 20 MeV
42 cells, doublet
pole-tip fields \(\approx 0.2\ T\)
apertures \(\approx \phi 40\ mm\)
37cm cell length
16m circumference
1.3GHz RF
1 cavity every other cell

- Launched in the frame of Neutrino Factory R&D
- An experimental model of muon accelerators
- International collaboration:
  BNL, CERN, FNAL, LPSC, STFC, J. Adams Inst., Cocksoc Inst., TRIUMF
- Recollection:
  1999: principle of linear FFAG optics, FNAL
  2001: first e-model meeting, BNL
  2006: project funded by “British Accelerator Science and Radiation Oncology Consortium”,
  3.5 years: 04-2007 / 09-2010, £5.6M budget
- Construction started at Daresbury, 04/2007, first beam planned summer 2009
  Beam due Autumn 2009
• Construction at Daresbury Lab. started in 2007
• Commissioning started in 2010
• “Serpentine” acceleration demonstrated in 2011

<table>
<thead>
<tr>
<th>EMMA parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy range</td>
</tr>
<tr>
<td>number of turns</td>
</tr>
<tr>
<td>circumference</td>
</tr>
<tr>
<td>Lattice</td>
</tr>
<tr>
<td>No of cells</td>
</tr>
<tr>
<td>RF frequency</td>
</tr>
<tr>
<td>No of cavities</td>
</tr>
<tr>
<td>RF voltage</td>
</tr>
<tr>
<td>RF power</td>
</tr>
<tr>
<td>Rep. rate</td>
</tr>
</tbody>
</table>
12 A bestiary of FFAG design studies

A *very limited* excerpt of what can be found in our Labs... See the bibliography for more, in particular the FFAG workshops.

• nuSTORM FFAG decay ring, J.-B. Lagrange et al. [IPAC2016]

◊ Neutrinos from STORed Muon beam (nuSTORM), the simplest implementation of a neutrino factory: pions are directly injected into a racetrack storage ring, where the circulating muon beam is captured.

◊ The racetrack nuSTORM FFAG lattice with zoom on the straight section (top left) and on the arc section (right).

◊ The muon flux is key to successful neutrino experiment, so, FFAG optics allows ring with large momentum acceptance, $\sim \pm 20\%$. 

straight section to avoid activation in the arc.
• Vertical FFAG [S. Brooks, PRST AB 16, 084001 (2013)]
  ◊ Vertical scaling optics was devised by K. Ohkawa (once known as the “smokatron”),
  ◊ Re-investigated recently [S. Brooks, prst-ab]

Field on closed orbit in a scaling VFFAG magnet:

\[ B_0 \exp(ky) \]

Momentum dependence of vertical orbit position:

\[ y = \frac{1}{k} \ln \frac{p}{p_{inj}} \]

Path-length is constant. Relativistic motion is isochronous.
• A linear FFAG proton-driver design [S. Ruggiero, BNL, 2004]

◊ A 3-stage linear FFAG cascade, as a NuFact p-driver

◊ Linear FDF FFAG triplet

◊ Neutrino factory p-driver parameters:

<table>
<thead>
<tr>
<th></th>
<th>Ring 1</th>
<th>Ring 2</th>
<th>Ring 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy, Inj. (GeV)</td>
<td>0.4</td>
<td>1.5</td>
<td>4.5</td>
</tr>
<tr>
<td>Extr. (GeV)</td>
<td>1.5</td>
<td>4.5</td>
<td>12</td>
</tr>
<tr>
<td># of turns</td>
<td>1800</td>
<td>3300</td>
<td>3600</td>
</tr>
<tr>
<td>cycle time (ms)</td>
<td>6</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>Circumf. (m)</td>
<td>807</td>
<td>819</td>
<td>831</td>
</tr>
<tr>
<td># cells</td>
<td>136</td>
<td>136</td>
<td>136</td>
</tr>
<tr>
<td>cell length (m)</td>
<td>5.9</td>
<td>6</td>
<td>6.1</td>
</tr>
<tr>
<td>h</td>
<td>136</td>
<td>138</td>
<td>140</td>
</tr>
<tr>
<td>RF freq. (MHz)</td>
<td>36-46</td>
<td>46-49.7</td>
<td>49.7-50.4</td>
</tr>
<tr>
<td>E gain / turn (MeV)</td>
<td>0.6</td>
<td>0.9</td>
<td>2</td>
</tr>
</tbody>
</table>

◊ Operation mode variant considered for ring #1, for ∼MW beam power in GeV range:
- broad-band, few MHz RF (JPARC style), cycled,
- repetition rate >100 Hz,

◊ Even higher rep. rate. (toward CW) based on
- “harmonic number jump”,
- using high frequency fixed-frequency RF.
Requires cavity with transverse RF voltage profile.

◊ Fast resonances crossing:

\[ Q_x : 40 \rightarrow 19, \quad Q_y : 38 \rightarrow 9. \]
• Pumplet lattice [G. Rees, RAL]

- A non-linear, non-scaling type of FFAG, isochronous
- A scheme investigated for a 20 GeV, 4 MW proton driver for the neutrino factory

○ The many knobs (field non-linearities) allow isochronism

Lattice for 8 to 20 GeV / 16 turns / 123 cell ring:

\[
\begin{align*}
O & \quad bd(-) & \quad F(\pm) & \quad BD(+) & \quad F(\pm) & \quad bd(-) & \quad O \\
\end{align*}
\]

\[
B_{bd}(x) = -3.456 - 6.6892 x + 9.4032 x^2 - 7.6236 x^3 + 360.38 x^4 + 1677.79 x^5
\]

\[
B_{BF}(r) = -0.257 + 16.620 r + 29.739 r^2 + 158.65 r^3 + 1812.17 r^4 + 7669.53 r^5
\]

\[
B_{BD}(x) = 4.220 - 9.659 x - 45.472 x^2 - 322.1230 x^3 - 5364.309 x^4 - 27510.4 x^5
\]

Allows insertion straights - advantages:
1. easier injection and extraction,
2. space for beam loss collimators,
3. RF gallery extending only above the insertions, not above the whole ring,
4. 4-cell cavities usable, thus reducing, by a factor of four, the total number of rf systems.

○ Field profile in BF and BD:

○ Beam trajectory in the tune diagram:
- Toward CW [C. Johnstone, FNAL]

Quasi-isochronous optics,
- based on SC dipoles and featuring
  - alternating-gradient with non-linear radial field profile
  - optimized magnet-edge contour
- Allows near-crest (serpentine) acceleration, based on SCRF

- Principle 6-cell lattice used for numerical beam dynamics studies:
  - 0.33 to 1 GeV at a rate of 10∼20 MV/turn

- Numerical beam dynamics studies show
  - large transverse dynamical acceptance
  - currents in 20 mA range with no transverse beam growth doable (OPAL simulations)
• Serpentine acceleration in scaling lattice, FFFAG [E. Yamakawa et al., KURRI]

○ The lattice $\gamma_{tr} = \sqrt{1 + K}$ is set to be in the acceleration range: beam $\gamma$ is accelerated in transition region, time of flight is parabolic

○ This allows using fixed RF-frequency acceleration in variable $\beta = v/c$ regime - i.e., case of non-relativistic beam, suitable for proton acceleration.

○ Experimental demonstration performed with an electron prototype (Japan, 2012):

- small e-beam ring
- 160 keV $\rightarrow$ 8 MeV
- F-D-F scaling triplet lattice at transition gamma (764 keV)
- RF freq. 75 MHz ($h=1$), 750 kV/gap

○ An ADS equivalent has been designed (NIM A 716 (2013))

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$-value</td>
<td>1.45</td>
</tr>
<tr>
<td>Equivalent mean radius at 200 MeV [m]</td>
<td>3</td>
</tr>
<tr>
<td>Equivalent mean radius at 1 GeV [m]</td>
<td>5.9</td>
</tr>
<tr>
<td>Stationary kinetic energy below transition [MeV]</td>
<td>360</td>
</tr>
<tr>
<td>rf voltage [MV/turn]</td>
<td>15 ($h=1$)</td>
</tr>
<tr>
<td>rf frequency [MHz]</td>
<td>9.6($h=1$)</td>
</tr>
</tbody>
</table>
And today ...

More going on!

THANK YOU FOR YOUR ATTENTION