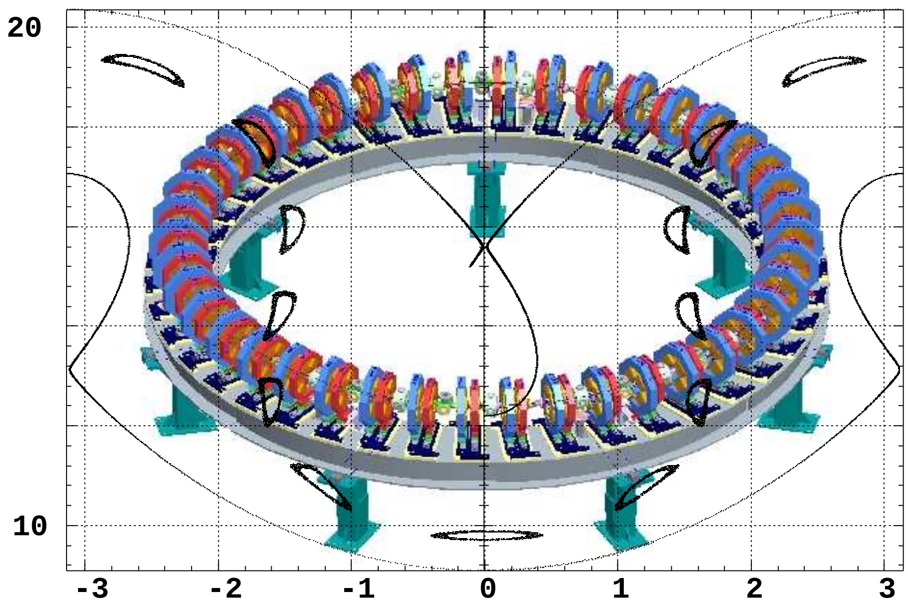
ZGOUBI GALAXY, AND THE REST

(Or : "Where The Answer Is Not Going To Just Be '42' ")

(Interestingly, SATURNE is Zgoubi genitor)



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Contents

1/ **I'll be commenting the version of Zgoubi that I maintained myself, over the years.**

I won't discuss developments done by (the many) other groups/people.

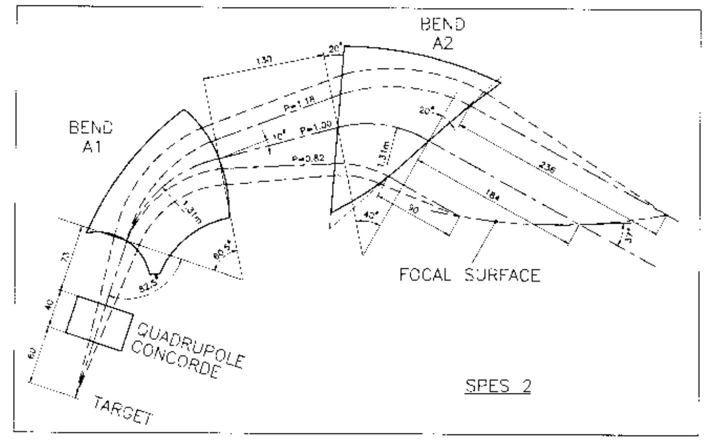
2/ **It is available on a development site, together with its "Users' Guide" and its graphic/analysis interface "zpop", and many operational examples**

http://sourceforge.net/projects/zgoubi/

3/ A lot of articles and other technical reports, of which many give many details on the examples I am going to show, can be found on the DOE OSTI site

http://www.osti.gov/bridge/

... **was written in 1972, at SATURNE, Saclay, for a big nuclear physics spectrometer, SPES2, by J. C. Faivre and D. Garreta**



- **The equation of motion in magnets writes**

$$d(m\vec{v}) \;=\; q\,\vec{v}\times\vec{b}\,dt$$

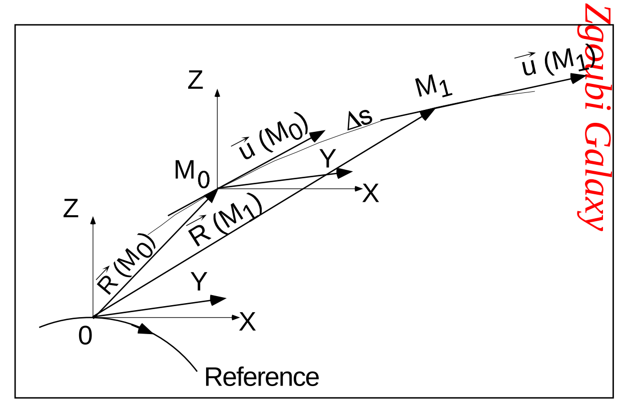
Introduce reduced notations, $\vec{u}=\frac{\vec{v}}{v},\quad \vec{B}=\frac{\vec{b}}{B\rho}$**, that yields :**

$$\boxed{\vec{u}' = \vec{u}\times\vec{B}}$$

- **Solved using truncated Taylor expansions of** $\vec{R}$ **and** $\vec{u}=\vec{v}/v$ **:**

$$\vec{R}(M_1)\approx\vec{R}(M_0)+\vec{u}(M_0)\,\Delta s+\vec{u}'(M_0)\,\frac{\Delta s^2}{2!}+\ldots+\vec{u}'''''(M_0)\,\frac{\Delta s^6}{6!}$$

$$\vec{u}(M_1)\approx\vec{u}(M_0)+\vec{u}(M_0)\,\Delta s+\vec{u}''(M_0)\,\frac{\Delta s^2}{2!}+\ldots+\vec{u}'''''(M_0)\,\frac{\Delta s^5}{5!}$$



• Over 45+ years... oodles of magnetic elements have been installed

WHAT YOU DREAM TO SIMULATE :	*KEYWORD :*
Semi-analytical models :	
Cyclotron	CYCLOTRON, DIPOLE, DIPOLES
Decapole	DECAPOLE, MULTIPOL
Dipole(s), spectrometer	BEND, DIPOLE[S][-M], MULTIPOL, QUADISEX
Dodecapole	DODECAPO, MULTIPOL
FFAG magnets	DIPOLE[S], FFAG, FFAG-SPI
Multipole	MULTIPOL, QUADISEX, SEXQUAD
Octupole	OCTUPOLE, MULTIPOL, QUADISEX, SEXQUAD
Quadrupole	QUADRUPO, MULTIPOL, SEXQUAD
Sextupole	SEXTUPOL, MULTIPOL, QUADISEX, SEXQUAD
Solenoid	SOLENOID
Helical dipole	HELIX
Field maps :	
1-D, cylindrical symmetry	BREVOL
2-D, mid-plane symmetry	CARTEMES, POISSON, TOSCA
2-D, no symmetry	MAP2D
2-D, polar mesh	POLARMES
3-D	TOSCA
include time (*a 4D map ?!?*)	Any of the above. (*yes, e.g., using TOSCA*)

• ... as well as many accelerator design and beam dynamics procedure

WHAT YOU DREAM TO DO :	*KEYWORD :*
Store coordinates (in view of "gnuplot" turn-by-turn for instance)	FAISTORE
Fit	FIT
A scan of optimal parameters	FIT + REBELOTE
Forking	GOTO
Store particle data across magnets (to "gnuplot" B field, why not)	IL=1 !
Compute a transport matrix, or Twiss parameters	MATRIX
Switch-on in-flight decay	MCDESINT
Generate a Monte Carlo particle bunch	MCOBJET
Generate sample particles	OBJET
Simulate a ring and other do-loop	REBELOTE
Vary magnet currents, RF system parameters etc.	SCALING
Switch-on spin tracking	SPNTRK
Switch-on synchrotron radiation	SRLOSS
Call your system for rescue !	SYSTEM

And more ... in "Zgoubi Users' Guide" (and its Index)

AN EXAMPLE OF A "KEYWORD" : 'MULTIPOL'

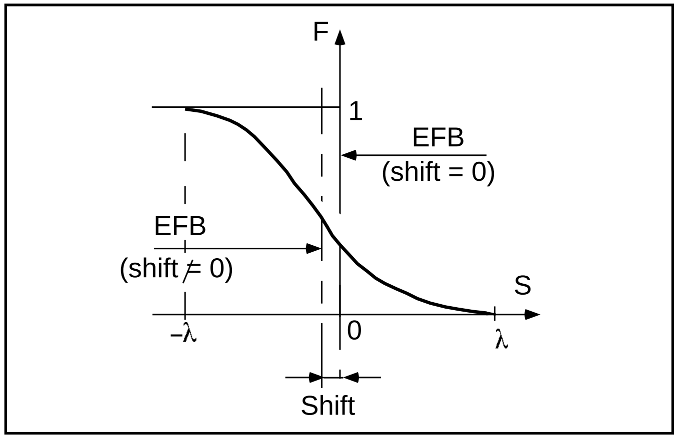
- **Field and derivatives as needed in the Taylor series**

$$\frac{\partial^{i+j+k} \vec{B}_n(X,Y,Z)}{\partial X^i \partial Y^i \partial Z^k} \qquad i+j+k = 0 \textbf{ to } 4 \tag{1}$$

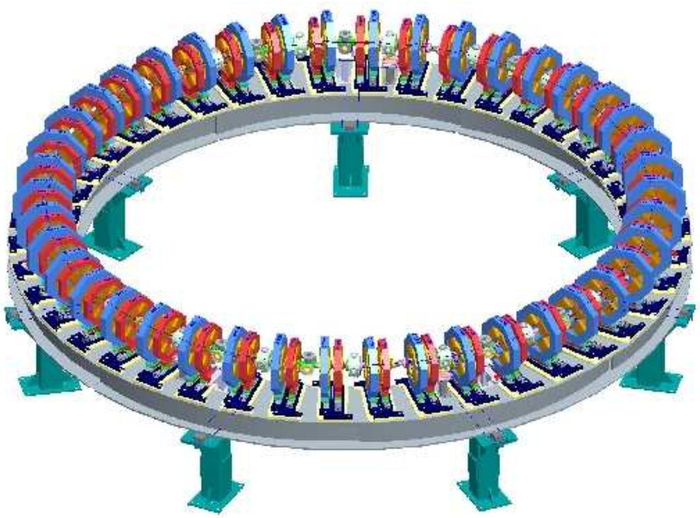
are obtained by differentiation of the scalar potential

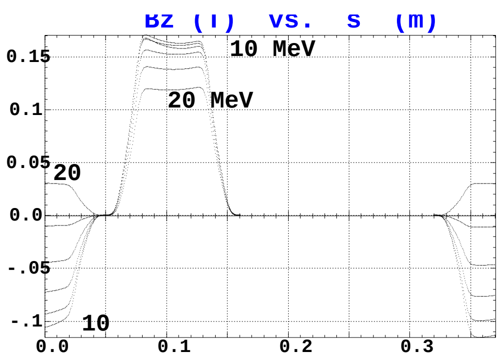
$$V_n(X,Y,Z) = (n!)^2 \left(\sum_{q=0}^{\infty} (-1)^q \frac{G^{(2q)}(X)(Y^2+Z^2)^q}{4^q q!(n+q)!} \right) \left(\sum_{m=0}^{n} \frac{sin\left(m\frac{\pi}{2}\right) Y^{n-m} Z^m}{m!(n-m)!} \right) \tag{2}$$

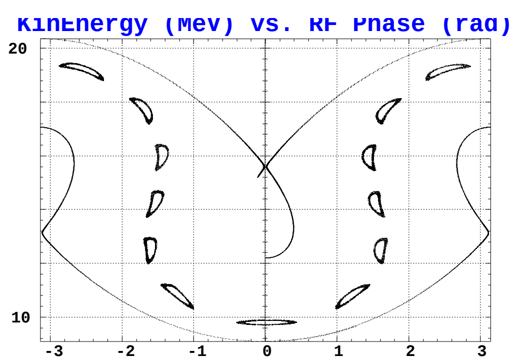
- $G(s)$ **is a longitudinal form factor which simulates the "field fall-off"**



EXAMPLE (2005+) – Virtual EMMA FFAG / ON-LINE MODEL



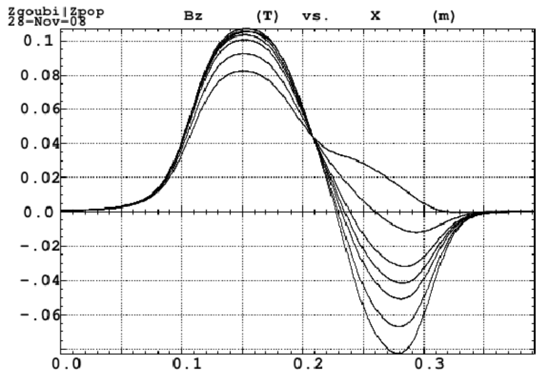




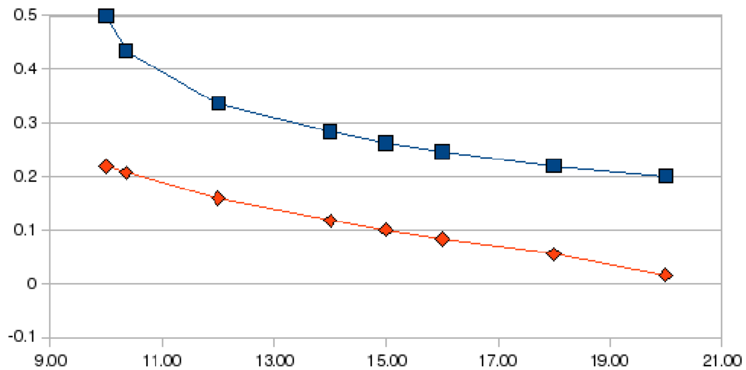
Zgoubi input data file - excerpt :

```
'MARKER' RingInj BegRing                         start of ring. Injection point
'MULTIPOL' QD                                    start of first cell
0
7.5698 5.3 0. -2.49324 0 0 0 0 0 0 0 0
1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0.1
2 0. 3.404834122312866 0.
'MARKER' BPM2 off                                BPM location
'DRIFT' sd
5.00
'MULTIPOL' QF
0
5.8782 3.7 0. 2.47708 0 0 0 0 0 0 0 0
1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0.1
2 0. 0.7513707181808552 0.
'DRIFT' ld
8.
'CAVITE'                                         programmable RF cavity
7
0.736669 1.3552e9                                Orbit length, RF frequency
70e3 0.                                          Voltage, relative phase
'MARKER' BPM1 off                                BPM location
'CHANGREF'                                       cell orientation - wrt. next one
0. 0. -8.571428571429                            end of first cell
-------------
next 41 cells
-------------
'REBELOTE'                                       multiturn tracking
150 0.2 99
'END'
```

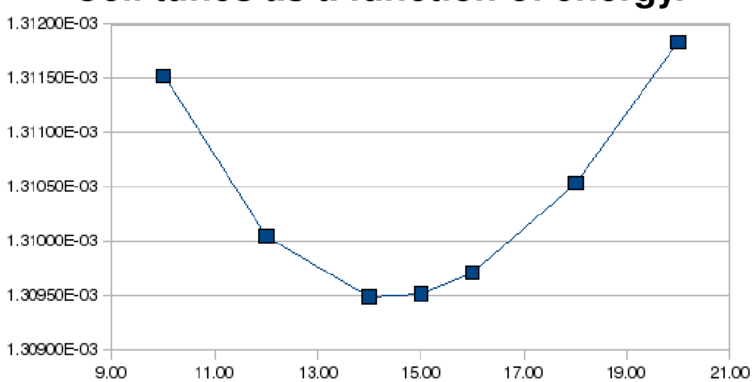
EMMA cell, using field map '02611DF.table' / compare with 'DIPOLES'



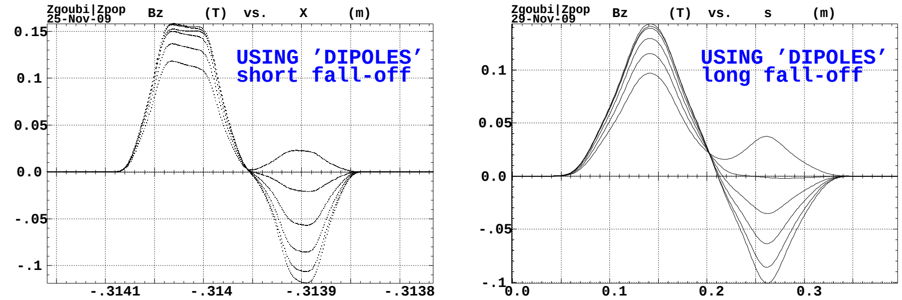
Field on orbits, 10, 12, 14, 15, 16, 18 and 20 MeV.



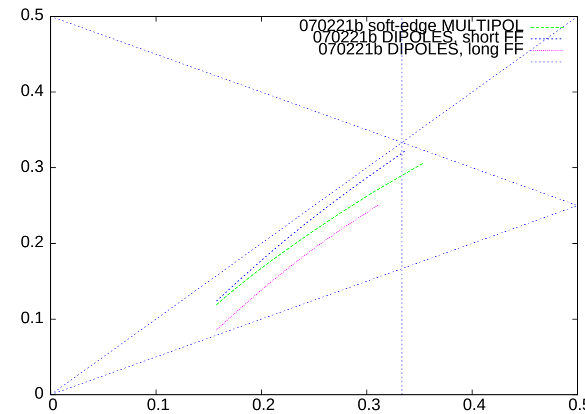
Cell tunes as a function of energy.
Cell tunes as a function of energy.



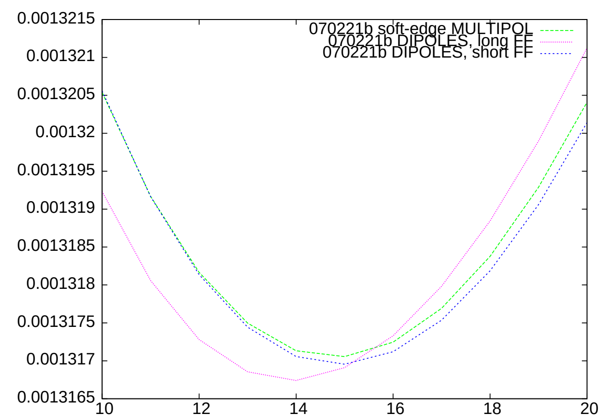
Time of flight as a function of energy.



Field on orbits, 10, 12, 14, 15, 16, 18 and 20 MeV.



Cell tunes as a function of energy.



Time of flight as a function of energy.

NOTE : 'TOSCA' WILL SUPERIMPOSE FIELD MAPS FOR YOU :

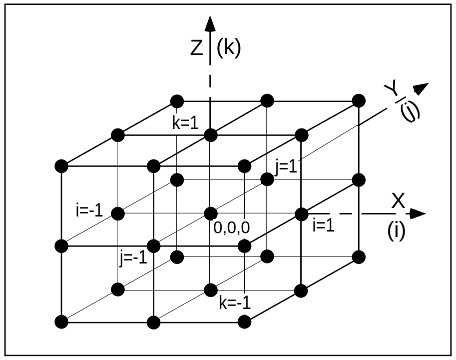
Zgoubi input data file - excerpt :

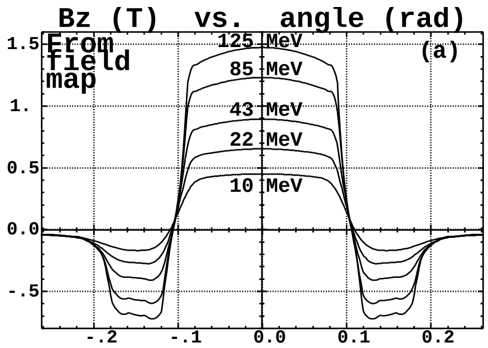
```
EMMA CELL, USING FIELD MAPS
'OBJET'
+5.171103865921708e+01
2
1 1
-4.83 1.38E+02 0. 0. 0. 0.7 '0'
1
'PARTICUL'
0.51099892 1.60217653e-19 0.0 0.0 0.0
'PICKUPS'
1
#E
'FAISTORE'
b_zgoubi.fai #E
1
'TOSCA'
0 0
-10. .1 .1 .1
QPOLES HEADER_8
961 161 1 15.2 1. 1.                    ! IZ=1 & mod.mod2=15.2 ⟹ two 2D maps / up to 5
b_both-20130204a-f-off.table            ! F-quad is off, D-quad is on
b_both-20130204a-d-off.table            ! F-quad is on, D-quad is off
0 0 0 0
2
.1
2 0 0 0
'CHANGREF'
0. 0. -8.57142857152
'FAISCEAU'
'MARKER' #E
'MATRIX'
1 11
'FAISCEAU'
'END'
```

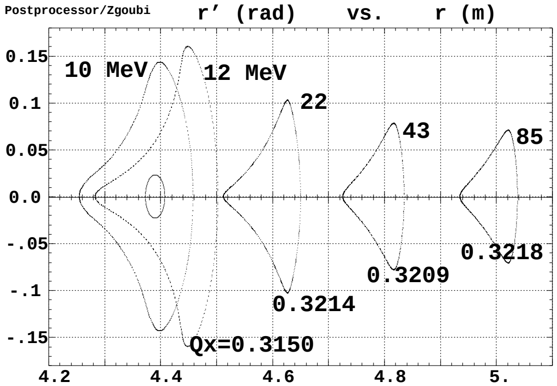
EXAMPLE (∼2005) - 'FFAG' and 'TOSCA' keywords

A simulation of a 10-cell 150 MeV FFAG ring based on a scaling FFAG dipole triplet cell.

- **Using 'TOSCA' keyword and an OPERA field map of the dipole triplet :**

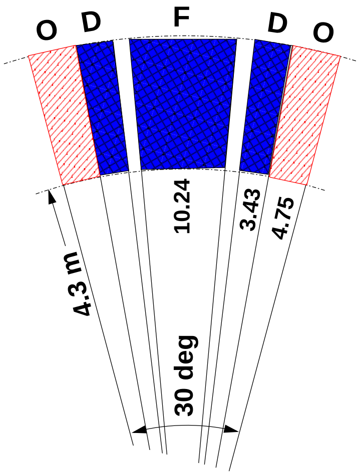


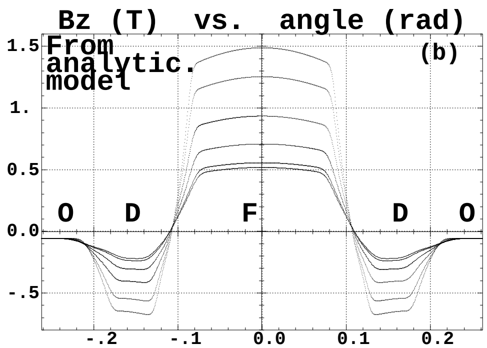


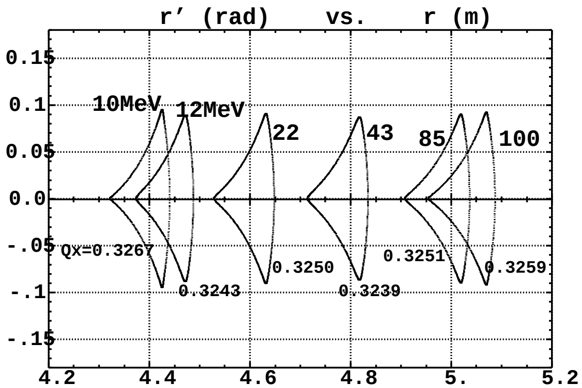


- **Using 'FFAG' keyword ('DIPOLES' would do as well) : superposition of dipole fields** [NIM A 547, Lemuet, Méot]

$$B_z(r,\theta) = \sum_{i=1,N} B_{z0,i}\, \mathcal{F}_i(r,\theta)\, \mathcal{R}_i(r)$$







Exemples available in sourceforge → code → exemples/KEK150MeVFFAG/analyticalModel, ./OPERAMap.

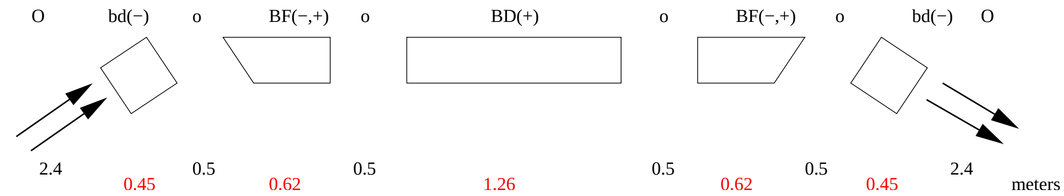
Zgoubi input data file, analytical model :

```
FFAG triplet. 150MeV machine
'OBJET'
1839.090113 150MeV
5
0.001 0.001 0.001 0.001 0.001 0.0001
486.802 0. 0.0 0. 0. 0.562925 50MeV
'FFAG' #START
0
3 30. 540. 1
6.465 0. -14.308348 7.25 0. 0.                DIPOLE #1 : ACNT, dum, B0, K,dummies
4. 000                                        EFB 1 : lambda, iop=data option f
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
1.715 0. 1.E6 -1.E6 1.E6 1.E6
4. 000                                        EFB 2
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
-1.715 0. 1.E6 -1.E6 1.E6 1.E6
0. -1                                         EFB 3 : inhibited by iop=0
0 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0.
15. 0. 17.16 7.58 0. 0.                       DIPOLE #2 : ACNT, dum, B0, K,dummies
4. 000                                        EFB 1
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
5.12 0. 1.E6 -1.E6 1.E6 1.E6
4. 000                                        EFB 2
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
-5.12 0. 1.E6 -1.E6 1.E6 1.E6
0. -1                                         EFB 3
0 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0.
23.535 0. -14.308348 7.25 0. 0.               DIPOLE #3 : ACNT, dum, B0, K,dummies
4. 000                                        EFB 1
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
1.715 0. 1.E6 -1.E6 1.E6 1.E6
4. 000                                        EFB 2
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
-1.715 0. 1.E6 -1.E6 1.E6 1.E6
0. -1                                         EFB 3
0 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0.
2 4.                                          IRD(=2, 25 or 4), resol(=¿step/*)
.5                                            integration step size (cm)
2 0. 0. 0. 0.
'MATRIX'
1 11
'END'
```

Zgoubi input data file, OPERA map :

```
150MeV FFAG
'OBJET'
1839.090113 150MeV
5
.001 .001 .001 .001 .001 .0001
444.15 0. 0.0 0. 0. 0.273042677097 'o'    12MeV Brho=502.1500877
'TOSCA' #START
0 20
-1.e-3 1. 1. 1.
HEADER_8 FFAG 150MeV
301 121 41 20
k75v113my021f45500d2700.table             OPERA field map
0 0 0 0
2
.0125
2
0. 0. 0. 0.
'MATRIX'
1 11
'END'
```

AN ISOCHRONOUS MUON FFAG
(G. REES, , RAL, ∼2004)



- **bd and BD multipoles :**

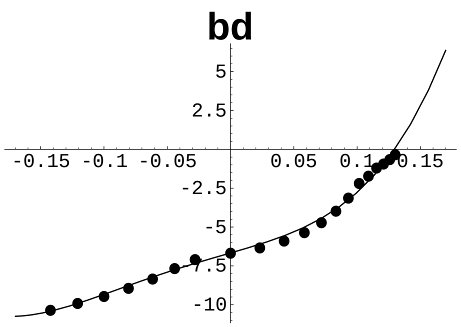
The magnets' gradients are constitutive of the design data, they are approximated using 4th degree polynomials.

$$B_{bd}(x) = -6.66771 + 23.5565r\ x + 11.9699\ x^2 + 926.188\ x^3 + 4952.98\ x^4$$
$$B_{BD}(x) = -9.723 - 51.5803\ x - 697.091\ x^2 - 33956.1\ x^3 - 241808\ x^4$$

The gradients are integrated to get the multipole coefficients of the field.

$$B_{bd}(x) = \mathbf{b_{bd0}} - 6.66771\ x + 11.7783\ x^2 + 3.98996\ x^3 + 231.547\ x^4 + 990.6\ x^5 \quad (3)$$
$$B_{BD}(x) = \mathbf{b_{BD0}} - 9.723\ x - 25.7902\ x^2 - 232.364\ x^3 - 8489.03\ x^4 - 48361.5\ x^5 \quad (4)$$

Gradient profiles K (m^{-2}) vs. x (m)

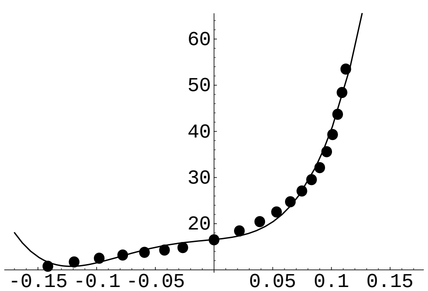


After finding the dipole coefficients $\mathrm{b_{bd0}}$, $\mathrm{b_{BD0}}$ with the matching procedure (next slide), the field and its derivatives are derived from the classical multipole modelling of the form.

$$\vec{B} = \mathbf{gr\vec{a}d} V_n \ \ \textbf{with} \ \ V_n(s,x,z) = (n!)^2 \left(\sum_{q=0}^{\infty} \frac{(-)^q\, G^{(2q)}(s)(x^2+z^2)^q}{4^q q!(n+q)!} \right) \left(\sum_{m=0}^{n} \frac{\sin\left(m\frac{\pi}{2}\right) x^{n-m} z^m}{m!(n-m)!} \right) \quad (5)$$

with $G^{(2q)}(center)$ representing the (derivatives of) the fringe field form factor.

- **BF sector magnet :**



The same gradient matching procedure is applied to obtain

$$B_{BF}(r) = \mathbf{b_{BF0}} + 16.5655\ r + 12.612\ r^2 + 86.4359\ r^3 + 2987.43\ r^4 + 13647.1\ r^5$$

Transform from BF cylindrical frame into Zgoubi Cartesian frame, using

$$\partial B_z/\partial X = (1/r)\partial B_z/\partial\theta,\ \ \partial B_z/\partial Y = \partial B_z/\partial r,\ \ \partial^2 B_z/\partial X^2 = (1/r^2)\partial^2 B_z/\partial\theta^2 + (1/r)\partial B_z/\partial r,\ \ etc.$$

Z-derivatives and extrapolation off mid-plane yield the 3-D $\vec{B}$ model

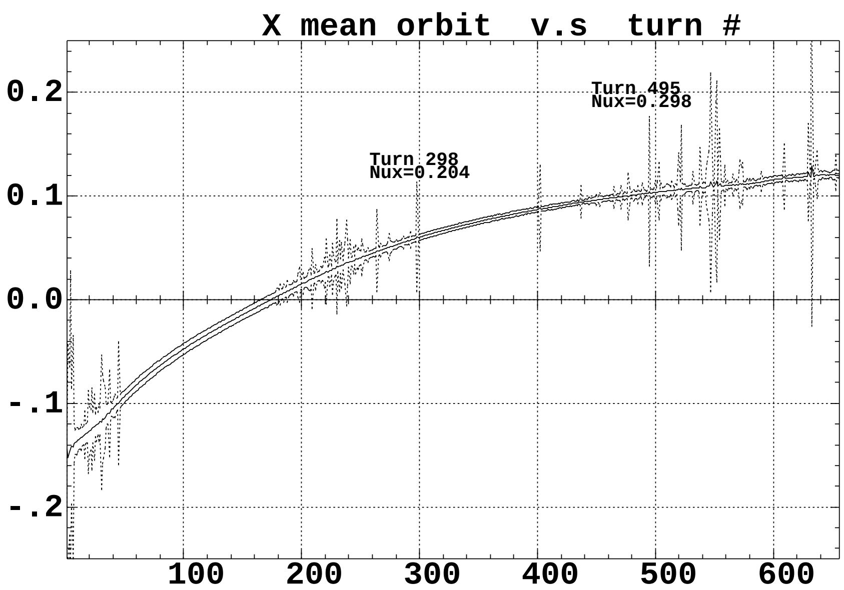
$$\vec{B}(X,Y,Z) \quad , \quad \partial^{i+j+k}\vec{B}/\partial X^i \partial Y^j \partial Z^k$$

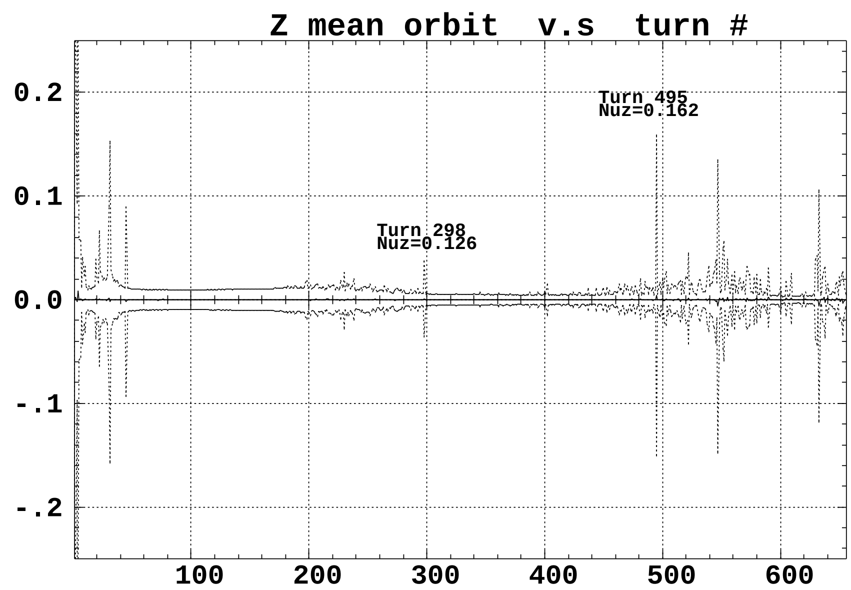
Optical sequence of the isochronous cell in Zgoubi input data file :

```
'MULTIPOL'       bd
  00
  45 100.00 -3.45374050E+01   -66.6771 117.783 39.8996 2315.47 9905.97 0. 0.0 0.0 0.0
 0. 0. 9. 4.  1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 0. 0. 9. 4.  1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
.5 step  bd
1 0. 0. 0.
'DRIFT'
50.
 'DIPOLES'    BF
  00
 1  1.463414634   24.274311920375e2           nbmag  AT/deg, RM/cm
 0.731707317  0. -2.25637704 5 -1782.12892 -32935.7018 -5479274.31  -4.59698831E+09 -5.09757397E+11
 0. 0.                                       EFB 1
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 0.731707317    0.  1.E6  -1.E6  1.E6  1.E6
 0. 0.                                       EFB 2
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
-0.731707317    0.   1.E6  -1.E6  1.E6  1.E6
 0. 0.                                        EFB 3
 0  0.      0.      0.      0.      0. 0.  0.
 0.  0.   0.    0.    0. 0. 0.
 0    2  64.
 .5 step  BF                                         step
 2   2.42294098E+03     0. 2.43432058E+03  0.   24.228e2  24.3458e2
'DRIFT'
50.
'MULTIPOL'       BD
  00
  63. 100.00  4.21503506E+01  -97.23 -257.902 -2323.64 -84890.3 -483615 0. 0. 0. 0.
 0. 0. 9. 4.  1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 0. 0. 9. 4.  1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
.5 step  BD
1 0. 0. 0.
```

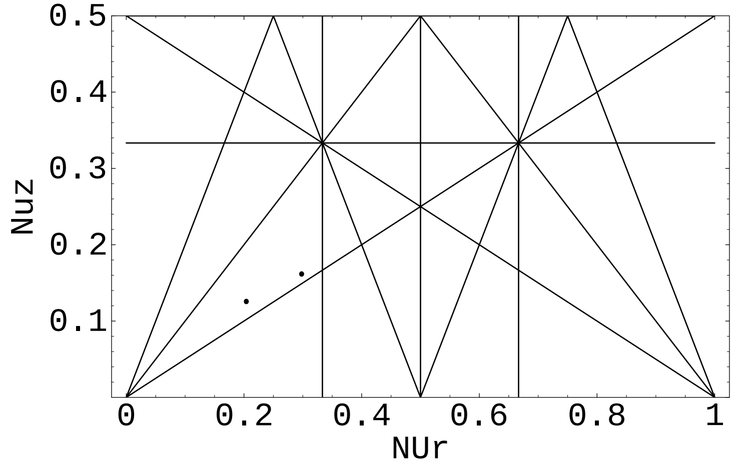
• **Tracking 10^4 muons, over 700 turns (a few muon life-time), in G. Ree's isochronous FFAG**

Correlation of beam losses and tunes :



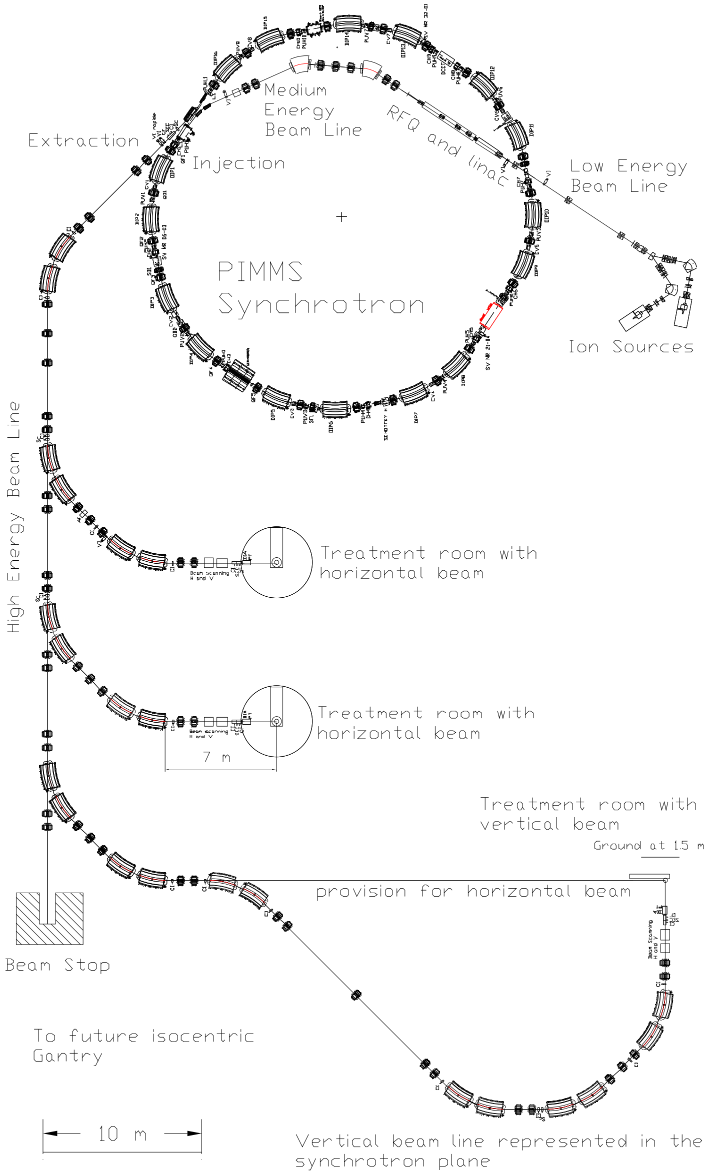


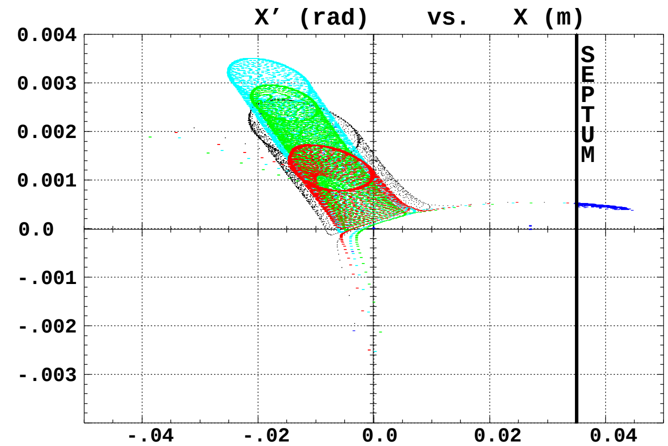
◇ Two tunes at which beam losses are observed :

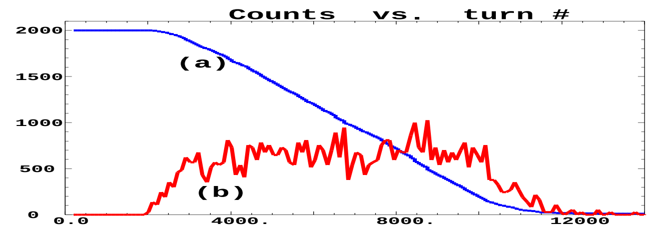


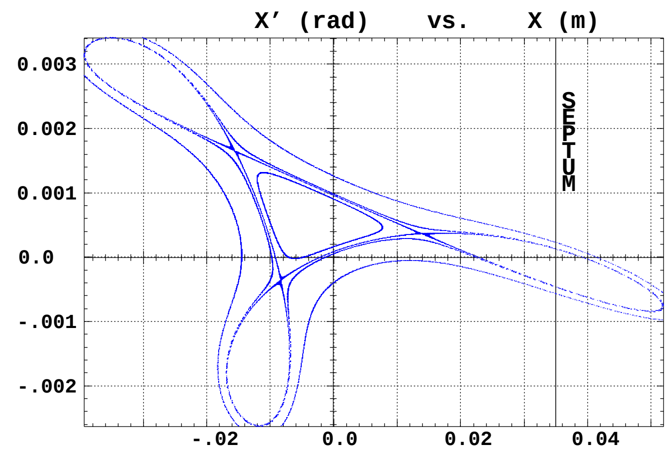
EXAMPLE (∼2000) – SLOW EXTRACTION FROM A MEDICAL CARBON SYNCHROTRON

• **Main difficulties : (i) motion near separatrix, (ii) slow process, ∼0.1 second(s) ⇒ $>> 10^5$ turns tracking.**







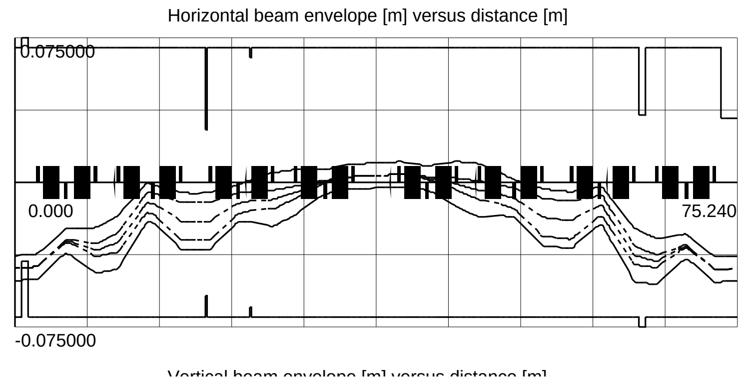


EXAMPLE 16-TURN INJECTION IN A MEDICAL CARBON SYNCHROTRON

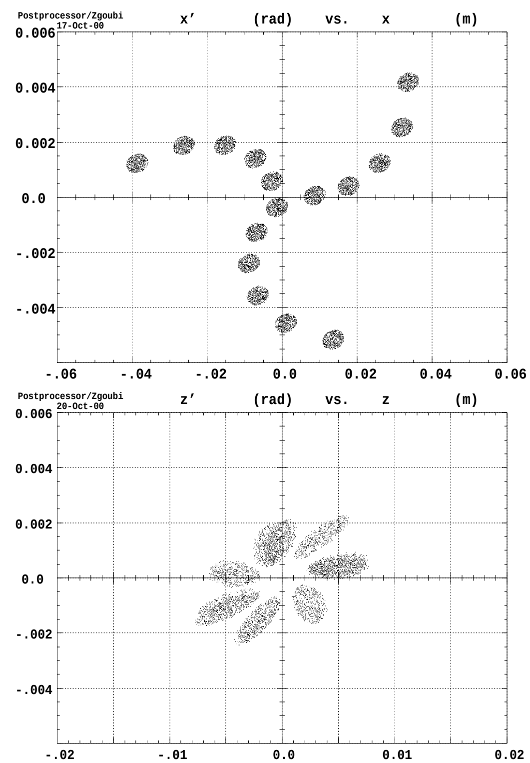
Zgoubi input data file, using 'SCALING'

```
****** Extraction, C6+
'MCOBJET'       Monte Carlo object, C6+ 120MeV/u
1000.
3
40
2 2 1 1 1 1
0. 0. 0. 0. 0. 0.997
0. 8.562 7.143E-6 1
0. 2.848 7.143E-6 1
0. 1. 4.E-6 1
123456 234567 345678

'SCALING'
1 2
MULTIPOL XR_
3
0 1 1
1 2000 999999
BETATRON
4
0. 0. 1 1
1 2000 2001 999999

'FAISTORE' lmnt# 106 179 206
b_xtract4Dp.fai ESE_ICOL
1
'BETATRON'      Start of ring
2.5e-6
'DRIFT' DRIF SS_MR_01_03
75.0000
'COLLIMA' ESi_COL
1
1 5.8 3.7 1.7 0. 1.7-5.8=-4.1, 1.7+5.8=7.5
'DRIFT' DRIF ES_INJECTION
60.0000

'INCLUDE'      ! the rest of the ring (one separate file, "carbonRing.inc")
1
carbonRing.inc

'DRIFT' DRIF SS_MR_01_02
109.4000
'FAISCEAU'      End of ring
'REBELOTE'
99999 0.1 99
'END'
```

- **'SCALING' commands the orbit bump**



Horizontal injection bump.



Carbon injection, 16 turns injected, observed at injection septum.

3 THE ELECTRIFICATION OF ZGOUBI

... intervened in the early 1990s, motivated, as usual, by on-going R/D tasks.

- **When both $\vec{e}$ and $\vec{b}$ are non-zero, the complete equation is solved,**

$$\boxed{(B\rho)'\vec{u} + B\rho\,\vec{u}' = \vec{e}\,/\,v\ +\ \vec{u}\times\vec{b}}$$

One can then push the rigidity, with the same method of (truncated) Taylor series

$$(B\rho)(M_1) \approx (B\rho)(M_0) + (B\rho)'(M_0)\Delta s + ... + (B\rho)''''(M_0)\frac{\Delta s^4}{4!} \tag{6}$$

and the time of flight,

$$T(M_1) \approx T(M_0) + \frac{dT}{ds}(M_0)\,\Delta s + \frac{d^2T}{ds^2}(M_0)\,\frac{\Delta s^2}{2} + \frac{d^3T}{ds^3}(M_0)\,\frac{\Delta s^3}{3!} + \frac{d^4T}{ds^4}(M_0)\,\frac{\Delta s^4}{4!} \tag{7}$$

• **A list of the electrostatic elements :**

WHAT YOU DREAM TO SIMULATE :	*KEYWORD :*
Semi-analytical models :	
2-tube (bipotential) lens	EL2TUB
3-tube (unipotential) lens	UNIPOT
Decapole	ELMULT
Dipole	ELMULT
Dodecapole	ELMULT
Multipole	ELMULT
N-electrode mirror/lens, straight slits	ELMIR
N-electrode mirror/lens, circular slits	ELMIRC
Octupole	ELMULT
Quadrupole	ELMULT
R.F. (kick) cavity	CAVITE
Sextupole	ELMULT
Skewed multipoles	ELMULT
Field maps :	
1D, cylindrical symmetry	ELREVOL
2-D, no symmetry	MAP2D_E

• A list of the *MAGNETO-ELECTROSTATIC* elements :

WHAT YOU DREAM TO SIMULATE :	*KEYWORD :*
Semi-analytical models :	
Decapole	**EBMULT**
Dipole	**EBMULT**
Dodecapole	**EBMULT**
Multipole	**EBMULT**
Octupole	**EBMULT**
Quadrupole	**EBMULT**
Sextupole	**EBMULT**
Skew multipoles	**EBMULT**
Wien filter	**SEPARA, WIENFILT**

EXAMPLE (TRIUMF, 1990) - BNL's TWO-STAGE 800-MeV/c KAON BEAMLINE, USING TWO *WIENFILTERS*

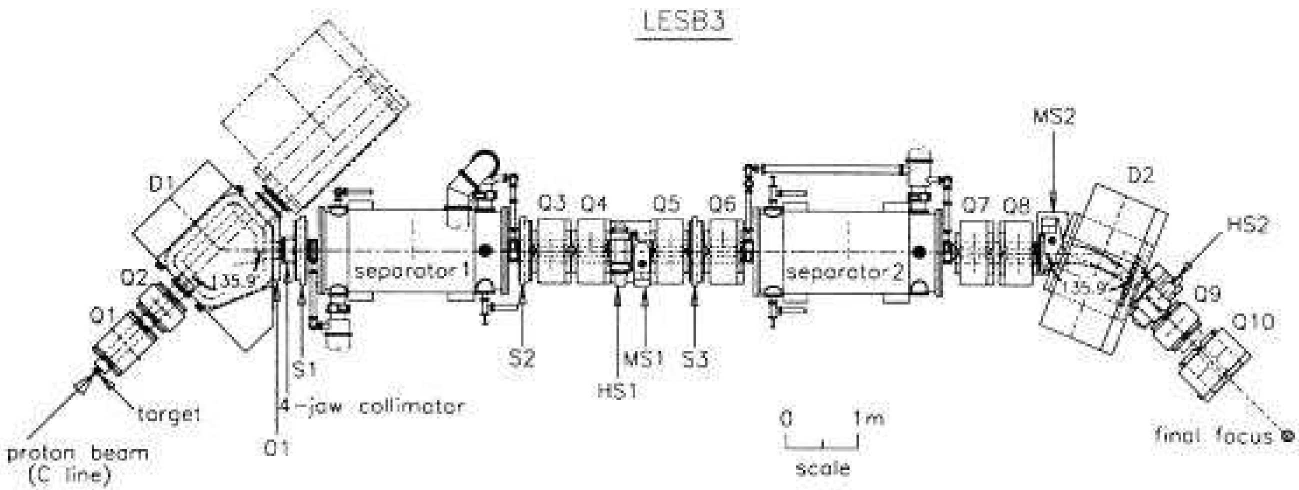


Fig. 1. Layout of LESB3 beamline.

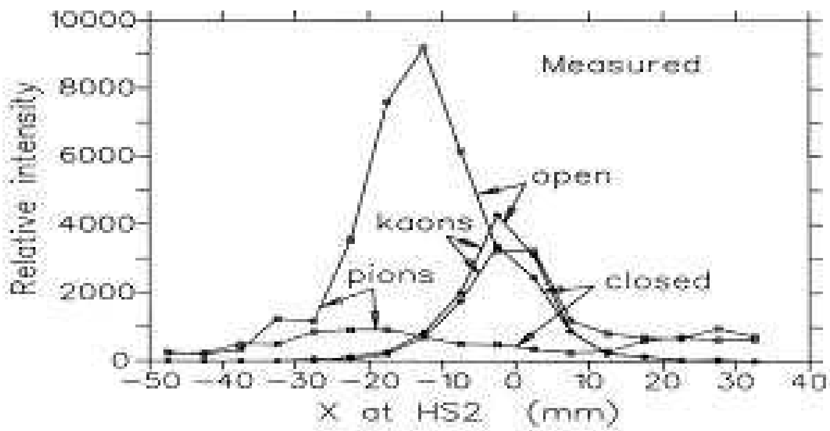


Fig. 16. Measured kaon and pion distributions at HS2 with the four-jaw collimator open and upper-right jaw closed. Compare with Fig. 10(a).

4 SPIN TRACKING

... was installed in late 1980s for a partial Siberian snake project at the 3 GeV ring SATURNE, Saclay.

- **Equation of spin precession :**

$$\frac{d\vec{S}}{dt} = \frac{q}{m}\vec{S} \times \vec{\Omega}, \quad \textbf{with} \quad \vec{\Omega} = (1+\gamma G)\vec{b} + G(1-\gamma)\vec{b}_{//}$$

- **Normalize as earlier**

$$\boxed{\vec{S}\,' = \vec{S} \times \vec{\omega}}$$

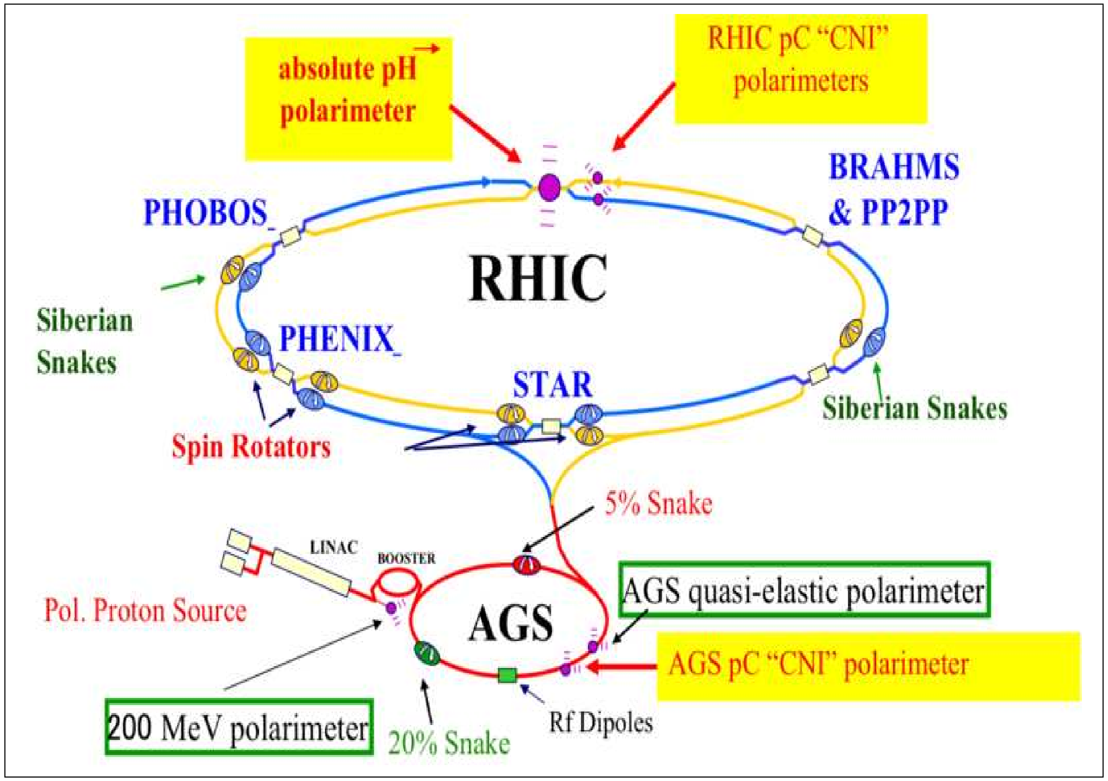
same form as $\vec{u}' = \vec{u} \times \vec{B}$ **!**

It is solved using the outcomes of the particle ray-tracing.

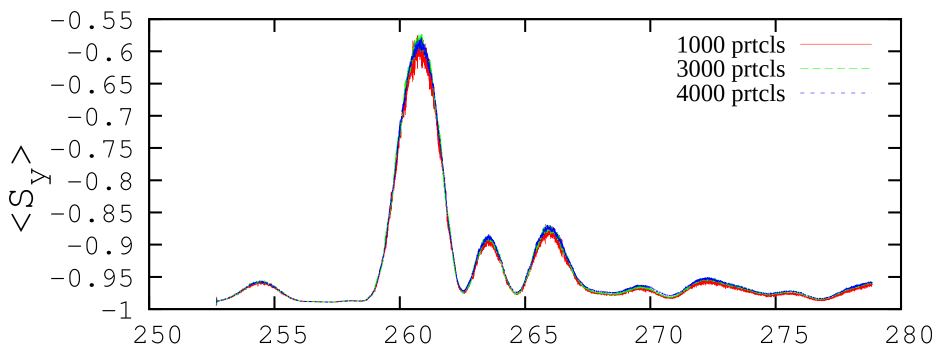
- **and so, the truncated Taylor expansion that pushes** $\vec{S}$ **:**

$$\vec{S}(M_1) \approx \vec{S}(M_0) + \frac{d\vec{S}}{ds}(M_0)\,\Delta s + \frac{d^2\vec{S}}{ds^2}\,(M_0)\frac{\Delta s^2}{2} + \frac{d^3\vec{S}}{ds^3}\,(M_0)\frac{\Delta s^3}{3!} + \frac{d^4\vec{S}}{ds^4}\,(M_0)\frac{\Delta s^4}{4!}$$

EXAMPLE (2009...) - RHIC STUDIES (and, GROUND FOR eRHIC)

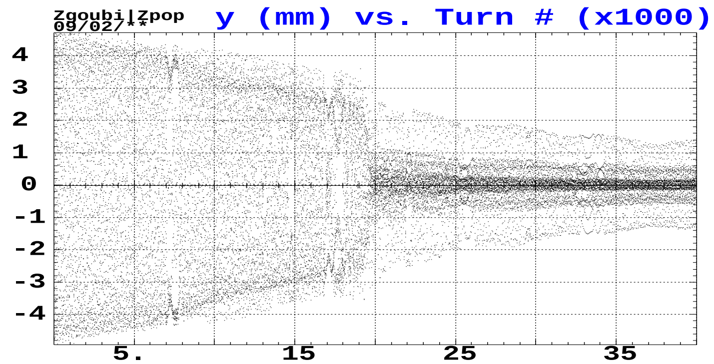


Polarization studies in RHIC - 10^5 turn runs :



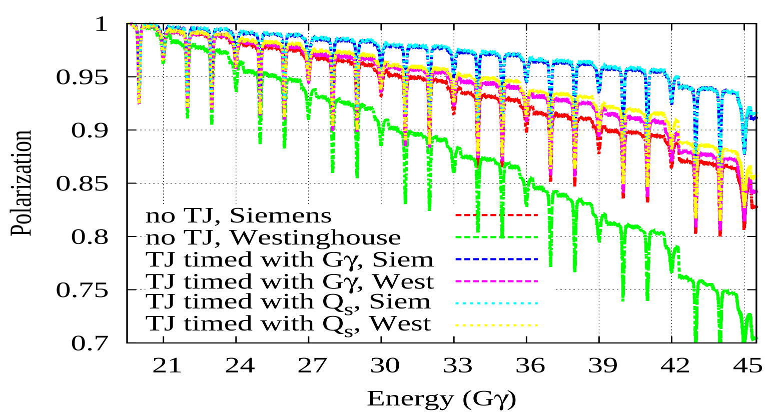
Average polarization as a function of energy at traversal of the snake resonance $G\gamma = 231 + Q_y$.

BD studies in the AGS with snakes :



Horizontal excursion from injection to transition energy. 5 particles.
~40000 turns, 20 min. CPU.

Polarization transport in AGS :



3000 particles tracking, 40000 turns.
Exploring machine setting conditions.

5 SYNCHROTRON RADIATION - ENERGY LOSS

Installed in $\sim$2000, for emittance increase studies along the linear collider (NLC) BDS.

• **The energy loss is updated after each integration step $\triangle s$, in a classical manner, accounting for two random processes :**

- probability of emission of a photon

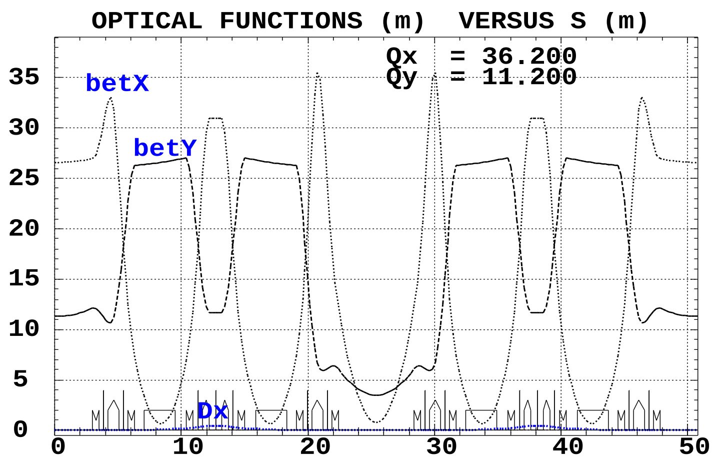
- probability of the photon energy

EXAMPLE (2009) – SYNCHROTRON RADIATION DAMPING IN RINGS

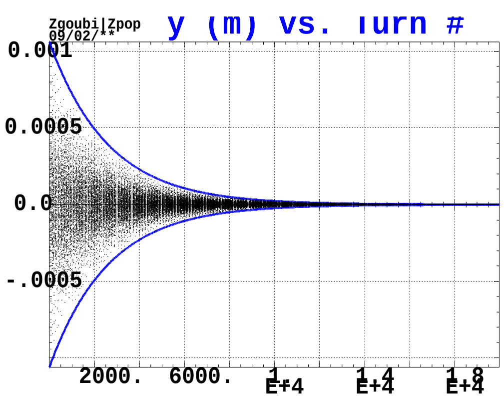
Consider ESRF Chasman-Green super-cell.

Interest : all-analytical understanding.

16 cells ring, 812.6 m, 64 bends.



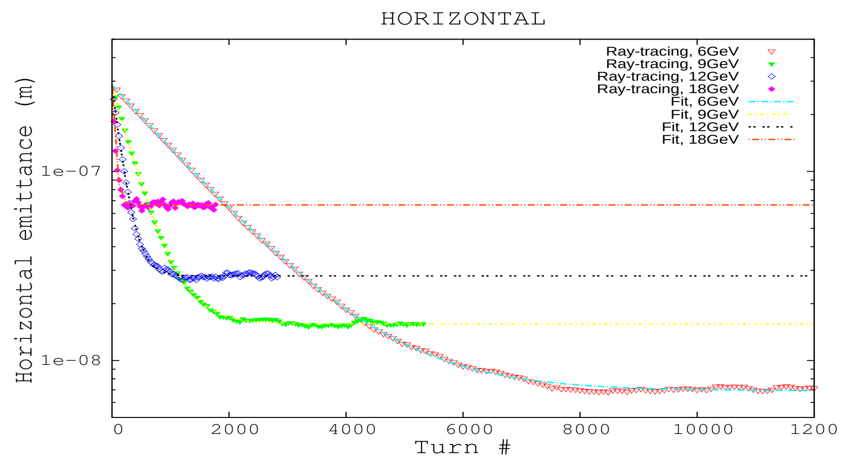
Principle :



Damping of vertical motion over 20000 turns (left), single particle is tracked. Its vertical invariant (right) decreases towards zero.

Emittance damping :

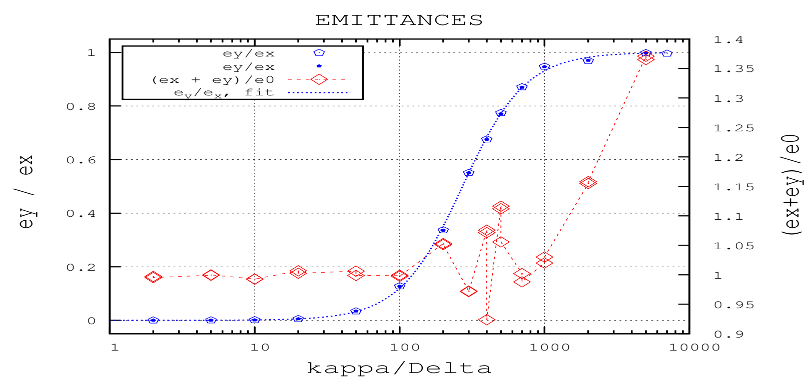
$$\epsilon(t) = \epsilon_0\, e^{-t/\tau} + \epsilon_{equil.} \left(1 - e^{-t/\tau}\right)$$



$\tau_x \approx 1300$ **turns.**

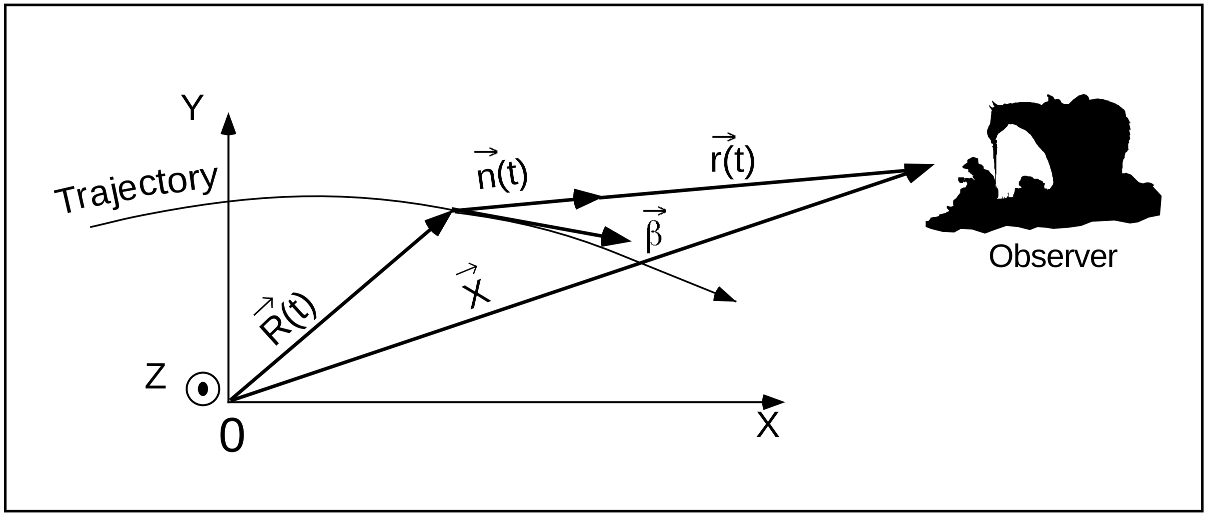
Coupling :

$$\frac{\epsilon_y}{\epsilon_x} = \frac{\kappa^2}{\kappa^2 + \Delta^2}, \qquad \epsilon_x + \epsilon_y = \epsilon_0.$$



SYNCHROTRON RADIATION - SPECTRAL-ANGULAR DENSITY

- **Installed in 1994 for the study of deleterious interference effects at the LEP beam diagnostics mini-wiggler.**



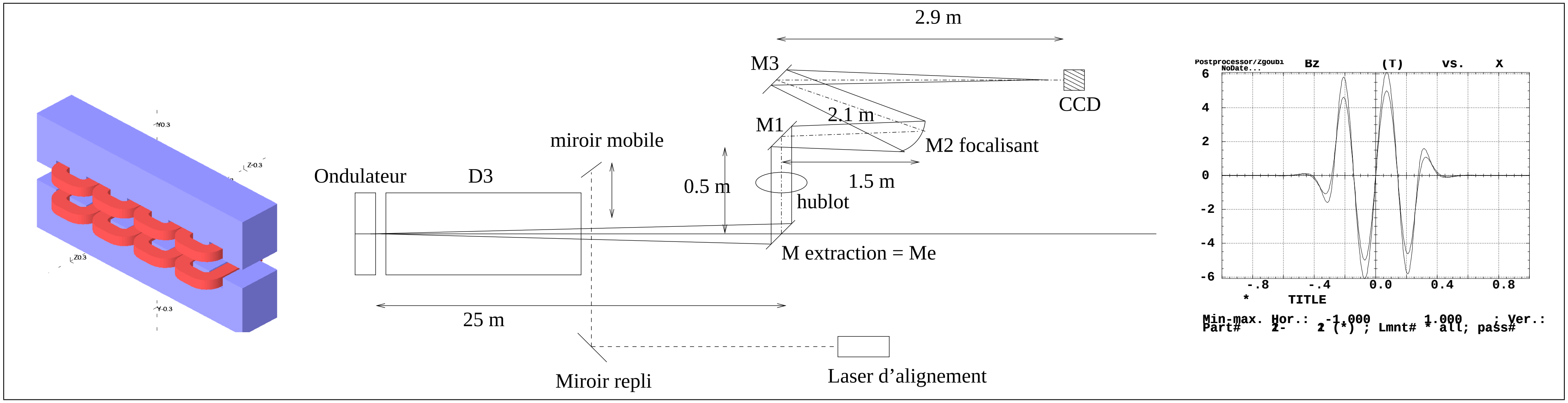
- **The ray-tracing provides the ingredients to compute**

$$\vec{\mathcal{E}}(\vec{n},\tau) = \frac{q}{4\pi\varepsilon_0 c}\,\frac{\vec{n}(t)\times\left[\left(\vec{n}(t)-\vec{\beta}(t)\right)\times d\vec{\beta}/dt\right]}{r(t)\left(1-\vec{n}(t)\cdot\vec{\beta}(t)\right)^3}, \qquad \mathcal{B} = \vec{n}\times\vec{\mathcal{E}}/c$$

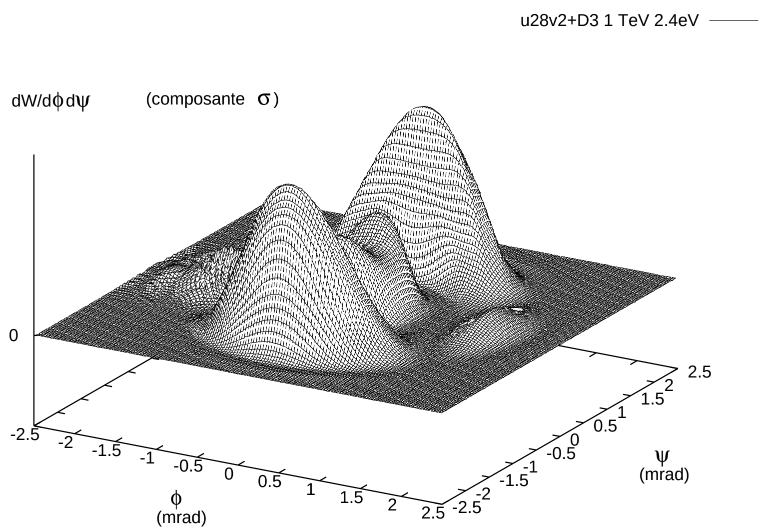
- **The electric field of the radiation is then Fourier transformed,**
  so yielding the spectral angular energy density :

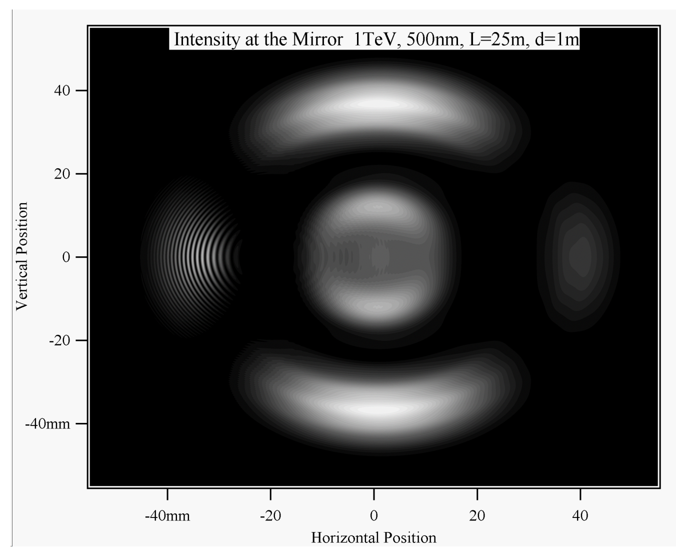
$$\partial^3 W/\partial\phi\,\partial\psi\,\partial\omega = 2r^2\left|FT_\omega\left(\vec{\mathcal{E}}(\tau)\right)\right|^2/\mu_0 c$$

EXAMPLE (2000) – DESIGN OF THE LHC SR DIAGNOSTICS INSTALLATIONS



• **LHC undulator is against a long dipole. The optical system is drawn from LEP's.**





• **Intensity emitted (horizontal component) by 1 TeV protons, λ = 500 nm, with a distance d = 1 m between the two sources, simulated with Zgoubi (left) and with SRW (right).**

7 SPIN DIFFUSION

- **Tightly inspected for eRHIC, yet ...**

... essentially a spin-off of what precedes! **Comes for free**

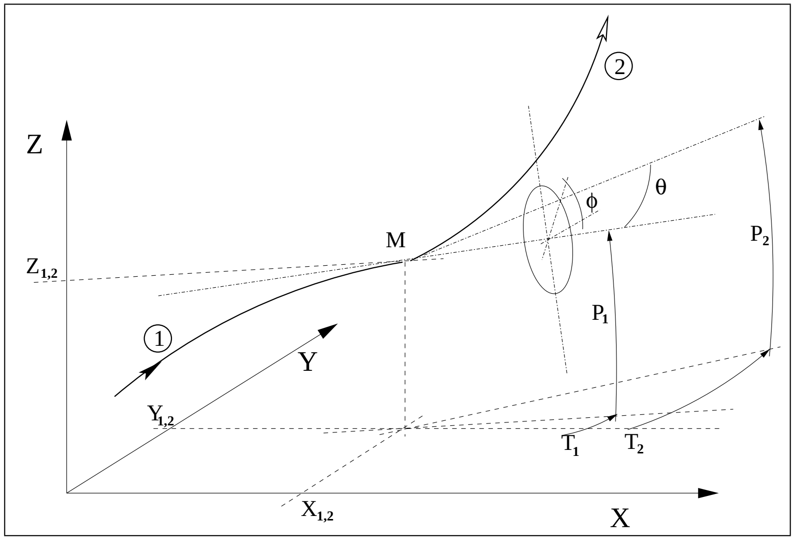
$$\left.\begin{array}{r}\textbf{SPIN DYNAMICS}\\ \textbf{+}\\ \textbf{STOCHASTIC ENERGY LOSS BY SR}\end{array}\right\} \Longrightarrow \textbf{SPIN DIFFUSION}$$

We are working on that, at the moment,

in relation with the eRHIC project R/D studies at BNL.

8 IN-FLIGHT DECAY

... was installed for eta meson spectrometry at SATURNE, Saclay, late 1980s.



Parent particle, kinematics ingredients needed :

- **lifetime** $\tau_\pi = \gamma_\pi \tau_\pi^*$,
- **decay law** $N(s) = N_0 e^{-\eta s/p_\pi}$ ($\eta = m_\pi \;/\; c\tau_\pi^*$)
- **momentum,** $\vec{p_\pi}$

Daughter particle, kinematics ingredients then derived :

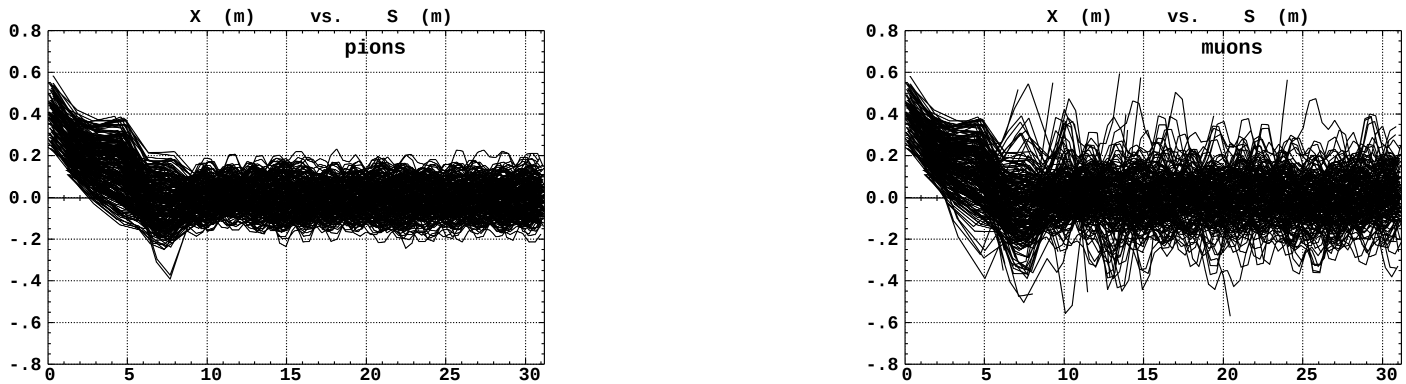
- ***com* energy** $E_\mu^* = (m_\pi^2 + m_\mu^2)/2m_\pi$
- **momentum** $\vec{p_\mu^*} = (m_\pi^2 - m_\mu^2)/2m_\pi \; \vec{u}$,
- **lab. energy** $E_\mu = \gamma_\pi (E_\mu^* + \beta_\pi p_\mu^* \cos\theta_\mu^*)$

Monte Carlo procedure sorts at random :

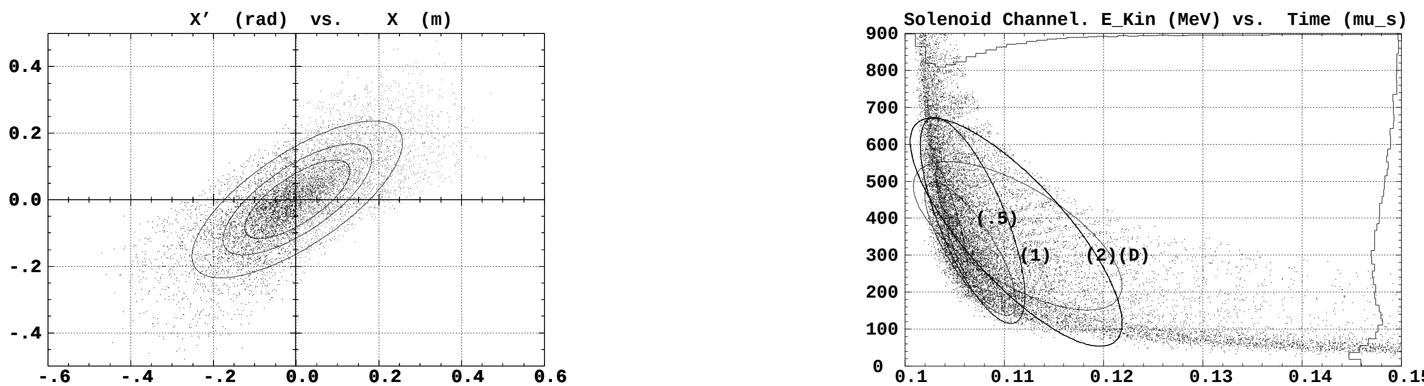
- $\theta = arcos(1 - 2R)$, R **random uniform in** $[0, 1]$
- $\phi = 2\pi R$, R **random uniform**
- **flight distance :** $s = -p_\pi/\eta \ln R$

EXAMPLE (2005+) – Neutrino Factory design studies

- **Objective : optimize transmission efficiency of a FODO pion collection channel, $\pi \rightarrow \mu + \nu$.**
  This used the `'FIT'` keyword, constraint is : maximize transmission through phase-space ellipses with given surface, at the downstream end of the line.



Sample rays in the AG Channel. Left : outermost pions. Right : decay muons.



Left : x-x' phase space at line end, $4 \cdot 10^4$ initial pions from MARS distribution.
Right : Time-energy.

9 THE FITTING PROCEDURE

Two methods installed, in 1985, and (Scott) 2007.

An indispensable tool for

- **preliminary adjustments (orbit, tunes ...)**
- **optimizations (higher order dynamics as DA, transmission efficiency ...)**

FIT CONSTRAINTS :

Trajectory coordinates, at any location

A number of quantities deduced from trajectory coordinates, e.g. :

- **first and higher order transport coefficients**
- **beam's α, β, emittances**
- **particle transmission efficiency,**
- **Spin coordinates**
- **etc.**

In the case of periodic structures :

- **closed orbits**
- **tunes, chromaticites, anharmonicities**
- **Spin closed orbit**
- **etc.**

FIT VARIABLES : any data

Zgoubi input data file, EMMA :

```
'MARKER' RingInj BegRing                      start of ring. Injection point
'MULTIPOL' QD                                               start of first cell
0
7.56987 5.3 0. -2.493246 0 0 0 0 0 0 0 0
0. 0. 1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
0. 0. 1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0.1
2 0. 3.404834122312866 0.
'MARKER' BPM2 off                                                  BPM location
'DRIFT' sd
5.00
'MULTIPOL' QF
0
5.87824 3.7 0. 2.477081 0 0 0 0 0 0 0 0
0. 0. 1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
0. 0. 1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0.1
2 0. 0.7513707181808552 0.
'DRIFT' ld
8.
'CAVITE'                                                    accelerating cavity
7
0.736669 1.3552e9                                   Orbit length, RF frequency
70e3 0.                                               Voltage, relative phase
'MARKER' BPM1 off                                                  BPM location
'CHANGREF'                                     cell orientation - wrt. next one
0. 0. -8.571428571429                                         end of first cell
'REBELOTE'                                                   multiturn tracking
150 0.2 99
'END'
```

10 CONCLUSION

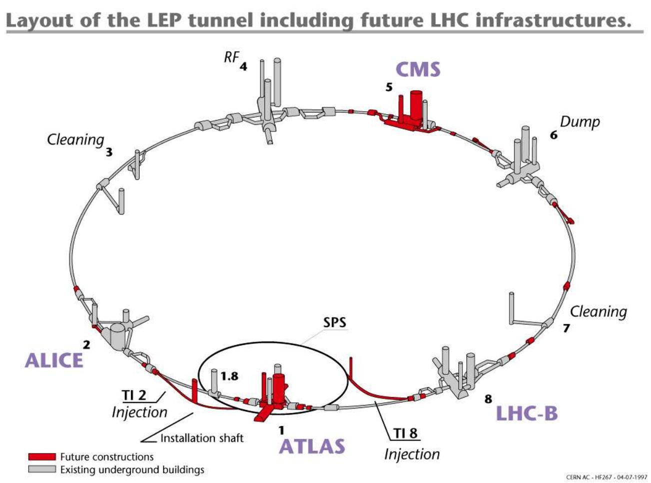
THERE IS MORE : CBETA

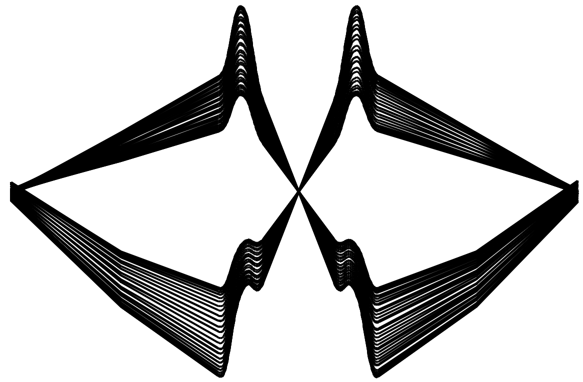
This FFAG workshop, on Saturday

THANK YOU FOR YOUR ATTENTION

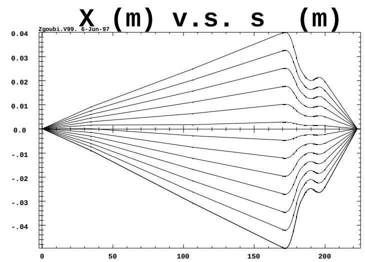
BACKUP SLIDES

EXAMPLE (∼mid-1990s) – Dynamic aperture tracking in LHC

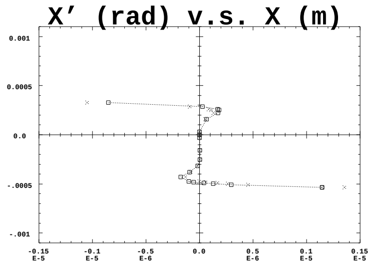


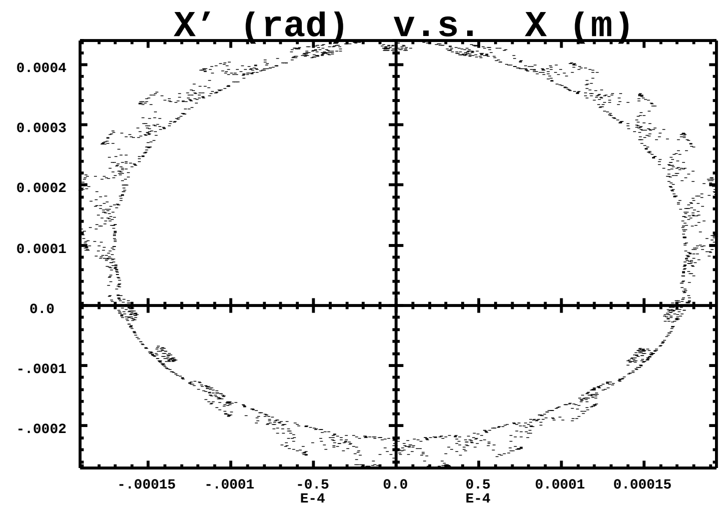


To be fulfilled : (i) → preserve beam size at IP, (ii) dynamic aperture.



Point-to-point imaging, $-30\,\sigma_{x'} < x'_{IP} < 30\,\sigma_{x'}$.
Accounting for b_{10} and tilted Xing plane at IP5.





Dynamic aperture in presence b_{10}.
≈10,000 turns / 1600 magnets/turn.

EXAMPLE (2000) – SR INDUCED EMITTANCE INCREASE IN THE LINEAR COLLIDER BEAM DELIVERY SYSTEM

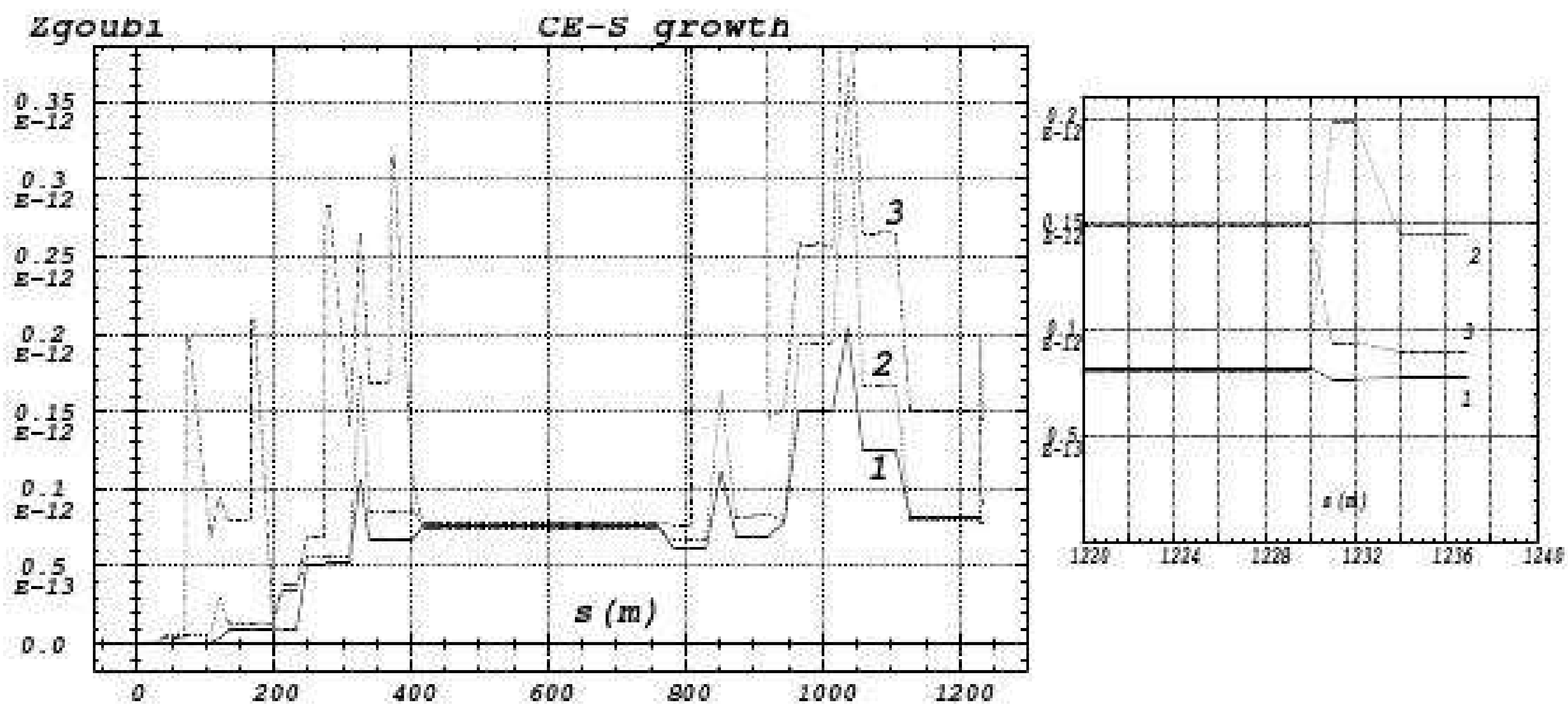
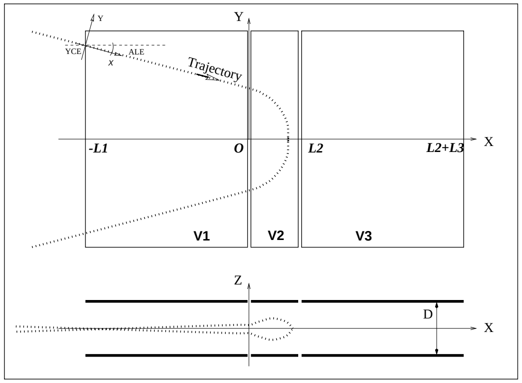


Figure 9: Horizontal CE-$\mathcal{S}$ variation $(\mathcal{S}_x/\pi(s) - \mathcal{S}_x/\pi(0))$ along TESLA-bds as obtained from the ray-tracing of 2 10^4 particles, in various cases of SR simulation (resp^{ly} 1, 2 and 3 in Table 1) :
- solid line : zero initial emittances, sextupoles off ;
- dashed line : initial emittances $\epsilon_x = 10^{-11}$, $\epsilon_z = 10^{-14}$ m.rad, sextupoles off ;
- dotted line : initial emittances $\epsilon_x = 10^{-11}$, $\epsilon_z = 10^{-14}$ m.rad, sextupoles are excited.

The last case shows a strong overshoot (cut out on the Figure) in the $s \approx 850$ m region due to chromatic distortions (see page 16) ; this effect appears also in the low-β quad and FF region zoomed on the right plot (broken lines are due to particle coordinates being saved only at optical element ends).

EXAMPLE (EARLY 2000s) - ELECTROSTATIC TIME-OF-FLIGHT RING SPECTROMETER

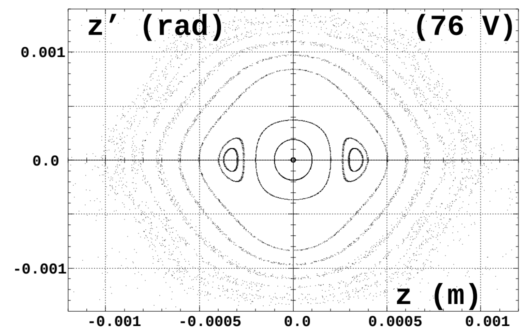
The simulation uses a single, highly non-linear element : 3-electrode parallel plate condenser 'ELMIR'



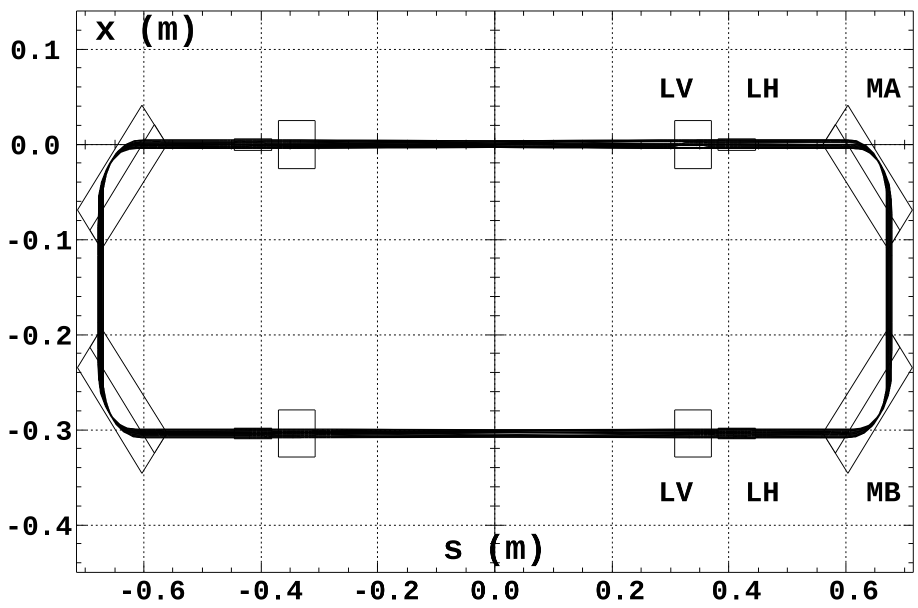
$$V(X,Z) = \sum_{i=2}^{3} \frac{V_i - V_{i-1}}{\pi} \arctan \frac{\sinh(\pi(X - X_{i-1})/D)}{\cos(\pi Z/D)}$$

Typical plate voltage 50-100 Volts.

DYNAMICAL ACCEPTANCE



ELECTROSTATIC RING



SEPARATION OF 2 DIFFERENT MASSES AFTER 100 TURNS